ILLUSTRATING THE MATHEMATICS STANDARDS

The following examples of student work illustrate achievement at the mathematics standards for years 6, 7, and 8.

Soak It Up

The task in this illustration was part of a science investigation into the absorbency of sponges. The class was working on a unit called Fibres and Fabrics from *Making Better Sense of the Material World*. As part of the unit, the students identified material properties that contribute to absorbency and completed a lesson on fair testing.

This task focuses on the mathematical problem solving that took place within the investigation. It relates to achievement objectives for Number and Measurement from the mathematics and statistics learning area in *The New Zealand Curriculum.*

Soak It Up

Your class is investigating which of four different types of sponge is the most absorbent. The four sample sponges vary in size.

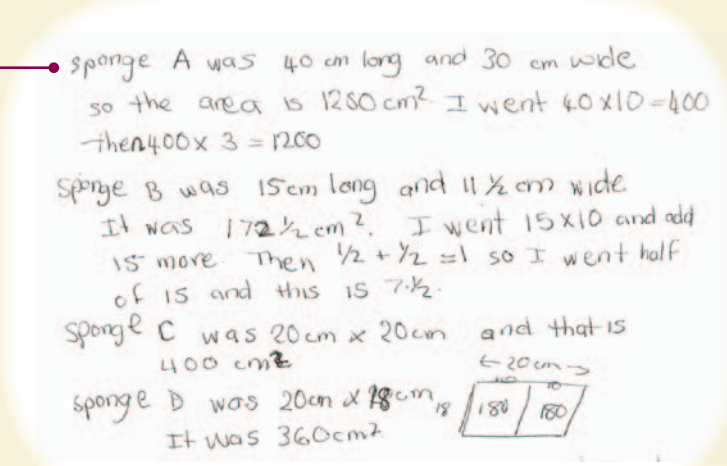
1. **a.** *Find the volume of each sponge.*
   b. *Record how much water each sponge absorbs.*
2. *Which sponge is the most absorbent?*

Some features of students' work used to make judgments in relation to the mathematics standards are described below.

Soak It Up

New Zealand Curriculum: Level 3	*Mathematics Standard: By the end of year 6*
In solving problems and modelling situations, students will:	
Number and Algebra • *use a range of additive and simple multiplicative strategies with whole number, fractions ... (number strategies)* • *know basic multiplication and division facts (number knowledge)* *Geometry and Measurement* • *use linear scales and whole numbers of metric units for length, area, volume and capacity ...* • *find areas of rectangles and volumes of cuboids by applying multiplication (measurement)*	*Number and Algebra* • *apply additive and simple multiplicative strategies flexibly to:* – *combine or partition whole numbers, including performing mixed operations ...* *Geometry and Measurement* • *measure ... the attributes of objects, choosing appropriate standard units* • *use arrays to find the areas of rectangles and the volumes of cuboids, given whole-number dimensions*

Jessica measured the length and width of each sponge in centimetres and used her understanding of place value when finding areas. She used place value partitioning for sponge B ($15 \times 11\frac{1}{2} = 15 \times 10 + 15 \times 1 + 15 \times \frac{1}{2}$) and also showed her understanding of halves. She used doubling and halving ($20 \times 18 = 10 \times 18 + 10 \times 18$) for sponge D.


sponge A was 40 cm long and 30 cm wide so the area is 1200 cm². I went 40 x 10 = 400 then 400 x 3 = 1200

Sponge B was 15cm long and 11 ½ cm wide It was 172½ cm². I went 15 x 10 and add 15 more Then ½ + ½ = 1 so I went half of 15 and this is 7½.

Sponge C was 20cm x 20cm and that is 400 cm²

sponge D was 20cm x 18cm It was 360cm²

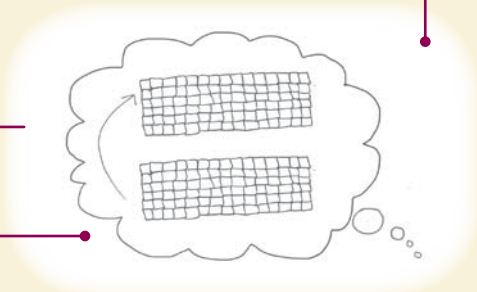
The teacher noted that Jessica accurately measured the amount that each sponge absorbed. Then she based her conclusions about absorbency on the surface area of each sponge.

Sponge A is three times bigger than C, but it doesn't hold three times as much water. C and D are more than twice the size of B, but they both hold less water. So B is best.

After a prompt from the teacher, Jessica recognised that, to investigate absorbency, she needed to find the volume of each sponge. She began by calculating the volume of sponge B as an array.

Jessica started by doubling the height (thickness).

B is one and a half centimetres thick, so I imagined I cut the sponge in half and put the second half on top, so it is 3 centimetres thick.



So now the area is 15 cubes long times 6 cubes wide. Each cube is 1 centimetre thick, so that's 3 layers. There would be 15 x 6 = 90 cubes on the top layer. Two rows is 180, and another 90 is 270. So it will be 270 cubic centimetres.

Jessica rounded the $11\frac{1}{2}$ centimetre width to 12. When she doubled the height (from $1\frac{1}{2}$ to 3), she halved the width (from 12 to 6). She used her measurements to get an array of 15 x 6 x 3. She visualised her array as cubes.

Discussion

This task provides some of the evidence needed to show that Jessica is achieving at curriculum level 3 and the year 6 standard in Number and Measurement. She was able to measure the length and width of sponges in centimetres and to find the surface area of each sponge and the volume, using arrays, of one sponge. She has demonstrated that she is able to apply additive and simple multiplicative strategies flexibly. This suggests that she is working at the Advanced Additive stage of the Number Framework.

Soak It Up

New Zealand Curriculum: Level 4	*Mathematics Standard: By the end of year 7*
In solving problems and modelling situations, students will:	
Number and Algebra • *use a range of multiplicative strategies when operating on whole numbers (number strategies and knowledge)* *Geometry and Measurement* • *use appropriate ... metric units for length, area, volume and capacity ...* • *convert between metric units, using whole numbers and commonly used decimals* • *use side or edge lengths to find the ... volumes of cuboids (measurement)*	*Number and Algebra* • *apply additive and multiplicative strategies flexibly to whole numbers ...* *Geometry and Measurement* • *measure ... the attributes of objects, using metric ... measures* • *make simple conversions between units, using whole numbers* • *use side or edge lengths to find the ... volumes of cuboids, given whole-number dimensions*

Awatea measured thickness in millimetres; she knew she needed to convert this to centimetres when finding volume.

Sponge B
15 cm long 12 cm wide 1½ cm thick
15x10 = 150 15x2 = 30 150 + 30 = 180
½ of 180 = 90 so it is 180 + 90 = 270
Water amount 130 mls

I know that 2 millimetres is the same as $\frac{2}{10}$ centimetres. So 5 layers of sponge A would be $\frac{2}{10} \times 5 = 1$ centimetre thick.

Awatea used fractional knowledge, place value strategies, e.g., 15 x 12 = [15 x 10] + [15 x 2], and rounding and compensation, e.g., 20 x 18 = [20 x 2] – [20 x 2], to find the volume of each sponge.

Sponge D
20 cm long 18cm wide 5mm thick
I just took away 40 cm³ from 400 +
Then I just had to half it because 5mm
is ½ cm. So it was 180 cm³
Water amount 105 mls.

Sponges A and D are the most absorbent.

Sponge	Volume	Water	Per cubic cm
A	240 cm³	140 mls	0.58 mls
B	270 cm³	130 mls	0.48 mls
C	400 cm³	110 mls	0.27 mls
D	180 cm³	105 mls	0.58 mls

Awatea used a calculator to find out the amount of water per cubic centimetre that each sponge held. The teacher noted that Awatea initially divided the volume by millilitres of water but then realised her error and reversed her calculation.

Discussion

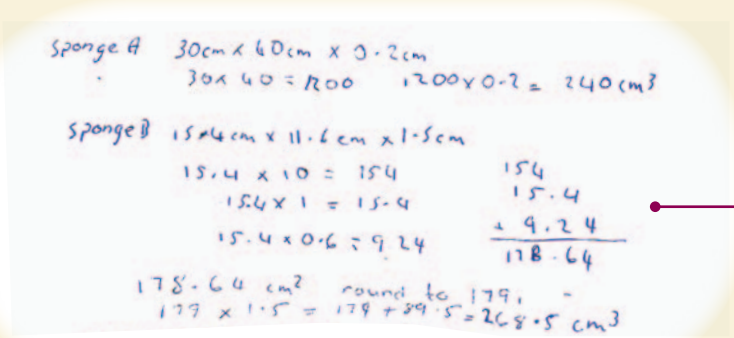
This task provides some of the evidence needed to show that Awatea is achieving at early curriculum level 4 and the year 7 standard in Number and Measurement. She was able to measure the dimensions and find the volume of each sponge. She has demonstrated that she is able to apply additive and multiplicative strategies flexibly with whole numbers. This suggests that she is working at the Advanced Multiplicative stage of the Number Framework.

Soak It Up

New Zealand Curriculum: Level 4	*Mathematics Standard: By the end of year 8*
In solving problems and modelling situations, students will:	
Number and Algebra • *use a range of multiplicative strategies when operating on whole numbers* • *understand addition … of … decimals …* • *know the equivalent decimal … forms for everyday fractions (number strategies and knowledge)* *Geometry and Measurement* • *use appropriate … metric units for length, area, volume and capacity …* • *use side or edge lengths to find the … volumes of cuboids (measurement)*	*Number and Algebra* • *apply … multiplicative strategies flexibly to whole numbers … and equivalent fractions (including decimals …)* *Geometry and Measurement* • *use metric … measures* • *use side or edge lengths to find the … volumes of cuboids*

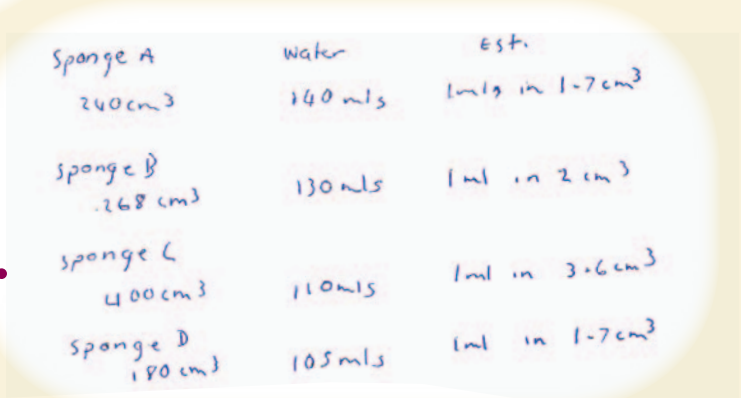
For sponge A, 1200 x 0.2, I just did two-tenths of 1 200 in my head … one-tenth is 120, so two-tenths is 240.

Chad converted between fractions and decimals.



Chad used his fractional and decimal knowledge to work out the volume of each sponge. He multiplied and added using decimals.

To determine which sponge was the most absorbent, Chad decided to find out how many cubic centimetres of each sponge would be needed per millilitre of water.



Chad collated his information and estimates into a table and came to a conclusion based on his findings.

So sponges A and D are the most absorbent.

Discussion

This task provides some of the evidence needed to show that Chad is achieving at curriculum level 4 and the year 8 standard in Number and Measurement. He was able to find the volume of each sponge, using metric measures fluently. He has demonstrated that he is able to multiply and add decimals and to use multiplicative reasoning to find a common measure (cubic cm per mL). This suggests that he is working at the Advanced Multiplicative stage of the Number Framework.