

Title Balancing with tidy numbers

I am learning to solve addition problems by using the strategy of converting one number to a tidy number, then correcting the other number.

Prerequisite Knowledge

Place value understanding of tens and ones

The equals sign is a balance, not an instruction to work something out

Equipment

Ice-block sticks – a mix of bundles of ten and individual sticks.

Using Materials

Put out ice-block sticks to show the addition $39 + 16$, and discuss with students easy ways of doing this problem. If it does not naturally arise, suggest that one way of making the problem easier is to move one ice-block stick from the 16 to the 39 as this will make 40. (The ten individual sticks can at this point be replaced by another bundle of ten). Get students to focus not only on the creation of the 40, but also on working out how many are left in the other pile now one has been moved. The problem can then be solved by working out $40 + 15$.

Other combinations to try:

$$18 + 43$$

$$15 + 27$$

$$38 + 23$$

After several problems ask students what this would look like with numbers, and work students through a way of showing the movement of some from one pile to the next, referring back to the numbers in the piles as needed.

Notation like $\overset{\leftarrow}{39} + 16 = 40 + 15$ would suffice
 $= 55$

Using Imaging

Ask students to explain how they would solve these problems using ice-block sticks (if they had enough)

$$87 + 45$$

$$119 + 68$$

$$59 + 129$$

For the final pair, ask students how they solved the problem. The point of the discussion is to highlight that it does not matter which of the number is made into a tidy number, and by initiating the discussion, the different ‘tidying’ used by different students should come to light. Continue the discussion by asking if it matters which number you make tidy in the other problems, and how you work out which is the best number to tidy.

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Using Number Properties

Provide students with larger numbers to work with. For example:

$$168 + 39$$

$$189 + 203$$

$$997 + 648$$

The discussion points for these questions may include that a number could be tidied by making it smaller rather than larger, that it doesn't matter how many are 'moved' providing the other number is properly balanced, and that this will work to tidy numbers to the nearest hundred or thousand as well.

Generalising from Number

By the time students have worked through the examples and discussions above, they should have already generalized the property being introduced. Students should realize that either number can be tidied, that different amounts can be added or subtracted to tidy, and that the trick is to make sure the other number is properly balanced.

At this stage introduce box equations to continue the generalizing process, as this looks more at the structure of what is happening to the numbers on either side of the equals sign. For example

$$9 + 5 = 10 + \square$$

$$48 + 15 = 50 + \square$$

$$35 + 47 = 32 + \square$$

$$57 + 64 = \square + 61$$

Note that in doing this, students need to look at the problem differently than when numbers alone are being used. For example:

$$\overleftarrow{39} + 16 = 40 + 15 \quad \leftarrow \quad \text{working on one side of the equals side only}$$

$$9 + \overrightarrow{5} = 10 + \overrightarrow{\square} \quad \leftarrow \quad \text{working across the equals sign}$$

These problems can cause confusion initially with some students who maintain the 'old method', as they tend to go 'the wrong way' when compensating.

Some students may also solve the problem by calculating. For example:

$$\begin{array}{r} 9 + 5 = 10 + \square \\ 14 \quad 14 \quad \square = 4 \end{array}$$

Challenge these students to try to work out what goes in the box, without calculating.

Further generalizing can be introduced by changing the structure of the problem. For example, by putting the box on the left hand side of the equals sign as in

$$\square + 17 = 61 + 20$$

Two different shapes can also be introduced, and a discussion of the relationship between the numbers in these 'boxes' discussed. For example, in the problem $24 + \bigcirc = 23 + \square$, the number in the box must be one more than the number in the circle.

If students have met the letter n being used as 'some number' before, and have explored how this is used to describe numbers, this latter example can be extended to include variables.