

Swimming lengths

Purpose:

The purpose of this multi-level task is to engage students in using their knowledge of fractions and decimals, and an ability to use operations on these, to solve a problem.

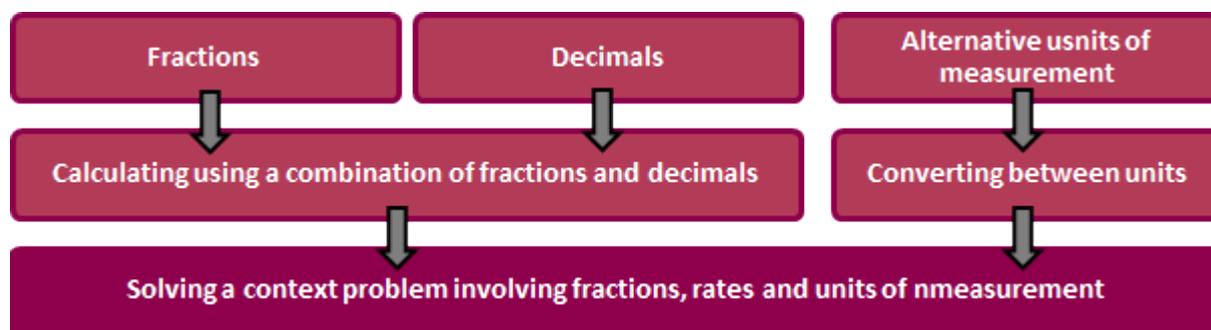
Achievement Objectives:

NA5-3: Understand operations on fractions, decimals, percentages, and integers.

NA5-5: Know commonly used fraction, decimal, and percentage conversions.

Description of mathematics:

The background knowledge and skills that need to be established before and/or during this task are outlined in the diagram below:



This task may be solved with guidance, breaking the problem into a series of smaller calculations or by the student independently constructing a method of solution. The approach should be chosen in sympathy with students' skills and depth of understanding.

Activity:

Task: A community swimming pool that was built before 1967 is 55 yards in length. It needs to be shortened to 50 m (accurate to the nearest $\frac{1}{100}$ of a metre) to be able to be used for official events.

A retired swim coach has said: "55 yards, less one foot, plus one inch, less half an inch should make it right."



One inch is $\frac{1}{12}$ of a foot and one foot is $\frac{1}{3}$ of a yard and there are 0.9144m in one yard.

Would his suggestion be within the official requirements for 50 ± 0.01 m?

The procedural approach

The student is able to calculate, with guidance, the original length and the adjustment recommended, in metres, to solve a problem.

Prompts from the teacher could be:

1. How many metres is 55 yards?
2. Calculate how much of a metre, one foot is.
3. How many metres is '55 yards less one foot'?
4. Calculate how much of a metre, one inch is.
5. How many metres is '55 yards less one foot plus one inch'?
6. How much of a metre is half an inch?
7. How many metres is '55 yards less one foot plus one inch less half an inch'?
8. Do the suggested alterations to the pool fit within the official requirements?

The pool is 55 yards $\times 0.9144 = 50.292$ m
less one foot $\frac{1}{3} \times 0.9144 = 0.3048$
 $50.292 - 0.3048 = 49.9872$ m
plus one inch $\frac{1}{12} \times 0.3048 = 0.0254$
 $49.9872 + 0.0254 = 50.0126$ m
less half an inch $\frac{1}{2} \times 0.0254 = 0.0127$
 $50.0126 - 0.0127 = 49.9999$ m
The coach makes the pool 0.0001 m too small
but that is $\frac{1}{10000}$ of a m, so less than $\frac{1}{1000}$ of a m
difference so it will be okay.

Successful conversion of length in yards to metres.

Understanding that a proportion can be found by multiplying by a fraction.

The conceptual approach (hide)

The student is able to calculate the original length and the adjustment recommended, in metres, to solve a problem.

A prompt from the teacher could be:

- Use the proportions given to convert the suggested adjustment in inches and feet, into yards.

Swimming pool needs to be 50 m. Shortened it will be:

$$55 - \frac{1}{3} + \frac{1}{3} \times \frac{1}{12} - \frac{1}{2} \times \frac{1}{3} \times \frac{1}{12}$$

$2 \times 3 \times 12 = 72$ $55 \times 72 = 3960$

$$= \frac{3960}{72} - \frac{24}{72} + \frac{2}{72} - \frac{1}{72} = \frac{3937}{72}$$

Conversion between fraction and decimal form.

$$= 54.6806 \text{ yards}$$
$$= 49.9999 \text{ m} \quad \times 0.9144$$

This is within $\frac{1}{1000}$ th of a m so is accurate enough!

The word problem is translated into a mathematical statement.