Mathematics in the New Zealand Curriculum
Second Tier

Strand: Geometry and Measurement   Thread: Measurement   Level: Three

Achievement Objectives:

- Use linear scales and whole numbers of metric units for length, area, volume and capacity, weight (mass), angle, temperature, and time.
- Find areas of rectangles and volumes of cuboids by applying multiplication

Important Teaching Ideas

Developing as a Measurer

The key idea of measurement at **Level One** is **comparison** within a chosen measurement attribute such as length, weight etc. Comparing, and therefore ordering, is the first step in a person becoming a mature measurer. Hence the language of measurement at this stage is 'longer', 'shorter', 'smaller', 'bigger', 'heavier', 'lighter', 'hotter', 'colder', etc.

Perception of the attributes of measurement (length, weight, time etc.) is fundamental and in fact precedes, or is developed in conjunction with, the comparative concepts. For example the **concept** of distance must be developed before any attempt to measure distance is made. Appropriate use of language such as 'shorter' and 'longer', 'further and 'closer' will develop and strengthen the concept of distance. The definitions of the measurement attributes of length, area, volume, capacity, weight, angle, temperature and time are given in the glossary. The metric system (SI) is also well covered in the glossary.

An important distinction is between **'direct comparison'** and **'indirect comparison'**. As an example, we could compare the lengths of two objects by placing them next to each other and thus observing which is the longer. That would be an example of direct comparison. Alternatively, we could take a piece of string the length of one object and place it against the other object and so compare their lengths that way. That would be an example of indirect comparison. It is important that children experience both forms of comparison at Level One. Indirect comparison is an important step towards the use of non-standard and standard measurement units and the use of measurement tools like rulers and jugs.

Students at level one should generalise the principles of counting to measurement. Counting describes the total number of objects in a set so measuring describes the total number of measures in a space. For example, a student measuring the width of a room using footsteps needs to understand that the count, 1,2,3,.., is describing the number of steps to that point not naming each step.

The key idea of measurement at **Level Two** is the use **of units** and 'devices' to measure length, area, volume etc. These units may be standard or non-standard. A non-standard unit is a unit of measurement that has been chosen by a group of people because it is convenient to use. Non-standard usually means that the unit is not universally accepted. Historically non-standard units have usually been decreed by or taken from body parts of an important person like a tohunga. A standard unit is one within the SI (metric) system, e.g. litre, metre, that are universally accepted if not always used, e.g. USA still uses pounds and miles.

So for length, examples of non-standard measures would be paces, hand spans or pencil lengths; for area, exercise books or sheets of newspaper. Non-standard units should be selected from the student's environment and experiences. The use of non-standard units and whole numbers of standard units introduces the student to the concept of measurement units without requiring the rigour of understanding the metric system, particularly the multiplication and division required, e.g. 1000 metres is 1 kilometre. Further it gives the student the opportunity to develop skills of estimation which are required for operating effectively in the adult world.

In measuring the length of, for example, a desk, students can choose an appropriate unit and see how many of those units can be fitted into the length of the desk. Length is a continuous measure but in order to measure the length of a desk we need to partition the length into a sequence of the chosen units, counting how many of those smaller subdivisions will fit into the length of the desk. We may need to partition the unit itself and consider halves and quarters to finish the task. What we call the length of the desk is the combination of those units and part units. Measurement is a critical context for the connection of whole (units) to parts (fractions). Once again, it is the completing of such tasks and the appropriate discussion that is associated that will lead the students to such understandings.

Students at **Level Two** should be able to apply the addition and subtraction, simple multiplication and division understandings they have from number to measurement problems involving whole numbers of units, e.g. 6 metres. Most children will have encountered some basic metric units, such as metres, kilometres, litres and kilograms, without realising how they relate to one another. For example, they should be able to recognise that a 20 centimetre length could be cut into two ten centimetre lengths, or a nine centimetre and an eleven centimetre length, etc. They should realise that if two litres of water weighs two kilograms then ten litres should weigh ten kilograms.

At **Level Two** students should be asked to create their own measurement instruments. For example, they might be given a strip of paper and cubes to measure the length of many object thus promoting the creation of a ruler. They may be asked to develop a time measurement device having been shown examples of sandtimers, candle and water clocks, or asked to find the weight of other children given a see-saw.

Effective use of non-standard units will lead students naturally to an understanding of the need for standard units. This can be seen through the effective selection of activities that expose the two main reasons for having standard units. Firstly, there is the difficulty that arises through the use of non-standard units such as hand spans or pencils. That is, that there can be many different hand span or pencil lengths and so the measure of the length of an object will vary according to whose hand span or pencil is being used. Such an understanding can be developed through the use of comparison activities. For example, having the students measure the length of something using their pencils and then comparing results and discussing the reason for the variation in results.

Secondly, if we wish to communicate the result of a measurement to a person in another classroom, city, or country we will all need to be using exactly the same units. This understanding can be assisted by using 'fixed non-standard units' such as the length of a piece of A4 paper as opposed to 'variable non-standard units' such as pencil lengths. Through appropriate examples students will see the need for a single standard system such as the metric system.

Level Three measurement sees a strong focus on **standard units** and specifically the **metric system (SI)**. Appropriate measurement experiences at **Level Two** will have prepared students for the necessity of having standard units so that people all over the world can communicate measurement values and understandings to each other.

Students should be immersed in measurement experiences that are rich in investigation and the use of scales and instruments. As well as gaining a good feel for the size of the metric units, students need to know the names of the units, understand the prefixes and know the symbols for each unit. Teaching students to use scaled instruments such as rulers and protractors effectively, and to read graduated containers is important at this level.

Note that some of the standard units in use today are not actually SI units but can be used in conjunction with SI units. For example, the second is the standard unit of time but the minute and the hour are not actually part of the metric system. However as a society we accept kilometres per hour as a standard unit.

Level Three also sees the start of the use of relationships between length and area, or length and volume, for figures such as rectangles and cuboids. Cuboids are portions of space bounded by rectangles. For example, nearly all packaging boxes are cuboids. So students find equations for the area of a rectangle in terms of the lengths of its sides in whole numbers of units, and the volume of a cuboid in terms of the lengths of its edges in whole numbers of cubes.

Level Four sees a consolidation of understanding and use of the metric system and the further use of scales and measuring devices. Students need to be able use their multiplicative thinking and emerging understanding of decimals to convert between metric units such as grams and kilograms, millimetres and centimetres, most of which they should have met at Level Three. They reinforce their use of formulas to find the areas of rectangles and the volumes of cuboids and extend that to figures such as parallelograms and triangles.

At **Level Five** students should have achieved a level of maturity as measurers that enables them, when given a practical measurement problem, to develop and use a method of solution and discuss the degree of accuracy of the result. Students should apply their understanding of decimals to converting between measures of the same attribute, e.g. 1.276t = 1276kg (t means tonne, 1000kg).

They should be capable of finding ways of measuring the perimeters (circumferences) and areas of circles and using their measurement knowledge to determine the areas of figures that are a composition of known shapes.

In doing so students should connect their understanding of classes of geometric shapes to measurement. For example, a cylinder can be seen as an example of a prism, a solid with constant cross section. The volume of all prisms is found by multiplying the area of its cross section by its height hence the formula $v = \pi r^2 h$.

Teaching Measurement at Level Three:

Providing a rich learning experience for measurement at Level Three requires the involvement of students in measurement activities using the units of the metric system. Activities involving length measurement is the obvious place to start since the metre is the fundamental, one-dimensional unit of the metric system. However, as will be seen in the exemplars, the centimetre is the easiest unit for students to begin measuring with in order to avoid problems with decimal numbers.

Students can move to area activities using the square metre and the square centimetre, and gain a feeling for the area covered by a square kilometre using maps of their local district. Students need to gain a feel for and an understanding of the metric system and its units. This also involves understanding of which unit is appropriate for a given task, e.g. measuring the area of a netball court in square metres not square centimetres, and that measurement is always a compromise between accuracy (using smaller units) and practicality (effort required). Physically making units like a square metre, cubic metre or kilogram is foundational in these units becoming useful benchmarks for students.. Practical activities are vital in this respect, such as having students walk a kilometre and estimate one minute. Also important to discuss ways of approaching measurement problems,and requiring students to select appropriate scale-based instruments to solve them. To this end, discussion is an important part of measurement.

Students should connect their emerging understanding of place value to the connecting units for the same attribute. They should know relationships such as 100 centimetres is the same as 1 metre so 500 centimetres is 5 metres, 1000mL equals 1 litre so 500 ML is half a litre. In doing so they should begin to understand that all units are multiples or divisions of a fundamental unit. For example, kilometres are 1000 times 1 metre and millimetres are 1 tenth of 1 tenth of 1 tenth (1 thousandth) of a metre.

The history of the metric system from Napoleon France is interesting and could be investigated using the internet. D7 "Mathematics Across Cultures" in the old SM4 supplementary series contains a funny story about this history. In particular, the use of water in the connection between capacity and weight (the litre and the kilogram) is an important connection to make and should be the focus of classroom exploration.

Exemplars of Student Performance:

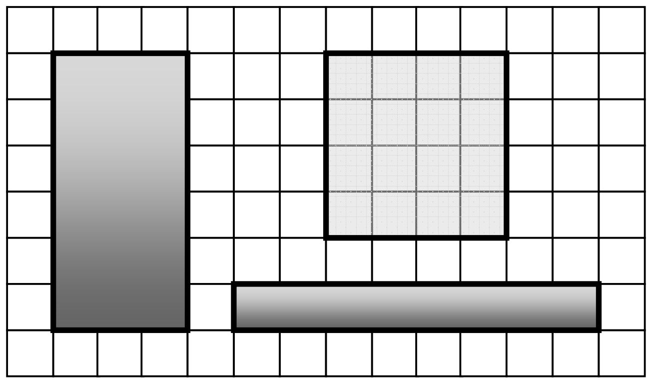
Exemplar One: Students are introduced to the metric system by measuring lengths in centimetres.

Students are shown a metre ruler as the basic unit of length in the metric system. They are shown the centimetre and learn that there are100 centimetres in a metre. (Language connections such as a century being 100 years or, in cricket, 100 runs, help to make sense of the language.) The notation for centimetre (cm) and metre (m) should be given. Students are then given 10 cm length measuring instruments which have the centimetres marked on them. These could be strips of cardboard marked out in centimetres, or they could be marked place value rods. Students are required to find the length of many things in the classroom. These lengths should range from about 5 cm to 50 cm. Adding and subtracting lengths can be set as problems.

For example, "What would be the total length of two exercise books and a new pencil?"

"How much longer is the long side of your exercise book than the short side?"

Exemplar Two: Students are introduced to area in the metric system by using squared paper.
Students are supplied with paper which is partitioned into centimetre squares. The students would have been involved in making and discussing many length measurements in centimetres prior to this activity. The concept of area as the measure of the covering of a region needs to be discussed. The square centimetre is examined, its name discussed and symbol (cm^2) given.
The squared paper has several different shapes drawn on it. All of the shapes involve whole square centimetres only. Students find the area of each shape by counting the number of squares inside the boundary. They are encouraged to find efficient ways to count the number of square units, using multiplication wherever Questions can then be asked such as:
" What would be the total area of these two shapes?"
"Can you draw a rectangle whose area in 15 square centimetres?"
"How many ways can you draw a rectangle whose area is 24 cm^2?"



Exemplar Three: Students are introduced to volume in the metric system by using place value blocks.
Students explore volume in the metric system through the use of place value blocks. They examine the 1cm cube and learn its name (cubic centimetre) and its symbol (cm^3). They create various shapes of given volume and compare and monitor each other's shapes. They look at the volume of the 100s block (the flat) and stack ten of those to make a block with a volume of 1000 cm^3. They realise that the volume of the place value cube is 1000 cm^3. Given boxes, place value block units, rods, flats, and cubes, they explore the capacity of different-sized boxes.
As an extension students could take a place value 1000s cube and use it to displace water which is then poured into a measuring container. From this they should be able to see that 1 litre is equivalent to 1000 cm^3.

Exemplar Four: Students extend their knowledge of area in the metric system by measuring with metre squares.
Students are given newspaper, a metre ruler, scissors and tape. They use these to create a square of side one metre. They discuss this as a unit of measure and learn its name (square metre) and it symbol (m^2). They are then given regions for which they are to find the area in square metres. Regions that are to be measured need to be free of obstacles such as desks and chairs. Students can collectively place their sheets of paper to cover the regions being measured. They report their findings by both writing the results using the unit symbol and verbally using the word.

Exemplar Five: Students are introduced to the kilogram as a measure of weight in the metric system.
Students are introduced to the kilogram as a measure of weight by being given a plastic bottle containing one litre of water. The students should have already met the litre as 1000 cm^3. This important connection between weight and capacity needs to be emphasized. It would be an appropriate place to discuss a little of the history of the metric system – especially the connection between capacity and weight. (The French as a colonising nation wanted their colonisers to have ready access to measurement systems and so, knowing that water was available universally, defined the weight of one litre of water to be one kilogram.)
Students discuss the name of the unit (kilogram) and are introduced to the symbol (kg). They feel and guess the weight of a variety of objects, always being able to refer to the plastic bottle as a standard. They record their guesses in whole numbers, quarters and halves. The objects are then weighed on a set of scales and the true weights are compared with their estimates.
Later, when the kilogram has been understood the gram (g) can be introduced. There are 1000 grams in a kilogram. The gram is approximately the weight of a small paperclip. The students could then be given a collection of objects which weigh less than a kilogram and they could make their estimates in grams of the weights of the objects and check using the weighing scales.

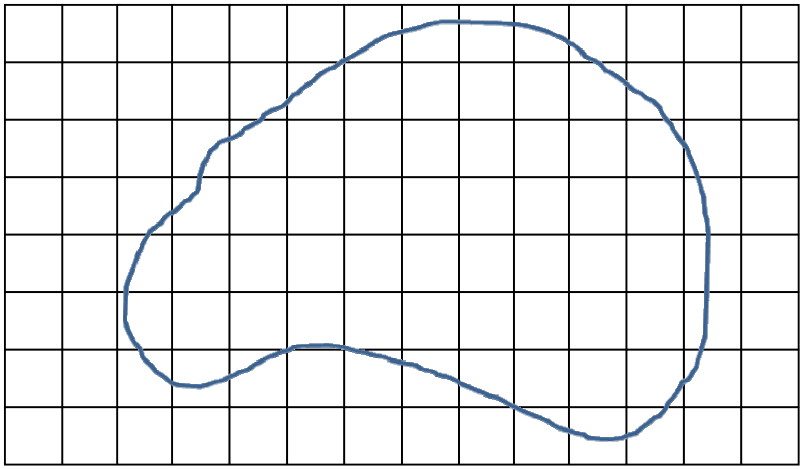
Exemplar Six: Students develop their understanding of time, in particular, a 10 second interval.
Students are given cardboard and other construction material, and stopwatches. They design a marble run that takes 10 seconds for a marble to descend from the start line to the finish line. This activity requires some logical thinking on the part of students and the investigation of some physics concepts. In the process the students will develop a feel for the time interval of 10 seconds as well as concepts about speed and slope.

Exemplar Seven: Students explore the relationship between the volume of a cuboid and the lengths of its edges.
Students create a collection of cuboids from blocks such as multi-link cubes. They record the number of blocks that each cuboid has as edge lengths and also the total number of blocks that make up the cuboid. From that they may see the relationship that the volume of the cuboid (in blocks) is the product of the length of the width, the depth and the height. A good way to reason this relationship is to build the cuboid one level of blocks at a time. So, for example, if we were intending to build a cuboid that was to be five blocks wide, three blocks deep and four blocks high, we could construct the bottom layer of 15 blocks and discuss why it has 15 blocks. We then add another layer of 15 blocks and see that the number of blocks is the product of 3, 5 and 2. Continuing in this way it should be easy to see that the volume of the cuboid is the product of its width, depth and height.

Exemplar Eight: Students develop an understanding of the size of a cubic metre.
Students build a cubic metre. This could be done by taping together 12 one-metre rulers. Dowel which fits into blocks is excellent. The size of the cubic metre is surprising to most students and discussion could take place about how many people, lunchboxes, schoolbags, etc.. could fit into it. Students estimate how many of the cubic metres they could fit into their classroom. If they have already explored the relationship between the volume of a cuboid and its edge lengths they could measure the room and check their estimates.

Exemplar Nine: Students make connections between cubic metres, litres and tonnes.
Students begin by building a cubic metre as in Exemplar Eight. They need a collection of at least ten place-value block cubes (1000s) which they place along one edge of the cubic metre. (They need to have already explored the 10cm by 10cm by 10 cm cube and know that its volume is one litre.) From this they can see that if they were to fill the whole of the bottom layer with the cubes they would require 100 cubes. They can also see that to fill the whole cubic metre they would need ten layers and therefore 1000 cubes. So a volume of one cubic metre is the same as a volume of 1000 litres.
If they are already aware that the weight of one litre of water is one kilogram then they can deduce that a cubic metre of water weighs 1000 kg, which is the same as one tonne.

Exemplar Ten: Students measure the area of a curved region on centimetre - squared paper.
Students are given a curved region overlaid on a grid of centimetre squares. They are asked to think of a way in which they could determine the area of the curved region. They decide to count the squares that are covered by the region. Discussion needs to take place as to what will happen to squares that are partly covered and partly not covered. Generally, people count squares that are half in or more as being in, and those that are less than half in as being not in. If asked how the accuracy of the result may be checked students might realise that the sum of the ‘in’ squares and the ‘out’ squares should equal the total number of squares in the grid.



Exemplar Eleven: Students scale distances off maps

Students are given a map with a simple scale. A map of the school buildings at a scale of one centimetre to five metres would be good. Students could be given questions such as: “How long is the middle classroom block?”; “Mark on the map the point which is 20 metres from the flagpole and 15 metres from the staffroom.”; “How many such points are there?”etc.

The exercise could be made more interesting by having some ‘buried treasure’ points marked on the map and the students have to scale them from the map and go out and locate them.

Exemplar Twelve: Students read classroom temperatures and build time series graphs.

A thermometer is placed in the classroom and, each day, the students read and record the temperature at 9:00 a.m., 12:00 noon, and 3:00 p.m. They graph the day’s temperatures on a chart in different colours for each of the three readings. Above the readings they write their assessment of the day’s weather.

Students could also measure the length of shadow cast by a one metre ruler at these times. They should display this time series data about temperature and shadow length using a line graph. Students should discuss the trends they notice, e.g. “The shadows get shorter until 1:00pm then begin getting longer again.”

Useful resources

Figure It Out:
Measurement Level 3: Length (pp. 1-3, 18-19, 21), Area (pp.5-9, 22), Volume (pp.6-7, 14-15, 20, 23), Mass (pp.9, 12, 24), Time (pp.4, 10-11, 13), Temperature (pp.8,16-17).
Geometry and Measurement, Year 7-8, Link: Defining Attributes (pp.10-11), Estimation (p. 15), Metric Units (p. 16-17), Length (pp.12-13), Area (pp.18), Mass (pp.12-13,14), Time (pp.12-13, 24), Angle (p.19-21).
Numeracy Project Book 9: Teaching Number through Measurement, Geometry, Algebra, and Statistics, pages 3-15.
nzmaths.co.nz units (This website is sponsored by the Ministry of Education)
http://www.nzmaths.co.nz/node/385 (Length: Stepping Out)
http://www.nzmaths.co.nz/node/383 (Length: Stepping Out)
http://www.nzmaths.co.nz/node/157 (Length & Statistical Investigations: Paper Planes L3)
http://www.nzmaths.co.nznode/411 (Length & Number: Perplexing Perimeters)
http://www.nzmaths.co.nz/node.412 (Length & Statistical Investigations: Giant Mystery)
http://www.nzmaths.co.nz/node/422 (Area: How much room?)
http://www.nzmaths.co.nz/node/435 (Volume: Rainbow Jelly)
http://www.nzmaths.co.nz/node/436 (Volume and Number: Slosh, Dribble and Plop)
http://www.nzmaths.co.nz/node/437(Volume: Boxing On)
http://www.nzmaths.co.nz/node/438 (Volume: Oranges L3)
http://www.nzmaths.co.nz/node/470 (Volume and Number: Marshmallows)
http://www.nzmaths.co.nz/node/799 (Mass: Supermarket Shopping)
http://www.nzmaths.co.nz/node/203 (Angle: Measuring Angles)
http://www.nzmaths.co.nz/node/205 (Angle: Simple Angles)
http://www.nzmaths.co.nz/node/818 (Temperature: Weather dot com)

Digital Learning Objects (These are accessed through the Ministry of Education Digi-Store and are the result of a collaborative project run by The Learning Federation, Australia)
http://www.nzmaths.co.nz/learningobjects/315/3

Other Website links:
http://nlvm.usu.edu/en/nav/category_g_2_t_4.html