Mary knows that the 5th triangular number is 15 because it needs 15 counters to make the triangle.

But she doesn’t know which of these expressions is equal to the nth triangular number, $T(n)$.

Which are and which aren’t and why?

1. $T(n) = 1 + 2 + 3 + 4 + \ldots + (n – 3) + (n – 2) + (n – 1) + n$

2. $T(n) = \frac{1}{2}n(n + 1)$

3. $T(n) = 1 + 3 + 5 + \ldots + (2n – 5) + (2n – 3) + (2n – 1)$

4. $T(n) = n^2 – (n – 1)^2 + (n – 2)^2 – (n – 3)^2 + \ldots + 4^2 – 3^2 + 2^2 – 1^2$

5. $T(n) = [(n + 1)^2 + n^2 + (n – 1)^2 + \ldots + 12] – [(n^2 + 2) + ((n – 1)^2 + 2) + ((n – 2)^2 + 2) + \ldots + (12 + 2) + 2]$

6. $T(n) = T(n -1) + n$ and $T(1) = 1$