Nim: How do you make sure you always win? Teachers' notes

Curriculum Links

This problem solving investigation requires a student to work with patterns and relationships in both additive and multiplicative ways. While they can work through trial and error, a high ability student should be encouraged to work in a systematic and logical way, considering the relationships between the numbers in the whole set and the sub sets being taken away in each turn.

Background

Game theory is a large and diverse area of mathematics, and specifically logistics, that can be of interest to many high ability students. Games in mathematical learning also have many purposes.

"There are several educationally useful ways of incorporating games into mathematics lessons. Games can be used as lesson or topic starters that introduce a concept that will then be dealt with in other types of activities. Some games can be used to explore mathematical ideas or develop mathematical skills and processes and therefore be a main component of a lesson. Perhaps the most common use of games is for practice and consolidation of concepts and skills that have already been taught. ***Yet another way to use games is to make them the basis for mathematical investigations***."

Jenni Way (http://nrich.maths.org/2546)

The purpose of this investigation is to discover how the game ***works*** and how to express this mathematically in both words and numbers while using diagrams to represent the choices and actions within the game itself.

The strategy to always win **this version** of Nim is that you should always take the second turn and no matter how many the other person chooses you should make the combined total taken by both players be four. In this case, the game will last five turns and you will always take away the last counter.

A takes 3, B takes 1 (total 4)
A takes 1, B takes 3 (total 4)
A takes 2 B takes 2 (total 4)

The reason this works is that when each player can take 1, 2, or 3 counters, it is **always** possible for the second player to make the combined total taken equal four, regardless of how many counters the first player takes.

If the starting pile has 25 counters then the winning strategy will be different. You always want to leave a multiple of four counters in the pile after your turn. Therefore, the player that goes first should win by taking one counter to leave 24, and then for each subsequent turn following the same strategy as described above.

If the game is modified so that each player can only take either 1 or 2 counters, then the second player can always make the combined number of counters taken be three (not four). Now the player that leaves a multiple of three counters at the end of their turn should always win.

Suggestions

Students should initially be left to experiment with the game and record any observations about how it works. If students begin to get stuck or are persisting with trial and error and not gaining insight into the patterns and relationships within the game, support them through questioning to become more systematic, or engage with the prompts and suggestions on the student page.

The pdf recording sheet encourages students to represent their understanding in three ways.

There are two e-ako which provide an in-depth exploration of versions of Nim in e-ako maths (e-ako.nzmaths.co.nz). They are in the problem solving pathway, and are called *21 stones* and *More stones*.

Several variations of Nim are described here: http://nrich.maths.org/5794.

Others variations are found in the links given to students, and many more can be found by doing an internet search for “nim game”. Researching the history and variations of this game or other ancient games may be of interest to students as well as the development of new versions.