Mathematics in the New Zealand Curriculum
Second Tier

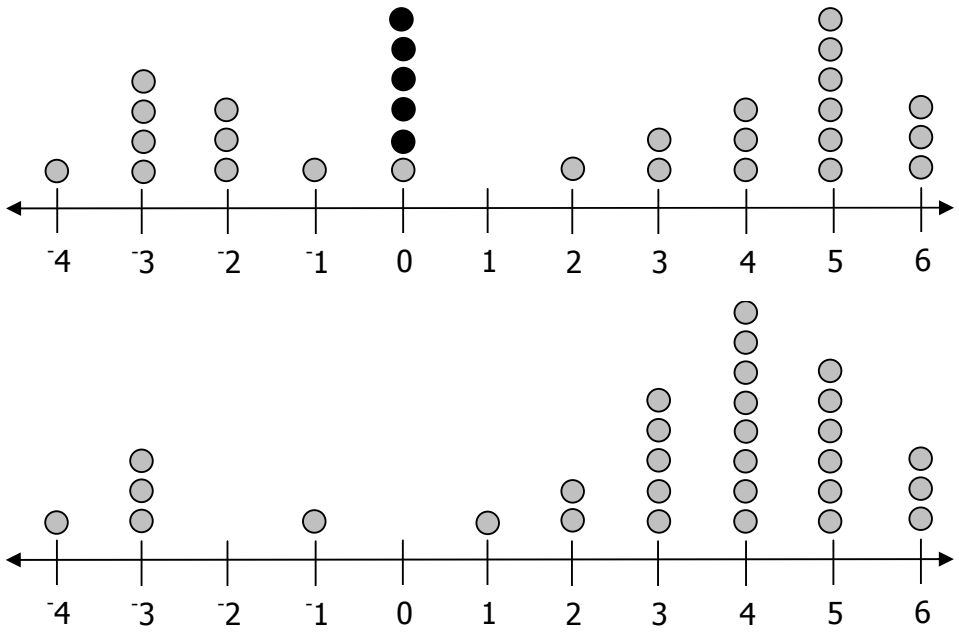
Strand: Statistical Thinking Thread: Statistical Investigations Level: Four

Achievement Objective: Conduct investigations using the statistical enquiry cycle:
- Determining appropriate variables and data collection methods;
- Gathering, sorting and displaying multi-variate category, measurement, and time-series data to detect patterns, variations, relationships and trends;
- Comparing distributions visually;
- Communicating findings using appropriate displays.

Exemplars of student performance:

Exemplar One: Healthy is as Healthy Eats

The student X investigates the question, "Are students eating healthier lunches at our school after the "Healthy Eating" unit than they did before it?" After visiting internet sites and talking to a dietitian X decides to measure the "healthiness" of a lunch in the following way. The food value of a lunch is calculated by; 3 points for whole grain breads or cereals, 2 points for vegetables or fruit, 1 point for dairy products or lean meat and 1 point also for water. Negative points are calculated by ¯1 point for soft drinks or foods with high sugar contents, ¯2 points for starch based snacks, and ¯3 points for fast foods like pies and hamburgers with high fat content. Zero points are given to no lunch or where positive and negative points balance out.
X selects a sample of thirty students from different classes in the school, getting a mix of gender and ages. She records the food items in each lunch, either purchased at school or brought from home, for each student on a day before the unit and on a day one week after the unit. She then calculates the healthy points total for each lunch. To analyse the data the student creates several data displays but decides that dot plots best show the before and after distributions.



Total healthy points scores of lunches **before** unit

● No lunch

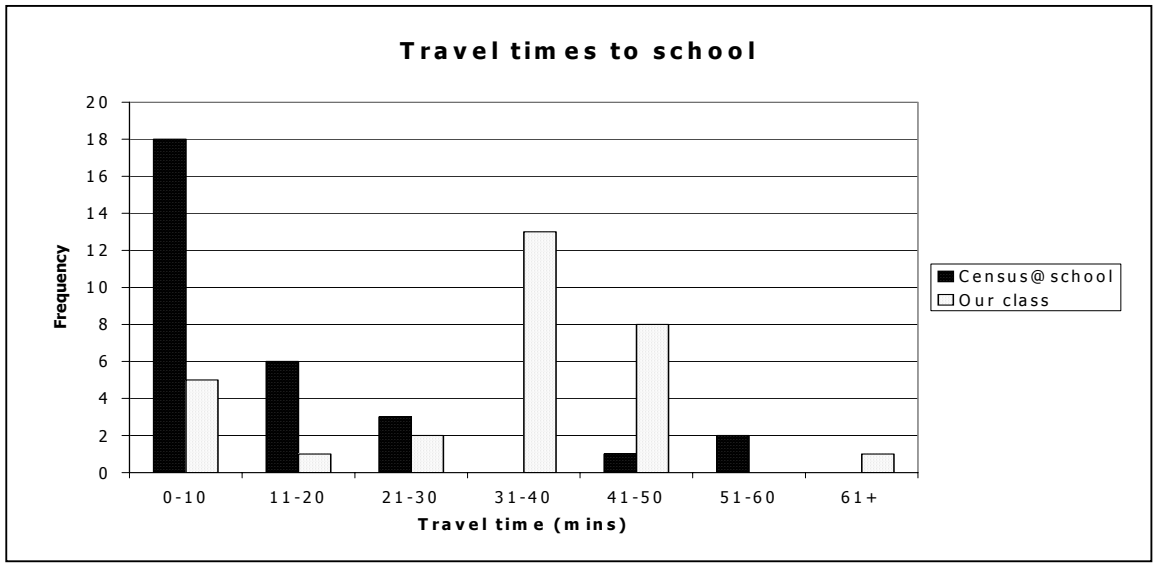
Total healthy points scores of lunches **after** unit

The student notes that the shapes of the distribution show that many more scores are at the positive end of the scale in the after sample than the before sample. She notices that using centrality (middle) is not a useful indicator for either distribution as both have significant clusters at the top and bottom of the scale.
In her conclusion she says that more students are eating healthier lunches after the unit than before. However, there still remains a group of students eating unhealthily after the unit. An interesting result is the disappearance of zero scores in the after sample. The fact that all students were bringing lunch after the unit explains this absence.

This exemplified Level Four because X determined the variable to measure ("healthiness of lunch"), and pre-empted Level Five by developing a measure for this variable. X selected an appropriate data collection process by sampling students from across the school rather than just her own class. She displayed the discrete numeric data (lunch ratings) using two dot plots and compared the two distributions (before and after) visually. X used the main clusterings of lunch ratings in the dot plots to analyse the data and communicated her findings using that display.

Exemplar Two: Getting There

The student E is given a complete set of census@school data for a randomly selected group of 30 students at his year level. This dataset is available electronically. He uses this data to make comparisons to a sample of 30 students in his own class. He brainstorms questions that might be answered from the data by considering the fields in the dataset, e.g. "Do students in Room 16 take longer to get to school than other students in New Zealand?"
E collects a set of matching data from his own class. He trials a range of different data displays to make comparisons between the random sample from *censusatschool* and the sample of students from his class.
E selects a double bar graph as a good representation to answer the question, "Do students in Room 16 take longer to get to school than other students in New Zealand?" He notices the modal (most common) travel times for the two samples are significantly different. The shapes of the distributions show that, overall, students in his class travel for longer times than students in the census@school sample.



E concludes that students in his class travel for longer than the national sample. He knows that most of his classmates travel to school by bus, and very few of them walk, and speculates that this is variation from the national data is typical of rural schools. E uses computer software to compare time taken in getting to school with method of getting to school for the censusatschool

sample. E notices that the students from the national sample who take a long time to get to school also travel by bus or car. This supports his conclusion. E wonders "Do people who travel to school by bus or car take more or less exercise than people who walk or bike to school?"

This exemplified Level Four because E started his investigation by accessing a multi-variate dataset (censusatschool sample). He posed questions for statistical investigation that involved comparison between samples. In answering his question, "Do students in Room 16 take longer to get to school than other students in New Zealand?" E replicated the censusatschool categories to achieve consistency of response . He chose an appropriate display (double bar graph) to compare distributions (censusatschool sample and his class sample), and used main clustering as an indicator of centrality in this comparison. His conclusion was supported with a suggested explanation for the variation between samples and he validated the explanation by establishing a relationship between travel methods and travel times in the censusatschool sample. E enacted the statistical enquiry cycle by posing questions for further investigation.

Exemplar Three: Beats Working

The student (K) reads an article that shows players in the All Whites soccer team are as fit as players in the All Blacks. She investigates the question, "Are rugby players fitter than soccer players?"

After some research the student decides that one measure for fitness is resting pulse rate, in beats per minute. K chooses to sample all the students from the top boys and girls soccer and rugby teams. The data is collected before each teams' practice. She presents the data as dot plots, bar charts, and stem and leaf graphs, using computer software. K decides that a stem and leaf graph gives her the clearest picture of the data as she also wants to detect any differences between girls (bold) and boys. In the bar chart each individual score is lost and comparing the distributions with dot plots is visually confusing unless two separate graphs are used.

Soccer Players		Rugby Players
6	11	
	10	0 **4**
	9	**2** 2 **6** 6
6 **4** 4 **0 0** 0 **0** 0	8	**0** 0 0 **4** 8 **8 8**
6 6 **6 2** 2 **2 2** 2	7	**2** 2 **6 6** 6 6
8 **8** 8 **4 0** 0	6	**0** 0 4 **4** 8 **8**
	5	2 **2 2** 6 6

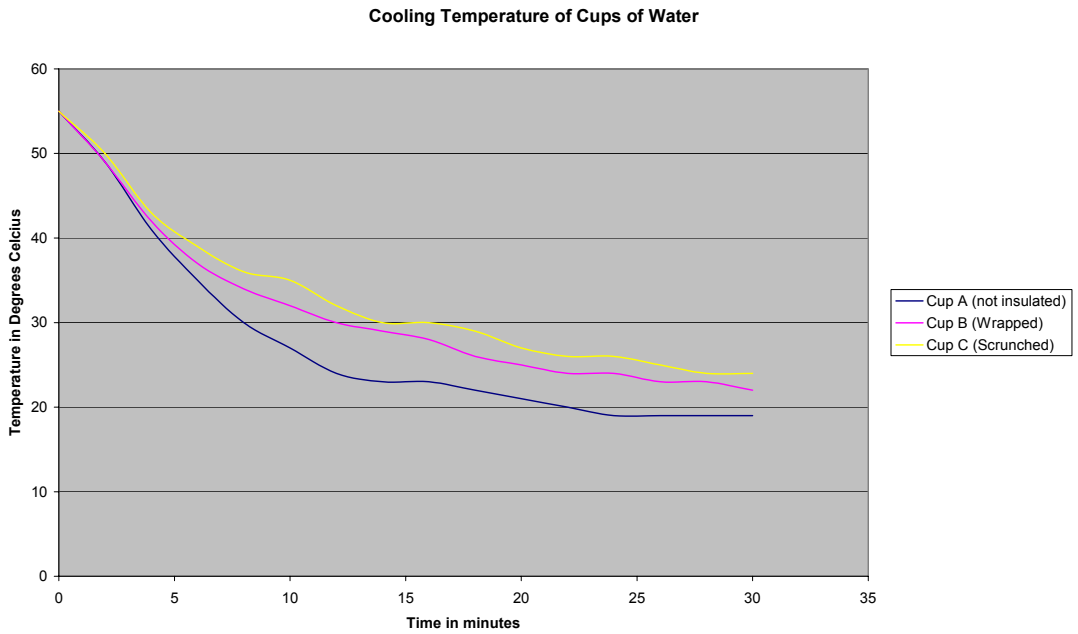
She notices that, excluding the player with a pulse rate of 116, there is a bigger spread in the pulse rates of rugby players than

the soccer players. She finds out that the soccer player with the 116 pulse rate is a goalkeeper. By looking at the middle (centrality) of the distributions she concludes that there appears to be little difference, with rugby players having a slightly higher middle pulse rate than soccer players. K also notes that there appears to be no clear difference between boys and girls. In her conclusion, she states that the fittest and least fit players in both sports are the rugby players. She suggests that this is because some rugby players rely more on strength than speed while other rugby players need to be very mobile. Nearly all soccer players require a reasonable level of fitness, except perhaps for the goalie. K wonders how the pulse rates of players from other sports would compare to these groups.

This exemplified Level Four because K's investigation focused on detecting patterns, variations, and relationships within and between the samples (rugby players and soccer players). She posed a question to investigate and organised her own data collection method electing to sample the elite players from her school rather than select a representative sample (Level Six). K used software to display the distributions in multiple ways before choosing a back-to-back stem and leaf graph as the most useful graph. She compared the distributions visually attending to both the spread and central clustering. She established that there was no obvious difference pattern by gender but she detected variation in spread by sport. K explained the outlier (pulse rate 116). K's conclusions related directly to patterns and relationships she found in the data and she suggested possible explanations for variations. She posed questions for further investigation.

Exemplar Four: Hot Stuff

A student (T) is investigating the effect of insulation on cooling. He conducts an experiment involving three cups of water filled from the hot tap, one not insulated, one wrapped in newspaper and the other surrounded by the same amount of screwed up newspaper. The temperature of each cup, in degrees Celsius, is measured at intervals of two minutes. T asserts that "The water in the insulated cups will cool more slowly than the water in the not insulated cup."
T enters the data into a spreadsheet and graphs the data from all three cups on a single line graph.

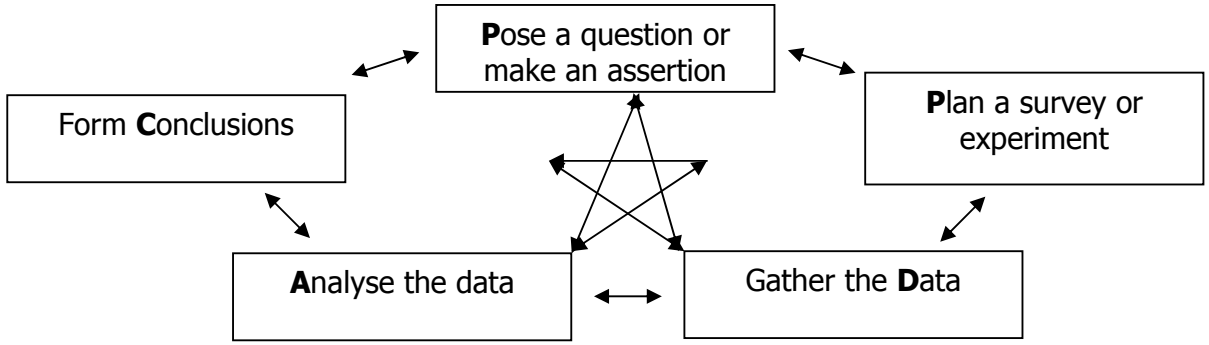


He notes that a trend in both graphs is for the temperature decline to "flatten" around the early twenties of degrees Celcius.
T wonders why this occurs and looks up the internet to find that the cooling limit will be the surrounding air temperature.
He observes that the insulated cups lose heat at a slower rate than the non-insulated cup and that the scrunched up paper is a better insulator than the wrapped paper. T suggests that is why many insulation products are fluffy with lots of air gaps.
He wonders how the different cups might behave as very cold water heats up to room temperature.

This exemplified Level Four as T used the statistical enquiry cycle to investigate a question with time series data. He determined the variables to measure (time and temperature), collected the data, and used an appropriate display to look for trends over time (line graph). Representing the data from all three cups on the same graph helped T to compare and contrast the trends. He explained the observed trends with reference to the context, and posed questions for further enquiry.

Important teaching ideas (working at):
Posing Statistical Questions or Assertions

Level Four marks an increase in students' sophistication in implementing the Statistical Enquiry Cycle (below). Statistical investigations begin with contexts that are relevant to the students. Contextual knowledge is critical to interpretation of the data.



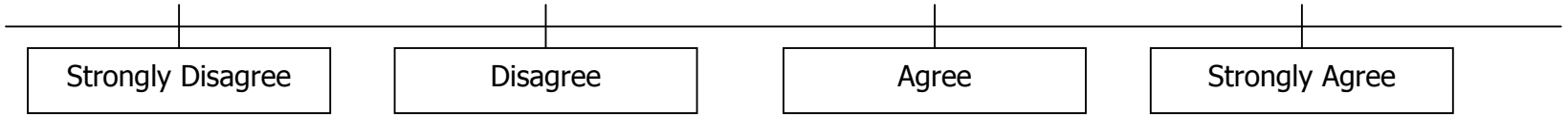
In the problem phase of the cycle, students need experience with both asking questions and making assertions that provoke the gathering of data. They also need experience in asking question about datasets that already exist. Assertions are statements that require validation, "Girls have better reflexes than boys." At Level Four students should be encouraged to ask statistical questions of three main types:

1. Summary questions, e.g. What is the height of an ten year old person?
2. Comparative questions or assertions, e.g. "Are eleven year old girls taller that eleven year old boys?"
3. Relationship questions, e.g. "Are people's foot-lengths related to their handspans?"

Through creating good questions students consider attributes, characteristics of people or things, that are of interest. The act of measuring or counting attributes creates variable. Variables are quantities that can take on different values. Some variables are discrete and can only take on certain values, e.g. Number of people who live in your household can only take up whole number values, like four people. Other variables are continuous and can take up any value within a range, e.g. A person's height. Continuous variables involve measurement. Note that usually these measurements round to the nearest unit.

Planning Investigations and Gathering the Data

In the planning phase of the cycle Level Four students need to consider what **measures,** and **criteria** they will use to answer the question or validate the assertion. For example, in considering the question "Do boys do more or less homework than girls?" students might **measure** the amount of homework in pages read or written, or time spent each night or week. They may need to define **criteria** for what constitutes homework, e.g. Work set by the teacher. In some cases scales must be constructed to measure variables.
For example, a Likert Scale is commonly used to measure attitudes. In response to a statement respondents choose one of the following options:



Student should also consider how to take a sample. This involves the concept of sampling from a population. A population is a group of people, objects or ideas that we are interested in finding out about. For practical reasons it is difficult to gather data from every member of the population. Commonly data is gathered from a selected part of the population, the sample, and the results are used to draw conclusions. An important issue is how much the results from a sample can be trusted to represent the whole population. Usually the results for larger samples are more reliable than smaller samples. At higher levels students investigate how to get representative samples that are more reliable predictors about the population and how taking many different samples (resampling) can give a more reliable prediction about the population.
Having decided on how they will gather their sample students need to consider what systems they will use to track respondents and ensure that the data is secure. For example, they might use a tally or two-way table to ensure the sample reflects their design for data gathering:

Age/Gender	Female	Male
5 years old	𝍸 𝍸	𝍸 𝍸
10 years old	𝍸 𝍸	𝍸 𝍸

Planning also involves considering the data collection process as well as the sample taken. At level four students need to be alert to the effects that inappropriate data collection can have on results. Important examples of this are constraint of response and bias. Limiting response categories while popular for making data processing easier can create distortion in that they force people to answer in certain ways or they mask potentially interesting information.
For example, consider the question:
"What do you usually eat for breakfast?"
Choose **one** option from:

a. Toast
b. Cereal
c. Bacon and eggs
d. Fruit
e. Other

Limiting the responses in this way masks the diversity of information about the foods eaten that may be of significant. The question does not allow respondents to indicate that their usual breakfast is a combination of these choices or if they do not eat breakfast at all. Bias is also generated by the phrasing of questions, in combination with the options provided. This is particularly common in attitudinal surveys.
For example, consider this question:

Every year many people are mauled by savage dogs. Dogs should be banned from populated areas.
Strongly Agree Agree Disagree Strongly Disagree

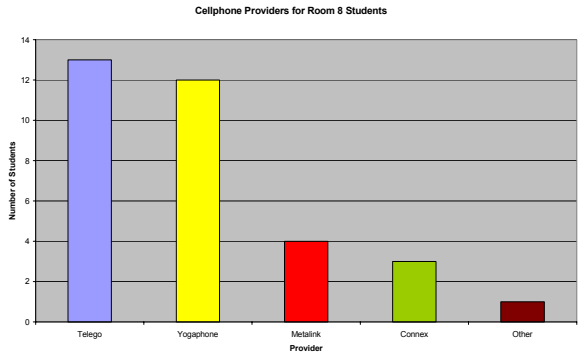
The opening statement is designed to persuade the respondent and there is no neutral position. The statement also simplifies a complex issue and terms like "populated" are open to interpretation.
Students also need to consider how they intend to record the data they get. Some data displays such as Stem and Leaf Graphs and Dot Plots allow the data to be displayed as it is collected. Usually a table is the best way to record data systematically as it allows multiple variables to be entered against each respondent. For example:

People	Gender	Age	Height	Favourite subject	Pulse rate…
Fred					
Wilma…					

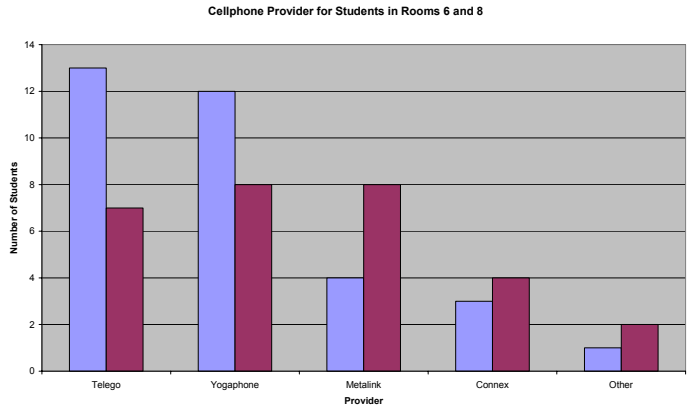
Data Analysis

In the analysis phase students should use appropriate technologies, including software, to select from a broad range of data displays to highlight patterns, trends and relationships. Patterns are about similarities and differences between samples or subgroups within a sample. This involves comparing and contrasting the main clustering in distributions considering shape ideas like middle, spread, and outliers (unusual occurrences). Trends are about patterns that develop over time, e.g. Less television watched in summer months than winter months. Relationships are about connections between variables and dimensions (categories), e.g. Taller people have longer feet, or Girls watch less television than boys.
It is expected that students will generate a variety of displays and critically evaluate the usefulness of each display. Implied in this is that they have access to computer technology to allow quick production of data displays from tables of data. Many applets for generating graphs are freely available on internet, e.g.
http://peabody.vanderbilt.edu/depts/TandL/mted/Minitools/Minitools.html,
http://nlvm.usu.edu/en/nav/category_g_3_t_5.html, http://illuminations.nctm.org/ActivityDetail.aspx?ID=146,
http://illuminations.nctm.org/ActivityDetail.aspx?ID=63, http://illuminations.nctm.org/ActivityDetail.aspx?ID=77,
http://illuminations.nctm.org/ActivityDetail.aspx?ID=60, http://illuminations.nctm.org/ActivityDetail.aspx?ID=78.
Other useful software includes MSExcel™, Tinkerplots™, and Fathom™.
Appropriate displays for Level Four include:

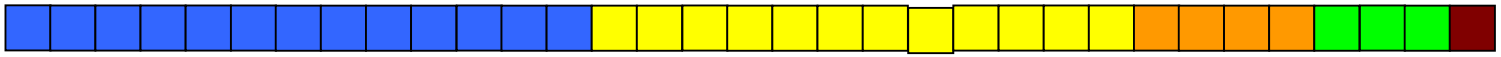
Category data displays:



Single Bar Graph

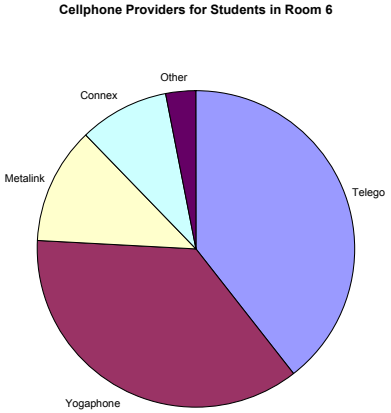


Double Bar Graph



Telego Yogaphone Metalink Connex Other

Strip Graph



Pie chart

Cellphone Provider for Students in Room 6	
Cellphone provider	Number of Students
Telego	13
Yogaphone	12
Metalink	4
Connex	3
Other	1

Frequency table

Discrete number data displays:

Minutes spent on cellphone in one week (Room 6 students)

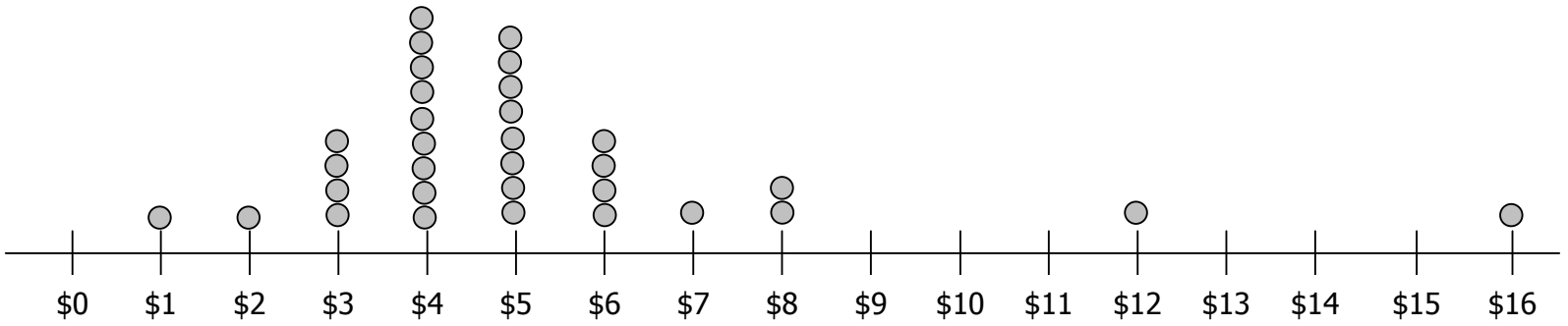
6	0 0
5	1 4 6
4	0 3 5 5 5 7 8 9
3	0 5 7 8
2	3
1	
0	0 0 0 0

Single Stem and Leaf Graph

Minutes spent on cellphone in one week (Room 6 and Room 8 students)

Room 8								Room 6							
						9	**6**	0	0						
							5	1	4	6					
					2	0	0	**4**	0	3	5	5	5	7	8	9
9	8	8	7	5	3	2	0	**3**	0	5	7	8				
			8	5	5	0	0	**2**	3							
							5	**1**								
								0	0	0	0	0				

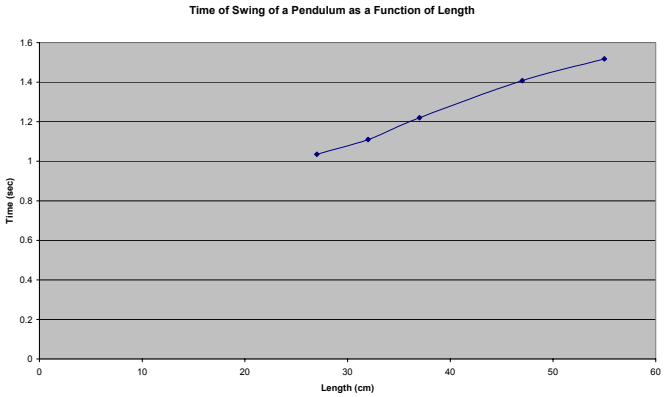
Back to Back Stem and Leaf Graph



Amount spent on one canteen lunch to the nearest dollar, May 12, 2006.

Dot plot (May also be used for continuous data where the dots a are placed within intervals, see homework example below)

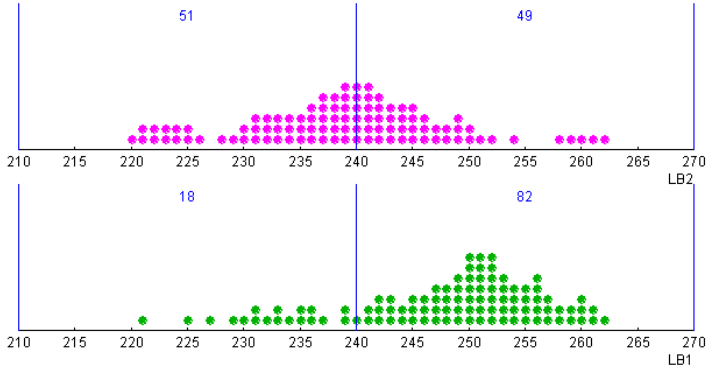
Continuous data displays:



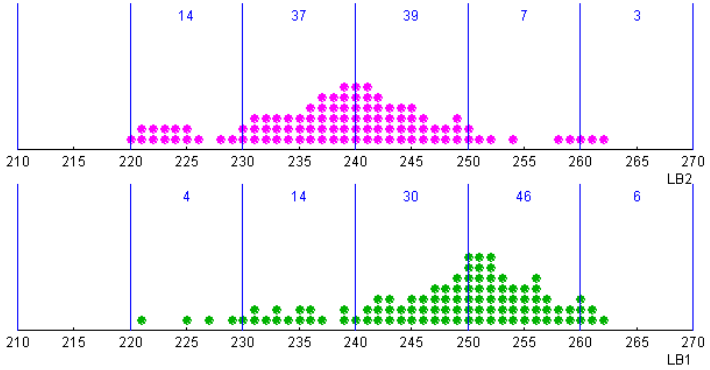
Line graphs

Students need to see the creation of displays as both a way to find patterns in the data and as a way to communicate those patterns to others. They need to select displays critically that show the patterns, trends and relationships of the particular data. For category data, some displays are excellent for showing differences between frequencies, e.g. bar charts and pictographs, while other displays show proportions, e.g. pie charts and strip graphs. With number data the selection of categories and intervals is vital to facilitate comparisons, e.g. choosing the stem values for a stem and leaf graph or selecting intervals for a dot plot.

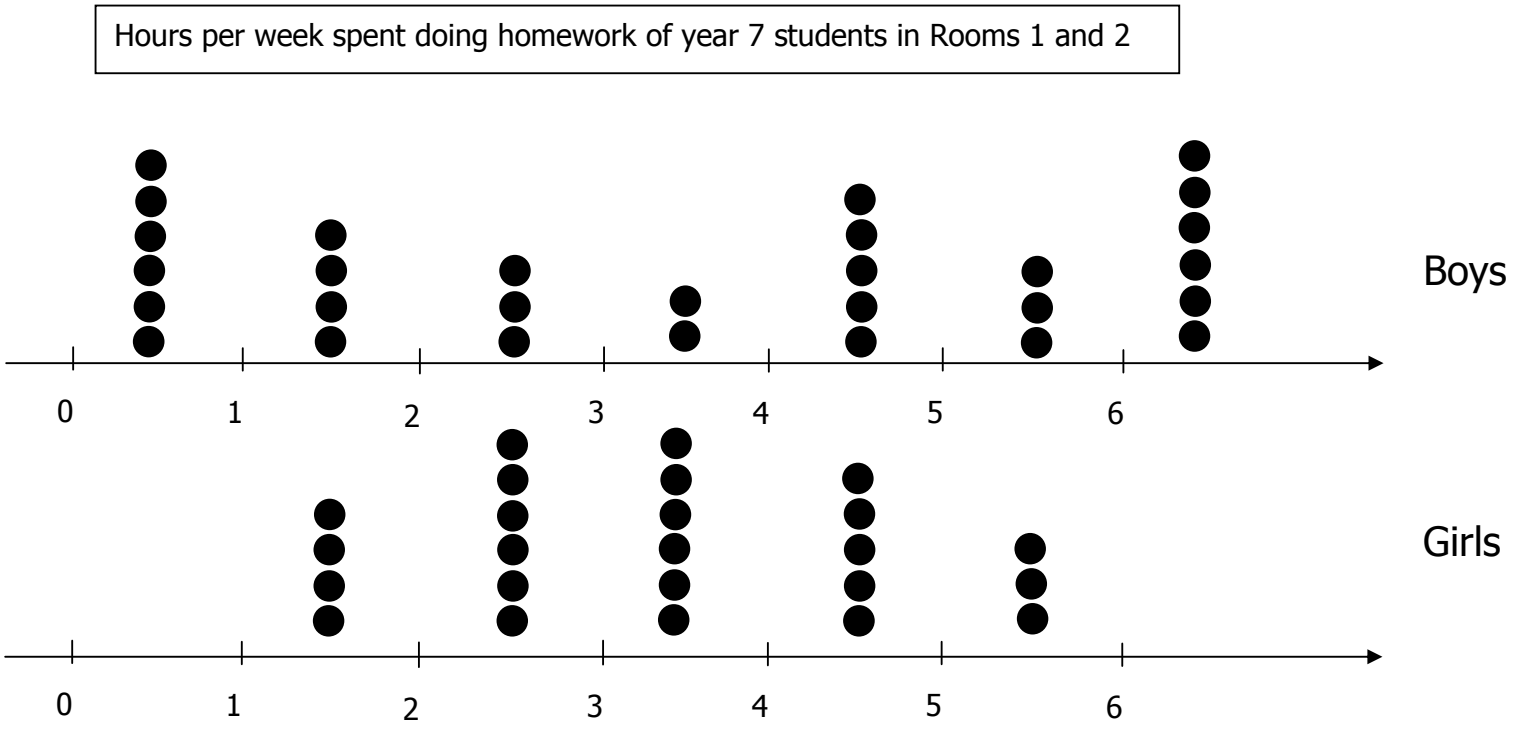
Creating intervals highlights differences between the shape and centrality of the distributions but creating intervals that are too small can hide that variation. The two dot plots below compare the height of corn plants under two different conditions, no fertilizer (LB2) and with fertilizer (LB1). Each distribution is partitioned into two intervals. This starkly reveals the difference between the distributions.



Compare this display with the same distributions divided into six intervals. The variation in frequency (number of datapoints) between comparable intervals reduces though, in this case, the patterns of fertilised corn growing higher is still clear.



Comparison of the distributions from different samples is critical for answering the relationship questions that arise from multi-variate datasets. This involves looking at the shape of these distributions considering centrality (the middle), spread, and clusters. For example, the dotplots below show the homework times for boys and girls in two year 7 classes. The spreads of these distributions shows that there is greater variation in the homework times for boys than girls. The centrality (middle) of both distributions is about the same but the shapes of the distributions show the data clustering in different places. This allows for comparative statements to be made, such as, "Most girls do between 2 and 5 hours homework per week whereas most boys do either less than 2 hours or more than 4 hours."



Conclusion

The conclusion phase of the Statistical Inquiry Cycle involves making decisions about how best to report findings to a particular audience. Statistical reports should include the original question, how the data was gathered, displays selected to show relationships, patterns or trends, and statements seeking to answer the original questions. Students at this level should acknowledge when no conclusion can be drawn from the data. Conclusion need to be interpreted in context and tentative explanations given for the findings, e.g. Listening to loud music while you are doing your school work may distract you and make it harder to remember things.
Conclusions should also lead to more "I wonder…" questions that initiate the inquiry cycle again. For example, the student who has investigated the effect of fertilizer on the growth of corn might wonder, "Will other conditions also create growth that is just as good?" or "Does this fertliser create better growth than other fertilizers I could buy?"

Useful resources
Figure It Out (Learning Media)
Statistics Level 3-4, pages 1-17
Statistics Year 7/8, Level 4, pages 1-16
Statistics Year 7/8, Level 4+, pages 1-16
[Comprehensive Teacher notes are provided for each student book. These notes have been distributed to schools and can also be accessed through http://www.tki.org.nz/r/maths/curriculum/figure/index_e.php
Numeracy Project Book 9: Teaching Number through Measurement, Geometry, Algebra, and Statistics, pages 41-52.
nzmaths.co.nz units (This website is sponsored by the Ministry of Education)
http://www.nzmaths.co.nz/node/161 (Statistical Investigations: Average Looking)
http://www.nzmaths.co.nz/node/162 (Statistical Investigations: Census at School)
http://www.nzmaths.co.nz/node/163 (Statistical Investigations: Time Use Survey)
http://www.nzmaths.co.nz/node/164 (Statistical Investigations: Typical Year 8)
http://www.nzmaths.co.nz/node/165 (Statistical Investigations: Measuring up)
http://www.nzmaths.co.nz/node/169 (Statistical Investigations: Sports Stats)
http://www.nzmaths.co.nz/node/166 (Statistical Investigations: Cars)
http://www.nzmaths.co.nz/node/167 (Statistical Investigations: Travel to School1)
http://www.nzmaths.co.nz/node/168 (Statistical Investigations: Travel to School 2)

http://www.nzmaths.co.nz/node/170 (Statistical Investigations: Marble Roll)
Census at School (This website is sponsored by Statistics New Zealand and The University of Auckland)
http://www.censusatschool.org.nz/
http://www.nzmaths.co.nz/node/162 (Statistical Investigations: Census at School)

Digital Learning Objects (These are accessed through the Ministry of Education Digi-Store and are the result of a collaborative project run by The Learning Federation, Australia)
http://www.nzmaths.co.nz/learningobjects/317/4

Other Website links:
http://illuminations.nctm.org/WebResourceList.aspx?Ref=2&Std=4&Grd=0
http://peabody.vanderbilt.edu/depts/tandl/mted/Minitools/Minitools.html
http://nlvm.usu.edu/en/nav/category_g_2_t_5.html