MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Book 5

Teaching Addition, Subtraction, and Place Value

Numeracy Professional Development Projects – Revised 2012

New Zealand Government

Effective Mathematics Teaching

The Numeracy Professional Development Projects assist teachers to become effective teachers of numeracy.

Effective teachers of numeracy demonstrate some distinctive characteristics.[1] They:

- have high expectations of students' success in numeracy;
- emphasise the connections between different mathematical ideas;
- promote the selection and use of strategies that are efficient and effective and emphasise the development of mental skills;
- challenge students to think by explaining, listening, and problem solving;
- encourage purposeful discussion, in whole classes, in small groups, and with individual students;
- use systematic assessment and recording methods to monitor student progress and to record their strategies to inform planning and teaching.

The focus of the Numeracy Professional Development Projects is number and algebra. A key component of the projects is the Number Framework. This Framework provides teachers with:

- the means to effectively assess students' current levels of thinking in number;
- guidance for instruction;
- the opportunity to broaden their knowledge of how children acquire number concepts and to increase their understanding of how they can help children to progress.[2]

The components of the professional development programme allow us to gather and analyse information about children's learning in mathematics more rigorously and respond to children's learning needs more effectively. While, in the early stages, our efforts may focus on becoming familiar with the individual components of the programme, such as the progressions of the Framework or the diagnostic interview, we should not lose sight of the fact that they are merely tools for improving our professional practice. Ultimately, the success of the programme lies in the extent to which we are able to synthesise and integrate its various components into effective mathematics teaching as we respond to the individual learning needs of the children in our classrooms.

1 Askew et al. (2000). *Effective Teachers of Numeracy*. London: King's College.

2 See also the research evidence associated with formative assessment in mathematics: Wiliam, D. (1999) "Formative Assessment in Mathematics" in *Equals*, 5(2); 5(3); 6(1).

Title: Book 5 – Mathematics
Numeracy Professional Development Projects
Published by the Ministry of Education.
PO Box 1666, Wellington 6140, New Zealand.

ISBN 978 0 7903 4023 4 (Print)
Item number 34023

ISBN 978 0 7903 4022 7 (Online)
Item number 34022

This book is also available on the New Zealand Maths website, at www.nzmaths.co.nz

Book 5

Teaching Addition, Subtraction, and Place Value

Contents

Teaching for number strategies

Introduction

Book 5: Teaching Addition, Subtraction, and Place Value will help teachers to guide the development of students' mathematical thinking through the stages of the Number Framework. Each stage presents a new set of key ideas that are directly linked to the New Zealand Curriculum and *Book 1: The Number Framework*. Each key idea is then explored through a series of relevant learning experiences.

The activities presented in this book are designed specifically to develop students' mental strategies in mathematics. This is done using the Teaching Model found in Book 3 of the series. For each stage, the book presents a set of strategies through the phases of:

- using materials
- using imaging
- using number properties.

Addition, subtraction, and place value

This book emphasises the vital link between place value and operations by incorporating place-value thinking into various activities. Indeed, it could have been called *Teaching Addition and Subtraction* ***through*** *Place Value*, because most of the mental strategies shown in the book link to either a "with tens" or a "through tens" strategy.

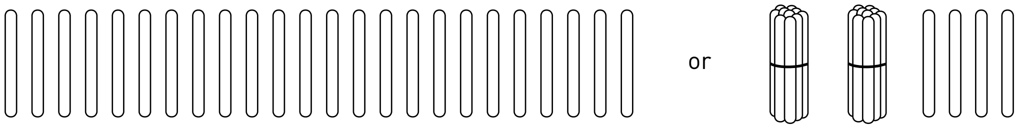
A good understanding of addition, subtraction, and place value is crucial for strategies in multiplication, division, fractions, algebra, and statistics. Students initially use counting to solve addition and subtraction problems. They then start to think strategically, first with smaller whole numbers, then with larger ones, and later with fractions, decimals, and integers.

There are two central ideas to place value: the place holder, zero, and the concept that if, as a result of addition or multiplication, the numeral in any place exceeds 9, then 10 of these units must be traded for one unit that is worth 10 times as much. Conversely, for subtraction or division, if a unit needs to be broken down, it must be traded for 10 units worth 10 times less.

The idea that once we have 10 of something we call this group one of something else is not at all straightforward for young students. However, understanding that ten "ones" are equivalent to one "ten", for example, is profoundly important.

Initially, students need to crack the language and symbolic code for naming and reading the number names from one to nine. Saying "ten, eleven, twelve" and writing "10", "11", and "12" may seem no different than counting from one to nine aloud, or writing the words and symbols for these numbers, but the difficulty of doing this should not be underestimated.

The core idea of place value can be summed up by saying that numbers greater than 9, whether spoken or written, have different *representations*. For example, 24 can mean 24 single objects, or 2 tens and 4 ones.



Students need repeated practice at bundling (grouping) place-value materials and connecting the model to the words and symbols. A powerful activity is the Read, Say, Do (times two) model. Students need to show that they understand the relationship between reading, saying, and doing (modelling) numbers – first from 10 to 19, then from 20 to 99, and finally for larger numbers.

For example, for the number 17, students need to be able to read, interpret, and show the following.

Read:

- 17 as "seventeen"
- seventeen as "17".

Say:

- seventeen or 17 as "seventeen"
- seventeen or 17 as "one ten and seven ones".

Do/model:

- seventeen or 17 as seventeen singles
- seventeen or 17 as one ten and seven ones.

Choosing wisely is the aim

Students learn to solve number problems mentally and to choose critically from the numeracy strategies at their disposal. They learn to discriminate between strategies according to the numbers involved in a particular problem. Group or class discussions in which students are required to justify their choice are ideal for promoting the idea of efficiency. An efficient strategy uses the minimum number of steps to solve a problem.

In applying these strategies, students use their knowledge of grouping and place value. Basic fact knowledge develops as they move up the stages of the Number Framework – this knowledge is essential for all more sophisticated strategies.

Communicating mathematical thinking

The ability to articulate mathematical processes and strategies is crucial to the development of confident and capable mathematicians. Students learn to understand and use mathematical language, texts, symbols, and diagrams to express their mathematical thinking in a range of contexts. Students who use counting strategies will do most of their communicating orally. As they move through the stages of the Number Framework, however, they need to be given opportunities to communicate in a variety of ways.

Teachers have a key role in developing students' ability to communicate orally and in writing. Listening to students, and knowing when to step in and out of discussions and when to press for further understanding, are vital if students are to develop the skills needed to communicate and justify their mathematical thinking.

Written recording can be used to reduce the mental load, to communicate ideas to others, and to provide a window into student thinking.

Teachers need to model a variety of ways of recording. Students may use words, diagrams, and symbols and then, as they progress to the higher stages, empty number lines, diagrams, calculators, dotty arrays, or ratio tables. Suggestions on the types of recording appropriate for each stage can be found at http://nzmaths.co.nz/written-recording

There are two aspects to communicating by using symbols: being able to use a symbol to appropriately represent a real-world situation, and understanding what a symbol requires you to do in a particular context.

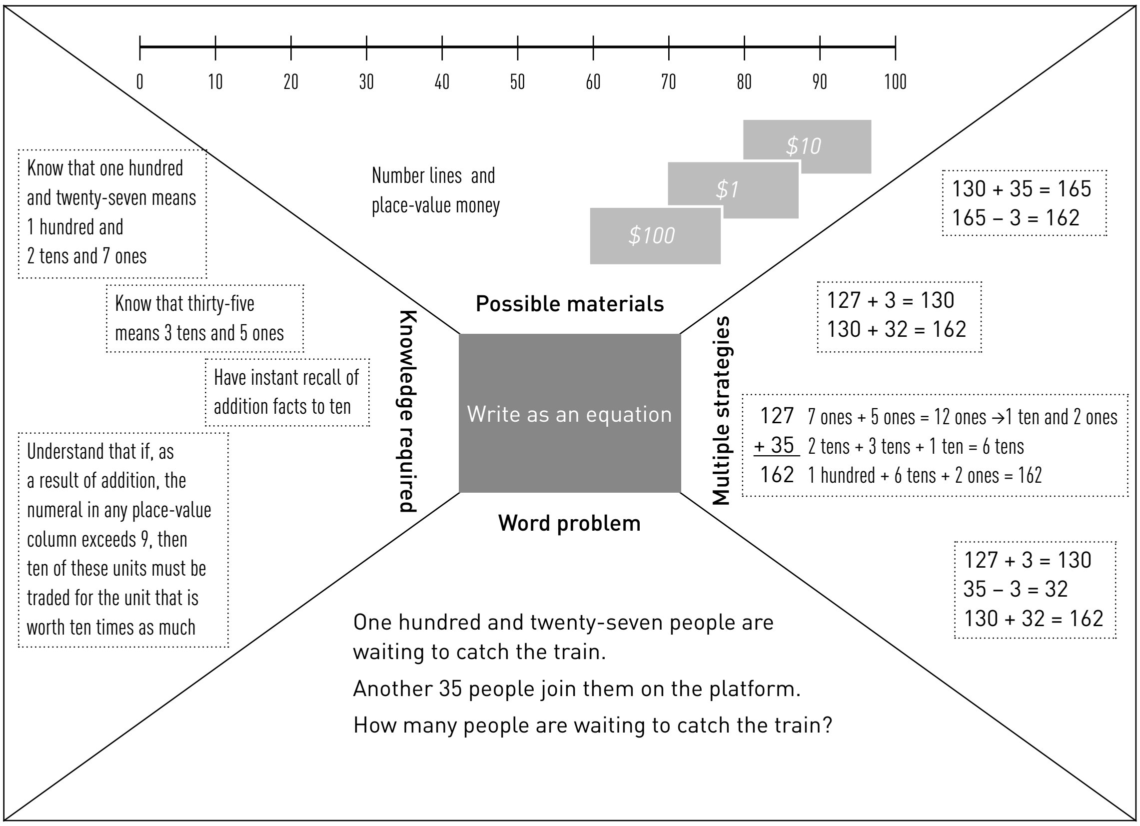
Most problems in Book 5 start with a word problem that gets transformed into an expression and is then solved. Word problems require students to choose which operation to use and how to record the process correctly.

Students need multiple opportunities to explore problem-solving processes. The teacher can ask questions instead of simply writing the relevant expression on the board or in the modelling book without discussion.

Good questions include:

- What operation do we need to use to solve this problem?
- How will we record what we have just done with the equipment?
- What symbol should we use when we are finding the difference between two amounts?
- Are there two ways we could record this problem?
- How could we record how [this student] just solved the problem so that it is easier for us all to understand?
- Can anyone write a new problem that can be solved using this strategy?

The best way to find out whether students understand what they have just recorded is to ask them to tell you a story about the expression they have written. Think boards (see below) are useful to show student understanding.



In recording, the transition from using just words to using words and numbers, then just numbers and symbols, requires a great deal of care on the teacher's part. Careful choice of words is critical for developing students' understanding of each symbol. Their ability to move from "and" to "plus" to the + symbol needs time to develop; folding back may be needed.

The equals sign

Initially, students understand the equals sign to mean "find the answer". For example, 2 apples plus 3 apples equals how many apples ($2 + 3 = \square$).

When part–whole thinking develops, a broader definition of "equals" is needed. For example, to work out 28 + 6 using a tidy number strategy, students need to understand that $28 + 6 = 28 + (2 + 4) = (28 + 2) + 4 = 30 + 4 = 34$.

Here, "equals" means "is the same as". An important development of this more general understanding is when secondary school students realise that, to find the solution of algebraic equations, they must understand that the equals sign also means "balance". So, in $3x + 11 = 11x – 5$, students must see "=" as meaning "is the same as", not "get the answer".

Ideas and resources to promote understanding of the equals sign as the balancing point in an equation can be found on the Assessment Resource Bank website:
http://arb.nzcer.org.nz/supportmaterials/maths/concept_map_algebraic.php#equality

Addition and subtraction as word problem types

There are important distinctions between the different types of addition and subtraction problems. Throughout Book 5, students are asked to *join* or *separate* or *combine* or *compare*. Number size varies as students move through the stages, but the structures remain the same.

Careful thought is required both when creating word problems and when modelling them so that the action taken accurately reflects the problem type. Tens frames are ideal for modelling the different problem types. *Join* and *separate* problems involve action. In *join* problems, elements are added to a given set; in *separate* problems, elements are removed from a given set. *Combine* and *compare* problems don't involve action: they involve making comparisons between two disjoint sets.

Start unknown is the problem type that students find most challenging, but once they are confident with the commutative property of addition, they can use this knowledge to change a problem from *start unknown* to *change unknown*. The table below summarises the problem types.

Problem types

Problem type			
Join (change: add to)	**Result unknown** Ana had 5 plums. Sam gave her 8 more plums. How many plums did Ana have altogether? 5 + 8 = □	**Change unknown** Ana had 5 plums. How many more plums does she need to have 13 plums altogether? 5 + □ = 13	**Start unknown** Ana had some plums. Sam gave her 8 more plums. Now she has 13 plums. How many plums did Ana start with? □ + 8 = 13
Separate (change: take from)	**Result unknown** Ana had 13 plums. She gave 5 to Sam. How many plums did Ana have left? 13 – 5 = □	**Change unknown** Ana had 13 plums. She gave some to Sam. Now she has 8 plums left. How many plums did Ana give Sam? 13 – □ = 8	**Start unknown** Ana had some plums. She gave 5 to Sam. Now she has 8 plums left. How many plums did Ana start with? □ – 5 = 8
Combine (part–part–whole)	**Whole unknown** Ana had 5 red plums and 8 yellow plums. How many plums did she have altogether? 5 + 8 = □	**Part unknown** Ana has 13 plums; 5 are red and the rest are yellow. How many yellow plums does she have? 13 – 5 = □	
Compare (difference)	**Difference unknown** Ana has 13 plums. Sam has 5 plums. How many more plums does Ana have than Sam? 13 – 5 = □	**Compare quantity unknown** Sam has 5 plums. Ana has 8 more plums than Sam. How many plums does Ana have? 5 + 8 = □	**Referent unknown** Ana has 13 plums. She has 5 more plums than Sam. How many plums does Sam have? 13 – 5 = □

Source: Young-Loveridge, J. and Mills, J. (2011). "Supporting Students' Additive Thinking: The Use of Equal Additions for Subtractions". *Set: Research Information for Teachers*, 1, 51–60.

Features of this book

The following diagram shows how the information in this book is organised.

Moving students from advanced counting to early additive

The important ideas are listed at the start of each section.

Introduction

Students who have mastered …

Ideas being developed

- Addition and subtraction problems can be solved by …
- Numbers can be partitioned …

Learning experiences that will advance each key teaching idea are listed here. Some students may not need all the learning experiences. Others may need to repeat them with different equipment before they fully understand the key ideas.

Learning experiences

Key teaching ideas	Learning experiences
1. Our number system is based on ten.	More ones
2. Basic fact knowledge …	Adding ones ar tens

Each key idea is indicated by a curriculum nautilus symbol. The key ideas replace learning intentions. They ensure clarity about the focus of the lesson. The learning intention can be developed with the students from the key idea as the lesson progresses.

Key idea 1: Our number system is based on ten

Mathematical knowledge required

Students need:

- instant recall of basic facts to 10
- knowledge of "10 and" facts (e.g., 10 + 4).

This is the knowledge the student needs in order to work successfully with this key idea.

Diagnostic snapshot

How many ten-dollar notes would I need to make $68?

Students who answer this with confidence …

The snapshot is used to find out what students already understand. The teacher can use this information to either move on to the next learning experience or build knowledge further as required.

Learning experience

■ More ones and tens

Equipment: $1, $10, and $100 notes (Material Master 4-9) …

Lists the materials needed for this learning experience.

Using materials

In pairs, one student shows two-digit numbers on the arrow cards. The other student reads that number, then models it in place-value money.

Example: $93, $47, $82, $99 …

Note that student–student talk is encouraged at every opportunity.

Learning experiences for emergent and one-to-one counting

Introduction

The key concept for one-to-one counting is that counting determines the number of objects in a set. Students are learning to count, with understanding, in order to identify "how many" in any set of objects. Learning experiences at this stage are designed to connect the counting words with the counting sequence (1–10) to correctly produce the number of a set.

The students need to be given multiple opportunities to read, count, and model numbers up to 10. The learning experiences listed in the table below will enable students to reliably count the number of objects, one to one, in structured and unstructured collections.

Counting principles

- Counting the number of objects in a set requires students to say all the counting words in the correct order, starting from one (one-to-one principle).
- Counting the number of objects in a set requires students to assign one word to each object in the set (stable-order principle).
- Counting the number of objects in a set requires students to recognise that the last number counted defines the number of the set (cardinal principle).
- The order in which objects are counted does not affect the outcome (order-relevance principle).[1]
- Any collection of objects can be counted, whether tangible or not (abstraction principle).

Recognising patterns to five instantly (by subitising) enables students to recognise small numbers quickly without explicitly counting them.

Learning experiences

Key teaching ideas	Learning experiences	Page*
1. Symbols/words for numbers in the range 1–10 are identified.	Lucky dip Number mat Lily pads Pipe cleaner numbers	11 11 11 11
2. The number word sequence for numbers in the range 1–10 is said accurately.	Counting as we go How many now? Loud and soft Clapping Walk the bridge Counting together Counting movements to nine	12 12 12 12 13 BSM‡ BSM
3. The symbols/words for numbers in the range 1–10 are matched to the number of objects in the set.	Match it up Magic caterpillar legs Petals and flower centres Feed the elephant Birthday cake	13 13 14 14 14
4. The sequence of numbers in the range 1–10 is ordered correctly.	Before and after Ordering numerals Up or down How many beans? Making a series to nine	14 15 15 15 BSM
5. Patterns for numbers 1–5 are recognised instantly.	Patterns to five, then ten How many different sets of five can you make? Fabulous fives	15 BSM 16

* Page numbers in these tables refer to pages in this book.

‡ Beginning School Mathematics

[1] Gelman, R., and Gallistel, C. (1978). *The Child's Understanding of Number.* Cambridge, MA: Harvard University Press.

Although the learning experiences have been matched to specific key ideas, they are all closely linked and can be used to complement each other. Because knowledge and strategy about one-to-one counting are developed simultaneously, no specific knowledge is identified for each of the suggested learning experiences.

Key idea 1: Symbols/words for numbers in the range 1–10 are identified

Learning experiences

■ Lucky dip

Equipment: Numeral cards 1 to 10 (Material Master 4-1); a container.

Using materials

Show the students a card and ask them what number it is. Encourage them to draw the number in the air and/or draw the number on the mat with their fingers. Repeat with further cards.

■ Number mat

Equipment: A large piece of cloth or PVC with the numbers 0 to 9 arranged randomly over it, or an A4 number mat for the students to use in pairs (Material Master 4-13).

Using materials

Lay the large number mat out in the middle of a group of students. Ask a student to jump on a number you have chosen for them. Get the students seated around the outside of the mat to draw the number on the floor in front of them with their fingers so that you can quickly see which students can identify the number correctly.

The students can play Twister by having three or four numbers called out consecutively. They must use their hands, feet, or head to touch each new number while leaving the other numbers untouched. Two or three students can be on the mat at the same time and can be given different sets of numbers to touch.

■ Lily pads

Equipment: Large numeral cards showing 0 to 9 (Material Master 4-3).

Using materials

Line up the numeral cards in order to create "lily pads". The students act as frogs and jump on specific numbers until "Stop" is called. Get the students who are seated around the outside of the lily pads to write the number on the floor in front of them with their fingers so that you can quickly see which students can identify the number correctly.

■ Pipe cleaner numbers

Equipment: Two pipe cleaners for each student; large numeral cards showing 0 to 9 (Material Master 4-3).

Using materials

Say a number to the students. (The size of the number will depend on the ability of the students.) Have the students make that number with their pipe cleaner(s). If they have trouble doing this, they can refer to the numeral cards and make their pipe cleaner number either beside or on top of the numeral card. This learning experience can be repeated with play dough.

Key idea 2: The number word sequence for numbers in the range 1–10 is said accurately

Learning experiences

Counting as we go

Equipment: Classroom objects to pass around.

Using materials

Have the students form groups and arrange themselves in circles. Nominate a student from each group to start and provide them with one object. The object is passed around the circle from student to student, and the number of passes is counted. The students count as far as they can or until the teacher calls "Stop".

Repeat the above activity, but count forwards or backwards from a number other than one.

Give all the groups the same starting number. Have all the groups count forwards. Play some music. When you stop the music, each student draws the group's current number in the air. Record the number that each group stopped at on the board or in the modelling book so the students can practise ordering the numbers from smallest to biggest.

How many now?

Equipment: Marbles or heavy counters or pebbles; a container.

Using materials

Have the students close their eyes and listen as you drop objects, one at a time, into the container. The students count aloud as they hear each object being dropped. At the end, ask how many objects are in the container and record the students' responses on the board or in the modelling book. The students can check their counts by emptying the container and counting the number of objects.

Repeat with students in pairs. One student does the dropping while the other student counts. Then get the students to swap roles.

Loud and soft

Equipment: Two hand puppets.

Using materials

Put one puppet on each hand. The first puppet speaks loudly, and the second speaks softly. Counting from one, make the puppets say the numbers alternately. Have the students count simultaneously with the puppets – loudly and then softly.

If you have a puppet that can squeak, get the students to close their eyes and count the squeaks silently or aloud as the puppet squeaks. Discuss how many squeaks the puppet made and record the count on the board or in the modelling book. Repeat by counting forwards or backwards to or from different numbers.

Clapping

Equipment: None.

Using materials

Get the students to count from one, clapping their hands in time with each count. Repeat by counting forwards or backwards to and from different numbers.

Change the learning experience by getting the students to count from one as they alternately clap their hands and slap their knees. The students then say the clapping numbers aloud and the slapping numbers silently (or vice versa). Repeat by varying the parts of the body used.

■ Walk the bridge

Equipment: Large numeral cards 1 to 10 (Material Master 4-3).

Using materials

Place numeral cards on the ground, in order from 1 to 10, to form a bridge. The students count aloud as one student steps across the numbers (like a bridge). The student who is walking the bridge can decide whether to walk forwards or backwards. The other students watch closely and produce the forwards- or backwards-counting sequence. When "Stop" is called, all the students who are off the bridge have to identify the number on which the walker has stopped and do that many star jumps (or another movement). Repeat by starting from different numbers.

■ Counting together

BSM: 3-1-21 (page 18)
Equipment: Class counting rhymes.

■ Counting movements to nine

BSM: 3-1-22 (page 19)
Equipment: Rope; drum; chairs; dandelions (or other real or artificial flowers).

Key idea 3: The symbols/words for numbers in the range 1–10 are matched to the number of objects in the set

Learning experiences

■ Match it up

Equipment: Numeral cards 1 to 10 (Material Master 4–1) and dot cards 1 to 10 (Material Master 5–11).

Using materials

Have the students place the dot cards face down in one row, and the numeral cards face down in a parallel row. The students take turns to turn over a card from each row, trying to match a numeral card with a dot card. While some students may need to count all the dots from one, others may instantly recognise some of the number patterns. If there is a match, the student keeps the pair. The game continues until all the pairs have been matched.
The game can be repeated by having all the cards mixed and face down in the middle of the group, rather than being in rows.

■ Magic caterpillar legs

Equipment: Clothes pegs; numeral cards 1 to 10 (Material Master 4-1); caterpillars (Material Master 5-6).

Using materials

Give each student blank caterpillars to which numeral cards have been added. Students with little number recognition should initially be presented with the 1, 2, and 3 numeral cards only.
Gradually increase the number size as the students' knowledge of the numbers increases. Have the students add the correct number of "legs" (i.e., pegs) to each of their caterpillars. They can then practise writing in the air, or on the floor, the number of legs of their caterpillars. The students can also order their caterpillars according to the number of legs each caterpillar has.

■ Petals and flower centres

Equipment: Numeral cards 1 to 10 (Material Master 4-1); counters or ice-block sticks.

Using materials

Give each student a flower (a numeral card). Have the students surround the centre of a flower with the correct number of petals (counters or ice-block sticks). They can then order their flowers according to which flower has the greatest or least number of petals. This learning experience can be repeated with spots on dogs, spots on butterflies, and/or lollies on an ice cream.

■ Feed the elephant

Equipment: Paper cups; counters or beans or Checkout Rapua material in the School Entry Assessment kit; laminated elephants (Material Master 5-7).

Using materials

Add single-digit numbers to the speech bubbles on the elephants and clip each elephant to a paper cup. In pairs, students take turns feeding their elephant the correct amount of food (counters, beans, etc.) while their partner checks the answer.

Students can then order the elephants according to how much each elephant ate. This experience can be used to develop "five and …" thinking. Using numbers from six to nine only, the students feed the elephant five yellow "bananas" (yellow counters or beans) and green coloured "vegetables" (green counters or beans) for the rest of the numbers.

■ Birthday cake

Equipment: Ice-block sticks; numeral cards 1 to 10 (Material Master 4-1); laminated birthday cake (Material Master 5-8).

Using materials

In pairs, have the students select numeral cards that tell the age of the student who is having the party. They then put the matching number of "candles" (ice-block sticks) on the birthday cake. This learning experience can be used to develop simple addition and subtraction of one.

Repeat the activity, but this time ask questions such as: "How many more candles will be on the cake next year when [name] is one year older?" or "How many fewer candles would have been on the cake last year when [name] was one year younger?" Discuss, and record the story problems on the board or in the modelling book.

Key idea 4: The sequence of numbers in the range 1–10 is ordered correctly

Learning experiences

■ Before and after

Equipment: Counters; numeral cards 1 to 10 (Material Master 4-1); game boards (Material Master 5-4).

Using materials

Have the students take turns to pull a card from the stack of cards. One at a time, they say the number that comes directly before or after the number they have pulled out and then cover this number on their game board. If a student cannot cover a square, the next student has their turn. The pulled-out card is replaced in the stack so that it can be pulled out again. The game ends when a student covers all their numbers.

■ Ordering numerals

Equipment: Numeral cards 1 to 10 (Material Master 4-1).

Using materials

Have the students order the numeral cards from 1 to 10, forwards or backwards. They can then practise saying the sequence as they point at the cards. For example, have them point to a number and then ask, "What number comes before that number?" or "What number comes after that number?"

■ Up or down

Equipment: Pegs; vertical number lines (Material Master 5-3).

Using materials

Ensure each student has a vertical number line from 1 down to 20. Have each student place a peg on 1 and a peg on 20 on their number line. Choose a student to think of a mystery number between 1 and 20. The rest of the students take turns to guess the number and ask the chooser if their number is up (less than) or down (more than) from the mystery number. The students move their pegs to narrow the range of answers until the mystery number is found. Repeat by choosing another student to select a mystery number.

■ How many beans?

Equipment: Plastic beans.

Using materials

In pairs, have each student pick up a handful of beans and, before counting them, say how many they think they have. Each student then counts their partner's beans. They compare who has got more and who has got less by lining up the beans side by side. Record the students' comparison stories on the board or in the modelling book – for example, "Brian had eight beans, and Margaret had six beans. Brian had more beans than Margaret."

■ Making a series to nine

BSM: 6-3-3 (page 8)
Equipment: Cards showing sets of one to nine objects; blank cards and stamps; numeral cards 1 to 9 (Material Master 4–1); an egg timer.

Key idea 5: Patterns for numbers 1–5 are recognised instantly

Learning experiences

■ Patterns to five, then ten

Equipment: Tens frames (Material Master 4-6); playing boards (Material Master 5-10); counters; dominoes and dice.

Using materials

Have a pile of tens frames with five or fewer dots on each. Flash a tens frame briefly. The students say aloud how many dots they see and model that pattern on one hand.
Once you are sure the students successfully know all their patterns to five, explore the numbers between six and ten. Use a pile of tens frames with five or more dots on each. Show one of the students a tens frame briefly. That player says how many empty spaces there are on the frame. If correct, the player moves their counter forwards that number of spaces on the playing board. Alternate turns. The first person to reach the end of their playing board is the winner.

Dominoes are also excellent for getting students to learn to instantly recognise patterns for the numbers 0–6 (standard set) and 0–9 (extended set). By playing the game, students learn to match without counting. Dice are similarly useful for the numbers 1–6.

■ How many different sets of five can you make?

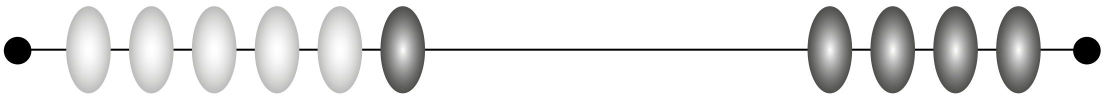
BSM: 5-3-54 (page 34).
Equipment: Cubes; nursery sticks; beans; three dice – one showing cubes, one showing nursery sticks, and one showing beans.

■ Fabulous fives

Equipment: Ten-bead string.

Using materials

Students practise making finger patterns for the numbers one to ten. The focus is on grouping, not counting. Begin with finger patterns the students will know, like one, two, five, and ten. Say, "Show me five fingers … Now, show me six. What did you do to make six? Show me ten fingers … How do you know it is ten?" (five and five). "Now, show me nine. What did you do to make nine?"



Using imaging

Students *image the count* when they are able to count physical objects in their minds.
Repeat this activity with the students hiding their fingers behind their backs. As they pretend that their fingers are tall soldiers standing at the castle gate, get them to image the finger patterns from one to five, and then from six to ten.

Moving students from one-to-one counting to counting from one on materials and by imaging

Introduction

The most basic strategy for solving addition and subtraction problems is to count from one. Students initially solve single-digit addition and subtraction problems by counting from one on physical materials before they begin to *image the count* (counting the physical objects in their minds). By the time students have mastered counting from one on materials and by imaging, they will confidently model and solve single-digit addition and subtraction problems, firstly by counting all the objects one to one on materials and then by mentally counting the objects. To *develop a mental image* of the objects being added or subtracted, students need first to talk about the image and describe the visual patterns that have been created.

Addition and subtraction problems should be presented to the students orally as stories and initially recorded by the teacher in words. Although the problems are presented in equation form throughout this book, we recommend recording them like this: "There were three apples in the basket, and Mum put two more apples in. How many apples were in the basket altogether?"

The combined use of digits and words can be introduced later as students show understanding of the key ideas. For example, "5 cats plus 4 cats equals 9 cats."

It is crucial that students begin to develop place-value understanding. Having the opportunity to practise bundling, say, nineteen objects into tens and ones, and to connect these physical models to the written symbols (and vice versa), will enable students to understand the critical place-value idea that one ten and ten ones are the same.

Ideas being developed

- Sets of objects can be joined physically on materials, and then all the objects can be counted *from one* to find the answer.
- A set of objects can be separated physically on materials, and then all the remaining objects can be counted *from one* to find the answer.
- The number of objects remains the same, regardless of spatial arrangement.
- There are many different representations for a set of objects. These can include: a finger pattern; a tens frame or dice pattern; the numeral itself; the word for the number; a place on a number line, number strip, or hundreds board; or a row of beads on an abacus. Students need to be exposed to different representations to help develop a flexible understanding of how a count can be tracked and how the number for a set can be recorded.

Learning experiences

Key teaching ideas	Learning experiences	Page
1. The number of objects in a set stays the same, regardless of spatial arrangement.	Animals on the farm	18
2. Addition and subtraction problems that involve *numbers up to five* can be solved by physically counting all the objects from one or by mentally counting the objects.	Adding and subtracting with one hand How many left outside? Mini skittles	19 BSM BSM
3. Addition and subtraction problems that involve *five as one of the numbers* can be solved by physically counting all the objects from one or by mentally counting the objects.	Murtles 5 and … Fly flip Using fives	21 21 22

Key teaching ideas	Learning experiences	Page
4. Addition and subtraction problems that involve *numbers up to ten* can be solved by physically counting all the objects from one or by mentally counting the objects.	Adding and subtracting with counters/hands Setting foot on Cigol Milking the cows	23 BSM BSM
5. Addition and subtraction problems that involve *ten as one of the numbers* can be solved by physically counting all the objects from one or by mentally counting the objects.	Making tens	25
6. Place value is developed by connecting physical models, words, and symbols.	Read, say, do (10–19) Our system Houses for earthlings Place-value Snap	26 BSM BSM BSM

Key idea 1: The number of objects in a set stays the same, regardless of spatial arrangement

Mathematical knowledge required

Students need to be able to:

- rote count fluently from one to ten
- read one to ten in words and 1 to 10 as numerals.

Diagnostic snapshot

> There are five dogs in their kennels at the vet's. Two of the dogs come out of their kennels to have a drink of water. How many dogs are there at the vet's?

Students who understand that there are still five dogs at the vet's are ready to move on to the next key idea. Otherwise, the following learning experiences will develop the idea.

Learning experiences

■ Animals on the farm

Use the copymasters from the unit *Counting on Counting*, found at www.nzmaths.co.nz
Equipment: Plastic farm animals; paper divided into two rectangles (representing paddocks on a farm).

Using materials

> There are eight farm animals on the farm. How many different ways could the farmer split the animals between the two paddocks?

Show the students the eight plastic farm animals and a piece of paper with two rectangles drawn on it to represent two paddocks. Let the students take the animals for a walk around the paddocks until "Stop" is called. Ask: "How many animals are in the first paddock, and how many are in the second paddock?"

Discuss the result – for example, three animals and five animals is the same as eight animals – and record it on the board or in the modelling book. Continue repeating the walk around the farm until all the pairs of numbers that add to eight have been found. Make sure the students understand that there are still eight animals altogether.

Continue to explore numbers from two to ten.

Using imaging

There are six cows on the farm. Two of the cows are in the first paddock, and four are in the second paddock.

Show the two cows and the four cows, and then cover the four cows with a piece of material. Say: "One of these cows joined her friends in the first paddock. How many cows are there on the farm altogether?"
The students solve the problem by imaging the cow moving to the first paddock. If necessary, have the students fold back to "Using materials" by moving one cow to the other paddock and counting how many cows there are altogether. Discuss why two cows and four cows is the same as three cows and three cows and record on the board or in the modelling book.

Continue to explore numbers from two to ten.

Key idea 2: Addition and subtraction problems that involve *numbers up to five* can be solved by physically counting all the objects from one or by mentally counting the objects

Mathematical knowledge required

Students need to be able to:

- rote count fluently from one to ten
- read one to ten in words and 1 to 9 as single numerals
- instantly recognise finger patterns for the numbers one to five.

Diagnostic snapshot

Rangi had three toy cars. He got some more cars for his birthday. Now Rangi has five cars. How many toy cars did Rangi get for his birthday?

Students who answer this problem confidently by imaging five as three and two are ready to move on to the next key idea. Otherwise, the following learning experiences will develop the idea.

Learning experiences

■ Adding and subtracting with one hand

Equipment: None. Alternatively, use pre-printed tens frames (Material Master 4-6), plastic counters, plastic beans in opaque containers, and number strip 1–10 or 1–20.

Using materials (result unknown)

Jayanti has three carrots, and she buys two more carrots. How many carrots does Jayanti have now?

Discuss and record $3 + 2 = \square$ as a word problem on the board or in the modelling book. The students model three fingers, and then two more fingers, on the same hand. Without counting to solve the problem, the students need to recognise that three fingers and two fingers equal five fingers. Record the solution to 3 + 2 on the board or in the modelling book.

Examples: Word problems and recording for: 1 + 4, 3 + 1, 5 – 2, 4 – 3, 2 + 3, 4 – 1 ...

Using imaging (result unknown)

> There are four carrots in the fridge and one carrot on the bench. How many carrots are there altogether?

Discuss and record 4 + 1 = $\square$ as a word problem on the board or in the modelling book. Have the students solve the problem by imaging the numbers. If necessary, have the students fold back to "Using materials" by modelling four fingers and one more finger on the same hand. Record the solution to 4 + 1 = $\square$ on the board or in the modelling book.

Examples: Word problems and recording for: 1 + 4, 3 + 1, 5 – 1, 4 – 3, 2 + 3, 5 – 2 …

Using materials (change unknown)

> When Pippa goes to sleep, she has three Easter eggs in her bedroom. While she is asleep, her mother gives her some more Easter eggs. She wakes up in the morning to find that she has five Easter eggs. How many eggs did her mother give Pippa?

Discuss and record 3 + $\square$ = 5 as a word problem on the board or in the modelling book. Encourage the students to act out the problem for themselves using counters. Record the solution to 3 + $\square$ = 5 on the board or in the modelling book.

Examples: Word problems and recording for: 1 + $\square$ = 4, 4 + $\square$ = 5, 5 – $\square$ = 2, 4 – $\square$ = 3 …

Using imaging (change unknown)

> Pippa's mother has five Easter eggs. She gives some of them to Pippa. Pippa's mother now only has two eggs left. How many did she give to Pippa?

Discuss and record 5 – $\square$ = 2 as a word problem on the board or in the modelling book. The students each put a hand behind their back and think about the missing number. If necessary, have the students fold back to "Using materials" by putting five counters in front of them and solving the problem. Record the solution to 5 – $\square$ = 2 on the board or in the modelling book.

Examples: Word problems and recording for: 1 + $\square$ = 4, 5 – $\square$ = 0, 4 – $\square$ = 2, 3 + $\square$ = 5 …

■ How many left outside?

BSM: 8-1-53 (page 28)

Equipment: 10 small toys; two boxes with opening doors; dice showing 1 to 6.

■ Mini skittles

BSM: 8-1-85 (page 29)

Equipment: Cuisenaire rods; ping-pong ball.

Key idea 3: Addition and subtraction problems that involve *five as one of the numbers* can be solved by physically counting all the objects from one or by mentally counting the objects

Mathematical knowledge required

Students need to be able to:

- rote count fluently from one to ten
- read one to ten in words and 1 to 10 as numerals
- instantly recognise, using a five strategy, finger patterns for the numbers one to ten.

Diagnostic snapshot

James has nine marbles. On the way to school, he drops some. When he gets to school, he finds that he has only five marbles left. How many marbles has he dropped?

Students who answer this problem confidently by instantly imaging nine as five and four, and then remove the four by imaging, are ready to move on to the next key idea. Otherwise, the following learning experiences will develop the idea.

Learning experiences

■ Murtles 5 and …

Equipment: Counters; wooden cubes labelled 0, 1, 2, 3, 4, 5; turtle game board (Material Master 5-9); and number strip 1–10 or 1–20.

Using materials (result unknown)

Each student has a turtle game board. In pairs, the students alternately toss a wooden cube labelled 0 to 5. Five is then added on to the number thrown. Students use materials (fingers) to physically show "five and …". Once they have added the "five and …", they put a counter on the matching number on their board. The winner is the first person to cover all the numbers on their turtle.

Using imaging (result unknown)

As above, but students are encouraged to image the "five and …" pattern (hands behind backs) and to describe the visual patterns that have been imaged. If necessary, have the students fold back to "Using materials" by modelling "five and …" on their fingers.

■ Fly flip

Equipment: Fly flips (Material Master 4-5).

Using materials (change unknown)

There are eight flies on this fly flip altogether. Five are on top and some are underneath. [The students can see the eight on the bottom and five flies on their side of the fly flip.] How many flies are on the other side?

Discuss and record $5 + \square = 8$ as a word problem on the board or in the modelling book. Encourage the students to model eight as five fingers and three fingers and then to work out that there must be three flies on the reverse side. Record the solution to $5 + \square = 8$ on the board or in the modelling book.

Examples: Repeat for the 9, 5, 6, 10, and 7 fly flips.

Using imaging

Show the students the eight-fly flip with three flies visible to them. Ask how many flies they can see and then what number they can see.

Discuss how many flies are hidden and record 3 + 5 = □ as a word problem on the board or in the modelling book. Have the students think about the solution and work out the answer without using their fingers or turning the fly flip over. If necessary, have the students fold back to "Using materials" by modelling three and five on their fingers. Discuss the answer and record the solution to 3 + 5 on the board or in the modelling book.

Examples: Repeat for the 9, 5, 6, 10, and 7 fly flips.

■ Using fives

Equipment: None.

Using materials (result unknown)

Ahorangi has $7, and she buys an ice cream for $2. How much money does Ahorangi have left?

Discuss and record 7 – 2 = □ as a word problem on the board or in the modelling book. Have the students model seven fingers and try to solve the problem by removing the hand showing two fingers and instantly recognising that five fingers remain. Discuss the answer and record the solution to 7 – 2 on the board or in the modelling book.

Examples: Word problems and recording for: 3 + 5, 8 – 3, 4 + 5, 10 – 5, 7 – 5, 5 + 5 ...

Using imaging (result unknown)

Tai has $8, and he buys an ice cream for $3. How much money does Tai have left?

Discuss and record 8 – 3 = □ as a word problem on the board or in the modelling book. Have the students think about the solution and work out the answer without using their fingers. If necessary, have the students fold back to "Using materials" by modelling eight on their fingers as five and three and removing the three. Discuss the answer and record the solution to 8 – 3 on the board or in the modelling book.

Examples: Word problems and recording for: 3 + 5, 8 – 3, 4 + 5, 10 – 5, 7 – 5, 5 + 5 ...

Using materials (change unknown)

Sally has nine jellybeans. On the way to school, she drops some. When she gets to school, she finds that she has only five jellybeans left. How many jellybeans has Sally dropped?

Discuss and record 9 – □ = 5 as a word problem on the board or in the modelling book. Have the students use their fingers to instantly make the finger pattern for nine and to work out that if five marbles are left, then four marbles were dropped. Discuss the answer and record the solution to 9 – □ = 5 on the board or in the modelling book.

Examples: Word problems and recording for: 10 – □ = 5, 5 + □ = 10, 9 – □ = 4, 3 + □ = 8, 5 + □ = 8 ...

Using imaging (change unknown)

Jono has eight rugby cards. At school he gives some away to his friends. When he gets home, he has three left. How many rugby cards did Jono give away?

Discuss, and record 8 – □ = 3 as a word problem on the board or in the modelling book. Have the students think about the solution and work out the answer by imaging the finger pattern for eight, using a five strategy. If necessary, have the students fold back to "Using materials" by modelling eight on their fingers and removing the five. Discuss the answer and record the solution to 8 – □ = 3 on the board or in the modelling book.

Examples: Word problems and recording for: 9 – □ = 5, 5 + □ = 7, 7 – □ = 2, 4 + □ = 9, 5 + □ = 8 ...

Key idea 4: Addition and subtraction problems that involve *numbers up to ten* can be solved by physically counting all the objects from one or by mentally counting the objects

Mathematical knowledge required

Students need to be able to:

- count fluently from one to ten
- read one to ten in words and 1 to 10 as numerals
- recognise instantly, using a five strategy, finger patterns for the numbers one to ten.

Diagnostic snapshot

> Hina has nine bottles of drink for her birthday party. Her naughty brother drinks two of the bottles. How many bottles of drink are left?

Students who answer this problem confidently by using a five strategy to image nine, remove two, and then recognise that seven remain are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Learning experiences

■ Adding and subtracting with counters or hands

Equipment: None. This learning experience can also be done using pre-printed tens frames (Material Master 4-6) and hands, and number strip 1–10 or 1–20.

"Crossing over" from one hand to the other requires the beginning of part–whole thinking, which is challenging for students who are counting from one on materials. For example, to work out 4 + 3, students put four on one column of a tens frame (or one hand), but then they may struggle to "see" the remaining three fingers split across the two columns or hands. When the students see this, they recognise that the answer to 4 + 3 is 7 because 4 + 3 is the same as 5 + 2.

Using materials (result unknown)

> Ioane has four bottles of drink in the cupboard, and he buys three more bottles for his birthday party. How many bottles of drink does Ioane have now?

Discuss and record 4 + 3 = □ as a word problem on the board or in the modelling book. Have the students solve the problem using counters.

If you are using pre-printed tens frames, provide a tens frame printed with four dots. Ask the students to point to where three more counters would go. It is important that students complete a column of five and then add two more in the other column. Discuss the answer and record the solution to 4 + 3 on the board or in the modelling book.

Examples: Word problems and recording for: 3 + 5, 2 + 5, 6 + 2, 2 + 8, 4 + 4, 7 + 2 …

Using materials (result unknown)

> Hohepa puts eight boiled eggs on the breakfast table. Moana and Hinewai eat one egg each. How many eggs are left?

Discuss and record 8 – 2 = □ as a word problem on the board or in the modelling book. Have the students solve the problem using physical materials like counters or pre-printed tens frames. If using pre-printed tens frames, it is important that students model the eight by completing a column of five and a column of three. Discuss the answer and record the solution to 8 – 2 on the board or in the modelling book.

Examples: Word problems and recording for: 6 – 3, 10 – 3, 7 – 6, 9 – 1, 6 – 5 …

Using imaging (result unknown)

> Ranjit has five jellybeans, and he gets three more jellybeans from a friend. How many jellybeans does Ranjit have now?

Discuss and record 5 + 3 = ☐ as a word problem on the board or in the modelling book. Out of the students' sight, build five on an empty tens frame. Show the students the tens frame and ask them to describe how you built it. Drawing pictures in the air may help. Add three, out of the students' sight, and ask them to explain how you built it. Ask how many counters there are altogether.
If necessary, fold back to "Using materials" by showing the students the hidden counters. Discuss the answer and record the solution to 5 + 3 on the board or in the modelling book.

Examples: Word problems and recording for: 4 + 5, 10 – 4, 6 – 5, 5 + 2, 10 – 8 …

Using imaging (change unknown)

> Carol has eight eggs in the cupboard. She uses some eggs to make a cake, and then there are only five eggs left in the cupboard. How many eggs did Carol use for the cake?

Discuss and record 8 – ☐ = 5 as a word problem on the board or in the modelling book. Have the students put both hands behind their backs and model the eight, then show how they would remove five to work out how many eggs were used. If necessary, fold back to "Using materials" (the students bring their hands out in front of them) to solve the problem. Discuss the answer and record the solution to 8 – ☐ = 5 on the board or in the modelling book.

Examples: Word problems and recording for: 9 – ☐ = 4, 2 + ☐ = 5, 8 – ☐ = 5, 2 – ☐ = 1, 3 + ☐ = 8 …

Using imaging (change unknown)

> Ana has seven dollars. She spends some money on an ice cream. Now she has four dollars. How much did Ana spend on her ice cream?

Discuss and record 7 – ☐ = 4 as a word problem on the board or in the modelling book. Hide a pre-printed tens frame with seven dots and ask the students to image the tens frame and to talk about the model they have imaged. Then have the students image the removing of three dots. If necessary, fold back to "Using materials" by showing seven dots and shielding three of the dots. Discuss the answer and record the solution to 7 – ☐ = 4 on the board or in the modelling book.

Examples: Word problems and recording for: 3 + ☐ = 7, 8 – ☐ = 6, 4 + ☐ = 9, 8 – ☐ = 6, ☐ + 4 = 8 …

■ Setting foot on Cigol

BSM: 9-1-14 (page 41)

Equipment: Box representing spaceship; nine objects representing astronauts.

■ Milking the cows

BSM: 9-3-13 (page 122)

Equipment: Toy animals.

Key idea 5: Addition and subtraction problems that involve *ten as one of the numbers* can be solved by physically counting all the objects from one or by mentally counting the objects

Mathematical knowledge required

Students need to be able to:

- rote count fluently from one to twenty
- read one to twenty in words and 1 to 20 as numerals
- write numerals from 1 to 20.

Diagnostic snapshot

> Scott has blown up two balloons, and he needs to blow up ten for his birthday. How many more balloons does Scott need to blow up?

Students who answer this problem confidently by working out that Scott still has eight (three and five) more balloons to blow up are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Learning experiences

■ Making tens

Equipment: Pre-printed tens frames (Material Master 4-6); counters; a blank card.

Using materials

> Jonathan has seven black lollies to put on top of some cupcakes, and he needs ten. How many more lollies does Jonathan have to get?

Record $7 + \square = 10$ as a word problem on the board or in the modelling book. Get out a tens frame that shows seven dots. Discuss why the answer is three. Put three counters on the tens frame. Record the solution to $7 + \square = 10$ on the board or in the modelling book.

Examples: Word problems and recording for: $9 + \square = 10$, $6 + \square = 10$, $5 + \square = 10$, $4 + \square = 10$, $8 + \square = 10$ …

Using materials

> Tai has ten marbles, and he gives four away to his friends. How many marbles does Tai have left?

Record $10 - 4 = \square$ as a word problem on the board or in the modelling book. Take a tens frame with ten dots on it and discuss which four can be removed. Cover these four with a blank card to show the dots that are left. Discuss why there are six left (five and one). Record the solution to 10 – 4 on the board or in the modelling book.

Examples: Word problems and recording for: 10 – 1, 10 – 7, 10 – 3, 10 – 5, 10 – 9, 10 – 8 …

Using imaging

> Margaret has ten stickers on her schoolbag, and two fall off in the rain. How many stickers does Margaret have left on her schoolbag?

Record $10 - 2 = \square$ as a word problem on the board or in the modelling book. Have ten on a tens frame, which the students can see but not touch. Ask the students to image taking two stickers off and to work out how many would be left.
Discuss what the students are imaging and how they are working out how many stickers are left. If necessary, fold back to "Using materials" by letting the students use their fingers, or remove two from the tens frame. Record the solution to $10 - 2 = \square$ on the board or in the modelling book.

Examples: Word problems and recording for: $3 + \square = 10$, $6 + \square = 10$, 6 + 4, 10 – 8, $1 + \square = 10$ …

Key idea 6: Place value is developed by connecting physical models, words, and symbols

Mathematical knowledge required

Students need to be able to:

- rote count fluently from one to twenty
- read one to twenty in words and 1 to 20 as numerals
- write numerals from 1 to 20.

Diagnostic snapshot

Write 17 on the board. Ask the students:

> Pio gets this number of lollies [pointing to 17 on the board without reading aloud] and packs the lollies into bags of ten. How many bags of ten lollies does Pio pack?

Students who answer this problem confidently, knowing that seventeen is packaged as one bag of ten with seven left over, are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Learning experiences

■ Read, say, do (10–19)

Allow students multiple opportunities to practise bundling ten to twenty objects into groups of ten and connecting the bundles to symbols and words so that they eventually understand that one ten and ten ones are the same.

Equipment: Sticks with rubber bands or beans and plastic bags; little whiteboards or paper.

Using materials

> Gary has fifteen lollipops. Get me that many lollipops (ice-block sticks). [Allow the students to count out fifteen loose sticks.] Gary's job at the lollipop shop is to pack ten lollipops together. Can you put ten lollipops together? How many loose lollipops has Gary got left over?

Look for students who have to count one, two, three, four, five and those who know instantly that there are five left over.

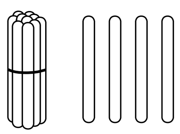
Examples: Repeat for numbers from ten to nineteen.

> Shada has this many lollipops [write and point to 17 on the board or in the modelling book without reading aloud]. If she packs ten lollipops in a bag, how many loose lollipops will Shada have left over?

Look for students who have to count one, two, three, four, five, six, seven and those who know instantly that there are seven left over.

Examples: Repeat for numbers from ten to nineteen.

> Caitlin has this many lollipops [draw or model one bundle of ten and four loose sticks on the board or in the modelling book]. Write in symbols and words how many lollipops Caitlin has.



Look for the students who are making the connection between the model, the word, and the symbol.

Examples: Repeat for numbers from ten to nineteen.

Using imaging

Make sixteen as a bundle of ten and six loose ones, then cover with a tea-towel or place in a lolly jar.

> I have sixteen lollipops in my lolly jar that I have packed into bags of ten. Write down how many loose lollipops there are in my lolly jar.

Look for the students who write six instantly, as a word or a symbol, or who need to count from one or from ten. If necessary, fold back to "Using materials" by writing "sixteen" or "16" on the board or letting the students see the model of sixteen you have made.

Examples: Repeat for numbers from ten to nineteen.

■ Our system

BSM: 9-1-9 (page 30)

Equipment: Calendar pages; nursery sticks and rubber bands.

■ Houses for earthlings

BSM: 9-1-10 (page 31)

Equipment: Nesting boxes; nursery sticks and rubber bands.

■ Place-value Snap

BSM: 9-1-48 (page 32)

Equipment: Cards showing the numbers 10 to 20.

Moving students from counting all to advanced counting

Introduction

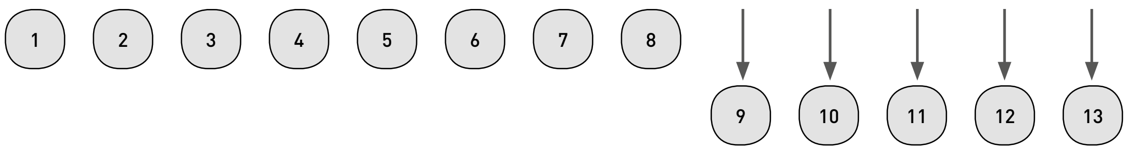
Students who have mastered advanced counting will confidently use counting on, or counting back, to solve addition and subtraction problems. Advanced counting is the most sophisticated of the counting stages and requires students to co-ordinate a number of counting concepts.
For example, to solve 18 + 5, students need to:

- start the count at nineteen and not eighteen
- say the next five numbers after eighteen accurately and fluently (i.e., say nineteen, twenty, twenty-one, twenty-two, twenty-three)
- know to stop when the last of the sequence of five numbers has been said
- know that as the last number said, twenty-three is the answer to 18 + 5.

It is crucial that students extend their place-value understanding in preparation for using part–whole thinking to solve addition and subtraction problems.

Ideas being developed

- The last number in a line of numbered objects gives the correct number in the set only when all the numbers are in the correct sequence and no numbers are missing.
- Counting on is the process of repeatedly adding on one.
- Counting back is the process of repeatedly subtracting one.
- When adding, the last number said defines the size of the set (cardinality).
- When subtracting, the number of objects that remain defines the size of the set (cardinality). For example, the solution to 13 – 5 requires the removal of five tiles starting at 13. So 13, 12, 11, 10, and 9 are removed, leaving 8 as the answer.



- Tracking of the smaller number being added or subtracted (typically on fingers) is allowed. It does not indicate that the student is still counting from one on materials.

Learning experiences

Key teaching ideas	Learning experiences	Page
1. Numbers can be added by counting on from the largest number in increments of one.	Can you count on? Number tiles The number strip The bears' picnic Change unknown Taking a group and counting on	BSM 29 30 31 31 BSM
2. Numbers can be subtracted by counting back from the largest number in increments of one.	Counting back	32
3. Objects can be counted by creating bundles of ten.	Read, say do (20–99) Ones and tens	33 33
4. Groups of ten can be added and subtracted by using simple known addition facts.	Ten stickers per packet Adding tens Subtracting tens	34 35 35
5. Addition is commutative, so the order of the numbers can be rearranged to make counting on easier.	The bigger number first	36

Key idea 1: Numbers can be added by counting on from the largest number in increments of one

Mathematical knowledge required

Students need to be able to:

- fluently and accurately count on from numbers between one and one hundred.

Diagnostic snapshot

Michael has forty-seven marbles and wins five more. How many marbles does Michael have now?

Students who answer this problem confidently by counting on from forty-seven are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea. Alternatively, the use of any part–whole thinking also indicates the student is ready to move on to the next key idea.

Learning experiences

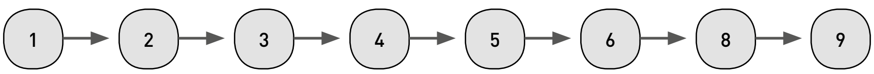
■ Can you count on?

BSM: 6-3-2 (page 7)

Equipment: A container of bottle tops or cubes; pieces of string.

■ Number tiles

Check that the students understand that the last number in a line of numbered objects gives the number in the set only if the numbers are in order and no number is missing. For example, there are not nine tiles below, because the 7 is missing:



Equipment: A set of small cards or tiles labelled 1 to 20.

Using materials

Fetu has nine lollies in one bag and two lollies in another bag. How many lollies does Fetu have altogether?

Record 9 + 2 = □ on the board or in the modelling book. Get the students to arrange tiles (in place of lollies) 1 to 9 in order. Shield numbers 10 and 11 in your hand. Ask what the numbers on the lollies (tiles) in your hand are and then what 9 + 2 equals. Then lay down tiles 10 and 11 to check that the answer is 11. Record the solution to 9 + 2 on the board or in the modelling book.

Examples (second number is five or less): Word problems and recording for: 4 + 2, 7 + 2, 12 + 2, 7 + 3, 12 + 3, 16 + 3 ...

Using imaging

Solve eight lollies plus four lollies.

Record 8 + 4 = □ on the board or in the modelling book. Turn the tiles numbered 1 to 8 face down to hide the numbers. Hold the next four tiles (9 to 12) in your hand and ask the students to image what numbers you have.
Ask what 8 + 4 equals. If necessary, fold back to turning the eight tiles over and add tiles 9 to 12 to the end. Some students may need to use their fingers to track on. Record the solution to 8 + 4 on the board or in the modelling book.

Examples (second number is five or less): Word problems and recording for: 12 + 2, 9 + 2, 11 + 3, 13 + 3, 8 + 4, 7 + 4, 11 + 5, 14 + 5, 18 + 2 ...

Using number properties

By increasing the size of the first number in addition problems, the students are encouraged to let go of using materials or imaging and concentrate on the properties of the numbers instead.

Examples (second number is five or less): Word problems and recording for: 29 + 4, 46 + 5, 63 + 5, 78 + 3, 34 + 4, 89 + 3 ...

■ The number strip

As with "Number tiles", the students must realise that the last number in a set of tiles gives the number of tiles only if each number has a counter on it. For example, there are not twenty counters shown below, because counters 6 and 10 are missing.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Equipment: A set of number strips labelled 1 to 20 (Material Master 5-1); two sets of different-coloured transparent counters.

Using materials

Gita has nine toy cars, and her mother gives her four more as a present. How many toy cars does Gita have now?

Record 9 + 4 = □ on the board or in the modelling book. Have the students place nine counters of one colour on the number strip from 1 to 9. The students have four counters of another colour. Before putting the counters on the strip, ask what numbers they go on and then what 9 + 4 is. Have the students confirm the answer by placing four counters on squares 10, 11, 12, and 13. Record the solution to 9 + 4 on the board or in the modelling book.

Examples: (second number is five or less): Word problems and recording for: 6 + 2, 9 + 2, 7 + 3, 9 + 3, 11 + 4 ...

Using imaging

> Work out seven toy cars plus four toy cars.

Record 7 + 4 = ☐ on the board or in the modelling book. Turn the number strip over and encourage the students to image seven counters. Then ask what squares the four counters will go on, and so what 7 + 4 is. Fold back, if necessary, by building one to seven in one colour and eight to eleven in another colour. Some students may need to count on with their fingers. Record the solution to 7 + 4 on the board or in the modelling book.

Examples: Word problems and recording for: 13 + 2, 11 + 2, 6 + 3, 8 + 4, 11 + 5, 12 + 2, 15 + 4 ...

Using number properties

Provide addition problems in which the first number has two digits and the second number is a single digit below five. This encourages the use of number properties.

Examples: Word problems and recording for: 75 + 2, 46 + 2, 49 + 2, 53 + 3, 87 + 3, 91 + 4, 67 + 4 ...

■ The bears' picnic

In this learning experience, the students need to see why counting on from the largest number is the more efficient method. Present problems in which the larger number varies between being first and second.

Equipment: A set of plastic bears (or counters) for counting; an opaque container.

Using materials

> Eight bears go into their cave. Two more bears arrive late to join the family. How many bears are there now in the cave?

Record 8 + 2 = ☐ on the board or in the modelling book. Hide eight bears in the container. Add Bobo Bear and then ask how many bears are now in the cave. Add Bernice Bear and then ask how many bears are now in the cave (encourage the students to use their fingers to track the counting on). Ask what 8 + 2 equals. Record the solution to 8 + 2 on the board or in the modelling book.

Examples: Word problems and recording for: 12 + 2, 7 + 2, 13 + 2, 7 + 31, 1 + 3, 14 + 3, 7 + 4, 11 + 4 ... (Change the order of some numbers so that the larger number comes second for approximately half the problems.)

Using imaging

Examples: Word problems and recording for: 11 + 2, 9 + 3, 4 + 12, 5 + 14, 7 + 4, 5 + 12 ...

Using number properties

Examples: Word problems and recording for: 39 + 4, 66 + 4, 5 + 33, 28 + 3, 5 + 87 ...

■ Change unknown

Equipment: Sets of counters or plastic beans.

Using materials

> A class is growing beans from bean seeds. When they leave school on Monday, six seeds have sprouted. When they come to school on Tuesday morning, they find that eight seeds have sprouted altogether. How many seeds have sprouted overnight?

Discuss why the problem amounts to solving 6 + ☐ = 8. Have the students solve the problem with the materials and then discuss their methods. Encourage the students to count on by pointing at their fingers and saying "seven, eight, so the answer is two". Record the solution to 6 + ☐ = 8 on the board or in the modelling book.

Examples: Word problems and recording for: 4 + ☐ = 6, 5 + ☐ = 6, 7 + ☐ = 8, 7 + ☐ = 9, 6 + ☐ = 8, 6 + ☐ = 9 ...

Using imaging

Examples: Word problems and recording for: 12 + □ = 14,   8 + □ = 10,   12 + □ = 15,   7 + □ = 11,
8 + □ = 11,   9 + □ = 12 ...

Using number properties

Examples: Word problems and recording for: 87 + □ = 89,   43 + □ = 45,   79 + □ = 83,   51 + □ = 54,
□ + 43 = 45,   □ + 56 = 59,   58 + □ = 63 ...

■ Taking a group and counting on

BSM: 9-3-57 (page 124)

Equipment: Feely bag and ten objects.

Key idea 2: Numbers can be subtracted by counting back from the largest number in increments of one

Mathematical knowledge required

Students need to be able to:

- fluently and accurately count back from numbers between one and one hundred.

Diagnostic snapshot

Kylie has sixty-two stickers in her sticker book and gives four away. How many stickers does Kylie have left in her sticker book?

Students who answer this problem confidently by counting back from sixty-two are ready to move on to the next key idea. Otherwise, the following learning experience will develop this key idea. Alternatively, the use of any part–whole thinking also indicates the student is ready to move on to the next key idea.

Learning experience

■ Counting back

The students need to understand that removing lollies from the smallest number does not solve the problem, because the last ordinal number in the line of objects does not give the correct number of objects remaining.

Equipment: A set of small cards or tiles labelled 1 to 20.

Using materials

Marama has eleven lollies and eats four. How many does Marama have left?

Record 11 – 4 = □ on the board or in the modelling book. Model the eleven lollies with tiles labelled in order 1 to 11. Ask which four lollies Marama would eat and how many would be left. Record the solution to 11 – 4 on the board or in the modelling book.

Examples: Encourage the students to track on their fingers which lollies get eaten using word problems and recording for: 9 – 2,   9 – 3,   12 – 4,   11 – 5,   19 – 3,   13 – 4,   11 – 4 ...

Using imaging

Examples: Word problems and recording for: 12 – 3,   19 – 2,   17 – 3,   8 – 2,   20 – 4,   15 – 5 ...

Using number properties

Examples: Word problems and recording for: 81 – 2,   90 – 2,   78 – 3,   62 – 4,   92 – 4 ...

Key idea 3: Objects can be counted by creating bundles of ten

Two-digit numbers can be modelled by using place-value material suitable for bundling into tens (e.g., ice-block sticks).

Mathematical knowledge required

Students need to be able to:

- read and write one to one hundred in words, and 1–100 as numerals
- decode words such as "sixty" as so many tens.

Diagnostic snapshot

> Your job at the factory is to bundle up packets of cards that have exactly ten in each packet. If you bundle this many cards [write 63 on the board or in the modelling book – *do not* say "sixty-three" aloud], how many packets of ten cards have you bundled?

Students who answer this problem confidently by understanding that sixty-three means six bundles of ten with three left over are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Learning experiences

■ Read, say, do (20–99)

See "Read, say, do (10–19)" on page 26.

Equipment: Sticks with rubber bands; beans and plastic bags; ones and tens place-value money.

■ Ones and tens

Many students will be able to read two-digit numbers but not realise that they represent ones and tens. This activity is designed to help them to learn this. In particular, many may not realise that "ty" at the end of words means "tens".
It is important that students learn to decode words like "sixty" as six tens.

Equipment: Materials suitable for bundling into tens (e.g., sticks with rubber bands around bundles of ten, or beans in lots of ten in film canisters or plastic bags).

Using materials

> Your job at the factory is to bundle up felt pens and send them to the shops. Each bundle has exactly ten felt pens in it. At the end of the day, you have to write down how many felt pens you have packed.

Give pairs of students about fifty items of loose place-value material and get them to create bundles of ten. Record the answers for all the pairs in a table.

Bundles	*Ones*

Examples: Have the students repeat the grouping and recording of objects in ones and tens with other place-value materials.

> Jerry has forty-three felt pens and Mark has thirty-four felt pens. Who has more felt pens?

Record 43 and 34 on the board or in the modelling book. Have the students model both forty-three and thirty-four with bundled materials and discuss why forty-three is more. Record "43 is more than 34" on the board or in the modelling book.

Examples: Word problems and recording for these pairs: Which number is larger: 56 or 65? 14 or 41? 25 or 52? 2 or 23? ...
(For this problem in greater depth, see "The Bubblegum Machine", *Connected* 2 1999.)

Key idea 4: Groups of ten can be added and subtracted by using simple known addition facts

Mathematical knowledge required

Students need to be able to:

- decode "ty" words as meaning "tens" (e.g., eighty means eight tens)
- decode ones and tens notation (e.g., thirty-six means three tens and six ones)
- recall addition and subtraction facts with answers up to 10 (e.g., 4 + 3,   6 + 2,   8 – 5,   9 – 2 …).

Diagnostic snapshot

> Tui and her grandfather go to the school fair to buy some trays of plants. There are ten plants in each tray. Tui buys five trays and her grandfather buys four trays. How many plants did Tui and her grandfather buy altogether?

Students who answer this problem confidently by using the addition fact 5 + 4 = 9 to work out 50 + 40 = 90 are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Learning experiences

The learning experiences that follow are important because they simultaneously help to develop addition, subtraction, and place-value concepts.

■ Ten stickers per packet

Equipment: A Slavonic abacus or materials suitable for bundling into tens (e.g., sticks with rubber bands around bundles of ten, or beans in lots of ten in film canisters or plastic bags).

Using materials

> There are ten stickers in each packet. Miranda has three packets and her grandfather buys her two more packets. How many stickers does Miranda have altogether?

Model 30 on the abacus by showing three rows of ten (or model on the bundled sticks or container materials). Ask the students how to model twenty. Discuss what has been modelled; for example, three packets (of ten) plus two packets (of ten).

Record "3 packets (of 10) + 2 packets (of 10)" on the board or in the modelling book. Below this, record 30 + 20. Discuss what basic fact will help to find the answer. Record the solution to 30 + 20 on the board or in the modelling book.

Examples: Word problems and recording for: 3 packets + 1 packet,   5 packets – 1 packet,
1 packet + 5 packets,   60 – 20,   80 + 10 …

Using imaging

> A packet contains ten lollies. If Matiu has seven packets and Andrew has one packet, how many lollies do Matiu and Andrew have altogether?

Record "7 packets of 10 + 1 packet of 10", and below this, 70 + 10, on the board or in the modelling book. Out of sight of the students, model 70 on the abacus. Ask them what the model looks like. Model 10 more. Ask the students what the model looks like now. Record the solution to 70 + 10 on the board or in the modelling book.

Examples: 5 packets + 3 packets,   5 packets – 1 packet,   4 packets + 2 packets,   10 + 50,   90 – 40,
30 + 20,   50 – 40,   50 + 10,   80 + 10 …

Adding tens

Equipment: The Slavonic abacus; ones and tens material (bundled and loose ice-block sticks); sets of ten beans in film canisters and loose beans; ice-cream containers or other material for shielding. This activity can be repeated using different kinds of material.

Using materials

> Hemi has thirty-nine marbles, and he buys another packet of twenty marbles. How many marbles does Hemi have altogether?

Record 39 + 20 = ☐ on the board or in the modelling book. Get the students to model 39 and then 20 on ones and tens materials. Have the students work out the answer using materials and then discuss the answer. Record the solution to 39 + 20 on the board or in the modelling book.

Examples: Word problems and recording for: 35 + 20, 42 + 10, 20 + 34, 30 + 34, 21 + 50, 40 + 27 …

Using imaging

> Work out 34 + 20.

Record 34 + 20 = ☐ on the board or in the modelling book and ask the students how they would build 34 and 20 separately. Discuss the idea that, because 3 + 2 = 5 and three tens and two tens equals five tens (30 + 20 = 50), 34 + 20 must be 54. (Instant recall of basic facts is needed.) Record the solution to 34 + 20 on the board or in the modelling book.

Examples: Word problems and recording for: 18 + 20, 30 + 24, 23 + 40, 13 + 50, 10 + 46 …

Using number properties

Examples: Word problems and recording for: 87 + 10, 78 + 2, 20 + 62, 80 + 17 …

Subtracting tens

Equipment: The Slavonic abacus; ones and tens material (bundled and loose ice-block sticks, sets of ten beans in film canisters and loose beans, ice-cream containers or other material for shielding). This activity can be repeated using different kinds of material.

Using materials

> Tania has forty-five lollies; and she gives twenty lollies away to her friends. How many lollies does Tania have left?

Record 45 – 20 = ☐ on the board or in the modelling book. Get the students to model forty-five on ones and tens materials. Have the students work out the answer using materials and then discuss the answer. Record the solution to 45 – 20 on the board or in the modelling book.

Examples: Word problems and recording for: 45 – 20, 52 – 10, 42 – 20, 35 – 30, 63 – 50, 48 – 30 …

Using imaging

> Work out 51 – 20.

Record 51 – 20 = ☐ on the board or in the modelling book and ask the students how they would model fifty-one. Then build and hide fifty-one. Ask the students how many tens there are and how many would be left if twenty were removed. Discuss the idea that because 5 – 2 = 3, then five tens take away two tens equals three tens, so 51 – 20 must be 31. (It is here that the instant recall of basic facts is needed.)

Examples: Word problems and recording for: 48 – 40, 53 – 50, 27 – 20, 64 – 10, 43 – 40, 57 – 50, 71 – 40 …

Using number properties

Examples: Word problems and recording for: 97 – 10, 78 – 30, 88 – 20, 90 – 30, 71 – 60 …

Key idea 5: Addition is commutative, so the order of the numbers can be rearranged to make counting on easier

Mathematical knowledge required

Students need to be able to:

- fluently and accurately count on from numbers between one and one hundred.

Diagnostic snapshot

Eva has three stickers. Tasha has six stickers. How many do they have altogether?

Students who answer this by confidently counting on from the larger number are ready to move on, as they understand the commutative principle. If they start counting on from the smaller number, the following learning experience will develop this key idea.

Learning experience

■ The bigger number first

Equipment: Counters, number strip 1–20, hundreds board.

Using materials

Theo has three stickers and Bas has eight stickers. How many stickers are there altogether?

Record 3 + 8 = ☐ on the board or in the modelling book. Ask students to choose one colour of counters to represent Theo's stickers and one colour to represent Bas' stickers. Model placing the three counters on the number strip covering the first three places and then counting on as you add the eight covering the next eight places. Discuss how there are 11 counters altogether and record the answer.

Let's change this problem. Bas has eight stickers and Theo has three stickers. How many stickers are there altogether?

Record 8 + 3 = ☐ on the board or in the modelling book. Ask students for their ideas about the difference between the two equations and how the second one can be solved.
Model placing the eight counters and then counting on as you add the three other counters. The students need to see why counting on from the eight is more efficient than counting on from the three and that the order of the addends does not affect the sum.

Examples: Word problems for 2 + 11,   4 + 7,   3 + 14 ...

Using imaging

The students who know how to count on from the larger number by rearranging the order of the addends can proceed straight to "Using number properties".

Using number properties

Pose a mixture of problems in which the larger number varies in its place in the equation. If there is difficulty with the larger numbers, fold back to "Using materials", working with a hundreds board.

Examples: Word problems and recording for: 2 + 94,   4 + 67,   78 + 5,   5 + 41,   2 + 65,   31 + 3 ...

Challenging examples with more than 2 addends: 67 + 2 + 3,   2 + 88 + 1,   3 + 78 + 4

Moving students from advanced counting to early additive

Introduction

Students who have mastered early additive part–whole thinking will confidently use part–whole strategies that utilise their place-value knowledge of tens. They will use doubles initially, then with and through tens strategies. These strategies are both more sophisticated than the use of doubles or fives strategies and generalise to larger numbers; for example, 9 + 8 = (9 + 1) + 7 and then 889 + 8 = 890 + 7.

Ideas being developed

- Addition and subtraction problems can be solved by part–whole strategies instead of counting.
- Numbers can be partitioned and recombined to make a ten to solve an addition or subtraction problem.
  9 + 5 = 9 + (1 + 4)
  = (9 + 1) + 4
  = 14
- Basic facts are essential when partitioning and recombining numbers.

Learning experiences

Key teaching ideas	Learning experiences	Page
1. Our number system is based on ten.	More ones and tens	38
2. Basic fact knowledge can be used to add and subtract tens.	Adding ones and tens Subtracting ones and tens	38 39
3. Numbers can be rearranged and combined to make ten.	Make ten (working with ten)	40
4. Addition and subtraction problems can be solved by partitioning one of the numbers to go up or back through ten.	Adding in parts (working through ten)	41
5. Subtraction problems can be solved by going back through ten, partitioning numbers rather than counting back.	Subtraction in parts (subtracting back through ten)	42
6. Addition is associative, so addends can be regrouped to solve a problem more efficiently.	Compatible numbers	44
7. Change unknown problems can be solved by using place-value knowledge of tens and ones or by partitioning through tens.	Up over the tens (change unknown working through ten) The missing ones and tens Problems like 37 + ☐ = 79 (change unknown with tens) Problems like 67 – ☐ = 34	45 46 46 47
8. Subtraction can be used to solve difference problems in which two amounts are being compared.	Comparisons: Finding difference in data More comparisons: Comparing heights	48 49
9. Knowledge of doubles can be used to work out problems close to a double.	Near doubles	49
10. The equals sign represents balance.	A balancing act	50

Key idea 1: Our number system is based on ten

Mathematical knowledge required

Students need:

- instant recall of basic facts to 10
- knowledge of "10 and" facts (e.g., 10 + 4).

Diagnostic snapshot

How many ten-dollar notes would I need to make $68?

Students who answer this with confidence and can explain their reasoning are ready to move on to the next key idea. Otherwise, the following learning experience will develop this key idea.

Learning experience

■ More ones and tens

Equipment: $1, $10, and $100 notes (Material Master 4-9); arrow cards (Material Master 4-14).

Using materials

In pairs, one student shows a two-digit number on the arrow cards. The other student reads the number, then models it in place-value money. Students swap roles.

Examples: $93, $47, $82, $99 …

Key idea 2: Basic fact knowledge can be used to add and subtract tens

Mathematical knowledge required

Students need:

- instant recall of basic facts to ten
- knowledge of how a two-digit number is made.

Diagnostic snapshot

Wiremu has fifty-four cards, and he gets thirty-two more. How many cards does Wiremu have now?

Students who use their knowledge of basic facts to say 5 tens and 3 tens are 8 tens, and 4 ones and 2 ones is 6 ones, so 8 tens plus 6 ones is 86 are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Learning experiences

■ Adding ones and tens

The answers should not exceed ninety-nine, and in no problem should the ones add up to ten or more. For example, 56 + 32 is fine, but 56 + 87 and 45 + 28 are not. Problems like the latter two, where the ones add up to ten or more, should be delayed until the students have part–whole thinking strategies.

Equipment: Ones and tens material (bundled and loose ice-block sticks, film canisters with ten beans in each and loose beans, interlocking cubes, the Slavonic abacus, play money, place-value blocks, or tens frames).

Using materials

Start by using the Slavonic abacus to show sixty. Ask: "How many are here, and how do you know? Discuss this with your partner." Ask: "How do the colours on the abacus help us identify the groups of ten?" Try other examples. Have students discuss how they know how many tens there are without needing to skip-count in tens.

> Ray has $54, and he gets $25 for a birthday present. How much money does Ray have now?

Ask: "What expression do I write to record this problem? What operation is needed here?" Have students discuss with a partner and record. After discussion, record 54 + 25 on the board or in the modelling book.

Have the students model fifty-four and twenty-five using the chosen materials and group the ones and tens. Relate the problem to knowledge of basic facts: "How much is five tens and two tens?" "What is another way of saying five tens and two tens?" (50 + 20). "How much is four ones and five ones?" (4 + 5). Discuss the answer and record 54 + 25 = 79 on the board or in the modelling book.

Examples: Word problems and recording for: 45 + 22, 52 + 13, 42 + 25, 35 + 43, 53 + 25, 43 + 22 …

Using imaging

Examples: Word problems and recording for: 14 + 43, 31 + 25, 23 + 41, 24 + 25, 32 + 26, 38 + 21, 13 + 41, 25 + 23, 44 + 24 …

Using number properties

Examples: Word problems and recording for: 87 + 12, 73 + 26, 24 + 52, 16 + 62, 81 + 17 …

Challenging examples: Make connections to the previous money learning experience. The students will need to understand the meaning of three-digit numbers to do these: 241 + 21, 342 + 44, 643 + 21, 27 + 210, 303 + 44, 25 + 510 …

■ Subtracting ones and tens

The ones digit in the beginning number should be greater than or equal to the ones digit in the subtracted number to avoid a renaming problem. For example, 56 – 34 is fine, but 57 – 29 is not. Problems like the latter should be delayed until the students have part–whole thinking strategies.

Equipment: Ones and tens materials.

Using materials

Examples: Word problems and recording for: 45 – 21, 52 – 21, 42 – 32, 35 – 14, 63 – 61, 38 – 28 …

Using imaging

Examples: Instant recall of single-digit subtraction facts is needed here. Word problems and recording for: 46 – 24, 55 – 25, 43 – 12, 37 – 21, 34 – 13, 46 – 41 …

Using number properties

Examples: Word problems and recording for: 97 – 11, 78 – 38, 99 – 62, 56 – 46, 88 – 17 …

Challenging examples: The students will need to understand the meaning of three-digit numbers to do these: 240 – 20, 347 – 47, 443 – 21 …

Key idea 3: Numbers can be rearranged and combined to make ten

Mathematical knowledge required

Students need:

- instant recall of basic facts to ten.

Diagnostic snapshot

What is a really quick way to add up 60 + 30 + 40 + 70?

Students who can tell you that the answer is 200 because they paired up the numbers that add to 100 may be ready to move to the next key idea. Otherwise, the following learning experiences will develop this idea.

Learning experiences

■ Make ten (working with ten)

This learning experience focuses on adding three or more numbers by first making up pairs that add up to ten. This in effect uses the commutative property in a strategic way. For example,
5 + 8 + 5 = 5 + 5 + 8 = 10 + 8 = 18.

Equipment: Counters and blank tens frames.

Using materials

Tina catches six fish, Miriama catches seven fish, and Liam catches four fish. How many do they catch altogether?

Discuss and record 6 + 7 + 4 = ☐. Have the students model this with counters and three blank tens frames. Ask: "How can we add these efficiently? Can anyone see two of these numbers that add to make a full tens frame (ten)?" Get the student who notices to move their four-dot tens frame over so that it sits inside the six-dot tens frame.
"We have discovered that we can add the six and four first to give ten, which makes the answer (seventeen) obvious." Record the answer on the board or in the modelling book: (6 + 4) + 7 = 17.

Examples: Word problems and recording for a "make tens" strategy: 5 + 2 + 5,   9 + 5 + 1,   8 + 3 + 7 + 2, 3 + 5 + 5 + 7,   4 + 6 + 4 + 9 + 6,   3 + 6 + 4 + 9 + 7 ...

Using imaging

Examples: Word problems and recording for a "make tens" strategy: 6 + 2 + 4,   8 + 5 + 2,   8 + 1 + 9 + 2, 1 + 5 + 5 + 9,   4 + 6 + 4 + 9 + 6,   1 + 5 + 5 + 9 + 7 ...

Using number properties

Examples: Record these numbers on paper and cross out pairs that make a ten, or use an arrow to link the pairs: 8 + 6 + 4 + 7 + 2,   7 + 8 + 4 + 3 + 2,   5 + 2 + 3 + 7 + 8 + 5,   1 + 2 + 5 + 7 + 9 + 5 + 3 ...

Challenging examples (for students with strong place-value understanding}: 30 + 60 + 70 + 40,
20 + 70 + 80 + 30 + 40,   40 + 50 + 70 + 60 + 30 ...

Key idea 4: Addition and subtraction problems can be solved by partitioning one of the numbers to go up or back through ten

Mathematical knowledge needed

Students need:

- instant recall of basic facts to ten
- knowledge of ten and basic facts (e.g., 10 + 4).

Diagnostic snapshot

Darryl has forty-eight strawberries and picks seven more. How many strawberries does Darryl now have?

Those students who split the seven confidently to solve the problem as 48 + 2 + 5 are ready to move on to the next key idea of this stage. Otherwise, the following learning experiences will develop the idea.

Learning experiences

■ Adding in parts (working through ten)

Equipment: A magnetic whiteboard with two tens frames drawn on it and magnetised counters or two blank tens frames (Material Master 4–6) and counters; bead strings; student sheets of number lines by tens with empty number lines printed on the back (Material Master 5-12); pegs.

Because this is a key concept at this stage, two different pieces of equipment can be used. A new piece of equipment can be used on a different day to broaden the concept. Some students may get the concept very quickly, while others may benefit from multiple representations. Use the equipment that works best for your students.

Using materials with bead strings

Use a 30-bead string set-up in groups of five, using only two colours (e.g., yellow and red). The bead string should be strung so that beads may be moved along it (in other words, not too tightly). Each student could have a bead string.

Pio has eight oranges and six apples. How much fruit has Pio got altogether?

Record 8 + 6 = ☐ on the board or in the modelling book. Then say: "Show me the eight that Pio has on your bead strings. Pinch the cord in the gap after eight. How do you know you have eight?" Expect answers that link to the colour change shown as five and three.

Then say: "Pio gets six more apples. Hold your finger further along the string to show the six that are added on. Tell your partner what you notice about the colours in your six beads."

"What could we add first?" (two yellow beads). "So, 8 + 2 = 10. What do we have still to add?" (4). "What is 10 + 4 more? How could we record that?"

Record as 8 + 6 = (8 + 2) + 4 = 10 + 4 = 14.

Repeat activity using a number line and pegs.

Using imaging with bead strings

Andrea has seven polished stones, and she gets five more. How many polished stones does Andrea have now?

Say: "I am pinching the beads at seven. What would you see if you could see the bead string?"

Expect answers such as "I would see five red beads and two yellow ones".

"I am now adding the five beads. How many beads will I add first? I want to make a ten. What will I add next? How will we record that?" Invite students to do the recording. Look for recording that shows working through a ten as illustrated above.

Using materials with tens frames

See *Tens Frame: Bridging to Ten Strategy* on the NZ Maths website:
http://nzmaths.co.nz/sites/default/files/Animations/tens.swf

> Peter has eight oranges and six apples. How many fruit has Peter got altogether?

Record $8 + 6 = \square$ on the board or in the modelling book. Model eight on a tens frame and six on another tens frame.
Ask the students, without touching the tens frames, to discuss how they can add the six to the eight.
Invite a student who says that fourteen is the answer to come and demonstrate how they got the answer. Record on the board or in the modelling book:

$$\begin{aligned} 8 + 6 &= (8 + 2) + 4 \\ &= 10 + 4 \\ &= 14 \end{aligned}$$

Examples: Word problems and recording for: 5 + 6, 9 + 7, 8 + 5, 7 + 6, 8 + 7, 4 + 8, 3 + 9 …

Using imaging with tens frames

When using the tens frame, present a problem relevant to your students, such as:

> Tash has eight books and gets given six more. How many books does Tash have now?

Build the numbers on the tens frame behind a shield.
Say: "Tell your partner what the tens frame looks like behind this shield." "Now tell your partner what I am doing as I move things around to make ten. What does the tens frame look like now? How would we record what we just did on the tens frames?"
For those students who do not make the connections, have them fold back to showing the materials to work through the problem.

Using number properties

Examples: Word problems and recording for: 75 + 8, 9 + 48, 6 + 67, 75 + 7, 94 + 7, 9 + 89 …

Key idea 5: Subtraction problems can be solved by going back through ten, partitioning numbers rather than counting back

Mathematical knowledge required

Students need:

- instant recall of basic facts to ten
- knowledge of ten and basic facts (e.g., 10 + 4).

Diagnostic snapshot

> I have thirty-six strawberries, and I eat eight of them. How many strawberries do I have now?

Those students who split the eight confidently and solve the problem as 36 – 6 – 2 are ready to move on to the next key idea. Otherwise, the following learning experience will develop this key idea.

Learning experience

■ Subtraction in parts (subtracting back through ten)

Equipment: A magnetic whiteboard with two tens frames drawn on it and magnetised counters (or use two blank tens frames [Material Master 4-6] and counters); bundled sticks (ones and tens) or beans in film canisters (ones and tens) or a 30-bead string; student sheets of number lines by tens with empty number lines printed on the back (Material Master 5-12).

Using materials

> Rangi has fourteen mussels and eats six of them. How many mussels does Rangi have left?

Record 14 – 6 = ☐ on the board or in the modelling book. Model ten and four counters on the tens frames. Ask the students, without them touching the tens frames, to say what remains when six are removed. Invite a student who says that eight is the answer to come and demonstrate how they got the answer.

Typically, a student will remove four from the four frame and then two from the ten frame to leave eight. Sometimes they will remove six from the ten frame to leave four, and then add four and four to give eight. While this second method is correct, it is not to be encouraged in this learning experience as it does not model going back through ten.

Record 14 – 6 = 8 on the board or in the modelling book. As students become more confident repeat the learning experience with a number line.

Examples: Word problems and recording for: 15 – 6, 21 – 5, 16 – 9, 12 – 4, 23 – 8, 11 – 4, 13 – 6 ...

Using imaging

When using the tens frame, present problems relevant to your students, such as:

> Chris has fifteen books and returns seven to the library. How many books does Chris have now?

Build the numbers on the tens frames behind a shield.

Say: "Tell your partner what the tens frames look like behind this shield." "Now tell your partner what I am doing as I remove counters from one of the tens frames. What will I remove first? What do the tens frames look like now? How would we record this?"

Step one: 15 – 7 = (15 – 5) – 2

Step two: 10 – 2 = 8.

Using materials

> Tara has $34 and spends $5. How much money does Tara have left?

Ask: "How would we record this problem?" Have the students discuss the suggestions.

Record 34 – 5 = ☐ on the board or in the modelling book. Let the students model thirty-four with bundled sticks or beans in film canisters. Look for the students who break a ten to get the answer. Discuss how the students get the answer twenty-nine.

Examples: Word problems and recording for: 45 – 6, 33 – 5, 46 – 7, 32 – 7, 36 – 8, 41 – 4, 33 – 6 ...

Using imaging

> Work out 43 – 5.

Record 43 – 5 = ☐ on the board or in the modelling book. Model forty-three on sticks or beans out of sight of the students. Ask the students to image removing five and describe what is happening. Look for the students who partition the five into three and two, take the three away, and then break a ten to take the final two away, leaving thirty-eight. Fold back, if necessary, to "Using materials". Record 43 – 5 = 38 on the board or in the modelling book.

43 – 5 = (43 – 3) – 2
    ↓
  2 + 3

Examples: Word problems and recording for: 35 – 7, 41 – 9, 26 – 8, 31 – 5, 32 – 8, 31 – 3, 25 – 7 ...

Using number properties

Examples: Word problems and recording for: 75 – 8, 83 – 8, 96 – 9, 61 – 5, 65 – 7, 51 – 6, 81 – 9 ...

Key idea 6: Addition is associative, so addends can be regrouped to solve a problem more efficiently

For example, 5 + 3 + 6 – 8 can be solved by first adding 5 + 3 to get 8, then removing the 8, which then leaves 6.

Mathematical knowledge required

Students need:

- instant recall of basic facts to ten.

Diagnostic snapshot

> Epa had $26. She was given $65 more on her birthday and got $24 from washing cars. She then spent $50. How much did Epa have left?

Look for students who can see that the problem is much easier to solve by regrouping the addends as $26 + $24 = $50 and then realising that $50 – $50 = 0, so Epa has $65 left. The students who can see this may be ready for the next key idea. Otherwise, the following learning experiences will help develop this key idea.

Learning experiences

■ Compatible numbers

Equipment: Counters.

Using materials

> Tina has six tomatoes, Miriama has two tomatoes, and Liam has three tomatoes. They use nine tomatoes for a salad. How many tomatoes are left?

Discuss how to record the problem, then record 6 + 2 + 3 – 9 = □ on the board or in the modelling book. Have the students model piles of six, two, and three counters. Discuss which two piles add to nine and remove them to leave the pile with two counters. Record 6 + 2 + 3 – 9 = 2 on the board or in the modelling book.

Examples: Word problems and recording for: 5 + 2 + 5 – 10,   9 + 5 + 1 – 6,   8 + 2 + 7 – 9,   4 + 5 – 9, 3 + 5 + 5 – 8,   4 + 6 + 4 + 3 – 7 …

Using imaging

Examples: Word problems and recording for: 4 + 2 + 5 – 9,   8 + 5 + 2 – 7,   10 + 2 + 7 – 12,   10 + 5 – 15, 3 + 2 + 6 – 5,   2 + 6 + 4 + 3 – 7 …

Using number properties

Examples: Record these numbers on paper and cross out the pairs and the numbers subtracted to get the answers: 8 + 6 + 4 – 10,   7 + 8 + 2 – 9,   7 + 3 + 3 + 7 – 10 – 10,   1 + 2 + 5 + 2 – 7 – 3, 9 + 3 + 4 – 7 – 2 …

Key idea 7: Change unknown problems can be solved by using place-value knowledge of tens and ones or by partitioning through tens

Mathematical knowledge required

Students need:

- instant recall of basic facts to ten
- knowledge of ten and basic facts (e.g., 10 + 4).

Diagnostic snapshot

I had $17 and I went over to see my Nana on my birthday and I came back with $99. How much money did I get at my Nana's house?

Students who answer this problem confidently are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Learning experiences

■ Up over the tens (change unknown working through ten)

Equipment: Animal strips (Material Master 5-2).

Using materials

A farmer has twenty-eight cows. She buys some more cows at the stock sale. Now she has thirty-three cows. How many cows did the farmer buy?

Record 28 + □ = 33 on the board or in the modelling book. Tell the students to model twenty-eight cows with two ten-animal strips and an eight-animal strip. Tell them you want them to make a complete ten by modelling thirty animals. Get the students to predict what animal strips they will need to complete the problem. Look for the students who recognise that the 2-strip will take them to 30 (28 + 2 = 30), and then the 3-strip will take them to 33 (30 + 3 = 33). Ask how many animals were added on altogether: 2 + 3 = 5.
Record 28 + 5 = 33 on the board or in the modelling book: (28 + 2) + 3 = 33.

Examples: Word problems and recording for: 7 + □ = 13, 26 + □ = 31, 8 + □ = 13, 18 + □ = 23, 25 + □ = 32, 17 + □ = 24 …

Using imaging

I had eighteen fish and I got some more. Now I have twenty-four. How many more fish did I get?

Model eighteen animals on the number line.
Say: "If I want to use the idea of going up and through a ten, which animal strip would I use first? Talk to your partner. Tell your partner what you would do next."
Record 18 + □ = 24 on the board or in the modelling book. Look for students who use a 2 strip to make 20, then a 4 strip to make 24 and recognise that the answer is 2 + 4 = 6. Record 18 + 6 = 24 on the board or in the modelling book: (18 + 2) + 4 = 24.

Examples: Word problems and recording for: 17 + □ = 23, 16 + □ = 25, 28 + □ = 35, 14 + □ = 23, 35 + □ = 41, 23 + □ = 31 …

Using number properties

Examples: Word problems and recording for: 77 + □ = 83, 56 + □ = 65, 68 + □ = 75, 54 + □ = 63, 65 + □ = 71, 22 + □ = 31 …

■ The missing ones and tens

Equipment: Ones and tens materials (e.g., bundled and loose ice-block sticks, ten beans in film canisters and loose beans, interlocking cubes, the Slavonic abacus, play money, place-value blocks, or tens frames); student sheets of number lines by tens with empty number lines printed on the back (Material Master 5-12).

Using materials

> Hinewai is planning a big party for thirty-four friends. Everyone needs a can of drink at the party, but she only has twenty-one cans in her cupboard. How many cans of drink will Hinewai have to buy at the shop?

Ask: "How many does Hinewai need in total? [Thirty-four.] How many cans does Hinewai already have? [Twenty-one.] How can we record that?"
Discuss why this amounts to solving 21 + □ = 34 and record on the board or in the modelling book.
Have the students use ones and tens materials to solve the problem and then discuss their solutions. Record the answer on the board or in the modelling book. As students' understanding develops, repeat the learning experience using a number line.

Examples: Word problems and recording for: 15 + □ = 36, 21 + □ = 36, 20 + □ = 46, 31 + □ = 37, 42 + □ = 52, 34 + □ = 46 ...

Using imaging

Examples: Word problems and recording for: 25 + □ = 46, 31 + □ = 41, 30 + □ = 41, 11 + □ = 18, 42 + □ = 52, 4 + □ = 36 ...

Using number properties

Examples: Word problems and recording for: 25 + □ = 86, 31 + □ = 91, 20 + □ = 85 ...

Challenging examples: The students will need to understand the meaning of three-digit numbers to do these: 241 + □ = 248, 320 + □ = 345, 643 + □ = 647, 99 + □ = 109 ...

■ Problems like 37 + □ = 79 (change unknown with tens)

Equipment: $1 and $10 play money (Material Master 4-9).

Using materials

> Kumar has $37 and is saving to buy a skateboard costing $79. How much does Kumar need to save?

Discuss the problem and record 37 + □ = 79 on the board or in the modelling book. In pairs, have the students solve the problem with play money. As a group, discuss why the answer is 42. For example, three tens and four tens make seven tens (remember the four tens); and 7 + 2 = 9, so the answer is four tens and two ones, which makes forty-two. Record 42 on the board or in the modelling book.

Examples: 27 + □ = 69, 12 + □ = 67, 75 + □ = 95, 61 + □ = 83, □ + 26 = 96, □ + 51 = 82, □ + 36 = 59 ...

Using imaging

> Marama has $62 and needs a total of $89. How much does Marama need to save?

Record 62 + □ = 89 on the board or in the modelling book. Model $62 (or use a number line) out of sight and ask the students to image what to do next. Record the answer on the board or in the modelling book. Fold back to "Using materials" if necessary.

Examples: Word problems and recording for: 37 + □ = 79, 12 + □ = 69, 75 + □ = 95, 61 + □ = 83, 36 = □ = 77, 23 + □ = 49, 68 + □ = 98 ...

Using number properties

Examples: Word problems and recording for: 17 + □ = 99, 4 + □ = 89, 56 + □ = 99, 58 + □ = 88, 48 + □ = 59 ...

■ Problems like 67 – □ = 34

The problem type is change unknown and involves subtraction. The problems should be presented to the students as word problems using the change unknown structure, not as expressions.

Equipment: Ones and tens materials (e.g., bundled and loose ice-block sticks, ten beans in film canisters and loose beans).

Diagnostic snapshot

> Joshua had sixty-seven newspapers in his delivery bag. After he had delivered the papers on Brown Street, he had thirty-four left in his bag. How many papers did Joshua deliver?

Students who use their knowledge of basic facts and place value to solve the problem are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Using materials

> At her party Erena provides sixty-seven biscuits. At the end of the party, she has thirty-four left. How many biscuits did Erena's guests eat?

Record 67 – □ = 34 on the board or in the modelling book. The students model 67 as six tens and seven ones using place-value equipment and experiment to see why 34 will be left. The connection the students need to make is that the problem can be solved by working out 67 – 34 or by solving 34 + □ = 67. Record the answer on the board or in the modelling book.

Examples: Word problems and recording for: 86 – □ = 61, 67 – □ = 21, 63 – □ = 17, 500 – □ = 46 …

Using imaging

> Jeremy had sixty-seven marbles at the start of the game. After playing with his friends he has only thirty-four left. How many marbles did Jeremy lose to his friends?

Record 67 – □ = 34 on the board or in the modelling book. Model 67 using place-value equipment and then shield. Ask the students to image a suitable next step. Record all steps. Remove the shield for those who still need the support of materials.

Examples: Word problems and recording for: 45 – □ = 13, 89 – □ = 37, 55 – □ = 26, 300 – □ = 168

Using number properties

Examples: Word problems and recording for: 84 – □ = 61, 77 – □ = 25, 42 – □ = 26, 200 – □ = 156 …

Key idea 8: Subtraction can be used to solve difference problems in which two amounts are being compared

Up to now, "subtraction" has meant "to take away"; it now has a second meaning – "find the difference". This is a more difficult concept for students to understand and requires students to compare sets to find the difference.

Word problems need to be constructed very carefully to ensure that the context accurately reflects the type of problem being practised. If you are unsure, refer to the problem type examples at the beginning of this book.

Mathematical knowledge required

Students need:

- instant recall of basic facts to ten.

Diagnostic snapshot

> Tui has $78, and Jack has $84. How much more money does Jack have than Tui?

Students who can link the problem to either $78 + □ = $84 or $84 – $78 = □ and then solve it are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

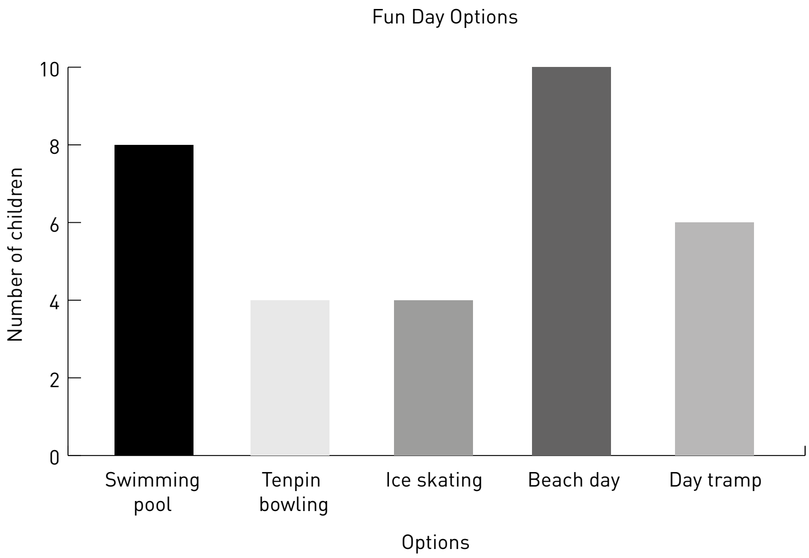
Learning experiences

The New Zealand Curriculum encourages the application of number strategies in meaningful contexts. Statistics and measurement are ideal contexts for exploring the key idea of using subtraction to find the difference. Some examples are given here, but word problems can be created from practical activities done in the classroom.

■ Comparisons: Finding difference in data

Equipment: TrueOrFalseCM4 (available on the NZ Maths website: http://nzmaths.co.nz/sites/default/files/TrueOrFalseCM4.pdf).

Using materials



Say: "Look at the graph above that some students have created. Build their graph with interlocking cubes if you need to, or use this presentation to find the differences between the preferred activities." Here are some questions to ask:

- "How many more students chose the beach than those who chose ice skating?"
- "How many more students chose the swimming pool than those who chose a day tramp?"
- "What is the difference between the number of students who wanted to go ice skating and the number who wanted to go to the swimming pool?"
- "Which activities had the same number of students wanting to do them?"

Examples: Comparison word problems and recording for: 8 + 3, 11 – 2, 7 – 3, 11 – 5, 7 + 3, 4 + 5, 10 – 8 ...

Using imaging

Sam built a tower behind this screen, using two interlocking blocks. Matiu makes a tower six blocks higher than Sam's. I am now building a second tower that is six blocks taller than Sam's tower. How tall is the tower that Matiu built?

Examples: Comparison word problems and recording for: 12 – 4, 7 – 6, 11 – 2, 4 + 5, 2 + 11, 10 – 4 ...

Using number properties

Examples: Comparison word problems and recording for: 28 + 6, 42 – 4, 71 – 6, 31 – 2, 44 + 5, 32 + 11, 60 – 4 ...

■ More comparisons: Comparing heights

Equipment: Counters or interlocking cubes of different colours, set up in groups of five of each colour.

Using materials

Kalila and Holly are measuring the height of the plants they have grown. Kalila's plant is fourteen cubes high, and Holly's is eight cubes high. How much taller is Kalila's plant than Holly's?

In pairs, one student is Kalila and the other Ngaire. They build fourteen and eight units high, with interlocking cubes. Look to see which students make the connection that the problem is solved by working out either $14 - 8$ or $8 + \square = 14$. Record whichever of these arises in the discussion on the board or in the modelling book. Discuss why the answer is six, and record this on the board or in the modelling book.

Examples: Comparison word problems and recording for: $3 + \square = 7$, $5 + \square = 10$, $9 + \square = 13$, $6 + \square = 11$, $\square + 8 = 12$ ...

Using imaging

Kelly has a plant eight cubes high, and Tony has a plant three cubes high. How much taller is Kelly's plant than Tony's?

Shield parallel lines of eight and three interlocking cubes. Ask the students to image the extra ones on Kelly's line and how many there are. Record $3 + 5 = 8$ or $8 - 3 = 5$ on the board or in the modelling book, depending on how the students solve the problem.

Examples: Comparison word problems and recording for: $10 + \square = 17$, $11 + \square = 19$, $12 + \square = 16$, $16 + \square = 20$ ...

Using number properties

Tara's skipping rope is 100 cm long. Lila's skipping rope is 23 cm shorter than Tara's. How long is Lila's skipping rope?

In groups, have the students discuss how to compare the two lengths. Discuss why the answer is found by $100 - 23$. Record $100 - 23 = 77$ on the board or in the modelling book.

Examples: Comparison word problems and recording for: $77 + \square = 81$, $38 + \square = 42$, $56 + \square = 64$, $68 + \square = 77$, $\square + 87 = 95$ ...

Key idea 9: Knowledge of doubles can be used to work out problems close to a double

Mathematical knowledge required

Students need:

- instant recall of doubles to ten.

Diagnostic snapshot

I had $448, and I was paid $452. How much did I have in total?

Students who answer this with confidence and can explain their reasoning using knowledge of doubles may be ready to move on to the next key idea. Otherwise, the following learning experience will develop this key idea.

Learning experience

■ Near doubles

Equipment: Play money (Material Master 4-9).

Using materials

Work out 153 + 147.

Record 153 + 147 = □ on the board or in the modelling book. Show $153 and $147 with play money. Ask the students to talk to their partners about what they could do to make this problem easier to solve. Share ideas from the group. Discuss the fact that transferring $3 gives $150 and $150, so the answer is $300. Record the answer on the board or in the modelling book.

To work out 79 + 79, Harry works out $80 + $80 with play money first and then says: "79 + 79 must be 158."

Discuss Harry's method. Record the answer on the board or in the modelling book.

Examples: Word problems and recording for: 149 + 149, 352 + 348, 350 + 352, 104 + 96 ...

Using imaging

Examples: Word problems and recording for: 440 + 439, 248 + 252, 349 + 350, 202 + 198 ...

Using number properties

Examples: Word problems and recording for: 150 + 149, 448 + 452, 47 + 53, 502 + 498 ...

Key idea 10: The equals sign represents balance

In other words, the expression on the left-hand side of the equals sign represents the same quantity as the expression on the right-hand side of the equals sign.

Mathematical knowledge needed

The students need:

- instant recall of basic facts to ten.

Diagnostic snapshot

Jack and Matiu have the same amount of money. Jack has $105 in the bank plus some in his pocket to put in the bank. Matiu has $103 in the bank and $43 in his pocket. How much has Jack got in his pocket? What equation would we write to show this problem?

Students who can record the problem as 105 + □ = 103 + 43 and then solve it confidently are ready to move on to the next key idea. Otherwise, the following learning experience will develop this key idea.

Learning experience

■ A balancing act

Many students think the = sign means "get the answer". This is, after all, what "equals" means on a calculator. Here the equals sign is extended from the current meaning of "find the answer" to include: "The left-hand side (of the equals sign) equals (balances) the right-hand side."

Equipment: Counters and tens frames and paper to represent bags. Alternative materials are balances and weights, double-sided counters, Cuisenaire rods, or animal strips.
Because this is an important concept, a number of pieces of equipment can be used. Choose the equipment that works best for the students and is most readily available.

Using materials (Cuisenaire rods or animal strips)

Use the 1 (white), 2 (red), 3 (lime green), 4 (crimson), 5 (yellow), 6 (dark green), 7 (black), 8 (brown), 9 (blue), and 10 (orange) rods. Choose one of the large Cuisenaire rods per pair of students. As an example, they may take the 10 (orange rod) and put several smaller rods underneath it. They then record the number sentence that matches what they have done.

If they have placed a 5 (yellow), a 3 (lime green), and a 2 (red), they would record 10 = 5 + 3 + 2 and read it as "Ten is the same as five plus three plus two."
Have them explore other combinations that are equal to ten. They then remove one of the small rods and show it to another pair. The other pair records what they can see. If they removed the 2 rod from the previous example, they would write 10 = 5 + 3 + □. The pair would then look at the number sentence and identify the missing number needed to balance the equation.

Using imaging (with Cuisenaire rods or animal strips)

> Behind this card, I am using a pair of rods. I will choose a 4 rod and a 6 rod. I am now picking up a 3 rod to go underneath the 4 and the 6 rod. What rod will I need to select next so that the total length of the two pairs of rods will be the same? How could we record that? (4 + 6 = 3 + □) Read this as "four plus six is the same as three plus [seven]."

Using materials (with scale balances)

A scale balance can also be used to show equality. If this is not available, use a coat hanger and string bags, or yoghurt pottles with handles tied to each end. Marbles can be used in the pottles.

Examples: 4 + 3 = 5 + □, 5 + 1 = 4 + □, 6 + 1 = 5 + □, 5 + 2 = 4 + □ …

Using imaging (with scale balance)

Alternatively, an image of a scale balance can be used and a different combination of blocks placed on each side to build number sentences that balance.
Activity AL 7111 from the Assessment Resource Bank supports this idea – see the ARB web page at: http://arb.nzcer.org.nz/supportmaterials/maths/concept_map_algebraic.php#equality

Using materials (double-sided counters)

Have the students work in pairs. Both get eight double-sided counters. They check to agree that they are both starting with the same number of counters. Partner B takes some of their counters and hides them under a piece of card. Partner A throws their counters on the floor and records what they see (e.g., "I have five red and three yellow counters"). Partner B places the counters that are still in their hand down on the floor, all with the red side showing, and says: "I had four red counters in my hand. How many yellow counters must be hiding?"
They discuss how they could record this. After discussion with the teacher, they record 5 + 3 = 4 + □. Partner B produces their counters from under the card and checks that what they have is represented by what they have recorded: 5 + 3 = 4 + 4. They read: "five plus three is equal to four plus four".
Repeat with other amounts.

Using imaging (with double-sided counters)

> I have seven counters in my hand. I throw them down behind this card. Four are red, and three are yellow. Record the equation that shows this. Now I throw another seven counters, and this time five are red. How many must be yellow? How do we record this? (4 + 3 = 5 + □)

Examples: Word problems and recording for: 2 + 4 = □ + 3, 5 + 5 = □ + 6, 5 + □ = 4 + 3, □ + 6 = 6 + 3, □ + 6 = 8 + 2, 1 + 7 = 4 + □ …

Using number properties

> Work out 67 + □ = 66 + 44.

Discuss why the answer is one less than 44 (because 67 is one more than 66 and balance has to be maintained). Record 43 in the square.

Examples: Word problems and recording for: 42 + 38 = □ + 39, 55 + 35 = □ + 34, 105 + □ = 103 + 43, □ + 65 = 67 + 33, □ + 180 = 190 + 38, 1 020 + □ = 1 010 + 67 …

Challenging examples: 442 – 38 = □ – 39, 585 – 35 = □ – 34, 105 – 56 = 103 – □, □ – 65 = 667 – 62, □ – 280 = 790 – 290 …

Moving students from early additive to advanced additive

Introduction

The learning experiences in this section build on the early additive mental strategies that use place value, partitioning or splitting numbers into tens and ones, and rounding and compensation. Students who are transitioning into this stage will already have an understanding of how these mental strategies work. The transition to this level of thinking provides them with the opportunity to make connections or generalise their use of strategies when working with larger numbers. Students will be able to compare strategies in order to select the most efficient method and justify their reasoning. For example, 995 + 76 would be best solved by using a rounding and compensating strategy rather than a written method or a place-value strategy.

It is important to give students the opportunity at the start of each new learning experience to solve the problem their way. Students can be selected to share their solution, while others in the group can say if they have solved the problem in a similar way. Students will then see that a variety of mental strategies can be used.

This initial sharing is also a valuable formative assessment opportunity to see which strategies a particular group of students are most confident with and who is confident with the strategy that best fits the focus of the learning experience.

Ideas being developed

- Addition and subtraction of whole numbers require partitioning and recombining.
- The mental strategies used to add and subtract multi-digit numbers are predominantly based on either a standard partition into place-value units, known as a place-value strategy, or using tidy numbers and a rounding and compensation strategy.
- The fact that addition and subtraction are inversely related can be used to solve problems efficiently. For example, $6001 - 5998 = \square$ can be solved as $5998 + \square = 6001$, and $3 + \square = 6001$ can be solved as $6001 - 3 = \square$.

Learning experiences

Key teaching ideas	Learning experiences	Page
1. Introduction to using the number line to solve change unknown problems.	Jumping on the number line	54
2. 10 tens make one hundred and 10 hundreds make one thousand.	How many ten-dollar notes? How many tens and hundreds?	55 56
3. Solve addition and subtraction problems using place value.	Addition and subtraction on the number line Problems like □ + 29 = 81	56 57
4. Solve addition and subtraction problems by using rounding and compensating	When one number is near one hundred Problems like 73 – 19 = □ Problems like 23 + □ = 71 Problems like □ + 29 = 81	58 59 60 60
5. Addition and subtraction are inversely related.	Don't subtract – add!	61
6. Solve subtraction problems with the mental strategy of equal adjustments.	Equal additions	62
7. Choosing wisely.	Mixing the methods – mental exercises for the day Some problem sets	63 63
8. Using the standard written form to solve addition and subtraction problems.	A standard written form for addition Decomposition – a written form for subtraction Large numbers roll over Mental or written?	64 65 66 66

Note: While students are encouraged to solve the problem in any way they select, an identified strategy is highlighted within each key idea as the most efficient. Once you have identified the students who have or have not solved the problem using a particular strategy, you can decide which students need to stay with the teaching group for further teaching and scaffolding and which students can work independently.

Key idea 1: Introduction to using the number line to solve change unknown problems

As this is the first time students are formally introduced to the number line, all students should work through the problems to gain an understanding of its usefulness. They will also need to become familiar with the associated recording conventions.

Mathematical knowledge required

Students need to be able to:

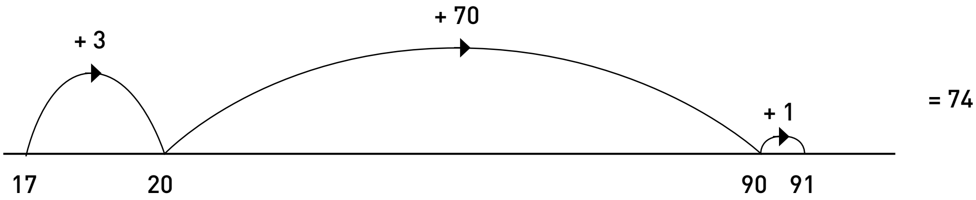
- make up to a ten (e.g., 33 + □ = 40)
- solve 30 + □ = 70 through 30 + 40 = 70, or three tens plus four tens equals seven tens, but not as increments of ten (i.e., 30 → 40 → 50 → 60 → 70).

Learning experience

■ Jumping on the number line

Equipment: A large class number line (Material Master 4-8); student sheets of number lines by tens with empty number lines printed on the back (Material Master 5-12).

Students are introduced to the number line and use it to solve a change unknown problem such as 17 + □ = 91. The calculation involves the use of "tidy" numbers. Tidy numbers usually end in a zero (multiples of ten and a hundred, and so on) and are useful numbers to jump to on a number line when solving addition and subtraction problems.



In the "Using materials" phase of the learning experience, the number line is a tool for the students to work with. They use a line that has the numbers from 0 to 100 marked on it. The jumps are recorded, and the answer is calculated.

In the "Using imaging" phase, the number lines used only have decades marked on them, or they are empty lines with no numbers. Here the number line becomes an image for the students to think *with* rather than a tool to work *on*. They solve the problem in their head and then record their thinking on the line.

Using materials

> Freddo the Frog sits at 28 on the number line. He wants to visit his friend at 81. How far does he have to jump to get there?

Record 28 + □ = 81 on the board or in the modelling book. Suggest that Freddo will first jump to 30 because it is a tidy number. Show this jump with an arrow, and ring the jump of two. Discuss how far Freddo has to go. Some students will jump by tens to 80 and then go one more. Some will jump fifty then one more, and a few will jump fifty-one directly to 81. Show these jumps with arrows and ring the numbers.

In all cases, focus attention on how the ringed numbers always give the answer fifty-three. Discuss which way is best. Have the students do individual work while you observe their methods.

Examples: Give the students the first sheet from Material Master 5-12 and get them to write the following seven problems down against each number line: 39 + □ = 61,   48 + □ = 81,   57 + □ = 85, 29 + □ = 78,   18 + □ = 60,   27 + □ = 93,   36 + □ = 90.

Have the students do the problems and then discuss the answers as a whole group.

Using imaging

> Freddo the Frog jumps from 18 to 73 on the number line. How far is this?

Discuss recording this as 18 + □ = 73. Draw a large, empty number line on the board or in the modelling book and discuss where to place the 18 and 73. Without adding 30, 40, 50, 60, and 70 to the empty number line, discuss how Freddo could jump from 18 to 73 in only two steps. Record the answer on the board or in the modelling book.

Examples: Get the students to turn over their sheet to use the seven empty number lines. Get them to write the following seven problems down against the number lines: 29 + □ = 62,   58 + □ = 93, 27 + □ = 86,   29 + □ = 78,   48 + □ = 70,   29 + □ = 83,   46 + □ = 83.

Using number properties

Examples: Word problems and recording for: 19 + □ = 62,   36 + □ = 94,   58 + □ = 84,   39 + □ = 75, 37 + □ = 75,   25 + □ = 81 …

Key idea 2: 10 tens make one hundred, and 10 hundreds make one thousand

Mathematical knowledge required

Students need:

- knowledge of the ten times table.

Diagnostic snapshot

Tickets to the zoo cost $10 per child. Mrs Jones has $855. How many children can Mrs Jones take to the zoo?

Ask the students how they solved the problem. Look for the students who understand that ten $10 equals $100. If a student's response is to say they removed a zero, ask for another solution. Those students who understand that there are 80 tens in eight hundred may be ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Learning experiences

■ How many ten-dollar notes?

In a base ten numeration system, ten of any unit is equivalent to the next larger unit. For example, ten tens equal one hundred, and ten hundreds equal one thousand. The same quantity can be represented with a different unit. For example, 240 can be viewed as two hundreds and four tens, twenty-four tens, or two-hundred-and-forty ones.

Through composing and decomposing numbers into larger or smaller units, students develop an understanding of the value of each unit as well as how it ranks in size in comparison to the other units.

Equipment: Play money $1, $10, $100 (Material Master 4-9).

Using materials

Mrs Jones takes her class to the circus. She has $237 to pay for admission. Admission is $10 per person. She has twenty-five students in her class. Does she have enough money?

Record 237 on the board or in the modelling book and discuss the meaning of the digits. Have the students solve the problem in groups with play money. Ask: "How many tens are there in this number?" "How many tens are needed for the twenty-five students?" "Is there enough money?" Discuss the answer (no) and record it on the board or in the modelling book.

Examples: Word problems and recording for: $167 for 13 students, $203 for 41 students, $203 for 21 students, $199 for 18 students, $167 for 17 students …

Using imaging

Boxes of chocolates cost $10 each. Can Charlotte buy a box for everyone if she has: $125 for 12 students, $283 for 29 students, $200 for 19 students, $314 for 31 students, $438 for 45 students …?

Using number properties

Examples: Word problems and recording for: $867, $701, $327, $991, $563 …

■ How many tens and hundreds?

Equipment: Play money $100, $1,000 (Material Master 4–9).

Using materials

The Bank of Mathematics has run out of $1,000 notes. Alison needs to withdraw $2,315. How many $100 notes does Alison get?

Record $2,315 on the board or in the modelling book and discuss the meaning of the digits. Have the students solve the problem in groups with play money. Ask: "How many hundreds could you get by exchanging the thousands?" Discuss which notes are irrelevant (the tens and the ones). Ask: "How many $100 notes will Alison get from the bank altogether?"
Discuss the answer and record it on the board or in the modelling book.

Examples: Word problems and recording for: $2,601, $3,190, $1,555, $1,209, $2,001, $1,222, $2,081 ...

Using imaging

Tickets to a concert cost $100 each. How many tickets can you buy if you have $3,578?

Record $3,578 on the board or in the modelling book. Shield three one-thousands, five one-hundreds, seven tens, and five ones. Ask the students to describe their image of the money. Discuss how many $100 notes they could get by exchanging the thousands. Discuss which notes are irrelevant (the tens and the ones). Record the answer on the board or in the modelling book.

Examples: Find and record the number of hundreds in: $1,608, $2,897, $2,782, $3,519, $3,091, $4,000 ...

Using number properties

Examples: Find and record the number of hundreds in: 3 459,   8 012,   9 090,   6 088,   3 280,   5 823,   7 721,   2 083 ...

Key idea 3: Solve addition and subtraction problems using place value

Mathematical knowledge required

Students need:

- knowledge of addition and subtraction facts.

Diagnostic snapshot

Sione had $56 and got another $37 for his birthday. How much money did Sione have altogether?

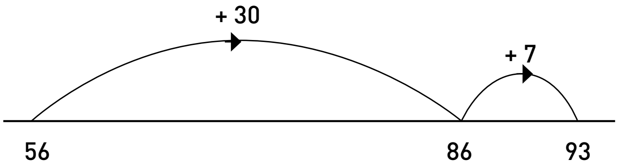
Students who answer this problem confidently using a place-value strategy are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Learning experiences

■ Addition and subtraction on the number line

Equipment: A large class number line (Material Master 4-8); student sheets of number lines by tens with empty number lines printed on the back (Material Master 5-12).
Students need to develop an understanding of place-value mental strategies by using equipment that allows for grouping as well as a continuous model such as a number line. The number line model preserves the magnitude of a number and shows where it fits among other numbers. It also provides a strong image for students to work with and is particularly useful for solving subtraction problems.

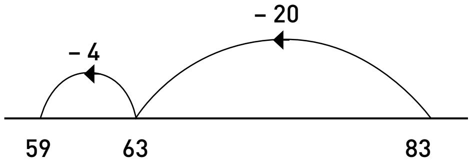
Addition and subtraction on a number line involve keeping the first number whole and partitioning the second. For example, 56 + 37 = □ can be solved in the following way:



Likewise, on a number line, 83 – 24 = □ can be shown as:

Step 1: 83 – 20 = 63

Step 2: 63 – 4 = 59

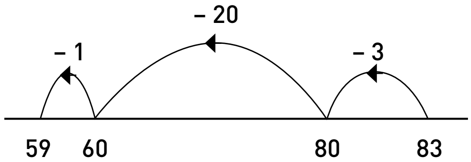


Alternatively:

Step 1: 83 – 3 = 80

Step 2: 80 – 20 = 60

Step 3: 60 – 1 = 59



Using materials

> On sports day, there were fifty-seven senior students and forty-five junior students competing in different events. How many students were competing altogether at the sports day?

Have the students solve the problem by using empty number lines. Ask: "How might you show this problem on a number line?" Record strategies on the board or in the modelling book.

Examples: Word problems and recording for: 68 + 26, 37 + 24, 158 + 46, 72 – 34, 143 – 26, 251 – 33 ...

Using imaging

Students rely on the number line image to support their thinking.

Examples: Word problems and recording for: 85 – 56, 78 + 27, 86 + 47, 42 – 25 ...

Using number properties

Students can jot down a number to support short-term memory.

Examples: Word problems and recording for: 528 – 86, 728 + 34, 456 + 127, 563 – 125 ...

■ Problems like □ + 29 = 81

This learning experience involves solving a start unknown problem. The student develops the understanding that the problem can be solved by changing the order of the addends (commutative property): □ + 29 = 81 to 29 + □ = 81. (If they do not understand this idea initially, work with smaller numbers.) Problems should be presented as word problems using the start unknown structure, not as equations. The students can then solve them using a place-value strategy.

Using materials

Leyla has some cows. She buys thirty-eight more cows. Now she has seventy-two cows. How many cows did Leyla have to start with?

Here the unknown number comes first. Discuss why this problem is recorded as □ + 38 = 72. Discuss how reversing the problem to 38 + □ = 72 makes it easy to solve by using a place-value strategy on a number line. Record the answer on the board or in the modelling book.

Examples: Word problems and recording for: □ + 33 = 83, □ + 18 = 43, □ + 9 = 62, □ + 23 = 72, □ + 21 = 88, □ + 25 = 70 ...

Using imaging and number properties

Examples: Word problems and recording for: □ + 32 = 71, □ + 16 = 43, □ + 45 = 62, □ + 43 = 92, □ + 31 = 85, □ + 55 = 72 ...

Key idea 4: Solve addition and subtraction problems by using rounding and compensating

Mathematical knowledge required

Students need to be able to:

- add a two-digit number to a multiple of a hundred (e.g., 400 + 62 = 462)
- subtract a multiple of ten or a hundred from a number (e.g., 833 – 100 = 733).

Diagnostic snapshot

On Monday 1 008 newspapers were sold, and on Tuesday, another 996 newspapers were sold. How many newspapers were sold altogether?

If the student solves the problem using the steps shown below, they are ready to move to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Step 1: 1 008 + 1 000 = 2 008 (they have rounded to 996 to 1 000 by adding 4)

Step 2: 2 008 – 4 = 2 004 (they have compensated by subtracting the 4)

The rounding and compensating strategy makes a calculation easier by rounding a number in the problem to a tidy number and then compensating by adding and/or subtracting the amount the number was rounded by, as shown in the example above. This strategy is appropriate to use when the problem has a number that is close to a multiple of ten, one hundred, one thousand, and so on. Students need to be on the lookout for such opportunities in problems rather than always relying on using a place-value strategy.

Learning experiences

■ When one number is near one hundred

Equipment: A large class number line (Material Master 4-8); student sheets of number lines by tens with empty number lines printed on the back (Material Master 5-12); $1, $10, $100 play money (Material Master 4-9).

Using materials

Work out $99 + $78.

Record 99 + 78 on the board or in the modelling book. Draw an empty number line on the board. Show on the number line why 100 + 78 is one too many. Record the answer on the board or in the modelling book.

Examples: Draw empty number lines and work out: 56 + 99, 67 + 9, 98 + 63, 38 + 298, 123 – 99, 456 – 98 ...

Using imaging

Work out $175 – $99.

Encourage the students to image the number line.

Examples: Word problems and recording for: 141 – 98, 55 + 98, 97 + 86, 18 + 298, 603 – 198, 299 + 456 ...

Using number properties

Examples: Word problems and recording for: 834 – 99, 456 – 98, 297 + 198, 818 + 698, 1 000 – 797, 1 200 – 998 ...

■ Problems like 73 – 19 = □

Present the problems in a story context that involves paying for something at a shop when you don't have the exact value in notes. Look for students who recognise that they need to pay a little bit more (rounding), but then receive a little bit back in change (to compensate).

Equipment: Place-value money is a powerful piece of equipment for this type of problem.

Using materials

Anne has $72 in her wallet. She buys a new DVD for $39. How much money does Anne have left in her wallet?

Provide the students with access to $1, $10, and $100 notes. In pairs, get students to act out the shopping experience. One student in each pair is the shopper with seven $10 notes and two $1 notes, while the other is the shopkeeper. The role play should show why the $39 is rounded to $40 and why the shopkeeper returns $1 in change.

Examples: Use place-value money and the context of shopping for the following examples: 72 – 59, 98 – 79, 45 – 18, 66 – 48, 72 – 17, 103 – 88 ...

Using imaging

Chandra has $83 in her wallet. She buys a new computer game for $58. How much money does Chandra have left?

Students image $83 in place-value money. They image Chandra using six of her ten-dollar notes to pay the shopkeeper, leaving $23 in her wallet. They then image Chandra receiving two dollars back as change, which when added to the $23 leaves $25 in her wallet. If this proves too difficult, fold back to "Using materials" by physically doing the actions with the money. Record the answer on the board or in the modelling book.

Examples: Word problems and recording for: 82 – 29, 68 – 19, 65 – 28, 96 – 28, 182 – 47 ...

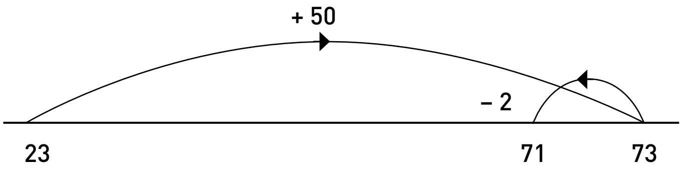
Using number properties

Examples: Word problems and recording for: 123 – 18, 228 – 19, 91 – 28, 596 – 89, 312 – 9, 991 – 87 ...

■ Problems like 23 + □ = 71

Equipment: A large class number line (Material Master 4-8); student sheets of number lines by tens with empty number lines printed on the back (Material Master 5-12).

This is a change unknown problem that requires students to use their understanding of place value and basic facts to make a tidy number jump on the number line. Because the jump goes beyond the targeted number, students are then required to make compensation.



Step 1: 23 + 50 = 73

Step 2: 73 – 2 = 71

Step 3: 50 – 2 = 48, so 23 + 48 = 71

This is a new use of the number line. All students need to learn how to jump a tidy number and then compensate by subtracting a small amount.

Using materials

> Rapata had thirty-four trading cards. He was given some more for his birthday, and then he had sixty-two trading cards. How many cards did Rapata get for his birthday?

Record 34 + □ = 62 on the board or in the modelling book. On the class number line, make a jump of 30, from 34, to 64. Discuss why a jump of two back is required and why the answer is 28. Record 34 + 28 = 62 on the board or in the modelling book.

Examples: Solve these problems: 24 + □ = 82, 52 + □ = 90, 25 + □ = 94, 13 + □ = 71, 12 + □ = 91, 54 + □ = 72, 45 + □ = 83, 14 + □ = 83 ...

Using imaging and using number properties

Students use the number line as a tool to think with, not to record on.

Examples: Word problems and recording for: 24 + □ = 82, 55 + □ = 93, 24 + □ = 92, 12 + □ = 61, 82 + □ = 101 ...

■ Problems like □ + 29 = 81

This learning experience involves solving a start unknown problem. The student develops the understanding that the problem can be solved by changing the order of the addends (commutative property): □ + 29 = 81 to 29 + □ = 81. (If they do not understand this idea initially, work with smaller numbers.) Problems should be presented as word problems using the start unknown structure, not as equations. The students should solve them using an appropriate mental strategy.

Equipment: None.

Using number properties

> Taine has some cows. Taine buys twenty-nine more cows. Now he has eighty-one cows. How many cows did Taine have before?

Here the unknown number comes first. Discuss why this may be written as □ + 29 = 81. Discuss reversing the problem to 29 + □ = 81 and solve it. Students can solve these problems by choosing wisely from the different methods they have learned.

Examples: Word problems and recording for: □ + 38 = 72, □ + 33 = 83, □ + 18 = 43, □ + 9 = 62, □ + 23 = 72, □ + 21 = 88, □ + 25 = 70 ...

Key idea 5: Addition and subtraction are inversely related

Students develop the understanding that problems such as $34 + \square = 51$ and $51 - 34 = \square$ have the same answer. This means that being able to use addition to solve a subtraction problem can provide a better and easier alternative.

Mathematical knowledge required

Students need:

- instant recall of addition facts to ten.

Diagnostic snapshot

Aziz has $112. He buys a skateboard for $95. How much money does Aziz have left?

Ask the students to record the steps they took to solve the problem. Students who solve the problem by adding up from 95 are ready to move on to the next key idea. Otherwise, the following learning experience will develop this key idea.

Learning experience

■ Don't subtract – add!

Equipment: Place-value money $1 and $10 (Material Master 4-9).

Using materials

Moana has $63 and gives her brother Henare $59. How much money does Moana have left?

The students pair off. One student acts as Moana. This student gets $63 as six tens and three ones. The other student is to be paid $59. They discuss how this can be done. Usually, Moana gives Henare five tens, and then realises that she must swap the remaining ten-dollar note for 10 ones. Have a group discussion. Discuss the solutions to $63 - 59$ and $59 + \square = 63$. Ask: "Why are both answers the same?" Record the answer on the board or in the modelling book.

Examples: Word problems and recording for: 80 – 78, 83 – 77, 63 – 57, 33 – 26, 50 – 49, 23 – 18 …

Using imaging

Mike has $92, and he spends $87. How much does Mike have now?

Record $92 - 87 = \square$ on the board or in the modelling book. Shield $87 and ask the group of students how much more money is required to make $92. Discuss why you have added $3 to $87 to make $90 and then added $2 more to make $92, giving the answer $5. Discuss the link to 92 – 87. Fold back to "Using materials" if necessary. Record the answer on the board or in the modelling book.

Using number properties

Sefina has $73, and she spends $28. How much money does Sefina have left?

Discuss why this problem is the same as $28 + \square = 73$.

Examples: Word problems and recording for: 81 – 18, 75 – 39, 93 – 57, 103 – 88, 110 – 89, 64 – 38, 243 – 226, 345 – 309, 871 – 818 …

Key idea 6: Solve subtraction problems with the mental strategy of equal adjustments

The equal adjustments (equal additions) strategy involves solving a subtraction problem by adding the same amount to both numbers. For a problem such as 445 – 398, the fact that 398 is very close to a tidy number (400) suggests that a useful way of solving it is by equal additions – in this case, of 2.
The problem then becomes 447 – 400, and the answer is obviously 47.

Mathematical knowledge required

Students need:

- knowledge of addition and subtraction facts to ten.

Diagnostic snapshot

> Epa has $445 in her bank account, and her younger brother, Rangi, has $398. How much more money does Epa have?

Students who answer this problem confidently by making an equal adjustment (adding 2 to both numbers) and then solve the problem easily as 447 – 400 are ready to move on to the next key idea. Otherwise, the following learning experience will develop this key idea.

Learning experience

■ Equal additions

Equipment: Place-value money (Material Master 4-9).

Using materials

> Christine has $367 in her bank account, and her younger sister, Julie, has $299. How much more money does Christine have?

Have the students make piles of $367 and $299. Ask: "Now, suppose that their Nana gives Julie $1 to give her a tidy amount of money. To be fair, Nana has to give Christine $1 as well."
Discuss why 367 – 299 has the same answer as 368 – 300. Record 367 – 299 = 68 on the board or in the modelling book.

Examples: Word problems and recording for: 367 – 299, 546 – 497, 662 – 596, 761 – 596, 334 – 95, 567 – 296 …

Using imaging

> Brian has $483 in his bank account, and his younger brother, Tom, has $397. How much more money does Brian have?

Students image piles of $483 and $397. Ask: "How much money would Nana give to both boys to make this an easy subtraction problem?" Discuss why 483 – 397 has the same answer as 486 – 400. Record 483 – 397 = 86 on the board or in the modelling book.

Examples: Word problems and recording for: 257 – 199, 676 – 498, 562 – 496, 763 – 396, 434 – 195, 762 – 598 …

Using number properties

Examples: Word problems and recording for: 900 – 298, 701 – 399, 760 – 96, 905 – 96, 507 – 296, 865 – 590, 1 000 – 396 …

Key idea 7: Choosing wisely

Learning experiences

Mixing the methods – mental exercises for the day

At this stage, offering students a regular daily dose of mental calculation is strongly recommended. It is a good idea to record only one problem on the board or in the modelling book at a time and not allow the students to use pencil and paper. Make sure they all have time to solve each problem. Don't allow early finishers to call out the answer.

For example, take the problem 73 – 29 = □. Prompt the students to look carefully at the numbers before deciding how they might solve this. The following are possible strategies:

- Equal adjustments: solved by adding 1 to both numbers, so 74 – 30 = 44.
- Rounding and compensating: 73 – 29 becomes 73 – 30 = 43, then 43 + 1 = 44.
- Reversibility: adding up from 29 to 73, so 1+ 40 + 3 = 44.
- Place value: partitioning the 29, so 73 – 20 = 53 → 53 – 3 = 50 → 50 – 6 = 44.

When recording the strategies the students selected to use, ask: "What is it about the numbers that made you choose the strategy you used?" and "Which of these strategies is the most efficient? Why?"

If the students have used place-value strategies to solve the problem, they may need to revise equal adjustments and rounding and compensating.

As well as presenting problems for the students to solve as equations, it is also important to present them as word problems – for example: "The children had to blow up 182 balloons to decorate the school hall. By playtime, they had blown up 26. How many more did they still need to blow up?"

Some problem sets

Record the problems on the board or in the modelling book in the horizontal form.

Set 1:

45 + 58	67 + □ = 121	8 001 – 7 998	26 + □ = 52
81 – 67	456 + 144	789 – 85	□ + 58 = 189
33 + 809 + 67 + 91			

Set 2:

28 + 72	191 + □ = 210	7 001 – 21	39 + □ = 77
234 – 99	6 091 + 109	2 782 – 15	□ + 123 = 149
616 + 407 – 16 + 93			

Set 3:

999 + 702	287 + □ = 400	2 067 – 999	45 + □ = 91
771 – 37	316 + 684	709 – 70	□ + 88 = 200
7 898 – 6 000 – 98 – 100			

Set 4:

38 + 128	14 + □ = 101	9 000 – 8 985	102 – □ = 34
800 – 33	78 + 124	4 444 – 145	□ + 8 = 1 003
4 700 – 498 + 200 –2			

Set 5:

405 + 58	880 + □ = 921	8 789 – 7 678	80 – □ = 41
701 – 96	8 888 + 122	781 – 45	□ + 48 = 789
6 000 – 979 – 11 – 10			

Key idea 8: Using the standard written form to solve addition and subtraction problems

Using the written form requires the student to move flexibly between the quantity value of a number and its place value (e.g., 120 is twelve tens; 14 is fourteen ones and one ten and four ones).
The conventions of the written form require teaching. Numbers are recorded vertically, with the hundreds, tens, and ones lined up in columns. The calculation starts from the right, and if renaming is required, the new unit is recorded above the number with the same unit value (e.g., when ten ones have been renamed as one ten, the "1" is recorded above the tens).
Written forms are needed when numbers get too large to handle mentally. However, it is important that students can also estimate the answer. Students who have efficient part–whole strategies will find this straightforward.

Mathematical knowledge required

Students need:

- instant recall of basic addition facts to twenty
- knowledge that 10 tens = 100, 10 hundreds = 1 000, and 10 thousands = 10 000.

As the following two learning experiences introduce students to an extended way of recording the standard written form, it is important that all students work through both methods.

Learning experiences

■ A standard written form for addition

Equipment: Place-value money; pencil and paper.

Using materials

Work out 235 + 386.

Each step in solving and recording the problem needs to be connected to the actions and exchanges taking place with the money. Record the following on the board or in the modelling book:

				10		100 10		
235	→	200 + 30 + 5	→	200 + 30 + 5	→	200 + 30 + 5		
+ 386	→	300 + 80 + 6	→	300 + 80 + 6	→	300 + 80 + 6	→	621
				1		600 + 20 + 1		

Ensure place-value language is used (e.g., "Five ones plus six ones equals eleven ones, and ten ones are exchanged for one ten"). Make the exchange of ten of a smaller unit for one of a larger unit explicit through the use of equipment. Connect the actions and exchanges of the money to the recording of the renamed unit in the new column.

Examples: 234 + 478,   459 + 328,   308 + 536,   889 + 67…

Using imaging

Work out 235 + 487 using the extended written form demonstrated above.

Examples: 484 + 468,   279 + 326,   508 + 536,   89 + 557,   367 + 902,   78 + 970 ...

Using number properties

Examples: Find the answers by using the standard written forms, e.g., 235 + 487:

$$\begin{array}{r} {\scriptstyle 11} \\ 235 \\ +\ 487 \\ \hline 722 \\ \hline \end{array}$$

Using place value self-talk, discuss how five and seven produce two ones and one ten, and twelve tens produce two tens and one hundred.

Examples: 404 + 478,   4 079 + 2 327,   588 + 4 536,   59 + 4 556,   3 268 + 8 902,   78 + 970 ...

■ Decomposition – a written form for subtraction

The decomposition of a larger unit into a smaller unit needs to be made explicit with equipment and developed alongside the written recording. The process of exchanging to create new units is referred to as "renaming".
A temporary non-standard partitioning of a number is required for subtraction. The standard partitioning of, for example, 856 is 800 + 50 + 6, but to subtract, say, 138 (see the calculation below), 856 is decomposed into 800 + 40 +16.
The common error is for the student to subtract the smaller number in each column and disregard whether it is the number that is to be subtracted or the number to be subtracted from. To help prevent this error, have the students read the number they are subtracting in full (e.g., 138 as one hundred and thirty-eight, which is then partitioned as one hundred, three tens, and eight ones).

Equipment: Play money (Material Master 4–9).

Using materials

There were 856 containers on the ship, and 138 were unloaded onto the wharf. How many containers were left on the ship?

Record 856 – 138 = □ on the board or in the modelling book:

$$\begin{array}{rcl} & & \quad\ \ {\scriptstyle 40\ \ 16} \\ 856 & \rightarrow & 800 + \not{5}\not{0} + \not{6} \\ -\ 138 & \rightarrow & \underline{100 + 30 + 8} \\ \hline 718 & & 700 + 10 + 8 \ \ = 718 \end{array}$$

Get the students to model the 856 with place-value money. Ask: "Have we got enough ones to solve the problem?"
Because there are only six ones, there are not enough to take eight away from. A ten is exchanged for ten ones and added to the six ones, so the problem can now be solved.
This calculation process is worked through with examples that also require the exchanging of one hundred for ten tens, and one thousand for ten hundreds. In more complex cases, both a ten and a hundred may need to be exchanged in the same problem.

Examples: 456 – 259,   1 034 – 439,   781 – 678,   625 – 432,   5 385 – 2 749...

Large numbers roll over

When ten of any unit is formed in an addition problem, the next larger unit is created. In numbers such as 995 or 9 998, this can involve multiple exchanges that go beyond the next-ranked larger unit. It is the situation seen on an odometer when a car that has travelled 9 999 kilometres travels one more kilometre. The number rolls over. In subtraction problems, the number rolls back (e.g., 10 003 – 4).

Equipment: Play money: $1, $10, $100, $1000, $10 000 (Material Master 4-9).

Using materials

Work out $9,993 + $9.

Record $9,993 + $9 = ☐ on the board or in the modelling book. With play money, model $9,993 and $9. Discuss why the twelve single dollars must be swapped for a $10 note and two $1 notes. Discuss why nine tens plus the extra $10 note makes ten tens, which must be swapped for a $100 note.
Continue these swaps until there is a single $10,000 note and two single dollars. Record the answer of $10,002 on the board or in the modelling book.

Work out $10,003 – $4.

Record $10,003 – $4 on the board or in the modelling book. Using play money, break the $10,000 down to ten $1,000 notes, break the $1,000 note down to ten $100 notes, and so on until there are thirteen single dollars. Record the answer of $9,999 on the board or in the modelling book.

Examples: Word problems and recording for: $9,988 + $19, $6 + $52,994, $116 + $9,884,
$40,003 – $7, $20,000 – $100, $999 + $1,004, $1,001– $45, $50,003 – $5 …

Using number properties

Examples: Word problems and recording for: 8 992 + 9,   6 + 12 996,   16 + 6 684,   44 503 – 7,
18 900 + 102,   99 + 12 099,   50 + 6 150,   102 003 – 5 …

Mental or written?

Students who have mastered advanced additive, early multiplicative part–whole thinking should be deciding whether to use a written form or a mental strategy for each problem. Generally, they will choose the written form when presented with numbers that cannot easily be rounded or split. In some problems, the difficulty of keeping track of the numbers can be overcome by jottings that support quick mental calculation.

Equipment: None.

Using number properties

Which is the better way to solve these problems: mentally or by using a standard written form?

- 997 + 1 234
- 4 546 – 2 788

Discuss why 997 + 1 234 is easy to solve mentally by using a rounding and compensating strategy such as 1 000 + 1 234 = 2 234 → 2 234 – 3 = 2 231.
However, solving 4 546 – 2 788 mentally will be beyond most students, so the standard written form is an appropriate choice.

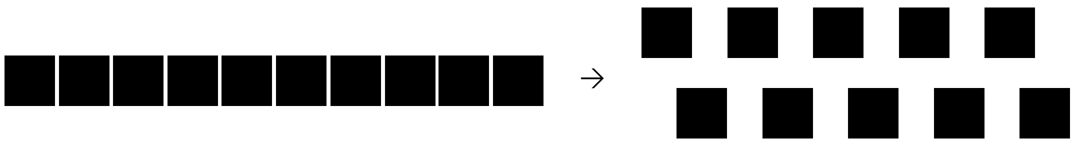
Examples: Use a mixture of addition and subtraction problems (Material Master 5-13) and problems that the students have written.

Moving students from advanced additive to advanced multiplicative

Introduction

Students who have mastered advanced multiplicative, part–whole thinking will confidently choose between, and apply, a range of strategies to solve addition and subtraction of decimal numbers, fractions, and integers. By looking carefully at the numbers presented in the problems, students will flexibly choose and use mental strategies that were developed at the early multiplicative, part–whole stage.

Decimal numbers should be introduced using material with the same characteristics as the place-value material used to introduce whole numbers; that is, a unit that can be easily broken into ten parts. A good example is a bar (below) made from ten interlocking cubes wrapped in thin card. The bar represents one but is easily broken into ten tenths.



Wrapped Unwrapped

Before students are introduced to decimal fractions and try to solve problems that involve decimal numbers, they need to thoroughly understand fractions. Students need to know that fractions arise from division and to understand, for example, that when five wholes are divided by six, each of the resulting parts equals five-sixths of one whole. The difficulty this presents to most students should not be underestimated, but mastering this idea will increase their ability to understand decimal numbers.

See *Book 7: Teaching Fractions, Decimals, and Percentages* to develop students' ideas about fractions.

Ideas being developed

- Adding or subtracting fractions requires an understanding of equivalent fractions.
- Decimal fractions are special cases of fractions, having denominators that are always a power of ten.
- Addition and subtraction problems involving decimal numbers can be solved with the same mental strategies as those used for problems with whole numbers.
- The same "ten for one" or "one for ten" canons that apply to whole numbers apply to decimal numbers.
- Decimal fractions are vital for solving problems of measurement and sharing problems in the real world.
- Integers are introduced through contexts such as bank balances and temperature.

Learning experiences

Key ideas	Learning experiences	Page
1. To add or subtract fractions, they must be renamed to have a common denominator.	Comparing apples with apples	Book 7
2. Decimal fractions arise out of division.	Introducing decimal fraction place value	69
3. The "ten for one" and "one for ten" canons apply when adding and subtracting with decimal fractions (one-decimal-place fractions).	Adding with decimal fractions Subtraction with tenths How can two decimals so ugly make one so beautiful?	71 71 Book 7
4. Subtraction can produce negative numbers.	Dollars and bills Dropping and rising temperatures Bucket balance	73 73 74

Key idea 1: To add or subtract fractions, they must be renamed to have a common denominator

When adding or subtracting fractions, the key idea is that fractions with different denominators must be renamed to have a common denominator. Then problems involving the addition and subtraction of fractions can be solved with whole-number knowledge.

Mathematical knowledge required

Students need to:

- be able to recognise equivalent fractions
- instantly recall multiplication facts up to ten times ten
- know common factors
- be able to convert improper fractions to mixed numerals.

Diagnostic snapshot

Students who can correctly answer problems such as $\frac{3}{4} + \frac{2}{5}$ and $\frac{8}{6} - \frac{3}{8}$ can move to the next key idea. For students who do not change a fraction or fractions to like denominators, or who add the numerators and the denominators, the following learning experience will develop this key idea.

Learning experience

■ Comparing apples with apples

See *Book 7: Teaching Fractions, Decimals, and Percentages* (page 65).

Key idea 2: Decimal fractions arise out of division

Although it is common practice to introduce the decimal point to students through money, this is unwise – in money amounts, the point is not a true decimal point but a device that separates two whole numbers, the dollars and the cents. We strongly recommend that when students first start working with decimal numbers and come across the point, the following definition is used:

The digit to the immediate right of the decimal point is in the tenths column.

Extending this definition to two decimal place numbers, we have:

The digit that is two places to the right of the decimal point is in the hundredths column.

We strongly recommend that initially teachers and students read decimal numbers such as 34.7 as "thirty-four and seven tenths", not "thirty-four point seven".

Mathematical knowledge required

Students need to be able to:

- instantly recall division facts up to ten times ten.

Diagnostic snapshot

> Henry has six bars of chocolate to share among five friends. How much chocolate does each of Henry's friends get?

Students who answer this problem confidently by sharing out one whole bar to each friend, then unwrapping the last bar to give each friend another two-tenths of a bar, are ready to move on to the next key idea. However, if students give the answer "one and one fifth" or "six fifths" (both correct, but not expressed in terms of parts of ten), the following learning experiences will develop this key idea.

Learning experiences

■ Introducing decimal fraction place value

Equipment: Sets of ten connected interlocking cubes wrapped in paper (chocolate bars).

Using materials

> Hohepa has five bars of chocolate to share among two friends. How much chocolate does each friend get?

Provide the students with five chocolate bars per group. Let them share out the bars. Watch to see who shares two whole bars to each friend first or who has to unwrap all five bars. Listen to see who then shares out the remaining bar, saying "five tenths" or "one half".

Discuss and record the result on the board or in the modelling book as "5 ÷ 2 = 2 wholes + 5 tenths". Discuss why the answer as a decimal number is two wholes and five tenths and not two wholes and one half.

Students who instantly give the answer "two and a half" have probably not transferred the whole-number place-value idea that, when being divided, any unit that needs to be broken up must always be broken into ten pieces, regardless of the divisor. The fact that these new pieces are now tenths needs careful teaching.

Examples: 8 ÷ 5 = □ wholes + □ tenths, 7 ÷ 5 = □ wholes + □ tenths, 2 ÷ 5 = □ wholes + □ tenths, 1 ÷ 2 = □ wholes + □ tenths …

Using imaging

Megan has four bars of chocolate to share among five friends. How much chocolate does each of Megan's friends get?

Place four chocolate bars in the middle of the group, where the students can see but not touch. Listen to see who talks the language of place value, breaking each whole into ten tenths and then sharing the forty tenths created using a known basic fact (i.e., 40 ÷ 5).

Students who are unable to solve the problem by imaging may need to fold back to "Using materials". Confirm whether the problem is related to place value or recall of division facts.

Examples: 1 ÷ 2 = □ wholes + □ tenths,   6 ÷ 4 = □ wholes + □ tenths,   4 ÷ 5 = □ wholes + □ tenths, 7 ÷ 2 = □ wholes + □ tenths …

Using number properties

Students need to repeat this kind of problem until they can predict the answer and explain their reasoning without using or referring to materials.

Examples: 10 ÷ 4 = □ wholes + □ tenths,   4 ÷ 10 = □ wholes + □ tenths,   3 ÷ 10 = □ wholes + tenths, 9 ÷ 2 = □ wholes + □ tenths …

Key idea 3: The "ten for one" and "one for ten" canons apply when adding and subtracting with decimal fractions (one-decimal-place fractions)

The place-value ideas that apply to whole numbers also apply to decimal numbers. Numbers always come in the canonical form; in other words, the maximum number in any column is 9. To proceed with a calculation, temporary non-canonical forms are usually necessary, but the final answer must be turned back into its canonical form.

The *canonical form* is defined as the standard, conventional, and logical way of writing numbers. For example, in order to calculate 4.9 + 3.5, nine tenths and five tenths are added, resulting in the non-canonical answer "fourteen tenths". This non-canonical form must then be transformed back into its canonical form – namely, "one whole and four tenths". The calculation can then be completed by adding the wholes.

Mathematical knowledge required

Students need to:

- instantly recall addition and subtraction facts to twenty
- know that one tenth of one tenth equals one hundredth
- know that one tenth of one hundredth equals one thousandth.

Diagnostic snapshot

Lalita has 3.9 metres of white ribbon and 4 tenths of a metre of black ribbon for the school sports day. How much ribbon does Lalita have altogether?

Watch for students who answer:

- 3.9 + 4 tenths = 7.9 (incorrect)
- 3.9 + 4 tenths = 3.13 (incorrect)
- 3.9 + 4 tenths = 4.3 (correct).

Students who answer this problem confidently, by recognising that ten tenths need to be swapped for one whole, are ready to move on to the next key idea. Otherwise, the following learning experiences will develop this key idea.

Learning experiences

■ Adding with decimal fractions

Equipment: Sets of ten connected interlocking cubes wrapped in paper.

Using materials

Wiremu has 1.9 metres of rimu and 2.5 metres of pine for the cupboard he is building. How much wood does Wiremu have altogether?

Have the students model 1.9 as one wrapped bar (a whole) and nine small pieces (tenths), and 2.5 as two wrapped bars and five small pieces (tenths). Adding them together creates three wholes and fourteen tenths. By the canon of place value, ten of these tenths are rewrapped to give one whole: 1.9 + 2.5 = 4.4 metres of wood.

Notice whether students understand that the number after the decimal point always represents tenths, that the maximum digit in any place is 9, and that ten tenths must be replaced by one whole.

Examples: 1.8 + 2.7, 2.6 + 1.6, 1.4 + 0.8, 3.5 + 2.5, 0.6 + 1.7, 5.8 + 0.8 …

Using imaging

Work out 2.7 + 3.4.

Place two wrapped bars and seven small pieces, and then three wrapped bars and four small pieces, where the students can see but not touch them. Listen to see who talks the language of place value as they swap ten tenths for one whole, leaving one-tenth remaining, and then add the one whole to the other wholes. Students who are unable to solve the problem by imaging may need to fold back to "Using materials".

Examples: 2.2 + 1.9, 4.7 + 3.6, 2.9 + 1.9, 0.6 + 1.6, 3.2 + 0.8, 2.6 + 1.5 …

Using number properties

Students need to repeat this kind of problem until they can predict the answer and explain their reasoning without using or referring to materials.

Examples: 10.4 + 10.9, 18.5 + 1.7, 0.4 + 20.8, 15.8 + 2.3 …

When you make up your own examples, you should generally make sure there are ten or more tenths in the addition so that ten tenths must be swapped for one whole. In other words, 3.3 + 1.9 is suitable, but 3.4 + 4.4 is not.

■ Subtraction with tenths

Equipment: Sets of ten connected, interlocking cubes wrapped in paper (chocolate bars).

Using materials

Hina has 4.3 metres of fabric and uses 1.7 metres to make a skirt. How much fabric does Hina have left over?

Students model 4.3 as four wrapped bars (wholes) and three small pieces (tenths). Students may do 4.3 – 1 = 3.3 at first. Then they may unwrap one bar, add the ten tenths to the three tenths, and remove seven tenths to leave six tenths: 4.3 – 1.7 = 2.6. Alternatively, they may unwrap one bar first to take away the seven tenths and then take away the one whole.

Notice whether students understand that the number after the decimal point always represents tenths, that the maximum digit in any place is 9, and that to perform subtraction calculations, it is usually necessary to break one whole into ten tenths.

Examples: 4.6 – 1.7, 3.3 – 1.6, 5 – 3.6, 2 – 0.4, 3.6 – 2.9, 2.3 – 1.8 …

Using imaging

Work out 1.5 – 0.8.

Place one wrapped bar and five small pieces where the students can see but not touch them. Listen to see who talks the language of place value as they break the one whole into ten tenths and then subtract the eight tenths from the fifteen tenths. Students who are unable to solve the problem by imaging may need to fold back to "Using materials".

Examples: 3 – 2.9,   3.3 – 0.6,   7.2 – 3.5,   9.2 – 7.3,   11 – 1.6,   12.8 – 0.9 ...

Using number properties

Students need to repeat this kind of problem until they can predict the answer and explain their reasoning without using or referring to materials.

Examples: 15.8 – 8.9,   15.1 – 1.8,   16 – 3.9,   10.3 – 0.8 ...

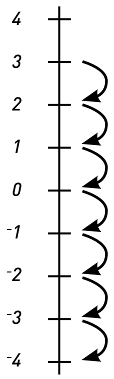
When teachers make up their own examples, the digit in the tenths column of the number being subtracted needs to be greater than the digit in the tenths column of the number it is being subtracted from. Thus 3.3 – 1.7 is suitable, but 5.6 – 3.4 is not.

■ How can two decimals so ugly make one so beautiful?

See *Book 7: Teaching Fractions, Decimals, and Percentages* (page 45).

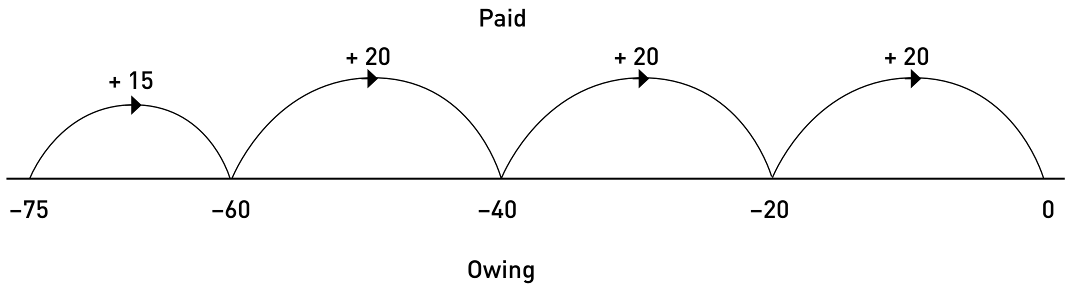
Equipment: Decimats (Material Master 7–3); scissors; calculators.

Key idea 4: Subtraction can produce negative numbers



Some problems that produce negative numbers are easily solved – for example, "The temperature is 3 degrees. It falls by 7 degrees. What is the new temperature?" Using a vertical number line (i.e., a thermometer), students can easily see that $3 - 7 = {}^{-}4$.

Money problems require somewhat different thinking, with the students understanding that a negative balance means owing money. For example: Dylan puts a Star Wars Lego set on layby. The set costs $75. He puts a deposit of $15 on it. How much does he owe? He pays $20 each payday. Track his payments on a number line until he owes nothing ($0) and can take the set home.



Mathematical knowledge required

Students need to:

- understand that there is a set of numbers called integers that are expressed as positive if greater than 0 and negative if less than 0.

Four types of problem are explored in this key idea. We suggest you work through all four types with the students. Note that certain models or representations work better with certain problem types:

- Type 1: from an integer, subtract a positive integer

  $^{+}5 - {}^{+}7 = {}^{-}2$
- Type 2: to an integer, add a positive integer

  $^{-}5 + {}^{+}7 = {}^{+}2$
- Type 3: to an integer, add a negative integer

  $^{+}5 + {}^{-}7 = {}^{-}2$
- Type 4: from an integer, subtract a negative integer

  $^{-}5 - {}^{-}7 = {}^{+}2$

Learning experiences

■ Dollars and bills (type 1: from an integer, subtract a positive integer)

Equipment: Place-value money.

Using materials

Put one student or a pair of students in charge of recording purchases at the layby counter. Give them a pile of $1 and $10 notes and the modelling book.

> Margaret has $8. She wants to buy a birthday present for her Mum that costs $15, so she goes to the layby counter. She asks to put the item on layby and deposits her $80. Ask the layby clerk to record the transaction and tell her how much she owes. A negative integer will show how much money she owes and a positive integer will show how much money she has.

Record $^{+}8 - {}^{+}15 = \square$. Discuss how this shows that Margaret started with $8, purchased a $15 item, and now owes $7. We show that the $7 is owed by recording it as a negative integer: $^{-}7$. Margaret owes $7. When she comes in to pay the rest, the clerk will record: $^{-}7 + {}^{+}7 = 0$. She will owe nothing.

Examples: Pose pocket money and bills stories for: $^{+}5 - {}^{+}7$, $^{+}10 - {}^{+}13$, $^{+}2 - {}^{+}5$, $^{+}9 - {}^{+}12$, $^{+}5 - {}^{+}8$, $^{+}5 - {}^{+}11$, $^{+}12 - {}^{+}25$ …

Using imaging and number properties

Students need to repeat this kind of problem, at first by imaging the money, until they can predict the answer and explain their reasoning without using, or referring to, materials. Pose problems about a student's saving and spending.

Examples: $^{+}48 - {}^{+}99$, $^{+}150 - {}^{+}225$, $^{+}1\,000 - {}^{+}1\,450$, $^{+}995 - {}^{+}1\,025$ …

Students who are unable to solve the problems by imaging and/or number properties may need to fold back to "Using materials".

Note: These problems could also be solved using the context of temperature.

■ Dropping and rising temperatures (type 2: to an integer, add a positive integer, and type 3: to an integer, add a negative integer)

The aim here is for students to become comfortable moving up and down a vertical number line (thermometer) to reflect the operation of adding an integer. Adding a positive integer is a rise and adding a negative integer is a drop. For example, if the temperature at Mt Ruapehu was $^{-}2$ degrees in the morning and rose by 5 degrees it would be recorded as $^{-}2 + {}^{+}5 = {}^{+}3$. If the temperature had dropped by 5 degrees, it would be recorded as $^{-}2 + {}^{-}5 = {}^{-}7$.

Equipment: Vertical number lines (thermometers).

Using materials

Ask students to imagine they are visiting friends in Dunedin during the winter, when temperatures are often below 0.

> In Dunedin the morning temperature was $^{-}3$ degrees. If the temperature fell 4 degrees by lunchtime, what is the new temperature?

Using a vertical number line, have students record the starting point on the thermometer and the direction they will need to move (down).

Record this as: $^{-}3 + {}^{-}4 = {}^{-}7$.

> Now imagine that the temperature at the same time the next day is $^{+}4$. How much has the temperature changed in 24 hours?

Using the same number line, ask students record the starting point ($^{-}7$) and the direction they will need to move (up).

Record: $^{-}7 + \square = {}^{+}4$.

How have they figured out how many degrees the temperature has risen? Will this be a positive or a negative integer?

Examples: Pose temperature problems for the thermometers, making sure that students record the rises and falls as additions of integers and use the number lines to track the movements. They can also use red and blue pens to indicate adding positive (red) integers and adding negative (blue) integers.

Using imaging and number properties

Students need to practise these types of problem by imaging the movement up and down a thermometer until they can successfully use part–whole thinking to work back and forth across the 0 point on the number line.

Examples: $^{-}30 + {}^{-}50$, $^{-}14 + {}^{+}12$, $^{-}99 + {}^{+}107$, $^{+}5 + {}^{-}37$, $^{-}204 + {}^{-}102$, $^{-}204 + {}^{+}102$ …

■ Bucket balance (works for all types, but is especially useful for type 4: from an integer, subtract a negative integer)

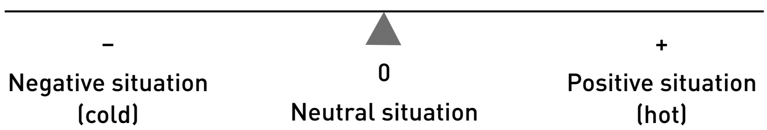
The key idea here is that subtracting a negative integer has the same effect as adding a positive integer. For example, $^{+}5 - {}^{-}7 = {}^{+}12$.

Rather than memorising a rule such as "two negatives make a positive" (which will mislead them), students can see from this learning experience *why* this happens.

Equipment: A small bucket or opaque container; blue and red unifix cubes.

Using materials

Ask the students to imagine that the bucket is filled with water. The red unifix cubes are units of heat (or "hot cubes") and the blue unifix cubes are units of cold (or "cold cubes"). Draw a horizontal line on the board or in the modelling book. Place a small triangle at the middle so that the drawing represents a simple balance.



Explain that if there are an equal number of hot cubes and cold cubes (pairs), the situation is represented by 0 (there is a perfect balance between hot and cold). If there are more hot cubes than cold cubes, there is a *positive situation* or warmer water. If there are more cold cubes than hot cubes, there is a *negative situation*.

Place 10 red cubes and 10 blue cubes in the bucket. Agree that the starting situation is neutral or 0. Look at the following equations and show us what would happen to the 'water' in the bucket if you carry out the operation. Act out the equation with the model to find or prove your answer.

(1) $0 - {}^{+}2 = \square$ (2) $0 + {}^{+}2 = \square$ (3) $0 + {}^{-}2 = \square$ (4) $0 - {}^{-}2 = \square$

For the first equation, listen for explanations and demonstrations that include the following idea/action: starting at neutral and taking away two hot cubes tips the balance to the cold side and the result is described as a ${}^{-}2$ situation (that is, there will be two more cold cubes than hot cubes in the bucket).

For the second equation, listen for explanations and demonstrations that include the following idea/action: starting at neutral and adding two hot cubes tips the balance to the warm side and the result is described as a ${}^{+}2$ situation (that is, there will be two more hot cubes than cold cubes in the bucket).

For the third equation, listen for explanations and demonstrations that include the following idea/action: starting at neutral and adding two cold cubes tips the balance to the cold side and the result is described as a ${}^{-}2$ situation (that is, there will be two more cold cubes than hot cubes in the bucket).

For the fourth equation, listen for explanations and demonstrations that include the following idea/action: starting at neutral and taking away two cold cubes tips the balance to the warm side and the result is described as a ${}^{+}2$ situation (that is, there will be two more hot cubes than cold cubes in the bucket).

Discuss with the students what they notice about the answers.

Pose new sets of four problems with different starting points (for example, add a positive, add a negative, subtract a positive, subtract a negative).

Examples: The water in the bucket is at a ${}^{-}3$ situation, so we have 3 more cold cubes than hot cubes. Now model ${}^{-}3 + {}^{+}4$, ${}^{-}3 + {}^{-}4$, ${}^{-}3 - {}^{+}4$, ${}^{-}3 - {}^{-}4$ …

The water in the bucket is at a ${}^{+}5$ situation …

Using imaging and number properties

Students will need practice working with the addition and subtraction of integers. The imaging of the bucket model can be especially helpful, as it is easy to imagine a starting point and the way the balance shifts as they add or withdraw elements of heat and cold.

Students who cannot solve these problems by imaging or number properties should fold back to manipulating blocks in the bucket model. They need to realise that it doesn't matter how many actual cubes are in the bucket, it's the balance or difference between the two sets of cubes that we are interested in.

Examples: ${}^{-}25 - {}^{-}34$, ${}^{+}42 - {}^{-}3$, ${}^{-}99 - {}^{-}1$ …

Students who are able to solve the problems found in this section can be extended by exploring the activities in *Book 8: Teaching Number Sense and Algebraic Thinking*.

Moving students from advanced multiplicative to advanced proportional

Introduction

Proportional reasoning is the mathematics of relationships and comparisons.

Unlike comparisons based on addition or subtraction (for example, "A is 12 more than B"), comparisons based on proportional reasoning (for example, "A is 120% of B") are based on multiplicative thinking.

For further learning experience, see *Book 7: Teaching Fractions, Decimals and Percentages*, *Book 8: Teaching Number Sense and Algebraic Thinking*, and *Book 9: Teaching Number through Measurement, Geometry, Algebra and Statistics.*

Ideas to be developed

To become generalised proportional thinkers (levels 5 and 6 of the curriculum), students need to give meaning to and connect these ideas:

- part–whole comparisons
- quotients
- measures
- operators
- rates and ratios
- probability.

Part–whole comparisons involve finding the multiplicative relationship between part of an area or set and the whole. For example: What part of a floor has green tiles? What part of the day do I spend sleeping?

Quotients are the answers to whole-number division problems. (Quotients are fractions.) It is important that students recognise that 3 ÷ 4 is an operation, while ¾ (the quotient) is the fraction that *results* from the operation.

In a **measurement** context, a fraction is the answer to questions such as: "How many times does this fraction (or ratio) fit into that fraction (or ratio)?"

As **operators**, fractions perform operations on other numbers; for example, ⅓ x 12 = ☐.

Rates involve a multiplicative relationship between two variables, each of which has a different unit of measurement; for example, kilometres per hour (km/h). **Ratios** are a special type of rate, in which the unit of measurement is the same for each variable; for example, 1 shovel of cement to 5 shovels of builders' mix (1:5).

Probability involves the part–whole relationship between how often an event occurs and the sum total of all possible events. Probability can be experimental (based on data) or theoretical (based on properties). Probability relationships can be observed over time as patterns, but they do not prescribe individual outcomes.

Advanced proportional thinkers are able to move flexibly between these six ideas, between representations, and between different fraction forms ($^a/_b$, decimals, and percentages).

For example, they know that a constant rate can be expressed as number pairs and represented as a straight-line graph. They know that ⅛ = 0.125 = 12.5% and that they enter 35% into a calculator as 0.35. They know that what sets percentages apart from "regular" fractions is that the denominator is standardised at 100, which means that percentages can easily be compared without the need to first find a common denominator.

Advanced proportional thinkers keep the context in mind. They are able to recognise the type of situation in which, for example:

- a rate is not constant (if a person can sprint 100 metres in 14 seconds, this does not mean they can run 1000 metres in 140 seconds)
- the product of the two variables is constant (if two boys can paint a fence in three days, it will not take four boys six days)
- a part–part ratio, not a part–whole ratio, is involved (for example, when comparing the number of successes to the number of failures).

Learning experiences

Key ideas	Learning experiences	Book/Page
1. Part–whole comparisons	Hot shots	Book 7, page 47
	Extending hotshots	Book 7, page 56
2. Quotients	Wafers	Book 7, page 16
3. Measures	Brmmm! Brmmm!	Book 7, page 68
	Reverse percentage problems	Book 8, page 44
4. Operators	Animals	Book 7, page 18
	Hungry birds	Book 7, page 22
	Birthday cakes	Book 7, page 26
	Fractional blocks	Book 7, page 28
	Extending hotshots	Book 7, page 56
5. Rates and ratios	Seed packets	Book 7, page 30
	Mixing colours	Book 7, page 50
	Extending hotshots	Book 7, page 56
	Extending mixing colours	Book 7, page 61
	Rates of change	Book 7, page 71
	Tree-mendous measuring	Book 7, page 76
	Ratios with whole numbers	Book 8, page 42
	Comparing by finding rates	Book 8, page 43
	Inverse ratios	Book 8, page 43
6. Probability	Coin tossing on the computer	Book 9, page 46
	The Monty Hall problem	Book 9, page 47
	Varying as you go	Book 9, page 49
	The two coin problem	Book 9, page 50
	Data gathering	Book 9, page 51
	The drawing pin problem	Book 9, page 51

■ Stacks and rubble

Equipment: Unilink cubes.

Using materials

Build a yellow stack of 3 cubes and a green stack of 6 cubes by putting down 1 yellow and 2 green and then adding 1 yellow and 2 green, 1 yellow and 2 green. Ask the students to describe what you are doing ("You are building two piles by adding one yellow and two green to the piles each time"). Ask them what they can tell you about the heights of the two stacks. They should observe that the green stack is twice as high as the yellow stack or that the yellow stack is only half the height of the green stack.

If they give an additive response ("There are three more green cubes [than yellow]"), add further yellow and green cubes in the same ratio and repeat the question until they see that as long as you keep adding cubes in this way, the green stack will always be twice the height of the yellow stack. In other words, they have found and described a *constant relationship*.

Explain that the mathematical way of describing this relationship is 1:2 ("1 to 2"), where the 1 is the stack of yellow cubes and the 2 is the stack of green cubes. Explain that 1:2, 2:4, 3:6, and so on are all equivalent ratios (because all say that the green pile is twice as high as the yellow pile) but that 1:2 is the simplest way of expressing this relationship. Relate equivalent ratios to equivalent fractions.

It is important that students understand that a ratio does not say *how many* cubes (or whatever) are involved.

Now swap the positions of the two piles and ask the students what happens to the ratio. It is important that they realise that, as the green stack is still twice the height of the yellow stack, whatever its physical position, the ratio must stay the same (and not be reversed).

Then ask the students to suggest what two stacks built in the ratio 2:1 would look like. Illustrate their ideas, using materials.

The important understanding is that the meaning of any ratio depends on the context (you have to know what the two numbers represent). So the two stacks of 3 yellow and 6 green cubes that we described using the ratio 1:2 ("1 yellow to 2 green") could equally well have been described using the ratio 2:1 ("2 green to 1 yellow").

Now make other pairs of stacks, using different colours of cube for each stack. In each case, build the two stacks by simultaneously adding blocks to each in a particular ratio. For example, 3:1, 2:4, 1:5, 4:1, 2:3, 3:4, 5:2. Swap the positions of the stacks, collapse the stacks into piles of cubes, put all the cubes into a single pile, and so on so that the students can see that the ratio exists independently of the physical position of the cubes.

Using imaging

Create further pairs of stacks of cubes, but this time do not build them by simultaneously adding cubes of each colour. The students describe the ratio by counting the number of blocks in the finished stacks. Encourage them to express each ratio in its simplest form.

Using number properties

Give students pairs of numbers but no blocks and get them to say or write the ratio, in each case in its simplest form.

Examples: 12 green and 24 blue, 18 and 6, 4 and 20, 1 and 8, 9 and 15, 8 and 10, 15 and 6, 32 and 8, 60 and 12, 2 and 11, 21 and 35, 35 and 14, 27 and 36, 18 and 81 ...

Acknowledgments

The Ministry of Education wishes to acknowledge the following people and organisations for their contribution towards the development of this handbook in its several editions.

Participants

The New Zealand Numeracy Project personnel – facilitators and principals, teachers, and children from hundreds of New Zealand schools who contributed to this handbook through their participation in the numeracy development projects.

Numeracy reference group

Professor Derek Holton, convenor (University of Otago), Professor Megan Clark (Victoria University of Wellington), Dr Joanna Higgins (Victoria University of Wellington College of Education), Dr Gill Thomas (Maths Technology Limited), Associate Professor Jenny Young-Loveridge (University of Waikato), Associate Professor Glenda Anthony (Massey University), Tony Trinick (University of Auckland Faculty of Education), Garry Nathan (University of Auckland), Paul Vincent (Education Review Office), Dr Joanna Wood (New Zealand Association of Mathematics Teachers), Peter Hughes (University of Auckland Faculty of Education), Vince Wright (University of Waikato School Support Services), Geoff Woolford (Parallel Services), Kevin Hannah (Christchurch College of Education), Chris Haines (School Trustees' Association), Linda Woon (NZPF), Jo Jenks (Victoria University of Wellington College of Education, Early Childhood Division), Bill Noble (New Zealand Association of Intermediate and Middle Schools), Diane Leggatt of Karori Normal School (NZEI Te Riu Roa), Sului Mamea (Pacific Island Advisory Group, Palmerston North), Dr Sally Peters (University of Waikato School of Education), Pauline McNeill of Columba College (PPTA), Dr Ian Christensen (He Kupenga Hao i te Reo), Liz Ely (Education Review Office), Ro Parsons (Ministry of Education), Malcolm Hyland (Ministry of Education).

Writers and reviewers

Peter Hughes (University of Auckland Faculty of Education), Vince Wright (University of Waikato School Support Services), Sarah Martin (Bayview School), Gaynor Terrill (University of Waikato School of Education), Carla McNeill (University of Waikato School of Education), Professor Derek Holton (University of Otago), Dr Gill Thomas (Maths Technology Limited), Bruce Moody (mathematics consultant), Lynne Petersen (Dominion Road School), Marilyn Holmes (Dunedin College of Education), Errolyn Taane (Dunedin College of Education), Lynn Tozer (Dunedin College of Education), Malcolm Hyland (Ministry of Education), Ro Parsons (Ministry of Education), Kathy Campbell (mathematics consultant).

Revised edition 2012

Gail Ledger (University of Auckland Faculty of Education), Carol Butel (University of Canterbury), Denise Carter (University of Otago College of Education), Peter Hughes (University of Auckland Faculty of Education), Associate Professor Jenny Young-Loveridge (University of Waikato), Chanda Pinsent (mathematics consultant).

Book 5

To be numerate is to have the ability and inclination to use mathematics effectively – at home, at work, and in the community.

ISBN 978-0-7903-4023-4