

How long is the bar?

Purpose:

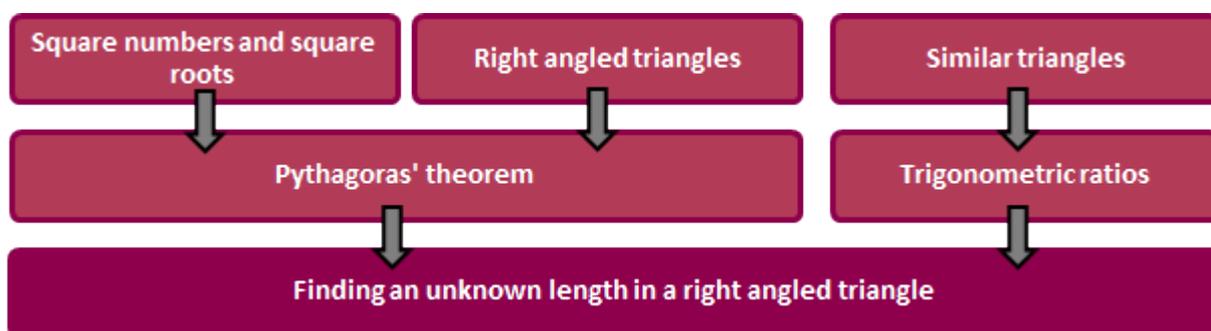
The purpose of this multi-level task is to engage students in using deductive steps, including the applications of Pythagoras' theorem to solve a problem.

Achievement Objectives:

GM5-10: Apply trigonometric ratios and Pythagoras' theorem in two dimensions.

Description of mathematics:

The background knowledge presumed for this task is outlined in the diagram below:



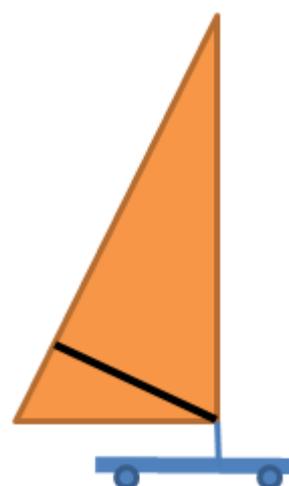
This task may be carried out with numerical exploration based on scale diagram or calculation of lengths, and/or by use of right angled triangle processes that have been established earlier. The approach should be chosen in sympathy with their skills and depth of understanding.

Activity:

Task: A land based windsurfer is being constructed by affixing a mast and sail to a skateboard. The sail has a bar fixed to the side of the mast and going across the sail as shown in black in the diagram.

The sail is a right angled triangle with a base of 1m and the length of the hypotenuse is 2m. The bar starts at the vertex of the sail as shown and meets the hypotenuse at right angles.

Find the length of the bar.



The arithmetic approach

The student is able to construct a scale diagram to solve the problem to a reasonable degree of accuracy and to use Pythagoras' theorem to verify their result.

Prompts from the teacher could be:

1. Construct a scale diagram of this problem. Take care to label all the right angles and all the lengths you know. Measure and calculate the length of the bar from this diagram.
2. Check your answer for the length of the bar, using Pythagoras' theorem. You may need to measure more sides to have all three sides of a right angled triangle that includes the bar.

How long is the bar?

1. Scale $10\text{ cm} = 1\text{ m}$
Answer = 0.87 m
2. Check with Pythagoras
Small triangle
Hypotenuse = 10 cm
Short Sides = 5.1 and 8.7 cm
 $5.1^2 + 8.7^2 = 101.7$
 $\sqrt{101.7} = 10.0846$
 $= 10\text{ cm (0 d.p.)}$

T: Did you expect these answers to agree?

S: If Pythagoras is true then they should agree because I did my scale drawing as carefully as possible. I guess it shows that it works.

The procedural algebraic approach

The student is able to solve a right angled triangle problem using trigonometric ratios and Pythagoras' theorem.

Prompts from the teacher could be:

1. Sketch a diagram of what you know.
2. Use your diagram to plan for what you will need to find out.
3. Apply trig ratios and/or Pythagoras to get the length of the bar.

T: I see you've crossed out some working here.

S: Yeah, I thought I could do everything with Pythagoras, but then I didn't have enough info, so I started again with getting an angle. Then I saw I could use my first answer and it all worked out from there.

How long is the bar?

~~$2^2 = 1^2 + x^2$
 $x^2 = 2^2 - 1^2 = 4 - 1$
 $x^2 = 3$
 $x = 1.732$ (2 d.p.)~~

Need angles!

$\sin x = \frac{1}{2}$
 $x = \sin^{-1} 0.5$
 $= 30^\circ$ exactly!

$1.732 \times \sin 30 = \frac{x}{1.732} \times 1.732$
 $x = 0.866$

ANSWER $x = 0.866$ m

Each step of the solution has been treated as a separate problem. This is evident in the use of x to stand for the unknown each time (angle or length)

T: Which x is this?

S: That's the final answer – the length of the bar.

The conceptual algebraic approach

The student is able to solve a right angled triangle problem with the greatest possible accuracy.

Prompts from the teacher could be:

1. Sketch a diagram of what you know.
2. Use your diagram to plan for what you will need to find out. To solve this problem as accurately as possible, it is best to avoid calculations that will involve rounding (ie writing square roots as a decimal, or using trigonometric ratios if these can be avoided).
3. Find the length of the bar.

T: Why did you cross out the 1.73?

S: Because it was rounded and I can just put $\sqrt{3}$ in my calculator and be exact later. So I decided to use the most accurate length.

The split in the diagram with y labelling the bar on both shows how the student is breaking the problem up into two parts

The crossing out and side working gives an insight into how the student is constructing their method of solution

The equating of the two expressions for y^2 shows the student has consistency in their application of pronumerals; that they see x and y as standing for particular values

$\sqrt{\frac{3}{4}}$ shows the algebraic steps taken to get to the answer. 0.866m gives a practical answer linking to the context. Providing both is useful.

$\sqrt{2^2 - 1^2} = \sqrt{3} \approx 1.73$
 $\left(\sqrt{3}\right)^2 = (2-x)^2 + y^2$
 $3 = 4 - 4x + x^2 + y^2$
 $1^2 = y^2 + x^2$
 $y^2 = 3 - 4 + 4x - x^2$, $y^2 = 1 - x^2$
 $-1 + 4x - x^2 = 1 - x^2$
 $4x = 2$
 $x = \frac{1}{2}$
 $1^2 = \left(\frac{1}{2}\right)^2 + y^2$
 $1 = \frac{1}{4} + y^2$
 $y^2 = \frac{3}{4}$
 $y = \sqrt{\frac{3}{4}} = 0.866 \text{ m}$