Mathematics in the New Zealand Curriculum
Second Tier

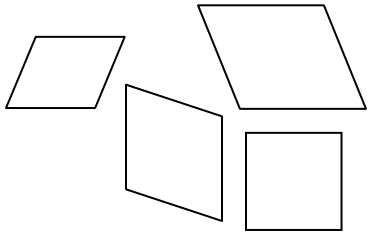
Strand: Geometry and Measurement Strand Thread: Shape Level: Three

Achievement Objectives:

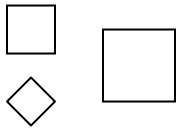
- Classify plane shapes and prisms by their spatial features.
- Represent objects with drawings and models.

Exemplars of student performance:

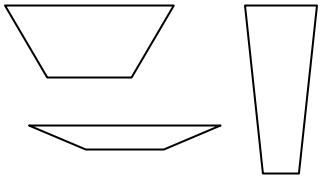
Exemplar One: The student is provided with a set of shapes containing squares, rhombuses and trapeziums. He/She sorts the shapes by their global features then defines the characteristics of each set. e.g.



"These are all rhombuses as they have two pairs of parallel sides, four equal sides with opposite angles the same."



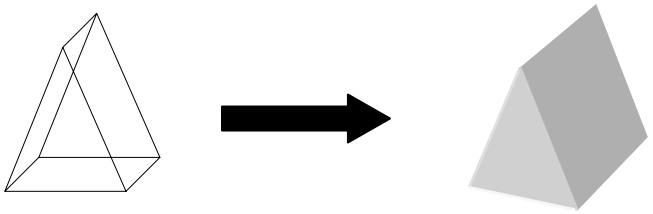
"These are all squares as they have two pairs of parallel sides, four equal sides and four right angles".



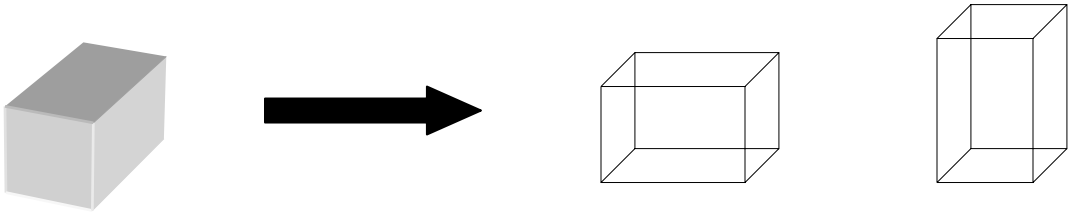
"These are all trapeziums as they have 4 sides, with only one pair of parallel sides"

This response shows achievement at Level Three because the student is able to describe all the sets of shapes by their spatial features. e.g. sides, angles, parallel.

Exemplar Two: Given a drawing of a triangular prism the student is able to construct a model of it using materials of their choice (e.g. paper, toothpicks, blutack™ etc).



Given a model of a different prism (e.g. a cuboid) the student is able to draw a representation of it viewed from different positions.



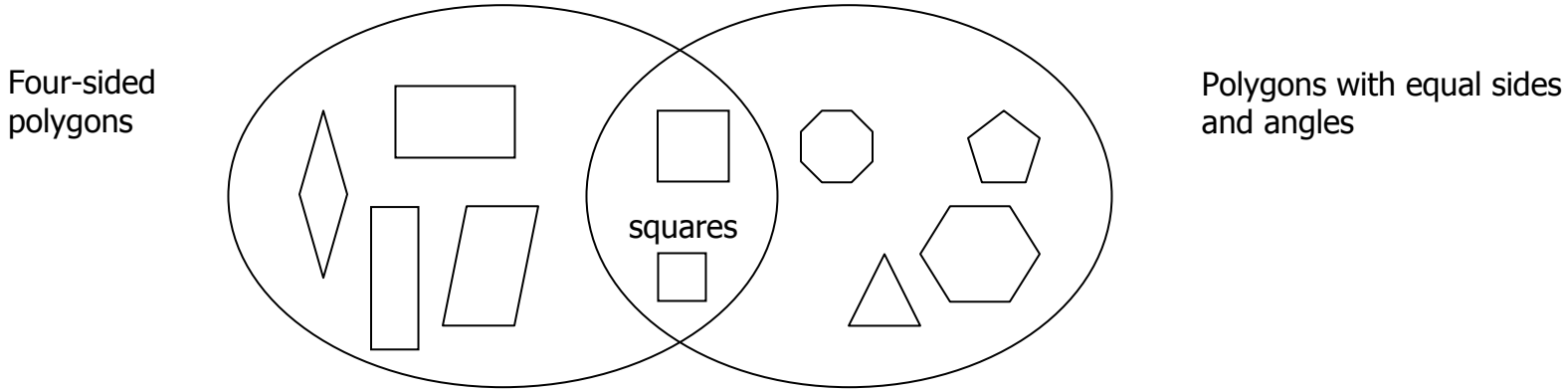
This response shows achievement at Level Three because the student is able to preserve the relationships between features (e.g. faces meeting at vertex, lengths of edges) when the models are shown in another representation.

Important teaching ideas (working at):

The focus of this level is defining plane (flat) shapes and simple solids (prisms) by their spatial features, The student should be able to use vocabulary such as corner, side, diagonal, line of symmetry, angle, parallel and radius to describe the properties of two dimensional shapes, and vocabulary such as face, edge, vertex/vertices to describe the properties of three dimensional solids. Students should be able to describe how the spatial features of an object relates to it's purpose, e.g. Why are cans cylindrical? They are stable when packed (tessellate almost) and are easy to make because of one curved surface and two flat surfaces.

Definitions capture the spatial features common to all members of a group of plane shapes or prisms, for example;
"All triangles have three straight sides". This means that any shape that has three straight sides is a triangle.

"Squares are four sided polygons with equal sides and angles".



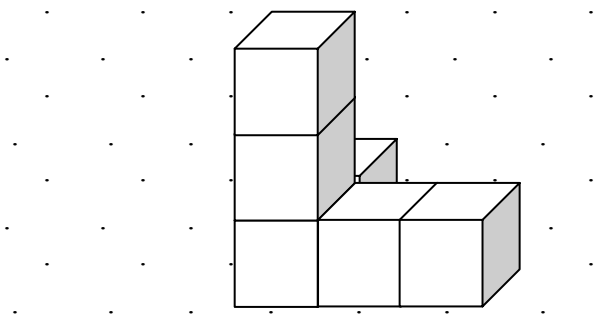
Shapes can be classified into classes and sub-classes, for example, rectangles are one sub-class of parallelograms but this type of formal classification is not required until level 4.

The key attribute of prisms is their constant cross section from which their name is derived, e.g. slices of a loaf of bread are rectangular and come from a rectangular prism (or cuboid). With justification a student should be able to identify which groupings a given shape or prism belongs to.

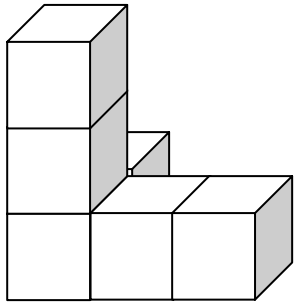
Students should be working towards refining definitions to eliminate unnecessary features. For example, investigate which attributes of a square (four equal sides, two pairs of parallel sides and four right angles) are not essential in it's definition i.e. a shape with four equal sides and four right angles must also contain two pairs of parallel sides. Minimal definition is a difficult idea at level three but it can be encouraged by listing all the properties of a shape then seeing which properties are subsumed by others, e.g. If a shape has four sides, how many corners must it have?

The student should be able to work from model to a representation as well as a representation to a model. This involves attending to features (e.g. faces meeting at corners, cubes along an edge) of the shapes/models. It also involves preserving the relationships between features when the models are shown in another representation. Students at this level should be able to construct models of prisms e.g. a toothpick and blutack^{TM} construction of a cube as well as construct shapes with given characteristics., e.g. construct a polygon with four equal sides. Students could use isometric

dot paper to draw cube models as well as make a model from an isometric drawing. They could also make models of objects and scenes represented in photographs and drawings by focusing on the plane shapes and other attributes found within them.

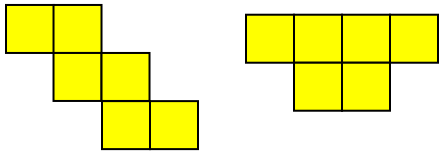


Isometric paper drawing



Cube model

Students should be able to predict the nets for pyramids and simple prisms such as cuboids.
e.g. Which of the following two nets will make a cube?

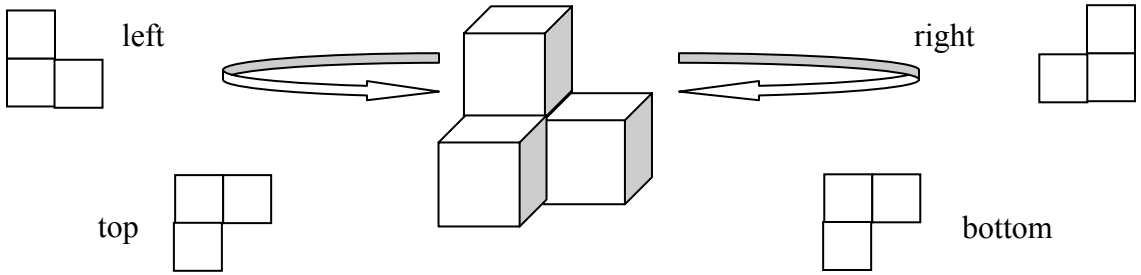


They should also be able to make the nets, and construct the solids from them.

At this level, students are working with more three dimensional shapes than just prisms to include cones and pyramids and should be developing the ability to predict the cross-sections of shapes by trying to image the internal manipulation of an object, e.g. What would the cross section of a cone, an apple, or a carrot be? How many different cross-section shapes can be made by cutting a cube with one straight cut? For examples like these students will need access to materials, like plasticine, so that can check their predictions by actually cutting a model of the shape.

Students could explore tangrams to reassemble shapes that have been partitioned by focusing on the features of the shape, e.g. angles, sides, symmetry.

Drawing and describing images of objects as viewed from different positions contributes to the development of spatial visualization. Students should have experience of manipulating, drawing and describing what solid objects as well as everyday objects look like from different viewpoints. Two contexts of spatial visualization need to be encouraged; considering the object moving relative to the person as well as the person moving relative to the object. For example, draw plan views of a model made from cubes and make a cube model from someone else's plan views.



Similarly, manipulating, drawing and describing what shapes and prisms look like following a reflection or rotation will help to develop this idea. The link with transformation can be developed further by exploring tessellations of shapes and predicting which regular shapes (equal sides) will tessellate. This could lead to students investigating what features of shapes make the tessellation possible with no gaps or overlaps. For further information refer to level four of Tesselation support material.

Useful resources

Figure It Out: Geometry Level 2-3, Pages 1-13, 24 (Learning Media, Wellington)
Figure It Out: Geometry Level 3, pages 1-12, 16-17 (Learning Media, Wellington)

nzmaths Website:
http://www.nzmaths.co.nz/node/207 (Shapes with Sticks)
http://www.nzmaths.co.nz/node/208 (Te Whanau Taparau)
http://www.nzmaths.co.nz/learningobjects/314/3 (Learning objects)

Virtual Library Website Manipulatives:
http://nlvm.usu.edu/en/nav/frames_asid_270_g_2_t_3.html (Attribute Blocks)
http://nlvm.usu.edu/en/nav/frames_asid_165_g_2_t_3.html (Congruent Triangles)
http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html (Geoboards)
http://nlvm.usu.edu/en/nav/frames_asid_129_g_2_t_3.html (Geoboards -Isometric)
http://nlvm.usu.edu/en/nav/frames_asid_170_g_2_t_3.html (Pattern Blocks)
http://nlvm.usu.edu/en/nav/frames_asid_195_g_2_t_3.html (Space Blocks)
http://nlvm.usu.edu/en/nav/frames_asid_289_g_2_t_3.html (Tangrams)

Illuminations Website Manipulatives:
http://illuminations.nctm.org/ActivityDetail.aspx?ID=70 (Geometric Solids)
http://illuminations.nctm.org/ActivityDetail.aspx?ID=72 (Shape Cutter)
http://illuminations.nctm.org/ActivityDetail.aspx?ID=27 (Patch Tool)
http://illuminations.nctm.org/ActivityDetail.aspx?ID=84 (Cube Nets)
http://illuminations.nctm.org/ActivityDetail.aspx?ID=125 (Isometric Drawing Tool)
http://illuminations.nctm.org/ActivityDetail.aspx?ID=35 (Shape Tool)
http://www.tangram.i-p.com/ (Tangram puzzles)

Book 9 Teaching Number through Measurement, Geometry, Algebra & Statistics, Learning Media, Wellington 2006. p16-21