Ruler and Compass Constructions: Illustrated Constructions

Session 1

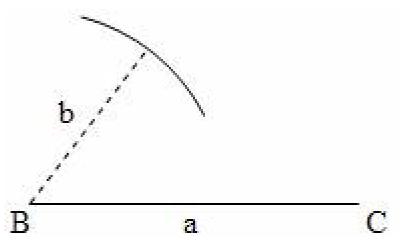
In this session we encourage students to experiment with their rulers and compasses to make up a variety of shapes. Constructions that groups might suggest are:

A. triangle with given side lengths a, b, c;

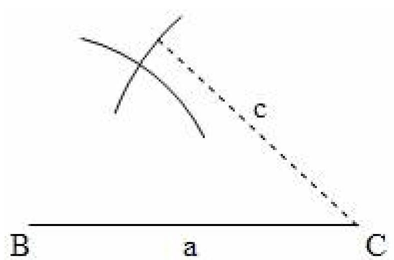
(i) draw a line segment, BC, of length a

(ii) set compass to a radius of b

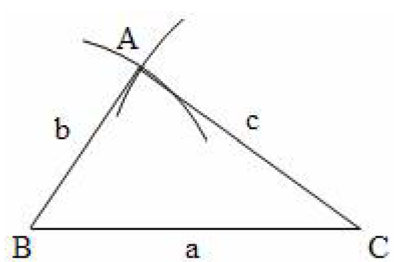
(iii) put the point of the compass at B and draw an arc



(iv) repeat (ii) and (iii) with a radius of c arc from C



(v) where the two arcs meet is the third vertex A of the triangle

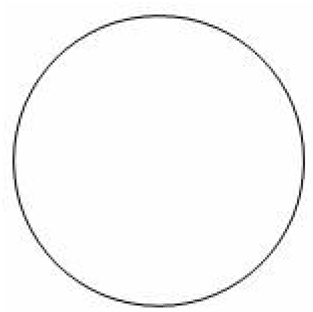


B. equilateral triangles of side length a;

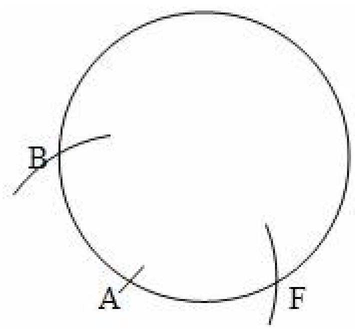
Repeat A with a = b = c.

C. construct regular hexagons of given side length

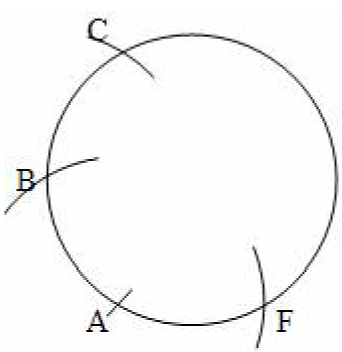
(i) draw a circle of radius equal to the given length



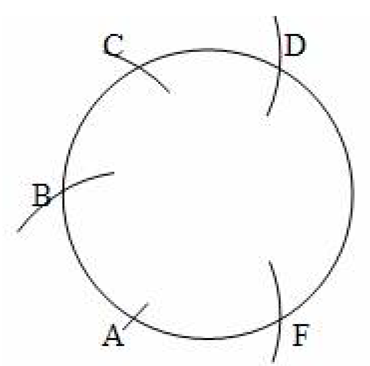
(ii) put the point of the compass anywhere on the circle (point A) and trace out two arcs that intersect the circle at B and F



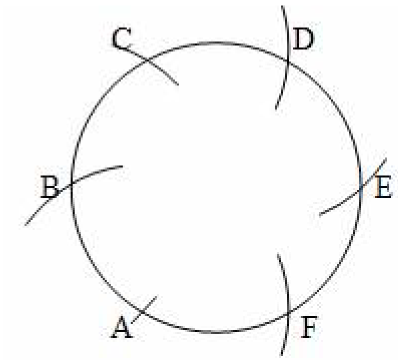
(iii) put the point of the compass at B and draw another arc that meets the circle at C



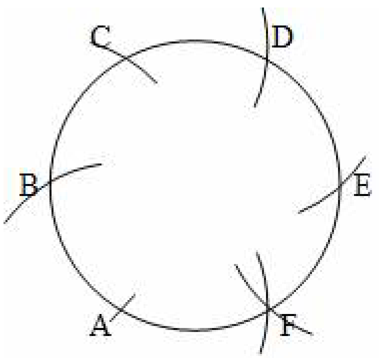
(iv) repeat from C to get point D



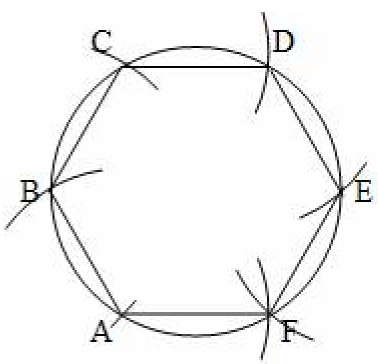
(v) repeat from D to get point E



(vi) repeat from E; the arc should meet the circle at F



(vii) this gives the six vertices of a regular hexagon ABCDEF

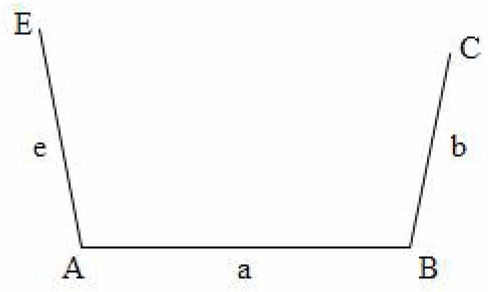


D. irregular pentagons with given sides;

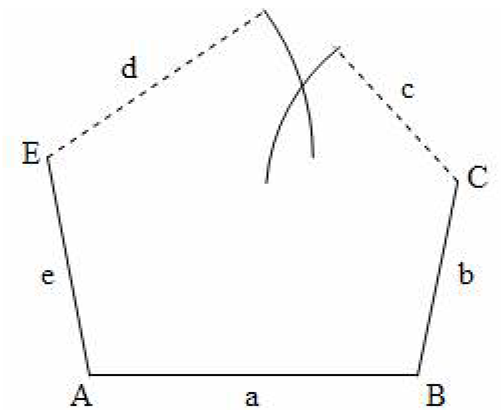
This construction takes advantage of the fact that there are many pentagons with given side lengths a, b, c, d, e.

(i) draw AB equal in length to a

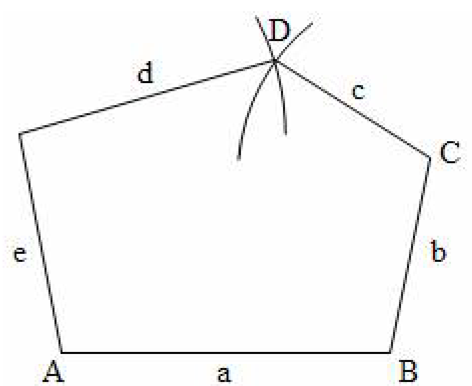
(ii) use the compass to mark out length e = AE from the point A and length b = BC from the point B



(iii) draw arcs of length d from E and c from C



(iv) these arcs meet at D



E. squares of given size

These can't be made accurately until they know how to construct a right angle.

Discuss where the problems with their constructions. What can go wrong?

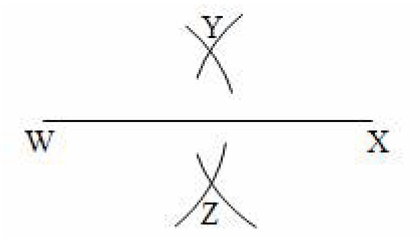
Session 2

A. Make a perpendicular bisector of a given line.

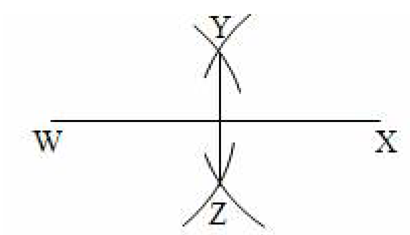
(i) draw a line WX with arbitrary length

(ii) open the compass to a radius greater than half of WX

(iii) draw arcs above and below the line WX with the point at W and arcs of the same length from X – these arcs meet at Y and Z



(iv) join Y to Z with the ruler – this line intersects WX at M.

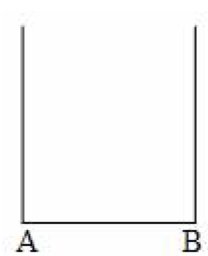


Note that YZ is perpendicular to WX and that M is the midpoint of WX.

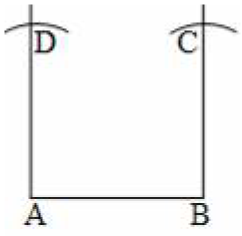
B. Square with a given side.

(i) mark off the given length on a straight line with end points A and B

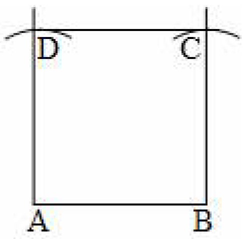
(ii) construct two perpendiculars, one at A and one at B



(iii) use the compass to mark off a length equal to AB on both of these perpendicular – this produces points C and D



(iv) use the ruler to join C to D

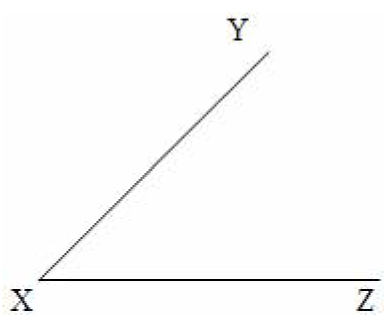


A similar approach can be taken to rectangles.

Sessions 3

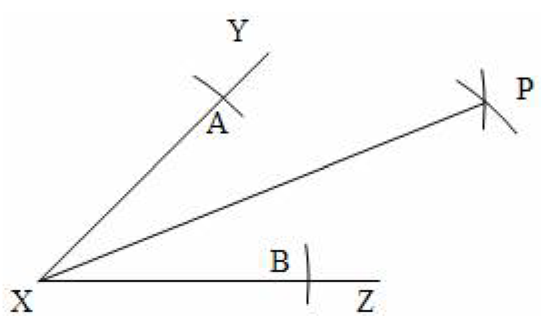
A. Bisecting a given angle.

(i) draw an arbitrary angle with sides XY and XZ

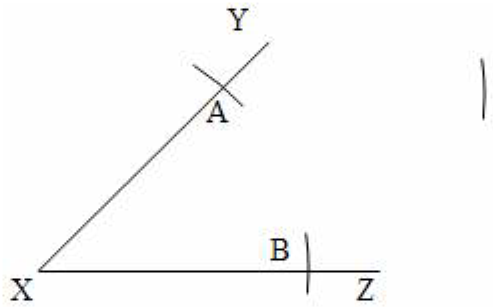


(ii) set the compass to any radius

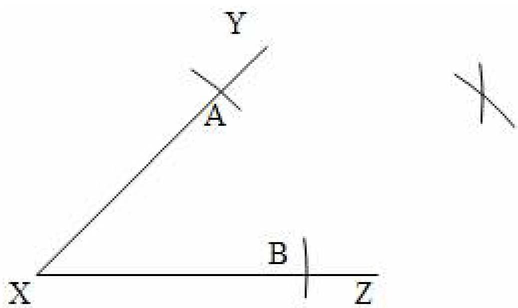
(iii) mark off two points A and B from X equal to that radius



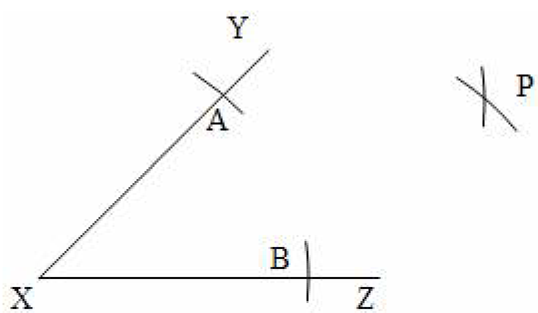
(iv) set the compass point at A and draw an arc



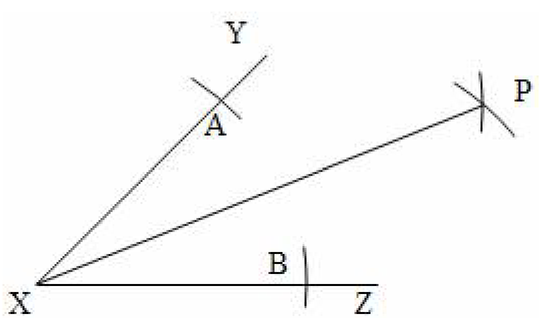
(v) set the compass point at B and draw an arc



(vi) these two arcs meet at P



(vii) draw the line PX – this line bisects the angle YXZ

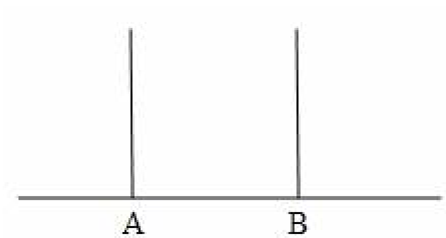


Session 4

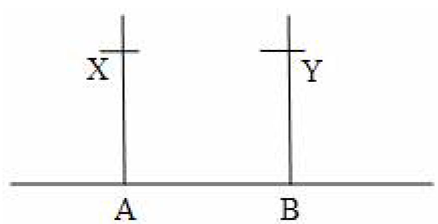
A. parallel lines.

(i) draw a line of arbitrary length

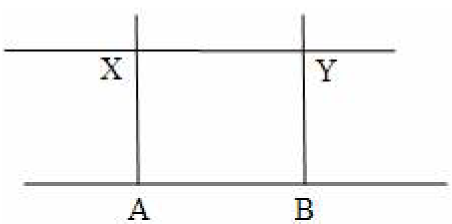
(ii) construct two perpendiculars at two chosen points on the line



(iii) mark off equal lengths from the line at points X and Y on these two perpendiculars



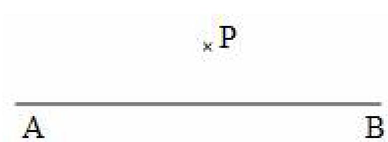
(v) join X to Y



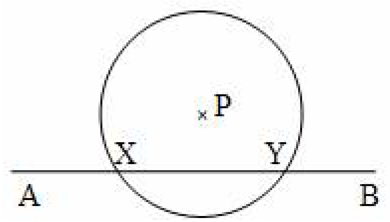
(vi) the line XY is parallel to the original line

B. parallel line through a given point not on the original line.

(i) assume that a line AB is drawn and a point P is chosen

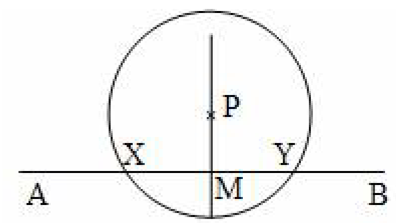


(ii) draw a circle centred at the given point, P, that intersects the given line at points X and Y

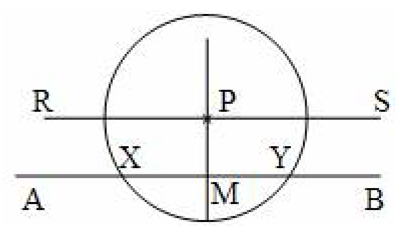


(iii) the perpendicular bisector, M, of XY will go through P

(iv) join MP



(v) construct a line RS through P that is perpendicular to MP

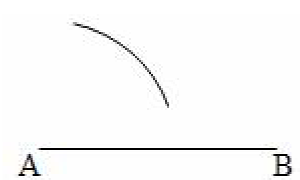


(vi) the line RS is parallel to AB

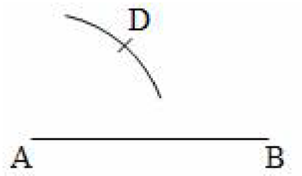
C. a parallelogram.

(i) draw a line AB that is the length of one of the sides of the parallelogram

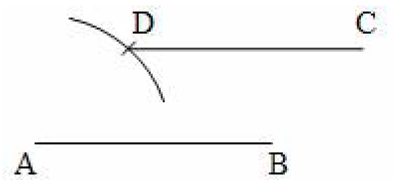
(ii) use a circle to draw an arc from A that is the length of AD



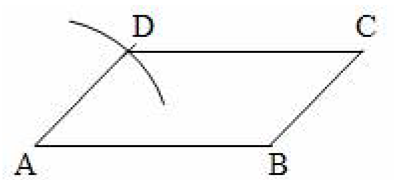
(iii) choose any point on that arc as D



(iv) construct DC parallel to AB and through D, the same length as AB



(v) join lines AD and BC to complete the parallelogram

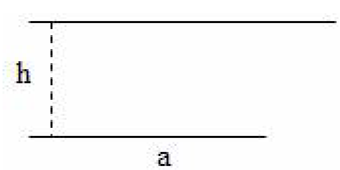


Note that there are many parallelograms with given side lengths. If one of the interior angles is specified, then a protractor will be needed to draw the parallelogram accurately.

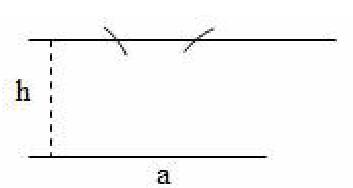
D.a trapezium.

(i) use Pythagoras Theorem to find the height, h, of the trapezium

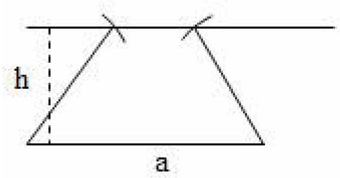
(ii) draw a line parallel to the side of length a and a distance h from it



(iii) use the compasses to mark off points on the new line that are the required distances from the end of the side of length a



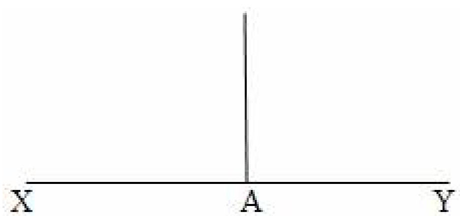
(iv) join the points to complete the trapezium



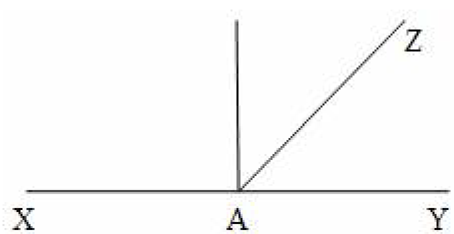
Session 5

(i) first note that a regular octagon has an interior angle of $\frac{(8-2)180^\circ}{8} = 135^\circ$

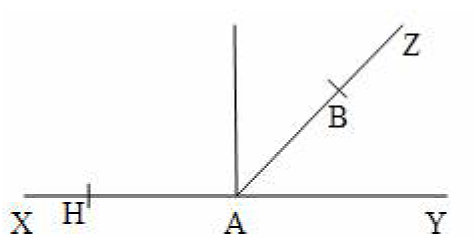
(ii) draw a straight line XY and bisect it at the point A



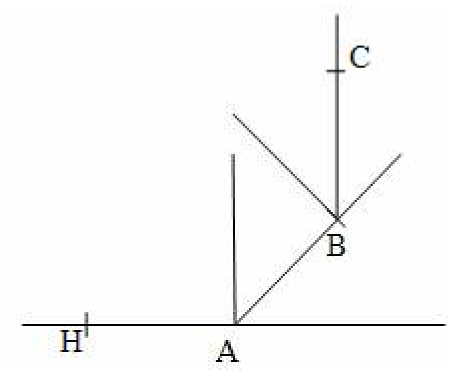
(iii) bisect the right angle at A to produce a line AZ



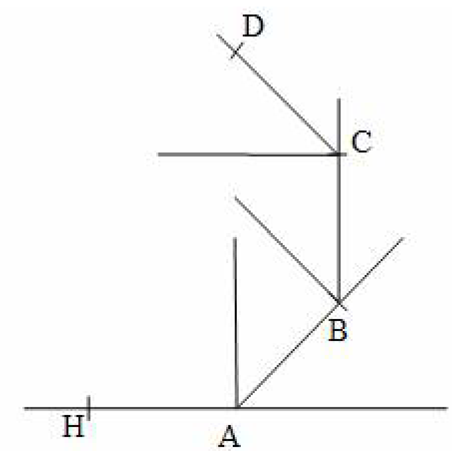
(iv) mark off lengths equal to the side of the octagon from the point A, one (H) on the line AX and one on the line AZ (B) (note that angle XAZ should be 135º)



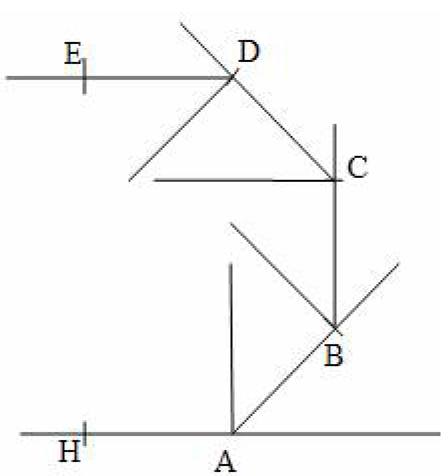
(v) repeat at B to get the new vertex C



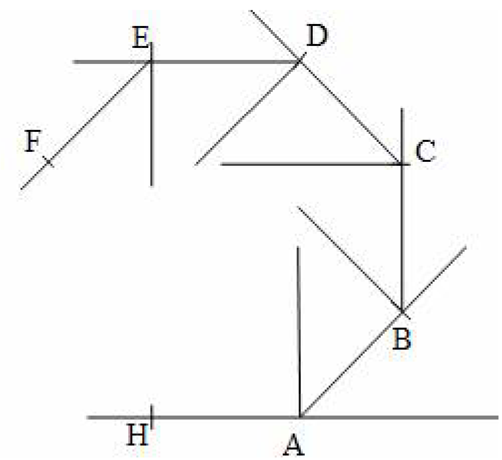
(vi) repeat at C to get the new vertex D



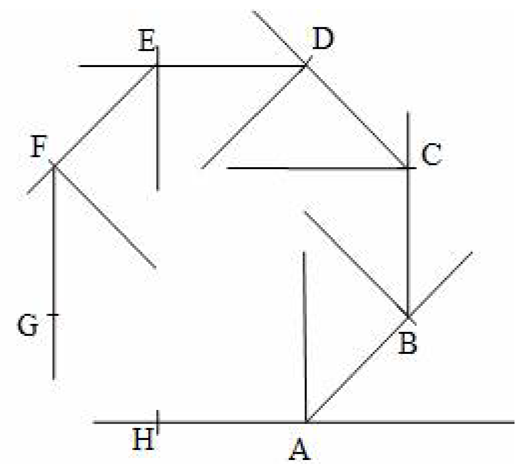
(vii) repeat at D to get the new vertex E



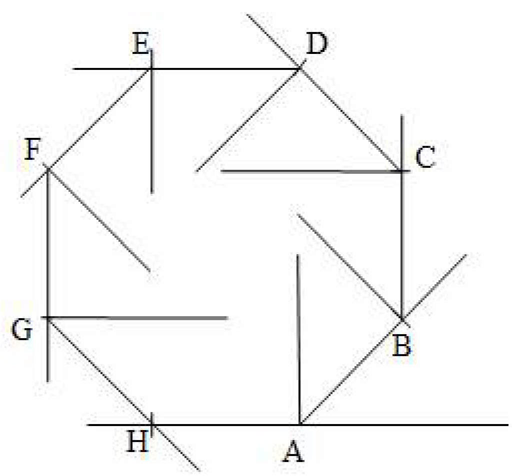
(viii) repeat at E to get the new vertex F



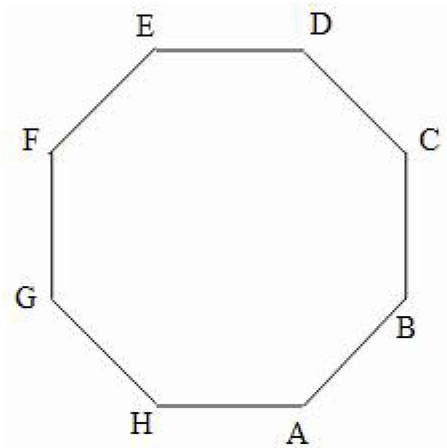
(ix) repeat at F to get the new vertex G



(x) repeat at G – the new vertex should coincide with H



(xi) ABCDEFGH is the required octagon.



Another way to do this is to first construct a square of side length $4 + 2\sqrt{2}$. Then construct lines through the vertices of the square that are perpendicular to the diagonals of the square. Using the compasses with the point on a vertex and radius 4, mark off two points on this line. This will give you two vertices of the octagon. Continue till you have the whole figure. This method is unlikely to be as accurate as the first method.