Renting a Car

Purpose:

The purpose of this multi-level task is to engage students in using their knowledge of linear relationships to solve a problem.

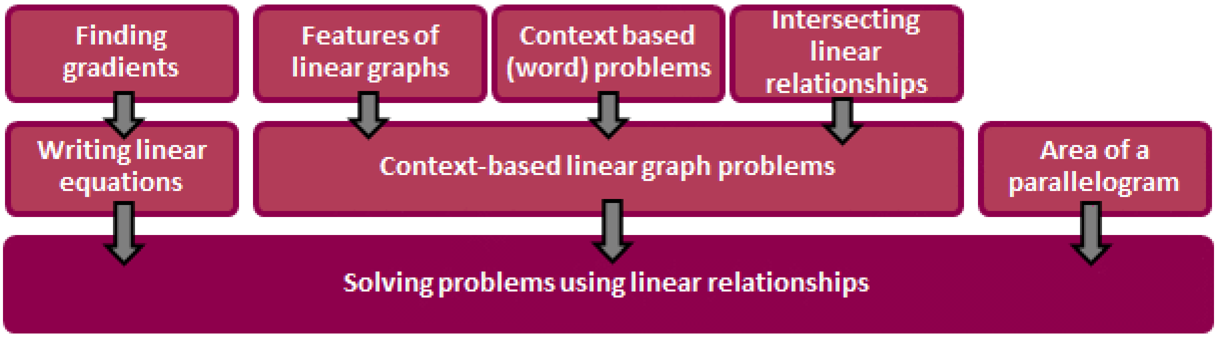
Achievement Objectives:

NA5-9: Relate tables, graphs, and equations to linear and simple quadratic relationships found in number and spatial patterns.

NA5-7: Form and solve linear and simple quadratic equations.

Description of mathematics:

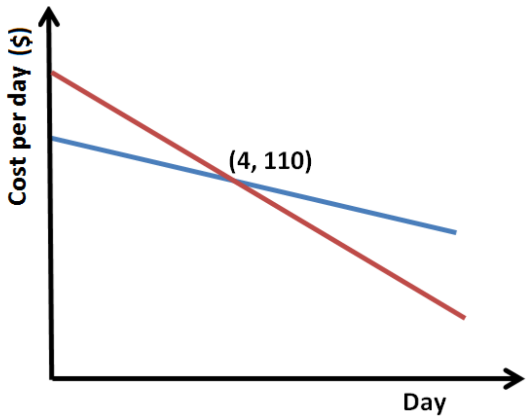
This background knowledge and skills that need to be established before and/or during this task are outlined in the diagram below:



This task may be carried out with numerical exploration, and/or by generalising with rules that have been established earlier. The approach should be chosen in sympathy with their skills and depth of understanding.

Activity:

Task: A car rental business has two rental schemes, red and blue for rentals up to ten days. These schemes are advertised in their brochures with this graph.



Use the following information to work out how many days of rental would carry the same total cost on either the blue or the red scheme.

The area under the graph gives the total cost of renting.

The schemes each follow a linear pattern, cutting the vertical axis at 120 and 170.

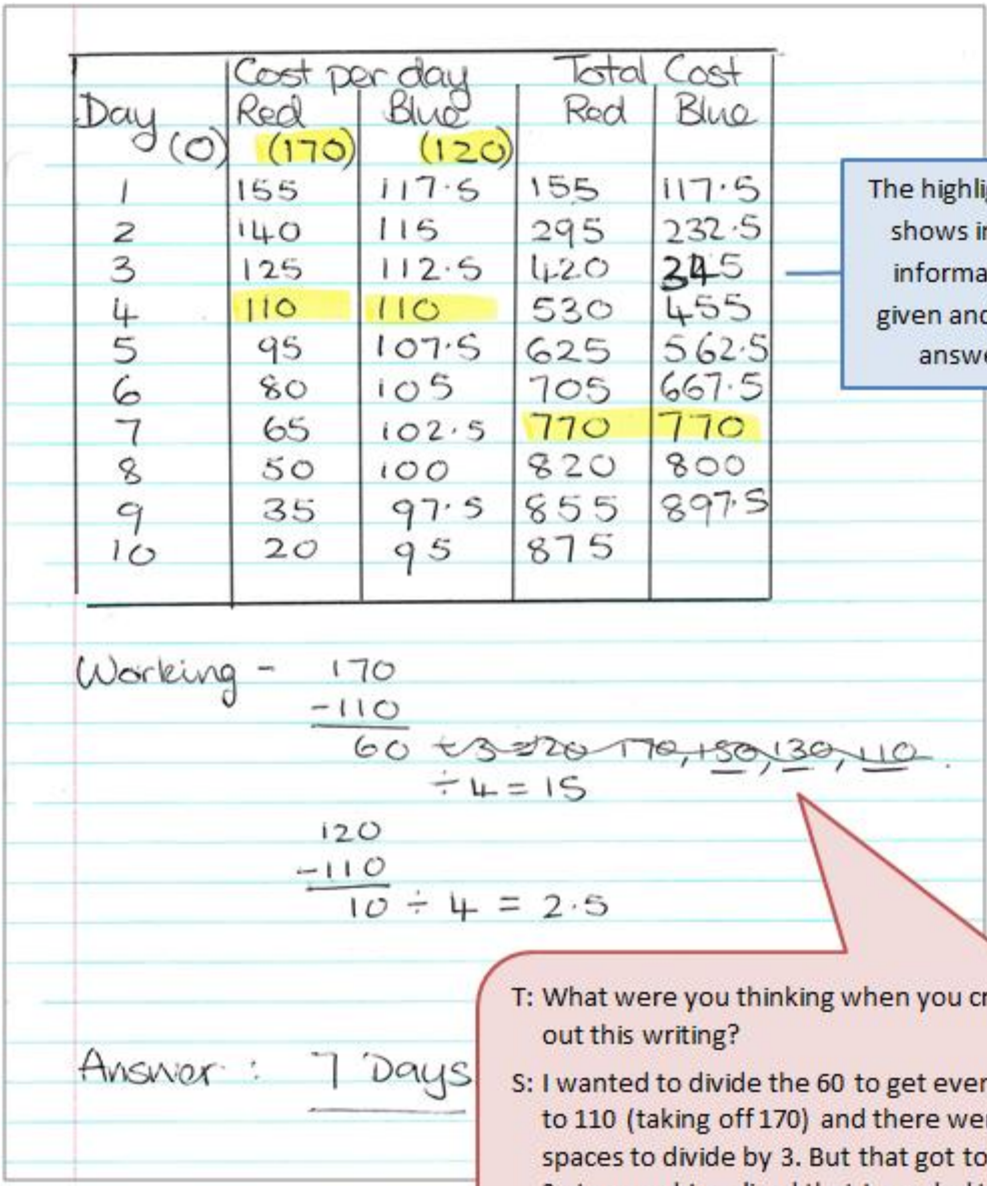
Both blue and red schemes cost $110 on the 4th day of rental.

The arithmetic approach

The student is able to interpret the graphical information given and to use a tool such as a table, to solve the problem.

Prompts from the teacher could be:

1. The lines start at 170 and 120 on the vertical axis, but this is for zero days. The charge for day one would be less.
2. Set up a table with the cost per day for each of the schemes, over ten days.
3. Write in the values you are given and use the information given to calculate the remaining values.
4. Extend your table to include a running total of the total cost of renting a car under each scheme.



Day	Cost per day Red	Blue	Total Cost Red	Blue
(0)	(170)	(120)		
1	155	117·5	155	117·5
2	140	115	295	232·5
3	125	112·5	420	345
4	110	110	530	455
5	95	107·5	625	562·5
6	80	105	705	667·5
7	65	102·5	770	770
8	50	100	820	800
9	35	97·5	855	897·5
10	20	95	875	

Working - 170
−110
60 ~~÷3 = 20 170, 150, 130, 110~~
÷ 4 = 15

120
−110
10 ÷ 4 = 2·5

Answer : 7 Days

The highlighting shows initial information given and final answer

T: What were you thinking when you crossed out this writing?

S: I wanted to divide the 60 to get even steps to 110 (taking off 170) and there were 3 spaces to divide by 3. But that got to 110 in 3 steps and I realised that I needed to go in 4 steps so I changed and divided by 4.

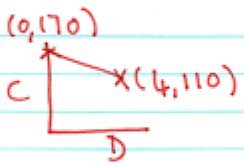
The procedural algebraic approach

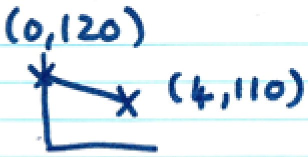
The student is able use algebraic techniques to solve the problem.

Prompts from the teacher could be:

1. Find the rules for each of the schemes, red and blue. Write these in the form
   C = gradient x D + starting value,
   where C is the cost per day and D is the number of days rented.
2. The lines start at 170 and 120 on the vertical axis, but this is for zero days. The charge for renting a car would start at day one. Use this information to find the area of the parallelogram made by each line, which represents the total cost of renting a car.
3. Find the day, D, when the total cost of renting a car is the same under either scheme.

Colour has been used to separate the two different linear relationships





$C = mD + 170 \quad m = \frac{-60}{4} = -15$

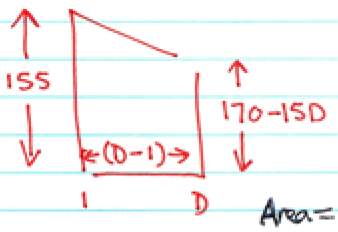
$C = -15D + 170$

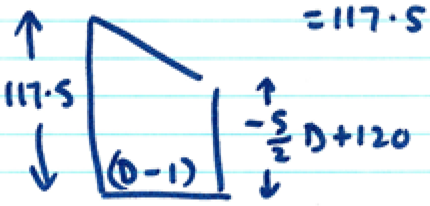
$D = 1, \; C = -15 + 170 = 155$

$C = mD + 120 \quad m = \frac{-10}{4} = -\frac{5}{2}$

$C = -\frac{5}{2}D + 120$

$D = 1, \; C = 120 - \frac{5}{2} = 117.5$





$\text{Area} = \frac{1}{2}(a+b)h$

$\cancel{\tfrac{1}{2}}(155 + 170 - 15D)\cancel{(D-1)} = \cancel{\tfrac{1}{2}}(117.5 + 120 - \frac{5}{2}D)\cancel{(D-1)}$

$155 + 170 - 117.5 - 120 = 15D - \frac{5}{2}D$

$\frac{87.5}{12.5} = \frac{12.5D}{12.5}$

$D = 7$

on day 7

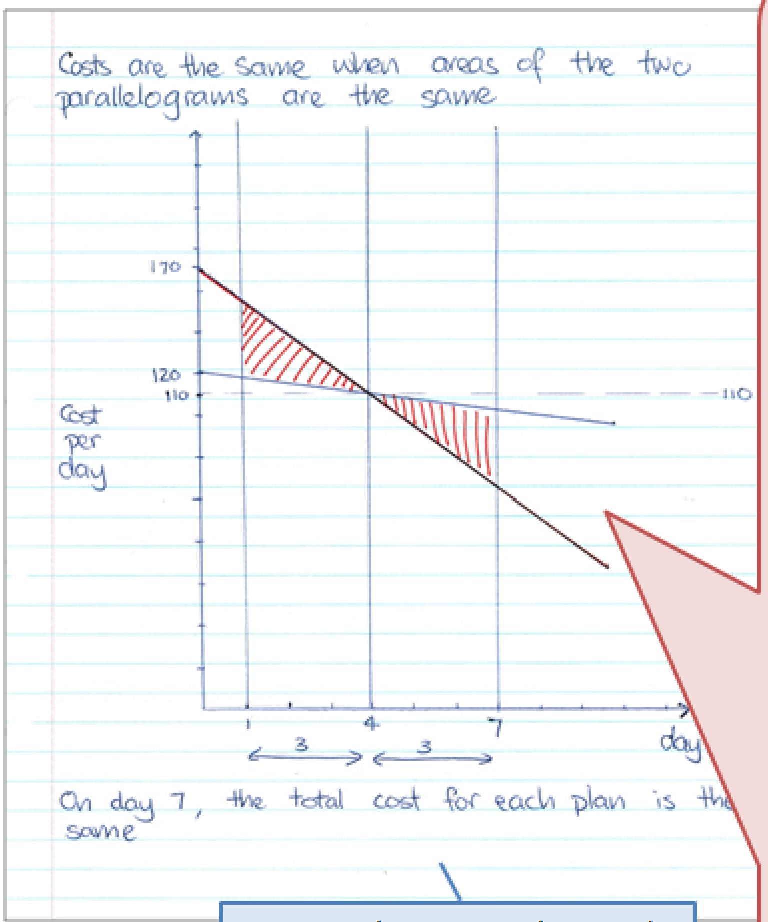
The key steps in working are here where the two expressions for area have been equated; showing understanding that the areas are equal. On the same line of working, common factors have been eliminated as the first step in solving for D.

The conceptual algebraic approach

The student is able devise an original approach, using mathematical techniques to solve the problem.

A prompt from the teacher could be:

The lines start at 170 and 120 on the vertical axis, but this is for zero days. The charge for renting a car would start at day one.



T: How did these shaded regions help you solve the problem?

S: I wanted to find the day that gave me equal areas. The graph makes two parallelograms. The red line is steeper, so it's got a shaded part more before day 4 and a shaded part less after day 4.

T: Tell me about the vertical line at day 1.

S: I was going to just start at the vertical axis, but then I realised that you wouldn't pay any rental until you'd hired the car for at least one day. So I started at day=1.

T: So how did you decide on day 7?

S: I used the symmetry in the graph. Because the slope of each line is steady, then I knew I should stop at 7 to get both of those triangles to be the same area.

An original, geometrical approach has been devised by the student