

Share the Profit

Purpose:

The purpose of this activity is to engage students in solving a problem involving fractions and proportional reasoning.

Achievement Objectives:

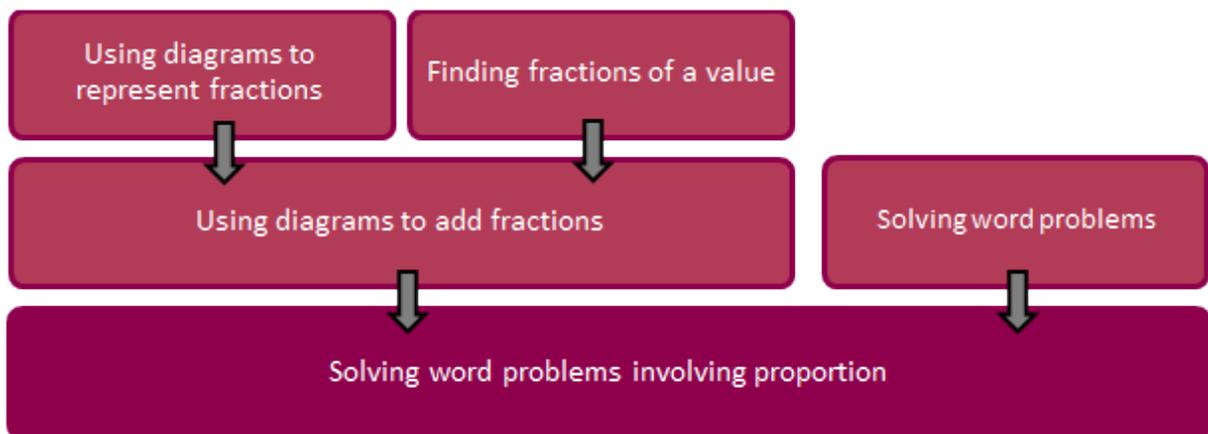
NA4-2: Understand addition and subtraction of fractions, decimals, and integers.

NA4-3: Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions, and decimals.

NA4-4: Apply simple linear proportions, including ordering fractions.

Description of mathematics:

The background knowledge and skills that need to be established before and/or during this activity are outlined in the diagram below:



Using diagrams to represent fractions

Shade in $\frac{1}{4}$ on this diagram.

Finding fractions of a value

Find $\frac{1}{3}$ of 210.

Using diagrams to add fractions

Shade in the following fractions to find their sum. $\frac{1}{3}$, $\frac{1}{2}$.

Solving word problems

Jane ate three times as many cherries as I did. I ate 14. How many did Jane eat?

Solving word problems involving proportion

I started my journey on a full tank of petrol, and when I ended it, the gauge read $\frac{3}{4}$ full. It took 9 L of petrol to fill up again. How much fuel does the tank hold?

This activity may be carried out with step by step guidance, or by allowing the student to follow their own method of solution. The approach should be chosen in sympathy with students' skills and depth of understanding.

Activity:

Three men run a business together.

The proportion of the profit they each get at the end of the month is based on the proportion of the funds they have invested in the business.

The April profit is shared out so that Al gets one third, Bob gets one half and Chuck gets one sixth.

In May, Des joins the group, investing half as much as Bob invested.

What share of the profit can Des expect to get?



The arithmetic approach

The student is able to use a value to divide onto the appropriate proportions in order to solve the problem.

Prompts from the teacher could be:

1. How are you going to go about solving this problem? Can you choose a value for the profit to divide between the men?
2. If Des had a share, how much would that be?
3. What would the profit had to have been for Des to also have a share?
4. What are the proportions of the profit shared between all four men?

A handwritten student solution on grid paper. At the top, a table lists shares and amounts:

A	$\frac{1}{3}$	2000	\$6000 Profit
B	$\frac{1}{2}$	3000	
C	$\frac{1}{6}$	1000	
D	?	1500	\$7500

Below the table, the student has calculated fractions of a total profit of 7500:

A	$\frac{2000}{7500}$	$\frac{20}{75}$
B	$\frac{3000}{7500}$	$\frac{30}{75}$
C	$\frac{1000}{7500}$	$\frac{10}{75}$
D	$\frac{1500}{7500}$	$\frac{15}{75}$

At the bottom, it says "Des gets $\frac{15}{75}$ ".

A red speech bubble contains the following dialogue:

T: It looks like you chose a value for the profit.
S: Yeah, it's easier to work out fractions of a number like 6000 which would cut up easily. I was going to do 1000 but the third would be mucky.
T: I see that you have crossed out the 6000.
S: Yeah, if Des gets some too then there must have been more so I made it up to 7500 with Des's bit.

The conceptual approach

The student is able to divide an unknown value into appropriate proportions in order to solve the problem.

Prompts from the teacher could be:

1. Consider the way April's profit is shared.
2. How much more would be needed for Des to have a share too?
3. How will May's profit need to be shared?

T: Tell me about how you added these fractions.

S: I pictured the fractions as parts of a pie because we usually use pies. They made one whole, but with Des we need more than one pie.

T: So what did you do then?

S: I didn't know how I could cut up the pie for all four of them so I used a number. I started with 30 but it didn't cut up nicely so I moved to 60.

profit = p

Al = $\frac{1}{3}$ of p
 Bob = $\frac{1}{2}$ of p
 Chuck = $\frac{1}{6}$ of p


 divide P into 6

$4+6+2+3=15$ ✓

$\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$

$\frac{2}{6} \quad \frac{3}{6} \quad \frac{1}{6} \quad \frac{6}{6}$

Des gets $\frac{1}{4}$ of p
 $\frac{1}{2}$ of $\frac{1}{2}$ $1 + \frac{1}{4} = \frac{5}{4}$

divide into ~~5~~ not ~~4~~
~~30~~ not 6.
 60

$\frac{1}{3}$ of ~~30~~⁶⁰ = ~~10~~ 20
 $\frac{1}{2}$ of ~~30~~⁶⁰ = ~~15~~ 30
 $\frac{1}{6}$ of ~~30~~⁶⁰ = ~~5~~ 10
 $\frac{1}{4}$ of ~~30~~⁶⁰ = ~~7.5~~ 15

$20+30+10+15 = 75$

Answer

Al = $\frac{20}{75} = \frac{4}{15}$

Bob = $\frac{30}{75} = \frac{6}{15}$

Chuck = $\frac{10}{75} = \frac{2}{15}$

Des = $\frac{15}{75} = \frac{3}{15}$

Answer