

Pizza pans

Purpose:

The purpose of this multi-level task is to engage students in using ratios to find a common ratio (scale factor).

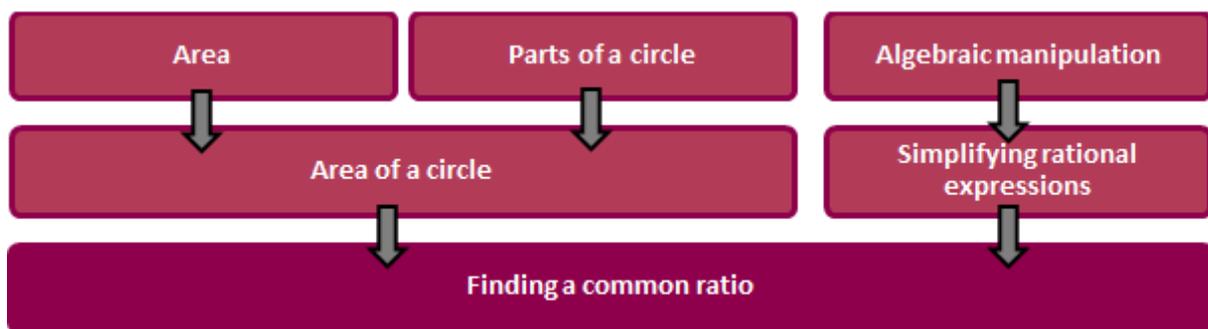
Achievement Objectives:

GM5-9: Define and use transformations and describe the invariant properties of figures and objects under these transformations.

NA5-4: Use rates and ratios.

Description of mathematics:

This background knowledge and skills that need to be established before and/or during this task are outlined in the diagram below:



This task may be 'scaffolded' with instructions that lead to a solution, and/or the students might be given a value for the area of the family pizza in order to have a concrete focus when they apply the process that leads to the solution. The approach should be chosen in sympathy with their skills and depth of understanding.

Activity:

Task: A family sized pizza is baked in a pan that has twice the base area of a standard pizza pan.

Find how much bigger the radius of the family pizza pan is than the standard (ie find the scale factor for this enlargement).



The arithmetic approach

The student is able to find a decimal scale factor, using concrete examples in their calculations.

Prompts from the teacher could be:

1. What if the family pizza has a radius of 20 cm? Work out the area of the family pizza.
2. What would the area of the standard pizza be?
3. What would the radius of the standard pan be?
4. How much bigger is the radius of the family pan than the standard?

Example of pizzas:

Family Radius = 20 cm

Then area = 400π
 $= 1257 \text{ cm}^2$

Standard area = 200π
 $= \pi r^2$

Standard radius is

$$r^2 = 200$$
$$r = \sqrt{200}$$
$$= 14 \text{ cm}$$

How much bigger is the radius of the family pizza pan?

$$\frac{20}{14} = 1.4 \text{ times bigger}$$

The procedural algebraic approach

The student is able to find an exact (surd) scale factor, using concrete examples in their calculations.

Prompts from the teacher could be:

1. What if the family pizza has an area of 100 cm^2 ? Work out the radius of the family pizza, r_1 .
2. What would the radius, r_2 , of the standard pan be?
3. How much bigger is the radius of the family pan than the standard?

family pizza eg 100 cm^2

$$100 = \pi r_1^2$$
$$r_1^2 = \frac{100}{\pi}$$

standard would be 50 cm^2

$$50 = \pi r_2^2$$
$$r_2^2 = \frac{50}{\pi}$$

$\frac{50}{\pi}$ is $2 \times \frac{100}{\pi}$

so $\sqrt{r_1^2} = \sqrt{2 \times r_2^2}$

$$r_1 = \sqrt{2 r_2^2}$$
$$r_1 = \sqrt{2} r_2$$

family radius standard radius

T: Tell me about your working here.

S: I made an equation because I could just put an equals sign where I had "is"!

T: And the square root signs?

S: I didn't have them at first, then I took the square root of each side.

The conceptual algebraic approach

The student is able to find an exact (surd) scale factor, using algebraic generalisations.

A prompt from the teacher could be:

Use the rule that you know for area of a circle in terms of the radius of each pan. Because the two radii are different, you should use different symbols for each.

The key step in solving this problem is here:

- using two different symbols for the two radii
- showing the standard area as a proportion of the family area.

Family Pizza : area = πR^2

Standard Pizza : area = $\frac{\pi R^2}{2} = \pi r^2$

$$\frac{\pi R^2}{2} = \pi r^2$$
$$R^2 = 2r^2$$
$$R = \sqrt{2} r$$

The family pizza is $\sqrt{2}$ times bigger radius than the standard.