Can 2 identical rectangles make cylinders with different volumes?

Teachers' Notes

Curriculum Links

This task involves working with the surface area of rectangles and circles and the volume of cylinders. These are Measurement and Geometry concepts are referred to in Level 3-5 of NZC. There are links to the Technological Modeling and design brief elements of the Technology curriculum. This task can be extended to the investigation of the materials and the cost of production as well.

Background

The student is asked to create two models and to compare them physically. They should also calculate the impact that the area of the base has on the volume of a cylinder. This task is accessible for students who are being introduced to calculation of volume as well as those for whom working with formulae is prior knowledge.

Students can make and fill the containers, and measure the volumes in various ways: filling each and pouring into a measuring cup or beaker; filling one and emptying it into the other container for comparison; or using calculations to find accurate measurements.

The area of the rectangle used for the sides of the containers is found by calculating:
Length x Width = 50 x 30 = 1500 sq cm

The volume of the containers is found by calculating the area of the circular base created and then multiplying this by the height of the container.

The area of the circle is
pi x radius squared (πr^2)

The volume of a cylinder is
$\pi r^2 \times h$

Once students have discovered that the shorter container has a greater volume, encourage them to try and generalise their findings: Why is this so? Would this always be the case no matter the size of the rectangle used for the sides? What parts of the formula are constant? What parts of the formula have the greatest impact on the volume (the impact of squaring a number)?

The advice to the cinema owner may not simply be based on the greatest or least volume, but is an open-ended task in which the student has to construct a argument. This may vary between students.

Suggestions

There is the opportunity to approach this task from a Technology curriculum perspective. Developing a brief and investigating the financial and ergonomic elements of packaging may be of interest to the student. The challenge to look at other shapes of containers (such as cylinders with elliptical bases, which fit more easily into your hands) and re-design the "popcorn at the cinema experience" is an engaging context that could have many trials and will contribute to the understanding of volume.

Students interested in the multiplicative relationships within formulae may be interested in exploring factorials as presented in this activity related to the picture book: *Anno's Amazing Multiplying Jar*
http://www.nzmaths.co.nz/resource/anno-s-mysterious-multiplying-jar

The following problem explores the relationship between volume and surface area:
http://www.nzmaths.co.nz/resource/a4-containers