Mathematics in the New Zealand Curriculum
Second Tier

Strand: Geometry and Measurement   Thread: Measurement   Level: Four

Achievement Objectives:

- Use appropriate scales, devices, and metric units for length, area, volume and capacity, weight (mass), temperature, angle, and time.
- Convert between metric units, using whole numbers and commonly used decimals.
- Use side or edge lengths to find the perimeters and areas of rectangles, parallelograms, and triangles and the volumes of cuboids.
- Interpret and use scales, timetables and charts.

Important Teaching Ideas

Developing as a Measurer

The key idea of measurement at **Level One** is **comparison** within a chosen measurement attribute such as length, weight etc. Comparing, and therefore ordering, is the first step in a person becoming a mature measurer. Hence the language of measurement at this stage is 'longer', 'shorter', 'smaller', 'bigger', 'heavier', 'lighter', 'hotter', 'colder', etc.

Perception of the attributes of measurement (length, weight, time etc.) is fundamental and in fact precedes, or is developed in conjunction with, the comparative concepts. For example the **concept** of distance must be developed before any attempt to measure distance is made. Appropriate use of language such as 'shorter' and 'longer', 'further and 'closer' will develop and strengthen the concept of distance. The definitions of the measurement attributes of length, area, volume, capacity, weight, angle, temperature and time are given in the glossary. The metric system (SI) is also well covered in the glossary.

An important distinction is between **'direct comparison'** and **'indirect comparison'**. As an example, we could compare the lengths of two objects by placing them next to each other and thus observing which is the longer. That would be an example of direct comparison. Alternatively, we could take a piece of string the length of one object and place it against the other object and so compare their lengths that way. That would be an example of indirect comparison. It is important that children experience both forms of comparison at Level One. Indirect comparison is an important step towards the use of non-standard and standard measurement units and the use of measurement tools like rulers and jugs.

Students at level one should generalise the principles of counting to measurement. Counting describes the total number of objects in a set so measuring describes the total number of measures in a space. For example, a student measuring the width of a room using footsteps needs to understand that the count, 1,2,3,.., is describing the number of steps to that point not naming each step.

The key idea of measurement at **Level Two** is the use **of units** and 'devices' to measure length, area, volume etc. These units may be standard or non-standard. A non-standard unit is a unit of measurement that has been chosen by a group of people because it is convenient to use. Non-standard usually means that the unit is not universally accepted. Historically non-standard units have usually been decreed by or taken from body parts of an important person like a tohunga. A standard unit is one within the SI (metric) system, e.g. litre, metre, that are universally accepted if not always used, e.g. USA still uses pounds and miles.

So for length, examples of non-standard measures would be paces, hand spans or pencil lengths; for area, exercise books or sheets of newspaper. Non-standard units should be selected from the student's environment and experiences. The use of non-standard units and whole numbers of standard units introduces the student to the concept of measurement units without requiring the rigour of understanding the metric system, particularly the multiplication and division required, e.g. 1000 metres is 1 kilometre. Further it gives the student the opportunity to develop skills of estimation which are required for operating effectively in the adult world.

In measuring the length of, for example, a desk, students can choose an appropriate unit and see how many of those units can be fitted into the length of the desk. Length is a continuous measure but in order to measure the length of a desk we need to partition the length into a sequence of the chosen units, counting how many of those smaller subdivisions will fit into the length of the desk. We may need to partition the unit itself and consider halves and quarters to finish the task. What we call the length of the desk is the combination of those units and part units. Measurement is a critical context for the connection of whole (units) to parts (fractions). Once again, it is the completing of such tasks and the appropriate discussion that is associated that will lead the students to such understandings.

Students at **Level Two** should be able to apply the addition and subtraction, simple multiplication and division understandings they have from number to measurement problems involving whole numbers of units, e.g. 6 metres. Most children will have encountered some basic metric units, such as metres, kilometres, litres and kilograms, without realising how they relate to one another. For example, they should be able to recognise that a 20 centimetre length could be cut into two ten centimetre lengths, or a nine centimetre and an eleven centimetre length, etc. They should realise that if two litres of water weighs two kilograms then ten litres should weigh ten kilograms.

At **Level Two** students should be asked to create their own measurement instruments. For example, they might be given a strip of paper and cubes to measure the length of many object thus promoting the creation of a ruler. They may be asked to develop a time measurement device having been shown examples of sandtimers, candle and water clocks, or asked to find the weight of other children given a see-saw.

Effective use of non-standard units will lead students naturally to an understanding of the need for standard units. This can be seen through the effective selection of activities that expose the two main reasons for having standard units. Firstly, there is the difficulty that arises through the use of non-standard units such as hand spans or pencils. That is, that there can be many different hand span or pencil lengths and so the measure of the length of an object will vary according to whose hand span or pencil is being used. Such an understanding can be developed through the use of comparison activities. For example, having the students measure the length of something using their pencils and then comparing results and discussing the reason for the variation in results.

Secondly, if we wish to communicate the result of a measurement to a person in another classroom, city, or country we will all need to be using exactly the same units. This understanding can be assisted by using 'fixed non-standard units' such as the length of a piece of A4 paper as opposed to 'variable non-standard units' such as pencil lengths. Through appropriate examples students will see the need for a single standard system such as the metric system.

Level Three measurement sees a strong focus on **standard units** and specifically the **metric system (SI)**. Appropriate measurement experiences at **Level Two** will have prepared students for the necessity of having standard units so that people all over the world can communicate measurement values and understandings to each other.

Students should be immersed in measurement experiences that are rich in investigation and the use of scales and instruments. As well as gaining a good feel for the size of the metric units, students need to know the names of the units, understand the prefixes and know the symbols for each unit. Teaching students to use scaled instruments such as rulers and protractors effectively, and to read graduated containers is important at this level.

Note that some of the standard units in use today are not actually SI units but can be used in conjunction with SI units. For example, the second is the standard unit of time but the minute and the hour are not actually part of the metric system. However as a society we accept kilometres per hour as a standard unit.

Level Three also sees the start of the use of relationships between length and area, or length and volume, for figures such as rectangles and cuboids. Cuboids are portions of space bounded by rectangles. For example, nearly all packaging boxes are cuboids. So students find equations for the area of a rectangle in terms of the lengths of its sides in whole numbers of units, and the volume of a cuboid in terms of the lengths of its edges in whole numbers of cubes.

Level Four sees a consolidation of understanding and use of the metric system and the further use of scales and measuring devices. Students need to be able use their multiplicative thinking and emerging understanding of decimals to convert between metric units such as grams and kilograms, millimetres and centimetres, most of which they should have met at Level Three. They reinforce their use of formulas to find the areas of rectangles and the volumes of cuboids and extend that to figures such as parallelograms and triangles.

At **Level Five** students should have achieved a level of maturity as measurers that enables them, when given a practical measurement problem, to develop and use a method of solution and discuss the degree of accuracy of the result. Students should apply their understanding of decimals to converting between measures of the same attribute, e.g. 1.276t = 1276kg (t means tonne, 1000kg).

They should be capable of finding ways of measuring the perimeters (circumferences) and areas of circles and using their measurement knowledge to determine the areas of figures that are a composition of known shapes.

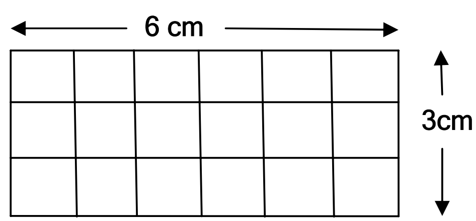
In doing so students should connect their understanding of classes of geometric shapes to measurement. For example, a cylinder can be seen as an example of a prism, a solid with constant cross section. The volume of all prisms is found by multiplying the area of its cross section by its height hence the formula $v = \pi r^2 h$.

Teaching Measurement at Level Four:

Providing a rich learning experience for measurement at Level Four requires the involvement of students in measurement activities using the units of the metric system in measurement problem solving situations. Constructing and using measurement devices, such as clinometers for measuring vertical angles, should be a part of their experience.
Students become more familiar with the metric system by converting between units, such as the centimetre and the millimetre. They thereby also become more familiar with the prefixes of the metric units (milli, centi, deci, kilo etc.)
They also extend their knowledge of the areas of rectangles to other plane figures such as the parallelogram and the triangle. It is important that students are not given formulas for areas or volumes of geometric shapes. Formulas, such as the area of a triangle being one half of the product of the length of the base and the length of the height, should be derived by the students from activities such as drawing shapes on squared paper.
The interpretation and use of scales, timetables and charts can involve and strengthen concepts of weight, temperature, time, and angle.

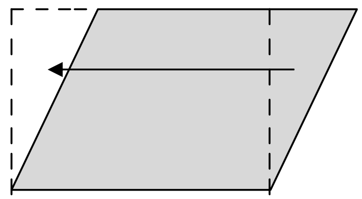
Exemplars of Student Performance:

Exemplar One: Students use squared paper to find expressions for the areas of rectangles, parallelograms and triangles.
Although this activity can be done on any paper, some students will find it easier to understand if it is done on squared paper. Students should be provided with the shapes drawn on the paper. Begin by confirming the Level Three finding that the area of a rectangle is the product of its length and its height.

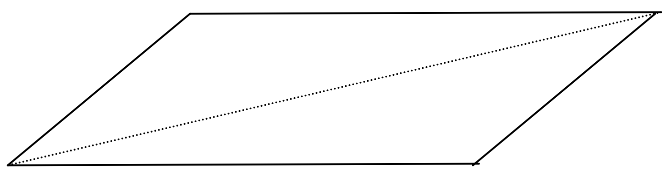


Area is 18 cm^2

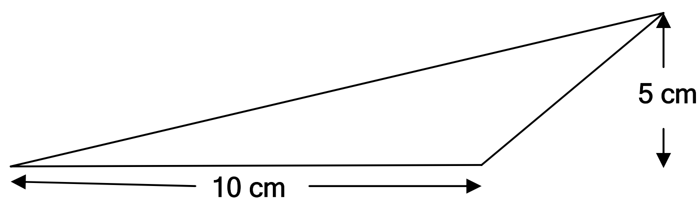
Then have the students explore the parallelogram and see if they can use their knowledge of the area of a rectangle to find the area of a parallelogram. Discussion should lead to the idea of cutting a right triangle off one end of a parallelogram and attaching it to the other end to form a rectangle. Students should actually do this with scissors and paper.



From this it should be obvious that the area of a parallelogram is the product of its length and its height. Students need to realise that the height is not the same as the edge length. (Note: It is a good idea to colour the edges of the parallelogram with four different colours so that students can see exactly what has become of the transformed edges.)
The area of a triangle, which is one half of the product of its length (base) and its height, is apparent from the fact that every parallelogram can be bisected (cut in half) to give two congruent triangles, and every triangle can be viewed as one half of an associated parallelogram.

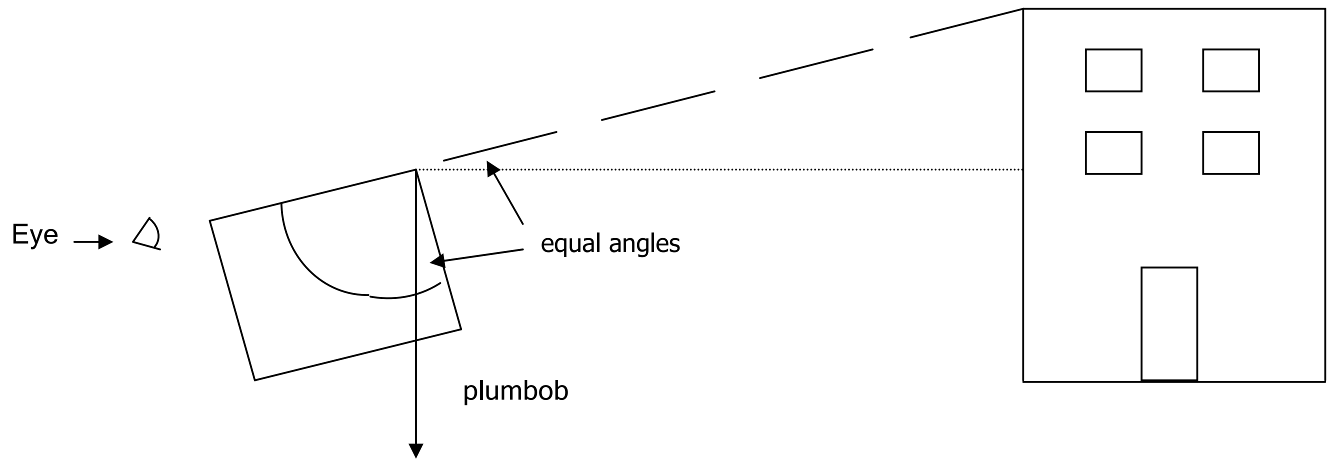


So the area of the following triangle is 25 cm^2

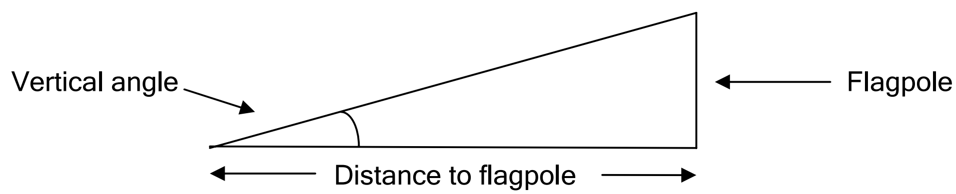


Exemplar Two: Students use clinometers to measure vertical angles and find heights of buildings, trees etc.

A clinometer is an instrument for measuring vertical angles. A simple clinometer can be made with a piece of strong cardboard, an enlarged photocopy of a protractor on light card, and a weight (such as a bolt) on a piece of string, to act as a plumbob. The photocopy of the protractor should be glued to the strong cardboard so that students can sight along the edge of the protractor.

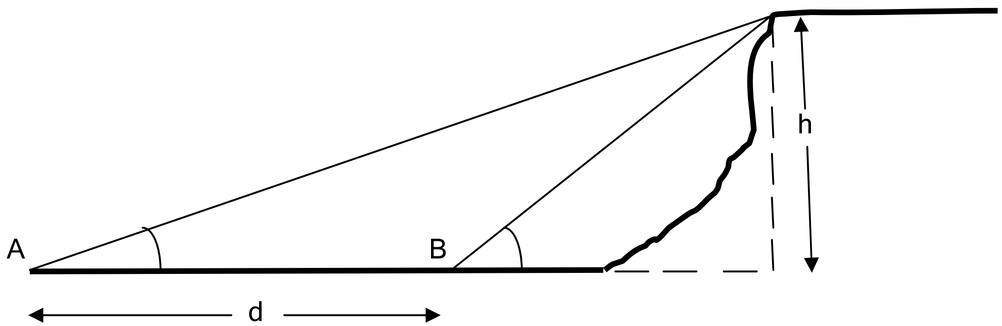


Students find the height of a flagpole as follows. They select a point a measured distance from the flagpole (20 or 30 metres would be good depending on the height of the flagpole). The ground needs to be reasonably horizontal. They use the clinometer to measure the vertical angle to the top of the flagpole. In the diagram this looks to be about 10 degrees. They then draw a scale diagram of their measurements.



They find the height of the flagpole by measuring the distance on the diagram and using the scale of the diagram to find the height of the flagpole above the level of their eye. They need to add their eye-height to the scaled distance to find the full height of the flagpole. Different students could draw the triangle to different scales. Finding that the scaled height is the same regardless of the size of the triangle is an important pre-trigonometry concept. This method can be used on outdoor education activities to find the heights of trees, cliffs etc.

Exemplar Three: Students find heights of objects whose bases are inaccessible.
This is an extension of Exemplar Two. Suppose students want to find the height of an inaccessible cliff, tree etc. They have a reasonably flat piece of ground not too far away and they measure the angles and the distance (d) shown in the diagram.



They again draw a scale diagram and use it to find h, the height of the cliff above the flat ground.

Exemplar Four: Students confirm their knowledge of the volume of a cuboid.
At Level 3 students used blocks of cubes to reason that the volume of a cuboid was found by multiplying the number of units of width, depth and height of the cuboid to obtain the number of units of volume of the cuboid. In this activity students confirm this finding and relate it more closely to other units of the metric system.
Students are supplied with a collection (perhaps six) of wooden or plastic cuboids of various sizes. These could be watertight containers, offcuts of wood etc. They measure the length in centimetres of the width, depth and height of each of the cuboids, and make a chart displaying those. Then they carefully immerse each cuboid in a pan full of water, and catch and measure the overflow for each one. They complete the chart as follows in the example.

Cuboid	Width (w) (cm)	Depth (d) (cm)	Height (h) (cm)	w x d x h (cm^3)	Volume of water (ml)
1	11	7	12	923	920
2	5.5	12	10.3	679.8	685
3	15.4	8.2	7.3	921.8	920
etc					

This activity reinforces knowledge of the volume of a cuboid as the product of its three edge lengths. It also reinforces knowledge of the millilitre as being equivalent to the cubic centimetre and the litre being 1000 cm^3.

Exemplar Five: Students use bus timetables to answer questions involving time.
Teachers can obtain copies of local bus timetables and set up questions around the information contained in them. Alternatively they could construct hypothetical tables such as in the example below.

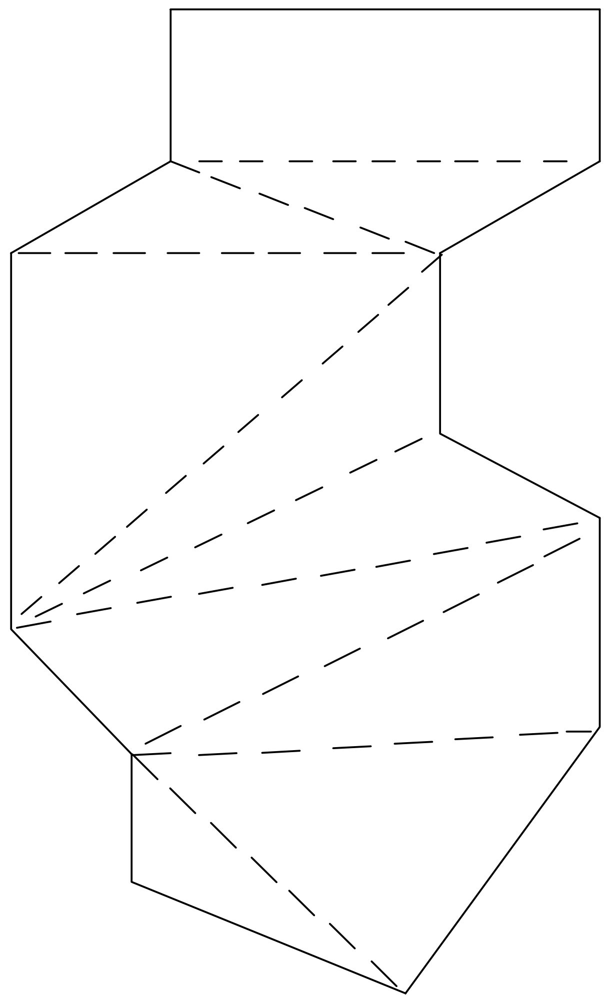
Bus Number — Time at which bus leaves depot A, B, C etc.

	A	B	C	D	E	F	A	
1	8:00	8:10	8:25	8:37	9:04	9:14	9:26	
	A	G	H	B	G	E	F	A
2	8:10	8:21	8:26	8:39	8:51	8:59	9:09	9:21
	A	G	H	D	E	G	A	
3	8:20	8:31	8:36	8:53	9:20	9:28	9:39	

Questions:
1. Which bus takes the shortest time to go from B to E?
2. What is the shortest time in which one can travel from H to A? Which bus is that?
3. How long does the trip from D to E take? Etc.

Exemplar Six: Students find the weight of water in cuboid containers
Students are given a variety of cuboid containers and weighing scales. They begin with a two-litre ice-cream container. They fill it with water and weigh it to confirm that a litre of water weighs one kilogram. They measure and weigh other containers to confirm the weight of one litre (1000 cm^3) of water as being one kilogram. Then they calculate the weight of water in larger containers such as fish tanks, water beds and swimming pools. Thinking out the volume of water in a swimming pool and taking an average depth is part of learning to think as a measurer. Students will be introduced to the tonne as 1000 kg, or the weight of one cubic metre of water.

Exemplar Seven: Students use their knowledge of area to find the areas of irregularly shaped regions.
Students are given a plan of a house that has non-rectangular regions. The house is to be carpeted throughout. Students find the floor area of the house. They use their knowledge of the area of a triangle as being one half of the product of the length of the base and the length of the height to determine the area.
Example:



Scale: One cm to one metre (1 cm:1m)

The house has been divided into a rectangle and nine triangles. Students find the area of each triangle by measuring on the plan the length of the base and the length of the height. They convert the lengths using the scale and then find their product which is the area of the triangle. This method can be used to find the areas of regions on maps and plans.

Exemplar Eight: Students measure their lung capacity.
This would be a good activity to do before and after an aerobic fitness programme. Each student is supplied with a reasonably large balloon. They take a deep breath and blow the balloon to the maximum capacity of their lungs, then seal off the balloon. The balloon is immersed in a large container completely filled with water and the overflow is measured. The overflow gives a reasonable measure of lung capacity.

Exemplar Nine: Students learn to interpret a map.
Students are given a simple map drawn to a scale which is easy for them to convert, for example, one centimetre to five metres. The map could be a map of a hypothetical region; however a map of the school grounds and buildings would be a good map to start with as the students know the actual region.
Questions about the map could be questions such as:

- What is the distance from our classroom door to the hall?
- What is the area of the sports field?
- What is the perimeter of the school boundary?
- There is hidden treasure at the spot marked X. Go outside and put a marker where you think the treasure is hidden.
- It has been decided to re-sow all the grassed region of the school. Find the area of the grassed region of the school by dividing the grassed region into triangles and scaling their length and height. (Note: Students need to convert the linear measurements according to the scale before multiplying them.)
- If grass seed costs $3.00 per square metre, how much will the grass seed cost for the whole school?

Exemplar Ten: Students develop an understanding of the size of a hectare and relate it to a rugby field.
Students are informed that a hectare is an area of ten thousand square metres. They are then asked to work out what sized square would have an area of one hectare. They should be able to reason that a square of edge length 100 metres has an area of one hectare. An appropriate region where the students can set out a 100 metre square is selected and the students use 30 metre tapes to mark out the square. Running or walking around the perimeter of the square would give them a feel for the size of the hectare. As an easy way of remembering they could remember that the hectare is the size of a squared rugby field of length 100 metres.

Useful resources

Figure It Out:
Measurement, Year 7-8, Book1: Length (pp.3-5, 10-11), Area (pp.8-11, 22-23), Volume (pp. 2, 6-7, 12-13, 14-15, 22-23), Mass (pp.6-7, 12-13, 14-15), Time (pp. 20-21,24), Rates (pp. 16-17).
Numeracy Project Book 9: Teaching Number through Measurement, Geometry, Algebra, and Statistics, pages 3-15.
nzmaths.co.nz units (This website is sponsored by the Ministry of Education)
http://www.nzmaths.co.nz/node/415 (Length: How Long is a Slinky)
http://www.nzmaths.co.nz/node/212 (Length and Transformation: Team Puzzles)
http://www.nzmaths.co.nz/node/414 (Length and Number: How Fast is Fast)
http://www.nzmaths.co.nz/node/423 (Area: What Goes Around)
http://www.nzmaths.co.nz/node/388 (Area and Algebra: You can count on Squares)
http://www.nzmaths.co.nz/node/424 (Area: Triangles)
http://www.nzmaths.co.nz/node/793 (Volume: Oranges L4)
http://www.nzmaths.co.nz/node/791 (Volume: Spaced Out)
http://www.nzmaths.co.nz/node/802Mass: Weighty Problems)
http://www.nzmaths.co.nz/node/209 (Shape and Measurement: Building with Triangles)
http://www.nzmaths.co.nz/node/213 (Shape and Measurement: Quadrilaterals)
http://www.nzmaths.co.nz/node/816 (Time: Time Zones)
http://www.nzmaths.co.nz/node/819 (Temperature: Cool Times)

Digital Learning Objects (These are accessed through the Ministry of Education Digi-Store and are the result of a collaborative project run by The Learning Federation, Australia)
http://www.nzmaths.co.nz/learningobjects/315/4

Other Website links:
http://nlvm.usu.edu/en/nav/category_g_3_t_4.html