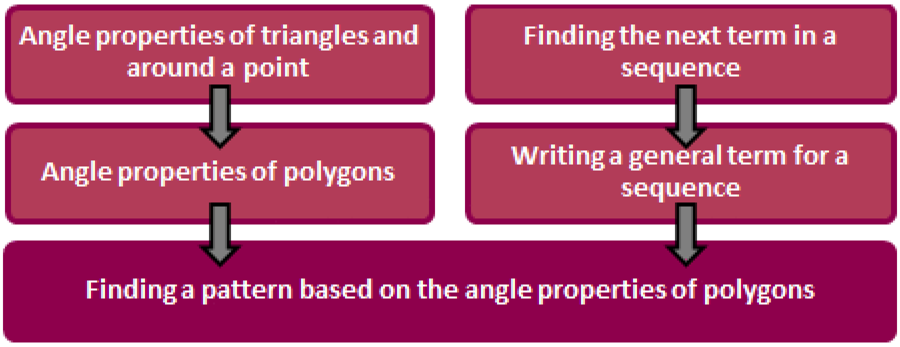
Pierced polygons

Purpose:
The purpose of this multi-level task is to engage students in an investigation applying geometric properties of polygons.

Achievement Objectives:
GM5-5: Deduce the angle properties of intersecting and parallel lines and the angle properties of polygons and apply these properties.

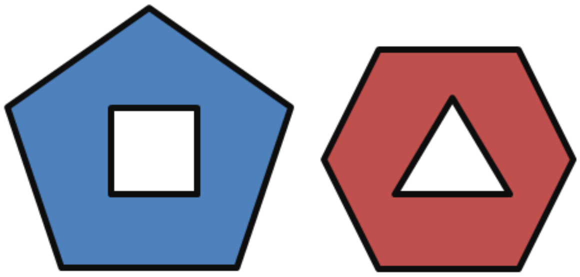
Description of mathematics:
The background knowledge presumed for this task is outlined in the diagram below:



The task can be presented with graded expectations to provide appropriate challenge for individual learning needs.

Activity:

Task: Investigate the sum of the internal angles of a regular polygon that has another regular polygon removed from its centre.

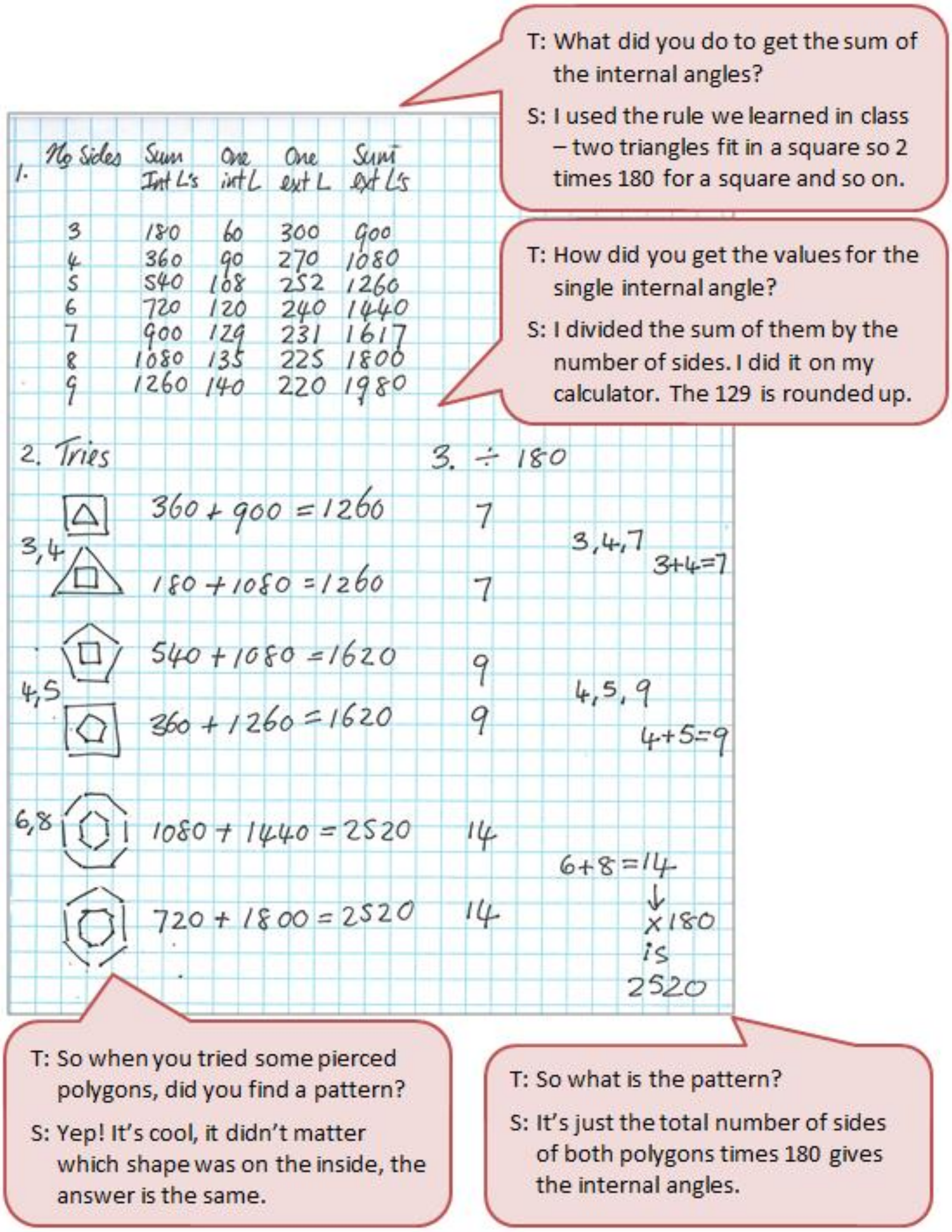


The arithmetic approach

The student is able to apply geometric properties of polygons to find solutions to the problem.

Prompts from the teacher could be:

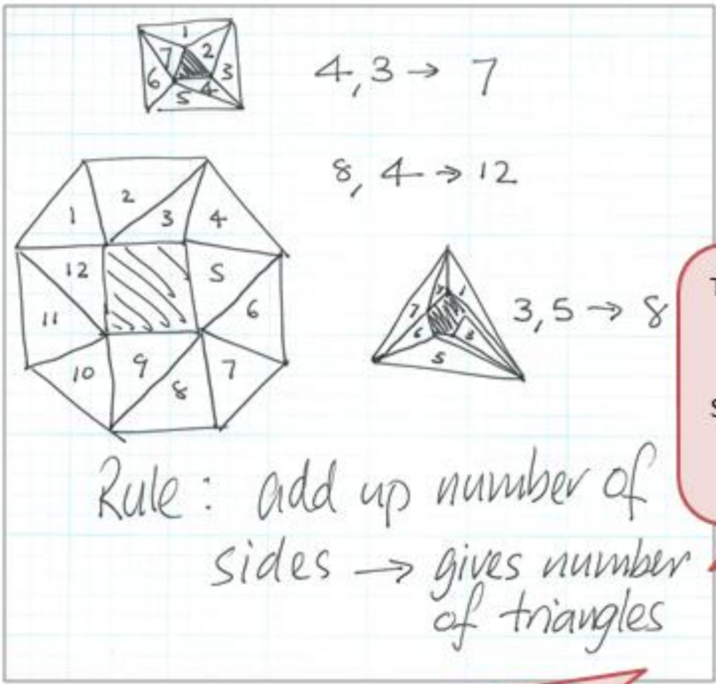
1. Set up a table for regular polygons that gives the sum of internal angles and the sum of the 'outside' reflex angles. (This is different to the sum of external angles)
2. Try a few pairs of polygons that make up a pierced polygon (eg a square with an equilateral triangle cut out and vice versa). What do you notice?
3. Divide your sum of all the internal and outside reflex angles by 180°. What do you notice?



The procedural algebraic approach

The student is able to apply geometric properties of polygons to find a pattern that can be used to solve the problem.

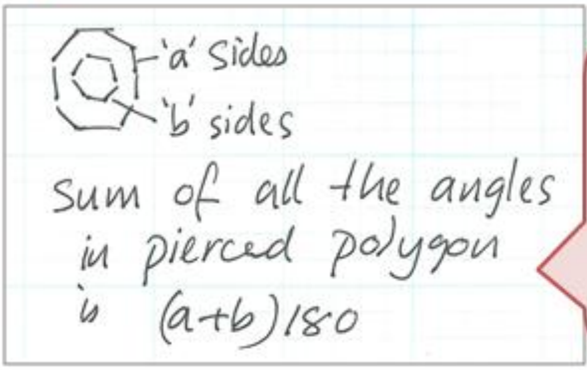
A prompt from the teacher could be to suggest approaching this problem of internal angles the same way that the class may have derived the rule for the sum of internal angles of a polygon – ie, to divide the shape into triangles, each being 180°.



T: So what can you say about the sum of all the angles inside a pierced polygon?

S: It's the number from my rule times 180 because that's how many degrees

T: Try writing that out for me if the outer polygon has a sides and the inner one has b sides.



T: That's great. Would this rule only work for regular polygons?

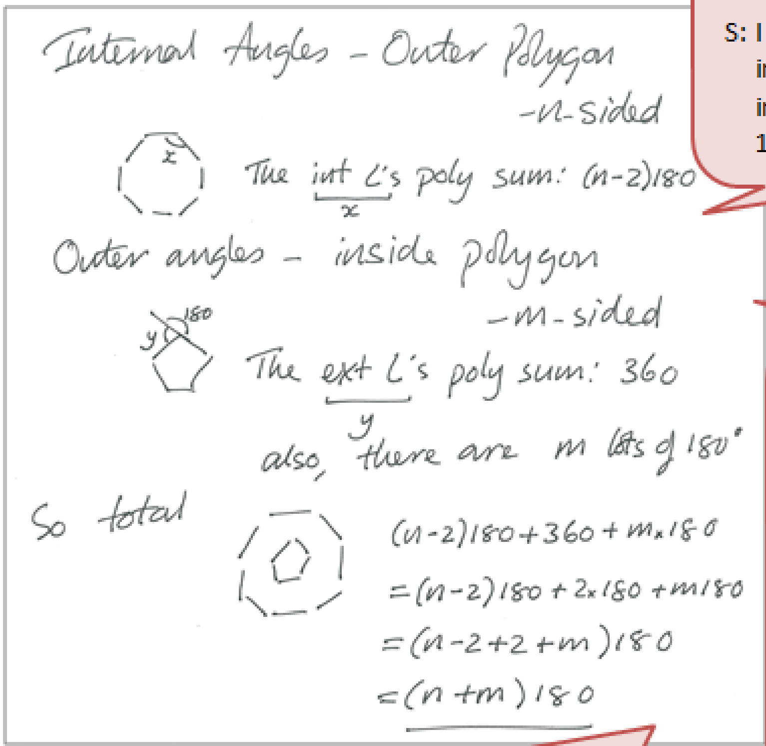
S: oh, no – I didn't just look at regular ones. It would work for any polygons, because any triangle has angles adding to 180.

The conceptual algebraic approach

The student is able to apply geometric properties of polygons to find a pattern that can be used to solve the problem.

T: What did you do to get the sum of the internal angles?

S: I used the rule we learned in class – two triangles fit in a square so two times 180 for a square and so.



T: I notice you have used y and m for your inside polygon.

S: Yes, I didn't want to get mixed up with the x and n from the outside one. They could be the same though, like a square inside a square.

T: Do you think this rule will work for irregular polygons?

S: It should do, because it just uses the sums of the angles and those are the same for regular and irregular polygons.