Mathematics in the New Zealand Curriculum
Second Tier

Strand: Number and Algebra   Thread: Patterns and Relationships   Level: One

Achievement Objectives:

- Generalise that the next counting number gives the result of adding one object to a set and that counting the number of objects in a set tells how many.
- Create and continue sequential patterns

Exemplars of student performance:
Exemplar One:
Lila is told that there are 18 cubes in a plastic icecream container. Two more counters go into the container and Lila is asked how many cubes are in it then. She replies, “20 because there were 18 and you put in two, that’s 19, 20.
Three cubes go from the container. Lila says, “There are now 17 cubes in the container, 19, 18, 17 (counting back with her fingers).”

Lila is demonstrating Level One achievement because she realizes that the next counting number tells her the result of adding one more object to an existing set, e.g. 14 + 1 = 15. She also realizes that the counting number before tells the result of removing one object from an existing set, e.g. 16 – 1 = 15.

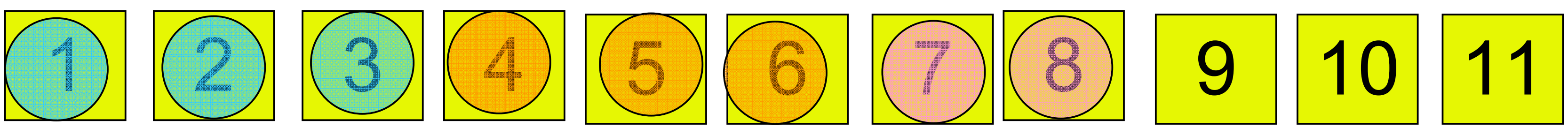
Exemplar Two:

Gerard is asked if there are enough bones for each dog to get one.

He counts the number of dogs then counts the number of bones.
Gerard replies, "There are seven dogs and eight bones. One bone will be left over or the biggest dog can have two bones."

Gerard is demonstrating Level One achievement because he uses counting to tell how many objects are in each set. He interprets each count as measuring the set and uses the counts as a basis for comparison. He knows a count of eight tells one more object than a count of seven.

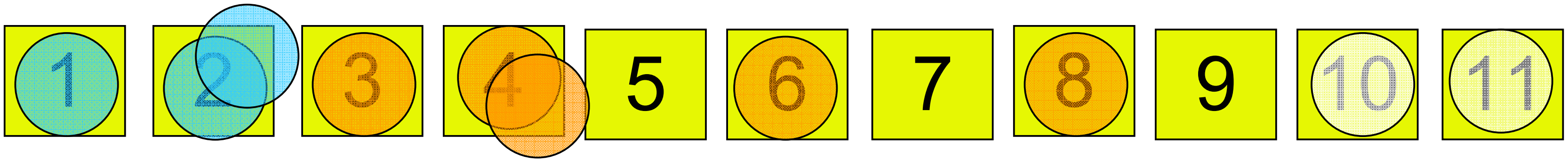
Exemplar Three:
Samantha counts out eight transparent counters of different colours. Her teacher says, "If you put the counters onto the strip one at a time, what number will the last one be on?" Samantha replies, "Number eight". Her teacher invites her to check her prediction by placing the counters:



The teacher asks, "If you had put the pink counters on first what number would the last counter cover?"
Samantha is certain that the last counter, whatever colour it is, would cover eight."

Samantha's thinking shows Level One achievement as she knows that the last number in the counting sequence tells how many objects are in the set, irrespective of which objects are counted first.

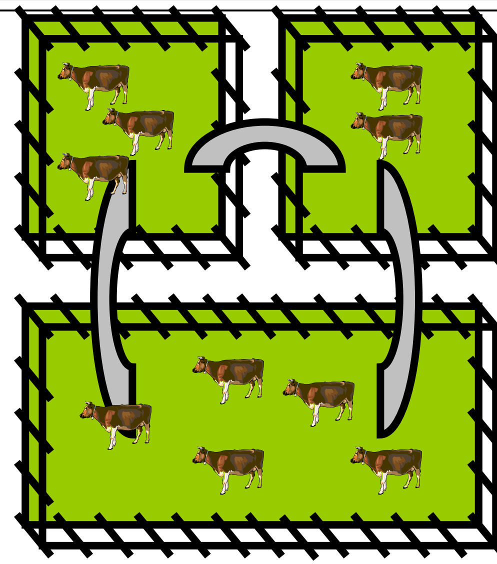
Exemplar Four:
Ollie has to look at this number strip and say how many counters there are without counting them up.



He says, "There are ten counters because there are three gaps and two extra counters. That will mean one gap is left. The last strip number is eleven so there are ten counters."

Ollie's answer shows Level One achievement because he recognizes that the counting sequence works if each number is used only once and the numbers are in order. He is able to compensate for missing numbers in the sequence and double counting.

Exemplar Five:
Bana counts the number of animals on this farm. He says, "There are ten animals."

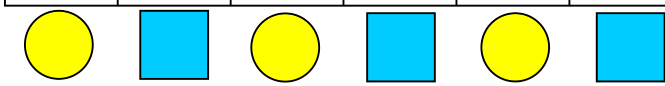


His teacher moves some of the cows from one paddock to another then takes one cow away (in a way that Bana can easily see).
He asks Bana, "How many cows are on the farm now?"
Bana replies, "Nine cows because you took one away."
The teacher moves some of the cows from one paddock to another and places two more cows on the farm.
He asks, "How many cows are on the farm now?"
Bana replies, "Now there are eleven cows. You put on two more."

Bana's thinking shows achievement at Level One because he recognizes that the number of cows stays unchanged no matter how they are arranged and he applies the number before/after when one cow is taken away or added.

Exemplar Six:
Esther describes how this pattern continues, "It goes yellow circle then blue square then yellow circle again."

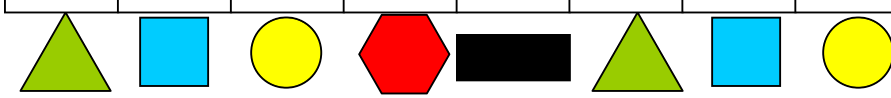
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**



Asked to predict what shape would be below number fifteen she skip counts in two's, "2, 4, 6, 8, 10, 12, 14, 16."
Esther says that the shape will be a yellow circle because a blue square is on 14 and 16.

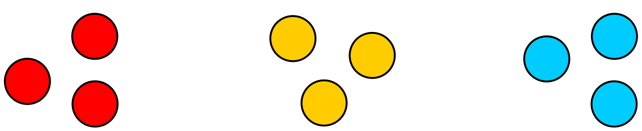
Esther predicts that the next objects in the pattern below will be, "Red hexagon, black rectangle, green triangle,…"
Asked what numbers she thinks the black rectangle will be on if the pattern continues she replies, "5, 10, 15, 20, 25,…

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**

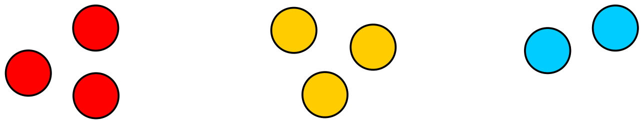


Esther's thinking shows she has achieved Level One because she can use skip counting to predict members of an ordinal counting sequence. This shows that she is able to establish a relation between the objects and the set of counting numbers.

Exemplar Seven:
Jim is briefly shown (3 seconds) this set of counters, and asked to recall what he saw.
He replies that he saw three sets of three counters and that is nine counters altogether (by counting the images in his head)



Next Jim sees this image briefly, and is asked what changed from the previous image.



He replies that one of the counters is missing from the three on the right so there must be eight counters now.

Jim's strategies show he has achieved Level One as he is able to hold an image of sets in his mind (subitise) and count the objects. He is also able to link his imaging with one less object using the number before concept.

Important Teaching Ideas

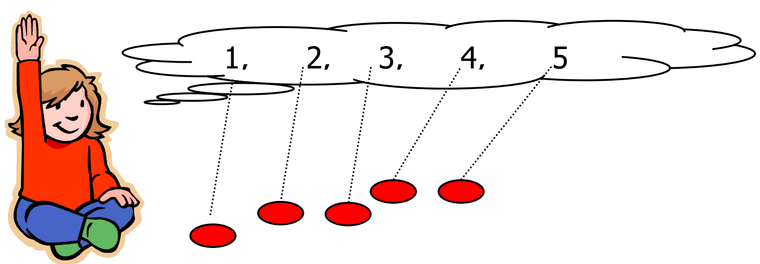
Two key ideas about counting connect at Level One. These ideas are:

1. Ordinality
2. Cardinality

Both ideas involve the concept of invariance (no change). Ordinality is about preserving the counting sequence of whole numbers. Adhering to ordinality means not leaving out numbers in the sequence, nor multiple counting any numbers. Rehearsal of the whole number counting sequences forwards and backwards, starting from different numbers, is crucial in developing the ordinal knowledge of numbers. This counting should be linked to number symbols (numerals) in a way that students can distinguish the numerals individually, e.g. "Point to the number eight".

Cardinality is about "how many." The number of items in a given set is invariant no matter how they arrange spatially, no matter the order of counting of the objects, and no matter what other attributes the objects have, e.g. size, colour, shape.

Counting is a mental activity not a physical one. Students match the objects or images of a set in one-to-one correspondence with a set of ideas in their mind. The ideas are the words and symbols of the forward and backward number sequence.

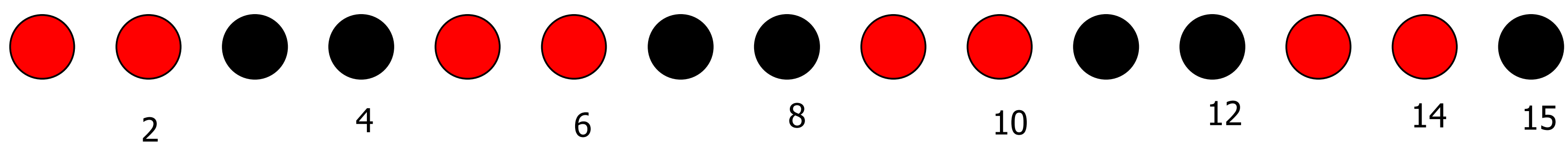


Students can therefore encounter difficulty in counting due to incomplete number sequences or an inability to match things in one-to-one correspondence. They can also create inappropriate generalizations about the purpose of counting. Students who are proficient at one-to-one counting may be considering that they are naming the objects, 1, 2, 3, … etc., rather than measuring the cardinality of the whole set. This naïve generalization must be contradicted by highlighting that the final count remains the same irrespective of the order in which the objects are counted or how they are arranged spatially (see exemplar three).

The connection of ordinality with cardinality of counting must be constructed by the student. Consider the situation where you count thirteen objects into a container with the students reciting the sequence as each object drops in. You then add another object or take an object out. You ask the students how many objects occupy the container. For students who do not know the number after or number before principle all the objects will need recounting. Highlight the connection between the number of objects before and how many are in the container now. Use a display of the number sequence that students are familiar with from rote counting, like a hundreds board or number strip, e.g. "You say there were 13 and now there are 14 (pointing to the numbers)" Operate on the "Goldilocks and the Three Bears" principle. This means that three examples of a pattern or property are sufficient for students to generalize. After three examples, students should be able to link adding one more object to a set to the next counting number. Follow a similar method for taking one object away.

Counting on and back to solve simple addition and subtraction problems occurs naturally if students understand the ordinal and cardinal nature of counting. However, students can learn to count on and back as an algorithm without understanding the principles about quantity that are in behind it. Students who fully understand counting should be able to do the following:

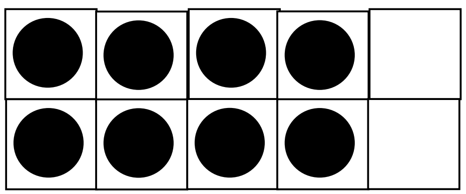
1. Compare two sets by counting rather than by one-to-one matching (see exemplar two)
2. Be unperturbed when a set of objects is partitioned into various subsets (see exemplar five)
3. Transfer counting to measuring. Note this requires the added dimension that units are constant, join exactly, and are parts of the attribute measured (for length we use a unit of length).
4. Apply simple skip counting, in twos, fives, and tens, to find the number of things in a set, even when the skip count isn't exact:

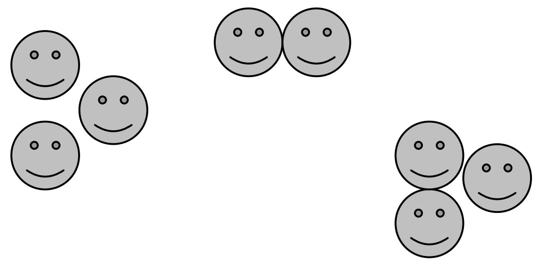


5. Be able to image sets of objects in structured and random arrangements and build onto or take away from those images (see exemplar seven)
6. Transfer counting to other strands, e.g. comparing the number of things in different categories on a data display, recognizing that an octagon has two more sides than a hexagon.

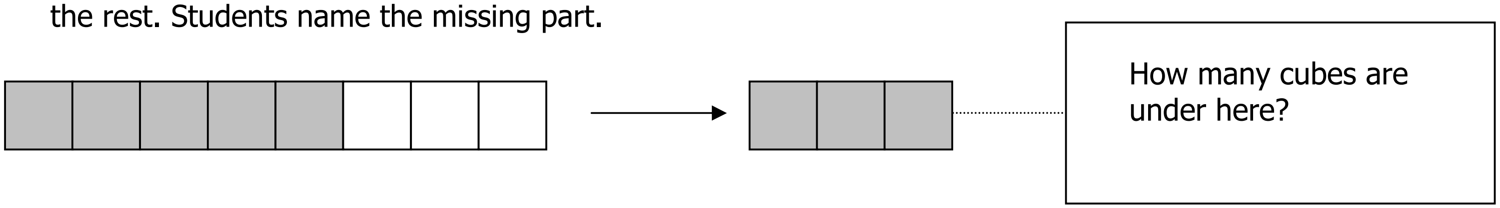
It is also important that students accumulate knowledge about individual numbers. This knowledge includes word-symbol association (e.g. "eight"- 8), visual patterns of that number (e.g. ten frame, fingers), partitions of the number (e.g. 4 and 4), and position of the number on the number line, e.g. 7, 8, 9. Capitalise on students' natural inclination to remember, by association. Link oral, symbolic and spatial representations of numbers. Early development of number symbols and place occurs in the part of our brain responsible for form, colour and location. The creation of images for individual numbers and the number line itself seem to be crucial. Suitable activities to develop these links include:

1. "Flashing" pattern cards and random collections, and asking students to capture an image of the objects then tell you how many.

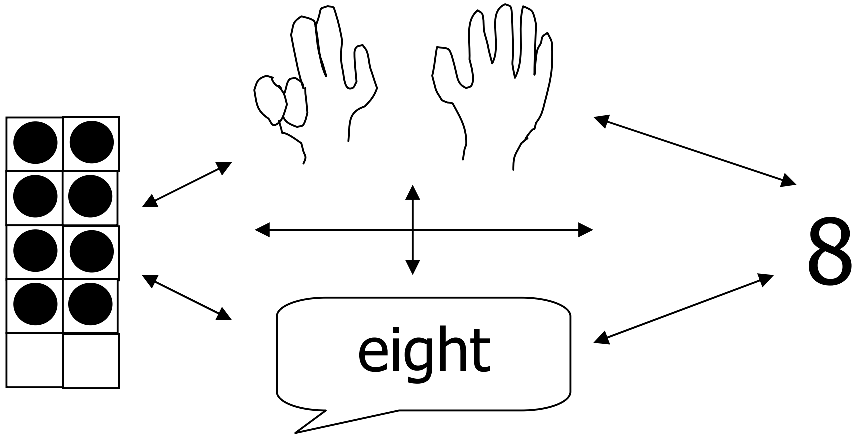




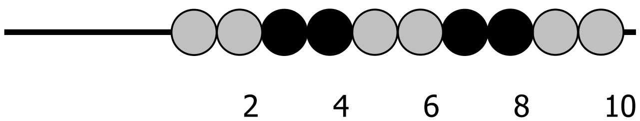
2. Making stacks of cubes. Using five-breaks to encourage knowing, rather than counting. Removing some cubes and hiding the rest. Students name the missing part.



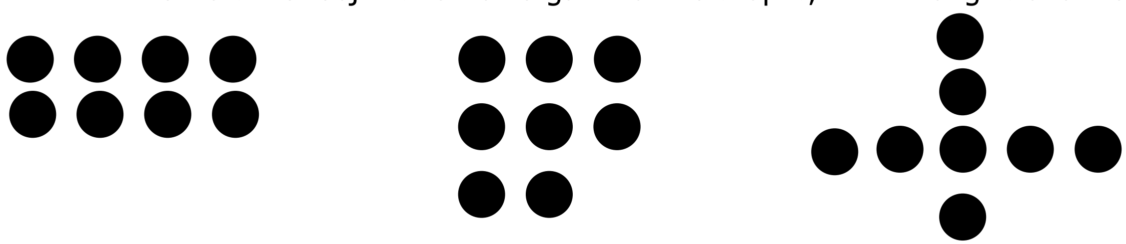
3. Connecting representations and creating displays for each number.



4. Skip counting and linking this to spatial displays of quantity



5. Experiment with how a set of objects can be organized into shapes, like rectangles and triangles:



Useful resources

Planning sheets for level one addition can be found on the nzmaths.co.nz website:
http://www.nzmaths.co.nz/node/1917

Numeracy Project Book 9: Teaching Number through Measurement, Geometry, Algebra, and Statistics, pages 30-40.
nzmaths.co.nz units (This website is sponsored by the Ministry of Education)
http://www.nzmaths.co.nz/node/215 (Number & Algebra: Counting on Counting)
http://www.nzmaths.co.nz/node/202 (Algebra: Pattern Makers)
http://www.nzmaths.co.nz/nod/204 (Algebra: Snakes and Scarves)
http://www.nzmaths.co.nz/node/206 (Algebra: Mary Mary quite contrary)
http://www.nzmaths.co.nz/node/211 (Number & Algebra :Ten in a Bed)

Digital Learning Objects (These are accessed through the Ministry of Education Digi-Store and are the result of a collaborative project run by The Learning Federation, Australia)
http://www.nzmaths.co.nz/learningobjects/313/1 (Learning Objects: Monster Choir)

Other Website links:
http://illuminations.nctm.org/ActivityDetail.aspx?ID=3
http://nlvm.usu.edu/en/nav/category_g_1_t_2.html
http://www.bbc.co.uk/education/mathsfile/gameswheel.html