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Findings from the New Zealand Numeracy Development Projects

2009

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Foreword

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Foreword 2010

In reading this foreword, you should bear in mind that it is written by an academic who has now retired from the education scene in New Zealand. What follows is in two parts: the first is a quick history of the Numeracy Development Projects (NDP), and the second is a look at some of the issues relating to the future of mathematics teaching and learning. The first part is essentially factual. The second part is my view of matters relating to mathematics education in the future.

There is always the concern that international tests of students' knowledge will lead to league tables that will send a conservative backlash into the education system of countries at the lower end of the tables. This could have happened after the 1996 TIMSS assessment, when New Zealand found itself lower down the international ladder than it had hoped to be. Fortunately, the desire for improvement led to searching for a better way to teach mathematics in New Zealand primary schools. In 1999, in line with recommendations from the 1997 mathematics and science taskforce, a numeracy group explored international ideas and ways to apply them to the improvement of primary mathematics education in New Zealand. They investigated the Count Me in Too numeracy project from New South Wales and used it as the basis for the 2000 NDP pilot study that was conducted in about 80 New Zealand schools. The results in these schools were evaluated in terms of student achievement and adjustments were made accordingly. A key feature of the development of the NDP has been the alignment and use of expertise in policy, research, mathematics, curriculum design, and teacher education.

The early years of the NDP included the Early Numeracy Project for years 0–3, the Advanced Numeracy Project for years 4–6, and the Intermediate Numeracy Project for years 7–8. Te Poutama Tau (the Māori-medium numeracy project) was first piloted in 2001 and extended to wharekura in 2006. The Secondary Numeracy Project, for years 9–10, was piloted in 2005. All these projects are now under the umbrella of the NDP, for New Zealand students from years 1 to 10. The nzmaths website (see www.nzmaths.co.nz/numeracy-project-information) provides up-to-date information and access to features of the NDP, including downloadable resource material.

Teachers have been supported by published material in the form of the NDP "pink" support material books, the Figure It Out series, and the nzmaths website. In some ways more importantly, teachers have also been strongly supported by professional development in their own schools.

The NDP's development has relied on research and evaluation, with over 100 papers published in previous compendia and in research journals. The research and evaluation have focused on aspects such as student achievement, professional practice, and the evaluation of various initiatives developed to support teachers. Raising student achievement in mathematics by increasing the capability, knowledge, and confidence of teachers has always been at the heart of the NDP. In recent years, school leadership, consolidation, and sustainability have been a key focus. It is important to point out that the NDP continue to be researched. A key aspect of NDP research is the data that is collected by teachers and entered in a central database (maybe the biggest of its kind in the world). This data is analysed in this volume by several authors. It enables us to have a very good knowledge of what students can do and what they might aspire to.

The Number Framework, which is at the heart of the NDP, helped inform the mathematics and statistics learning area of *The New Zealand Curriculum* (Ministry of Education, 2007). Unfortunately, data in this volume suggests that many students are not achieving at the higher levels of the curriculum. The big questions to me seem to be: How can we get all students to achieve at these levels? What help needs to be given to teachers and schools to ensure that these achievement levels are reached universally?

This brings me to another point. To date, the NDP have been a "work in progress". They have continued to develop. It is extremely important that mathematics education should continue to evolve and take note of the research that is being undertaken both in New Zealand and overseas. For example, current work in algebra in New Zealand should be acknowledged in an update of the Number Framework because algebra is a natural progression of the number that precedes it. There is also some suggestion in this volume that some of the NDP book material could be revised. It is important that mathematics education continues to progress as we learn more about students' mathematical development. And it is important that we are able to help teachers to understand these changes and see how to use that knowledge for the benefit of their students.

Providing effective professional development to teachers is an ongoing challenge. The NDP have highlighted the importance of a teacher's knowledge: knowledge of mathematics, knowledge of how to represent mathematical ideas, and knowledge of effective teaching practices. There seems to be an increasing concern about pre-service teachers' understanding and confidence in mathematics and how to teach it. I believe that more effort may need to be put into the professional development of all teachers as they begin their professional life.

From my perspective, three of the main items that underpin the philosophy of the NDP were essential to its success and are essential to effective mathematics education in the future. These are:

- Every student can do maths.
- Teachers are key figures in change.
- Understanding before algorithms.

An emphasis on letting students explore and absorb number sense, rather than teaching them learned algorithms without any understanding, seems to be the right way ahead for students to gain an understanding of number and, possibly more importantly, of liking and feeling comfortable with mathematics itself. At all costs, we should ensure that we never return to the hundreds of algorithms that have made mathematics a wasteland full of the rote learning of incomprehensible rules.

However, at the end of the day, the teachers are the key figures in our students' mathematical development. We need to support them in every way we can. I think that this is a basic tenet of the NDP, and it is one that needs to continue through the next 10 years of the development and consolidation of the NDP as we work towards the improvement of mathematics learning.

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The Performance of the Longitudinal Study Students in Year 8

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This paper reports on the achievement of the 78 year 8 students in the 2009 Longitudinal Study. The achievement pattern of these students against the Ministry of Education's numeracy expectations is different from that reported in previous years, with a smaller proportion of students achieving at or above expectation and therefore a higher proportion of students falling below expectation. This difference is largely attributable to the decline in performance of 8 students who regressed a strategy stage, moving from stage 7 in year 7 to stage 6 in year 8. While the results show that something changed for these students in terms of their experiences and success in learning mathematics, the cause of these changes is unclear. Although the longitudinal students in year 8 cannot be considered to be performing well in relation to the numeracy expectations, they are performing well in mathematics in relation to the national population, a result that highlights the differences between standards-based assessments and normalised assessment tools. In general, parent responses to a survey indicated a reasonable understanding of current approaches to teaching mathematics in New Zealand and a reasonably accurate understanding of their child's performance in mathematics.

Background

This Longitudinal Study forms part of a wider study that was established in 2002 to investigate the effectiveness and sustainability of the Numeracy Development Projects (NDP). The wider study focused on tracking the progress and achievement of students in a sample of schools over time in order to describe the longer-term impact of the NDP. The sample of schools changed to some extent over time, but replacement schools were selected in a way that ensured that the sample was as representative of the national population as possible. Findings have shown that the NDP continue to impact positively on the number strategies of students in NDP-focused schools in the years following the initial implementation of the NDP (Thomas & Tagg, 2004, 2005a, 2006, 2007). Results have also indicated that students in NDP-focused schools perform well on mathematics problems more generally, in addition to performing well on number problems (Thomas & Tagg, 2005b).

The Longitudinal Study has tracked the progress and achievement of a group of students from the wider sample for whom achievement information since school entry is available. Findings from the Longitudinal Study have shown that students performing well in years 1–5 are likely to continue to perform well in subsequent years (Thomas & Tagg, 2008), that the proportion of students within the longitudinal sample performing at the Ministry of Education's numeracy expectations is consistently high (Thomas & Tagg, 2009), and that students' perceived and actual abilities in mathematics are correlated (Thomas & Tagg, 2009). The longitudinal sample comprised 151 year 6 students in 2007 (Thomas & Tagg, 2008), 83 year 7 students in 2008 (Thomas & Tagg, 2009), and 78 year 8 students in 2009. It is these 78 students who are the focus of this paper.

Method

Materials

A written assessment was developed for the 2009 Longitudinal Study. The assessment included 41 items from the Progressive Achievement Test (PAT): Mathematics Test 5 (NZCER, 2006a), five attitude items, and five items designed to assess students' use of strategies to solve number problems. The number strategy items were based on items in the NDP Global Strategy Stage (GloSS) forms (Ministry of Education, n.d. 1). These items required students to record their answer to a word problem and to describe their method for solving the problem. The problem shown in Figure 1 is an example of one of the strategy items.

It takes 207 pegs to put up 9 tents.
How many pegs is that for each tent?

Figure 1. Example strategy item from student assessment

A written questionnaire was developed to collect information from a parent or caregiver of each student[1]. Questions focused on parents' confidence about supporting their child's mathematics learning, methods used by schools to inform parents of their child's progress in mathematics, parents' perceptions of their child's mathematical competence, and the mathematics that students were involved in outside of school hours. Most of the questions required a closed response, and opportunity was provided for parents to comment more generally.

Procedure

Letters were sent to the families of the 83 students who had participated in 2008, inviting them to participate in 2009. Information was provided about the purpose and nature of the study, and informed consent to be involved in the research was requested from the student and a parent. Consent forms were returned by 78 students and their parents.

Early in term 4, these 78 students were sent the assessment and their parents were sent the questionnaire. Both the students and their parents received an incentive to complete and return the forms; students received a book voucher, and parents received a petrol voucher. All 78 students returned completed assessments, and 77 of the parents completed and returned the questionnaire.

Participants

Table 1 provides demographic information for the 78 students included in the 2009 Longitudinal Study sample.

Table 1
Ethnicity and Gender of the 2009 Longitudinal Study Students

Ethnicity	Male	Female	Total
NZ European	19	37	72%
Māori	7	7	18%
Pasifika	1	2	4%
Asian	2		3%
Other	1	2	4%
Total	30	48	78

Note: Percentages do not all add up to 100 because not all the parents who responded answered all the questions.

[1] These participants are referred to as "parents" throughout the remainder of this paper.

More of the 78 students are female (62%) than male (38%). Nearly three-quarters (72%) of the students identified as New Zealand European, and 18% identified as Māori.

Information on the decile ratings of the schools that students attended in 2009 was available for 76 of the 78 students. In 2009, 7% of these students attended decile 1–3 schools, 43% attended decile 4–7 schools, and 50% attended decile 8–10 schools. As described previously (Thomas & Tagg, 2009), the self-selecting nature of the sample has resulted in a group that is not representative of the national population as a whole.

Analysis

Students' GloSS stages were estimated from their responses to the five strategy items. Students were initially rated at the highest stage for which they showed evidence of appropriate working with a correct answer. Examples for each stage are shown in Table 2. This initial rating was then evaluated in the light of all other evidence about the student, including strategy-stage ratings for the previous seven years and their performance on number items from PATs from the previous two years. Two researchers came to a consensus agreement about each student's rating.

Table 2
Example Evidence for Rating Students' GloSS Stage

Stage	**Problem**	**Example of response**
Stage 5: Early additive	27 + 19	I rounded up 19 to 20 by adding 1 which then 27 + 20 = 47 and then I subtracted 1 from 47. I then ended up with 46
Stage 6: Advanced additive	231 – 78	231 +22 78 +22 this makes the equation 253 - 100 = 153
Stage 7: Advanced multiplicative	207 ÷ 9	9×12 = 108 9×10 = 90 9×1 = 9 108+90+9 = 207 12+10+1 = 23
Stage 8: Advanced proportional	If 40 = 16*x*, what is 6*x*?	if 40 = 16 then 20 = 8 so ? = 6! well 6 is 3/4 of 8 so ? is 3/4 of 20 which is 15

Students' estimated strategy stages were analysed in relation to numeracy expectations. *The New Zealand Curriculum* (Ministry of Education, 2007) describes the stage or stages of the Number Framework that students are expected to be achieving by the end of each curriculum level. Building from this, the Ministry of Education developed a set of expectations for the achievement of students at the end of each year level (Ministry of Education, n.d. 2). These set out the expected level of achievement, as well as categories for students who are performing just below expected level ("cause for concern") and well below expected level ("at risk").

Students' achievement in the PAT was used as a measure of their overall mathematics ability. Students' scores were analysed in relation to the national reference sample in order to describe the relative achievement of the longitudinal sample. The relationship between students' scores in the PAT and their estimated strategy-stage ratings was also investigated.

Correlation coefficients were used as a measure of the relationship between two variables, for example, to determine the extent to which students' number ability and their ability in mathematics more generally are related. Where correlation coefficients are reported, values of 0.10 to 0.29 are described as small, values of 0.30 to 0.49 are described as moderate, and values of 0.5 to 1.0 are described as large (Cohen, 1988). T-tests were used to establish the significance level of differences between sub-groups of students. The results of these tests are reported along with their corresponding p-values.

Findings

The key research questions addressed in this section are:

- What is the longitudinal impact of the NDP on students' achievement on the Number Framework?
- How does the mathematics ability of year 8 students relate to their achievement on the Number Framework?
- To what extent do parents understand the NDP and their children's performance within them?
- How do the academic qualifications of parents relate to the performance of their children in mathematics?
- Does the mathematics that students complete outside of school hours impact on their performance?

What is the Longitudinal Impact of the NDP on Students' Achievement on the Number Framework?

Figure 2 shows the performance of the 78 students in the 2009 Longitudinal Study on the Number Framework and against expectations, over the eight years since school entry in 2002. Students' year 1–6 strategy-stage ratings were made by their teachers, while year 7 and 8 strategy estimates were based on the researchers' rating of the students' written responses to strategy items. The fact that students were not assessed using a one-to-one interview in years 7 and 8 may have impacted on the accuracy of results for these years.

8 (AP)					6%	14%	14%	24%
7 (AM)			5%	3%	18%	41%	44%	37%
6 (AA)	1%	8%	23%	41%	31%	28%	31%	33%
5 (EA)	10%	31%	44%	42%	32%	13%	12%	5%
4 (AC)	18%	35%	23%	12%	10%	3%		
3 (CA)	23%	14%	4%	3%	3%	1%		
2 (CA)	35%	9%						
1 (1–1)	8%	4%	1%					
0 (Em)	5%							
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8

Expected level Cause for concern At risk

Figure 2. Percentage of the 78 students in the Longitudinal Study sample at each strategy stage in years 1–8

From school entry to year 7, a high proportion of the students performed at or above the expected levels, with between 74% and 95% of the students meeting or exceeding expectations in these years.

The achievement pattern at year 8, shown in Figure 2, is different from the overall achievement pattern for these students in previous years. A smaller percentage of the students met the expectation in year 8 than in previous years (61%), with 38% below the expectation. The expectation at year 8 is advanced multiplicative, which requires students to solve multiplication and division problems with understanding. For example, students may solve the problem 24 x 6 by multiplying 25 by 6 and then subtracting 6.

The progression of students between stages in years 7 and 8 was investigated in an attempt to explain the observed changes in the achievement pattern. It was determined that the altered pattern of student achievement in year 8 was due, in large part, to eight students who had declined by one strategy stage, moving from stage 7 in year 7 to stage 6 in year 8. If, hypothetically, these eight students had remained at stage 7 in year 8, the picture would be more similar to previous years, with 71% of the students meeting or exceeding expectations and 28% below the expected levels.

In order to establish reasons for the unexpected decline in the strategy stage of these eight students, achievement information for three groups of students was compared. One of the groups contained students who were rated at stage 7 in year 7 and progressed to be rated at stage 8 in year 8 (nine students), a second group contained students who were rated at stage 7 in both years 7 and 8 (17 students), and the remaining group contained the eight students who declined from stage 7 to stage 6 between years 7 and 8. Figure 3 illustrates these groups.

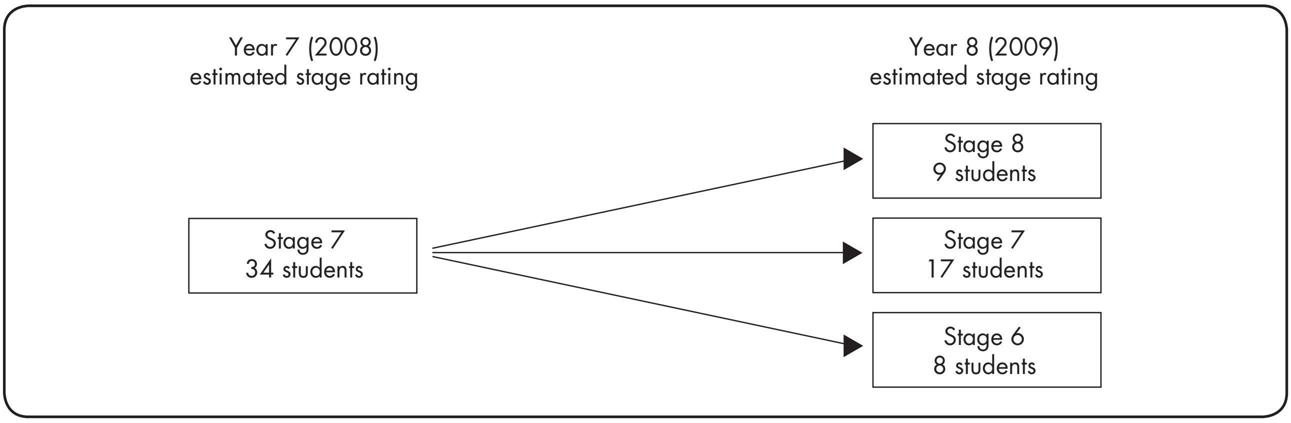

Figure 3. Change in estimated stage ratings between years 7 and 8 for the 34 students rated at stage 7 in year 7

Table 3 compares the previous achievement of the three groups of students. Note that PATs were carried out in years 7 and 8, while in year 6, knowledge information is based on stages of the Number Framework.

Table 3
Comparison of Previous Achievement for Three Groups of Students

Students' stage ratings Year 7 → 8	**N =**	**Year 6**			**Year 7**	**Year 8**
		Basic facts (mean knowledge stage)	Place value (mean knowledge stage)	Fractions (mean knowledge stage)	PAT (mean scale score)	PAT (mean scale score)
Stage 7 → 8	9	5.9	5.2	5.8	63.1	68.4
Stage 7 → 7	17	6.0	5.4	5.5	65.2	65.2
Stage 7 → 6	8	5.3	5.3	4.7	56.8	57.7

Note: Double-headed arrows show where the differences between groups are significant ($p \leq 0.05$).

On the whole, the students who declined in year 8 were more different from either of the other groups of students than they had been in previous years in terms of their achievement. In year 6, there were no significant differences between the number knowledge of the three groups. In year 7, there was a significant difference ($p \leq 0.05$) between the PAT-scale scores of the students who went on to decline in year 8 and those who advanced to stage 8. In year 8, there were significant differences ($p \leq 0.05$) between the PAT-scale scores of those students who declined and both the other groups of students. These results suggest that the eight students who declined in strategy stage also experienced declines in general mathematics ability from year 7 to year 8. More generally, these results, along with the strategy-stage information, suggest that something altered for these eight students in year 8 in terms of their success in learning mathematics.

The attitudes towards mathematics of the three groups of students were compared using their responses to the attitude items administered in years 7 and 8. These items are shown in Figure 4.

Circle the face that best describes how much you agree with each statement about mathematics.

	Agree a lot	Agree a little	Disagree a little	Disagree a lot
I usually do well in mathematics.				
I enjoy learning mathematics.				
I learn things quickly in mathematics.				
I think learning mathematics will help me in my daily life.				
I need mathematics to learn other school subjects.				

Figure 4. Student attitude items

Table 4 shows the mean response rating to two of the five attitude items for each of the three groups of students. Note that for the purpose of calculating means, "agree a lot" has been assigned a value of 1, "agree a little" has been assigned a value of 2, "disagree a little" has been assigned a value of 3, and "disagree a lot" has been assigned a value of 4.

Table 4

Mean Response to Attitude Items for Three Groups of Students in Year 8

Students stage ratings from year 7 to year 8	**Mean response rating to the statement "I usually do well in mathematics"**	**Mean response rating to the statement "I enjoy learning mathematics"**
Stage 7 → 8	1.78	1.78
Stage 7 → 7	1.71	1.76
Stage 7 → 6	2.25	2.88

Note: Double-headed arrows show where the differences between groups are significant ($p \leq 0.05$).

The attitude information in Table 4 also suggests that something changed in year 8 for the 8 students who declined a strategy stage. While there were no significant differences between the attitudes of the three groups of students in year 7, the students who declined a stage in year 8 were significantly less positive in response to the question "I enjoy learning mathematics" than both the students who remained at stage 7 ($p \leq 0.01$) and the students who progressed into stage 8 ($p \leq 0.05$). Similarly, the students who declined a stage were significantly less positive in response to the question "I usually do well in mathematics" than the students who remained stable ($p \leq 0.05$) in year 8. The assessment papers of the students who declined also suggested a significant change in the attitude of these students. Four of the eight did not attempt either of the advanced multiplicative problems in the year 8 test, although they had attempted them all the previous year.

These results suggest that the students who declined felt less positive about mathematics in year 8 than they had previously, particularly in terms of their enjoyment and ability. This is unsurprising given their declining results and is further evidence that the experiences of these students in learning mathematics were different in year 8 than in previous years. While the results show clearly that something changed in year 8 for these students in terms of their experiences and success in learning mathematics, the cause of these changes is unclear.

How Does the Mathematics Ability of Year 8 Students Relate to Their Achievement on the Number Framework?

Because the PAT includes items that focus on aspects of geometry, measurement, and statistics as well as number problems, students' scores on this test provide an estimate of their overall mathematics ability. In general, there was a strong correlation (r = 0.6) between the longitudinal study students' achievement in mathematics (as measured by their year 8 PAT stanine) and their achievement in year 8 against the Number Framework ($p \leq 0.05$). Table 5 shows the percentages of students performing above, at, and below numeracy expectations for each of three PAT stanine groups. Note that these stanine groupings are described as "above average" (stanines 7–9), "average" (stanines 4–6) and "below average" (stanines 1–3) in the teacher manual that accompanies the mathematics test (NZCER, 2006b, p. 13).

Table 5
Students' Year 8 Achievement against the Number Framework by Year 8 PAT Stanine Group

Year 8 PAT stanine	N =	Achievement against the Number Framework		
		Above	Expected	Below
7–9	20	50%	40%	10%
4–6	43	21%	42%	37%
1–3	15		20%	80%

The information in Table 5 shows the relationship between the students' achievement in mathematics and their achievement against the Number Framework. For example, 80% of the 15 students who were rated "below average" on PATs (stanines 1–3) were also below expectation on the Framework, and none of these students were above expectation against the Framework. Similarly, 90% of the 20 students who were "above average" on PATs (stanines 7–9) were also meeting or exceeding expectation against the Framework.

Because the PAT is a standardised assessment, the score profile of a group of students provides information about the achievement of the group in comparison with the PAT national reference sample. Figure 5 shows the profile of PAT results for longitudinal students in 2008 and 2009 alongside the national distribution of results.

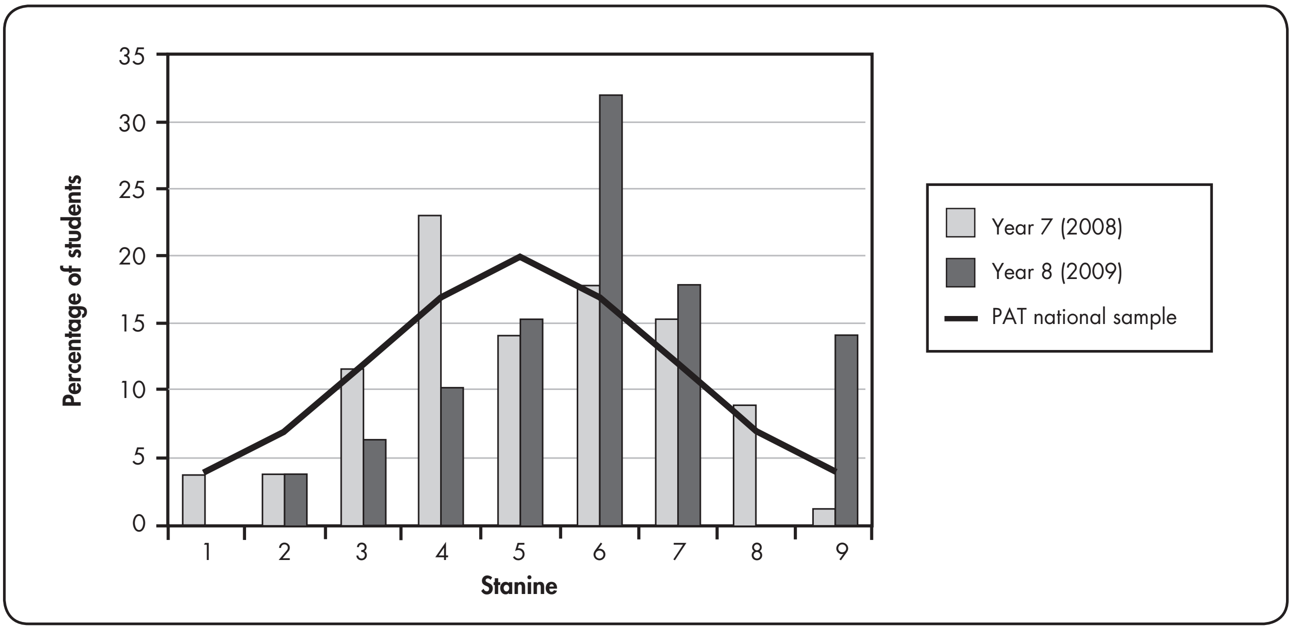

Figure 5. Longitudinal students' PAT results by stanine in years 7 and 8, compared with the PAT national sample

Note that the shape of the curve outlined by the longitudinal students' scores in 2008 is generally displaced to the right of the results for the national reference sample. This indicates that in 2008, the longitudinal students had a stronger performance than the national population on the PAT assessment. In 2009, the longitudinal students further outperformed the national reference sample, as illustrated in Figure 5.

The positive picture of the longitudinal students' PAT achievement in 2009 can be understood, alongside the relatively large percentage of students in the group not meeting expectation against the Number Framework (38%, Figure 2), as the difference between normalised and standards-based assessment. As a normalised assessment, the PAT provides a picture of achievement for students in year 8 in comparison with a national reference sample of actual student results. In contrast, the expectations for students' achievement against the Number Framework are not based on actual student results but on the New Zealand curriculum levels. As a group, the longitudinal students can be considered to be performing well in mathematics in relation to the national population; however only 61% are performing well in relation to the Ministry's numeracy expectations.

To what Extent Do Parents Understand the NDP and Their Children's Performance within Them?

The parent questionnaire asked respondents to indicate their level of agreement with a series of statements about the NDP and mathematics teaching in New Zealand. Table 6 shows these results.

Table 6
Parents' Level of Agreement with Statements about Mathematics Teaching in New Zealand

	Parents' level of agreement			
Statement	Strongly agree	Agree	Disagree	Strongly disagree
I have noticed that maths is taught differently now than when I was at school.	51%	47%	1%	
I believe the way I learned maths was better than the way it is now taught.	14%	29%	45%	4%
Maths in New Zealand schools focuses more on number than on the other areas of maths.	1%	43%	45%	2%
Current maths teaching in New Zealand is focused on developing children's thinking strategies for solving number problems.	27%	64%	5%	1%
Current maths teaching in New Zealand is focused on learning rules for solving number problems.	9%	70%	13%	3%
Children need to know their basic facts and times tables so they can use them to solve number problems.	73%	26%		

Note: Percentages do not all add up to 100 because not all the parents who responded answered all questions.

All but one of the respondents (99%) had noticed that mathematics is taught differently now than when they were at school, and the group were reasonably evenly split over whether current methods were better than previous methods. Forty-three percent of respondents agreed that the way they learnt mathematics was better than the way it is now taught, and 49% disagreed. All respondents agreed that students need to know their basic facts and times tables in order to use them to solve number problems.

The responses indicated a lack of understanding about the use of mental strategies versus the use of rules for solving number problems in current teaching programmes. Ninety-one percent of the respondents believed that current mathematics teaching in New Zealand is focused on developing children's thinking strategies for solving number problems, and 79% believed that current teaching is focused on learning rules for solving number problems. The large percentages of respondents agreeing with both these statements suggests that the respondents are unaware of the significant differences in approach between using number strategies and using rules and may regard some thinking strategies as rules for solving number problems.

Questionnaire responses indicated that parents feel reasonably well informed about their child's progress and achievement in mathematics. Eighteen percent of parents described themselves as very informed, while 53% believed they were moderately informed, and 29% identified themselves as minimally informed. Nearly all parents (97%) reported receiving information through parent-teacher interviews, with slightly fewer (91%) receiving information through school reports. Just over half of the parents (52%) had received information about their child's progress and achievement in mathematics through work samples or portfolios. Other methods of communication that were identified included email and phone communication with teachers, homework, and the results of assessments such as asTTle.

Parents' perceptions of the mathematics competence of their child were related to their child's achievement. There were moderate correlations between parent perceptions of competence and students' stanines on the PAT ($r = 0.33$) and parents' perceptions of competence and students' ratings against the Number Framework ($r = 0.35$). These correlations were significant at the 95% level ($p \leq 0.05$). Tables 7 and 8 provide more detail.

Table 7
Students Level of Competence as Perceived by Their Parents against Their Achievement in Year 8 PAT

PAT stanine	**Parents' Perceptions of Child's Mathematics Competence**		
	Minimally competent	Moderately competent	Very competent
7–9	10%[2]	21%	53%
4–6	50%	62%	33%
1–3	40%	17%	13%
Number of students	10	52	15

Table 8
Students' Level of Competence as Perceived by Their Parents against Their Achievement on the Number Framework

Achievement against Number Framework	**Parents' Perceptions of Child's Mathematics Competence**		
	Minimally competent	Moderately competent	Very competent
Above	20%[2]	19%	47%
Expected	10%	42%	40%
Below	70%	38%	13%
Number of students	10	52	15

[2] Due to rounding, some percentages do not total to 100.

Tables 7 and 8 indicate that most parents have a reasonably accurate understanding of their child's achievement. For example, 62% of parents who rated their child as "moderately competent" had children with average performance on PATs, and 70% of parents who rated their child as "minimally competent" had children who were below expectation against the Number Framework.

Contrary to this general pattern were three respondents who were very inaccurate in their perceptions. Two parents rated their child as very competent, while their child's performance was both below average on the PATs and below expectation against the Number Framework. Similarly, one parent believed their child was minimally competent, while their child's performance was above average on the PAT and above expectation against the Framework.

How Do the Academic Qualifications of Parents Relate to the Performance of Their Children in Mathematics?

Results indicate that the qualifications of parents were not related to their child's achievement in mathematics. There was no significant correlation between parents' highest educational qualification and students' PAT-scale scores.

Does the Mathematics that Students Complete Outside of School Hours Impact on Their Performance?

Fourteen percent of parents identified that their child had received mathematics tutoring outside of school. Seven of these 12 students had private tutoring, while the others received tutoring from a variety of sources, including franchised services such as Mathletics, Kip McGrath, and NumberWorks. Of the 12 students who had received tutoring, seven scored in the average PAT stanine range (4–6), three scored in the below-average PAT stanine range (1–3), and two scored in the above-average PAT stanine range (7–9). There was no significant difference between the PAT scale scores of those students who had received tutoring and those who had not.

Thirty-four percent of parents indicated that their child received mathematics homework most days, 61% identified that their child received mathematics homework occasionally, and 4% responded that their child never received mathematics homework. There was a small negative correlation ($r = -0.22$) between the amount of homework reported by parents and their children's PAT scale scores. This was significant at the 95% level ($p \leq 0.05$), indicating that those students whose parents reported a higher level of mathematics homework had slightly lower PAT scale scores than those students whose parents reported a lower level of mathematics homework.

Concluding Comment

The previous achievement of the students in the Longitudinal Study has been consistently high (Thomas & Tagg, 2008, 2009), and the changes to the achievement pattern of the students reported on in this paper were unexpected. While it should be noted that these results are based on a relatively small sample that was not randomly selected, the unexpected results highlight two issues. The first of these is the value of tracking students' achievement longitudinally so that unexpected results can be identified and further investigated in future years. The second issue highlighted is the dramatically different picture of achievement that can be obtained for the same group of students when the results of standards-based assessments are compared with results obtained from normalised assessment tools.

Acknowledgments

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A Decade of Reform in Mathematics Education: Results for 2009 and Earlier Years

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This paper reports on the impact of the Numeracy Development Projects (NDP) on the mathematics achievement of students whose teachers participated in the programme in the final year of the initial phase (2009). It also looks back at data gathered in years prior to this and presents an overview of the impact of the NDP on students' mathematics achievement, based on NDP data aggregated over several years (2003, 2005, and 2007) for almost one-quarter of a million students. Analysis of students' gains on the Number Framework for the additive and multiplicative domains shows that students made substantial progress in mathematics as a result of their teachers' NDP professional development. However, the absolute levels on the Framework attained by students were in many cases well short of the numeracy expectations for students at particular year levels stated in *The New Zealand Curriculum* (Ministry of Education, 2007) and in the *Mathematics Standards for Years 1–8* (Ministry of Education, 2009). The implications of these findings for further professional learning and development of teachers in mathematics are discussed.

Introduction

It is now almost a decade since New Zealand's Numeracy Development Projects (NDP) began as part of mathematics education reform. The focus has been on building the capacity of teachers to teach mathematics and, through this process, to raise the mathematics achievement of students across the board (Ministry of Education, 2001). The year 2009 marked the last year that teachers at primary and intermediate levels could participate in phase one of the NDP professional development programme. During this time, a revised curriculum document, *The New Zealand Curriculum,* embedded the Number Framework within the system of expected outcomes that apply to all students in New Zealand schools (Ministry of Education, 2007). This curriculum is now mandatory for all schools in New Zealand. Coinciding with the completion of the initial phase of the NDP was the release of a standards document, *Mathematics Standards for Years 1–8* (Ministry of Education, 2009). The standards document differs from the curriculum document in specifying the expected outcomes for students at the end of each year of schooling rather than for a two-year period (that is, curriculum level). A summary overview of these expectations in poster form shows the correspondence between the new mathematics standards and achievement outcomes in the curriculum, as well as the strategy stages on the Number Framework (Ministry of Education, 2010).

The introduction of standards is a contentious issue. There is some concern within the teaching profession about the impact of the standards on classroom teaching practices in mathematics. Pessimists worry about how the data will be used in the future and the extent to which the standards might inadvertently result in a shift in focus away from improving the quality of mathematics teaching to an overemphasis on assessment comparisons between students, teachers, and schools. Optimists see the potential benefits of making schools more accountable to their communities and the possibility that the standards could work to raise levels of student achievement.

Numeracy professional learning and development now focuses on in-depth sustainability of the NDP approach to teaching mathematics, with regular facilitator support for teachers in the schools involved. Facilitators are also supporting a large number of schools as they consider the implications of the mathematics standards document for their classroom programmes and assessment practices.

The evaluation research undertaken as part of the implementation of the initial phase of the NDP has enormous potential to assist in informing teachers and schools as they begin using the mathematics standards as a basis for reporting students' achievement to parents. A consistent finding has been that students at high-decile schools achieve higher stages on the Number Framework than students at middle-decile schools, who in turn reach higher stages than students at low-decile schools (Young-Loveridge, 2004, 2005, 2006, 2007, 2008, 2009). It is impossible to determine the extent to which these differences are a function of the characteristics of the students who attend schools at various decile levels, result from classroom mathematics programmes differing in quality, or are a consequence of lower expectations by teachers working in lower-decile schools. Ritchie's (2004) analysis of teacher career moves indicates that high-decile schools tend to attract teachers with more experience and qualifications than low-decile schools.

This paper reports on the impact of the NDP on the mathematics achievement of students whose teachers participated in the programme in the final year of the initial phase (2009). It also looks back at data gathered in years prior to this and presents an overview of the impact of the NDP on students' mathematics achievement, based on data aggregated over several years of the NDP (2003, 2005, and 2007). The data of almost one-quarter of a million students was aggregated to produce a summary picture of the impact of the NDP on students' mathematics achievement.

Method

Participants

For the first part of the study (the 2009 cohort), only students with complete data (that is, their teachers had submitted both initial and final assessment data to the Ministry of Education website) on the additive strategy domain and on the place value and basic facts knowledge domains were included in the analysis. (2009 was the last year in which teachers were given an opportunity to participate in the initial phase of the NDP professional learning and development programme.) The 2009 cohort consisted of 1645 year 1–9 students in 18 schools in the North Island. All of the students in years 1–6 of this cohort attended high-decile schools, while those at years 7–9 included a small proportion of students attending low-decile schools (between 7% and 11%), with the remainder at high-decile schools (see Table 1). The cohort included a disproportionately high number of boys (62%), particularly at years 7–9. The majority of students in years 1–6 were of European descent (75% to 87%). Students in years 7–9 included more Māori, Pasifika, Asian, and other ethnicities than for students in years 1–6.

Table 1
Percentages[1] of Students in Each Year Level as a Function of Gender, Ethnicity, and School-decile Band (2009)

Composition	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Overall
Number of students	*85*	*95*	*100*	*95*	*99*	*112*	*336*	*343*	*380*	*1645*
2009										
Gender										
Boys	42	52	57	54	52	58	69	68	64	*62*
Girls	58	48	43	46	49	42	31	32	36	*38*
Ethnicity										
European	87	80	81	77	79	75	51	45	40	*57*
Māori	6	7	7	7	6	7	10	10	34	*14*
Pasifika	1		2	1	1	2	9	10	6	*6*
Asian	1	1	1	5	8	3	17	19	12	*11*
Other	5	12	9	10	6	13	13	16	9	*11*
School-decile band										
Low decile (1–3)							7	8	35	*11*
High decile (8–10)	100	100	100	100	100	100	93	92	65	*89*

The second part of this study included participants with complete data (that is, initial and final) whose teachers participated in their first year of professional development for the NDP in either 2003, 2005, or 2007 (see Table 2). This mega-cohort consisted of close to one-quarter of a million children (n = 240 331). Because of the large numbers of participants in each of these three cohorts (n = 137 121, 54 935, 48 275, respectively), it was possible to look at consistencies from one year to another and to examine patterns for small sub-groups such as Pasifika students attending high-decile schools. (Note that only the percentages for European, Māori, and Pasifika students are included in Table 2 under ethnicity because they were the three largest subgroups within the cohort.)

Table 2
Percentages[2] of Students Who Participated in NDP in 2003, 2005, and 2007

Composition	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Total
Number of students	*17 189*	*18 265*	*19 508*	*18 774*	*18 050*	*18 775*	*12 997*	*11 340*	*2223*	*137 121*
2003										
School-decile band										
Low decile	33	35	36	34	34	34	41	42	56	*36*
Middle decile	37	36	35	38	38	39	43	42	27	*38*
High decile	31	29	29	28	28	28	15	16	17	26
Gender										
Boys	51	51	51	51	51	51	52	51	49	*51*
Girls	49	49	49	49	49	49	49	49	51	*49*
Ethnicity										
European	59	58	58	59	60	59	53	54	49	*58*
Māori	22	21	22	24	23	24	28	28	22	*24*
Pasifika	10	11	10	8	9	8	11	11	22	*10*

1 Percentages are rounded to the nearest whole number.

2 Percentages are rounded to the nearest whole number.

Table 2
Percentages[2] of Students Who Participated in NDP in 2003, 2005, and 2007 – continued

Composition	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Total
Number of students	*4 730*	*5 044*	*5 708*	*6 940*	*8 306*	*8 653*	*6 086*	*5 432*	*4 036*	*54 935*
2005										
School-decile band										
Low decile	15	16	17	21	22	21	13	16	11	*18*
Middle decile	46	45	43	42	39	39	56	55	52	*45*
High decile	39	39	40	37	39	40	32	29	37	*37*
Gender										
Boys	52	52	53	52	52	50	52	53	56	*52*
Girls	49	49	47	48	49	50	48	47	44	*48*
Ethnicity										
European	63	63	63	63	62	63	66	63	66	*64*
Māori	20	19	19	19	19	18	20	21	20	*19*
Pasifika	6	6	7	8	9	8	7	8	5	*7*
Number of students	*2525*	*4166*	*3969*	*4217*	*4360*	*4371*	*7352*	*10104*	*7211*	*48 275*
2007										
School-decile band										
Low decile	26	27	28	28	26	28	19	20	17	*23*
Middle decile	32	32	34	32	31	31	42	44	49	*38*
High decile	42	41	38	40	44	42	39	37	35	*39*
Gender										
Boys	52	52	51	53	52	52	53	52	47	*51*
Girls	48	48	49	48	48	48	48	49	53	*49*
Ethnicity										
European	54	56	56	55	57	58	59	62	60	*58*
Māori	22	23	24	23	22	23	22	21	20	*22*
Pasifika	13	12	11	12	10	10	9	8	8	*10*
Total 2003, 2005, 2007										*240 331*

Results

The results are reported in two sections: the first reports on the results for the cohort of students whose teachers participated in their first year of professional development for the NDP in 2009; the second section reports on analysis of aggregated data from 2003, 2005, and 2007, years in which large cohorts of students and their teachers were involved in the NDP.

2009 Cohort

It is clear from Table 1 (see earlier) that the 2009 cohort was quite atypical compared with larger cohorts in previous years (see, for example, Table 2). Compared with more representative cohorts, the 2009 cohort consisted mostly of students attending high-decile schools, apart from the year 9 students, one-third of whom came from low-decile schools.

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[2] Percentages are rounded to the nearest whole number.

Figure 1 presents the proportion of students at each year level who were at various stages on the Number Framework for the additive domain at the end of the year (for percentages, see Appendix A). Figures 2 and 3 present the proportion of students at each year level who, at the end of the year, were at various stages on the Number Framework for the multiplicative and proportional domains, respectively. Consistent with the Ministry of Education's numeracy expectations, approximately three-quarters of the students were at stage 4 on the additive strategy domain by the end of year 2, at stage 5 by the end of year 4, and at stage 6 by the end of year 6 (see Table 3).

The numbers of students at each year level in 2009 were fewer than 100, and all of the students in years 1–6 came from high-decile schools, so it was possible only to analyse the findings as a function of year level.

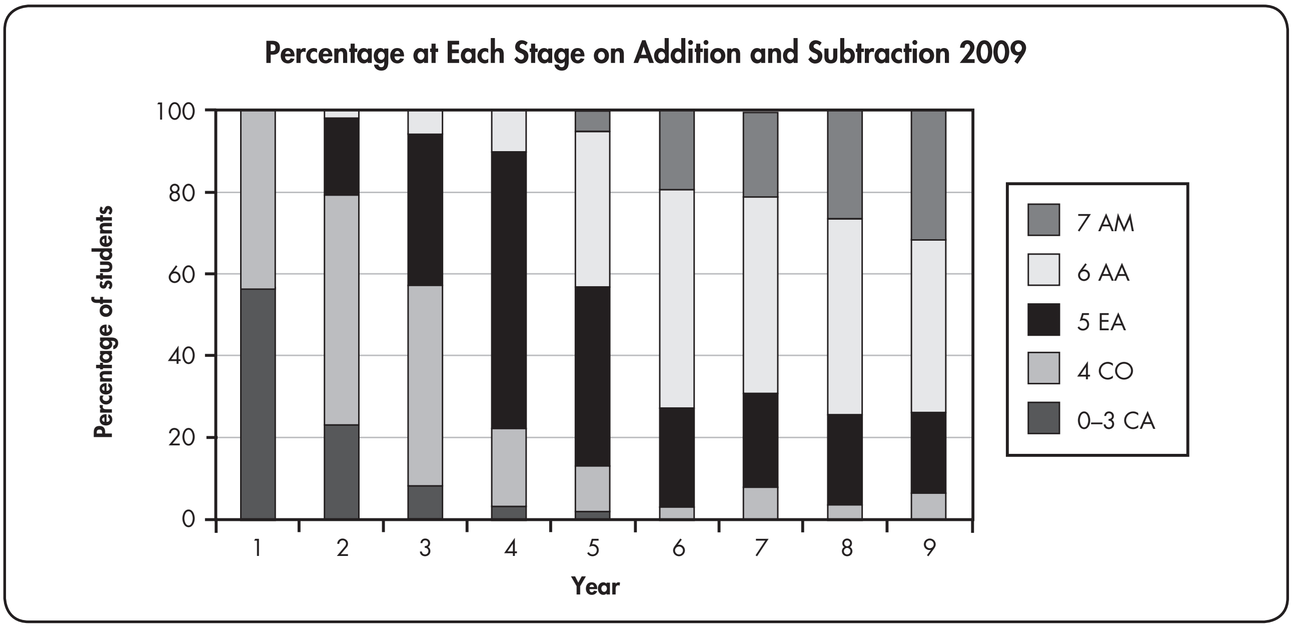

Figure 1. Percentages of students at each stage on the *additive* domain as a function of year level (2009)

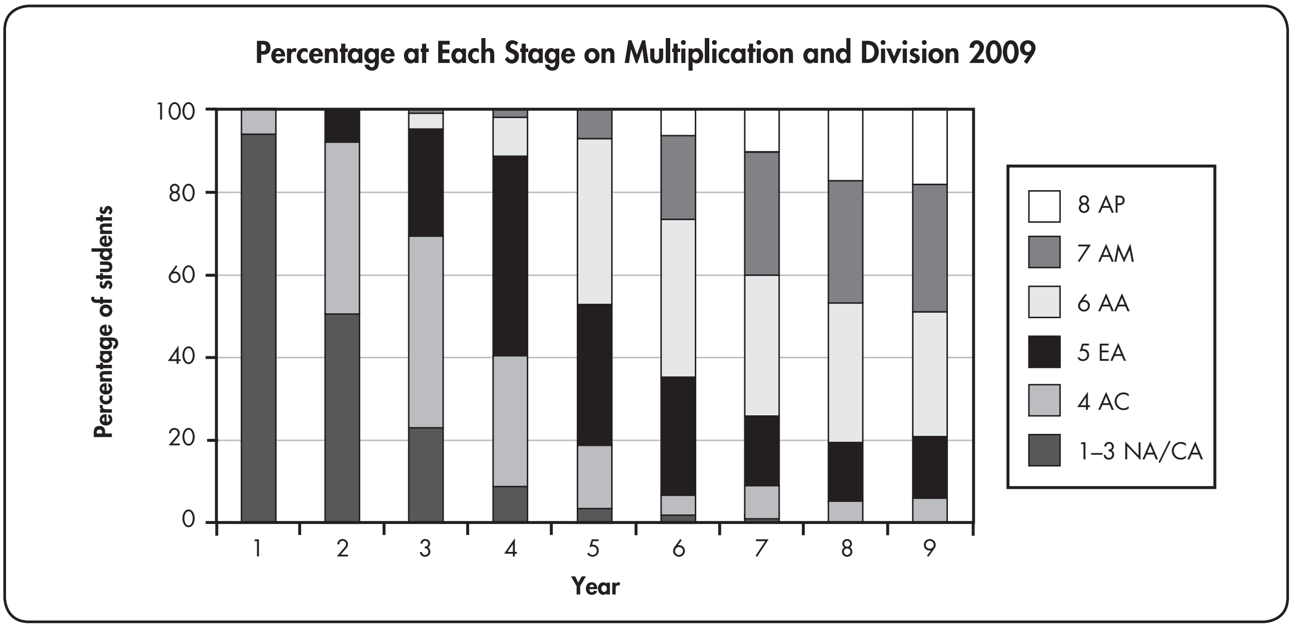

Figure 2. Percentages of students at each stage on the *multiplicative* domain as a function of year level (2009)

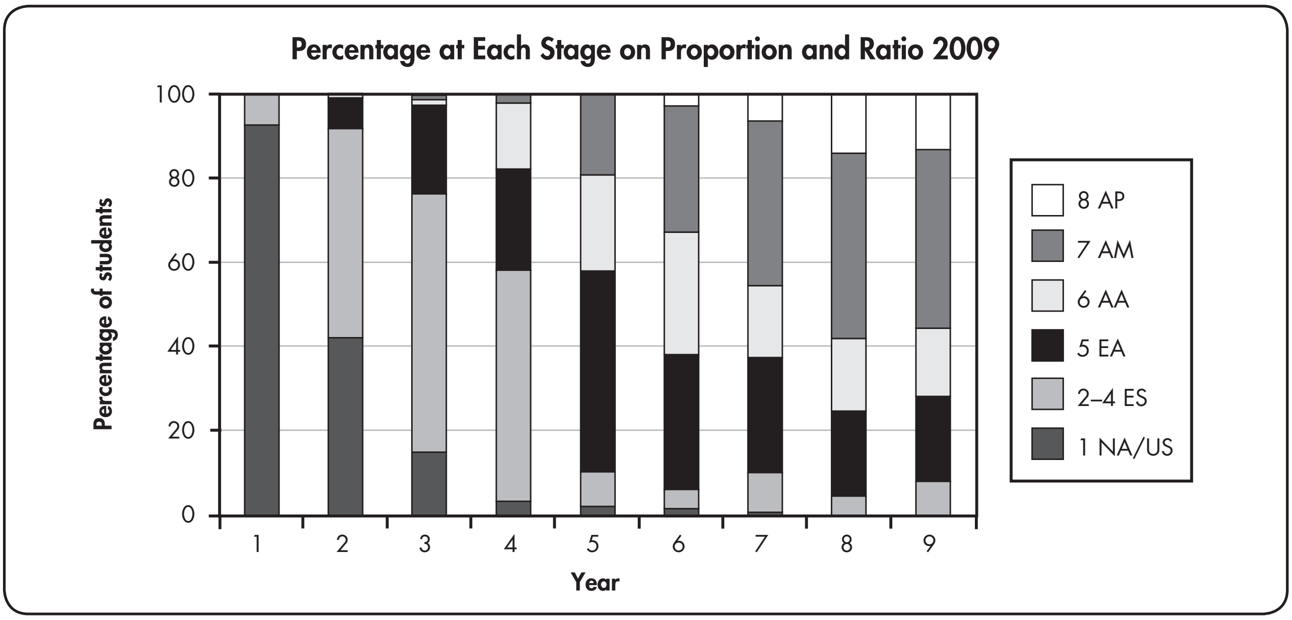

Figure 3. Percentages of students at each stage on the *proportional* domain as a function of year level (2009)

Table 3

Percentages[3] of Students Who Were at or above Particular Stages on the Number Framework as a Function of Year Level and Domain (2009)

Stage and domain, 2009 final	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
STRATEGIES									
Number of students	*85*	*95*	*100*	*95*	*99*	*112*	*336*	*343*	*380*
Additive domain									
Stage 4+	44	77	92	97	98	100	99	100	100
Stage 5+	0	21	43	78	87	97	92	97	93
Stage 6+	0	2	6	11	44	73	69	75	74
Stage 7+	0	0	0	0	5	20	21	27	31
Multiplicative domain									
Stage 4+	6	50	77	92	97	98	99	100	100
Stage 5+	0	8	31	60	82	94	92	95	94
Stage 6+	0	0	5	12	48	65	75	81	80
Stage 7+	0	0	1	2	7	27	40	47	49
Proportional domain									
Stage 5+	0	7	24	42	90	97	90	96	92
Stage 6+	0	1	3	18	42	64	63	76	72
Stage 7+	0	0	1	2	19	35	46	58	56

Aggregation of 2003, 2005, and 2007 Cohorts

Data from students with both initial and final data in 2003, 2005, or 2007 were aggregated to create a mega-cohort. Almost one-quarter of a million students between years 1 and 9 were included in this analysis. Tables 2 and 4 shows that the composition of this mega-cohort was very close to the composition of the New Zealand population, with approximately 30% of the students attending low-decile (1–3) schools, 30% attending high-decile (8–10) schools, and 40% attending middle-decile (4–7) schools. If anything, there were slightly more students at high-decile schools and slightly fewer at middle-decile schools for students in years 1–6. At the intermediate level (years 7 and 8), that pattern

3 Boxes indicate the expected stage at particular year levels in *The New Zealand Curriculum*

was reversed. Only at year 9 were there disproportionately fewer students from low-decile schools, balanced by more students from middle-decile schools. Hence, this mega-cohort could be considered to be representative of the total population.

Table 4
Percentages[4] of Students at Schools within Each Decile Band as a Function of Year Level (aggregated for 2003, 2005, 2007)

School Decile	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Low (1–3)	29	30	31	30	30	30	29	29	22
Middle (4–7)	38	37	37	38	37	37	46	45	46
High (8–10)	33	33	33	32	34	33	26	26	32
Number of students (246 165)	*24 931*	*27 947*	*29 720*	*30 576*	*31 475*	*32 526*	*27 286*	*27 998*	*13 706*

The Additive Domain

Overall

Figure 4 shows the percentages of students at each stage on the additive domain as a function of year level (see Appendix B for percentages). Table 5 shows the percentages of students who reached or exceeded stages 4 through to 7 on the strategy domains. It is clear from Table 5 that the actual percentages of a large representation sample of students whose teachers had completed one year of the NDP fell some way short of the Ministry's numeracy expectations (Ministry of Education, n.d.) and the mathematics standards (Ministry of Education, 2009, 2010). For example, just over half (57%) of the students were able to count on (stage 4) by the end of year 2. Almost two-thirds (63%) were able to use a limited range of additive part–whole strategies (stage 5) by the end of year 4. Just over one-third (38%) could use an extensive range of additive part–whole strategies (stage 6) by the end of year 6. A little over one-third (36%) could use an extensive range of multiplicative part–whole strategies (stage 7) by the end of year 8. It was interesting to note that a greater proportion of year 6 students were beginning to use a limited range of multiplicative part–whole strategies (stage 6) compared with those who could use an extensive range of additive part–whole strategies (stage 6; 53% compared with 38%).

Table 5
Percentages[5] of Students Who Were At or Above Particular Stages on the Number Framework as a Function of Year Level and Domain (2003, 2005, 2007)

Year Level	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Additive domain									
Number of students	*24 931*	*27 947*	*29 720*	*30 576*	*31 475*	*32 526*	*27 286*	*27 998*	*13 706*
Stage 4+	19	57	84	94	97	98	98	98	99
Stage 5+	2	14	41	63	75	84	86	90	92
Stage 6+	0	1	5	14	25	38	46	58	61
Multiplicative domain									
Number of students	*23 106*	*27 376*	*29 955*	*31 021*	*31 946*	*33 051*	*28 149*	*28 445*	*14 708*
Stage 4+	6	34	68	86	92	95	96	96	98
Stage 5+	1	7	26	50	67	79	83	88	91
Stage 6+	0	1	7	21	37	53	60	70	74
Stage 7+	0	0	1	3	9	19	25	36	41

4 Percentages are rounded to the nearest whole number.
5 Boxes indicate the expected stage at particular year levels in *The New Zealand Curriculum*.

Table 5 – continued

Percentages[5] of Students Who Were At or Above Particular Stages on the Number Framework as a Function of Year Level and Domain (2003, 2005, 2007)

Year Level	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Proportional domain									
Number of students	*23 116*	*27 391*	*29 910*	*30 994*	*31 931*	*33 025*	*28 122*	*28 417*	*14 698*
Stage 5+	0	5	21	42	60	72	77	84	88
Stage 6+	0	1	4	14	29	44	51	62	66
Stage 7+	0	0	1	3	9	19	25	36	47

Figure 5 (and Table 6) shows the average initial stage on the Number Framework for the additive domain and the gain from initial to final assessment for each year level (see Appendix C for averages for subgroups). On average, students in year 1 began the project halfway between stage 1 and stage 2. By year 9, the average initial stage was a little above stage 5. It is clear from Appendix C that there is a great deal of consistency from one cohort to another in the first couple of years of school. However, in 2005 and 2007 the average stage on the Framework from about year 3 was higher than in 2003. It is likely that this increase can be explained by the addition of another stage (stage 7) on the Framework for the additive domain, making it possible for students who could add and subtract with fractions and decimals as well as whole numbers to be awarded a higher stage. This increase became more pronounced at higher year levels as more students reached stage 7 on the Framework for the additive domain (see Appendix C). It is important to note that more than half of the mega-cohort consists of students in the 2003 cohort, and at that time, the Number Framework only went to stage 6 on the additive domain and to stage 7 on the multiplicative domain. This would have effectively lowered the mean stage slightly for students at higher year levels for whom a ceiling effect was probably operating in 2003.

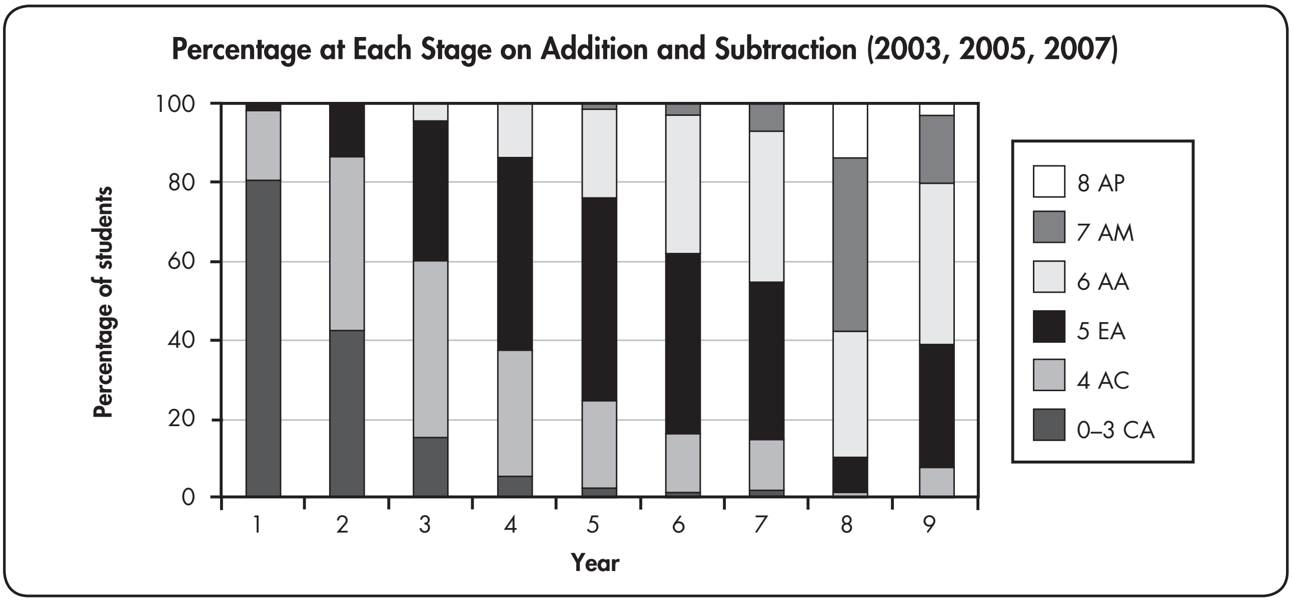

Figure 4. Percentages of students at each stage on the *additive* domain (2003, 2005, 2007)

[5] Boxes indicate the expected stage at particular year levels in *The New Zealand Curriculum.*

Table 6
Average Stage[6] on the Additive Domain of the Number Framework as a Function of Year Level (averaged for 2003, 2005, 2007)

Additive Domain	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Initial stage	1.45	2.44	3.46	4.16	4.54	4.81	4.97	5.16	5.18
Gain	1.06	1.01	0.76	0.53	0.46	0.47	0.42	0.44	0.40
Final stage	2.51	3.45	4.22	4.69	4.99	5.27	5.39	5.60	5.59

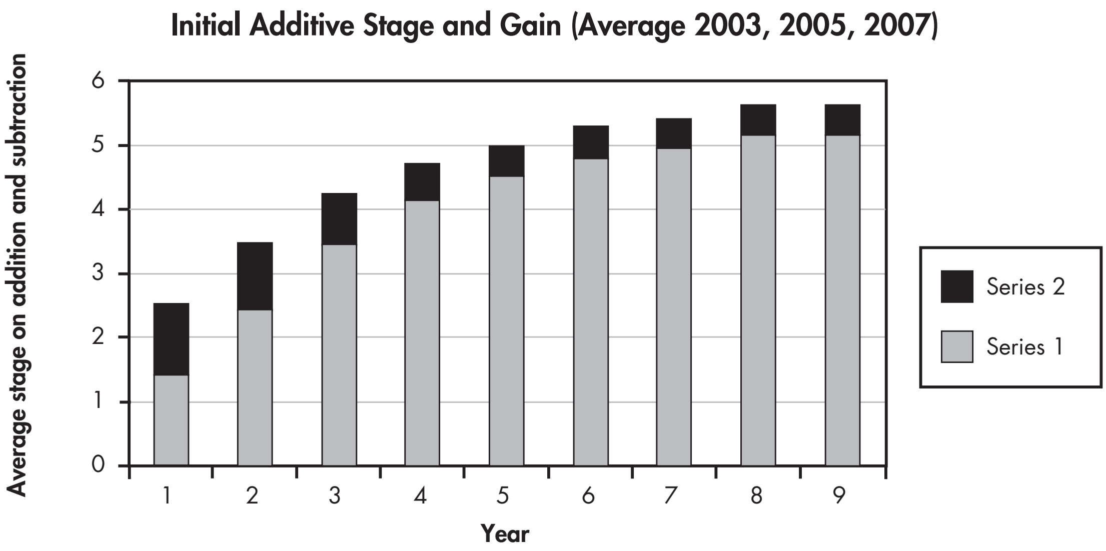

Figure 5. Average stage on the *additive* domain as a function of year level (averaged for 2003, 2005, 2007; Series 1: Initial stage; Series 2: Gain to final stage)

Gender

Appendix C shows the difference in average initial stage for boys compared with girls. Initially, the difference was very small and favoured girls in the first year of school. However, from the second year at school, boys gained a slightly higher average stage, and this average difference gradually increased from 0.01 to 0.19 by year 9. This could be explained by the Math-Fact Retrieval hypothesis, which shows that boys are able to access number facts more quickly and effectively than girls (Geary, 1999; Royer, Tronsky, Jackson, & Marchant, 1999a; Royer, et al., 1999b; Wigfield & Byrnes, 1999).

School decile

Students at high-decile schools reached higher initial stages on the Number Framework for addition and subtraction than those at middle- and low-decile schools (see Appendix C). By year 9, students at low-decile schools had not quite reached stage 5, on average, whereas those at high-decile schools were almost halfway between stage 5 and stage 6. The average advantage for high-decile over middle-decile schools was almost one-quarter of a stage (0.23) and just over half a stage (0.52) for high-decile over low-decile schools.

Ethnicity

Students of European backgrounds reached higher initial stages on the Number Framework for addition and subtraction than those of Māori and Pasifika ancestry (see Appendix C). By year 9, European students were well into stage 5 (mean = 5.29), while those of Māori and Pasifika descent

[6] Average stage is rounded to two decimal places.

were towards the end of stage 4 (means = 4.94 and 4.83, respectively). The average advantage for European students over Māori was just over one-third of a stage (0.37), while that for European over Pasifika students was just under half a stage (0.48). The direction of the differences as a function of ethnicity are consistent with those found in other studies of mathematics achievement, such as the Third International Mathematics and Science Study (TIMSS) (Garden, 1996, 1997, 1998) and the National Education Monitoring Project (NEMP) (Crooks & Flockton, 2002a, 2002b; Flockton & Crooks, 1998; Flockton, Crooks, Smith, & Smith, 2006). However, a major difference between this NDP research and other comparisons is that the effect sizes (standardised differences between groups) are smaller for the NDP than for the international comparisons based on group paper-and-pencil tests. Typically, the effect size for the European-Māori comparison on international studies is about three-quarters of a standard deviation and that for the European-Pasifika comparison is approximately one standard deviation (see Young-Loveridge, 2006). Analysis of ethnicity differences on initial Number Framework stage show them to be much smaller (approximately 0.17 for European–Māori comparisons and 0.29 for European–Pasifika comparison; see Young-Loveridge, 2006). The advantage of the individual orally administered NDP assessment over the paper-and-pencil group assessment is that the questions are read out by the teacher, so the mathematics is immediately accessible to the student. This contrasts with written mathematics assessment tasks, which may not be accessible to poor readers. Māori and Pasifika students have, on average, lower levels of achievement than European students, so they are likely to be more disadvantaged by the literacy demands of a written test. It has been suggested that the reason for the substantial reduction in ethnicity differences for NDP assessment is that the individual diagnostic interview by the student's own teacher provides a much more reliable (albeit more time-consuming) assessment of their mathematics than a paper-and-pencil test to an entire class (Young-Loveridge, 2006).

Ethnicity and school decile

The impact of ethnicity was examined separately for students from high- and low-decile schools. Students from all three major groups (European, Māori, Pasifika) reached higher stages on the Number Framework for the additive domain when they attended a high-decile school rather than a low-decile school. For all three groups, the advantage was about one-third of a stage, with the biggest difference found for Māori students (high-decile advantage = 0.38, on average). European students at high-decile schools reached the highest stage (mean = 5.42 at year 9), whereas Pasifika students at low- and high-decile schools were the lowest (mean = 4.72 and 4.81, respectively). Research on teacher movements in relation to the decile ranking of schools indicates that there is a tendency for more experienced teachers to be in schools with higher decile rankings (see Ritchie, 2004). Low-decile schools tend to have teachers with less teaching experience, on average. This, combined with the greater transience levels for students (and teachers), and lower expectations for students' achievement, may compound the disadvantage experienced by low-decile students.

Gain in Stage on the Additive Domain

Overall

The magnitude of the gain in stage on the addition and subtraction domain varied as a function of year level (see Appendix D). In the first two years of school (years 1 and 2), students gained one whole stage on the Number Framework, on average. In year 3, the gain was about three-quarters of a stage. Between year 4 and year 6, the gain was approximately half a stage. Between years 7 and 9, the gain was between one-third and half a stage. A possible reason for the change in size of gain across the primary years is that students can progress through the early (lower) stages on the Framework far more quickly and easily than they can through the later (upper) stages. What this indicates is that the steps on the Framework are not of equal size, and this is borne out by a statistical analysis (of logits; see Ward & Thomas, this volume).

Gender

Although boys began at a slightly higher initial stage on the Number Framework than girls, their gain was virtually identical, with boys gaining fractionally more than girls initially, and then from year 6, girls gained very slightly more than boys (see Appendix D). The differences ranged from 0.05 to –0.07 but averaged out to zero across all year levels.

School decile

There were very tiny differences in gain between students at schools differing in terms of decile and no consistent pattern across year levels (see Appendix D). On average, there was absolutely no difference in the size of the *gain* as a function of school decile even though there were clear differences in terms of initial stages.

Ethnicity

Students from all ethnic backgrounds made similar gains on the Number Framework for the additive domain, and any tiny differences cancelled each other out when averaged across year level (see Appendix D). One exception to this appeared to be year 7 and 8 Pasifika students in 2003, who made zero gains after their teachers' year of involvement in the NDP professional learning and development programme. However, in 2005 and 2007, the gains for Pasifika were approximately one-third of a stage on the Framework.

Ethnicity and school decile

The analysis of gains as a function of ethnicity and school decile revealed that in 2003, Pasifika students attending low-decile schools actually went backwards, losing, on average, one-tenth of a stage on the Number Framework for addition and subtraction. This exacerbated the high-decile advantage, which was between half and almost three-quarters of a stage. However, this aberration vanished in 2005 and 2007, suggesting that it may have been the result of something unusual happening in the particular intermediate schools that participated in the NDP in 2003 rather than a particular vulnerability for Pasifika students at the intermediate (year 7–8) level. It is also possible that later Schooling Improvement projects for low-decile schools had an advantageous impact on students' numeracy levels (for example, the Manurewa Enhancement Initiative; see Young-Loveridge, 2005).

Effect Sizes

An analysis of the gains was made in terms of effect size for each of the years 2003, 2005, and 2007, and then the average across these years was calculated (see Appendix E). The advantage of using effect size is that it standardises the magnitude of the difference between students' initial Number Framework stage and the final stage at the end of the year of NDP professional development. According to several writers (for example, Fan, 2001; Schagen & Hodgen, 2009), effect sizes of 0.2, 0.5, and 0.8 are considered small, medium, and large, respectively. However, Hattie (2009) argues that for educational outcomes, effect sizes of 0.40 and 0.60 should be considered medium and large, respectively. He argues further that quite small effect sizes may be important in some circumstances. According to Hattie, an effect size of 1.0 means that students receiving a particular intervention programme would outperform 84% of students not receiving that programme. Hattie suggests setting the bar at 0.40 in terms of a desirable effect size and using that as the benchmark to judge effects in education. He asserts that "any innovation, any teaching programme, and all teachers should be aiming to demonstrate that the effects on students' achievement should exceed d = 0.40" (p. 249; d means effect size). This corresponds to students receiving intervention outperforming 66% of students not receiving intervention.

The average effect sizes calculated in the present study for the gain in stage from the initial assessment at the beginning of the school year to the final assessment at the end of the school year were substantial:

approximately one standard deviation in the first two years of school and more than half a standard deviation in the following years of primary schooling (see Appendix E). In the intermediate years, effect sizes were closer to half a standard deviation and in year 9, were between one-third and half a deviation. All the effect sizes for the primary years (1–6) were well in excess of 0.40. At the intermediate level (years 7–8), the only effect sizes not to exceed 0.40 were those for low-decile students at year 7 (d = 0.39) and for Pasifika students at both years 7 and 8 (d = 0.36 and 0.37, respectively). When ethnicity was considered separately for each decile level, it was only the low-decile Pasifika students in years 7 and 8 (d = 0.34 and 0.32, respectively) for whom the effect size did not exceed the 0.40 level. At the year 9 level, boys and Māori students at high-decile schools had effect sizes less than 0.40 (d = 0.37 and 0.36, respectively).

Initial stage on the Multiplicative Domain

The analysis of data for the multiplicative domain was complicated by the fact that not all students were given the opportunity to solve multiplication and division problems and some were only given this chance at the end of the year but not the beginning. Hence a decision was made to include data only from students who had been assessed on the multiplicative domain at the beginning and end of the year of professional development. Very few year 1 students were given the opportunity to solve multiplication and division problems, so the analysis was confined to students in years 2–9 (see Appendix C).

Overall

On average, students in year 2 began on the multiplicative domain halfway between stages 3 and 4, while those at year 9 almost reached stage 6 (mean = 5.85) (see Table 6, Figure 6, and Appendix C).

Table 7
Average Stage on the Multiplicative Domain of the Number Framework as a Function of Year Level (averaged for 2003, 2005, and 2007)

Multiplicative Domain	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Initial stage	3.41	3.79	4.35	4.80	5.21	5.44	5.74	5.85
Gain	0.63	0.75	0.71	0.71	0.68	0.63	0.62	0.52
Final stage	4.04	4.54	5.06	5.51	5.89	6.07	6.36	6.36

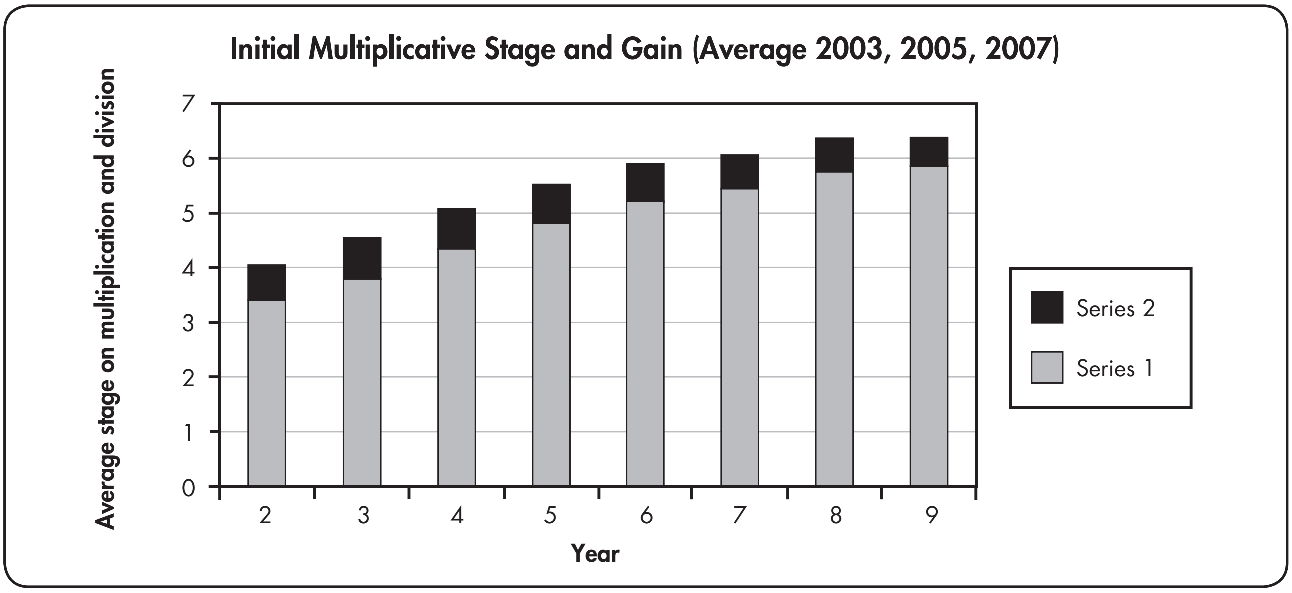

Figure 6. Average stage on the *multiplicative* domain as a function of year level (averaged for 2003, 2005, and 2007; Series 1: Initial stage; Series 2: Gain to final stage)

Gender

Boys did consistently better than girls on the multiplicative domain at all year levels and for all three cohorts (see Appendix C). On average, the boys were one-fifth of a stage higher on the Number Framework for multiplication and division than were girls.

School decile

As for the additive domain, students from high-decile schools reached the highest Number Framework stages on the multiplicative domain, with an advantage over those from middle-decile schools of almost one-quarter of a stage (0.24), while the high-decile advantage over low-decile was more than half a stage (0.54), on average (see Appendix C).

Ethnicity

European students reached stage 6, on average (mean = 6.05), whereas Māori and Pasifika students reached partway between stage 5 and 6 (mean = 5.52 & 5.33, respectively) (see Appendix C).

Gain on the Multiplicative Domain

Overall

On average, students annually gained between half and three-quarters of a stage on the Number Framework for multiplication and division (see Appendix D). The highest gains were for students in years 3–6 and the lowest at year 9.

Gender

Boys and girls made very similar gains on the multiplicative domain, with virtually no difference when differences were averaged across the three cohorts and over all year levels (mean = 0.01) (see Appendix D).

Decile

There was a small difference, with students from high-decile schools making slightly higher gains than those from middle-decile schools (mean difference = 0.04) and those from low-decile schools (mean difference = 0.09) (see Appendix D).

Ethnicity

European students made slightly higher gains than those of Māori (mean difference = 0.04) and Pasifika descent (mean difference = 0.07). However, these differences were very small and indicate that all ethnic groups benefited equally from participation in the NDP (see Appendix D).

Effect Sizes

An analysis of the gains on the multiplicative domain was made in terms of effect size (see Appendix E). Effect sizes on the multiplicative domain were not quite as large as for the additive domain. However, all were at least three-quarters of a standard deviation in years 2 and 3, while those for students in years 4–8 were at least half a standard deviation. Those for students in year 9 were between one-third and half a standard deviation. None of the effect sizes calculated was below Hattie's (2009) "hinge-point" of 0.40. However, it is important to remember that this analysis only includes students who were assessed on the multiplicative domain at the beginning and end of the year of NDP professional development. Students who were not initially assessed on the multiplicative domain (because the only strategy domain included in Form A is the additive domain) but by the end of the year had made sufficient progress to be assessed on Form B or C (which include both the multiplicative and proportional domains) were excluded from the analysis because of missing data.

Discussion

One of the notable findings of this analysis was the remarkable consistency across the three cohorts in initial stage on the Number Framework and the gain in stage on the Framework for the two domains examined. When the large sample size (almost one-quarter of a million students) is taken into account, it is clear that the findings have considerable weight. Although there were some differences between various subgroups (based on ethnicity and school decile level) in terms of initial stage on the Number Framework, it was encouraging to see that all subgroups made very similar gains in stage. When the effect sizes were examined in relation to Hattie's (2009) 0.40 hinge point, it was clear that the NDP professional development programme has produced substantial gains in terms of progress on the Framework. Teachers of students in the early years can be justifiably proud of the effect sizes of approximately one standard deviation that they produced on the additive domain. According to Hattie (2009), this corresponds to acceleration of between two and three years of education. He suggests that "teachers average an effect of d = 0.20 to d = 0.40 per year on student achievement" (p. 16). In his own work, the yearly effect in reading, mathematics, and writing from years 4 to 13 is d = 0.35. He argues that "teachers should be seeking greater than d = 0.40 for their achievement gains to be considered above average and greater than d = 0.60 to be considered excellent" (p. 17). It is clear that most of the gains from NDP professional development were above average, and many of these gains were excellent. Teachers have a lot to be proud of.

The next big question is, are these gains enough when considered in relation to desirable levels of achievement? Whether the data is examined in terms of percentages of students reaching or exceeding particular levels on the Number Framework or in terms of the average stage attained on the Framework, the picture is consistent. Students after the first year of the NDP professional development programme are not yet reaching the levels thought to be needed for them to reach an acceptable level of achievement by year 12 (the fourth year of secondary schooling) (see Ministry of Education, 2007, 2009, n.d.).

For example, data from year 6 students has shown that by the end of the first year in which their teachers participated in NDP professional development, fewer than half of the students at this level were at stage 6 or higher by the end of the year (between 41% and 49% over the period 2005–2008) (see Young-Loveridge, 2005, 2006, 2007, 2008, 2009). This is well short of the majority of students (75% to 80%) expected to be at stage 6 or higher by the end of year 6 (Ministry of Education,2007, 2009, n.d.). It should be noted that the proportion of year 5 students at stage 6 was considerably lower (between 22% and 34% over the period 2005–2008). Likewise, the percentages of year 8 students reaching stage 7 by the end of their teachers' first year of NDP were considerably less than half (between 28% and 42% over the period 2005–2008). An analysis of changes in students' end-of-year achievement levels for NDP schools that submitted data in two consecutive years (2007 and 2008) shows little, if any, gain from the first to the second year of the NDP, except for students in year 2 (see Young-Loveridge, 2009).

These findings may raise important questions for some people about the levels at which the expectations are set. However, there is clear evidence that the standards need to be set high, even though it may be some time before the majority of students meet the standards set for their particular year levels. The decision about what standard to expect at a particular year level is based on several important sources of evidence. One is research evidence showing that students need to be multiplicative thinkers if they are to engage meaningfully with instruction in algebra at secondary school (Brown & Quinn, 2006; Lamon, 2007; Wu, 2002). Hence, the evidence clearly shows that students need to be at stage 7 or higher on the Number Framework by the end of year 8.

Another reason for having the expectations at the current level comes from *The New Zealand Curriculum* (Ministry of Education, 2007). Unlike previous curriculum documents, the latest curriculum document is based on research evidence about progressions in students' thinking rather than on a collection of isolated topics to be covered by teachers. This curriculum also reflects recent reforms in mathematics education that have led to prioritising conceptual understanding over procedural knowledge and skills. The curriculum overall is designed to support the development of learners in being "confident and creative, connected, and actively involved" (Ministry of Education, 2007, p. 4) – in other words, innovative problem solvers who can confidently think through the solutions to new problems using the tools they have acquired. It is no coincidence that both *The New Zealand Curriculum* and the *Mathematics Standards* have the expectation that students at the end of year 8 need to be multiplicative thinkers. Although the national standards were written later, they were designed to align with the curriculum document (see Ministry of Education, 2007, 2009).

Another important source of evidence for expectations is the curriculum documents of other education systems, which reveal that the expectations for students at the end of year 8 in New Zealand to be advanced multiplicative thinkers (stage 7 on the Number Framework) are set somewhat later than in other jurisdictions. For example, in the United States, students at grade 5 (year 6) are expected to be "developing an understanding of and fluency with addition and subtraction of fractions and decimals" and in the following year (grade 6 [year 7]), "developing an understanding of and fluency with multiplication and division of fractions and decimals" as well as "connecting ratio and rate to multiplication and division" (NCTM, n.d.). These expectations are at least stage 7 on the New Zealand Number Framework – an expectation for the end of year 8. At grade 7 (year 8), the United States students are "developing an understanding of and applying proportionality", which matches stage 8 advanced proportional on the New Zealand Number Framework, an expectation for the end of year 9. In New South Wales, the expectation at grade 6 (year 7) is that a student operates with fractions, decimals, percentages, ratios and rates (Board of Studies NSW, 2006). It is not clear whether or not students in other jurisdictions actually meet the expectations described in their curriculum documents. However, the implications of such documents are that New Zealand students at the end of year 8 should ideally be working at stage 8 advanced proportional part-whole thinking. Only about 12% of year 8 students were advanced proportional thinkers by the end of the school year (see 2003, 2005, 2007 in Appendix B). Hence, it is very clear that New Zealand's Mathematics Standards need to be at least as high as they currently are.

Data from cohorts of students whose teachers were participating in their first year of NDP training provide an insight into the initial impact of teachers' professional development on students' achievement after just one year of participation. The Longitudinal Study, which continued to gather data from schools after their initial involvement in the NDP, was designed to provide a longer-term view. From 2002 to 2006, the NDP Longitudinal Study reported on the performance of students in a group of schools who continued to collect numeracy data in the years following their initial implementation of the NDP (Thomas & Tagg, 2005, 2006, 2007). Since 2007, the Longitudinal Study has tracked the progress of a cohort of students from these schools for whom numeracy data is available for every year of their primary schooling (Thomas & Tagg, 2008, 2009). The data from the Longitudinal Study in 2006 was used to inform the setting of expectations for students at particular year levels. In 2006, the Longitudinal Study sample contained disproportionately more students from high- and middle-decile schools and fewer from low-decile schools, raising some questions regarding the generalisability of these results to students from low-decile schools.

Summarising the evidence presented in this paper thus far, it is clear that teachers have made considerable progress in their understanding of how numeracy develops and what they can do to support their students' numeracy learning. However, it is also clear that there is still a long way to go. Teachers are up against a systemic problem that dates back to their own schooling and to that of their teachers (and probably also to that of their teachers before them). Researchers have noted that teachers tend to teach the way they themselves were taught when they were at school (see Grootenboer, 2001, 2002). Teachers need to realise that low levels of mathematics achievement are a consequence of problems with the entire education system rather than solely the responsibility of a few individual teachers. The problems go back generations and are the legacy of decades of mathematics teaching being procedural rather than conceptual, a pattern that has become deeply ingrained over time (see Skemp, 2006). Moreover, the beliefs, attitudes, values, and feelings that teachers have about mathematics that were formed when they were at school have probably had a substantial impact on their teaching. Although there are international mathematics education reforms calling for a focus on conceptual understanding instead of procedural skills, there is little, if any, writing about what an enormous challenge that represents for the teacher education process, both pre-service and in-service. It would be naive to think that one or two years of professional development could miraculously change teachers' attitudes, feelings, beliefs, and values as well as their understanding of mathematics. It seems likely that many teachers may need considerably more professional development if they are to acquire the deep and connected understanding that they need to move a greater majority of their students closer to the expected levels of achievement. It is important that this process of ongoing professional learning (including having higher expectations for students) continues throughout the primary years of schooling. Becoming an advanced multiplicative thinker (stage 7) by the end of year 8 is only possible if all the teachers in the years leading up to year 8 have had appropriate expectations of their students. This needs to be seen as an issue for all primary teachers to address, not just those working at the year 7 and 8 level. Given the amount of time needed to develop an understanding of the complexities of additive and multiplicative thinking, a considerable lead-up time is necessary in order to provide a strong foundation on which those ideas can be built. We must not underestimate the challenge of bringing about deep and lasting change in teachers' understanding of mathematics learning.

If we accept that further efforts are required to support primary teachers to the point where they are all confident and skilled teachers of mathematics, the question then is what does research evidence indicate about possible directions that might help to achieve that goal. One area that might be fruitful is the place of counting on the Number Framework. It has been suggested by Sophian (2007) that in many education systems there is an overemphasis on counting. Sophian argues that the focus on discrete quantity (i.e., "how many?") draws attention away from continuous quantity (i.e., "how much?") and could help to explain the difficulties that students worldwide experience with fractional quantity, which requires co-ordination of both "how many?" (e.g., the numerator) and "how much?" (e.g., the denominator). The NDP data analysis shows that students in the early years of school are using counting strategies for longer than is desirable, according to the expectations conveyed in the curriculum and the national standards (Ministry of Education, 2007, 2009). For example, the highest counting stage on the Number Framework (stage 4, advanced counting) is, on average, attained by students at the end of year 3 (see Table 6). Likewise, students do not reach stage 5 (early additive part-whole thinking), on average, until the end of year 5. The strong emphasis on counting in the NDP may send students the wrong message, resulting in some students being reluctant to move away from counting at any stage because of its proven reliability for solving problems with small numbers. It is of great concern that even at years 8 and 9, approximately 5% of students are still using counting to solve addition and subtraction problems (see Appendix A).

Such a prolonged period using counting strategies may occur because teachers do not begin early enough helping students to develop an understanding of part-whole relationships among numbers and of the use of partitioning and recombining strategies as a means for determining the answers to simple problems. Teachers could begin introducing ideas about the composition of numbers as wholes made of different parts right from the early days of school. For example, there is no reason why five-year-olds could not investigate different ways to "make" six and other small numbers. They could explore the way that six can be made of five and one, or four and two, or three and three. They might also be introduced to the idea of six as double three or as three pairs of two, the beginnings of ideas important for work with the multiplicative domain. They need to become familiar with many different representations of six, such as the stylised pattern of six dots on a dice arranged in a rectangular shape, a row of five dots plus an extra dot as represented on a tens frame, or one hand of five fingers plus the thumb of the other hand. They should be estimating and making predictions about how many would result from adding another one on or taking one away from that collection of six objects.

Another reason for prolonged counting may be that subitising (the instant naming of quantities, such as the stylised patterns presented on dot dice and dominoes) is not encouraged as an alternative way (apart from counting) of determining quantity for small collections of objects. There is considerable research to suggest that subitising for small sets precedes counting and can provide a valuable introduction to concepts of numerical quantity. Some researchers have suggested that subitising might suit certain students better than counting (e.g., Hannula, Rasanen, & Lehtinen, 2007; Willis, 2000). A distinction has been made in the literature between perceptual and conceptual subitising (Clements, 1999). The latter uses the initial patterns that are learned as a basis for building more complex patterns. Although quantities represented on ten-frames may come to be subitised eventually, using other representations such as double two (four) or double three (six) displayed on dot dice and dominoes might be easier to learn than a linear display of four dots/counters or a line of five plus an additional dot (see Clements, 1999). Support for part-whole strategy instruction could also come from building knowledge of basic facts for small numbers (for example, "one and one", "two and one", "two and two", "three and one") right from the beginning of schooling, instead of (or as well as) counting. (Note: the smallest basic fact assessed in NumPA is "2 + 3".)

The existence of so many micro-stages at the lower end of the Number Framework may suggest to some teachers that the steps between these stages are of the same magnitude as those higher up the Framework. Hence they do not appreciate the importance of moving through those early stages at a reasonably brisk pace. Another possibility is that the alignment between strategy and knowledge at corresponding stages may suggest to some teachers that students need to be able to meet all of the stage 4 knowledge requirements before being given instruction in stage 5 (early part-whole thinking) strategy ideas. Teachers holding that view would require students to show that they could read 3-digit numerals, order numbers in the sequence up to 100 both forwards and backwards, and know all combinations of single-digit addends for sums and differences up to 20 before they could be considered ready for instruction in stage 5 strategy (using part-whole strategies to join or separate two small collections). Yet it is possible to work with concepts of parts and wholes with far less knowledge than is required at stage 4. Could that knowledge be developed alongside instruction focused on early additive part-whole thinking?

There is now considerable evidence to support the idea that in order to succeed in mathematics, students need to develop an appreciation of the pattern and structure of number (Mulligan & Mitchelmore, 1997, 2009; Mulligan, Prescott, & Mitchelmore, 2004). Mulligan and colleagues found that students who were low achievers in mathematics did not appear to notice the regularities and structure in the mathematics that was presented to them. NDP support materials are consistent with the idea of helping all students to appreciate pattern and structure within the number system. However, the suggestions above may help to strengthen that process.

Evidence from research on the classroom practices of teachers in countries that scored high on international comparisons of mathematics achievement has shown that they spent considerable time focusing on making connections among mathematical ideas (Stigler & Hiebert, 2004). This contrasts markedly with the practices of teachers in the United States, who tended to turn most problems into procedural exercises. Students in the US scored significantly lower in mathematics than these countries (e.g., Czech Republic, Hong Kong, Japan, Netherlands). Components of the NDP such as the Number Framework, the diagnostic interview, and the books providing support materials are consistent with the idea of making connections among mathematical concepts, but such practices would require a good understanding of mathematics as well as a high level of familiarity with the complexities of the framework and the support materials. However, it is not clear just how much time is needed for teachers to develop a deep understanding of the NDP so that they can then help their students to make connections among mathematical ideas.

An important consideration with respect to students' progress in mathematics is the length of the NDP professional learning and development programme. The numeracy strategy policy determined that teachers would get support over two years in the NDP approach to teaching mathematics. However, research indicates that it takes considerable time to effect major change in approaches to teaching mathematics (Anthony & Walshaw, 2005; Lamon, 2007; Reid & Zack, 2010). In retrospect, it may be that teachers needed to receive support by facilitators for considerably longer in order to effect a more permanent change in their classroom mathematics practice. A research project with teachers in classrooms designed to explore the impact of the NDP approach on the teaching of addition and subtraction (see Young-Loveridge & Mills, this volume) and multiplication and division (Young-Loveridge & Mills, 2009a, 2009b) showed that many teachers stuck very closely to the printed NDP resources (the "pink" books), which describe examples of possible lessons that they could use (Ministry of Education, 2008c, 2008d). This could reflect the low levels of confidence that many teachers still have about their mathematics teaching, despite their involvement in NDP. Decisions about the ideal duration of NDP professional learning and development programmes were influenced by the need to balance the depth of impact with the provision of opportunity for as many teachers as possible. Over the decade in which the initiative was introduced to teachers, teachers in virtually all primary and intermediate schools were given the opportunity to participate. It would have been difficult to justify denying half of these teachers the opportunity for professional learning in order to give the other half twice as much support.

The issue of teachers' content (subject matter) knowledge (SMK) and pedagogical content knowledge (PCK) in mathematics has been the focus of considerable writing (Ball et al., 2005, 2008; Ward & Thomas, 2006, 2007, 2008, 2009). Evidence has clearly established the need for teachers working at the primary/elementary level to have greater understanding of mathematics and ways to support their students' mathematical development. International reforms in mathematics education have moved the emphasis away from drilling students in rules and procedures (instrumental learning) towards a deeper and more connected conceptual (relational) learning and understanding (Lambdin & Walcott, 2008; Skemp, 2006). Connected to this issue of teacher knowledge is the topic of classroom norms for mathematics instruction and the importance of getting students not just to explain their strategies, but also to critique them and justify them to each other. This is particularly important for students from low-decile schools who may not be familiar with argumentation processes. Hunter (2010) has shown how teachers in a low-decile school with many Maori and Pasifika students, who thought initially that they had high expectations for their students, later realised through their work with a teacher educator that their students were in fact capable of engaging in far more sophisticated mathematical argumentation and went on to achieve substantially higher on the Number Framework as a result.

Finally, it is important to think about the implications of the NDP data analysis for the implementation of the mathematics standards. The results presented here indicate that, in the initial implementation of the mathematics standards, many students in the middle and senior years may not reach the expected standards for their year level. Teachers need to understand that, at this point in time, the mathematics standards are *aspirational* for many students rather than *typical* and quite large numbers of students may be initially "below expectations". This should be recognised as a temporary situation that is inevitable while teachers are continuing to upskill themselves in mathematics content knowledge and pedagogical content knowledge. As part of that process, teachers of junior classes have the challenge of moving their students beyond counting more quickly than in the past by putting a greater emphasis on partitioning small numbers, by including subitising as another important knowledge domain in their classroom programmes, and by encouraging students to become familiar with the "basic facts" for small number combinations.

No one has ever claimed that mathematics education reform was going to be easy. Bringing about substantial change in the way mathematics is taught throughout the primary school (and beyond) is a huge challenge. This researcher believes that we have underestimated just how difficult that process is. As writers have recently pointed out, "elementary mathematics is anything but elementary" (Bahr & de Garcia, 2010, title page).

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Analysis of the Number Framework

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This paper reports on a Rasch analysis of the Number Framework. The analysis provides information on the relative difficulties of the stages and domains of the Framework and the relative extent of the stages within each domain. The analysis supports the validity of the Framework and the theory on which it was based. The stages within each domain describe increasing levels of difficulty, and the difficulty level of each stage across the domains is relatively consistent. While generally supporting the structure of the Framework, the analysis identifies several areas in which it requires refinement.

Background

The Number Framework is one of the key features of the Numeracy Development Projects (NDP). It describes a progression of student learning in terms of both the techniques that students need to develop in order to solve number problems (strategies) and the key items of number knowledge that they need to learn. The Framework is structured in domains, with each domain describing one aspect of students' ability with number. Each domain consists of a number of stages through which students progress as they gain competence (Ministry of Education, 2007).

The Number Framework was developed on the basis of research that investigated children's arithmetical thinking and found identifiable common progressions in the development of number concepts (Bobis et al., 2005). The development of the Framework was theoretically derived and aimed to ensure that each stage was distinct and the stages within each domain described increasingly sophisticated thinking. For example, stage 5 in the multiplicative domain (using known number facts to solve multiplication problems) involves thinking that is theoretically more sophisticated than the thinking in stage 4 in the same domain (using counting strategies). The Number Framework was also developed to ensure that students' stages across the domains are generally well matched. For example, a student at stage 4 on the additive domain (counting on to solve addition problems) is also theoretically likely to be at stage 4 on the multiplicative domain (skip-counting to solve multiplication problems) because both of these stages involve the use of sophisticated counting strategies. To maintain students' consistency across the Framework, some stages in some domains span more than one stage, for example, stages 2–4 in the proportional domain, and some domains omit the lowest stages. For example, stages 2–3 is the first stage of the multiplicative domain.

Limited empirical analysis of the Number Framework was carried out some years ago (Irwin, 2003; Irwin & Niederer, 2002), although the Framework has been updated since that time. This empirical analysis, along with qualitative feedback from the implementation of the NDP, suggested that the difficulty levels of the stages are not equally spaced across any domain and any one stage varies in difficulty across the domains (Irwin, 2003; Irwin & Niederer, 2002; Ward & Thomas, 2002). These factors have limited efforts to compare the progress of samples of students across a range of year levels and Framework stages (Ward & Thomas, 2008; Ward & Thomas, 2009).

This paper reports on an extensive empirical analysis that provides information on the relative difficulties of the domains and the extent of the stages within each domain. A one-parameter logistic (Rasch) item response model is used to place all the stages and domains of the Number Framework on a unidimensional scale. This scale, which is measured in units called logits (natural logarithm units), is used to describe the relative difficulties of the stages within and across each of the domains. It is important to be clear that the unidimensionality of the logit scale constructed under Rasch analysis does not imply that the scale measures a single psychological construct or a single skill. What it does mean, in the case of the Framework, is that, to the extent that the domains fit the scale, a similar cluster of cognitive characteristics and skills is likely to underpin performance across the various domains. For this reason, if the domains fit the scale well, progression through the stages of one domain would tend to be quite highly correlated with progression through the other domains.

Results are presented and discussed in three sections. The first section investigates whether the use of Rasch techniques is justified by the given data set. The second section describes the relative difficulties of the stages and domains of the Number Framework, using the estimated difficulty scores (in logits), and the final section uses transition points between adjacent stages to describe the relative extents of the stages within each domain.

Method

Participants

This paper analyses the results of 3742 students from the 2006 online numeracy database. These students were selected to be part of a multi-year dataset on the basis that it was possible to link their results over three consecutive years, from 2006 to 2008. The dataset was selected to ensure that each student had an entry for the additive strategy domain and for at least one of the other two strategy domains (multiplicative and proportional). Students were also required to have entries for the knowledge domains of both place value and basic facts because these were considered to be the most significant of the knowledge domains in terms of determining students' progress. Further analysis of the results of these students over consecutive years can be found in this volume.

Tables 1 and 2 provide demographic information on the students in the sample.

Table 1
Ethnicity and Gender of Participating Students

Ethnicity	**Male (%)**	**Female (%)**	**Total**
NZ European	50.8	49.2	2625
Māori	50.0	50.0	578
Pasifika	43.1	56.9	232
Asian	45.8	54.2	179
Other	52.3	47.7	128
All	50.0	50.0	3742

Table 2
Year Level and School Decile of Participating Students

Year level	Decile 1–3 (%)	Decile 4–7 (%)	Decile 8–10 (%)	Total
Years 0–1	9.1	25.6	65.3	582
Year 2	9.8	29.6	60.7	737
Year 3	11.9	31.6	56.5	798
Year 4	11.3	29.3	59.4	860
Year 5	15.2	26.8	58.0	343
Year 6+	14.6	27.7	57.7	383
All	11.5	28.9	59.7	3703

Note: 39 students were at schools with no decile rating (private schools).

Method

Student achievement data was collected by teachers who used the Numeracy Project Assessment (NumPA) and entered the data into the online numeracy database. The NumPA is an individual interview that provides information on students' knowledge and strategy stages. The stages against which students are assessed in the NumPA (Ministry of Education, 2008) predominantly correspond with those described by the Number Framework (Ministry of Education, 2007). Where there are discrepancies between the NumPA and the Framework, the stages of the NumPA are used in this paper.

Analysis

Initial analysis investigated whether the use of the Rasch model was appropriate. In particular, the Rasch model fits data to a unidimensional logistic scale, and it is important to ensure that the covariance structure of the data is approximately unidimensional. Correlations between the strategy and knowledge stage ratings of all students (Spearman's coefficients) were investigated and a principal components analysis (without rotation) was used to determine whether a logistic unidimensional scale would be a valid representation of the data.

Having established the approximate unidimensionality of the covariance matrix, difficulty estimates for each stage of each domain were calculated under a Rasch model, using a maximum-log-likelihood procedure. The Rasch model is called a one-parameter model because it produces a single model parameter for each item, which is an estimate of the difficulty of the item in terms of a position on the overall logistic scale. In fact, however, the model also produces an estimate of the ability of each student, again with respect to their position on the overall scale. Item difficulty and student ability are estimated on the same scale, and the difficulty of a stage is defined to be the ability level at which a student would have a 0.5 probability of "correct performance" on that item, that is, of being at that stage or higher. The logit scale constructed by the Rasch analysis is unidimensional, so the difficulty and range of the stages can be meaningfully compared across domains.

Probability functions describing the likelihood of students of any given ability being rated at each stage of each domain were constructed. This enabled the points at which there was an equal probability of being in either of two adjacent stages to be identified. These were characterised as transition points between the stages. These transition points were then used to determine the relative extent of stages on the overall scale.

Findings and Discussion

Factor Structure of the Domains

The Rasch model fits all of the domains to a unidimensional logistic scale. A consequence of this is that students' progression in all of the domains of the Number Framework is likely to be governed by one common set of cognitive processes or to reflect the nature and sequence of instructional programmes. Correlations between students' ratings in the three strategy and two knowledge domains, shown in Table 3, tend to support this, with medium to high correlations evident between all five domains.

Table 3
Correlations between Students' Stage Ratings in the Five Domains

	Additive	Multiplicative	Proportional	Place value	Basic facts
Additive	1	0.665	0.560	0.770	0.807
Multiplicative		1	0.633	0.585	0.651
Proportional			1	0.514	0.598
Place value				1	0.781
Basic facts					1

A principal-components analysis without rotation was conducted to test whether a single dominant component could explain the majority of the variance between students' ratings in the five domains. Table 4 shows the eigenvalues for, and the proportion of the total variance explained by, each component in descending order of magnitude.

Table 4
Eigenvalues of and Variance Explained by Each of the Principle Components

Component	Eigenvalues	Percentage of variance explained	Cumulative percentage
1	3.32	66.4	66.4
2	0.54	10.8	77.2
3	0.43	8.5	85.7
4	0.42	8.4	94.1
5	0.30	6.0	100.0

The finding of a single eigenvalue greater than 1, compared with much smaller eigenvalues for all other components, indicates there is one dominant underlying principal component that explains two-thirds of the total variance. This result confirms that the five domains can be meaningfully fitted to a unidimensional scale and therefore affirms the validity of the Rasch model in this instance. The second component, which explains approximately 11% of the variance, is also of interest because of the way in which the strategy and knowledge domains differentially load onto it. The loadings of each domain on each component are shown in Table 5.

Table 5
Estimated Loadings of Each Domain on Each of the Principal Components

	Component				
Domain	1	2	3	4	5
Additive	0.812	–0.297	0.454	0.065	0.208
Multiplicative	0.869	–0.163	0.002	0.092	–0.457
Proportional	0.817	–0.223	–0.461	0.165	0.205
Place value	0.758	0.597	0.066	0.249	0.048
Basic facts	0.813	0.138	–0.053	–0.563	0.031

All domains load highly on the first component, whereas the second component appears to separate the strategy and knowledge domains. The three strategy domains all load negatively on this component, and the two knowledge domains both load positively. This, to some extent, supports the theoretical distinction between these two sets of domains.

Relative Difficulty of the Stages and Domains

Difficulty estimates for each stage of each domain represent the point on the scale at which a student whose ability is equal to the difficulty of the stage has a 50% chance of being rated at that stage or higher. The probability of students being at the lowest stage or higher in any domain is 100%, so no difficulty estimates can be calculated for the lowest stage in each domain, that is, stage 0 on the additive domain, stages 2–3 in the multiplicative domain, stage 1 in the proportional domain, and stages 0–1 in the place value and basic facts domains. It should also be noted that the difficulty estimates at the extreme ends of the scale, in particular at the upper end, are not as reliable as those nearer the middle. This is because the accuracy of each estimate is related to the number of students at each stage and the sample was weighted to the bottom and middle of the scale. Table 6 shows the proportion of students at each stage of each domain.

Table 6
Proportion of Students Rated at Each Stage of Each Domain

	Domain				
Stage	Additive	Multiplicative	Proportional	Place value	Basic facts
0			16.2	1.0	37.9
1	3.3				
2	20.2			41.5	32.2
3	11.7		72.9 *	3.4	4.7
4	37.2	28.4		45.4	39.7
5	24.1	63.0	24.4	8.4	19.0
6	3.3	27.8	2.4	1.1	4.1
7	0.03	8.6	0.2	0.1	0.2
8	0.6	0.1	0.03	0.05	
No. of student results	2919	1333	1474	2890	2116

*This stage in the proportional domain is labelled "stages 2–4".

The difficulty parameter estimates for each stage within each of the five domains are shown in Figure 1. These parameters are estimated with respect to one overall scale, which is measured in logits.

The scale extends from stage 1 in the additive domain, which has a logit value of –6.13, to stage 8 of the place value domain, at 8.72 logits. Exact values for the difficulty estimates can be found in Appendix F.

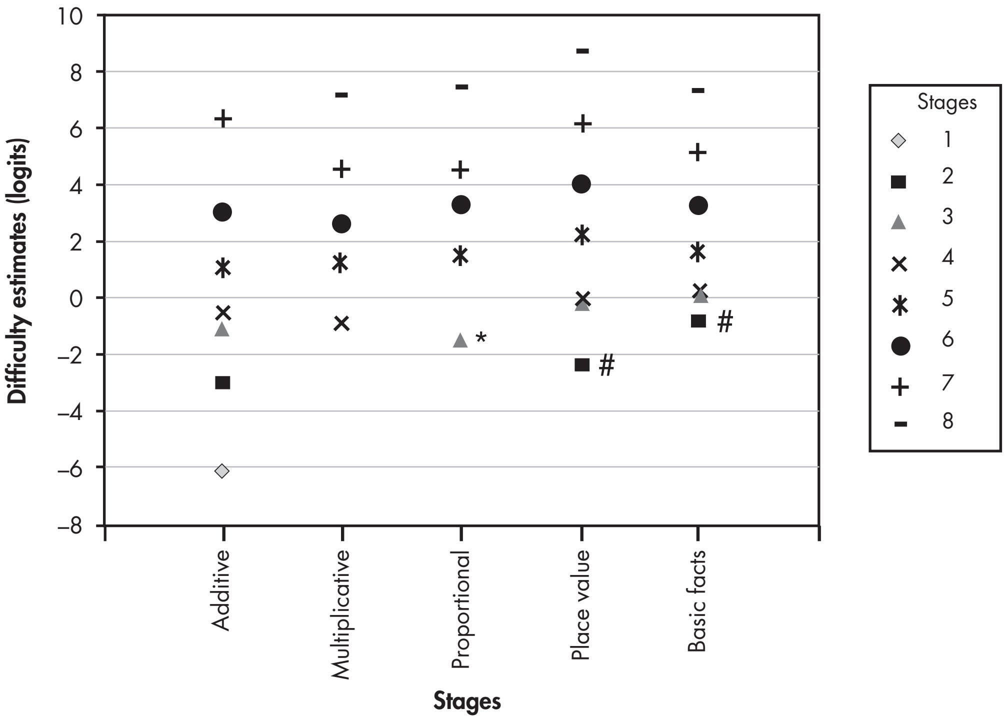

Figure 1. Estimated difficulty scores in logits for each stage of each domain

* This stage would be more accurately labelled as stages 2–4 (proportional domain).

These stages would be more accurately labelled as stages 0–1 (place value and basic facts domains).

The analysis provides evidence that the structure of the Number Framework is generally sound. That is, consecutive stages generally increase in difficulty within any domain, there is some consistency between the difficulties of any particular stage across the domains, and most stages of most domains cover a reasonable interval on the scale. Discrepancies in these general patterns are found in stages 3 and 4, particularly in the additive, place value, and basic facts domains. These discrepancies are discussed in detail in the following section.

Of particular interest is the extent to which each stage varies in difficulty across the domains of the Number Framework. Table 7 shows the difficulty range of stages 2–8 across the three strategy and two knowledge domains.

Table 7
Difficulty Range within Each Stage

Stage	Lowest logit estimate	Highest logit estimate	Difficulty range / logits
2	–3.00 (Additive)	–0.83 (Basic facts)	2.17
3	–1.48 (Proportional)	0.11 (Basic facts)	1.59
4	–0.94 (Multiplicative)	0.23 (Basic facts)	1.17
5	1.02 (Additive)	2.20 (Place value)	1.18
6	2.60 (Multiplicative)	3.98 (Place value)	1.38
7	4.50 (Proportional)	6.30 (Additive)	1.80
8	7.17 (Multiplicative)	8.72 (Place value)	1.55

In general, the stages are reasonably consistent in terms of difficulty across the domains, with a difficulty range of between 1.17 (stage 4) to 2.17 logits (stage 2). This result supports, to some extent, the theoretical relationships between the different domains of each stage of the Number Framework. Tables 8 and 9 provide short descriptions of stages 4 and 5. These are the two stages for which there is the most consistent difficulty across the domains.

Table 8
Relative Difficulty of Stage 4 across the Domains

Logit estimate	Domain	Description
0.23	Basic facts	Recalls doubles and teens facts
–0.09	Place value	Counts in tens
–0.53	Additive	Counts on to solve addition and subtraction problems
–0.94	Multiplicative	Skip-counts to solve multiplication and division problems

Within stage 4, basic facts is the most difficult domain and involves students recalling doubles and teens facts. The multiplicative domain is the least difficult at stage 4, with students skip-counting to solve addition and subtraction problems at this stage.

Table 9
Relative Difficulty of Stage 5 across the Domains

Logit estimate	Domain	Description
2.20	Place value	Knows tens in numbers to 1000 and tenths among whole numbers
1.61	Basic facts	Knows addition facts and multiplication facts for 2, 5, and 10
1.47	Proportional	Uses addition facts to solve proportional problems
1.29	Multiplicative	Uses repeated addition or known multiplication facts to solve multiplication and division problems
1.02	Additive	Uses partitioning strategies to solve addition and subtraction problems

Place value is the most difficult domain in stage 5, with students needing to know tens in numbers to 1000 and the number of tenths in whole numbers at this stage. The least difficult domain at stage 5 is the additive domain. This involves students using partitioning strategies to solve addition and subtraction problems.

Stage 2 is the least consistent in terms of difficulty across the domains. Table 10 provides brief descriptions of stage 2 within each of the three domains for which it is relevant.

Table 10
Relative Difficulty of Stage 2 across the Domains

Logit estimate	Domain	Description
–0.83	Basic facts	Instantly recalls facts to 5
–2.38	Place value	Counts in ones
–3.00	Additive	Counts from 1 on materials to solve addition and subtraction problems

Basic facts is the most difficult domain at stage 2, with students recalling addition facts to 5 at this stage. The least difficult domain at stage 2 is the additive domain, with students counting from one on materials to solve addition and subtraction problems.

Focusing on the lower end of the Number Framework, Table 11 shows the ten least difficult stages (excluding the lowest stage) across the five domains. Short descriptions of each of the stages are also included.

Table 11
Stages with the Least Difficulty across All Domains

Logit estimate	Domain	Description
–6.13	Additive, 1	One-to-one counting
–3.00	Additive, 2	Counts from one on materials to solve addition and subtraction problems
–2.38	Place value, 2	Counts in ones
–1.48	Proportional, 2–4	Shares objects equally to identify fractional amounts
–1.06	Additive, 3	Counts from one by imaging to solve addition and subtraction problems
–0.94	Multiplicative, 4	Skip-counting to solve multiplication and division problems
–0.83	Basic facts, 2	Instantly recalls facts to five
–0.53	Additive, 4	Counts on to solve addition and subtraction problems
–0.22	Place value, 3	Counts in fives
–0.09	Place value, 4	Counts in tens

The additive and place value domains dominate the lower end of the Number Framework, with four of the ten least difficult stages of the Framework placed in the additive domain and three in the place value domain. This reflects the content of these domains, both of which include stages focused on counting at the lower end (additive stage 1 and place value stage 2), while other domains start with more difficult skills, for example, recalling facts to five (basic facts domain). One interesting result is the large difference in the difficulty of these two stages, which are both based on students' ability to count. Stage 1 in the additive domain has a difficulty of –6.13 logits, 3.75 logits easier than stage 2 in the place value domain, which has a difficulty of –2.38. One possible explanation for this result is the nature of the tasks used to assess a student's stage within the NumPA. Students need to be able to form a set of eight counters to be rated at stage 1 on the additive domain, but they need to be able to count a given set of 14 dots to be rated at stage 2 on the place value domain. The later task could be regarded as more difficult, even though the descriptors for both stages identify students as counting one-to-one.

At the top end of the Number Framework, six stages have a difficulty estimate that is greater than six logits. These stages are shown in Table 12, along with brief descriptions of each stage.

Table 12
Stages with the Greatest Difficulty across All Domains

Logit estimate	Domain and stage	Description
8.72	Place value, 8	Knows hundredths, names decimals between decimals, converts percentages to decimals, and vice versa
7.45	Proportional, 8	Uses advanced strategies for solving proportional problems
7.33	Basic facts, 8	Knows common factors and multiples
7.17	Multiplicative, 8	Uses advanced strategies to solve multiplication and division problems involving decimals and fractions
6.30	Additive, 7	Uses advanced strategies to solve addition and subtraction problems involving decimals and fractions
6.14	Place value, 7	Knows tenths, orders decimals

The stages that involve using advanced strategies to solve number problems are clearly some of the most difficult stages of the Number Framework. Solving proportional problems is more difficult than solving multiplication and division problems, with relative difficulty scores of 7.45 and 7.17 logits respectively. In turn, solving multiplication and division problems is more difficult than solving addition and subtraction problems, which has a difficulty of 6.30 logits.

The Relative Ranges of Stages and Domains

The probabilistic nature of the Rasch model results in students being described as more or less likely to be at particular stages of the Number Framework, given their ability score, with any given ability score being related to several possible stage ratings. Probability functions showing the probability of a student being at a particular stage on the additive domain given any estimated ability are presented in Figure 2. The functions for the multiplicative and proportional domains can be found in Appendix G. Note that the x-axis in each graph represents both student ability and stage difficulty on the logistic scale. This measure relates to students' stage ratings in all five domains and is a consequence of the unidimensional nature of scales constructed by Rasch analysis.

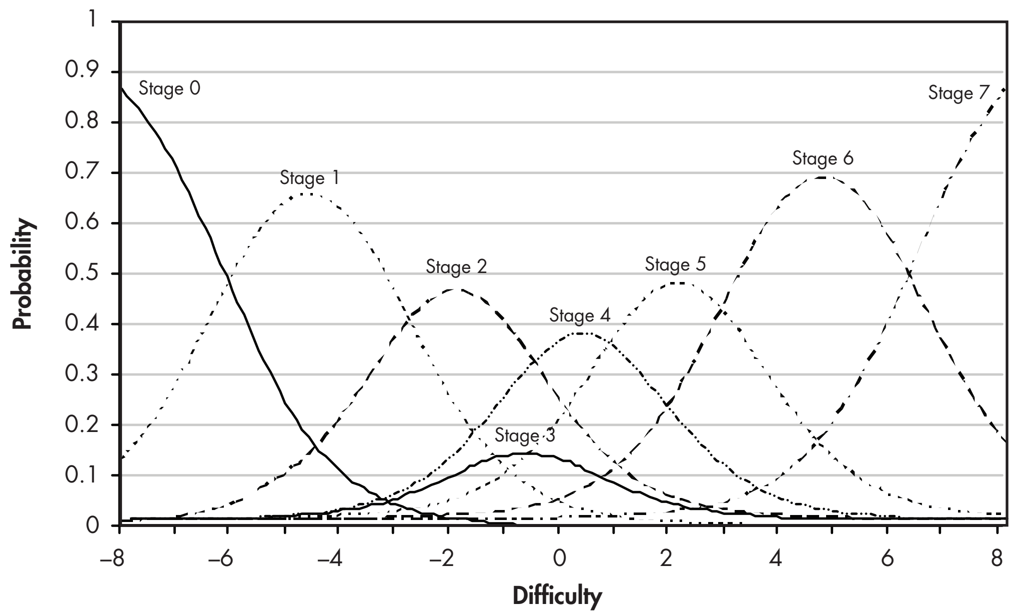

Figure 2. Probability functions for classification at each stage of the additive domain, conditioned on numeracy difficulty

In terms of the additive domain, it seems that there is very little to distinguish stage 3, which requires students to solve addition and subtraction problems by counting from one using imaging, from stage 4, which requires students to count on to solve addition and subtraction problems. There is a difference of just 0.53 logits in difficulty between these stages (Table 15, see Appendix F), and Figure 2 shows that there is never more than a probability of 0.15 that students will be rated at stage 3. This may be because imaging and counting on are so developmentally close that some students either pass very quickly through imaging or omit this stage entirely. An alternative explanation is that imaging is a distinct strategy stage, but it is largely unobservable because it is a cognitive procedure that occurs internally.

Figure 3 shows the probability functions of a student being at a particular stage on the place value domain, given any estimated ability.

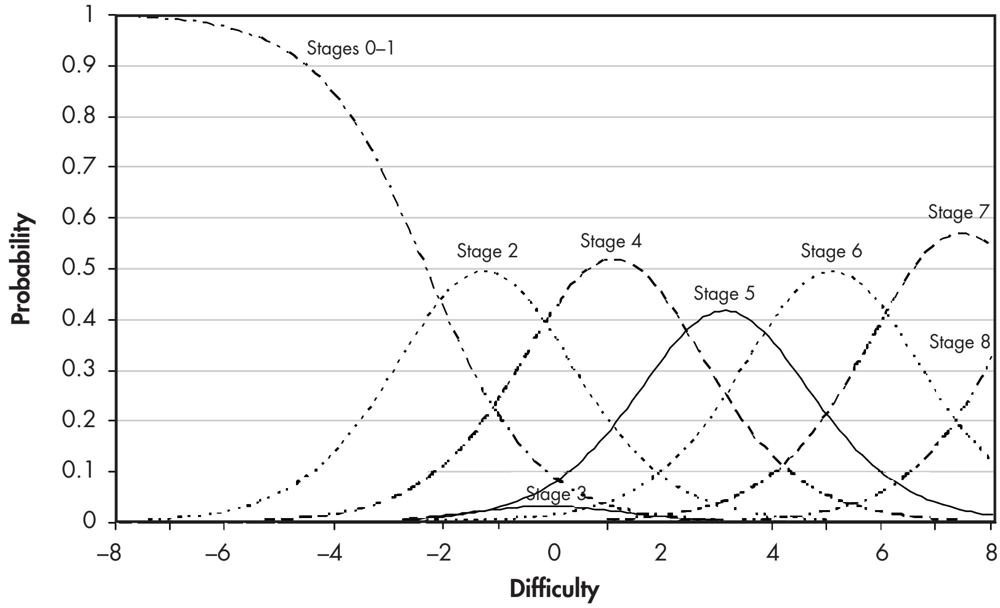

Figure 3. Probability functions for classification at each stage of the place value domain, conditioned on numeracy difficulty

In the place value domain, there is very little difference in terms of difficulty between students' ability to count in fives and ones at stage 3 and their ability to count in tens at stage 4, with just 0.13 logits difference between these stages. Stage 3 is also a very unlikely stage rating, with no ability at which there is more than a 0.05 probability of students being rated at that stage. This may reflect the fact that counting in tens precedes counting in fives in some instructional sequences, or it may be a reflection of the task used to assess knowledge at stages 3 and 4. It could be argued that the assessment task encourages students to count in tens rather than fives because strips of ten dots are consecutively added to a strip of four dots, with students asked to keep a track of how many dots there are in total.

Figure 4 shows the probability functions of a student being at a particular stage on the basic facts domain, given any estimated ability.

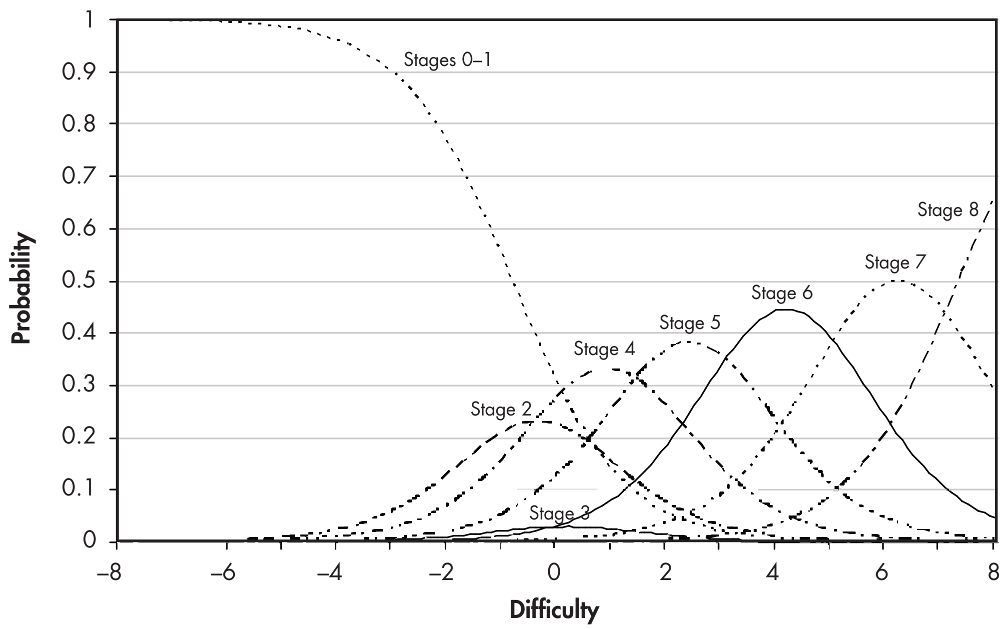

Figure 4. Probability functions for classification at each stage of the basic facts domain, conditioned on numeracy difficulty

The basic facts domain shows a slightly different pattern to that in the additive and place value domains, with three stages close in difficulty level. Stages 3 and 4 are very close in difficulty, with a difference of 0.12 logits between them and never more than a 0.05 probability of students being rated at stage 3. These stages require students to instantly recall facts to ten and to recall doubles and teens facts respectively. It is likely that the closeness of these stages reflects the fact that, in practice, these addition and subtraction facts are all taught together rather than in any structured sequence. Stage 2 in the basic facts domain is also never the most likely stage for students. It has a higher maximum than stage 3, with a maximum probability that students will be rated at stage 2 of just over 0.20. There is also more difference in difficulty between stages 2 and 3, with a difference of 0.94 logits (compared with the 0.12 logits between stages 3 and 4). In practical terms, this means students' ability to instantly recall facts to five (required to be rated at stage 2) is distinct from the highly-related abilities of recalling facts to ten and recalling doubles and teens facts, but overall these three stages are all very closely aligned.

The transition points between consecutive stages were identified as the ability scores at which it is equally likely for a student to be placed on either of two adjacent stages, rounded to the nearest 0.05 logits. For example, Figure 2 shows that students with a numeracy ability of 2.80 logits have a probability of just over 0.40 of being rated at stage 5 on the additive domain and an equal probability of being rated at stage 6 on that domain. Table 13 shows the transition points for the stages of the additive domain; the remaining four domains can be found in Appendix H. Note that stage 3 in the additive domain has been omitted from this analysis because it is not the most likely stage rating for students at any point on the estimated ability scale. Stage 3 in the place value domains and stages 2 and 3 in the basic facts domain were similarly problematic from the point of view of analysing transition points and were therefore omitted from the analysis; transition to the next stage was computed in lieu of the omitted stage.

Table 13

Transition Points on the Additive Domain

Transition between stages					
0–1	1–2	2–4	4–5	5–6	6–7
–6.05	–2.75	–0.75	0.80	2.80	6.20

Figure 5 shows the relative ranges of the stages within the five domains of the Number Framework, calculated using transition points. The highest stage within each domain is not included because the lack of a theoretical end-point to each domain means it cannot be calculated. Note that, because the logit values as estimated are subject to measurement error, the precise values should not be over-interpreted.

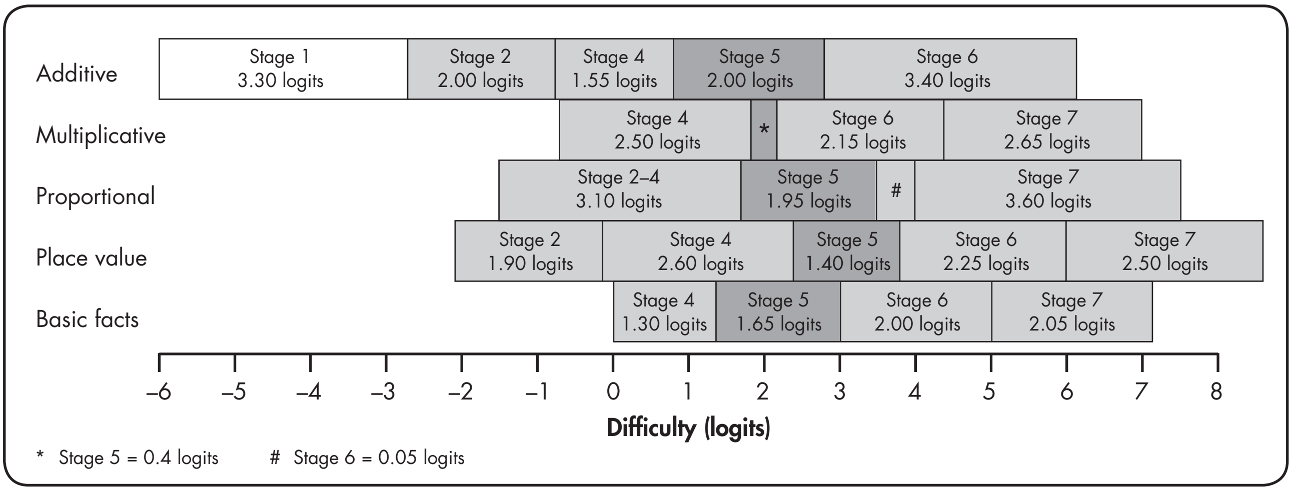

Figure 5. Range of stages in each of the five domains

The ranges of the stages vary from stage 6 in the proportional domain, which has the most limited range (0.05 logits), to stage 7 in the proportional domain, which has the greatest range (3.60 logits). While the difficulty level of each stage varies between the domains, there is no difficulty rating that spans more than two stages across the five domains. This indicates that the stages of the Number Framework are reasonably consistent across the domains. There is a median range of 2.03 logits across the stages and domains shown. It is of note that the stages within each domain with the greatest range shown are those at the highest end of the Framework. Four stages have a range of greater than 3.00 logits across one stage. These stages are described in Table 14.

Table 14
Stages of the Number Framework with Values Greater than 3.0 Logits

Domain and stage	Description
Proportional, 7	Uses advanced strategies to solve proportional problems involving ratios, percentages, and fractions
Additive, 6	Uses advanced strategies for the addition and subtraction of whole numbers
Additive, 1	Counts one-to-one
Proportional, 2–4	Shares objects equally to identify fractional amounts

Stage 7 in the proportional domain and stage 6 in the additive domain have two of the greatest ranges shown, and both involve the use of advanced strategies to solve number problems. The range of these stages can most likely be attributed to the variety of strategies that can be used to solve number problems and the complexity of the learning that results. The fact that stage 7 in the multiplicative domain, which requires students to use advanced strategies to solve multiplication and division problems, also has a relatively large range supports this notion. Stage 1 in the additive domain also has a relatively large range, with students needing to master the skill of one-to-one counting to be rated at this stage. The complexity of this stage most likely lies in the fact that students need to co-ordinate knowledge of the number sequence with the skill of one-to-one matching in order to count successfully.

Two stages are particularly limited, with ranges of less than 0.50 logits; these are stage 5 in the multiplicative domain and stage 6 in the proportional domain. The limited ranges of these stages can most likely be attributed to their transitory nature because both represent a partial move towards multiplicative reasoning on the part of the student. Stage 5 in the multiplicative domain requires students to use repeated addition or known multiplication facts to solve multiplication and division problems and can be seen as a transitory step between stage 4, which involves using skip-counting procedures, and stage 6, which involves using only multiplication facts to solve such problems. Stage 6 in the proportional domain has a similar transitory nature. It requires students to use addition and multiplication facts to solve proportional problems, which is a step between stage 5, which involves using addition facts, and stage 7, which involves using multiplication facts to solve proportional problems. Students will quickly limit their use of counting or additive strategies to solve number problems once they see the power of multiplicative relationships, and this accounts for the minimal values of these two transitory stages.

Conclusions

To a large extent, this analysis confirms the validity of the Number Framework and supports the theory on which it was based. The stages within each domain describe increasing levels of difficulty, and the difficulty level of each stage across the domains is relatively consistent.

As well as generally supporting the structure of the Number Framework, the analysis identifies several areas in which the Framework would benefit from refinement. These are:

- Stage 3 in the additive domain, counting from one by imaging to solve addition and subtraction problems, should be removed from the Framework because it is very close in difficulty to stage 4, advanced counting.
- Stages 2, 3, and 4 in the basic facts domain should be combined to include instant recollection of facts to five, facts to ten, and doubles and teens facts in one stage because these are all of similar difficulty.
- Stage 5 in the multiplicative domain and stage 6 in the proportional domain should be removed from the Framework because they are transitory stages that describe the initial development of multiplicative strategies.

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The Development of Students' Ability in Strategy and Knowledge

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This paper reports on the progress of primary-school students in the use of strategy and knowledge as their numeracy skills develop over time. In particular, Rasch analysis was used to investigate the relationship between growth in strategy and growth in knowledge. Results support the contention that students require an initial body of knowledge to solve number problems using strategy and that this body of knowledge is accumulated rapidly during their first three years at school. Students progress at a reasonably constant rate through the stages of the strategy domains from years zero to eight. In comparison, students progress more quickly through the stages of the knowledge domains in the first three years of school, after which time, growth in knowledge slows significantly. A regression analysis highlights the pedagogical importance of strategy development. Results show that practising known strategies assists students to develop both new items of number knowledge and also increasingly sophisticated strategies for solving number problems.

Background

The Numeracy Development Projects (NDP) are focused on improving student achievement in mathematics by improving the professional capability of teachers. They are a major initiative of the New Zealand Government, first implemented in the year 2000 and ongoing since that time (Bobis et al., 2005).

A theoretical underpinning for the NDP is provided by the Number Framework (Ministry of Education, 2007). The Framework describes a progression of student learning, both in terms of the techniques that students need to develop in order to solve number problems (strategies) and the key number facts that they need to learn (knowledge). This distinction between strategy and knowledge is consistent with the theoretical distinction between procedural and declarative systems in cognitive psychology (see, for example, Cohen and Squire, 1980).

The Number Framework is structured in domains, with each domain describing one aspect of students' ability with numbers. Each domain consists of a number of stages through which students progress as they gain competence. Teachers participating in the NDP are required to assess their students against the Framework both before and after a period of focused instruction.

Over the course of the NDP, a large volume of student achievement data has been collected and analysed, with results used to guide successive improvements. Analysis has focused on describing the patterns of student progress and achievement each year and comparing them with patterns in previous years (see, for example, Young-Loveridge, 2007, 2008, 2009). Smaller-scale studies have also investigated aspects relating to the teaching and learning of number concepts and skills (see, for example, Higgins & Bonne, 2009).

Analysis of student achievement data has been limited to some extent by the theoretical nature of the Number Framework. Some empirical analysis of the Framework has been carried out previously (Irwin, 2003; Irwin & Niederer, 2002) and this, along with qualitative feedback from the implementation of the NDP, has suggested that the difficulty levels of the stages are not equally spaced across any domain and that stages that are theoretically aligned actually vary in difficulty across the domains (Irwin, 2003; Irwin & Niederer, 2002; Ward and Thomas, 2002). This has limited efforts to compare the progress of students across a range of year levels and Framework stages (Ward & Thomas, 2008; Ward & Thomas, 2009).

This is the second of two papers in this volume by Johnston, Thomas, and Ward that report on an extensive empirical analysis of the Number Framework. The first paper reports on an item-response (Rasch) analysis of the relative difficulties of the stages and domains of the Framework and the relative extent of the stages within each domain. This work analysed the structure of the Number Framework within the context of a single scale, although a factor analysis showed some support for the division of the Framework into strategy and knowledge scales. Included in the strategy scale were the additive, multiplicative, and proportional domains and, in the knowledge scale, the place value and basic facts domains. The present paper expands on that analysis using the same dataset. The two scales are used to describe the development of students' abilities in strategy and knowledge over time and to investigate the relationship between growth in strategy and growth in knowledge.

Method

Participants

This paper presents an analysis of Number Framework data from 3742 students over three consecutive years, from 2006 to 2008. Students were included in the dataset on the basis that it was possible to link their results over these years. All students in the sample had results for the additive strategy domain and for at least one of the other two strategy domains (multiplicative and proportional). All students in the sample also had results for the two knowledge domains of place value and basic facts, which were considered the most significant of the knowledge domains in terms of determining progress.

Tables 1 and 2 provide demographic information about the students in the sample.

Table 1
Ethnicity and Gender of Participating Students

Ethnicity	Male (%)	Female (%)	Total
NZ European	50.8	49.2	2625
Māori	50.0	50.0	578
Pasifika	43.1	56.9	232
Asian	45.8	54.2	179
Other	52.3	47.7	128
All	50.0	50.0	3742

Table 2
Year Level and School Decile of Participating Students

Year level	Decile 1–3 (%)	Decile 4–7 (%)	Decile 8–10 (%)	Total
Years 0–1	9.1	25.6	65.3	582
Year 2	9.8	29.6	60.7	737
Year 3	11.9	31.6	56.5	798
Year 4	11.3	29.3	59.4	860
Year 5	15.2	26.8	58.0	343
Year 6+	14.6	27.7	57.7	383
All	11.5	28.9	59.7	3703

Note: 39 students were at schools with no decile rating (private schools).

Method

Student achievement information was collected by teachers using the NDP assessment (NumPA) and entered into an online numeracy database. The NumPA is an individual interview that provides the basis for judgments of students' knowledge and strategy stages on the Number Framework.

Analysis

Students' Number Framework stages were available for four time points: initial 2006 (time one), final 2006 (time two), final 2007 (time three), and final 2008 (time four).

Item difficulty estimates were calculated for each stage on each of the three strategy (additive, multiplicative, and proportional) and two knowledge (place value and basic facts) domains. Logit estimates under a Rasch model were calculated on the basis of students' stage ratings at time one, using a maximum-log-likelihood procedure (see Johnston, Thomas, & Ward, this volume, for further information on this analysis).

Student-ability estimates in both strategy and knowledge at each time point were calculated for each student. These were calculated simultaneously with item difficulty estimates at time one and normalised to a mean of zero and a standard deviation of one. In order to calculate *growth* in ability, the ability estimates were recalculated for each subsequent time interval using the initial item-difficulty estimates, without normalisation.

The analysis is presented in two sections:

1. The mean student ability in strategy and knowledge (in logits) for all students in each year level, over the final three time points, is analysed. This allows the way in which strategy and knowledge growth varies over the years of schooling to be described and the relationship between them to be illustrated.
2. A linear regression analysis is used to estimate the proportions of unique and total variance on the strategy and knowledge scales at each time point, accounted for by prior performance on these scales. This analysis predicts students' later strategy and knowledge logit scores from their earlier strategy and knowledge logit scores and is used to investigate the extent to which earlier ability in each of strategy and knowledge predicts students' later performance on each of these scales.

Note that students who were rated at the top stage of every domain at any time point have been removed from some analyses. This is because students with maximum ratings theoretically have infinite ability estimates. The data used in those analyses therefore underestimate performance but are nonetheless useful for examining in detail the relationship between strategy growth and knowledge growth.

Findings and Discussion

The key research questions addressed in this section are:

- What is the relationship between growth in students' ability in strategy and growth in students' ability in knowledge?
- To what extent can students' prior ability in strategy and knowledge be used to predict their later ability in strategy and knowledge?

What Is the Relationship between Growth in Students' Ability in Strategy and Growth in Students' Ability in Knowledge?

Figure 1 shows students' mean ability scores, at each year level, for both strategy and knowledge. The mean scores include collated results for all students at each of the specified year levels across three time-points: final 2006, final 2007, and final 2008. The number of results represented at each year level varies: on the knowledge scale, from 259 sets of results for year 8 students to 2438 for year 2 students; on the strategy scale, from 307 for year 8 to 2433 for year 2 students. The standard error statistics provide an indication of the confidence levels of the measure.

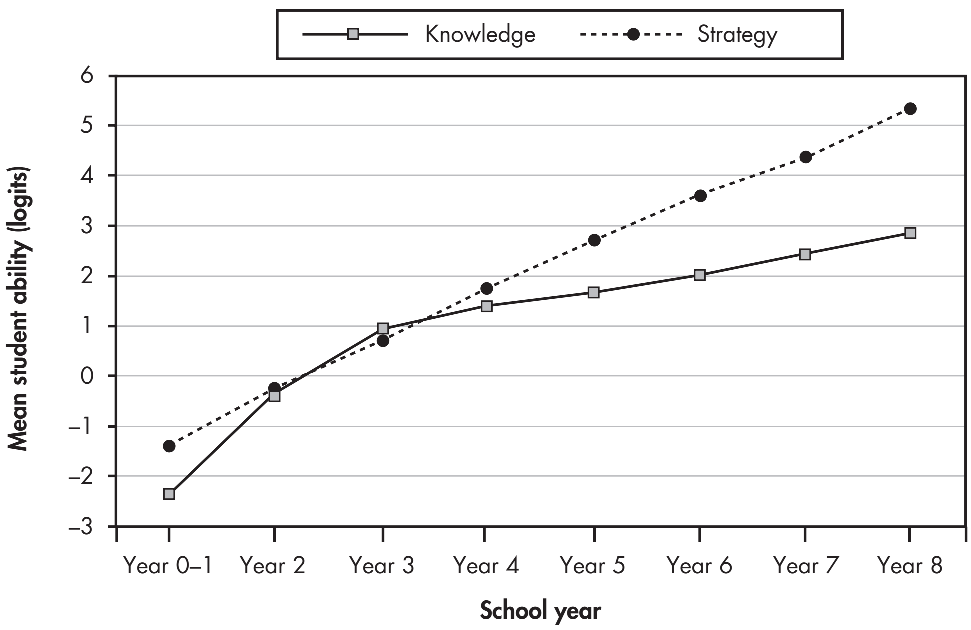

Figure 1. Mean strategy and knowledge ability estimates in logits at each primary school year. Error bars denote standard errors of the mean.

Figure 1 illustrates the average growth of students' ability in strategy and knowledge over the first eight years of schooling. On average, students make progress on the strategy scale at a reasonably constant rate over this period. As a comparison, students in the first three years of school make more rapid progress on the knowledge scale than they do on the strategy scale, but after year 3, they make slower progress in knowledge than they do in strategy. This result supports the notion that students require an initial body of knowledge that facilitates solving number problems using strategy and that this body of knowledge is accumulated rapidly during students' first three years of school.

An alternative explanation of the growth patterns in Figure 1 is that growth in the knowledge domains is subject to a ceiling effect, whereas growth in the strategy domains is not. If this were the case, it would be expected that, in later school years, greater proportions of students would be at the highest levels of the knowledge domains than of the strategy domains. Table 3 shows that a significant proportion of year 8 students were rated at the top stage of each domain in 2008. For example, just under one-third of students (28.3%) were rated at stage 8 of the multiplicative domain, while approximately one-third of students were rated at stage 8 of the place value and basic facts domains (30.9% and 31.7% respectively).

Table 3
Percentages of Year 8 Students at Each Stage of Each Domain in 2008, N = 375

	Domain				
Stage	Additive	Multiplicative	Proportional	Place value	Basic facts
1	0.3%				
2					
3					
4	1.9%	2.7%	2.4%*	4.5%	1.9%
5	7.2%	6.1%	9.6%	12.3%	5.1%
6	35.5%	22.9%	17.6%	22.4%	21.6%
7	40.5%	40.0%	45.9%	29.9%	39.7%
8	14.7%	28.3%	24.5%	30.9%	31.7%

* This stage in the proportional domain is labelled "stages 2–4".

These results indicate that, by year 8, substantial proportions of students are operating at the final stage of each domain, with the exception of the additive domain. However, the proportion of students at the penultimate stage of each domain except place value was greater than the proportion at the final stage. Also, the proportions of students at the final stages of the knowledge domains were not substantially greater than those at the final stages of the multiplicative or proportional domains. Furthermore, the large difference between the mean strategy and knowledge logits at year 8 is the continuation of a trend that began in year 4, at which point very few students were at the final stage of any domain. It is most unlikely, therefore, that the slowing of students' growth on the strategy scale relative to growth on the knowledge scale can be attributed to a ceiling effect.

To What Extent Does Prior Ability in Strategy and Knowledge Predict Later Ability in Strategy and Knowledge?

Students' strategy and knowledge ability estimates at any given time are closely related. For example, the correlations between students' logit estimates for strategy and their logit estimates for knowledge varied between 0.47 at time point one and 0.62 at time point four.

The nature of the relationship between growth in strategy ability and growth in knowledge ability is complex because the strategy and knowledge constructs are correlated. Part of the variation in students' later strategy and later knowledge ability can be accounted for by shared variance in prior strategy and knowledge ability, a consequence of their correlation. Nonetheless, some of the variance in later measurements of both strategy and knowledge is *uniquely* explained by variance in prior ability in each of strategy and knowledge.

Regression analysis allows estimation of the extent to which previous ability in each of strategy and knowledge uniquely explains variance in later strategy and later knowledge. Furthermore, the variance can be partitioned into shared and unique components. The unique component for each scale estimates the influence of prior performance on that scale on later attainment, independent of the relationship between strategy and knowledge.

Table 4 shows the proportions of unique and total variance in students' ability on both strategy and knowledge scales accounted for by students' prior performance in both strategy and knowledge scales, across the four time points.

Table 4
Proportions of Unique and Total Variance on Later Strategy and Knowledge Performance Scales Accounted for by Prior Performance on Strategy and Knowledge Scales

		Dependent Measure					
		2008		2007		Final 2006	
Predictor		Strategy	Knowledge	Strategy	Knowledge	Strategy	Knowledge
Initial 2006	Strategy	0.02	0.02	0.02	0.04	0.11	0.04
	Knowledge	0.02	0.01	0.03	0.02	0.04	0.10
	Total	**0.07**	**0.06**	**0.10**	**0.09**	**0.27**	**0.25**
Final 2006	Strategy	0.12	0.09	0.14	0.08		
	Knowledge	< 0.01	< 0.01	< 0.01	0.03		
	Total	**0.18**	**0.14**	**0.21**	**.20**		
2007	Strategy	0.23	0.09				
	Knowledge	0.01	0.06				
	Total	**0.39**	**0.31**				

It is a property of variance that it can be partitioned into unique and shared components, with the total variance being the sum of the partitioned variances. Due to this property, the unique and total variance can be used to calculate the shared variance. For example, the total proportion of variance in 2008 strategy ability accounted for by 2007 ability in both strategy and knowledge was 0.39. Of this, an estimated 0.23 was accounted for uniquely by strategy and 0.01 uniquely by knowledge. This leaves an estimated 0.15 of the variance in 2008 strategy ability explained by shared variance in 2007 strategy and knowledge ability.

Although the proportions of variance seem small, especially to those who are not familiar with their use to describe relationships, small amounts of variance can be statistically significant. One way to appreciate the size of the relationship is to use the proportion of variance to calculate a correlation coefficient, a statistic that may be more familiar. This is done by taking the square root of the variance. For example, the shared variance of 0.15 between students' 2008 ability in strategy and their 2007 ability in both strategy and knowledge represents a correlation coefficient of 0.39.

Figure 2 shows the proportions of the variance in students' ability in strategy and knowledge that can be accounted for by students' ability in strategy and knowledge at prior time points. Both shared and unique components are shown.

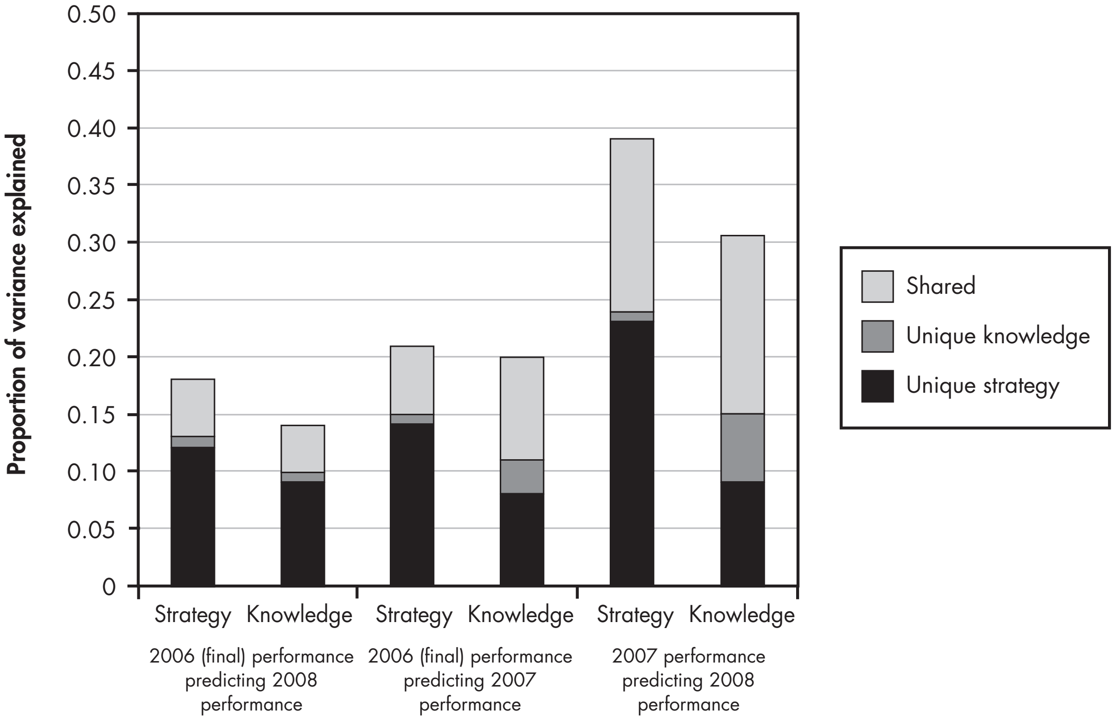

Figure 2. Percentage of variance in students' strategy and knowledge ability that is explained by prior performance

Figure 2 shows that students' ability in strategy at earlier time points is more predictive of their later ability in both strategy and knowledge than is their ability in knowledge at earlier time points. For example, in predicting students' 2008 performance from their 2007 performance, 23% of the variance in students' strategy ability is attributable to their prior performance in strategy, compared with the 1% of the variance attributable to their prior performance in knowledge. Similarly, 9% of the variance in students' 2008 knowledge ability is attributable to their 2007 strategy ability, compared with the 6% variance attributable to their 2007 knowledge ability. It is unsurprising that students' earlier ability in strategy is an important predictor of their later ability in strategy. However, the finding that strategy ability is more predictive of later knowledge ability than is prior knowledge ability is more surprising. This finding highlights the pedagogical importance of strategy development.

As shown in Figure 2, the proportions of variance explained by students' prior performance increase where the intervals between prior and predicted performance are smaller. For example, 39% of the variance in the 2008 strategy performance can be accounted for by the 2007 performance, compared with the 18% of the variance that can be accounted for by the 2006 final performance. This result reflects the fact that more measurement error is involved in prediction over longer time intervals than over shorter time intervals.

Figure 2 also shows that the proportions of variance explained over the same time period are larger as the cohort gets older. For example, students' 2007 strategy ability predicts 23% of the variance in their 2008 strategy ability, compared with their late 2006 strategy ability, which predicts just 12% of the variance in their 2007 strategy, even though these intervals are both one year apart.

One explanation for this is maturational; that is, it is possible that the predictive relationships become stronger as students' development advances. This might happen because older children have less variable assessment data than younger children and, also, as students advance on the developmental continuum, between-student variability in the strategy and knowledge that is most foundational for further progress becomes more consistent. A second potential explanation for the stronger predictive relationships in later measurements over time periods of equal length is that, at later time intervals, students have had longer exposure, on average, to teachers trained in the NDP.

The analysis presented here does not conclusively identify one of these explanations as more likely. However, there is considerably more variation in the age distribution within each time interval than there is between time intervals. This is because the age of participants in the research varied over about six years, whereas the longitudinal comparisons spanned only two-and-a-half years. This indicates that the second (non-maturational) explanation may be more likely, but further analysis would be required to produce any firm conclusions in this regard.

NDP teacher materials describe the relationship between students' strategy and knowledge development as shown in Figure 3. In this representation, students' current use of strategy drives the acquisition of knowledge through use and students' current knowledge provides the foundation upon which new strategies can be developed. Both of these processes are shown as equally important.

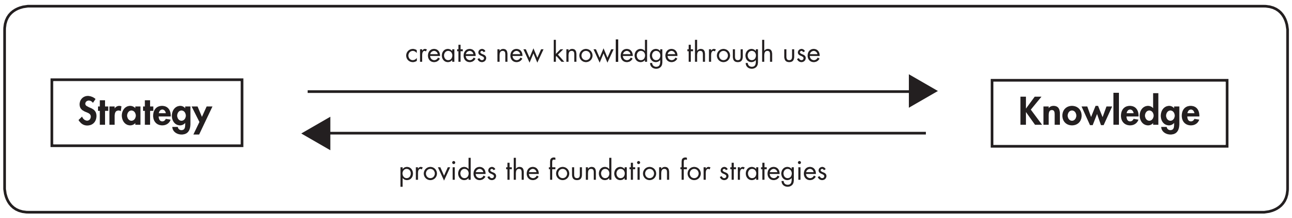

Figure 3. Relationship between strategy and knowledge development, as depicted in NDP materials (Ministry of Education, 2007, p. 1)

The findings of this paper emphasise the significance of the top link in Figure 3. Results indicate that students' use of strategy drives the development of their knowledge to a greater extent than students' developing knowledge drives their use of increasingly sophisticated strategies. This may be because students who know a body of facts are not necessarily able to use these facts effectively to solve number problems and thus increase their strategy ability. In contrast, students who are competent in using particular strategies to solve number problems will be able to apply these strategies in order to develop new items of number knowledge. Furthermore, to the extent that performance on the strategy and knowledge scales reflect procedural and declarative cognitive systems respectively, the strategy scale might embody processes that are less subject to retrieval effort or to forgetting, whereas the knowledge scale might be more sensitive to deliberate retrieval mechanisms.

This finding suggests that focusing teaching on developing students' ability in strategy will be more beneficial in terms of students' overall future achievement than focusing on developing students' ability in knowledge. However, the dissociation between knowledge and strategy should not be over-stated; the regression analyses show that variance shared between prior attainment in knowledge and strategy is just as predictive of later performance as unique variance in strategy. Furthermore, from a pedagogical point of view, the importance of knowledge should not be underestimated. All strategies require some knowledge as a pre-requisite for their use. For example, students will not be able to count (strategy stage 1) unless they know the forward number word sequence accurately. Similarly, students will not be able to use the strategy of skip-counting to solve multiplication problems (stage 4 in the multiplicative domain) unless they know skip-counting sequences reliably. Both strategy and knowledge are required to solve number problems, and effective pedagogy appropriately emphasises both of these aspects of students' development.

Rather than emphasising the pedagogical importance of strategy over knowledge, the findings emphasise the importance of providing opportunities for students to practise using the strategies they know to solve number problems. There is evidence that practising known strategies will assist students to develop new items of number knowledge and also facilitate the development of increasingly sophisticated strategies for solving number problems.

Concluding Comment

This research uses an entirely different approach to those used previously in analysing student achievement data from the NDP. The use of Rasch analysis techniques enables the Number Framework to be analysed empirically. Investigating the relationship between students' growth in strategy and their growth in knowledge has provided interesting insights into the relative importance of the two complementary aspects of the Framework. The findings highlight the value of empirical techniques for guiding effective classroom instruction.

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Supporting the Additive Thinking of Year 5–6 Students: The Equal Additions Strategy

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This study set out to investigate what actually happens in classrooms as teachers use the Numeracy Development Projects (NDP) resource materials to build students' repertoire of part–whole strategies for solving addition and subtraction problems. Nine teachers each worked with a group of students in their class who were judged to be ready for instruction in advanced additive thinking. They were observed teaching equal additions as a possible strategy for solving comparison problems in subtraction. On the whole, the teachers stuck pretty closely to the Equal Additions lesson described in NDP Book 5 (pp. 38–39). Despite their commitment to conceptual teaching, many of the teachers ended up teaching the equal-additions strategy in a procedural manner and did not encourage their students to weigh up the efficiency of this strategy when compared with other possible solution strategies. The availability of more background information about the key ideas underpinning the particular lesson and information about different problem structures for addition and subtraction problems would have benefitted the teachers.

Introduction

It is almost a decade since the national Literacy and Numeracy Strategy was launched (see Ministry of Education, 2001). The strategy was designed to raise students' achievement in mathematics (and language) and build teachers' professional capacity to teach mathematics (and language) (Ministry of Education, 2001). The Numeracy Development Projects (NDP) are a major government initiative designed to address both these components in relation to numeracy learning (Anthony & Hunter, 2005; Bobis, Clarke, Clarke, et al., 2005; Young-Loveridge, 2004, 2005, 2006, 2007, 2008, 2009). The Number Framework and associated diagnostic assessment tools were designed to help teachers make decisions about the nature of students' knowledge and understanding in relation to a sequence of learning progressions in number (Ministry of Education, 2008a, 2008b). A major goal for students is for them to understand the concept of additive composition, the way numbers can be split into parts and recombined (part–whole thinking) to solve problems more easily. Initially, students need to learn about this within the context of addition and subtraction processes. A distinction is made between those who are just learning to split and recombine single-digit quantities using a limited range of strategies (stage 5 early additive part–whole thinkers) and those who can do this with multi-digit quantities, making flexible use of a full range of strategies (stage 6 advanced additive part–whole thinkers).

The release of *The New Zealand Curriculum* (Ministry of Education, 2007) and *Mathematics Standards for Years 1–8* (Ministry of Education, 2009) has embedded the Number Framework within the system of expected outcomes that apply to all students in New Zealand schools. The goal is that by the end of year 4, students are able to split and recombine small numbers (stage 5), and by the end of year 6, students are able to choose from a range of strategies to solve multi-digit addition and subtraction problems (stage 6). The recent mathematics standards poster (Ministry of Education, 2010) indicates that students in the year prior to this should have an emerging capability (that is, early stage 5 by the end of year 3 and early stage 6 by the end of year 5). Hence it is vital that teachers have the necessary personal understanding of mathematics and of how to teach these mathematical ideas to students (pedagogical content knowledge [PCK]) so that they are able to support their students' learning in this way.

The NDP professional development programme for teachers working at the year 5–6 level was for just one year. That may have been enough for teachers to get their heads around the components of the NDP approach, such as the Number Framework, the assessment tools, the teaching model, and the published resources to support their teaching. However, it is questionable whether, within one year, teachers would also have been able to develop a deep and connected understanding of the complexity and subtleties of addition and subtraction as well as shift from a *calculational* to a *conceptual orientation* (Philipp, 2007). Virtually all primary schools have now had the opportunity to participate in NDP professional development programmes. The Ministry of Education's emphasis now is on in-depth sustainability of the NDP approach to teaching mathematics.

Data from year 6 students has shown that by the end of the year in which their teachers participated in NDP professional development, fewer than half of the students at this level were at stage 6 or higher (between 41% and 49% over the period 2005–2008; see Young-Loveridge, 2005, 2006, 2007, 2008, 2009). This is well short of the majority of students (75%–80%) expected to be at stage 6 or higher by the end of year 6 (Ministry of Education, n.d.). The proportion of year 5 students at stage 6 is considerably lower (between 22% and 34% over the period 2005–2008).

Several writers have emphasised the importance of students having an appreciation of the structure of mathematics (Bobis, Mulligan, & Lowrie, 2008; Lambdin & Walcott, 2007; Mulligan & Mitchelmore, 1997, 2009; Mulligan, Prescott, & Mitchelmore, 2004; Young-Loveridge & Mills, 2009a, 2009b). Particularly relevant to additive thinking is place value (the meaning of a digit within a multi-digit numeral by virtue of its position or place within the number). The base-ten structure of the number system dictates that 9 is the largest digit in any position within a multi-digit number. Once there are more than nine objects, the digit in the adjacent position (for example, tens) must "roll over" and the digit in the "ones" position begins incrementing again from zero. Students must learn about what combination of numbers makes ten (for example, 3 and 7) and then generalise this information to larger numbers to make other whole decades (for example, 53 and 7 makes 60, or 53 and 27 makes 80). Eventually, they also learn that when 1 is added to 99, another "rollover" occurs to create 100 and the digits in the "ones" and "tens" positions go back to zero.

This study set out to investigate what actually happens in classrooms as teachers use the NDP resource materials to build students' repertoire of part–whole strategies for solving addition and subtraction problems.

Method

Participants

Nine teachers (seven female and two male) of year 5–6 students from four schools ranging in decile rank from 2 to 10 participated in the study. One teacher from each school had previously worked with the researchers, and that teacher agreed to ask the other teacher/s working at the same level if they would agree to be involved in the study. Teachers' classroom experience ranged from two to approximately 25 years. Experience in working with the NDP approach ranged from two to about seven years. The focus of this paper is mostly on teachers A, B, C, and D (with 6–25 years of classroom teaching and 5–6 years of teaching with the NDP approach), whose transcribed lessons were available at the time of writing.

Procedure

Students were given a paper-and-pencil assessment prior to their teachers teaching the first lesson (Savings Hundreds; see Ministry of Education, 2008c, p. 32), then a similar assessment after the third lesson (A Balancing Act; Ministry of Education, 2008c, p. 40). The lesson that is the focus of this study is Equal Additions (see Ministry of Education, 2008c, pp. 38–39), the second of the three lessons observed. During the lesson, the teacher wore a portable digital audio-recorder attached to a flexible belt, with a lapel microphone to pick up their language to the students (and some responses from students who were close to the teacher). Students wore name tags so that the researchers could identify particular students in their observational records. The researchers observed the lesson and noted non-verbal (contextual) information that could assist with the interpretation of the transcripts of audio-recordings. Actions with materials, written recording in the group recording book, and students' individual mathematics books were photographed to capture some of this non-verbal information. Although the initial lesson, Saving Hundreds, is not the focus of this paper, it should be noted that the experience of working with missing addend problems, such as $\$287 + \square = \$400$, helped to familiarise the students and their teacher with what it was like to be observed and recorded by the researchers. Saving Hundreds also introduced the use of paper money, materials that were essential for the Equal Additions lesson. Another important aspect of the Saving Hundreds lesson was the exchanging of ten $1 notes for one $10 note, and vice versa, and this provided an important introduction to the contexts used for the Equal Additions lesson.

In the Equal Additions lesson, the first scenario is as follows:

> Problem: "Debbie has $445 in her bank account, and her younger sister Christine has $398. How much more money does Debbie have?"
>
> Make piles of $445 and $398. "Now suppose that Grandma gives Christine $2 to give her a 'tidy' amount of money. To be fair, Grandma gives Debbie $2 also." Discuss why 445 – 398 has the same answer as 447 – 400 and then record 445 – 398 = 47 on the board or modelling book.

The book then provides other examples of equations that can be turned into word problems and solved using materials (for example, paper money). It then gives further examples of equations that can be turned into word problems and solved using number properties.

Results

The transcripts for the Equal Additions lesson were analysed according to the content of each teacher's lesson. Most of the teachers drew students' attention to the learning intention for the lesson (see Figure 1). Teacher C appeared to teach this particular lesson very effectively. An examination of the transcript revealed several key features of teacher C's lesson that were not found to the same extent in the other teachers' lessons. These features are described in following sections.

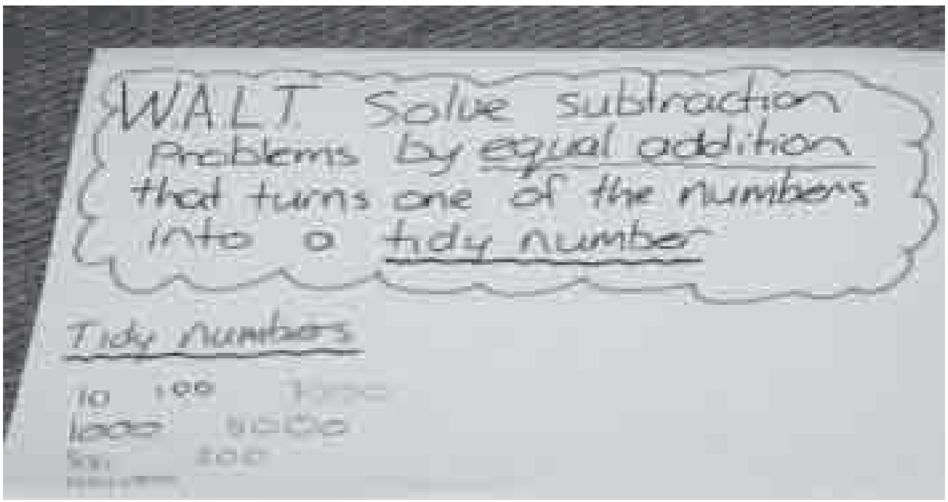

Figure 1. Teacher B's group recording book showing the learning intention (W.A.L.T.: We Are Learning To)

Personalising the Problem for the Students

Some teachers (for example, teacher C) changed the names of the students in the problem to those of students in the group they were working with and got the group to act out the scenario with a different person responsible for each denomination of the money. With subsequent problems, the names of other students in the group were used. Most of the teachers rotated responsibility for managing each denomination of the money so that all students became involved in the problems (see Figure 2).

Fairness

Teacher C emphasised the idea of fairness as a justification for giving the same amount of money to the two students who represented the subtrahend (the number being subtracted) and the minuend (the number from which the subtrahend is subtracted; note that the teacher did not actually use the terms minuend and subtrahend). Teacher B also commented on the importance of fairness as a reason for using equal additions to solve the problem. His scenario had Grandma giving $2 to each child as a gift that had to be kept fair. Teacher C asked the students to suggest a reason why Grandma might give $2 to one child and accepted the suggestion that it was for mowing the lawn, so to give $2 to the other child (who had apparently done no work) did not seem quite fair. This highlighted the importance of making sure that contexts are plausible.

Figure 2. Teacher A's group recording book and money used in the lesson

Proving that the Difference Remained Constant

At one point, teacher C checked to see if the students believed that the difference between the minuend and the subtrahend after the equal additions was the same as it had been before:

> Would that difference between these two numbers [445 and 398] have been 47?

Some students disagreed, expressing doubt about the magnitude of the difference being unaffected by the equal additions process. Teacher C then asked:

> So what's the difference between those two numbers? What do you think, S?

S (C4) responded "49". Teacher C then asked another student, C (C3), who responded "47".

Teacher C then got the students to check the difference by building the subtrahend up until it equalled the minuend and to keep track of how much money they added. The student holding the initial $398 was given $2 by the bank, and that amount ($2) was then recorded. The resulting ten $1 notes were taken away and traded for one $10 that was added to the pile of nine $10 notes. The resulting ten $10 notes were then taken away and traded for one $100 note, which was then added to the pile of three $100 notes. The students were then asked how much money they would need to get to $445. All appeared to agree that another $45 was needed to build up from $400 to $445. Teacher C then commented:

> Let me get this straight. The difference between this amount here [398] and this amount here [445] was $47. Nana gave you both $2 more, and the difference between those two amounts [400 and 447] is …

One student responded, "It would be exactly the same."

Later, one student raised the possibility of turning the minuend into a "tidy number" rather than the subtrahend. Teacher C supported the student in investigating whether it mattered which number was turned into a tidy number. Eventually, the student came to recognise that it was only when the subtrahend (the amount being taken away) was made into a tidy number that the problem became easy to work out (see Figure 3).

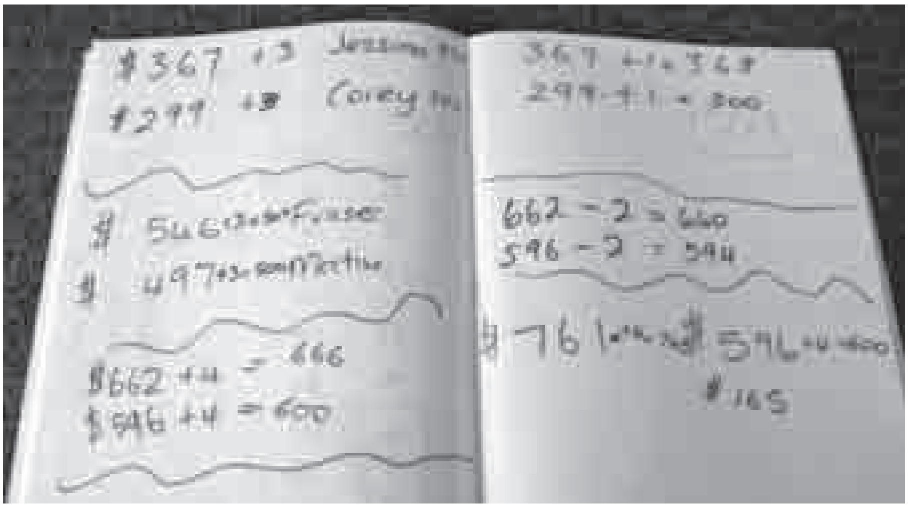

Figure 3. Teacher C's group recording book with students' recording of equal additions

Not all the teachers realised that making the subtrahend tidy was the most efficient way to apply the equal additions strategy. It became clear during the lesson that teacher D did not understand why making the minuend a tidy number would be less helpful than making the subtrahend a tidy number. This was evident in her statement that:

> They're both right, so it doesn't matter which number we turn into a tidy number.

Although teacher D was technically correct, adding 5 to both numbers (which rounded 445 up to 450 and 398 to 403) meant that the subtraction of 403 from 450 still involved the challenge of regrouping and so was not as easy to work out as 400 from 447. Hence, making the subtrahend tidy makes the subtraction problem much easier to work out.

Consistency of Language and Mathematical Structure

Although it was not clear whether teacher C understood about different problem structures, she was very careful in her use of mathematical language and special terminology with the students.

Teacher D took the first example from the book (see description above), which was structured as a *compare* (difference) problem and turned it into a *separate* problem. She said:

> Right, I had $445 right, ... K asked me if she could have a loan of $398 and being the giving, caring person that I am, I said sure. How much money did I have left over? Right, I want you to think about the tidy numbers, using tidy numbers.

One student (S: D1) was concerned that if 2 was added to one number, it needed to be taken off later. Teacher D explained to S (D1):

> S, what I think you've been confused with is if we did it to one of these numbers, if we added 2 to one number, then yes, we do have to take it away – but we did it to both numbers. If we just added 2 to 398 and 445 the same, then yes, we would have to take that 2 away, but because we do the same treatment to both numbers, the gap remains the same.

Several students commented that they had got lost, so teacher D then decided to bring the lesson to a close because they had run out of time for further explanations. (Note: In the post-lesson interview, when teacher D was asked what she would do to follow up on the lesson at a later date, she did not mention an intention to clarify the confusions that emerged during the Equal Additions lesson.)

Teacher B introduced his lesson by sharing with the group how, in preparing for this lesson, his own mathematics had been extended.

> This is one of these really cool exercises. Now I mentioned before that, since doing this, my understanding of maths has really improved. What this next lesson is [is an] actually really cool lesson for an area that I think we've got a bit of a weakness in as a class, looking at one particular type of operation. Now, so what we are going to do is, we're going to look at ... how to solve subtraction problems by equal addition that turns one of the numbers into a tidy number.

Teacher B then asked the students, "What sort of problem are we looking at?"

W (B1) suggested a missing addend structure: 398 plus what equals 445. Teacher B would not accept the missing addend structure because the learning intention focused on subtraction. He said:

> Oh okay, so, W, you've gone for that first one, reversing strategy, so you've gone for 398 plus what equals 445, yeah. If we just look at the learning intention, which is to solve subtraction problems by using equal addition, are we using a subtraction problem here?

One student answered "No". Teacher B continued:

> Is this still a good strategy? Yep, but we're going to look at just using subtraction, so what problem am I going to write down here to show subtraction?

Another student suggested 445 take away 398. Teacher B affirmed that response:

> Nice, so we are going to use 445 take away 398 equals ... We think it might be 47? [suggested earlier by one of the other students]

Teacher B then asked whether adding 2 to both numbers would change the answer. Some students thought it would increase the answer by 4, but others believed that the answer was still the same. Teacher B tried to get those students to explain why:

> What are we actually looking at, we're looking at what? ... So when I give the answer, what's the answer? Okay, if we take the answer, we say, let's say that's 47, we're happy that it's 47. What does the 47 actually mean?

One student suggested "Numbers?", to which teacher B responded, "Excellent, nice, okay."

There is further discussion, but it does not appear to produce what teacher B is wanting, so he explained:

> Okay, with subtraction, we're really looking at the difference between these two numbers, so the difference between 445 and 398 is 47, so we're just looking at difference. So the numbers here, you can change the numbers either way and it's not going to affect the outcome. Does that make sense?

At least one student agreed, but another was concerned about what happens if different amounts are added to different numbers. Teacher B responded:

> Ah now, good question. Will that affect the answer if you're not adding the same amount to each side, because you're looking at difference, but that's a good question, that's a very good question.

He then gave them another problem, but he did not stick consistently to either a *compare* or a *separate* structure.

> Okay, let's have a look. I'm going to give you another problem. Here you go. This time, W's got 367 apples and D would like to have some. He thought he could probably eat 299 apples 'cause he's sort of feeling a little bit hungry, he hasn't eaten for a while. So W started off with 367, and he is going to give D 299 of those 'cause he's quite generous. Now can you predict, now thinking about using that equal addition, will that help us solve the problem?

At least one student responded "Yes". Teacher B responded:

> Keeping in mind that we're looking at the difference between these two numbers, not necessarily the numbers themselves.

One student suggested that the answer was 68. When asked by teacher B how he did it, he responded:

> I gave each of them one more.

Teacher B then pressed for understanding:

> So just while we are doing this, but with F adding on one more, have we changed the difference between the numbers?

The students responded with both "Yes" and "No". Teacher B asked:

> We've changed the numbers, but have we changed the difference between the two numbers?

This time, the students knew they were expected to answer "No".

The Importance of Recording the Process Clearly

Teachers differed in the clarity of their recording. Teacher A recorded the equal-additions process vertically so that the adjusted equation was shown horizontally below the original equation (see Figure 4).

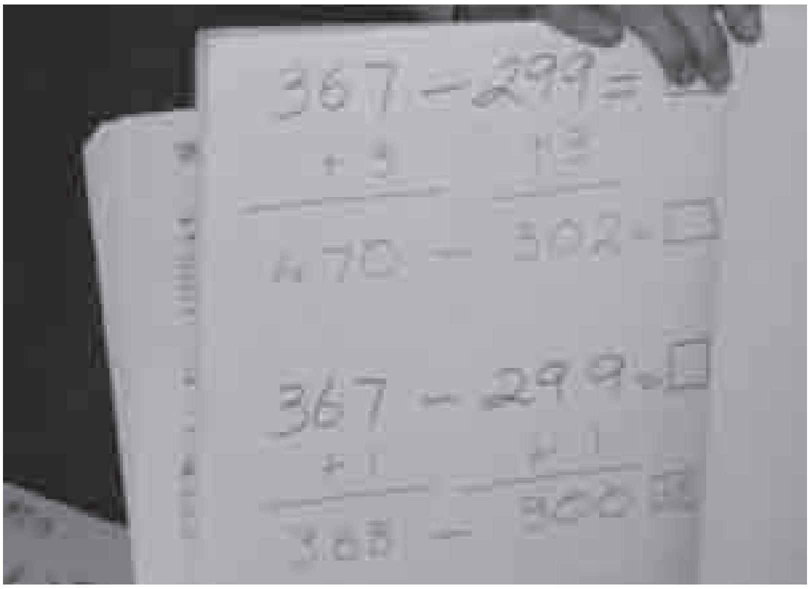

Figure 4. Photograph showing the "tidying" of the minuend (above) and the subtrahend (below) by Teacher A

In the Equal Additions lesson, the adjustments were so small (less than nine) that place value was not usually an issue. However, in the Saving Hundreds lesson, the researchers noticed how important it was to record each rounding-up step separately; for example, rounding up the "ones" digit to the nearest ten (7 + **3** = 10), then rounding the "tens" digit up to the nearest hundred (290 + **10** = 300), then working out how many more hundreds were needed (300 + **100** = 400). Finally, the three amounts needed to be added up to work out the total difference (**3** + **10** + **100** = **113**). It was important for following the step-by-step process that these three amounts were recorded separately. The researchers noticed that some teachers simply inserted the subsequent digits to the left of the one before, so that 3 became 13, which then became 113. This meant that it was not possible then to go back later and work out how the difference of 113 had been found; that is, it was the result of rounding 7 up (by 3) to 10, then rounding 290 up (by 10) to 300, and finally rounding 300 up (by 100) to 400.

Related to the need for care in recording was also the care needed in working with the materials. The researchers noticed one teacher who did not adhere to the rule that there could never be more than nine of any single denomination of the money. That teacher arranged for the person with the eight $1 notes to be given two more $1 notes to make ten altogether but did not ask the person who then had ten $1 notes to exchange them at the bank for one $10 note. The result was that the person with the ten $1 notes was asked to give them ("the ten") to the person holding the eight $10 notes (the "tens person"). The person with $10 notes then had a mixture of denominations, potentially a very confusing situation for students trying to understand place value.

Another important place value issue was the need to ensure that when students described the addition or subtraction process, they referred to the total value of the digits that they were operating with rather than just their "face" value. For example, with 900 – 298 (adjusted to 902 – 300), they needed to describe the subtraction of "three hundred from nine hundred" rather than "three from nine".

Discussion

The teaching of addition and subtraction was far more problematic than the researchers had anticipated. This seemed to be partly the result of teachers having a relatively superficial understanding of addition and subtraction processes and partly because Book 5 (Ministry of Education, 2008c) did not contain enough information to help teachers understand the underlying rationale behind each lesson. There was nothing in the description of the lesson to indicate that the problem structure for the missing-addend problems presented in the Equal Additions lesson was based on subtraction as difference (that is, the *compare* structure) rather than as "take away" (that is, the *separate* structure) (see Carpenter et al., 1999). None of the teachers seemed to be aware of the different problem structures that were possible for addition and subtraction, despite the fact that there has been considerable published work on classifying addition and subtraction problems according to "the types of action or relationships described in the problems" (Carpenter et al., 1999, p. 7). The classification scheme provides a structure for teachers to select problems that they could use for instruction and assists in the interpretation of students' solution strategies. Four classes of problems have been identified, including *join*, *separate*, *part–part–whole*, and *compare*. Join (addition) and separate (subtraction) both involve action, such as putting the objects in two collections together (join) or removing some objects from a collection (separate). Part–part–whole involves the static relationship between a collection and the parts that make up the collection. Compare involves comparisons between two different collections.

Another aspect of the classification focuses on which quantity is the unknown. For join or separate problems, the unknown quantity could be the start, the change, or the result. Traditionally, teachers have mainly used result-unknown problems, such as 8 + 5 = $\square$. The missing-addend problem above ($287 + $\square$ = $400) is an example of a join-change-unknown problem (that is, an addition problem where the second addend is unknown). The NDP resources for addition and subtraction provide problems where the unknown is in different positions, but there is nothing to help teachers understand the classification system or why it is important for students to be given problems from each of the different categories (Ministry of Education, 2008c). For part–part–whole problems, the unknown quantity could be either a part or the whole. For compare problems, the unknown quantity could be the difference, one of the compared collections, or the so-called "referent" collection (Carpenter et al., 1999). The equal-additions strategy provides students with a very elegant solution strategy that is suitable for solving difference-unknown problems from among the various compare structures. It is also valuable for solving separate problems of the result-unknown type (for example, $9 - 6 = \square$) and any of the other problem types in which a part is unknown (for example, join problems of the change-unknown type).

It was interesting to note that on the post-lesson assessment, very few students chose to use the equal-additions strategy to solve the compare problem or one of the two subtraction equations, despite the subtrahend being close to a whole decade or century. This may have been because their teachers did not appreciate that the main point of the lesson was to understand that subtraction could be about difference (rather than "take away") and instead focused on making tidy numbers. Hence, neither teachers nor students valued the equal-additions strategy as a very simple and efficient way to solve subtraction problems with certain numbers. Unfortunately, the students had simply learned a rule for adding the same amount to both sides because they had been told to do so in order to round one quantity up to a tidy number and make the problem easier to solve. This is quite contrary to the goals of the NDP, which are to teach mathematics conceptually rather than procedurally. Most students continued to use a "bridging through ten/hundred" or the "reversibility" strategy to solve the problems in the post-lesson assessment. The students tended to have one favourite strategy that they tried to use for all problems, regardless of its suitability. This raises questions about whether students are actually being empowered to draw on a flexible range of strategies, or whether, instead, the formal written algorithm is simply being replaced by other algorithms that are applied equally mindlessly (Kamii, 1994; Pesek & Kirshner, 2000; Skemp, 2006). Teacher B commented on this in his interview:

> Interesting, G, even when you've been through and all the demonstration, still chose to use his own strategy, so yeah, it was interesting that he reverted back to something he felt comfortable with, so whether he didn't understand or didn't feel comfortable with what we were doing, I'm not too sure.

It would have been good to see more discussion among students about their solution strategies (Whitenack & Yackel, 2002). Hunter (2005, 2006) argues that students need to be encouraged to question each other to check understanding and to convince each other. Teachers tended to control their lessons quite tightly, but this may have been partly because they had been asked to teach a particular lesson as part of the research and they wanted to make sure that the lesson went the way they had anticipated it would. A similar comment could be made about the recording process: students were given few opportunities to record their solution strategies in the group recording book because their teachers tended to do it for them.

Teachers' and students' understanding of the role of the equals sign was an issue, and this was highlighted when the teachers taught a lesson (A Balancing Act) designed to help them appreciate the idea that the expression on the left-hand side of the equals sign should be the same as (or equivalent to) the expression on the right-hand side. For example, the learning intention for A Balancing Act is stated as: "I am learning that the answer on the left of the equals sign is the same as the answer on the right of the equals sign" (Ministry of Education, 2008c, p. 40). Just prior to the learning intention, the book states that "the equals sign means that the total on the left of sign is the same the total on the right." The researchers observed that this learning intention seemed to lead directly to teachers asking the students to calculate the answer for the expression on each side of the equals sign. They did not ask the students to look at the expressions on the left and right of the equals sign and to decide on the missing value by looking at the change in values from left to right and the adjustment to the missing value needed to compensate for that change. This became particularly problematic when the numbers became larger. For example, in the problem $442 - 38 = \boxed{} - 39$, it is not necessary to calculate that 442 – 38 is 404 in order to work out that if 38 is subtracted on the left but one more than 38 (that is, 39) is subtracted on the right, then the missing value must be one more than 442 to ensure that the difference between the two quantities on the left-hand side is the same as the difference between the two quantities on the right. Most of the teachers asked the students to calculate the answer on each side of the equation. Students need to be alerted to look at what adjustments need to be made to the missing value to compensate for changes in the values of quantities on the left and right of the equals sign. The researchers found that very few of the teachers suggested drawing arrows from the left of the equation to the right and recording the size of the change between a quantity on the left and a closely related quantity on the right.

A common problem for many of the teachers was to allow the steps in the process of recording addition or subtraction to "run on" so that the equation was no longer true and therefore did not balance. Teachers (and students) tended to see the equals sign as signalling that the result was following rather than as a statement about the equivalence of the sides of the equation. This became particularly obvious in the subsequent lesson on A Balancing Act. For example, for a problem asking "If John has $360 and Troy has $298, how much more money does John have than Troy?", it was common for students to first record "298 + 2 = 300". Then, instead of starting a new line and adding 60 onto 300 to get 360 (300 + 60 = 360), they recorded the second step of the process straight after the first step, thus violating the requirement of equality within the equation because the first part of statement was no longer true. That is, they recorded the process as 298 + 2 = 300 + 60 = 360. In doing so, they failed to recognise that 298 + 2 does ***not*** equal 360.

Teachers tended to stick very closely to the lesson description in the book, including beginning with the difference between two 3-digit numbers. None of them began with single-digit quantities to get across the initial idea of difference or the comparison between two quantities. None used materials like multilink blocks to model two towers of different heights (for example, 5 compared with 7) to show how the difference in height between the two towers would be unaffected by adding two blocks to each tower (7 compared with 9). None used a number line to show how the distance between two numbers on a number line is constant as long as the same quantity is added to both numbers. Even though Book 5 (Ministry of Education, 2008c) is a reasonably substantial book, there are relatively few lessons that deepen students' knowledge of partitioning or part–whole strategies to give them a full range of possible strategies for solving addition and subtraction problems (that is, to support them in becoming advanced additive part–whole thinkers). This is a particular problem for teachers who treat the NDP resource books as prescriptive (rather than illustrative).

It was interesting to note that none of the teachers mentioned the possibility of solving problems by equal subtraction rather than by equal addition if the minuend could more easily be turned into a tidy number by subtracting one or two from both numbers. It would have been helpful if Book 5 had given some examples of problems that could be solved efficiently by equal subtraction (for example, 489 – 206 = (489 – 6) – (206 – 6) = 483 – 200 = 283). That would encourage teachers to see the difference between the two numbers as being emphasised, as opposed to subtracting digit by digit across the three place value positions.

To support numeracy teaching, the NDP have provided the Number Framework, which shows the learning progressions within particular domains, along with diagnostic assessment tools and a collection of resource materials. It may appear that teachers have everything they need in terms of resources to teach numeracy. However, what is missing from the NDP resource materials is the theoretical underpinning and variety of possible structures for addition and subtraction problems. What is also missing for a lot of teachers is a deep and connected understanding of the mathematics required for teaching with a conceptual emphasis; in other words, they lack pedagogical content knowledge (PCK) in mathematics (Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Goya, 2006; Moch, 2004).

It is generally assumed by teachers that helping students learn to add and subtract is a very straightforward process. However, this overlooks the enormous complexity of these operations. We believe that there is a lack of appreciation of the challenge for teachers of coming to understand at a deep conceptual level what addition and subtraction with whole numbers is all about. As Bahr and de Garcia (2010) point out in the title of their book, "Elementary mathematics is anything but elementary."

Conclusions

A number of key recommendations have emerged from this research:

- Teachers need to build a deeper, more connected understanding of addition and subtraction processes as part of developing greater pedagogical content knowledge in mathematics.
- Teachers would benefit from being given additional support material outlining the variety of problem structures for addition and subtraction to which their students should be exposed.
- The purpose of the illustrative lessons in the NDP "pink" books need to be clearly articulated so that teachers understand the reason for including that particular lesson in their classroom mathematics programme.
- The learning intentions for some lessons need to be reworded to indicate their purpose more accurately. For example, the learning intention for the lesson entitled A Balancing Act needs to be reworded to avoid implying that students need to find the "answer" in order to solve the problem. Students need to be alerted to look at what adjustments should be made to the missing value to compensate for changes in the values of quantities on the left and right of the equals sign.
- It would be helpful for many teachers to have additional notes about the key ideas they need to understand before they teach the lessons (for example, the idea that subtraction can be about comparing quantities to find the difference between them).
- An understanding is needed that the *compare* structure for subtraction is about *difference* rather than about "take away" or separating a collection into parts.
- It is important that teachers are careful in the way they demonstrate their recording of equations, ensuring that expressions on either side of an equals sign are in fact equal. Hence the steps involved in solving a problem need to be recorded separately, on a new line for each step.
- Students need to be encouraged to discuss the usefulness of particular strategies to solve particular problems (or problems with particular numbers). The goal should be about making connections between different strategies and about considering the efficiency of particular strategies.
- Students should feel comfortable about questioning their peers and about justifying to them the particular strategies they have chosen to use. This is an important aspect of the mathematical communication that is now emphasised as part of mathematics education reform.

- Giving students responsibility for recording in a group recording book (rather than the teacher doing all the recording) is important to help engage the students in the problem-solving process.
- The so-called "modelling book" needs to be renamed the "group recording book" to avoid implying that it is a place where the teacher provides good examples or "models" to students. Students should be encouraged to take ownership of the group recording book to record their strategies and the discussions they have about the efficiency of those various strategies.
- The emphasis needs to be on building conceptual understanding rather than being trained in particular procedures. The dangers of simply replacing one algorithm with another needs to be made clear to teachers.
- The equal-additions strategy needs to be introduced to students using small (single-digit) quantities before moving on to larger numbers. The process needs to be modelled initially with small quantities, using linkable blocks to demonstrate that adding the same number to two quantities does not change the difference between them.
- The number line could also be used with students to show the constancy of the difference between two numbers when the same quantity is added to each number. A cardboard strip with clip-on pegs can be used to record the initial difference between the two numbers and then used again to check that the difference between those numbers has not changed after the same quantity is added to each number.
- Students need to be encouraged to explore the issue of whether tidying the subtrahend or the minuend is the most efficient strategy when using equal additions.
- When teachers use paper money to model addition and subtraction problems, they need to make a special point of emphasising that there can never be more than nine notes of any one denomination and that every ten notes of a particular denomination need to be exchanged for one note of the next denomination (for example, ten $1 notes for one $10, ten $10 notes for one $100 note).
- Students need to be encouraged, when describing operations, to refer to a digit in terms of its total value rather than simply its face value (for example, in 398, the 9 should be referred to as 90 and the 3 as 300).

Acknowledgments

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Evaluation of Te Poutama Tau: Māori-medium Numeracy Project, 2003–2009

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Te Poutama Tau (the Māori-medium component of the Numeracy Development Projects [NDP]) has been implemented in a large number of Māori-medium schools from 2002 to 2009. Te Poutama Tau is based on the Mahere Tau (the Number Framework), in which students progress through stages of learning. The considerable corpus of student achievement data collected since Te Poutama Tau began provides information on longitudinal patterns of student performance. The patterns of performance and progress for Te Poutama Tau across 2003–2009 are generally similar, with some small variations from year to year, for particular age groups and in the various components of each domain. There do not appear to be any differences in performance in gender, which could be partly due to the type of oral diagnostic tool used to assess student progress. Previous studies (Trinick & Stevenson, 2007, 2008) have shown that boys do as well as girls in these types of numeracy assessments. In Te Poutama Tau, there appears to be significant differences in student performance between individual schools and between schools with different decile ratings. Overall, the most positive student progress is linked to the class within a school.

Background

Te Poutama Tau (the Māori-medium numeracy project) has been a significant professional development programme for Māori-medium mathematics educators. Māori medium includes kura kaupapa Māori[1], kura ā-iwi (tribal school), and Māori immersion classes in English-medium schools. Te Poutama Tau was initially developed from a pilot in 2002 (Christensen, 2003) and was then fully implemented in a wider range of schools from 2003 to 2009. From 2010, the focus of Māori-medium professional mathematics development for years 1–8 will be on the implementation of the whanaketanga pāngarau (Māori-medium mathematics national standards, Ministry of Education, 2010).

Te Poutama Tau is based on the Number Framework developed for New Zealand schools (Ministry of Education, 2007), which provides a clear description of the key concepts and progressions of learning for students. The two major domains of the Number Framework are the strategy and knowledge domains. The strategy domain is made up of three components: addition and subtraction, multiplication and division, and proportion. The knowledge domain is made up of various components, including numeral identification, forward and backward counting, basic facts and grouping, and fractions. These key domains and the concepts that underpin them have now been fully integrated into the re-developed pāngarau curriculum (*Te Marautanga o Aotearoa*, Ministry of Education 2009) and the recently developed draft whanaketanga (Ministry of Education, 2010). The teaching, learning, and assessment resources and practices developed in Te Poutama Tau will provide valuable ongoing support to teachers and students.

Te Poutama Tau data has provided a significant corpus of data for analysis and investigation. Analyses of student achievement data gathered every year from 2002 has provided a valuable source of information to teachers, schools, and numeracy facilitators supporting Te Poutama Tau.

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[1] kura kaupapa Māori: full Māori-immersion school that follows the philosophies of Te Aho Matua

This paper is in two major parts: Part A reports on the results of the 2009 Te Poutama Tau project, and Part B reports on longitudinal patterns of student performance. The research focuses on the following questions:

- What are student patterns of performance and progress for 2009?
- How do patterns of performance and progress compare from 2006 to 2009?
- Is there a relationship between students' language proficiency and their progress on the Mahere Tau (the Number Framework) in the years 2003–2009?
- Are there any differences in performance between gender, schools, and schools with different decile ratings?
- Have student patterns of progress in Te Poutama Tau improved over time?
- What are the main numeracy variables that impact on students' progress through the stages of the Mahere Tau?

Part A: An Evaluation of Te Poutama Tau 2009

Participants

The following summaries of the data were restricted to only those students with both initial and final test results. There was data for 900 students in the 2009 sample, with the largest number of students being in years 3 and 4 (121 and 124 students, respectively) and the lowest in years 9 and 10. As noted, these students are from kura kaupapa Māori, kura a-iwi, and Māori immersion classes in English-medium schools.

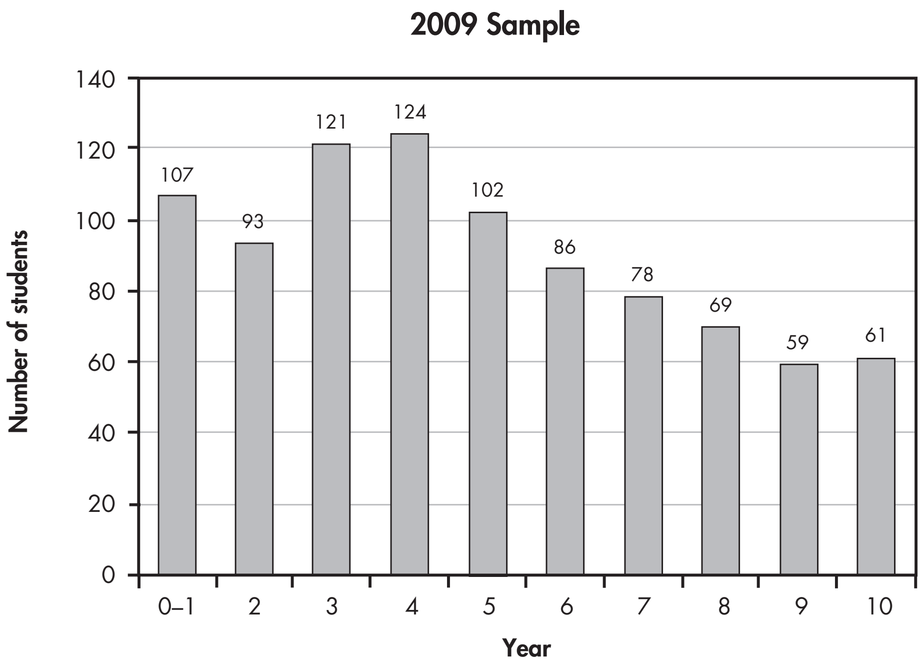

Figure 1. Distribution of Te Poutama Tau students across year levels, 2009

Student Achievement and Year Level in 2009: Final Stage Score

Strategy domain

The stack graph below (Figure 2) shows the mean final stage that each year group tested at in 2009. In general, the older the year group, the higher the final stage at which the students tested. For example, at year 3, students' mean final stage for addition and subtraction was above stage 4 and less so for the other components of the strategy domain. The year 9 students had the highest mean final stage for the strategy components. It is not clear why there are dips at year 8 and year 10. The year 10 dip could be partially explained by the ceiling effect, that is, students having already reached the highest stage. This dip patterns are explained more in the next section on longitudinal trends and patterns.

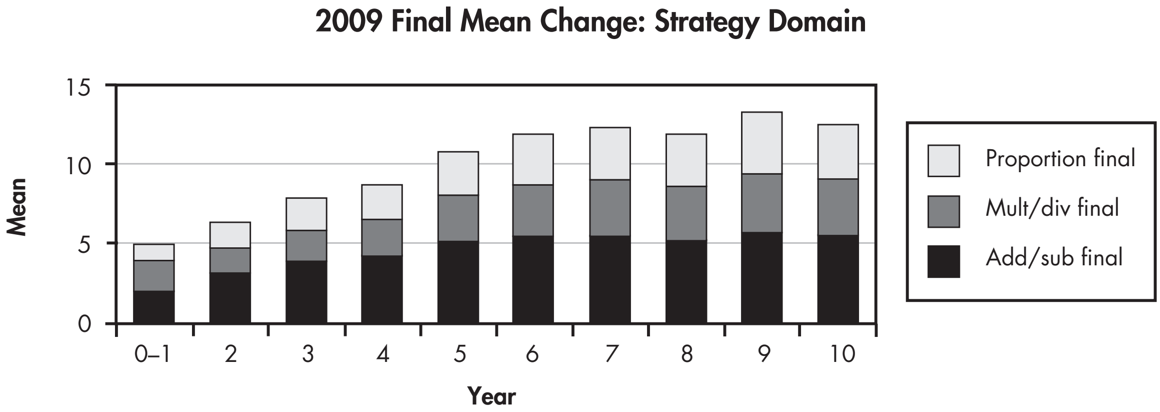

Figure 2. Student achievement and year level: strategies, 2009

Knowledge domain

The knowledge domain tests (Figure 3) tend to follow a similar pattern to those of the strategy domain (Figure 2). With the exception of NID (numerical identification), there is continuous improvement over time. The minimal change in NID after year 4 can be explained by the fact that there are only four stages and NID is subsumed into other components for the two tests beyond Uiui A (test A). The data has also been complicated by teachers entering 0 for the older students' stage gain in NID, when in fact they should have left this as a blank entry; that is, the students were already at stage 4, the highest stage for NID. For those not familiar with the acronyms, BNWS (backward number word sequence) requires students to count backwards and identify numbers before a given number. FNWS (forward number word sequence) requires students to count forward and name numbers after a given number.

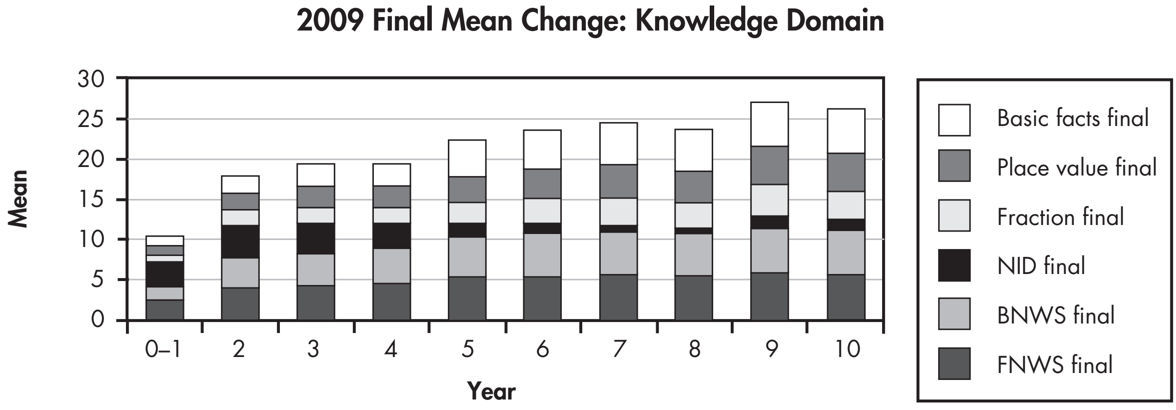

Figure 3. Student achievement and year level: knowledge, 2009

Change in Scores

Strategy and knowledge domains

The following section examines improvement in students' mean stage[2] between their initial and final tests for each domain of the Mahere Tau. Figure 4 shows a general trend for smaller improvements between initial and final at higher year levels. However, in common with previous years, there is a dip in performance for students in year 3, in which students made lower gains in the test scores than in years 4 and 5 or years 0–1 and 2. A number of variables need to be considered when interpreting the results, including the increasing complexity of the stages (higher levels are more complex), the ceiling effect, and the number of years that students have been in Te Poutama Tau.

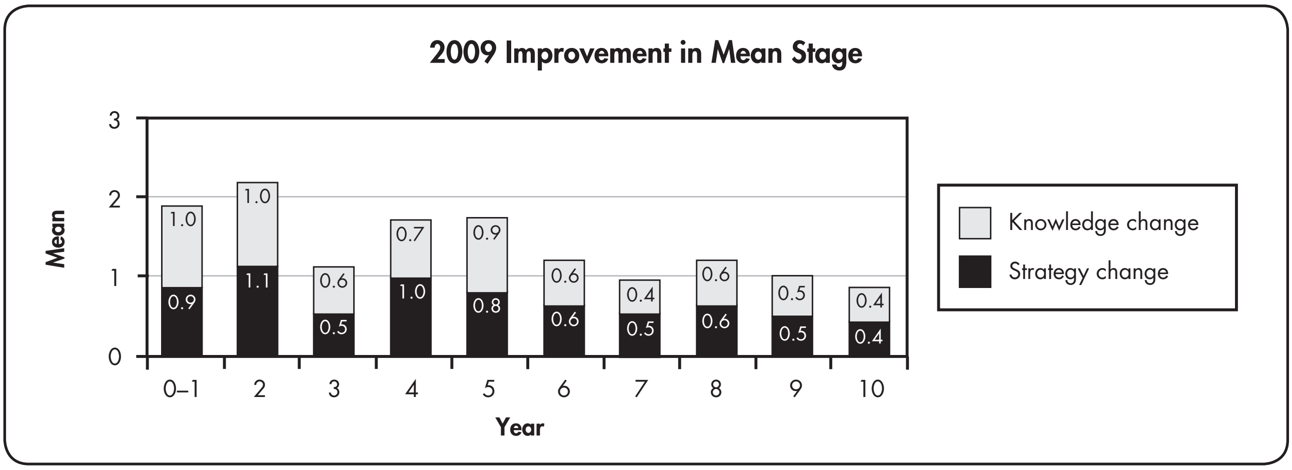

Figure 4. Mean change in students' scores in the strategy and knowledge domains

To ascertain whether this apparent improvement (Figure 4) was a property of the test or of the year level of the students, the mean change in the strategy and knowledge domains was compared by initial test score (see Table 1). This table shows that there is a clear trend for a smaller change in test performance with increasing initial score (except for a slightly smaller test performance for a stage 0 initial score compared with those scoring a 1 on the initial test), with no dip in performance in the middle of the scale. This was expected because of a ceiling effect. Students who were at stage 1 made the greatest stage gain for both strategy (1.37) and knowledge (1.46). Note that students tested initially at stage 1 are of different ages. The implications for teaching, learning, and assessment of these changes are discussed further in the summary section in relation to whanaketanga (national standards). Stage 7 and 8 data is not included for two reasons: stage 6 is the highest stage for addition and subtraction, stage 7 for multiplication and division, and stage 8 for proportion; secondly, the changes at stages 7 and 8 are very small.

Table 1
Mean Change in the Strategy and Knowledge Domains by Initial Test Score, 2009

Stage students initially tested at	0	1	2	3	4	5	6
Mean strategy change	1.17	1.37	1.11	0.67	0.36	0.23	0.17
Mean knowledge change	1.22	1.46	1.06	0.68	0.41	0.30	0.11

[2] This was an average of all the tests in each domain.

Changes: Strategy domain

This section examines the disaggregated components of the strategy domain. Figure 5 shows the variation in the mean gain for the tests in the strategy domain across the year levels. As discussed previously, there was a dip in performance at year 3, although the general trend was for a smaller change in the upper years. The majority of students at years 0–1 were tested using Uiui A. This test does not contain multiplication and division and proportion questions. Consequently, changes in these areas do not show. There are significant mean stage gains at year 2, which is consistent with previous years (Trinick & Stevenson, 2008, 2009).

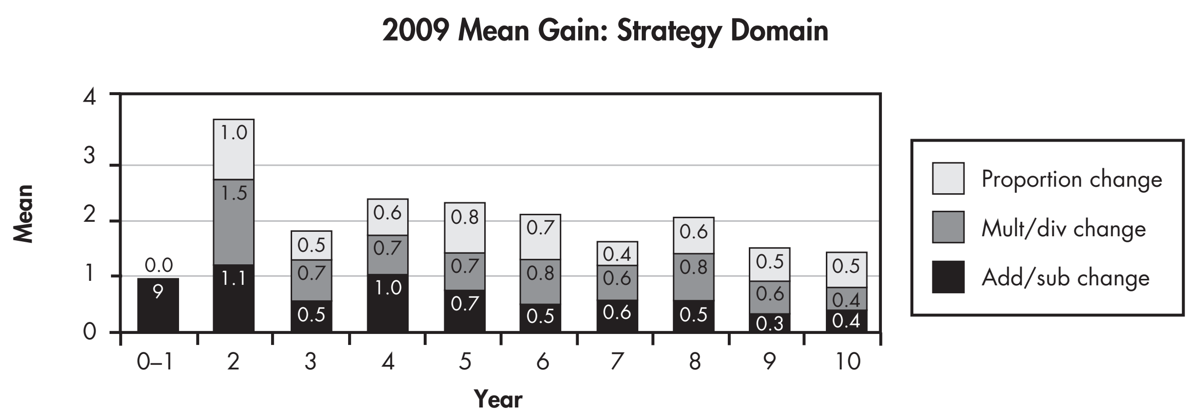

Figure 5. 2009 mean change across the components of the strategy domain

Changes: Components of the knowledge domain

As with the strategy domain, there was a trend for smaller improvements in the initial scores with higher year levels (given that higher years equated to a higher initial score and a consequent ceiling effect). Again, the year 2 students made the greatest stage gain. The implications that this may have on student progress in the whanaketanga are discussed in the summary section.

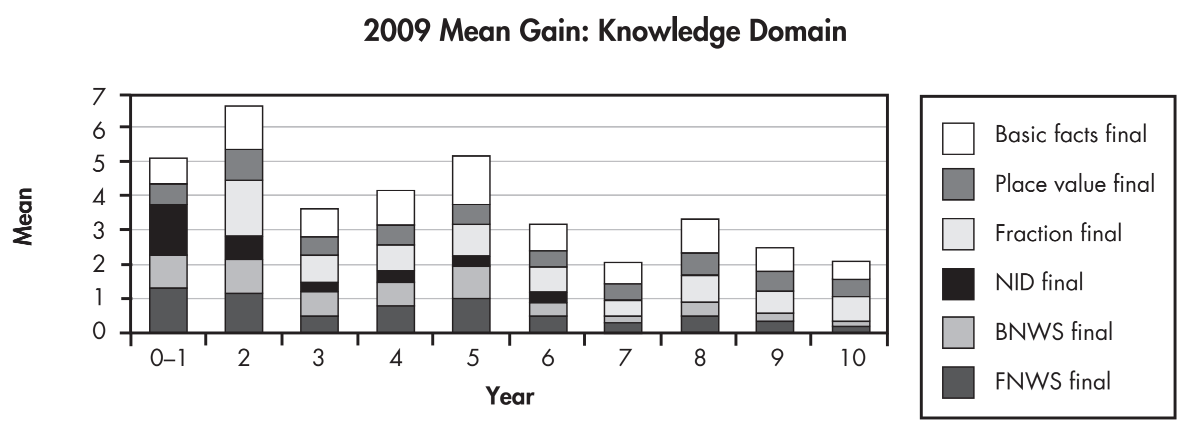

Figure 6. 2009 mean change across the components of the knowledge domain

Part B: Longitudinal Patterns of Progress

As noted, a considerable corpus of data has been collected that enables a range of general statements to be made about student achievement in Te Poutama Tau and the factors that affect performance. This section examines patterns of performance over six years of implementation of Te Poutama Tau and only includes the data of students who have participated in the project for at least two years. Most of the data collected is for the earlier year groups. Initially, the Te Poutama Tau professional development programme focused on the earlier years, with some shift to years 8–10 in the last few years of the project.

Student Patterns of Progress over Time, 2006–2009

Final Stage Score: Strategy and Knowledge

The line graphs below (figures 7–8) show how students' final stage score on the strategy and knowledge domains of the Mahere Tau has changed over time. Overall, there was little difference between the sample years, with a consistent trend upwards in final score as the students progress through year levels. This seems to indicate that students in each year group have performed at about the same level on the strategy final scores over the four years shown in the graph (2006–2009). There was a slightly higher performance for those in year 8 in 2006, with those (most likely the same) students also doing well in year 9 in 2007 and year 10 in 2008. (See the next section for a discussion of overall change in both strategy and knowledge.)

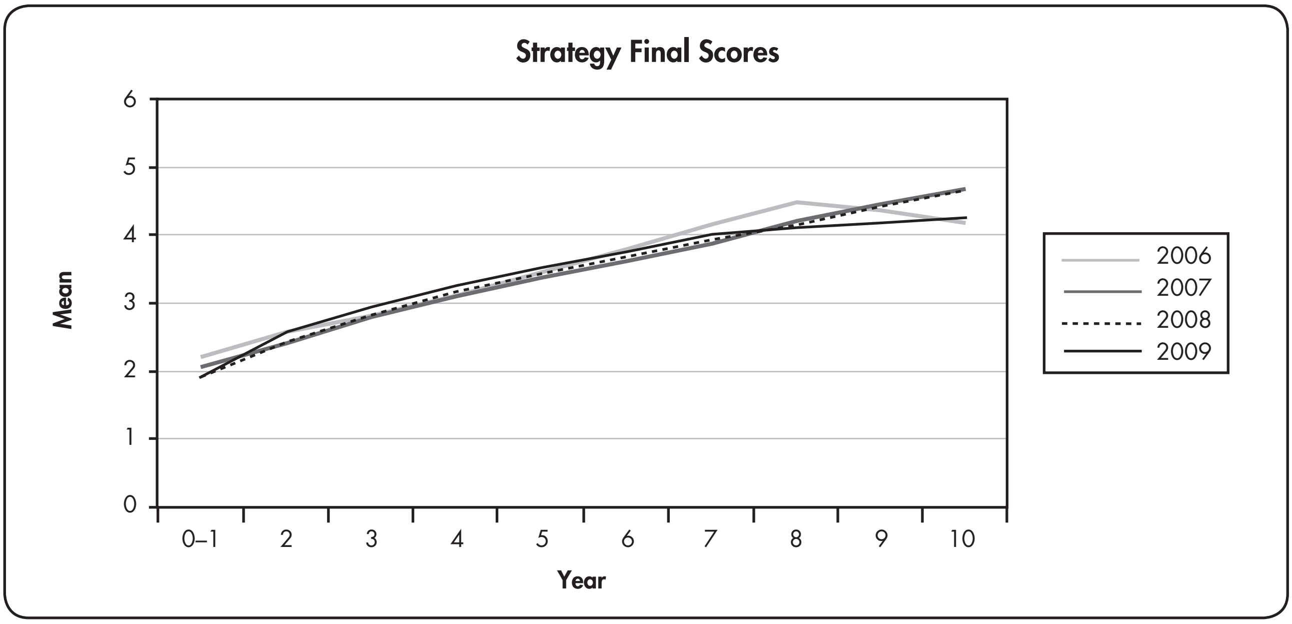

Figure 7. Line graph showing how strategy scores (estimated by Lowess method[3]) changed for the different year groups by sampling year

For the knowledge scores, there are only minor variations over the sample years 2006–2009. Also indicating that overall students have performed at the same levels on the final knowledge tests over the four years presented below.

[3] Using the Lowess command in Stata: Lowess carries out a locally-weighted regression of *y*var on *x*var, displays the graph, and saves the smoothed variable.

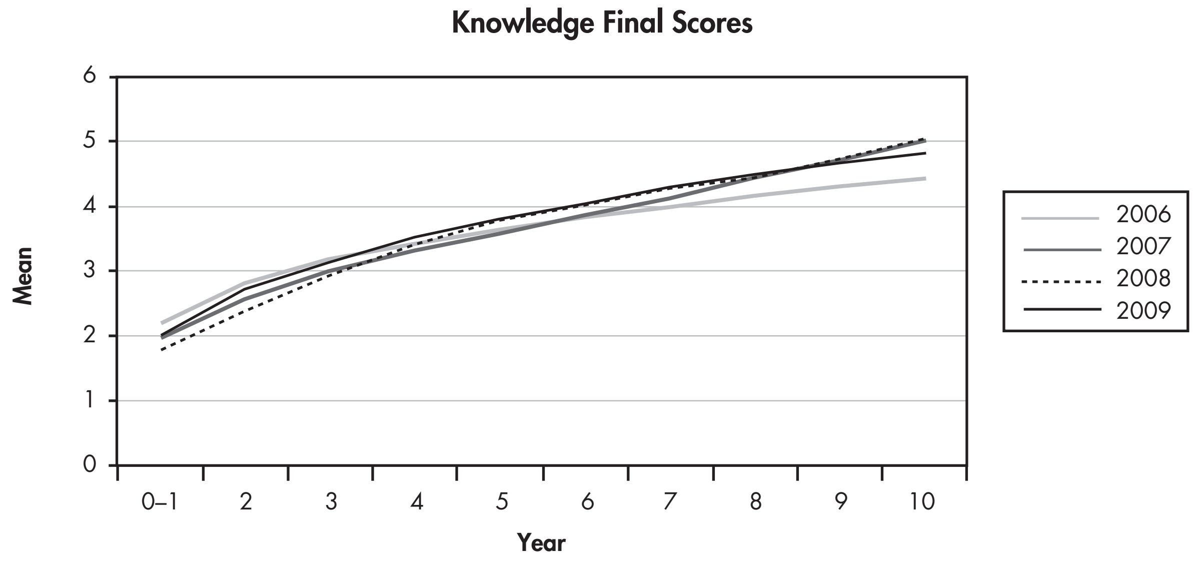

Figure 8. Line graph showing how knowledge scores (estimated by Lowess method) changed for the different year groups by sampling year

Changes in Mean Score: Strategy and Knowledge

Figures 9 and 10 show the size of the mean stage gain for each year group when comparing their final scores with their initial scores. In examining students' performance on the strategy domain from the initial tests, Figure 9 shows that there has been a consistent pattern in the mean change over the last three years (2007–2009). The older the students are, the less change in the mean scores. This probably has a lot to do with the complexity of the higher stages. As the strategies become more complex, student mean score is reduced accordingly. There was a slight improvement in performance for years 9 and 10 for the 2007 data (probably due to the cohort of students who performed well in the 2006 year 8 final strategy scores).

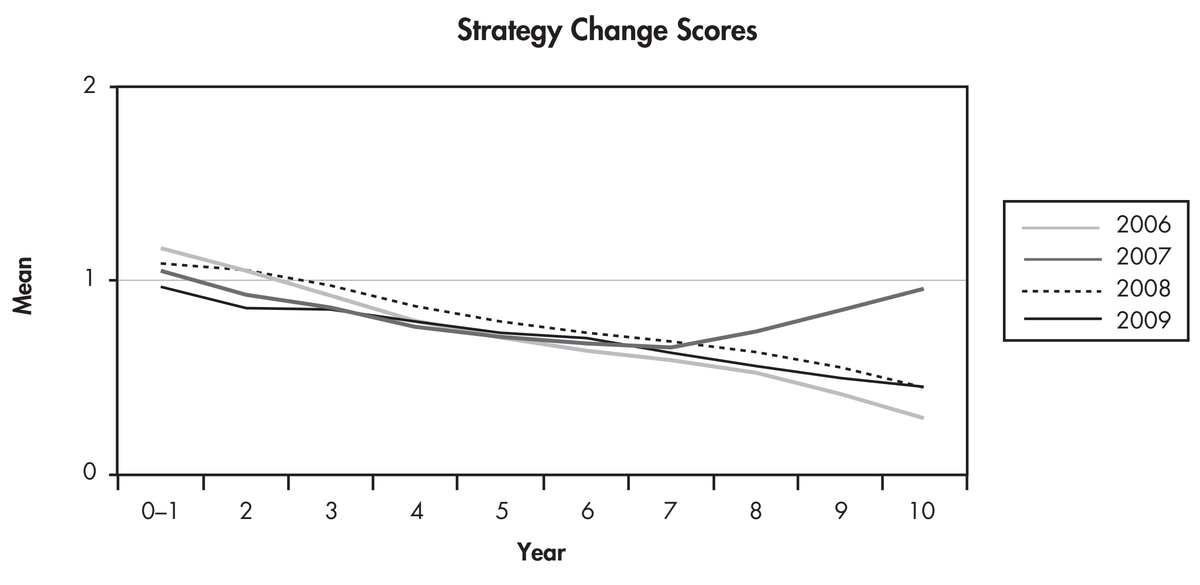

Figure 9. Line graph showing how strategy scores (estimated by Lowess method) changed for the different year groups by sampling year

Figure 10 indicates that students made marginally less improvement on the knowledge domain in 2009 than in the previous two years; however, given that students' final knowledge test results for 2009 (Figure 8) had been similar to the final knowledge test results for 2007 and 2008, this indicates that students were beginning the year at a higher stage level in 2009 (with less effort required to reach results typical of previous years).

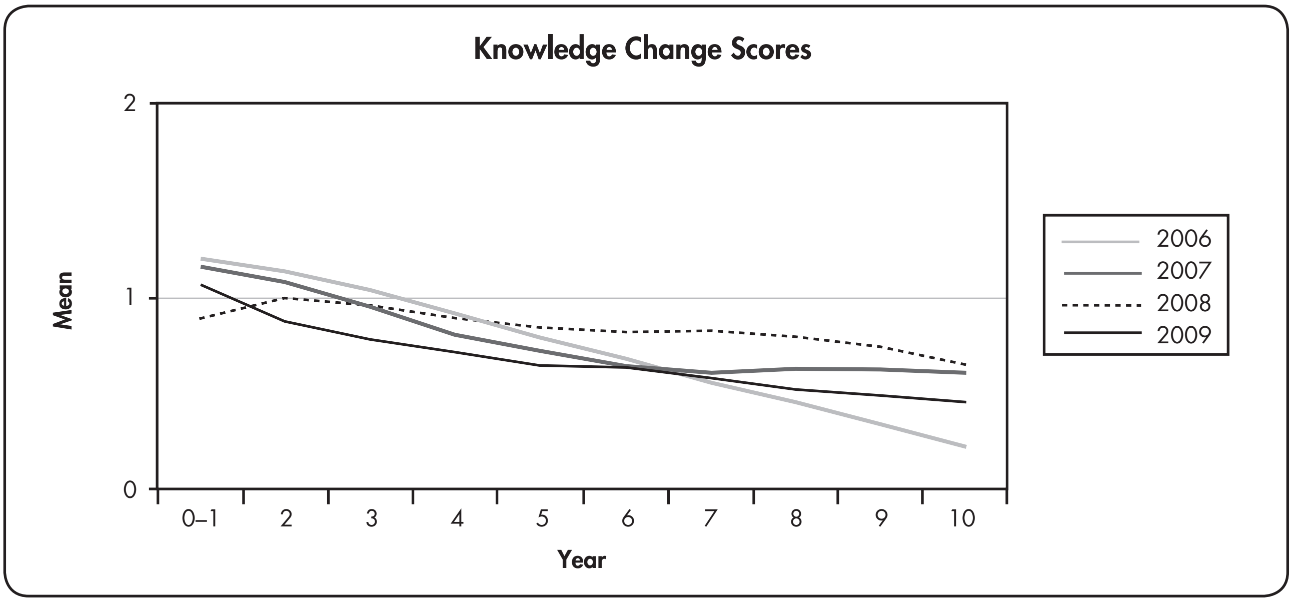

Figure 10. Line graph showing how knowledge scores (estimated by Lowess method) changed for the different year groups by sampling year

Te Reo Māori Language Proficiency of Students, 2003–2009

Mean Stage Changes According to Language Proficiency: Strategy and Knowledge

Māori-medium education, in particular kura kaupapa Māori, is based on the premise of revitalising te reo Māori. The data shows there is a wide range of language ability for the students in Te Poutama Tau. It is important to understand the relationship between students' language proficiency and their performance on the Mahere Tau.

The majority of the students who had data entered into the Te Poutama Tau database were classified by their teachers as being either "he matatau" (proficient) or "āhua matatau" (reasonably proficient). These ratings rely on the teacher's knowledge of the student's language ability, which is drawn from several indicators, including their oral and written work and their participation in the various communicative activities of the class and school. It is difficult to know how accurate these proficiency ratings are. There is little, if any, moderation between schools to develop a level of consistency, nor are there currently available any standardised tests to measure students' fluency.

For the mean strategy change (Figure 11), with the exception of years 8 and 10 there was a small performance benefit associated with better language proficiency. However, at year 8 and marginally so at year 10, the students rated less proficient made better stage gains. It is not clear why the less proficient students at year 8 do marginally better than their more proficient cohort.

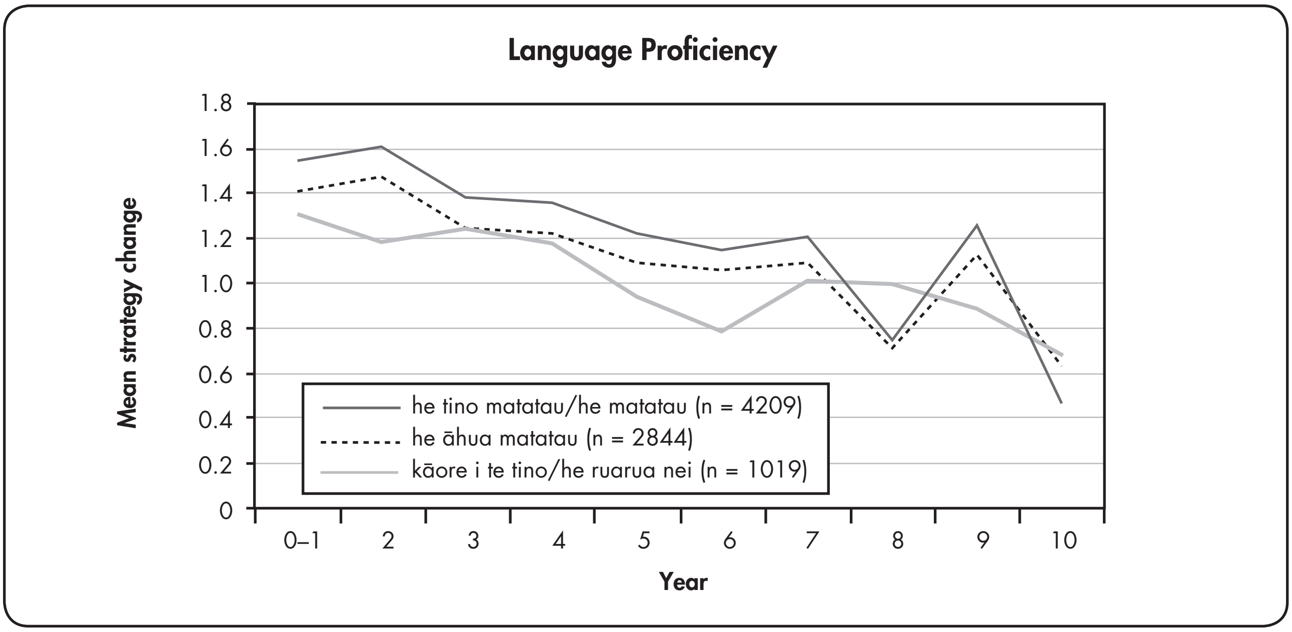

Figure 11. The language proficiency of the students for mean strategy change, 2003–2009

Similarly, for mean knowledge change (Figure 12), those students at year 8 rated as kāore i te tino matatau (not very proficient) did slightly better than the other ratings. The difference between students rated as very proficient (he tino matatau) and not very proficient (kāore i te tino matatau) is less pronounced for knowledge than it is for strategy (see Figure 11). This can be partly explained by the notion that students are not required to verbalise their ideas in the knowledge domain as they are in the strategy domain. For example, students are not asked to explain how they arrived at their answer. Therefore, the less proficient students are only having to translate single words in their minds if they are thinking in English and replying in Māori.

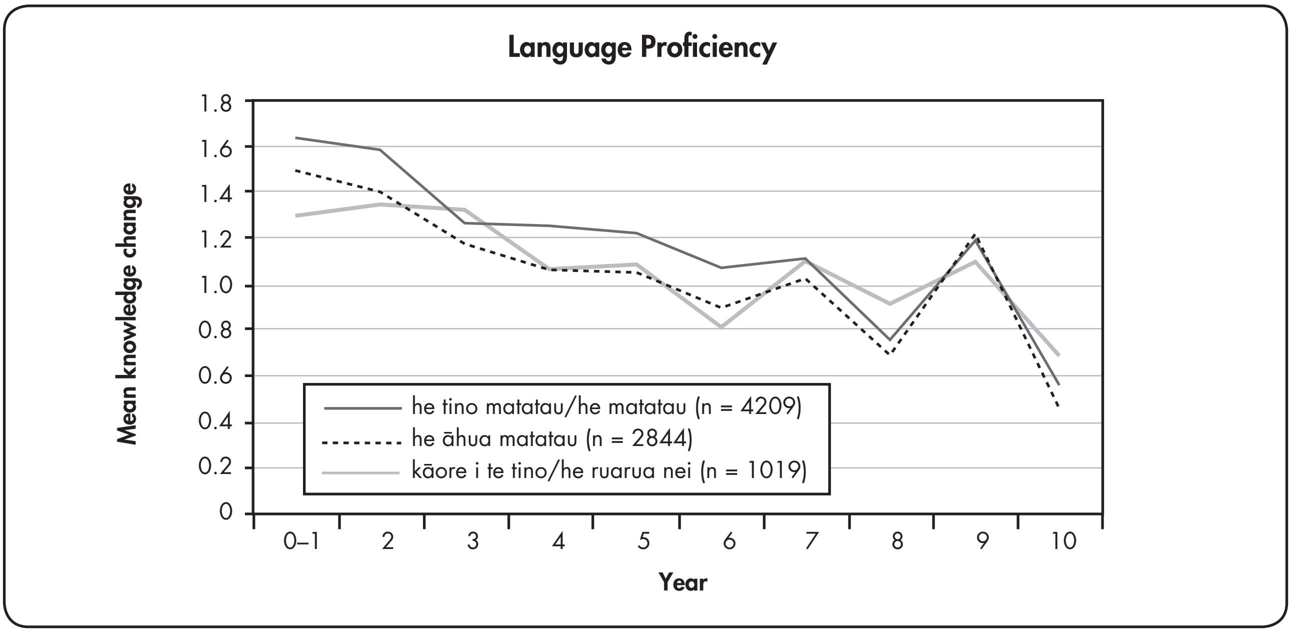

Figure 12. The language proficiency of the students for mean knowledge change, 2003–2009

Language proficiency patterns also closely mirror the results in changes in scores across the strategy and knowledge domains. The dip in performance at year 8 and year 10 corresponds to improved performance by those less proficient in comparison with their more proficient peers. This requires more research.

Difference in Performance between Gender, School, and Decile Rating

Differences in Gender

No difference in performance between genders was detected. This is in contrast to the English-medium NDP, in which girls who began the projects at lower Number Framework stages appeared to make slightly better progress than boys who began at the same stage, but the opposite pattern was found at higher Framework stages, with more boys progressing to a higher stage than girls (Young-Loveridge, 2004).

Individual School Performance

The following graphs show patterns of performance across a selection of schools that were in Te Poutama Tau for two years. The majority of schools participated in the project for no more than one year. The graphs show changes in student performance between the final tests over two years. Schools that have significantly wide bars are the schools that have small numbers, creating wider confidence intervals. The dot on the line indicates the mean.

When individual schools' data was analysed, there were significant differences in the range of performance from school to school. The dot on the error bar is the mean. The line itself shows the degree of certainty that the mean is accurate –we are 95% certain the actual mean lies along that line (with longer lines indicating more uncertainty and shorter lines more certainty). When there is no line, but only a dot, this indicates that the dot represents 95% of scores. There was also a significant range of performance within some schools. The error bar graphs (figures 13 and 14) show that generally most schools had students who were improving from one year to the next. Other than decile and performance in the test, this data cannot explain the impact on student progress of variables such as teacher training, home environment, or teacher expectations. Case studies of individual schools may ascertain the impact of various variables that affect student achievement and explain the differences in performance.

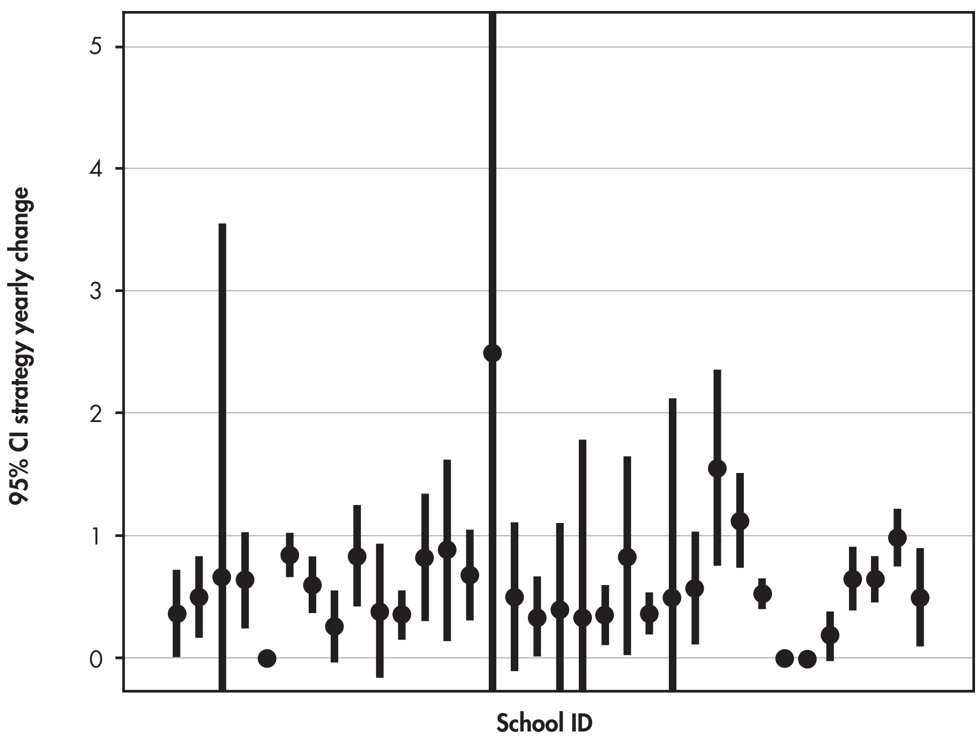

Figure 13. Error bar plot showing how change in performance for strategy over two years varied for participating schools

The mean change for knowledge generally follows the pattern for strategy. Noticeably, there is a wide range of performance in some schools. As discussed earlier, the smaller schools are probably those with the longer bars because any errors in measurement will have a larger impact when the sample is smaller. However, differences in performance can be made by comparing the mean change.

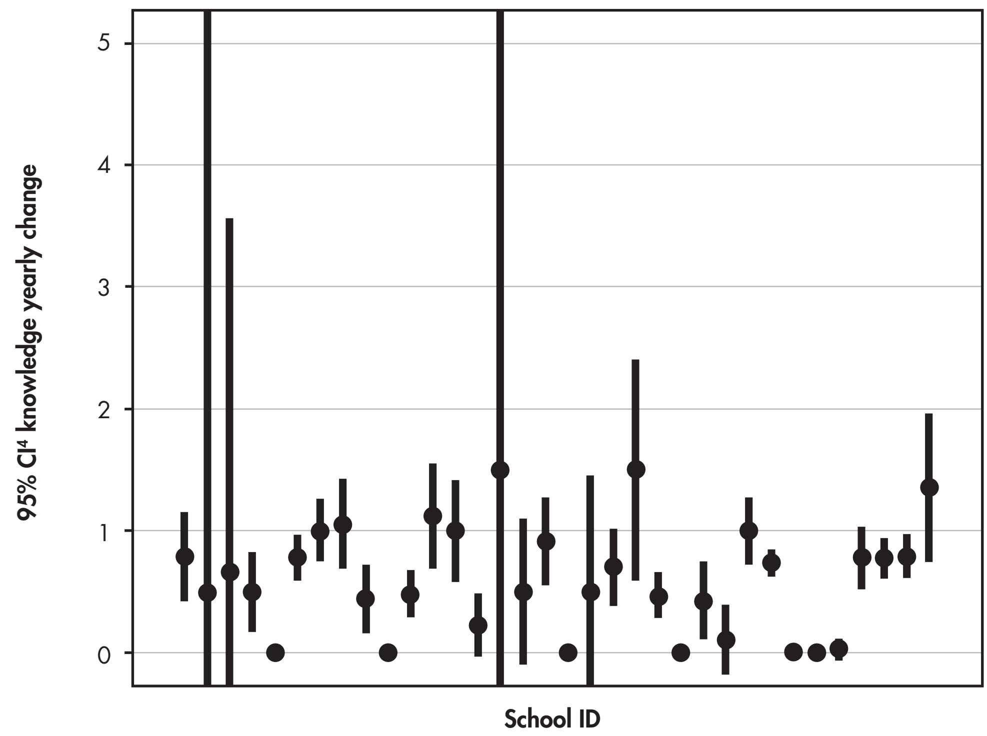

Figure 14. Error bar plot showing how change in performance for knowledge over two years varied for participating schools

Differences in Schools According to Decile

There is a significant difference between schools according to their decile rating (see figures 15 and 16). Students in higher decile schools made better progress over time than those in lower decile schools. All schools that participated in Te Poutama Tau from 2003 to 2008 have been included in the data set. The data was calculated on the basis of the schools' 2009 decile rating. Some schools' decile rating changed up (for example, from decile 2 to decile 3) or down (for example, from decile 2 to decile 1). The majority of the schools that have participated in Te Poutama Tau are decile 1–3. There are a couple of schools above decile 5, but their data is not enough to give a reliable picture.

[4] CI: Confidence Interval, that is, 95% of any means calculated from the sample of this population will contain the true mean.

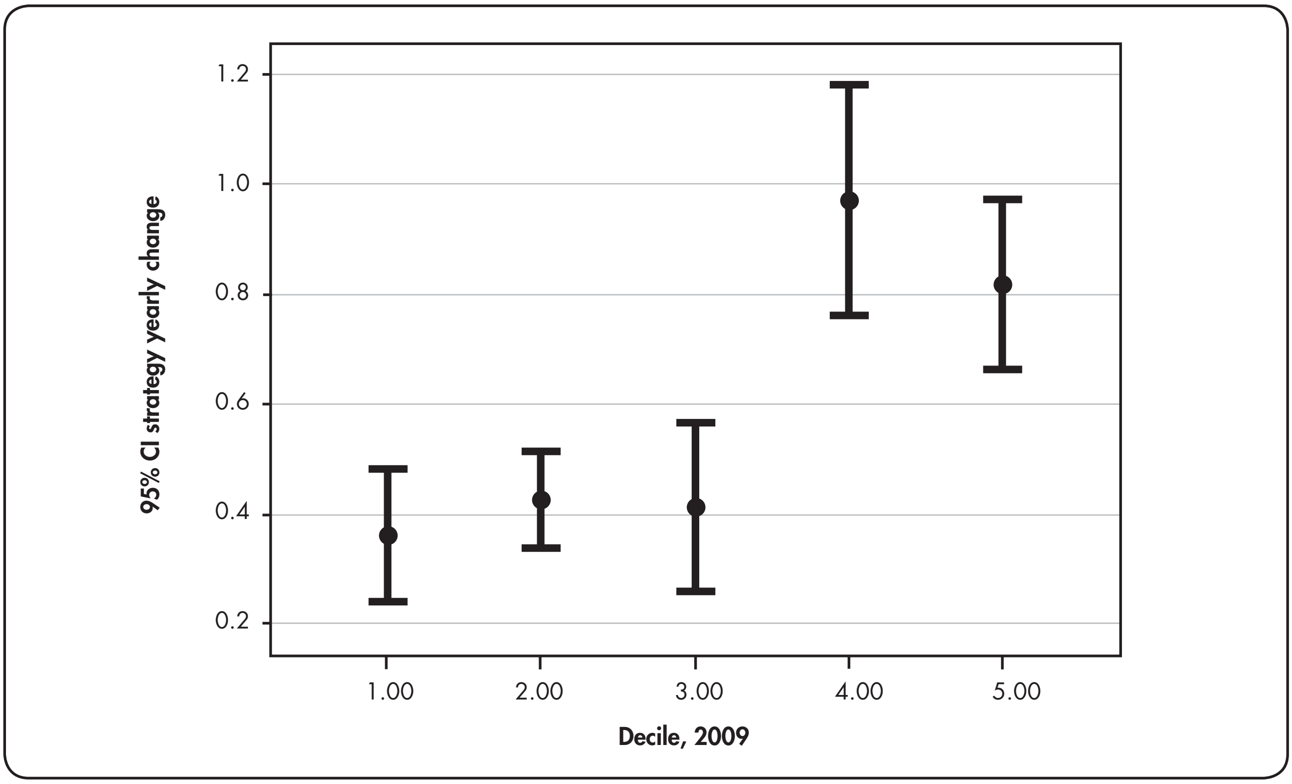

Figure 15. Performance in strategy component according to school decile

Similarly, in the knowledge domain, the students in higher decile schools, particularly decile 4, achieved higher mean stage gains than those in deciles 1–3.

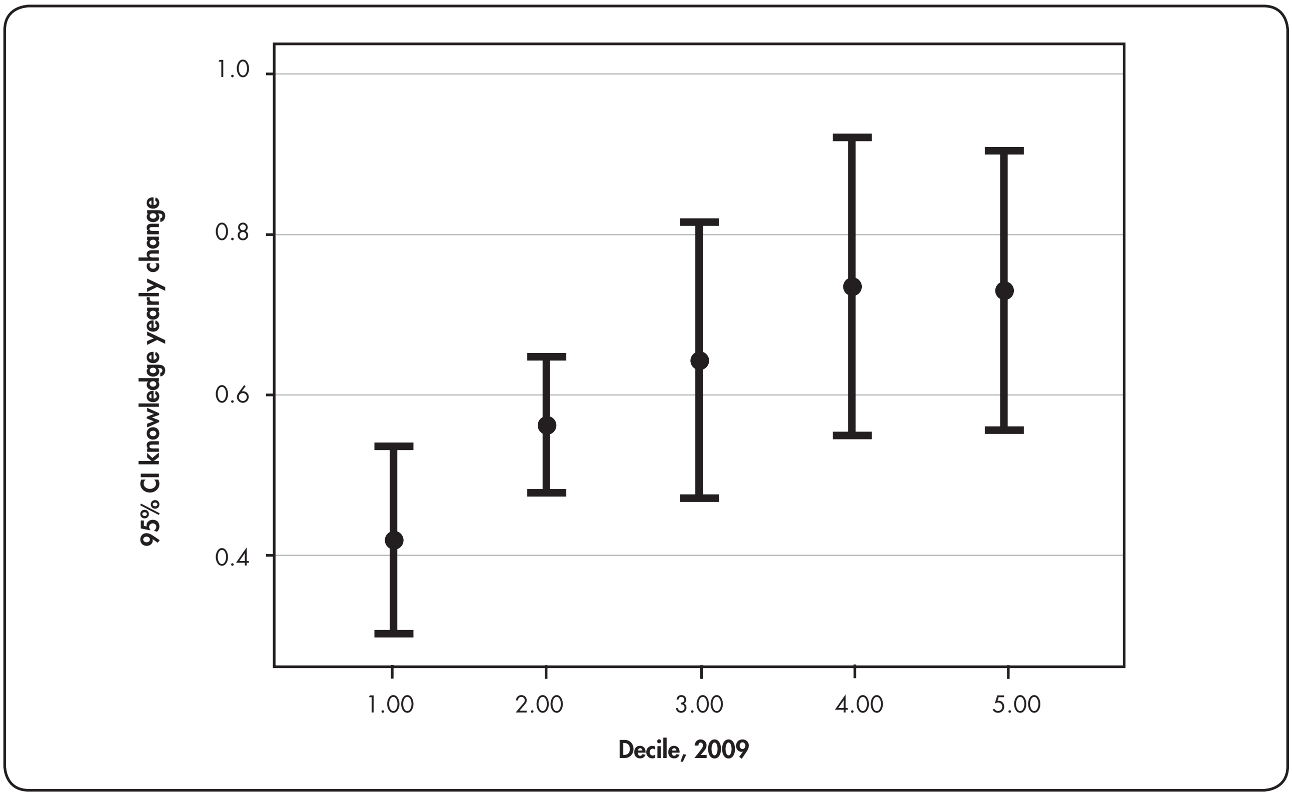

Figure 16. Performance in knowledge component according to school decile

Numeracy Factors that Impact on Students' Progress through the Stages of the Mahere Tau

To assess changes in students' performance over time, two variables were created that measured the change in each student's performance on the final strategy and final knowledge score from one year to the next. The majority of students had only one year of data and were not used in the analysis (this is a function of actual participation in the study and the difficulty in matching students in different data sets). Just over 1000 students had data for at least two years. The numeracy factors found significant were knowledge and/or ability in addition, multiplication, proportion, and basic facts. The following graphs (figures 17–20) show how final score varied over the year for two identified performance groups: those that had improved from their previous year, and those who had made no change.

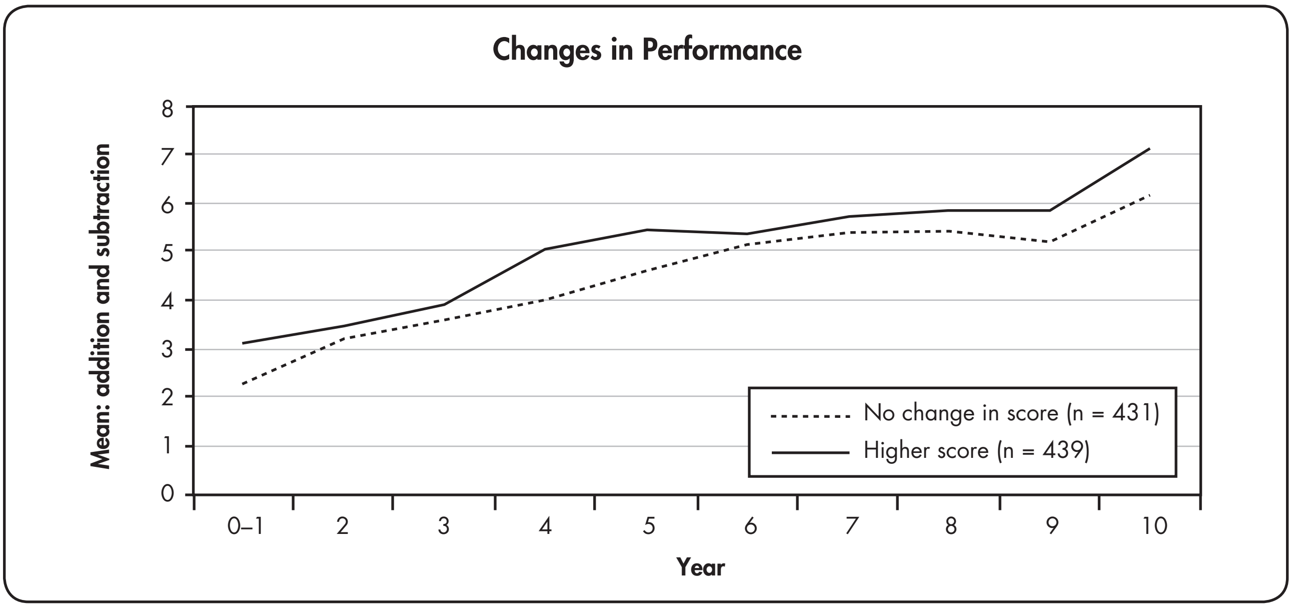

Figure 17. Line graph showing how final score in addition and subtraction varied over year level

Student performance in the other components of the strategy domain, that is, multiplication and division and ratio, was very similar to their performance in addition and subtraction.

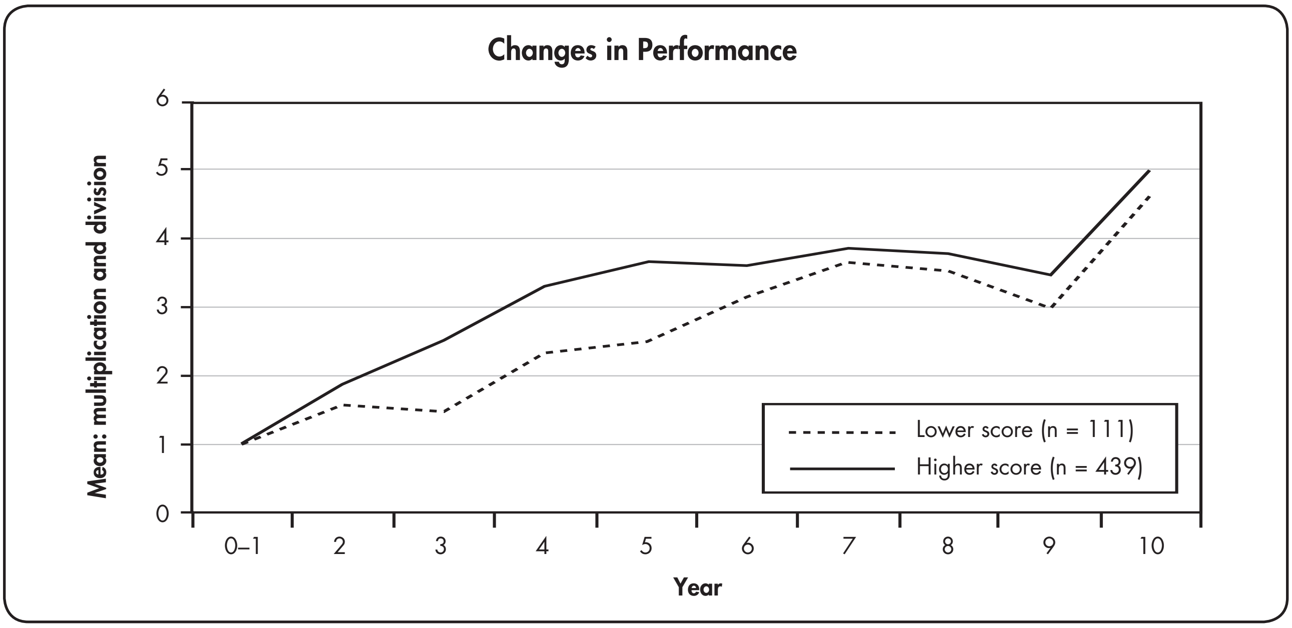

Figure 18. Line graph showing how final score in multiplication and division varied over year level

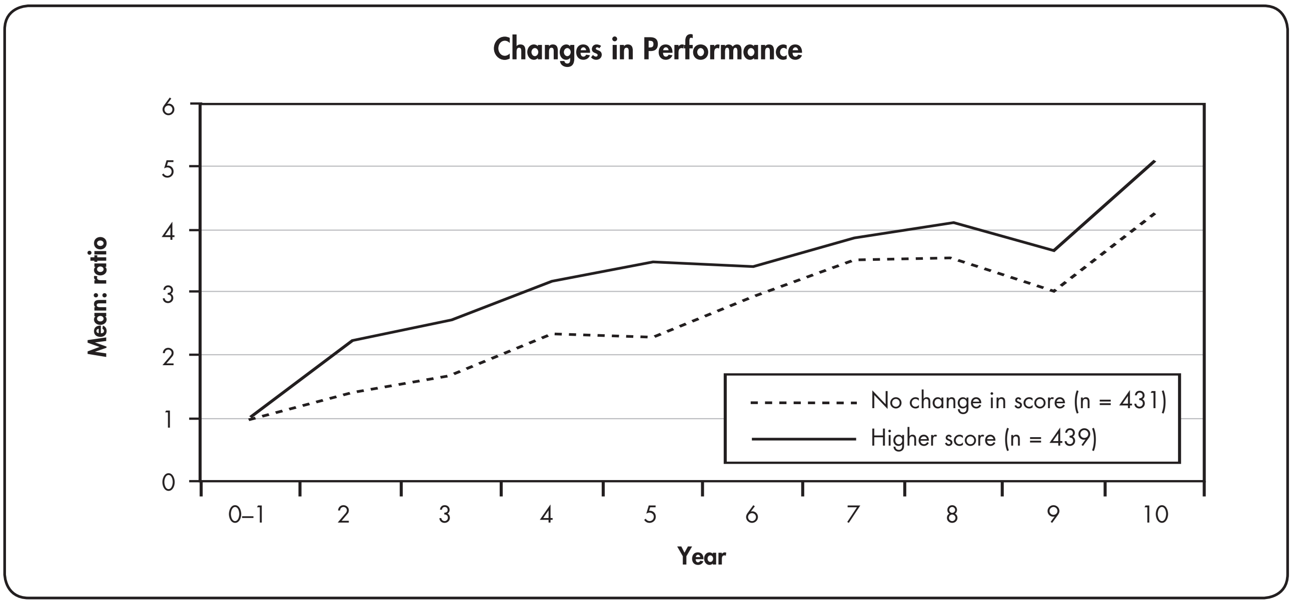

Figure 19. Line graph showing how final score in ratio varied over year level

The greatest difference in performance has been in the strategy components. The differences are not so significant in the knowledge domain, illustrated for basic facts in Figure 20. For the strategy questions, students are required to verbalise their mental strategies. Linguistically and cognitively, this is more challenging than the knowledge domains. Knowledge consists of the information that a student can instantly recall in order to think.

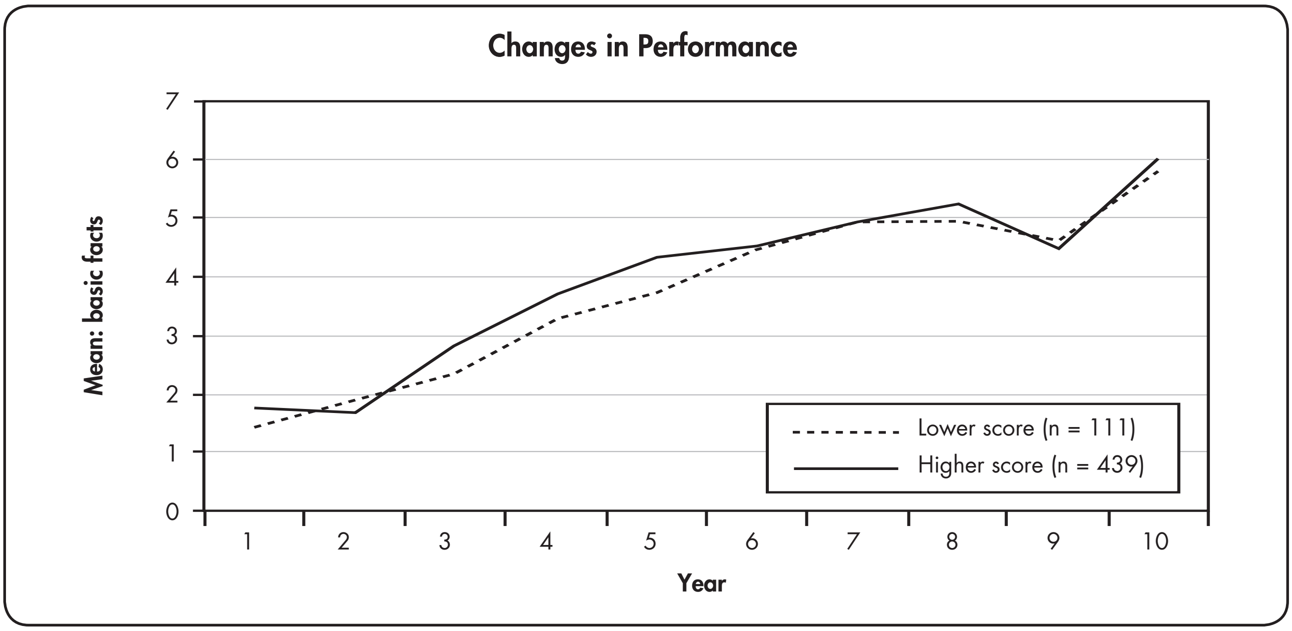

Figure 20. Line graph showing how final score in basic facts varied over year level

Summary

The patterns of performance and progress across the years 2006–2009 for Te Poutama Tau are generally similar, with some small variations from year to year with particular age groups and in the various components of each domain (see figures 8–11). In general, year 2 students made the greatest stage gain, with diminishing gains the older the students get. This may have implications for how teachers and parents interpret student progress in the whanaketanga (Māori-medium mathematics standards) because the conceptual elements of the Mahere Tau (including the stages) are a big component of each whanaketanga. In practice, younger students may make more rapid progress, but with a corresponding slow-down as the mathematics becomes more complex, leading to assumptions by parents (and others) that their child is not doing as well as in previous years. This pattern will need to be communicated to parents and to teachers because currently the standards are criteria referenced, not norm referenced.

There does not appear to be any difference in performance by gender, but there appears to be differences between kura and between kura with different decile ratings (see figures 15 and 16). There are significant differences in the mean stage gain between individual schools and classes and significant differences in performance in individual schools (see figures 13 and 14). Overall, the analysis found that nested class within school[5] was significant for all the dependent variables. This means that a particular class or classes in a school had the most positive results in terms of student progress and performance. The school's aggregated data may not have been as positive. This result is unsurprising, given that a number of researchers argue that the single largest influence on a student is the teacher (Hattie, 2003).

There are Māori-medium kura and classes rated higher than decile 5. But even in this narrower decile band (schools in New Zealand are rated from 1 to 10), there is a significant difference between the performance on the Mahere Tau of the higher deciles (3–5) and the lower deciles.

The researchers cannot explain from this data why there are differences between student and school performance. As discussed earlier, this study does not examine all the complex factors that impinge on student performance.

The main mathematical variables that impact on student progress through the stages of the Mahere Tau include student knowledge and understanding of addition and subtraction, multiplication and division, proportion, and basic facts. In terms of student performance and progress, a significant variable to consider is that students who initially tested at a higher stage maintained that advantage (see also Trinick & Stevenson, 2008).

Further Research

There is a content dip in student progress data at year 8. Why is this so? This is also linked to the converse trend where students who are rated as less proficient do not make a dip at year 8 (see Figure 11). Why do students who are rated as more proficient generally make better progress except for those in years 8 and 10?

Acknowledgments

He mihi tēnei ki a koutou katoa e tautoko mai i te kaupapa o Te Poutama Tau. He nui ngā āhuatanga o te whakaako i roto i te reo Māori kua puta mai i te 2002 ki te 2009. E hia kē ngā mano ākonga kua mahia te kaupapa nei. Nō reira, e ngā ākonga, e ngā pouako, e ngā kaitakawaenga, tēnā koutou katoa.

[5] A generalised linear model (using the SPSS statistical package) was used to account for the effect of class and school on how students performed on the strategy and knowledge tests, that is, where student performance is related to their belonging to particular class (i.e., teacher) and overall class performance is more similar to other classes within their school than classes in other schools.

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Māori Students' Views on Mathematics Equipment

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Mā te whakarongo ka mārama,
mā te whakaputa ka hua.

The literature indicates a range of factors that can affect whether and how students utilise equipment when learning mathematics. Any mathematics learning in Māori-medium schools in New Zealand is closely linked to the development of appropriate Māori language. This issue of language has contributed to the complexity of learning mathematics for these students. In Māori-medium classrooms, as with other classrooms in New Zealand, the use of equipment when learning mathematics is promoted (Higgins, 2005; Ministry of Education, 2008a, 2008b). Research indicates that equipment in many forms can be beneficial for supporting the development of mathematical ideas when used appropriately. The use of ICT also adds another dimension to learning mathematics. This paper focuses on the views of students in Māori-medium schools regarding the use of equipment in mathematics. Their perspectives, as major stakeholders in education, have been sought in order to consider how these might, or might not, align with those offered within the current literature. The findings here indicate that, while students in Māori-medium schools think equipment is useful for learning mathematics, most choose not to avail themselves of it for a number of reasons. The use of ICT to support the learning of mathematics is also an area that requires greater scrutiny and commitmment if we are serious about students in Māori-medium education enjoying the benefits that such involvement can bring.

Background

In New Zealand, more than 11 500 students attend Māori-medium schools in order to to be educated through the medium of te reo Māori (Ministry of Education, n.d.). All curricula, including mathematics, are expected to be taught and learned in this language at least until year 4 or 5 (Harrison, 2009; May & Hill, 2005). This has meant that the development of mathematics resources, vocabulary, and discourse in te reo Māori has become an imperative (Barton, 2008). The Ministry of Education (1996, 2005, 2008a, 2008b) has responded to this need by supporting the development of curriculum documents and teaching materials in te reo Māori.

There is some evidence that the use of resources and materials helps students to become actively involved in their learning (Lowrie, 2004; Moyer, 2001; Warren & Cooper, 2008). Materials can demand attention, thereby helping students to focus and channel their actions for mathematics learning (McDonald, Le, Higgins, & Podmore, 2005).

The use of concrete materials has also been advocated as a practice to help promote student thinking and discussion in mathematics (Higgins, 2005). Effective use of such materials has to be supported by the development of appropriate vocabulary and discourse (Booker, Bond, Sparrow, & Swan, 2004). Such expansion of vocabulary and discourse in te reo Māori has proved to be a challenge in Māori-medium classrooms (Fairhall, Trinick, & Meaney, 2007).

Using equipment accompanied by appropriate discourse can have a positive effect on Māori students' experiences in mathematics learning situations (Holt, 2001). The implementation of the Numeracy Development Projects (NDP) and Te Poutama Tau promotes a teaching model in New Zealand schools that requires appropriate discussion and use of equipment so that mathematical ideas can be explored and understood (Higgins, 2005). These pedagogical practices support a major goal of

Māori-medium education, which is to promote academic excellence through te reo Māori (Ministry of Education, 2008b). This includes the learning of mathematics.

The use of equipment does not necessarily guarantee understanding of mathematics. However, when used appropriately, it can support greater achievement (Bobis, Mulligan, Lowrie, & Taplin, 1999). Learners who interact with equipment must construct the relationship between the resource and the mathematics for themselves (Delaney, 2001). The quality of thinking required to construct mathematical ideas is largely dependent on the appropriate use of equipment (Kamii, Lewis, & Kirkland, 2001).

Askew (2004) supports the notion that equipment is a tool to help organise mathematical thinking. It can act as a stimulus to support such thinking and learning. When used effectively, it can help students to engage with the mathematics and offers them a medium to demonstrate their thinking (Anthony & Walshaw, 2007). The use of concrete materials helps learners to create connections between reality and mathematical ideas (Bonotto, 2002). Cultural artefacts, including photographs and brochures, may also influence students' approaches to solving mathematics problems because they add to the authenticity of the task. Learning for Māori can include use of natural resources such as those located in the forest and bush (Mackay, 2000). The type of artefact that teachers make available to students in mathematics can affect their achievement and performance (Higgins, 2005; Nunes, Light, & Mason, 1993).

English (2004) offers a view that artefacts for teaching and learning mathematics can be symbolic or involve inscriptions. Such artefacts can include diagrams, models, equations, and notations. These support the notion of "thinking spaces" that can help to organise mathematics learning (Askew, 2004). Higgins, Wakefield, and Isaacson (2006) suggest using a modelling book as a collective space for recording notations. The modelling book then becomes part of the cache that is available for students to access and reflect upon.

Many mathematical concepts are not immediately obvious. Equipment can often be arranged to represent the structure of a mathematical concept (English, 1999). For example, place value blocks can be used to represent tens and units in number. The equipment creates a visual representation of a mathematical idea that provides an opportunity for students and teachers to discuss and make their thinking explicit (Booker, Bond, Sparrow, & Swan, 2004). Therefore, the use of materials in learning situations can support students to recognise and understand mathematical ideas. This can help them to become proficient abstract thinkers (Ding, Anthony, & Walshaw, 2009).

When students have free access to equipment, they can become more selective and diverse in their use of the materials (Moyer & Jones, 2004). Learners who see materials as items to be easily accessed and used as they desire are not as likely to feel that using resources is for less knowledgeable students (Delaney, 2001).

The use of electronic tools to support students' learning in mathematics has been advocated for some time (Chval & Hicks, 2009; Ploger, Klinger, & Rooney, 1997; Seeley, Hagelberger, Schielack, & Krehbiel, 2005; Thomas, 2006; Zevenbergen & Lerman, 2004). Information and communication technology (ICT) for enhancing students' learning of mathematical ideas has been explored with some positive effects (Calder, 2009). Calculators, for example, can eliminate tedious computations and enhance learning by allowing more time for students to explore challenging and interesting mathematics (Chval & Hicks, 2009). Other digital technologies can influence students' engagement with mathematics tasks and help students understand mathematics in alternative ways (Calder, 2009). However, exploring mathematics with ICT is dependent on the skills of the teacher, access to appropriate professional development, and the availability of necessary resources (Zevenbergen & Lerman, 2004).

The New Zealand Curriculum (Ministry of Education, 2007) has the expectation that schools will explore the use of ICT to facilitate new and different ways of learning in all curriculum areas, including mathematics. Utilised to its fullest extent, e-learning (learning supported by or facilitated by ICT) can provide a vehicle to achieve Māori aspirations in education (Neal, Barr, Barrett, & Irwin, 2007). *Te Marautanga o Aotearoa* (Ministry of Education, 2008b) implies that using ICT will complement traditional ways of learning. Recent initiatives by the Ministry of Education have included the development of digital learning objects to support the learning of mathematics in te reo Māori.

Method

Participants

This study focuses on the responses of 61 year 5–8 Māori students in four schools. Two schools were kura kaupapa Māori catering for students from years 0–8. Another kura catered for students from years 1–15, and the wharekura catered for students from years 0–13. Three of the kura had participated in Te Poutama Tau, the Māori-medium equivalent of the NDP, for some years prior to this study. Twenty of the students were from a decile 1 kura, 21 from a decile 2 kura, and 20 from a decile 5 kura. Teachers were asked to select students across a range of mathematical ability. Thirty-eight students were female, and 23 were male.

Table 1
Composition of the Students by Kura and Year Level

Kura	Year 5	Year 6	Year 7	Year 8	Total
1*	2	2	0	6	10
2*	2	3	3	2	10
3*	4	4	8	4	20
4	0	0	12	9	21
Number of students	8	9	23	21	61

* Te Poutama Tau participants

Procedure

Schools were asked to nominate year 5–8 students from a range of mathematics levels. These students were interviewed individually for about 30 minutes in te reo Māori or English (their choice) in a quiet place away from the classroom. They were told that the interviewer was interested in finding out about their thoughts regarding their learning of pāngarau/mathematics.

The questions this paper focuses on are part of a larger collection that the students were asked to respond to. Other questions have been previously analysed and discussed elsewhere (Hāwera & Taylor, 2008, 2009; Hāwera, Taylor, Young-Loveridge, & Sharma, 2007).

The questions analysed here are the following:

- I a koe e mahi ana i āu mahi pāngarau, ka whakmahia e koe ētehi taputapu pērā i te pirepire, te porotiti rānei? Ka whakamahia mō te aha?
  (When you do mathematics, do you use equipment like beads and counters? What are they used for?)
- Ki ōu whakaaro, ka āwhina te taputapu i te tangata i a ia e ako ana i te pāngarau?
  (Do you think that using equipment helps people to learn mathematics? How?)

- Pēhea ōu matimati? Ka whakamahia ērā mō te ako pāngarau?
  (What about your fingers? Do you use those for learning mathematics?)
- Pēhea ngā tātaitai? Ina hiahia koe, ka taea e koe te tiki tātaitai hei āwhina i a koe? Ka whakamahia ēnei mō te aha?
  (What about calculators? When you want to, are you able to get calculators to help you? What do you use these for?)
- Pēhea ngā rorohiko? Ka whakamahia ēnei mō te ako pāngarau?
  (What about computers? Do you use those when learning mathematics?)

Results

- I a koe e mahi ana i āu mahi pāngarau, ka whakmahia e koe ētehi taputapu pērā i te pirepire, te porotiti rānei? Ka whakamahia mō te aha?
  (When you do mathematics, do you use equipment like beads and counters? What are they used for?)

Use equipment	Use equipment sometimes	Don't know	Don't use equipment
16	9	1	35

Of the 61 students, 16 stated that they use equipment when learning mathematics, nine said that they use equipment sometimes. One student said she didn't know. Thirty-five said that they don't use equipment because they:

don't need any	don't want any	were not offered any
19	12	4

Some reasons given for these three categories were:

- don't need any
  Nā te mea i te wā i tōku kura tuatahi, i whakaako rātou te mahi pāngarau tino kaha ki ahau, anā ināianei ka mārama tino tika. (K410f5[1]) (Because when I was at my first school, they worked hard to teach me mathematics, and now, it's really clear to me.)
- don't want any
  … mēnā ka mahi rātou he taputapu ka ako rātou mā ēnā, arā, ā te wā ka tupu rātou me whiwhi rātou i ērā anō. (K39m6) (… if they use equipment, and learn by that, when they get older they will need equipment again.)
  …'cause I just work my stuff out on the paper (K61m7)
  … me whakamahi i tō mātou hinengaro. (K16f6) (… we should use our minds.)
  … moumou wā te whakamahi taputapu. He tere ake mēnā ka kore e mahi te taputapu. (K36m7) (… wastes time using equipment. It's quicker to not use equipment.)
- not offered any
  … kāre ka pātai ngā māhita. (K18f8) (The teacher doesn't ask me.)

Twenty-five students stated they use equipment for:

number	games	finding an answer	measurement	don't know
10	5	5	4	1

[1] K = kura, 410 = the 4th group out of the 61 students and the 10th student in that group, f = female, and 5 = year level.

These students provided a range of contexts in which they used equipment. The largest group (10) viewed equipment as a support for learning number ideas, such as grouping, counting, and using them for operations. One student said:

> Um, put them in groups and you just count over it and stuff [laughs] (K63f7)

Five students mentioned games, some of which included the use of dice. One response was:

> Ētahi wā mō ngā kēmu pāngarau i roto i ngā pukapuka (K47f7) (Sometimes for maths games in the books)

Four students mentioned using equipment within the context of measurement. For example:

> … sometimes with geometry you use protractors … (K68m8)

Five students offered the broad idea that equipment could be used to find an answer. One of these said:

> Nā te mea, ka āwhina i ahau i te wā kāre au ka mōhio i ngā whakautu (K19f8) (Because it helps me at the time when I don't know the answer)

- Ki ōu whakaaro, ka āwhina te taputapu i te tangata i a ia e ako ana i te pāngarau?
  (Do you think that using equipment helps people to learn mathematics? How?)

While many students indicated previously that they did not use equipment themselves for learning mathematics, nearly all of the students (60 out of 61) thought that using equipment can help people to learn mathematics. Of these, 54 were unequivocal about its value.

When asked how equipment best helped people, students' responses indicated three main categories. These were:

counting regarding number ideas	don't know	as a support
27	17	16

Twenty-seven stated that equipment was helpful for use in number and operations. Two students said:

> … ka āwhina i tō rātou roro mō ngā mahi tāpiri me ngā whakarau (K40f5) (… because it helps their brain for addition and multiplication)
>
> Ka kaute i ngā taputapu (K12m8) (Can count the equipment)

A further 17 thought equipment was useful but were not able to say how or why.

Sixteen students stated that using equipment helped to support their learning by making the mathematics accessible. For example:

> … ka tiki koe i ētahi pātene and purua i roto i ētahi rōpū like whā me te rima anā ka tāpiri … taea koe te tāpiri, engari kore me ō ringa. (K42m7) (… you get some buttons and put them into a group like 4 and 5 and add … you are able to add, but without your fingers.)

- Pēhea ōu matimati? Ka whakamahia ērā mō te ako pāngarau?
  (What about your fingers? Do you use those for learning mathematics?)

Twenty-two students said they did not use their fingers, and one did not know. Thirty-eight students said that they used their fingers to help them to learn mathematics. Reasons given for use were:

times tables	number	find answer; for hard ones; speed	don't know
19	11	6	2

Of the 38 students who stated that they used their fingers for learning mathematics, 19 mentioned using them for their times tables. Within this group of 19, 11 made specific reference to the nine times tables. For example:

> I te wā ko ngā iwa whakarau (K48f7) (At the time when it's the nine times tables)

Eleven students referred to using their fingers for counting, adding, and other operations. One said:

> Mō te kaute i ngā nama … me ngā honohono (K11f5) (For counting … and adding)

The other eight gave a range of responses for their use, including:

> I te wā he uaua ngā pātai (K45m5) (When the questions are difficult)

- Pēhea ngā tātaitai? Ina hiahia koe, ka taea e koe te tiki tātaitai hei āwhina i a koe? Ka whakamahia ēnei mō te aha?

  (What about calculators? When you want to, are you able to get calculators to help you? What do you use these for?)

While 42 students said they had access to calculators, two of these said that they did not need them. Those who used them said they did so:

for checking answers	for hard ones	for mathematics	when the teacher said
20	8	10	2

Half of this group saw the calculator as a checking device that they used to affirm their answers. These responses included:

> … āe, kia mea he tika ngā whakautu, i te wā kua mutu koe i ō mahi (K10m8) (… yes, to check whether the answers are correct, when you have finished your work)

Eight students recognised that calculators were a tool to support them when the mathematics was difficult and made responses such as:

> … for um bedmas and … hard questions and stuff (K50f7)

Five said that they were used for "doing mathematics", while another five named specific contexts for calculator use such as percentages, fractional number work, and times tables. One student stated:

> … for those statistics, algebra, and fractions … And, um, percent, percentages and all that (K53f8)

Two students deferred to authority such as the teacher or textbook to determine when a calculator could be used. One of these students said:

> If we ask Matua and he says yes, then we can use them. (K69f7)

Nineteen out of the 61 students said that they did not access calculators for mathematics learning. The reasons given for this indicate the following categories:

inhibit learning	don't know	is cheating	only for teacher	don't want to	not sure where they are
7	5	4	1	1	1

Inhibit learning

> Nā te mea kāore ka ako, mēnā kei te whakamahi tātaitai (K24m7) (Because you won't learn if you use calculators)
>
> … kāre au ka ako. (K39m6) (… I won't learn.)

Is cheating

… nā te mea … he cheater koe (K11f5) (… because … you'll be a cheater)

Only for the teacher

Kāo, kāo, mā Whaea noa iho te tātaitai, engari me whakamahi koe i tō roro. (K41f6) (No, no, the calculator is only for the teacher, but you should use your brain.)

Don't want to

Nā te mea kāre au e pīrangi (K19f8) (Because I don't want to)

Not sure where they are

Kua hīkoi atu. Kāore au e mōhio kei whea rātou. (K611m8) (They've walked. I don't know where they are.)

- Pēhea ngā rorohiko? Ka whakamahia ēnei mō te ako pāngarau?
  (What about computers? Do you use those when learning mathematics?)

Yes	No
13	48

Five of the 13 students used computers to play games or to complete activities. This was mostly when other work had been completed.

I te wā kua oti ngā mahi, kei roto rā tētahi rorohiko ka purei i ngā kēmu pāngarau. (K26f8) (When the work is completed, there is a computer inside. [We] can play mathematics games.)

… i te wā wātea ka haere mātou ki runga ki te tākaro i ngā kēmu pāngarau. (K46m7) (… when we are free, we go there to play mathematics games.)

Forty-eight out of 61 did not appear to have the opportunity to explore mathematics within a digital context.

… i ētahi wā ka mahi, ka hanga mahi-kāinga ki runga rā, koirā noa iho. (K25f7) (… sometimes we make our homework on it, that's all.)

Yip … at our computer class … we do all this maths and writing and everything. … We go there every Friday afternoon. (K66f7)

Discussion

Data gained from the 61 students has been organised into two distinct categories. These categories are (i) physical and (ii) electronic.

The physical category of equipment use has been divided into two subsections. The first was where students thought that equipment could be used as a mathematics support to find a solution. This group was clear that equipment could be used for counting and for operations that required them to combine groups. Students explained that equipment could be used to physically and visually model mathematical thinking. This is not surprising given that the use of equipment in mathematics classrooms in New Zealand has long been advocated as part of teacher pedagogy (Anthony & Walshaw, 2007; Higgins, 2005).

There was a large cohort of students who used their fingers as equipment to support computation. Half of these students stated that they used fingers to work out their times tables. Students from years 5–8 who are reliant on their fingers for computation are of concern because they are showing that they are unable to recall their basic facts or times tables instantly or to readily access a suitable mental strategy for computation. Research indicates that for students to become multiplicative thinkers, they need to be able to easily make use of mental connections between numbers and associated patterns rather than rely on physical prompts. This expertise should be developed by the end of year 6 so that students are able to extend their multiplicative capabilities (Young-Loveridge, 2008). Te Poutama Tau evaluative data indicates that there is a dip in students' progress around years 5–6 (Trinick & Stevenson, 2008). If one-third of students are using their fingers for multiplication in an additive way, this could be inhibiting their development as multiplicative and algebraic thinkers (Irwin & Britt, 2007).

It is interesting that even though the majority of these students were very positive about the potential use of equipment when learning mathematics, less than half said that they actually use equipment themselves. They offered a variety of reasons for not using equipment for their mathematics learning. The main reason they gave was a perception that they did not need or want it. Some of these students expressed a view that the use of equipment would encourage a reliance on this practice when older. Others voiced a need to be able to image or become an abstract thinker when learning mathematics, so equipment use for them was perceived as a constraint. Some begrudged the time taken to model mathematical ideas with equipment and viewed that process as an inefficient use of their mathematics learning sessions, while others indicated that they had developed notational strategies that precluded the need to use physical material to explore and explain their thinking.

The perceptions expressed here are somewhat surprising given the emphasis placed on the use of equipment in the teaching model promoted in Te Poutama Tau for when new ideas are being introduced. It would appear that the point of using equipment when exposed to new ideas in mathematics seems to be outside the paradigm of some middle- or senior-school learners. If the use of equipment for exploring new mathematical ideas in classrooms is important, the purpose and value of this approach needs to be made more explicit. Students bring their own perceptions about equipment use that may impede their willingness to engage with the mathematical ideas that are being introduced in conjunction with materials. Students who are thinking in this way may be disadvantaging themselves by not appreciating the possibilities that using equipment can offer. This view can have consequences for these learners and impact on their mathematical reasoning and subsequent understanding (Anthony & Walshaw, 2007; Booker, et al., 2004; Young-Loveridge, 2008).

Of interest also is that while a quarter of the cohort indicated that equipment could help people to learn mathematics, they were not able to articulate how it might do so. Learners who are not clear about how the use of equipment can impact on learning mathematics may be less inclined to avail themselves of these resources. Consequently, constructing the intellectual links between equipment and mathematical ideas may be impeded. In turn, this could affect their engagement and understanding of some mathematical concepts.

The second subsection of the physical category was where some students considered that equipment could be used as a means to an end. That is, equipment could be used to play games. Equipment in these instances appeared to have little value to the students for supporting their conceptual mathematical thinking. Perhaps students could be encouraged to reflect upon these mathematics activities so that the intended mathematical ideas can be clearly exposed and appreciated.

The second category of equipment that students commented on was electronic. While many stated that they had access to calculators for mathematics, it would seem that these students did not perceive them to be tools for the exploration and investigation of mathematical ideas. The calculator had a summative focus designed to provide feedback to the learner regarding a specific solution. Its prime purpose was for checking answers and doing "hard ones".

With regard to computer use, the vast majority of these students shared that they had no experience with this medium for their learning of mathematics. Those who did stated that they used computers for games and not for any planned, systematic mathematics learning. When students are not able to make use of computers more broadly to help them learn mathematics, it may be because they do not have access to them or teachers are unsure about how to maximise the learning opportunities that computers can offer. This is a concern when one considers the electronic age that these students live in and their facility with digital technologies outside the school environment. Opportunities for students to explore mathematics in alternative ways are therefore being compromised (Calder, 2009; Murphy, Bright, McKinley, & Collins, 2009; Zevenbergen & Lerman, 2004). This situation can only be affecting the potential for mathematics achievement by learners in Māori-medium schools (Ministry of Education, 2008b; Neal et al., 2007).

The researchers in this study wonder if opportunities for using digital technologies may also be limited because software can demand that students are required to read, understand, and employ the English language. Such material therefore may be deemed inappropriate by some Māori-medium schools. Students attend Māori-medium schools so that they can become fluent bilingual New Zealand citizens. When and how the English language is introduced into the curriculum is part of a carefully considered plan that is determined by each school and its community. If students are to take advantage of digital technologies, they need to be able to do so in ways that support the development of the learning of mathematical ideas in te reo Māori. Government initiatives (which now include digital learning objects written in te reo Māori) potentially offer Māori-medium students an avenue for exploring mathematics. However, the level of reading and comprehension skills needed to access the mathematics will need careful consideration.

Worthwhile use of equipment is dependent on the vocabulary and discourse that surrounds mathematics learning experiences, so language and mathematics learning are inextricably bound (Fairhall et al., 2007; Barton, 2008). Dialogue in te reo Māori is required to discuss the mathematical ideas represented by equipment. This situation is made more complex by the need for rapid development of appropriate vocabulary. Māori language for making sense of mathematics is constantly being reviewed, refined, and developed. This is a result of the long hiatus in the planned development of te reo Māori in New Zealand. Using equipment for supporting the learning of mathematics in Māori-medium education has contributed to the demand for appropriate language and hence contributes to the complex nature of learning mathematics in these situations.

Conclusion

This study shows that while students in Māori–medium schools have some appreciation of the benefits of using equipment for mathematics learning, they do not necessarily perceive this as essential for their personal learning situation. The value that students place on the relationship between equipment and learning mathematical concepts may need to be more aligned with those espoused within Te Poutama Tau. Teachers may need to emphasise more explicitly the use and value of equipment when introducing new mathematical ideas to learners.

It was noted that many year 7–8 students in this study were dependent on using their fingers as equipment for computation. Whilst using fingers for computation can sometimes be appropriate, this practice should be diminishing by years 7–8 if students are to become competent multiplicative thinkers.

Calculators and computers were underutilised resources for mathematics learning. Planned and appropriate experiences for Māori students using electronic technologies to enhance their mathematics learning needs to be supported. This could only assist with increasing achievement in mathematics for Māori learners (Ministry of Education, 2008b).

Learning mathematics in Māori-medium education is facilitated through te reo Māori. The use of computer programs designed to support mathematics learning can require a significant knowledge of the English language. Computer programs dedicated to Māori-medium mathematics learning need to support the planning of meaningful and appropriate ICT experiences for students in this context.

The views of these students are significant for mathematics educators in the twenty-first century. If we are genuine about greater achievement in mathematics for learners in the medium of Māori, it behoves us to take their perspectives seriously and act accordingly.

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Secondary Numeracy Project Students’ Development of Algebraic Knowledge and Strategies

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This report presents results from the third year of an investigation into students’ algebraic thinking. A written assessment tool that was a development of the diagnostic assessment tools used in the previous two years was employed. In this study of 1007 year 9–11 students, two written tests were used to investigate knowledge of aspects of algebra and the strategies that students use to solve equations. Overall, results were similar to those reported last year (Linsell, 2009). A relatively small proportion of students (21.9%) were able to solve equations by the most sophisticated strategy of using transformations, and a similar proportion (18%) were restricted to the primitive strategies of guess and check, use of known basic facts, or counting strategies, or had no successful strategy. As in the previous study, there were strong associations between knowledge of aspects of algebra and the most sophisticated strategy a student could use. There were significant differences in achievement between year levels, with good progress apparent in the three schools that participated in the study.

Introduction

The first two years of this study investigated the range of strategies that year 7–10 students use to solve linear equations and the knowledge associated with the use of these strategies (Linsell, 2008, 2009). There was, however, no attempt at representative sampling of any population, so findings could not be readily generalised. The diagnostic tool used in the previous two years was an oral interview, which was very time-consuming to administer and unlikely to gain widespread use by secondary school teachers.

Tagg and Thomas (2009) raised concerns about the lack of progress in number knowledge and the strategies of year 10 students in Secondary Numeracy Project (SNP) schools. They found very few significant gains in any of the strategy and knowledge domains. Furthermore, although the proportion of students in the top stages of the multiplicative and proportional domains increased slightly, there was little change in the proportion of students rated as at risk. Tagg and Thomas postulated that the teachers were more conservative in their judgments at the end of year 10 compared with year 11. However, other possibilities were that year 10 students genuinely did not make any progress or that the progress they made was not being measured by the assessment tools used in the SNP.

The aim of this present study was to use a written assessment tool to document the strategies that students in SNP schools use to solve equations and the knowledge of algebra associated with the use of these strategies.

The research questions addressed in this study were:

- Can a written assessment tool be used to diagnose a student’s most sophisticated strategy for solving linear equations?
- For students in schools participating in SNP, what strategies do they use for solving linear equations and what algebraic knowledge do they have?
- What progress do students make between year 9 and year 11?

Method

This study examined the types of linear equations that students were able to solve and made inferences about the most sophisticated strategy that each student was able to employ. Relationships between students' strategies for solving equations and their knowledge of aspects of algebra were examined. Two written tests were employed. The first test, to assess algebraic knowledge, did not permit the use of calculators, while the second test, to assess strategies for solving equations, required calculators.

Subjects

The study took place in three schools that have had a strong involvement in the SNP. These schools were nominated by the SNP national coordinator, following a meeting of regional facilitators. All were coeducational schools and represented a range of deciles and geographical locations. One was low decile, one mid decile, and one high decile. Two schools were from the North Island and one from the South Island. Testing took place at the end of term 3 and the beginning of term 4 2009.

1200 students from the three schools (400 from each school) were sought to take part in the study. Approximately 135 students at each year level were wanted. Each school used some form of streaming, so five or six complete classes at each year level were selected as being representative of the school.

Not all teachers managed to administer both tests correctly, either not doing a test or failing to provide a calculator for Test B. A total of 1018 students' results were returned. Within the classes who returned data, 11 students did not agree to participate and were excluded from analysis. The demographics of those included in the analysis were as follows:

Table 1
Number of Students Participating by School

	School A (high decile)	School B (mid decile)	School C (low decile)
Year 9	113	107	135
Year 10	86	118	134
Year 11	97	128	89
Total	296	353	358

Table 2
Number of Students Participating by Gender

	Female	Male	Total
Year 9	172	182	355
Year 10	173	165	338
Year 11	157	155	314
Total	502	502	

Three students did not specify gender.

The sample of 1007 participating students is fairly representative of the national population in terms of year level, decile, and gender.

Diagnostic Tool

The diagnostic tool was a development of those used in previous studies (Linsell, 2008, 2009; Linsell, Savell, Johnston, et al., 2006). Test A (see Appendix I) was very similar to the knowledge test previously used, but Test B (see Appendix J) was a written test rather than the oral interview used in the earlier studies.

Test A retained the previous structure of five sections, examining knowledge of algebraic notation and convention, knowledge of arithmetic structure, knowledge of inverse operations, acceptance of lack of closure, and understanding of equivalence. However, some redundant questions were removed and two questions based on Steinle, Gvozdenko, Price, et al.'s (2009) work on knowledge of algebraic notation and convention were added. Two classic questions on acceptance of lack of closure (Küchemann, 1981) were also added.

Test B was designed to determine the most sophisticated strategy for solving equations that each student could use. Previous work (Linsell, 2008, 2009) found that students used a range of strategies for solving equations, with a clear hierarchy of sophistication (see Table 3).

Table 3
Hierarchy of Strategies

Rank	Strategy
1	No successful strategy
2	Primitive: guess and check, counting, known basic facts
3	Inverse operations
4	Partially working backwards
5	Working backwards
6	Transformations (equation as object)

For any given question, students use a limited range of strategies. In the 2008 oral interview, the researchers were able to ask students what strategy they had used, but for the 2009 written test, it was crucial to design the questions so that a correct answer indicated the minimum strategy that must have been used. For example, $3n = 18$ may be solved by a wide range of strategies, including inverse operations, guess and check, known basic facts, or even a counting strategy. However, a correct solution to $29n = 205.9$ indicates that at least an inverse operation must have been used.

Test B was designed so that questions 1 and 2 could be solved by primitive strategies (guess and check, known basic facts, counting), inverse operations, or by treating the equation as an object to be transformed by doing the same thing to each side. Questions 3–5 required inverse operations or treating the equation as an object. Questions 6 and 7 could be solved by guess and check, partially working backwards, fully working backwards, or treating the equation as an object. Question 8 was a poorly designed question that was difficult, yet could possibly be solved by guess and check. Questions 9–11 required fully working backwards or treating the equation as an object. Question 12 could be solved by treating the equation as an object but could also be solved by guess and check. Questions 13–16 could be solved only by treating the equation as an object.

Data Analysis

Raw answers from each student to each question were entered into a spreadsheet. These answers were scored as correct/incorrect. The data was then cleaned by examining all the incorrect answers in order to decide whether they were acceptable alternative answers. For example, in Test A, question 16, an answer of $n \times 2$ was considered an acceptable alternative to $2n$.

From the algebraic knowledge test (Test A), each student was assigned a score between 0 and 4 for each of: knowledge of algebraic notation and convention; knowledge of arithmetic structure; knowledge of inverse operations; acceptance of lack of closure; and understanding of equivalence. A score of 0 was classified as very poor, 1 as poor, 2 as average, 3 as good, and 4 as very good. Relationships between each of these measures and the most sophisticated strategy used were later examined.

The first stage in the analysis of Test B was to determine the difficulty of the equations. Rather than using simple proportions, Rasch analysis was used to determine item difficulty (Wright & Masters, 1982). These scores permitted each question to be examined for whether it appeared useful for its intended purpose.

Students cannot always use a given strategy and frequently make mistakes, so a rubric was used to make a judgment about whether a student had used at least a given strategy:

- If a student gave a correct answer to either question 1 or 2 (see Table 4), this was seen as an indication that they had used at least a primitive strategy.
- If a student gave a correct answer to one or more of questions 3, 4, or 5, this was seen as an indication that they had used at least the strategy of inverse operations.
- If a student gave a correct answer to either question 6 or 7, this was seen as an indication that they had used at least the strategy of partially working backwards.
- If a student gave a correct answer to one or more of questions 9, 10, or 11, this was seen as an indication that they had used at least the strategy of fully working backwards.
- Finally, if a student gave a correct answer to one or more of questions 13, 14, or 15, this was seen as an indication that they had treated the equation as an object to be transformed by doing the same thing to each side.

The judgment of what was the most sophisticated strategy used by each student was based on the following criteria:

- If a student did not successfully solve any equation, they were judged to have no viable strategies.
- If one or more correct answers indicated that they had used a primitive strategy, then this was judged to be their most sophisticated strategy.
- If a student met the criteria for using a primitive strategy *and* they solved one or more equations requiring inverse operations, then inverse operations was judged to be their most sophisticated strategy.
- If a student met the criteria for using inverse operations *and* they solved one or more equations requiring partially working backwards, partially working backwards was judged to be their most sophisticated strategy.
- If a student met the criteria for partially working backwards *and* they solved one or more equations requiring fully working backwards, working backwards was judged to be their most sophisticated strategy.
- If a student met the criteria for working backwards *and* they solved one or more equations requiring transformations / equation as object, transformations / equation as object was judged to be their most sophisticated strategy.

It was possible, of course, that a student's answers might not have fitted neatly into the criteria above. For example, a student might have correctly solved an equation requiring the use of transformations / equation as object but did not meet the criteria for working backwards. In these cases, the original scripts were examined for evidence of working and/or types of errors in order to make a judgment regarding the most sophisticated strategy used.

The significance of differences between proportions was estimated using Pearson's χ^2, and effect sizes were estimated using odds ratios (OR) (Steinle & Stacey, 2005).

Results

Item Difficulty

As in last year's study, there was a huge variation between questions in the number of students able to solve them, with some questions being much harder than others (see Table 4).

Table 4
Item Difficulty

Question number	Equation	Percentage of students with correct responses	Rasch score (item difficulty)
1	$n - 3 = 12$	86	–3.2
2	$3n = 18$	81	–3.2
3	$n + 4.6 = 11.3$	76	–2.6
4	$29n = 205.9$	59	–1.2
5	$\frac{n}{26} = 11.5$	55	–0.9
6	$4n + 9 = 37$	64	–1.7
7	$3n - 8 = 19$	61	–1.5
8	$4(n - 3) = 21$	29	1.3
9	$16n + 78.2 = 147$	45	–0.2
10	$\frac{n + 12}{4} = 18$	41	0.1
11	$2.8 + \frac{n}{4} = 8.2$	27	1.5
12	$5n - 2 = 3n + 6$	31	1.0
13	$12n + 2 = 8n + 15$	22	2.0
14	$2n - 3 = \frac{2n + 24}{5}$	8	4.5
15	Solve for n $an - 3p = 5r$	10	4.1

Item difficulty generally corresponded well with the hierarchy of strategies proposed last year (Linsell, 2009) and the intended minimum strategy that each question was designed to elicit. Questions 1 and 2 were designed to be able to be solved by primitive strategies and were easy. Questions 3, 4, and 5 were designed to be able to be solved by inverse operations and were harder than questions 1 and 2. Questions 6 and 7 were designed to be able to be solved by partially working backwards and were harder than questions 1, 2, and 3 but easier than questions 4 and 5. Question 8 was not a good question. It had been designed to be solved by working backwards, but a number of students' working showed that it was sometimes solved by guess and check. Questions 9, 10, and 11 were designed to be able to be solved by working backwards and were harder than all the previous questions other than question 8. Question 12, with unknowns on both sides, could be solved by guess and check as well as by the use of transformations / equation as object. In spite of this, the question was as hard, or harder, than the questions that required working backwards. Questions 13, 14, and 15 were designed to be able to be solved by the use of transformations / equation as object and were harder than all the previous questions.

Most Sophisticated Strategies Used

If the hierarchy of strategies proposed last year (Linsell, 2009) is correct, then any student able to employ a sophisticated strategy should also be able to solve equations that require only less sophisticated strategies. For example, a student able to solve equations that require the strategy of working backwards should also be able to solve equations that require only partially working backwards, those that require only inverse operations, and those that require only primitive strategies.

It was found that 92.9% of students fitted the above model. Of the 7.1% who did not fit the model, most successfully solved an equation that required the strategy of partially working backwards but did not solve any equations that required the use of inverse operations. The scripts of all students not fitting the model were re-examined for any working that might provide evidence of what strategies had been used. Students had been remarkably compliant in showing working, and it was not difficult to determine the strategies used.

There were two distinct groups of students who could successfully solve an equation that required the strategy of partially working backwards but who could not solve any equations that required the use of inverse operations. One group could not correctly use inverse operations and used guess and check to solve the equations that were designed to require the strategy of partially working backwards. This group was classified as having primitive strategies as their most sophisticated. Students in the other group were using some inverse operations when solving the equations designed to require the strategy of partially working backwards but made computational errors in all the questions designed to require the use of inverse operations. This group was classified as having partially working backwards as their most sophisticated strategy. The most sophisticated strategy used by the remaining students who did not fit the model was determined in a similar manner.

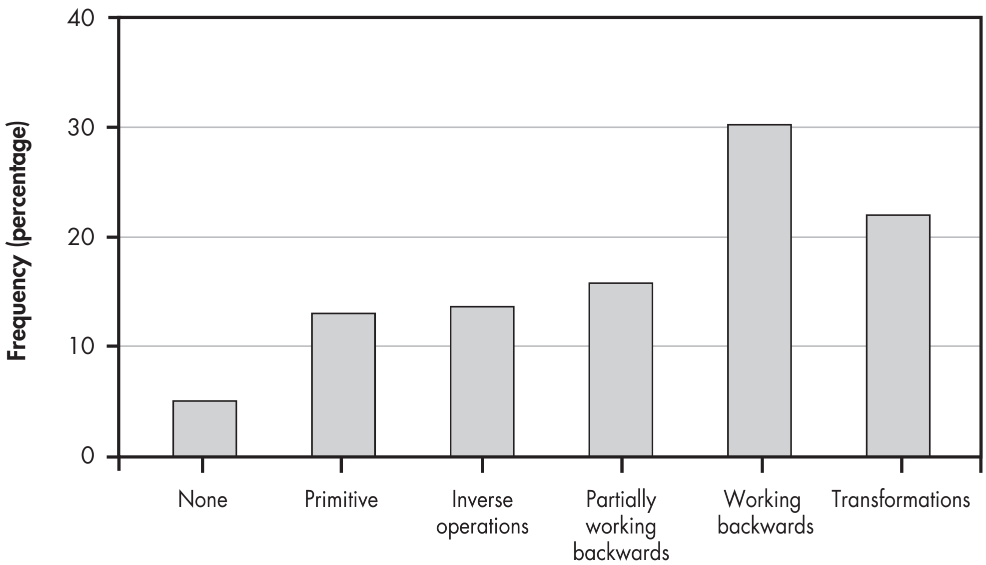

Figure 1. Most sophisticated strategy used

The proportion of students able to solve equations by performing transformations was fairly low (21.9%), and a similar proportion of students (18.0%) were not even able to use inverse operations. Sixteen percent of students were classified as partially working backwards and not able to fully work backwards. Many of these students, and also those restricted to primitive strategies, are likely to obtain correct solutions to standard textbook equations involving integer solutions.

There were highly significant differences between year levels in the most sophisticated strategies used ($\chi^2 = 53.3$, d.f. = 10, $p < 0.001$), as shown in Figure 2.

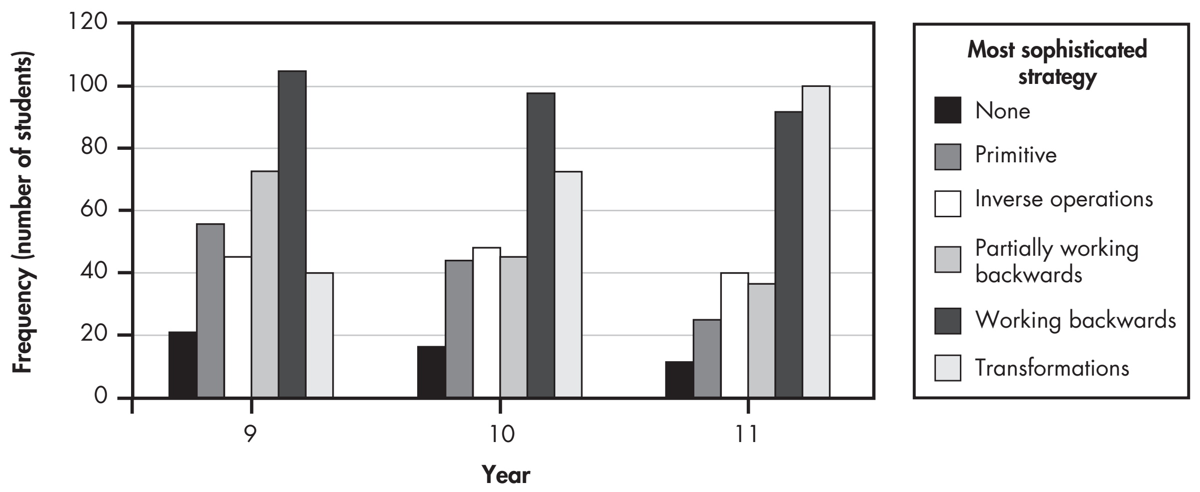

Figure 2. Most sophisticated strategy by year level

The proportion of students able to solve equations by transformations rose from 11.8% in year 9, to 22.5% in year 10, to 32.7% in year 11. Effect sizes were calculated using odds ratios. The chances of a year 10 student using transformations compared with a year 9 student were 2.2 times as great (95% confidence interval 1.4–3.3), and the chances of a year 11 student using transformations compared with a year 10 student were 1.7 times as great (95% confidence interval 1.2–2.4).

Conversely, the proportion of students restricted to primitive or no strategies decreased from 22.7% in year 9, to 18.7% in year 10, to 12.1% in year 11. The chances of a year 10 student using a primitive or no strategy compared with a year 9 student were 0.6 times as great (95% confidence interval 0.4, 0.9), and the chances of a year 11 student using a primitive or no strategy compared with a year 10 student were 0.8 times as great (95% confidence interval 0.5–1.2). However, 1.0 falls within the 95% confidence interval, so it cannot be concluded that there is any significant decrease in the use of primitive or no strategies between years 10 and 11.

Algebraic Knowledge

Knowledge of algebraic notation and convention, knowledge of arithmetic structure, knowledge of inverse operations, acceptance of lack of closure, and understanding of equivalence were examined by year level.

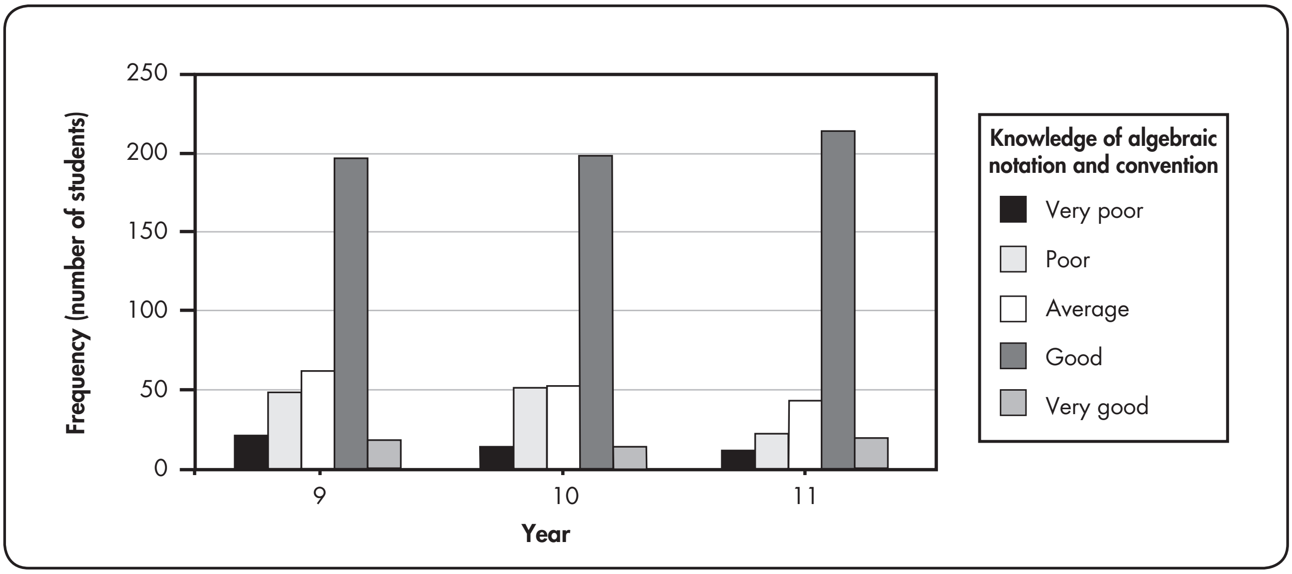

Figure 3. Knowledge of algebraic notation and convention by year level

There was little change in knowledge of algebraic notation and convention between years 9 and 11. Most students had good knowledge but answered question 4 incorrectly, thinking that in the equation $x + y = 16$, x and y had to have different values. The chances of a year 10 student having good or very good knowledge of algebraic notation and conventions compared with a year 9 student were only 1.1 times as great and not significantly different. However, the chances of a year 11 student having good or very good knowledge of algebraic notation and conventions compared with a year 10 student were 1.6 times as great (95% confidence interval 1.2–2.3).

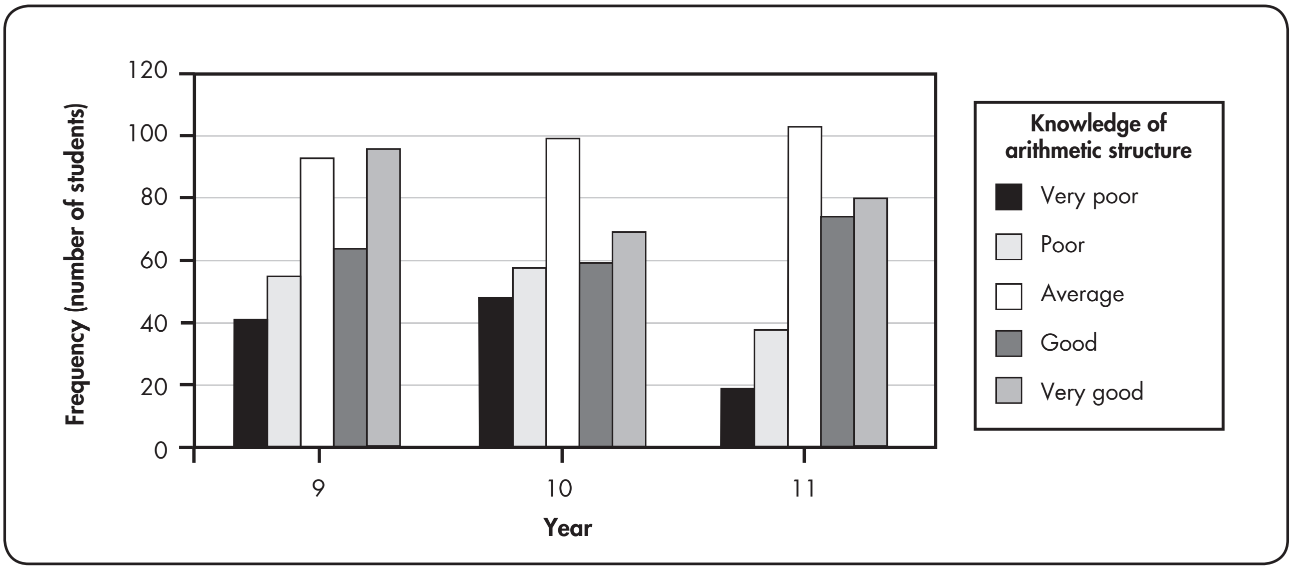

Figure 4. Knowledge of arithmetic structure by year level

There was only a small improvement in knowledge of arithmetic structure between years 9 and 11. However, many students attempted to evaluate expressions by working from left to right, even in year 11. The chances of a year 10 student having good or very good knowledge of arithmetic structure compared with a year 9 student were 0.7 times as great and not significantly different. However, the chances of a year 11 student having good or very good knowledge of arithmetic structure compared with a year 10 student were 1.5 times as great (95% confidence interval 1.1–2.1).

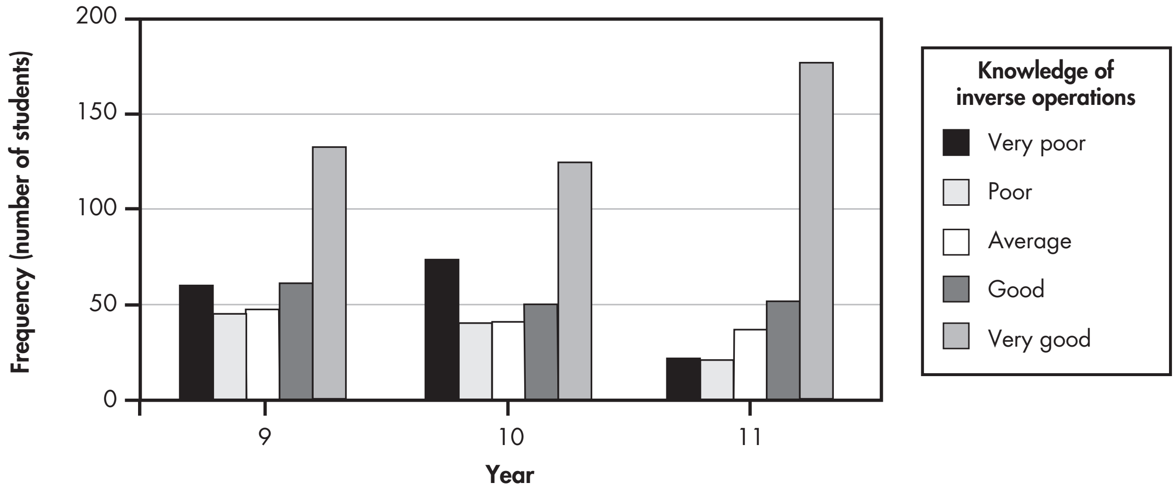

Figure 5. Knowledge of inverse operations by year level

Students' knowledge of which operations were the inverses of each other was generally good, and this improved from year 9 to year 11. The chances of a year 10 student having good or very good knowledge of inverse operations compared with a year 9 student were 0.9 times as great and not significantly different. However, the chances of a year 11 student having good or very good knowledge of inverse operations compared with a year 10 student were 2.5 times as great (95% confidence interval 1.8–3.5).

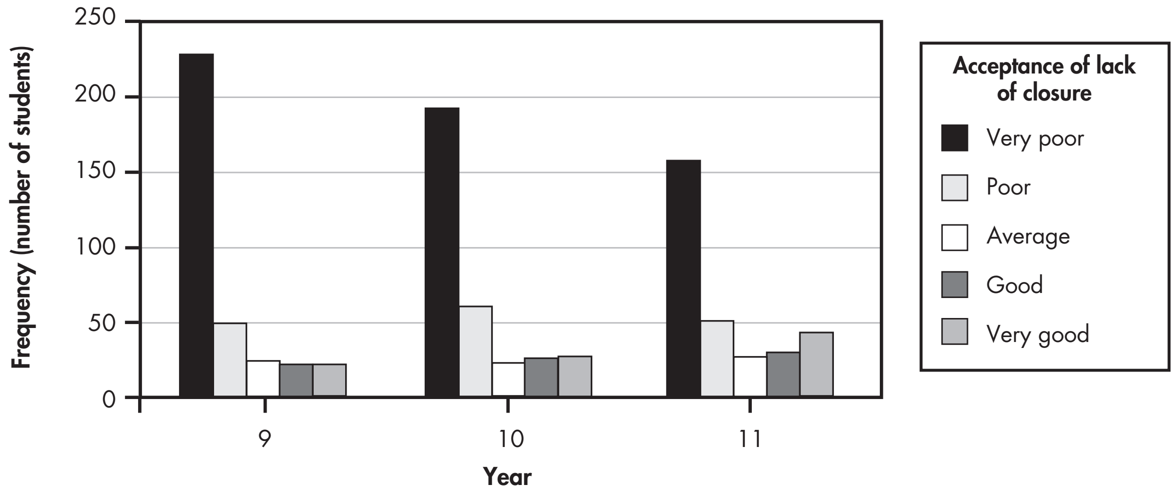

Figure 6. Acceptance of lack of closure by year level

Students' acceptance of lack of closure was generally very poor, although this did improve slightly between year levels. Most students seemed unable to accept that the answer to a question could be an algebraic expression and instead attempted to calculate a specific value, even though they had insufficient information to do so. For example in question 16, instead of the answer $2n$ for the perimeter, many students gave the answer 34, often drawing in a 17th side of the incomplete figure in the diagram. The chances of a year 10 student having good or very good acceptance of lack of closure compared with a year 9 student were 1.4 times as great and not significantly different. However, the chances of a year 11 student having good or very good acceptance of lack of closure compared with a year 10 student were 1.6 times as great (95% confidence interval 1.1–2.4).

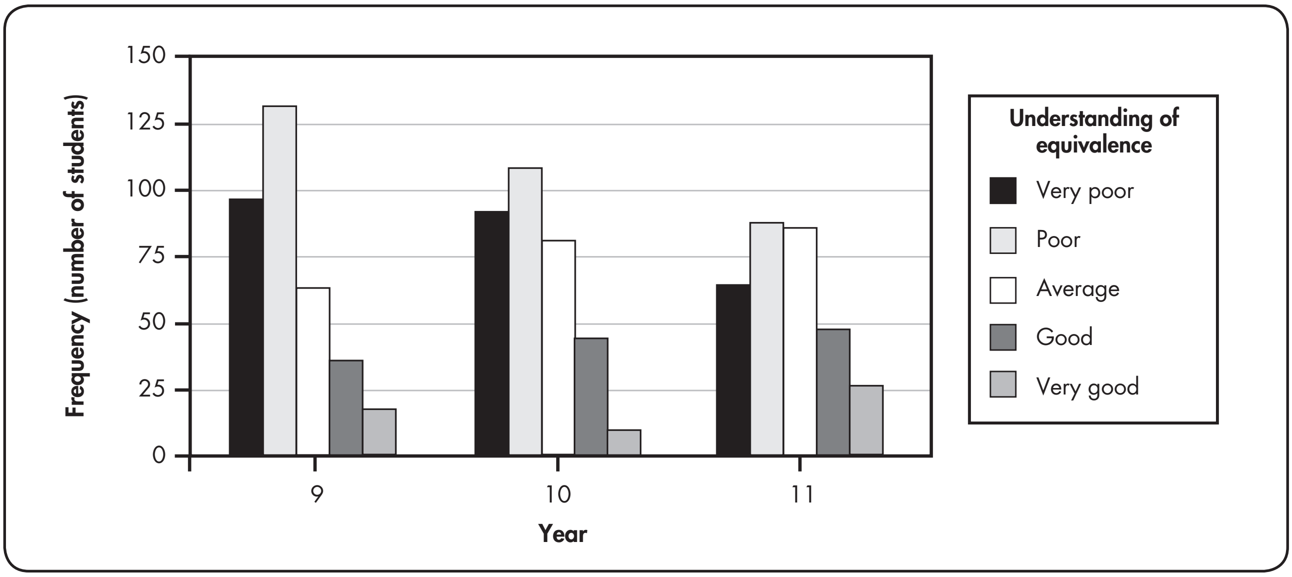

Figure 7. Understanding of equivalence by year level

Students' understanding of equivalence was also poor, although again this did improve slightly between year levels. Many students treated the equals sign as an instruction to evaluate the expression to the left of it and wrote, for example, $4 + 5 = 9 + 3$ when answering the question $4 + 5 = \square + 3$. The chances of a year 10 student having good or very good understanding of equivalence compared with a year 9 student were 0.9 times as great and not significantly different. However, the chances of a year 11 student having good or very good understanding of equivalence compared with a year 10 student were 1.8 times as great (95% confidence interval 1.2–2.6).

Prerequisite Knowledge and Strategy

The relationships between each student's prerequisite knowledge and the most sophisticated strategy that they were able to employ was then investigated. These relationships between knowledge and highest algebraic strategy are shown in figures 8–12.

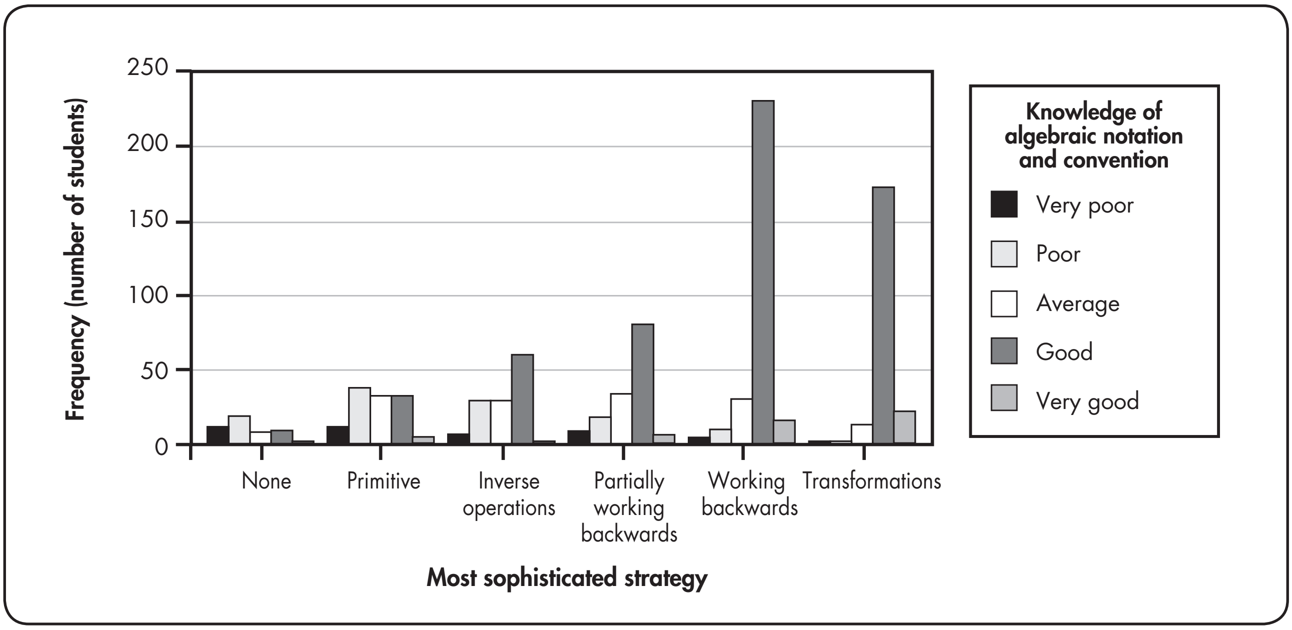

Figure 8. Knowledge of algebraic notation and convention by most sophisticated strategy

There was a strong association between the most sophisticated strategy that a student could use and their knowledge of algebraic notation and convention. Students using the more sophisticated strategies had much better knowledge than those using less sophisticated strategies.

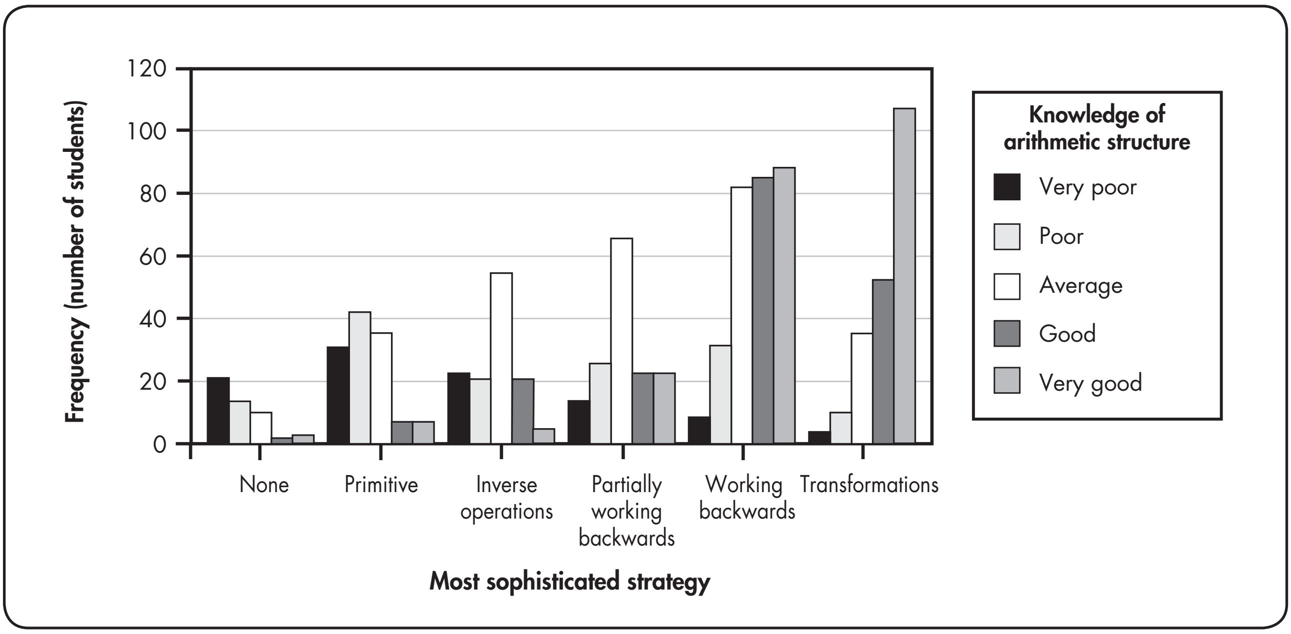

Figure 9. Knowledge of arithmetic structure by most sophisticated strategy

There was also a strong association between the most sophisticated strategy that a student could use and their knowledge of arithmetic structure. Again, students using the more sophisticated strategies had much better knowledge than those using less sophisticated strategies.

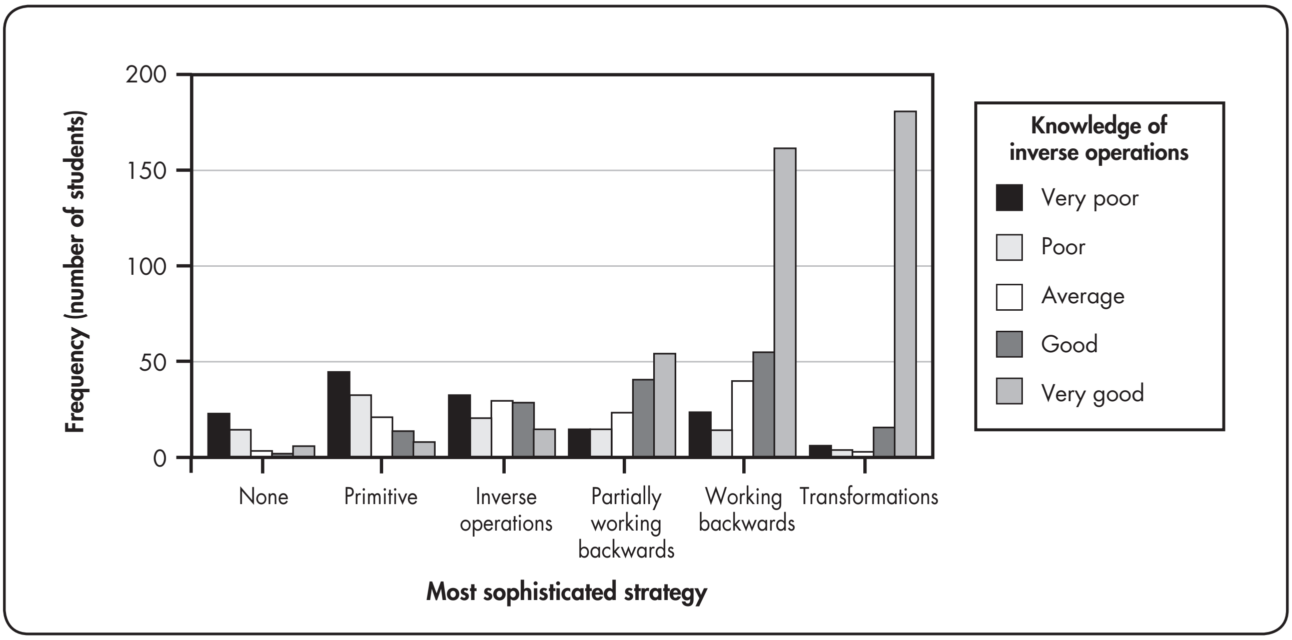

Figure 10. Knowledge of inverse operations by most sophisticated strategy

Not surprisingly, there was a strong association between knowledge of which operations were inverses of each other and the use of strategies that required the use of inverse operations. Nearly all students who were able to work backwards or use transformations had a good or very good knowledge of inverse operations.

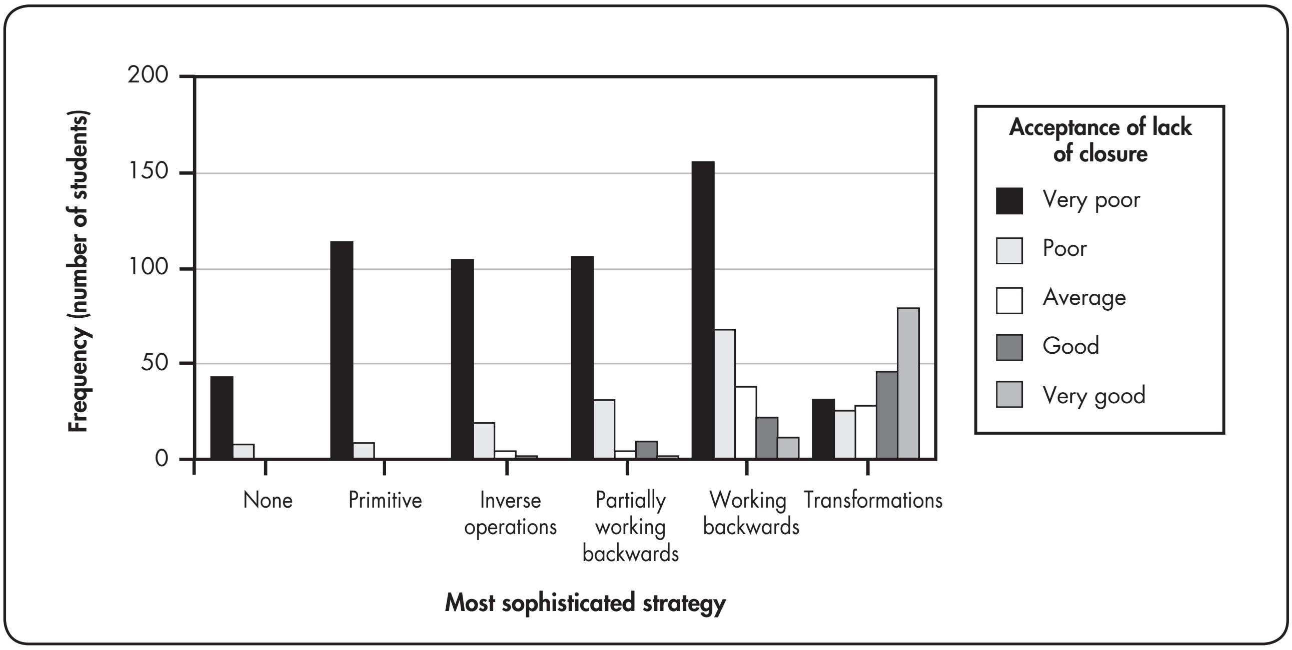

Figure 11. Acceptance of lack of closure by most sophisticated strategy

There was a very dramatic association between the most sophisticated strategy that a student could use and their acceptance of lack of closure. The major difference was between those students able to use transformations and all other students. The proportion of students able to solve equations by transformations was 10.8% for students with a very poor to average score for acceptance of lack of closure and 74.1% for students with a good or very good score. Effect sizes were calculated using odds ratios. The chances of a student with a high score for acceptance of lack of closure using transformations compared with a student with a low score were 23.7 times as great (95% confidence interval 15.8–35.8).

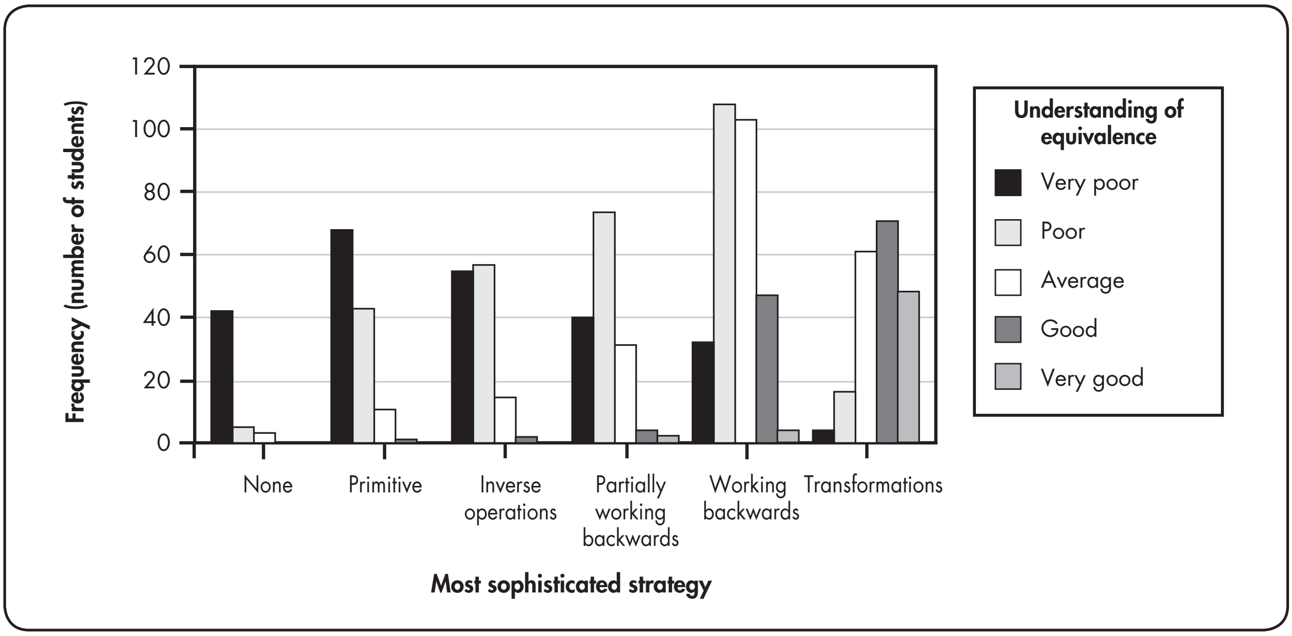

Figure 12. Understanding of equivalence by most sophisticated strategy

Similarly to acceptance of lack of closure, there was a strong association between the most sophisticated strategy that a student could use and their understanding of equivalence. Again, the major difference was between those students able to use transformations and all other students. The proportion of students able to solve equations by transformations was 11.8% for students with a very poor to average score for understanding equivalence and 66.5% for students with a good or very good score. Effect sizes were calculated using odds ratios. The chances of a student with a high score for understanding equivalence using transformations compared with a student with a low score were 14.9 times as great (95% confidence interval 10.2–21.7).

Discussion

This study was undertaken because of concerns about the apparent lack of progress of year 10 students in SNP schools (Tagg & Thomas, 2009). It had three foci:

- efficacy of a written assessment tool for diagnosing algebraic strategies
- algebraic knowledge and strategies used by students in SNP schools
- progress made by students in SNP schools.

Efficacy of Written Assessment Tool

There are strong grounds for concluding that the written assessment tool described in this study can be used for diagnosing the most sophisticated strategy that a student can employ for solving equations.

Many of the questions were the same as questions used in Linsell's (2009) study, in which an oral-interview technique was used. The range of strategies used for those questions was determined by talking to the students about their thinking, so it is possible to be confident that the range of strategies that students must have used for the same questions in this study was the same. Other questions from the earlier study were less useful for inferring which strategy had been used because many students solved the equations using guess and check. For example. $5n - 2 = 3n + 6$ was intended to require use of transformations, but most students solved it by guess and check. In the current study, an additional question, $12n + 2 = 8n + 15$, was included because it was highly unlikely that students would be able to guess the non-integer solution. Similarly, equations with non-integer solutions were included to differentiate between partially working backwards and fully working backwards and also to differentiate between primitive strategies and the use of inverse operations.

Secondly, Rasch analysis provided an objective measure of the difficulty of equations. The Rasch scores for item difficulty were very consistent with the strategy inferred for each question and the hierarchy of sophistication of strategies. The exceptions to this were the Rasch scores for the questions intended to require the use of partially working backwards. These questions were easier than questions requiring just inverse operations because students restricted to guess and check were still able to solve the questions intended to require the use of partially working backwards.

Finally, 92.9% of students fitted the model of meeting the criteria for all easier strategies as well as meeting the criteria for their most sophisticated strategy. The reasons for the remaining 7.1% of students not fitting the model were apparent in their recorded working. Some students used guess and check for the more difficult questions rather than the strategy that the questions were designed to elicit. However, other students made computational mistakes in all the easier questions, even though their correct solutions to more difficult questions demonstrated that they were able to use sophisticated strategies. To avoid misdiagnosing these students, it is recommended that any revised test provide sufficient opportunities for students to show evidence of using each strategy.

It should be stressed that the written test and data analysis was designed to find any evidence of students using sophisticated strategies, so a correct solution to just one question was considered evidence. This does not mean that the student had fully mastered the strategy. In a revised test, as well as providing sufficient opportunities for students to show evidence of using each strategy, criteria could be used for demonstrating mastery, for example, more than one question correct in each category.

Algebraic Knowledge and Strategies of SNP Students

The strategies used by SNP students and the knowledge that they displayed is largely consistent with the findings from last year's study (Linsell, 2009), although performance was slightly better.

Even though this year's study involved year 9–11 students, rather than the previous study's year 7–10 students, still only a small proportion of students (21.9%) were able to solve equations by the most sophisticated strategy of using transformations. This strategy is required when solving equations with non-integer solutions and unknowns on each side. However, this is comparable with 6% of students in the previous study and with Sfard's (1991) suggestion that use of this strategy is beyond the grasp of many students. A significant proportion of SNP students are clearly perceiving equations as objects and performing transformations on them in order to solve them. It would appear that for many of these students, it is not merely a learned procedure because they were able to solve an equation with a less routine structure ($2n - 3 = \frac{2n + 24}{5}$) and also rearrange $an - 3p = 5r$. Two factors may have contributed to the better performance of students in this study. Firstly, the age range of students is higher, including year 11 students and excluding year 7 and 8 students. Secondly, the present study used a sampling procedure designed to provide a representative sample of the population studied, whereas the previous study was designed to explore the range of thinking in the population without any attempt at representative sampling.

It is of some concern, however, that 18% of students in the present study were restricted to no strategy or primitive strategies and could not even use inverse operations to solve one-step equations. This compares with 14% in the previous study. This decrease in apparent performance may have been due to changes in the questions this year, for example, $n + 4.6 = 11.3$ was used instead of $n + 46 = 113$. The questions were changed in order to reduce the possibility of using known facts or counting strategies, but students' lack of confidence with decimals may have prevented them from using inverse operations on them. However, the relatively large proportion of students unable to use inverse operations is a concern.

The algebraic knowledge displayed by the students was also fairly similar to last year's study. Knowledge of algebraic notation and conventions was again fairly good, although the inclusion of questions from Steinle et al. (2009) elicited the same response as in their study. A large proportion of students appreciated that in the equation $x + x + x = 12$, x could take only the value 4 but believed that in $x + y = 16$, x and y had to have different values. This misunderstanding of the use of letter symbols to represent specific unknowns or variables may have profound effects on students' further learning. Steinle et al. (2009, p. 298) suggest that "mathematics lessons containing algebra are rendered incomprehensible; these students are trying to learn procedures, without meaning, carried out on letters with the wrong meaning." However, in spite of Steinle et al.'s concerns, this misconception did not have a major impact on the most sophisticated strategy that a student was able to employ. Students were able to solve equations by using transformations even though they believed that in $x + y = 16$, x and y had to have different values. It would seem that the broader conceptual issue of what a letter symbol is representing did not impact on students' ability to treat equations as objects. Apart from students' responses to that one question, there was a strong correspondence between their knowledge of algebraic notation and conventions and the most sophisticated strategy that they were able to employ.

Knowledge of arithmetic structure was similar to that reported last year, with many students reading expressions from left to right. Knowledge of which operations were the inverses of each other was also similar to last year's study, with about a quarter of the students having poor or very poor knowledge. For both of these aspects of knowledge, there was a strong correspondence with the most sophisticated strategy that a student was able to employ.

Students' understanding of equivalence was not as good as their other prerequisite knowledge. Only 18.1% of students had a good or very good understanding, which is similar to last year's study. Many students wrote 9 in the empty box for numerical equations such as $4 + 5 = \square + 3$ and were unable to answer algebraic equations such as $13x + \square = 12$, given that $13x + 2 = 9$. The correspondence between students' understanding of equivalence and their most sophisticated strategy was extremely strong. The effect size, of very poor to average understanding of equivalence compared with good and very good understanding, on whether students could solve equations by using transformations was huge (OR = 14.9). Although this study cannot differentiate between cause and effect, it would seem likely that a good understanding of equivalence is a prerequisite for solving equations using transformations.

This year's findings about students' acceptance of lack of closure were somewhat different from last year's. In the current study, 74.9% of students demonstrated a poor or very poor acceptance of lack of closure and only 17.5% were classified as good or very good. Almost certainly, the reason for this difference was the inclusion of two questions (14 and 16) from Küchemann's (1981) classic study. These questions could not be answered procedurally and required acceptance that the answer to a question could be an algebraic expression. Many students attempted to calculate a specific value, even though this was not possible. The correspondence between students' acceptance of lack of closure and their most sophisticated strategy was extremely strong. The effect size, of very poor to average acceptance of lack of closure compared with good and very good acceptance, on whether students could solve equations by using transformations was also huge (OR = 23.8). As was argued for understanding equivalence, it would seem likely that a good acceptance of lack of closure is a prerequisite for solving equations using transformations. Acceptance of lack of closure is described by Kieran (1981, p. 319) as the "ability to hold unevaluated operations in suspension". Developing this acceptance requires generalisation by students. Given the results of this study, it would seem that many students do not perceive general solutions to problems. Many authors regard generalisation as the essence of algebra, or even of all mathematics. "A lesson without learners having the opportunity to express generality is not a mathematics lesson" (Mason, Graham, & Johnston-Wilder, 2005, p. ix).

Progress of SNP Students

Between year 9 and year 11, there was clear progress by the students in terms of the most sophisticated strategy that they were able to use. The proportion of students able to solve equations by transformations rose from 11.8% in year 9, to 22.5% in year 10, to 32.7% in year 11. The odds ratios calculated showed that there was a significant effect size between year 9 and year 10 and also between year 10 and year 11. It can be concluded, therefore, that significant numbers of students are learning to use this strategy in both year 10 and year 11. For any student, to move from a most sophisticated strategy of working backwards to using transformations is indicative of considerable conceptual development (Filloy & Sutherland, 1996). The reported lack of progress of year 10 students on the Number Framework (Tagg & Thomas, 2009) may be, at least in part, due to the tool being used to measure progress. Assessment of students on the Framework examines their computational strategies, but no new computational strategies are required for solving equations. Given that above level 4 of the curriculum, the focus of the Number and Algebra strand moves from number to algebra, it can be argued that the Number Framework is no longer an appropriate framework for measuring progress.

The proportion of students restricted to primitive or no strategies decreased from 22.7% in year 9, to 18.7% in year 10, to 12.1% in year 11. The odds ratios calculated showed that there was a significant effect size between year 9 and year 10, but not between year 10 and year 11. The lack of demonstrable effect between year 10 and year 11 for these less able students is of some concern. Although the more able students are making progress between year 10 and year 11, it would seem that the less able may not be. Looking at the raw test scripts of the students, it appeared that by year 11, many of the less able students had given up and were not even attempting to solve the equations.

Progress on the aspects of algebraic knowledge (notation and convention; inverse operations; arithmetic structure; lack of closure; equivalence) between years 9 and 11 was less clear. There was no significant increase in any of the aspects of knowledge between year 9 and year 10. This echoes Tagg & Thomas's (2009) report of lack of progress of year 10 students on the Number Framework. However, the chances of year 11 students having good or very good knowledge compared with year 10 students was about one-and-a half to two times as great for each of the aspects.

Conclusions

A written diagnostic tool appears to be a very viable means of assessing the most sophisticated strategy for solving equations that a student is able to employ. A scoring rubric that requires students to demonstrate use of less sophisticated strategies as well as of their most advanced strategies obviates the possibility of mis-diagnosing students who have used guess and check for the more difficult equations. However, the present test could be improved by providing students with more opportunities to demonstrate use of each strategy. A more demanding scoring rubric could be used to evaluate students' mastery of strategies rather than just use of strategies.

The sampling used in this study provides a good picture of the algebraic knowledge and strategies used by students in SNP schools. The proportion of students able to use the most sophisticated strategy of solving equations by transformations was only about 20%. Use of this strategy was strongly associated with understanding of equivalence and acceptance of lack of closure.

There was clear evidence of progress between year levels. The proportion of students able to use the most sophisticated strategy of solving equations by transformations rose from year 9 to year 10 and from year 10 to year 11. However, there were indications that the less able students made little progress in acquiring new strategies between year 10 and year 11. There was also no clear evidence of learning of algebraic knowledge between year 9 and year 10.

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Performance of SNP Students on the Number Framework

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This paper analyses the results of students in schools participating in the Secondary Numeracy Project (SNP) in 2009, with the aim of quantifying the impact of the SNP on their number knowledge and strategies. The findings indicate that the performance of students in 2009 first-year SNP schools is similar to that of students in previous years, with mean gains of around half a stage on each domain of the Number Framework. Demographic factors impact on performance, with students from high-decile schools and New Zealand European students performing better than other students. An analysis of a small group of schools with three years of data on year 9 students suggests that the impact of the SNP in those schools was greater in the first year than in subsequent years.

Background

The Secondary Numeracy Project (SNP) began in 2005, with a pilot project implemented in 42 schools. Since then, new schools have participated each year. 2009 represents the fifth year of implementation of the SNP. The SNP forms part of the Numeracy Development Projects (NDP), which have been a focus of mathematics education in New Zealand since the Count Me In Too pilot project in 2000. The SNP shares many features in common with the other parts of the NDP. The Number Framework (Ministry of Education, 2008) provides the basis for understanding the likely development of student ability in number knowledge and strategies. An individual diagnostic interview is carried out with students to determine their position on the three strategy domains of the Framework, and a whole-class written knowledge test is carried out to assess students' knowledge against four knowledge domains. Teachers in participating schools are provided with professional development in the form of workshops, in-class modelling and observation, and peer mentoring. One key feature of the SNP is the role of the in-school facilitator (ISF). Professional development is provided by an existing member of the school's mathematics department, who receives training to become the ISF. The ISF continues teaching within the school but receives a time allowance to train and support other teachers.

This paper analyses two sets of results. The first are those of students in schools participating in the SNP in 2009, with the aim of quantifying any improvement made in their number knowledge and strategies. The second set of results is from a small group of schools with results for year 9 students from 2007, 2008, and 2009. The results of year 9 students in these schools are analysed to identify patterns in their performance. The research questions addressed in this paper are:

- Is the SNP continuing to have an impact?
- Which groups are benefitting most?
- What is the pattern of performance of students in schools with three years of data?

Method

Participants

The results reported in this paper were obtained by downloading all data from schools participating in the SNP from the online numeracy database on 16 January 2010. Schools participating in the SNP for the first time in 2009 were required to enter both initial and final data on the three strategy domains and four knowledge domains of the Number Framework for their year 9 students. Complete results were available for 3820 year 9 students in 30 schools participating for the first time.

Table 1 comprises a breakdown of the year 9 students in first-year SNP schools, included for analysis by gender and ethnicity. In first-year schools, approximately two-thirds (65%) of the students were of New Zealand European origin, 17% identified as Māori, and 5% identified as Pasifika. This compares with national proportions of year 9 students in 2009 of approximately 56% New Zealand European, 23% Māori, and 9% Pasifika (Ministry of Education, 2010). There were more female students than male in the 2009 SNP sample, while nationally in 2009 there were slightly more male (52%) than female (48%) students in year 9.

Table 1
Profile of SNP Students by Ethnicity and Gender

Ethnicity	Male	Female
NZ European	70%	61%
Māori	18%	17%
Pasifika	2%	8%
Asian	5%	9%
Other	5%	5%
Total	1666	2154

Table 2 presents the decile profile of year 9 students in first-year schools in the five years in which the SNP has been implemented. The profile of year 9 students in 2009 is quite different from that from previous years, with over half (54%) of the students coming from high-decile schools, compared with a previous maximum of 38% in 2007. Fifteen percent of year 9 students in first-year schools in 2009 came from low-decile (1–3) schools, with the remaining 31% coming from medium-decile (4–7) schools.

Table 2
Profile of Year 9 SNP Students in First-year Schools by Decile Group for2005–2009

	2005	2006	2007	2008	2009
Low decile	11%	12%	8%	36%	15%
Medium decile	52%	61%	54%	51%	31%
High decile	37%	27%	38%	13%	54%
Total	3975	5807	5093	2468	3820

Schools that first participated in the SNP prior to 2009 were not required to collect initial data on their students in 2009, but some chose to continue to do so. Four schools were identified that collected both initial and final data for year 9 students in 2007, 2008, and 2009. The four schools that entered year 9 data for all of 2007, 2008, and 2009 provided complete data for a total of 480, 479, and 340 year 9 students for those years respectively. Details are provided in Table 3.

Table 3
Numbers of Students in Schools with Complete Year 9 Data for 2007, 2008, and 2009

	2007	2008	2009
School 1 (low decile)	110	112	96
School 2 (medium decile)	70	66	40
School 3 (medium decile)	163	164	118
School 4 (medium decile)	137	137	86
Total	480	479	340

The numbers of students with complete data available were lower for all four schools in 2009 than in 2007 and 2008. It is likely that this reflects the fact that the SNP professional development is provided to schools for two years. In 2009, these schools were no longer required to enter data for all students, and it appears that data was therefore not entered for some classes.

Analysis

T-tests were used to compare the means of variables (gender and year level) with only two categories, and an ANOVA (analysis of variance) was used to compare the means of variables (decile group and ethnicity) with three categories. Where overall differences are described between groups, this has been verified to at least the 1% significance level, either by the T-test or by a post-hoc analysis using Tukey's honestly significant difference test. In addition, differences in percentages of students at particular levels of each domain of less than 5% and differences in mean stages of less than 0.2 are not reported because these differences are not considered to be of practical significance. It needs to be noted that, in some instances, significantly different mean gains and effect sizes may be smaller than other gains and effect sizes shown that are not significant due to differences in sample size. In all tables, rounded percentages are presented. Percentages less than 0.5% are therefore shown as 0%, and where there are no students represented, the cell is left blank. Due to rounding, percentages in some tables may not add up to 100. Effect sizes, where used, have been calculated by dividing the average difference between two groups by the pooled standard deviation of the two groups. Effect sizes of 0.2 are considered "small", effect sizes of 0.4 are "medium", and effect sizes of 0.6 or higher are "large" (Hattie, 2009). For the purposes of this paper, effect sizes of 0.2 or less are described as small, effect sizes between 0.2 and 0.6 are described as medium, and effect sizes of 0.6 or higher are described as large.

Findings

The findings of this research are reported under three headings that explore aspects of the effectiveness of the SNP. The first section addresses the question "Is the SNP continuing to have an impact?" Specifically, it looks at how the performance of students in schools participating for the first time in 2009 compares with that of students from first-year schools in previous years. The second section asks "Which groups are benefitting most?" This section compares the impact of the SNP in 2009 on various demographic subgroups of year 9 students. The final section analyses the results from schools with three years of data. The year 9 performance of students from four schools in their third year in the SNP is compared with that of year 9 students from the same schools in 2007 and 2008.

Appendices K and L provide a detailed breakdown of the percentages of students rated at each stage of the seven domains of the Number Framework.

Is the SNP Continuing to Have an Impact?

The annual NDP research reports have consistently shown that students in schools participating in the SNP make progress in numeracy as measured on the Number Framework (Tagg & Thomas, 2006, 2007, 2008, 2009). When considering the concept of progression, there are two aspects that need to be addressed. The first is the achievement level of a student at a given point of time. The second is the degree or amount of progress made by a student over a particular period of time. Both of these aspects are addressed in this section.

Tables 4 and 5 show the initial and final percentages of year 9 students at each stage of the multiplicative and proportional domains. The final results of year 9 students from 2005 to 2008 are provided for comparison (Tagg & Thomas, 2009).

Table 4
Performance of Year 9 Students on the Multiplicative Domain

	Initial					Final				
Stage	**2005**	**2006**	**2007**	**2008**	**2009**	**2005**	**2006**	**2007**	**2008**	**2009**
0–3: Counting from one	2%	2%	3%	2%	1%	0%	0%	1%	1%	0%
4: Advanced counting	12%	14%	10%	16%	11%	6%	5%	4%	7%	4%
5: Early additive	27%	28%	26%	27%	24%	16%	16%	14%	18%	14%
6: Advanced additive	34%	32%	36%	36%	38%	32%	35%	35%	34%	35%
7: Advanced multiplicative	20%	18%	20%	16%	21%	30%	29%	31%	31%	33%
8: Advanced proportional	5%	6%	5%	3%	6%	16%	14%	14%	9%	15%
Number of students	3975	5807	5093	2468	3820	3975	5807	5093	2468	3820

Table 5
Performance of Year 9 Students on the Proportional Domain

	Initial					Final				
Stage	**2005**	**2006**	**2007**	**2008**	**2009**	**2005**	**2006**	**2007**	**2008**	**2009**
0–3: Counting from one	1%	1%	1%	2%	1%	1%	0%	0%	1%	0%
4: Advanced counting	16%	17%	14%	21%	15%	6%	6%	6%	11%	7%
5: Early additive	29%	31%	30%	35%	32%	23%	24%	22%	27%	24%
6: Advanced additive	17%	17%	18%	14%	16%	17%	19%	18%	17%	17%
7: Advanced multiplicative	31%	30%	32%	24%	32%	41%	38%	41%	36%	38%
8: Advanced proportional	5%	4%	5%	4%	5%	12%	12%	13%	9%	14%
Number of students	3975	5807	5093	2468	3820	3975	5807	5093	2468	3820

On both the multiplicative and proportional domains, the proportions of 2009 year 9 students at the top stages increased while the corresponding proportions at the lower stages decreased. The numeracy expectations cited on the nzmaths website (Ministry of Education, n.d.) indicate that students at the end of year 9 rated at stage 7 (advanced multiplicative) or stage 8 (advanced proportional) are "achieving at or above expectations". The proportion of students rated as attaining these stages increased over the year from 27% to 48% on the multiplicative domain and from 37% to 52% on the proportional domain. These students can be considered to be meeting the expectations for year 9. The proportion of students rated as early additive (stage 5) or below decreased from 36% to 18% on the multiplicative domain and from 48% to 31% on the proportional domain. The expectations categorise students rated at stage 5 or lower at the end of year 9 as "at risk" and describe them as "sufficiently below expectations that their future learning in mathematics is in jeopardy".

While the proportions of students reaching at least stage 7 were higher in 2009 than in 2008 on both domains, it is important to note that the proportions of students rated at these stages were also higher at the initial assessment. It is also worth noting that at least part of this difference can be attributed to the differences in the decile profiles of the students. Previous findings (for example Young-Loveridge, 2007) have shown that students from high-decile schools are likely to reach higher stages on all domains of the Number Framework.

It is important to examine the progress made by students between the initial and final assessments. Figures 1 and 2 show the percentages of year 9 students gaining stages on the multiplicative and proportional domains in 2005 to 2009, broken down by initial stage. Consistent with results from previous years, approximately half of the students not initially rated at the top stage of each domain gained at least one stage between the initial and final assessments. In general, students with lower initial stages were more likely to make progress, although, consistent with previous years, a slightly higher proportion of students initially rated at stage 6 on the proportional domain made gains (59%) than of students initially rated at stage 5 (49%).

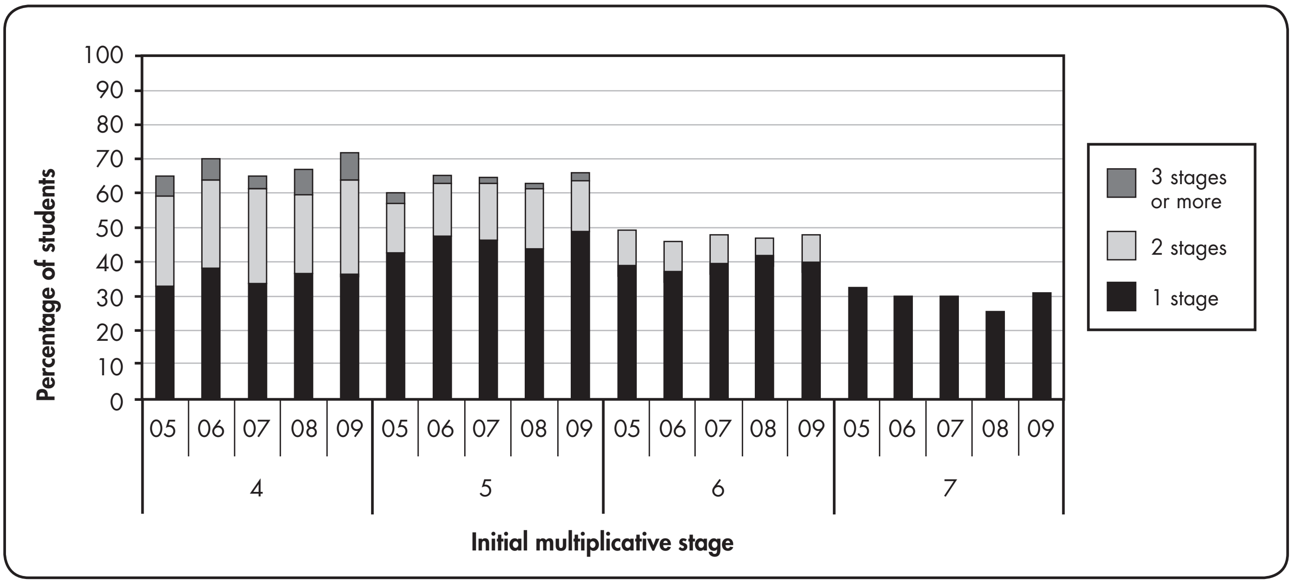

Figure 1. Number of stages gained by initial multiplicative stage for year 9 students in 2005 to 2009

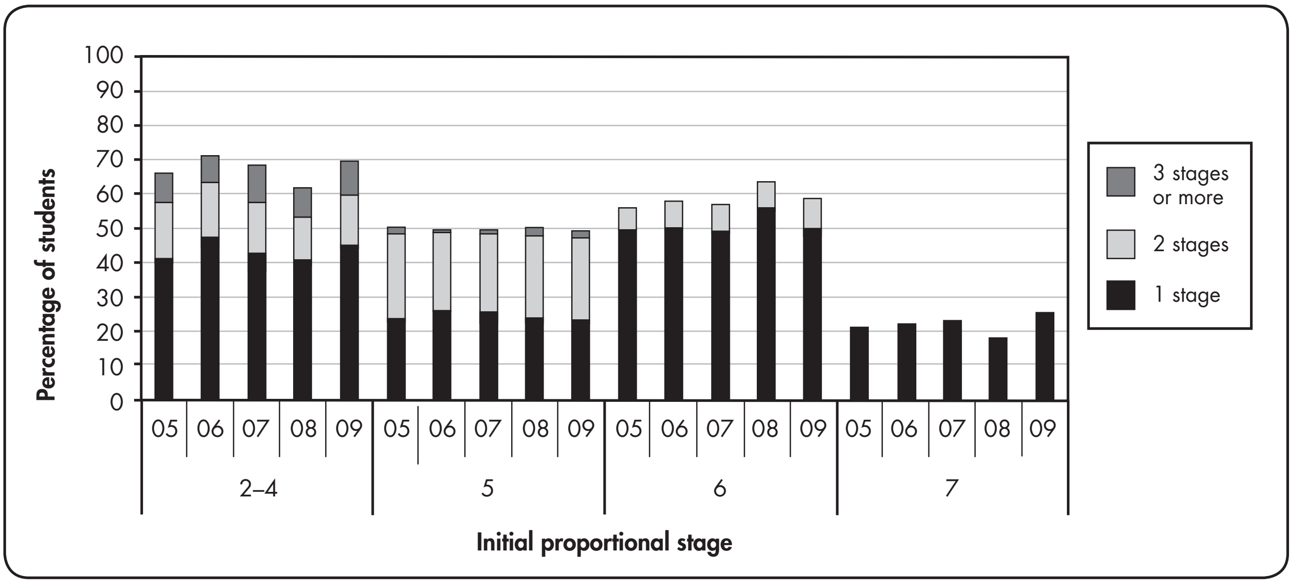

Figure 2. Number of stages gained by initial proportional stage for year 9 students in 2005 to 2009

The mean initial and final stages for the seven domains and effect sizes for the differences are reported in Table 6 to give a measure of the magnitude of the progress made by year 9 students in 2009.

Table 6
Effect Sizes for Comparisons of Initial and Final Scores of Year 9 SNP Students

Domain	Initial mean	Final mean	Difference	Effect size
Additive	5.52	6.01	0.49	0.54
Multiplicative	5.83	6.39	0.56	0.52
Proportional	5.61	6.22	0.62	0.44
FNWS	5.56	5.76	0.20	0.36
Fractions	5.60	6.14	0.55	0.49
Place value	5.69	6.31	0.62	0.53
Basic facts	5.85	6.31	0.46	0.51

The results show that the students made mean gains of around half a stage on all domains except on the forward number word sequence domain and that the effect sizes for the impact of the SNP on these students on each domain was medium in all cases. The lowest mean gain (0.2) and corresponding effect size (0.36) was for the FNWS domain. The lack of improvement on this domain has been discussed previously (Tagg & Thomas, 2009) and is primarily due to a ceiling effect, with over half (61%) of students rated at the top stage of the domain at the initial assessment and therefore unable to make progress. At the top end of the Number Framework, stages are equivalent to levels of the curriculum, so gains of half a stage are in line with expected progress of approximately one level every two years of schooling.

Which Groups Are Benefitting Most?

The results from the SNP in previous years (Tagg & Thomas, 2006, 2007, 2008, 2009) have shown a consistent pattern of comparative performances between demographic subgroups. Tables 7 and 8 show the mean initial and final stages of year 9 students in first-year SNP schools on the multiplicative and proportional domains broken down by gender, ethnicity, and decile group. Mean gains and effect sizes for the differences are included to indicate the magnitude of the impact of the SNP.

Table 7
Effect Sizes for Gains Made on the Multiplicative Domain by Demographic Subgroups of Year 9 Students

	Mean initial stage	Mean final stage	Gain	Effect size
Male	5.89	6.44	0.55	0.50
Female	5.79	6.35	0.56	0.54
Low decile	5.30	6.03	0.73	0.71
Medium decile	5.79	6.30	0.51	0.47
High decile	6.00	6.54	0.54	0.52
NZ European	5.92	6.45	0.52	0.50
Māori	5.42	6.06	0.64	0.62
Pasifika	5.36	6.02	0.66	0.64
Total	5.83	6.39	0.56	0.52

Table 8
Effect Sizes for Gains Made on the Proportional Domain by Demographic Subgroups of Year 9 Students

	Mean initial stage	Mean final stage	Gain	Effect size
Male	5.66	6.27	0.61	0.42
Female	5.56	6.19	0.62	0.46
Low decile	4.80	5.64	0.84	0.62
Medium decile	5.52	6.16	0.63	0.44
High decile	5.88	6.42	0.54	0.41
NZ European	5.76	6.33	0.56	0.41
Māori	4.95	5.74	0.79	0.56
Pasifika	4.98	5.70	0.72	0.53
Total	5.61	6.22	0.62	0.44

A T-test indicated that, at the final assessment, the mean multiplicative stage of males was higher than that of females ($p < 0.01$). On the proportional domain, no difference was found between genders. ANOVA tests with post-hoc analysis were carried out to compare the means of students by ethnicity and decile group. On both the multiplicative and proportional domains, the mean stage of New Zealand European students was higher than that of both Māori and Pasifika students ($p < 0.01$). Similarly, on both domains, the mean stage of students from high-decile schools was higher than that of students from medium-decile schools ($p < 0.01$), with both being higher than that of students from low-decile schools ($p < 0.01$). This pattern of comparative performance of demographic subgroups is consistent with that found in previous years (Tagg & Thomas, 2006, 2007, 2008, 2009). While all subgroups made mean gains of at least half a stage on both domains, in general, the subgroups with lower mean initial stages tended to make greater mean gains and have larger effect sizes. The effect sizes for the impact of the SNP on low-decile, Māori, and Pasifika students were large on the multiplicative domains, while for low-decile students, the effect size was also large on the proportional domain. All other effect sizes were medium. The greater progress made by those subgroups with lower initial scores may also be explained by a ceiling effect for higher-scoring students and by the fact that the gains made by students tend to be greater at the lower stages of the Number Framework (Tagg & Thomas, 2009).

Table 9 summarises the effect sizes for gains made on all seven domains by demographic subgroups.

Table 9
Effect Sizes for Gains Made by Demographic Subgroups of Year 9 Students

	Additive	Multiplicative	Proportional	FNWS	Fractions	Place Value	Basic Facts	N =
Male	0.50	0.50	0.42	0.33	0.51	0.52	0.51	1666
Female	0.57	0.54	0.46	0.38	0.47	0.54	0.52	2154
Low	0.84	0.71	0.62	0.22	0.37	0.33	0.48	570
Medium	0.46	0.47	0.44	0.34	0.47	0.52	0.45	1185
High	0.52	0.52	0.41	0.43	0.55	0.61	0.57	2065
NZ European	0.49	0.50	0.41	0.39	0.52	0.54	0.50	2482
Māori	0.61	0.62	0.56	0.25	0.36	0.43	0.41	668
Pasifika	0.77	0.64	0.53	0.26	0.53	0.60	0.65	200
Total	0.54	0.52	0.44	0.36	0.49	0.53	0.51	3820

Table 9 shows that, while the effect sizes for the impact of the SNP were at least medium for all subgroups on all domains, the impact on the various subgroups varies between the domains of the Number Framework. While the effect sizes on the three strategy domains appear to be larger for students from low-decile schools than for those from medium- and high-decile schools, the reverse was true for the knowledge domains, with the effect sizes appearing to be larger for students from high-decile schools. Similarly, while Māori and Pasifika students appear to have higher effect sizes on the strategy domains than those for New Zealand European students, the effect sizes on the knowledge domains for New Zealand European students appear larger than those for Māori students. The largest effect sizes found were on the additive domain for students from low-decile schools (0.84) and Pasifika students (0.77). The smallest effect sizes were on the FNWS domain for students from low-decile schools (0.22) and Māori students (0.25).

What Is the Pattern of Performance of Students in Schools with Three Years of Data?

This section analyses the results of the four schools that entered initial and final year 9 results in 2007, 2008, and 2009. While the results represent three different cohorts of students, they give an indication of the impact of teachers in the years following the initial implementation of the SNP. The numbers of students with complete data entered in the database were lower for all four schools in 2009 than in 2007 and 2008. The reasons for the lower numbers of students are unknown; because of uncertainty regarding the completeness of data entered, interpretation of these results should be viewed with caution.

Table 10 compares the initial and final profiles on the multiplicative domain of year 9 students in 2007, 2008, and 2009.

Table 10
Performance on the Multiplicative Domain of Year 9 Students in Schools with 2007, 2008, and 2009 Data

	Initial			Final		
Stage	**2007**	**2008**	**2009**	**2007**	**2008**	**2009**
0–3: Counting from one	4%	1%	1%	0%	0%	0%
4: Advanced counting	11%	9%	6%	4%	3%	4%
5: Early additive	23%	21%	25%	16%	13%	11%
6: Advanced additive	35%	38%	38%	30%	36%	32%
7: Advanced multiplicative	23%	23%	24%	35%	29%	36%
8: Advanced proportional	5%	7%	7%	16%	19%	17%
Number of students	480	479	340	480	479	340

While the final profiles for the three years are relatively similar, higher proportions of year 9 students were rated at the lower stages of this domain in 2007 than in 2008 or 2009. Thirty-eight percent of students were rated at stage 5 or below in 2007 at the initial assessment, compared with 31% in 2008 and 32% in 2009.

Table 11
Performance on the Proportional Domain of Year 9 Students in Schools with 2007, 2008, and 2009 Data

	Initial			Final		
Stage	**2007**	**2008**	**2009**	**2007**	**2008**	**2009**
0–3: Counting from one	3%	0%	0%	0%	0%	0%
4: Advanced counting	16%	14%	17%	4%	5%	5%
5: Early additive	29%	27%	28%	24%	22%	23%
6: Advanced additive	16%	22%	27%	18%	20%	24%
7: Advanced multiplicative	34%	33%	22%	40%	33%	34%
8: Advanced proportional	2%	5%	6%	15%	20%	14%
Number of students	480	479	340	480	479	340

Table 11 shows that in 2007 at the initial assessment, 48% of students were rated at stage 5 or below on the proportional domain, compared with 41% in 2008. The 2009 cohort were placed between the two, with 45% of students initially rated at or below stage 5. A higher proportion (20%) of students reached stage 8 in 2008 than in 2007 (15%) or 2009 (14%).

Effect sizes were calculated to investigate the impact of the SNP on year 9 students in each year. The results are shown in Table 12.

Table 12
Effect Sizes for the Impact of the SNP on Year 9 Students in 2007, 2008, and 2009

Year	Additive	Multiplicative	Proportional	FNWS	Fractions	Place value	Basic facts
2007	0.70	0.61	0.63	0.23	0.45	0.46	0.54
2008	0.58	0.48	0.43	0.12	0.41	0.43	0.12
2009	0.60	0.49	0.49	0.13	0.31	0.28	0.20

The effect sizes for the impact of the SNP on year 9 students in these schools in 2007 were large for all three strategy domains. A large effect size was also found on the additive domain in 2009. On all domains, the effect sizes for the impact of the SNP appear to be larger for students in 2007 than for either 2008 or 2009. The difference is particularly noticeable on the basic facts domain, where the effect size for students in 2007 was 0.54, compared with 0.12 in 2008 and 0.20 in 2009.

As noted above, differences in initial strategy profiles for the three years were noted for the multiplicative and proportional domains. ANOVA tests were carried out to compare the initial and final scores of the three groups of students on all domains. The results are shown in Table 13.

Table 13
ANOVA Tests Comparing Initial and Final Results for Year 9 Students in 2007, 2008, and 2009

Domain	Initial	Final
Additive	$p < 0.01$	$p < 0.05$
Multiplicative	$p < 0.01$	ns
Proportional	$p < 0.05$	ns
FNWS	ns	ns
Fractions	ns	ns
Place value	$p < 0.05$	ns
Basic facts	$p < 0.01$	$p < 0.05$

Significant differences ($p < 0.05$) were found between the three cohorts' initial results on all domains except the FNWS and fractions domains. A comparison of final results indicated significant differences on the additive and basic facts domain. Post hoc T-tests were carried out on those results where the ANOVA indicated significant differences. The results are shown in Table 14.

Table 14
Post-hoc T-tests Comparing Initial and Final Results for Year 9 Students in 2007, 2008, and 2009

	Initial			Final		
Domain	**2007 vs 2008**	**2007 vs 2009**	**2008 vs 2009**	**2007 vs 2008**	**2007 vs 2009**	**2008 vs 2009**
Additive	2008 ($p < 0.01$)	ns	2009 ($p < 0.05$)	2008 ($p < 0.05$)	ns	ns
Multiplicative	2008 ($p < 0 .05$)	ns	2009 ($p < 0.01$)	ns	ns	ns
Proportional	2008 ($p < 0.01$)	ns	ns	ns	ns	ns
FNWS	ns	ns	ns	ns	ns	ns
Fractions	ns	ns	ns	ns	ns	ns
Place value	ns	2009 ($p < 0.05$)	2009 ($p < 0.05$)	ns	ns	ns
Basic facts	2008 ($p < 0.01$)	ns	2009 ($p < 0.01$)	2007 ($p < 0.05$)	ns	2007 ($p < 0.05$)

As shown in Table 14, the T-tests indicated that the year 9 students in 2007 had significant lower mean initial ratings on the additive, multiplicative, and basic facts domains than students in 2008 and 2009. They also had lower mean initial scores on the proportional domain than the 2008 students and lower initial place value scores than the 2009 students. The only significant difference between the initial results of year 9 students in 2008 and 2009 was on the place value domain on which students from 2009 had higher mean initial scores. There were only two domains for which there was a difference at the final assessment. On the additive domain, students from 2008 had a higher mean final stage than students from 2007. The most noteworthy finding is that on the basic facts domain, the mean final stage of students from 2007 was higher than that of students from both 2008 and 2009. This is particularly surprising given that the mean initial stage of these students had been lower than that of the other two cohorts. The initial and final profiles of students in the three years were compared to investigate the differences. The results are shown in Table 15.

Table 15
Performance on the Basic Facts Domain of Year 9 Students in Schools with 2007, 2008, and 2009 data

	Initial			Final		
Stage	**2007**	**2008**	**2009**	**2007**	**2008**	**2009**
0–3: Facts to 10	2%	1%	1%	0%	1%	0%
4: Within 10, doubles, and teens	5%	3%	3%	2%	2%	2%
5: Addition, multiplication for 2, 5, 10	17%	17%	17%	11%	10%	12%
6: Subtraction and multiplication	43%	32%	36%	27%	39%	36%
7: Division	31%	44%	40%	52%	40%	44%
8: Factors and multiples	2%	4%	4%	8%	8%	6%
Number of students	480	479	340	480	479	340

The proportion of students rated as at least stage 7 on the basic facts domain at the initial assessment was lower in 2007 (33%) than in 2008 (48%) or 2009 (44%), while the proportion of students rated at those stages at the final assessment was higher (60%, compared with 48% in 2008 and 50% in 2009). This reflects the higher effect size indicated in Table 11 and the significant differences shown in tables 13 and 14.

There are several possible reasons for the apparent smaller impact of the SNP in the years following its initial implementation. It appears that the cohort of students starting year 9 in these schools in 2007 may have been poorer at mathematics than those in 2008 and 2009. Students' knowledge scores are based on the numbers of items answered correctly in a written test and are therefore less likely to be subject to variation. The fact that students in 2007 rated significantly lower than those in 2008 or 2009 on the basic facts domain would seem to support the idea that there was an actual difference in ability between cohorts. Alternatively, because the students' strategy scores are based on teacher judgment, it is possible that teachers may have rated students inconsistently in their first assessment at the start of 2007. Lower initial ratings would allow for greater gains and therefore the impression of a greater impact. It is also possible that there was a turnover of staff within these schools following the initial implementation of the SNP. Teachers joining a school after the initial professional development may have a lesser focus on the implementation of the SNP than those involved from the first year. As mentioned previously, there was data entered for fewer students in 2009 than in 2007 or 2008. It is uncertain whether the students for whom data was entered in 2009 are representative of that cohort as a whole. One other possible explanation for the lower initial numeracy stages of students in 2007 is that these students received less numeracy-focused mathematics instruction in primary school due to the timing of the rollout of the NDP. Without further investigation, it is not possible to be sure which, if any, of these factors have led to the apparent reduced impact of the SNP in the years following its initial implementation.

Concluding Comment and Key Findings

The findings from the SNP in 2009 indicate that it continues to have a positive impact on the achievement of year 9 students in first-year schools.

A comparison of the initial and final profiles of students on the multiplicative and proportional domains showed that the proportions of students at the lower stages decreased, while the proportions of students at the higher stages increased. About half of all year 9 students in first-year SNP schools attained numeracy stage 7 by the end of the year. An analysis of gains made by students from each starting stage showed similar results to those found previously, with students at lower starting stages tending to make greater gains. Students made mean gains of around half a stage on all domains except the FNWS domain, where a ceiling effect limited the mean gain to 0.2 of a stage. The effect sizes of these gains were medium in all instances (between 0.2 and 0.6), indicating that the impact of the SNP was substantial.

The performance of students was impacted on by demographic factors. Consistent with results from previous years, New Zealand European students had higher mean stages than Māori or Pasifika students, and students from high-decile schools had higher mean stages than students from medium- and low-decile schools.

An analysis of the results of a group of schools with data for year 9 students for 2007, 2008, and 2009 suggested that the impact of the SNP was greater in those schools in 2007 than in subsequent years.

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The Role of the Numeracy Lead Teacher in Promoting the Goals of the Numeracy Development Projects

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School-based leadership is a key element of the design of the Numeracy Development Projects (NDP) and reflects the contextualised nature of the professional development in classrooms and schools. The study reported on in this paper investigated who undertakes the role of lead teacher in schools, what the role entails, and how the NDP's goals of teacher learning and raising student achievement are promoted by the lead teacher. The study found that those in the role of numeracy lead teacher were likely to have other leadership roles in the school. Furthermore, many of those in the role of lead teacher shared this role with others. The role of the lead teacher appears to strongly support the NDP's goals when the role is shared with others and when a lead teacher also holds a designated leadership role in the school, such as deputy principal.

Background

School-based leadership is an important element of the design of the professional development associated with the New Zealand Numeracy Development Projects (NDP) (Higgins & Parsons, 2009). The NDP, underpinned by a contextualised perspective on teacher learning, uses the school and classroom as primary contexts for teacher learning. The role of lead teacher is one of three nested levels of leadership in the design; the others are facilitators external to the school and system-level leadership such as the national co-ordinators. This paper is part of a larger project on school-based leadership in numeracy (see also Higgins & Bonne, 2009). Previous investigations of school-based leadership in the NDP have examined the lead teacher role as a mechanism for sustaining the NDP in schools (Thomas & Ward, 2006) and leadership content knowledge (Higgins, Sherley, & Tait-McCutcheon, 2007). The focus of this paper is the actions taken by those in a lead teacher role and the ways in which the role is organised in relation to other instructional leadership roles in order to promote the goals of the NDP.

Method

To investigate the leadership role undertaken by lead teachers across a large number of schools, an anonymous survey was administered online using the software package, Qualtrics (Version 13171). The survey was developed alongside conjectures from exploratory studies carried out in 2008 (see Higgins & Bonne, 2009). Items were also informed by the leadership functions identified by Leithwood et al. (2007) and the dimensions of leadership described in Robinson, Hohepa, and Lloyd (2009).

Feedback on an initial draft of the survey items was sought from an experienced survey designer (F. Hodis, personal communication, 19 November, 2009). The final survey included items that asked respondents to rate the significance of their role in various aspects of implementing the NDP in their school, using a scale of zero to nine, with anchors of "not at all significant" at zero and "extremely significant" at nine. Respondents were asked to elaborate on these responses by listing specific examples of actions they had taken to do this. For instance, "Please rate the significance of your role in setting goals and expectations for numeracy" was probed further with "Please list specific actions you have taken to set goals and expectations for numeracy." Respondents were also asked a series of open-ended questions about leading the development of numeracy at their schools. A full list of survey items is included in Appendix M.

Participants

Numeracy co-ordinators from around the country were asked to provide email addresses for those with instructional leadership roles in numeracy in primary schools around New Zealand (this did not include intermediate and Māori-medium schools); typically, these were lead teachers of numeracy, principals, deputy and assistant principals, and syndicate leaders. The main criterion for school selection was that staff participated in school-wide numeracy development during 2009. Email addresses were provided for 134 school leaders from around New Zealand, who were then invited to complete the online survey. A total of 42 instructional leaders (31%) completed the survey, although not all respondents answered every question. Because the survey was anonymous, information about the distribution of leaders across different types of schools is unknown; neither is it known if this sample is representative of the population of school leaders in New Zealand primary schools. Due caution therefore needs to be taken when interpreting the findings of this study.

Responses to the question, "For approximately how many years have you been in your current leadership role?" are summarised in Table 1. While 23 (55%) of respondents had been in their current leadership role for more than two years, the remaining 19 (45%) respondents had worked in leadership roles in schools for less than two years. Eight of the 25 lead teachers had been in that role for less than two years.

Table 1
Number of Years in Current Leadership Role

Number of years	Number of respondents	Percentage of respondents
Less than 2	19	45
2–5	13	31
5–10	8	19
10–20	2	5
20 or more	–	–
Total	**42**	**100**

The lead teacher role is one of many leadership roles in a school, and those in the role also have other responsibilities such as classroom teacher, syndicate leader, or deputy principal. The roles of the survey respondents are presented in Table 2. It is important to note that these 42 leaders collectively represented just over double that number of roles within their schools (n = 87), with the 25 numeracy lead teachers holding a total of 65 roles between them. At one extreme of this multiplicity of roles, one respondent was a numeracy lead teacher as well as an assistant principal, a syndicate leader, and a classroom teacher – four roles in all. While it seems likely that some respondents may work in small- to medium-size schools where having multiple roles is common, it is interesting to note that in contrast, the eight principals who responded had none of the additional roles included in this survey (numeracy lead teacher, syndicate leader, or classroom teacher). More than half the 25 numeracy lead teachers (n = 14) also had responsibility for a class. Furthermore, the 18 leaders who were also classroom teachers had an average of almost three roles per teacher.

Table 2
Roles of 42 Respondents

	Numeracy lead teacher	**Principal**	**Deputy principal**	**Assistant principal**	**Associate principal**	**Syndicate leader**	**Teacher** (with responsibility for a class)	**Other**
Numeracy lead teacher	25	0	7	4	3	11	14	1
Principal	0	8	0	0	0	0	0	0
Deputy principal	7	0	10	0	0	4	4	0
Assistant principal	4	0	0	5	0	2	3	0
Associate principal	3	0	0	0	3	1	1	0
Syndicate leader	11	0	4	2	1	16	11	1
Teacher (with responsibility for a class)	14	0	4	3	1	11	18	1
Other	1	0	0	0	0	1	1	2
Total	**65**	**8**	**25**	**14**	**8**	**46**	**52**	**5**

Procedure

Early in December 2009, an invitation to complete the survey was emailed to the 134 email addresses provided, with a reminder email sent one week later to encourage addressees to complete the survey before the end of the school year. A second reminder was sent a week before the survey closed at the end of January 2010.

The Research Questions

The purpose of this research was to investigate the following questions:

- What actions do lead teachers take to support the goals of the NDP?
- What are the patterns of organisation of the lead teacher role?

Analysis

Two reports of the survey data were downloaded from Qualtrics (Version 13171) into QSR NVivo (Version 8); one version was a summary report showing the amalgamated data, and the second showed responses broken down into the respondents' role/s. The former gave a useful overview of all responses, while the latter allowed responses to be attributed to those in particular roles.

Responses using the ordinal scale were collated into tables by the report function of the Qualtrics software; some of these tables are included in the findings section of this paper.

Other survey items generated qualitative responses, which were analysed using QSR NVivo (Version 8). These responses were searched for words relating to leadership, such as: collaborat*, collective, follow*, group, hierarch*, individual, influen*, lead, network, power, shar*, team, together, role, and voice. This searching supported the identification of leadership themes emerging from the survey responses, which were then coded accordingly.

Findings

The findings of this study include indications that the relationship between the numeracy lead teacher role and other leadership roles in the school impacted on the lead teacher's perception of their effectiveness in their role. Furthermore, the ways in which the lead teacher role was shared with a colleague also influenced such perceptions. Specifically, the study found that those in the role of lead teacher were highly likely to have other leadership roles in the school. Eighteen leaders made comments that indicated that carrying out the role of the lead teacher seems to be easier when sharing the role with others; this seemed to be particularly so where a lead teacher (who was also responsible for a class) shared the role with a colleague who held a designated leadership role, such as deputy principal. The role of the lead teacher was found to align with the goals of the NDP of enhancing teacher knowledge and maintaining a school-wide focus on student achievement. The first part of this section presents evidence relating to the actions taken by lead teachers in support of the NDP's goals, while the second part outlines the different ways in which the lead teacher role was organised and the apparent effects this had on the influence of their actions.

The three areas in which lead teachers assigned the lowest ratings to the significance of their role were:

- communicating numeracy goals and expectations to the school community (five lead teachers rated this four or below)
- co-ordinating numeracy curriculum delivery across the school (five lead teachers rated this four or below)
- encouraging a closer match between students' experiences of numeracy outside school and the numeracy instruction they experience (nine lead teachers rated this four or below).

Other leaders rated the first two items more highly than did lead teachers. For example, six of the eight principals rated the significance of their own role in communicating numeracy goals and expectations to the school community as seven or above, and six of the ten deputy principals rated their own role in co-ordinating numeracy curriculum delivery across the school as seven or more. Responses from all leaders were very mixed for the third item. This suggests that the roles of various leaders generally overlap slightly to ensure that the different leadership responsibilities are all carried out.

Actions Taken by Lead Teachers to Support the Goals of the NDP

The actions taken by the lead teachers (n = 25), as reported in the questionnaire, all aligned with the goals of the NDP. Specific actions taken by lead teachers to set goals and expectations for numeracy included:

- encouraging teachers to include numeracy goals in their appraisal process and supporting them to achieve this
- taking a lead role in teacher development sessions
- collecting, analysing, and reporting student achievement data
- determining numeracy goals for students as part of annual strategic planning.

One lead teacher described the actions they had taken to support teacher development in numeracy as:

> Run staff meeting PD, organise maths advisor PD, organise staff to attend PD, observation and modelling in classrooms. Establishing teacher goals and reviewing these regularly. (Lead teacher/deputy principal/syndicate leader/teacher)

While the lead teacher often took a lead role in presenting teacher development in their school, their participation alongside their teacher colleagues also seemed important. As shown in Table 3, 28 of the respondents rated the significance of their role in participating in teacher learning and development as seven or above; this includes 17 of the 23 lead teachers who responded to this item.

Table 3
Responses to the Item, "Please rate the significance of your role in participating in teacher learning and development in numeracy"

Answer		**Response**	**%**
0 Not at all significant		0	0
1		1	2
2		0	0
3		2	5
4		0	0
5		5	12
6		6	14
7		6	14
8		10	24
9 Extremely significant		12	29
Total		**42**	**100**

In the data shown in Table 4, ten of the 14 respondents who rated the item as seven to nine were lead teachers. However, three lead teachers felt their role in this was not at all significant. Forty-one leaders responded to this item.

Table 4
Responses to the Item, "Please rate the significance of your role in challenging teachers to implement alternative strategies in order to improve student achievement"

Answer		**Response**	**%**
0 Not at all significant		3	7
1		1	2
2		2	5
3		2	5
4		8	20
5		5	12
6		6	15
7		6	15
8		1	2
9 Extremely significant		7	17
Total		41	100

Responsibility for challenging teachers to improve their practice was viewed by two principals as part of the lead teacher's role:

> The lead teachers and advisers have done that. (Principal)
>
> Delegated to DP and lead teacher. (Principal)

In support of the NDP's goal of raising student achievement, the lead teachers' actions included collecting, analysing, and reporting student achievement data and determining numeracy goals for students as part of annual strategic planning. In many schools, formulating a strategy for the collection of student achievement data was the responsibility of the lead teacher. Sixteen of the lead teacher respondents rated the significance of their role for this aspect as seven or above, along with 12 other respondents (see Table 5).

Table 5

Responses to the Item, "Please rate the significance of your role in deciding what school-wide numeracy achievement information will be collected"

Answer		**Response**	**%**
0 Not at all significant		3	7
1		1	2
2		2	5
3		2	5
4		2	5
5		0	0
6		4	10
7		9	21
8		7	17
9 Extremely significant		12	29
Total		**42**	**101 ***

*Due to rounding, not all percentages add up to 100.

Of the 29 respondents who rated their role in maintaining a focus on improving student achievement in numeracy as seven or above (see Table 6), 16 were lead teachers. Specific actions taken by lead teachers included maintaining a focus on student achievement results, ensuring regular cycles of classroom demonstrations, and observations and feedback. In response to the item, *Please list specific actions you have taken to set goals and expectations for numeracy*, 21 of the 40 leaders who answered the question commented that an analysis of student achievement data informs their school's goals and expectations.

> We review school-wide numeracy data in October of each year. This occurs at various levels – teacher, team, numeracy lead teacher, and principal. The analysis determines next year's target and focus for planning, professional development, spending, etc. (Lead teacher/associate principal)

Table 6
Responses to the Item, "Please rate the significance of your role in maintaining a focus on improving student achievement in numeracy"

Answer		Response	%
0 Not at all significant		0	0
1		0	0
2		1	2
3		1	2
4		0	0
5		5	12
6		6	14
7		6	14
8		10	24
9 Extremely significant		13	31
Total		**42**	**99 ***

*Due to rounding, not all percentages add up to 100.

Six lead teachers commented that they used national expectations for students at various year levels to inform the development of achievement expectations in their schools:

> Used Ikan and Gloss to form baseline data to systematically fill hot spots and group according to strategy level. End point data shows lift of achievement levels. Shared national expectations with staff, placed children within ranges, and developed next steps. Discussed and used maths standards in end of year reports to parents. (Lead teacher for numeracy, the arts, and environmental education/teacher)

Lead teachers also relied on support from senior management, who provided the conditions for school-wide goals and expectations to be established and achieved. The following quotes from two principals are examples of their roles in supporting the goal of teacher development in numeracy:

> Allocated staff meeting time, allocated teacher release for numeracy lead teachers, continued to have a numeracy curriculum team, insisted all teachers have numeracy PD goals, allowed all teachers time for maths observations. (Principal)

> All teachers have numeracy goals as part of their annual appraisal development goals. Annual reporting achievement targets for 2009 are numeracy based. (Principal)

Perhaps of some concern is the number of leaders who did not rate highly the significance of their role in promoting the importance of numeracy; the results in Table 7 show that 12 of the school leaders rated this as six or lower. Four of these respondents were in lead teacher roles. However, one reason for this may have been that numeracy was already highly valued in their schools and so did not need to be promoted.

Table 7
Responses to the Item, "Please rate the significance of your role in promoting the importance of numeracy"

Answer		Response	%
0 Not at all significant		0	0
1		0	0
2		0	0
3		2	5
4		0	0
5		5	12
6		5	12
7		11	26
8		6	14
9 Extremely significant		13	31
Total		42	100

For numeracy to be sustained in a school, one respondent commented that it needs to be constantly promoted:

> Numeracy needs to be kept as an area of focus even when new professional development initiatives are being undertaken, and my role is to ensure that this is managed in a positive way and is of benefit to teachers, students, and parents. (Lead teacher/deputy principal/classroom teacher)

Organisation of the Lead Teacher Role

The actions of the lead teacher presented in the previous section were possibly shaped by the organisation of the role in their school situation. The information shown in Table 2 highlighted that different combinations of roles are undertaken by lead teachers. Also, although not asked directly about the organisation of the lead teacher role, comments made by 12 respondents further illustrated leaders' perceptions of the many ways in which schools have organised that role. One comment related to the perceived advantages of having more than one lead teacher in a school:

> It's a very important role, I know, but it needs more of a voice, more of a collaborative, experienced team approach. (Lead teacher/syndicate leader)

One respondent remarked that having one lead teacher supported strong student achievement results in their school:

> Having one person with a coordinated big picture focus on mathematics provides a greater structure and focus for teaching and learning. This is making a difference and our results are affirming. (Lead teacher/associate principal, in response to the item, *Do you believe your actions in your leadership role have made a positive difference to student achievement in numeracy at your school?*)

Allocating the lead teacher responsibilities to two staff members appeared to give them greater influence, particularly when one is part of senior management, as is illustrated by the following three comments:

> There has been significant improvement in achievement, and staff practice is now consistent. Think [another lead teacher] and I have had a significant influence in [numeracy] as identified by the principal in his report to the board [of trustees]. (Lead teacher/syndicate leader/teacher)

> A great numeracy leadership team of DP and lead teacher (Principal)
>
> This is a key role for the 2 DPs to share. (Deputy principal in response to the item, *Please list specific actions you have taken to monitoring and evaluating numeracy teaching*.)

However, allocation of the lead teacher role to two people did not automatically guarantee that their influence would be strengthened. Where the role was clearly divided between two people, rather than their working collaboratively in their joint roles, sharing the lead teacher responsibilities might have the potential to weaken a lead teacher's influence. The following quotes illustrate divisions of the lead teacher responsibilities that do not seem consistent with a collaborative approach to carrying out the role:

> This is a shared responsibility, so I do not have a direct effect on [co-ordinating numeracy curriculum delivery across the school]. (Lead teacher/teacher)
>
> Not my role in the school. Other lead teacher responsible for this. (Lead teacher/teacher in response to the item, *Please list specific actions you have taken to challenge teachers to implement alternative strategies in order to improve student achievement*.)

Just as collaboration between those in lead teacher roles may serve to strengthen their leadership, this also seemed to apply to collaboration between, and overlapping of, various leadership roles, reflected in comments from senior management:

> I just question the numeracy co-leaders as to why they have selected the numeracy goals. They adjust if we jointly believe that is necessary. (Principal)
>
> Worked with leadership team, staff, and Numeracy Project providers to set school achievement levels. (Deputy principal)
>
> Worked with the numeracy lead teachers to determine the goals and targets and explain the expectations to staff. (Principal)

Respondents noted the following points in relation to the role of the lead teacher: time was needed in the role in order to become effective as a lead teacher, and the size of the lead teacher role should be acknowledged within the allocation of school resourcing of roles. For a small number of lead teachers, these factors contributed to a reluctance to continue in the role.

In responding to the item, *Do you believe your actions in your leadership role have made a positive difference to student achievement in numeracy at your school?*, three lead teachers, two of whom also led syndicates, commented on their relatively short time in the lead teacher role:

> As I have only been in the role for such a short time, this is not likely. The previous numeracy lead teacher, who had been in the role for two years, had a much greater influence. (Lead teacher/assistant principal/syndicate leader/teacher)
>
> I have not been in the role long enough to see this happen. (Lead teacher/syndicate leader/teacher)
>
> I am in the first year as a lead teacher. My role is beginning to develop further, and next year, I will have a significant role in numeracy in the school – as discussed during the questions. Then I feel my impact for most questions will be at the 8–9 level. (Lead teacher/teacher)

For one lead teacher, aware of the scope of the lead teacher's responsibilities, the role appeared to be significantly under-resourced:

> This is a very big role and needs the school to organise a release time component and probably [management unit] recognition. Currently it is an add-on to my regular job. (Lead teacher/assistant principal/teacher)

As stated earlier, eight lead teachers had had that role for less than two years. There are a number of possible reasons for this, including staff changes in a school or the previous lead teacher taking up a management team position. However, it is also possible that this is related to the role not always being well supported by school structures. Taking the above evidence into consideration together might suggest that the significant role of the lead teacher needs to be well resourced – in terms of both time and money – and shared with a colleague who can strengthen the impact of the role in order for lead teachers to effectively carry out their responsibilities in the longer term.

Discussion

The findings of this study should be interpreted with caution, due to characteristics of the sample that are unknown. The study indicated that the actions taken by lead teachers supported the dual goals of the NDP of ongoing teacher learning and development and ensuring a focus on student achievement. The influence of a lead teacher's actions appeared to be associated with how the role was organised in the school. A multiplicity of roles is typical for those taking a lead teacher role. This appears noteworthy in discussions about numeracy leadership in schools and needs to be considered when designing school-based professional development. The most powerful combination seemed to be to have two lead teachers, one of whom had classroom responsibilities and the other who had management responsibilities because this appeared to strengthen the influence of the lead teacher within the school. Where the lead teacher was situated within the school structures appeared to matter.

Future research could investigate the following:

- What impact does taking a multiplicity of roles have on lead teachers' influence on the development of numeracy instruction in a primary school?
- How might school size be associated with multiplicity of roles for lead teachers?
- Do lead teachers see their role as training for a career path involving other leadership positions?
- Are there differences between the level of influence of those in designated positions, and curriculum leadership roles?

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Actions Taken in Two Schools in Which Māori Students Achieve Well in Numeracy

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A focus on Māori students' achievement in numeracy is a key part of the Numeracy Development Projects. The study reported in this paper investigated the dispositional attitudes and processes underpinning two schools' focus on Māori students' achievement in English-medium settings. The study found common patterns in teacher and principal attitudes and beliefs about the nature of the student environment and relationships with the school community. Furthermore, deliberate actions to include community as well as to monitor and discuss Māori students' achievement, both individually and as a group, were central to the two schools' focus on Māori students' achievement.

Background

The Numeracy Project Assessment (NumPA) data provides teachers and instructional leaders with achievement information about groups of students as well as about individuals. In English-medium schools, this data can be used to inform the establishment of goals for the mathematics achievement of Māori students, in line with *Ka Hikitia: Managing for Success* (Ministry of Education, 2009a), which aims to improve learning outcomes for Māori students.

The research focused on leadership practices enacted in numeracy for Māori students in English-medium school settings, using the following eight dimensions of leadership identified by Robinson, Hohepa, and Lloyd (2009):

- Establishing goals and expectations
- Resourcing strategically
- Planning, co-ordinating, and evaluating teaching and the curriculum
- Promoting and participating in teacher learning and development
- Ensuring an orderly and supportive environment
- Creating educationally powerful connections
- Engaging in constructive problem talk
- Selecting, developing, and using smart tools.

Leadership is considered here to mean the set of leadership practices enacted across many roles in a school – not necessarily confined to the roles of numeracy lead teachers and senior management – and includes direct and indirect influences on teachers. The paper specifically focuses on the first dimension of establishing goals and expectations for Māori students, although other dimensions were also found to be present in the study schools.

Methodology

This case study covered two schools, with rolls of at least 30% Māori students, that were identified by facilitators as having strong achievement gains for Māori. Drawing on methods used in a study by Timperley and Phillips (2003), leaders at both schools were asked to nominate a meeting that specifically focused on achievement gains for Māori students in mathematics. Both members of the research team observed the meetings and took field notes, and the meetings were audio-recorded. Interviews with the principal, school-based mathematics leaders (senior management and numeracy lead teacher/s), and teachers were conducted after each meeting, with questions developed from the culturally responsive framework (Averill, Te Maro, Taiwhati, & Anderson, 2009). Teachers were observed teaching their students so that the researchers could identify which protocols were in use, and classroom environments were evaluated using the culturally responsive framework. In addition, photographs of the classroom and school environment and school materials relevant to a focus on Māori students' achievement were collected.

In order to increase the reliability of the study's findings, data was gathered using a range of methods. The dataset included the researchers' meeting notes, audio recordings of the meetings, transcripts of interviews, assessments of classroom environments using indicators of cultural responsiveness, and protocols in use as observed and as reported. In addition, the case study examined formal and informal processes and acts, including reporting to parents, that were responsive to Ministry of Education policy relating to Māori students' achievement, for example, national administration guidelines (Ministry of Education, n.d.), *Ka Hikitia* (Ministry of Education, 2008), professional standards (Ministry of Education, 1998), and mathematics standards (Ministry of Education, 2009b).

Research Question

This study investigated the question:

- What actions were taken in relation to goals for Māori students in schools in which they were making good progress?

Participants

Instructional leaders and teachers at two Wellington primary schools participated in the study. School A was decile 2 with 51% Māori students, and school B was a decile 4 school with 30% Māori students. Numeracy facilitators identified both schools as having strong achievement gains for Māori. At school A, two meetings were observed: one in which the numeracy faciliator led the school staff through data analysis focused on Māori students' achievement and a second meeting of four teachers in the middle syndicate (years 3–4) to discuss the achievement of Māori students in each class and the patterns across the syndicate compared with patterns of achievement across the school. At school B, six teachers from the junior syndicate were involved in the meeting. In total, interviews were held with seven teachers and two principals.

Procedures

A meeting that specifically focused on achievement gains for Māori students in mathematics – in both schools, a syndicate meeting – was nominated by school leaders. In the meetings, Māori students were identified as a group and their group's progress was compared with the cohort being examined. Individuals and suitable teaching strategies to move these students' learning were also discussed, based on prior teaching experiences with the student(s) in previous years by previous teachers and based on the Numeracy Development Projects' (NDP) identification of suitable strategies. Both members of the research team observed the meetings. Interviews were conducted after each meeting, with questions developed from the culturally responsive framework (Averill et al., 2009). Sample interview questions are included in Appendix N. Interviews were transcribed for analysis. The analysis used the culturally responsive framework (Averill et al., 2009) to support the identification of related themes, which were then coded accordingly. The report focuses on the statements made by the teachers in these two schools.

Findings

Both schools stressed their belief that, to raise Māori students' achievement in their schools, they needed to take deliberate action about goal setting and monitoring that achievement, as well as taking deliberate action to develop a culturally responsive school and classroom environment that respected te reo[1] and tikanga[2]. This section discusses each of these foci in turn.

1. Goal Setting and Monitoring Māori Students' Achievement

In both schools, there was a focus on Māori students' achievement that included the following actions:

- evidence of a collegial, mutually supportive, and comfortable dynamic
- focus on continual teacher development (for instance, sharing effective activities)
- interpretations of the Number Framework stages
- reference back to previous professional development
- setting timeframes for goals
- reflection of what worked well and why in terms of teaching strategies that worked well for their Māori students
- demonstration of in-depth understanding of the NDP (for instance, through making links to Ministry of Education targets, numeracy guidelines, and resources)
- discussion of specific identified students, including those at risk (to indicate that student achievement was seen as a shared responsibility)
- evidence in discussions of individual teachers' depth of knowledge about students, pedagogy, and the NDP.

The two schools considered themselves well placed for monitoring students' progress against the National Standards in terms of their current goal-setting and monitoring processes. Both schools considered that the principal was key to setting the school focus. As one member of senior management in school A explained:

> The principal has given us the focus of lifting Māori students' achievement … They've been the key person making those sorts of relationships [with teachers and with the community].

Similarly, at school B, the leadership was referred to as important:

> From the leadership point of view, actually focusing on Māori students' achievement is quite important, that people actually know that it's your focus. (Lead teacher, school B)

The study found that student achievement data was used as a starting point for discussion and action points for raising Māori students' achievement. In each school, school management played a key role in setting school expectations for raising Māori students' achievement through introducing practices that included:

- examining school-wide data in meetings as a regular school practice
- ensuring that the examination of data was sustained over a long period of time
- ensuring a simultaneous focus on individual students and on Māori students as a group
- organising data-focused meetings at the school, syndicate, and individual teacher level
- actively generating teacher ownership of the analysis of data.

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1 Te reo Māori: Māori language

2 Tikanga Māori: what is deemed to be the correct ways of carrying out tasks/duties

> The principal can say and write a whole lot of stuff, but [unless] the classroom teacher understands it, owns it, and takes responsibility for it, nothing will happen. … It's been hard because you were looking at the collection of data and storage of data on paper initially. It wasn't easy to actually do very much. A lot of the collection of data initially was seen as being collected for the principal because that was the perception. … So the mental move away from "We're only collecting this for the principal, who then gives it to the board of trustees", that was the definite pathway. (Principal, school A)

While tracking of individual student progress is important, it is the tracking of groups of students, such as Māori, that is seen to be the key to setting expectations for raising their achievement:

> So the two professional developments both involved considerable collection of data at particular times of the year and looking for things like progress of individual children, but I also wanted people to look at progress of groups and or lack of progress of groups. (Principal, school A)

> We've talked about looking at the achievement of all children, but specifically our Māori children, because that seems to be where in the past we've had the dips, the low levels of achievement. (Senior management, school A)

> Being one of the leaders, I can actually tell you what I do is I hold a meeting [with the teachers] and we go through the results … We target the children who are lower, and then we look to see, you know, where they stand, are they ESOL, are they Māori, are they Pasifika, and they're usually the same children who are targeted all through the school. (Senior management, school B)

Student achievement was also discussed with parents:

> Actually sitting down with parents and going through things like running records and oral responses … There are two [important things]: we want you to come and pick your time, and there is, we've got the information here for you. (Principal, school A)

> Yeah, well where the children are at and their GloSS results and their numeracy results. (Senior management, school B)

2. Creating a Culturally Responsive School Environment

Both schools actively pursued ways in which they could make their schools comfortable environments for Māori students and families/whānau. This was demonstrated, at the school level, by evidence from the research interviews that the schools actively explored ways to engage parents/whānau with students' mathematics learning, for instance, building working relationships with the local marae, and that teachers talked about their personal commitment to improving and using their knowledge of te ao Māori[3]. Both schools felt that they had been well supported by their numeracy facilitators in building these relationships.

An important part of establishing goals and expectations for Māori was engaging parents with students' mathematics learning. Both schools were proactive in organising opportunities for parents to experience the types of activities that students had in the classrooms. School A's "opening up the school communication" included "the whānau action group" and the link with the marae next door through kapa haka once a week, as well as "a big cultural day at the end of term 2" (Teacher 1, school A). The timing of the whānau action group sessions took into account parents' schedules:

> We coincided it with the time that they would be picking the kids up from school anyway, so we had it right at that time so it was convenient for them all to be able to come in and sort of stay in for a little while, and we did games. We taught them how to play numeracy games and played them with them and gave them little packs to take home as well, and we just made it parent-friendly and really easy for them to follow … and we also introduced them to the nzmaths website with the family part on it … we could show them [on the computer] how to get in there. (Teacher 1, school A)

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[3] Te ao Māori: pertaining to the Māori world/Māori viewpoint

Similarly, in school B, one teacher explained opportunities to work with parents:

> Workshop with parents, teaching the parents about numeracy and maybe showing them stuff on the computer … The parents would come in and we would provide them with dinner and look after their kids while we have the meeting and upskill them and particularly how they can help at home. (Teacher, school B)

Teachers also commented that they had made an effort to use te reo and tikanga Māori:

> I think we all try and do a bit of te reo and tikanga and probably just get more comfortable with the language and things like that ourselves. … Respecting the culture of other children, I guess seeing the language displayed around the school, connecting, like trying to use a bit of te reo Māori in whatever you are doing. (Senior management, school A)

At school B, one of the teachers talked about it as "valuing culture" and said:

> I think it makes Māori parents feel much more at home. It's a very open door policy, I think that's really important, that the parents are made to feel welcome and they come in and sit down ... I don't expect them to wait outside the classroom.

Another teacher at school B talked about "the use of language to enhance culture". She stressed that she was not talking about "supporting them by giving them extra things if you need to, but actually reflecting". She said:

> I always try and speak to [the Māori immersion teacher] in te reo in front of the children so that they see that there is another language and that it is perfectly acceptable to do that and respect it. (Teacher, school B)

Discussion and Implications

Deliberate actions to monitor and discuss student achievement data were central to both schools' focus on Māori students' achievement. By using data as a starting point for discussion in meetings, action points were identified for classes, syndicates, and the school as a whole. Alongside these action points, both schools actively explored ways to engage parents/whānau with students' mathematics learning, as well as pursuing ways in which they could make their schools more comfortable learning environments for students and parents/whānau. The teachers and principals believed that building links with the school community provided them with opportunities to discuss Māori students' achievement individually and as a group.

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Explorations of Year 8 to Year 9 Transition in Mathematics

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This paper reports on school transition systems and practices in mathematics when students move from one type of school to another between year 8 and year 9. The study used multiple case studies across five primary schools and four secondary schools in three different geographic regions. Data from questionnaires and interviews was collected from students, teachers, heads of mathematics, lead teachers in numeracy, and in-school facilitators. The data was analysed using a conceptual framework for examining transition. The first phase included 69 year 8 students. From this group, 30 students, including four gifted and talented, 12 Māori, and two Pasifika students participated in the second phase as year 9 secondary school students. Findings showed that the transition process from year 8 to 9 was not problematic for most of the targeted students. However, there was evidence of "fresh start" practices, including numeracy reassessments, and some mistrust between the sectors.

Introduction

Transition, the move from one stage of schooling and from one school to another, is an important milestone in students' education. This move from one context to another provides challenges socially, academically, and systemically. Key challenges for students moving from year 8 (full primary or intermediate school) to year 9 (secondary school) include making new friends, coping with a new school environment, differences in the organisation of the school day, multiple teachers (specialist subject teachers), and different learning and teaching approaches. International research has shown that students experience a decline in academic achievement (e.g., Anderson, Jacobs, Schramm, & Splittgerber, 2000; Galton, Morrison, & Pell, 2000) and, in particular, a negative effect on students' attitudes and motivation (Anthanasiou & Philippou, 2006).

In New Zealand, the Competent Learners @ 14 Project (Wylie, Hodgen, Ferral, et al., 2006) found no evidence that transition to secondary school negatively affects students' levels of performance. The 14 year old students in the study who were engaged in school and their learning were likely to be in positive learning environments where the pace of work was appropriate. They received constructive teacher feedback, and the work was both challenging and relevant. Further national research by Cox and Kennedy (2008) reveals that the "primary to secondary schooling transition is not the 'disaster' that is often feared" (Ministry of Education, 2008, p. 4). Their study followed a diverse group of just over 100 students from years 8 to 10 over the transition and focused on mathematics, reading, and writing (Cox & Kennedy, 2008). The data was based on Assessment Tools for Teaching and Learning (asTTle) results and included all mathematics strands. Students were assessed at four different times during years 8 and 9 and, to ensure consistency over time, the scope of the mathematics tests was relatively the same. Results showed that average student achievement in mathematics dropped, despite these students having fairly positive attitudes towards mathematics. There was also greater variability in students' mathematics scores; the gap between high- and low-achieving students widened once students were at secondary school.

The study reported on in this paper was preceded by a study of transition from year 6 to year 7 (Bicknell & Hunter, 2009). That study showed that the transition in numeracy between school levels was not smooth. There were challenges and limitations to the exchange of data and students' experiences in content, instruction, and teacher expertise (Bicknell & Hunter, 2009). Their findings are in agreement with the international literature (e.g., Anderson et al., 2000; Doig, Groves, Tytler, & Gough, 2005; Ward, 2000), which shows how different cultures, school systems, and pedagogical practices within the two sectors impede smooth transitions.

The practice of a "fresh start" approach is acknowledged as a key factor in the transition process. Secondary schools argue that they have greater specific academic expertise than the primary schools and are therefore in a better position to make decisions about students' achievement levels in a subject. They believe that records can be vague and sometimes misleading, and they prefer to use their own assessment methods and data (Galton, Morrison, & Pell, 2000). Galton and Hargreaves (2002) raise the question whether, if this is the case, curriculum continuity is taken seriously and is an achievable goal.

For gifted and talented students, fresh start practices can have an impact on their placement in specialised acceleration and enrichment programmes in mathematics. Bicknell's (2009) study provided evidence of this practice and the sense of mistrust between "sending" and "receiving" schools about gifted and talented students' levels of achievement in mathematics. The students in that study felt academically well prepared for the transition, although they found that there was a lack of mathematical challenge in their new schools. Curriculum continuity, pastoral care, and home-school communication were also recognised as issues. Appropriate programmes for gifted and talented students should incorporate both acceleration and enrichment (Ministry of Education, 2000), and students should have opportunities to participate in competitions (Riley & Karnes, 2007).

At secondary school, students face changes in the physical environment and differences in space, size, and their own position within the school. These classroom environment factors (freedom of movement, seating, and classroom display) can affect their identities as learners (Pointon, 2000). The move to secondary schools usually gives students opportunities to work in classes with students of similar abilities. Secondary schools commonly adopt streaming practices. Streaming (or within-grade level grouping by perceived ability) is seen as advantageous for high achievers and gives opportunities for students to be extended, enriched, and challenged (Forgasz, 2010).

The mathematics teacher in a secondary school is usually a specialist in their subject area, and the instructional framework is established through school-wide systems, mathematics schemes, teaching approaches, the role of homework, and the use of technologies and traditional resources, such as the textbook. These differences in instructional approaches and use of resources reflect the prevalent instructional philosophy. The changes in the teaching environment can mean a change from a student-centred to a more teacher-directed approach that requires more independent work by students (Schielack & Seeley, 2010). This need for the development of independent study and time management skills is an important part of preparedness for a smooth transition (Hawk & Hill, 2001). The Education Review Office (2006) found that there was a lack of focus on preparing students for the transition to secondary schools and that for diverse groups of students, there were "limited or no opportunities to develop awareness of their strengths and abilities" and the students were "at risk of being unprepared for the transition to secondary school" (p. 2).

Macfarlane, Glynn, Cavanagh, and Bateman (2007) argue that, for many students from non-dominant cultures (for example, in this study, the Māori and Pasifika students), the school climate does not enable them to be who and what they are while at school. These researchers and others (e.g., Bicknell & Hunter, 2009; Hawk, Cowley, Hill, & Sutherland, 2001; Hill & Hawk, 2000; Macfarlane, 2004, 2007) argue that, for Māori and Pasifika students to be in a socially and academically appropriate school setting, they need to be in a culturally responsive environment. Macfarlane and his colleagues explain that a culturally responsive environment for Māori and Pasifika students, based on the core components of rituals, relationships, and community, develops their sense of safety. Safety, they explain, "is taken to mean freedom to be who (individually) and what (collectively) we are" (2004, p. 69). In the study reported on in this paper, the researchers wanted to know how this diverse group of learners made the transition across sectors and how their cultural identity was maintained.

A framework developed by Anderson and colleagues (2000) was used to address the following research questions:

1. To what extent do the systems and structures, as they relate to the provision of mathematics education employed at a given sector, reflect the approaches taken at the following sector?
2. How do pedagogical provisions offered at consecutive sectors influence the mathematical achievement and identities of students, in general, and of Māori and Pasifika students and students identified as high achievers, in particular?

This framework consisted of three key concepts that influence school transitions. These three concepts for understanding and improving school transition are: preparedness, support, and transitional success or failure. "Preparedness is multidimensional and includes academic preparedness, independence and industriousness, conformity to adult standards, and coping mechanisms" (Bicknell & Hunter, 2009, p. 100). The concept of support relates to both tangible (for example, written information) and intangible factors. These intangible forms of support may be provided by parents, siblings, peers, and teachers. Transitional success or failure is indicated by grades, post-transitional classroom behaviour, relationships with peers, and academic orientation. This conceptual framework provided the basis for the data analysis and subsequent findings.

Research Design

The research reported in this paper was designed to examine the transition systems and structures in mathematics for students moving from year 8 to year 9. This research included a focus on assessment practices, achievement records, grouping practices, and the more specific provisions made for gifted and talented, Māori, and Pasifika students. In this study, the researchers wanted to investigate the ways in which the practices between the two sectors were similar, particularly those relating to mathematics. They also wanted to examine the communication between the sectors, the information provided about students, and the extent to which the secondary schools drew on this information. In addition, they wanted to know how the students viewed their movement across school sectors, their expectations about learning mathematics in the next sector, their preparedness for it, how successfully they managed the transition process, what changes they experienced, and how well they coped with these changes.

Several schools were invited to participate in the first phase of the research project. These schools were representative of full primary and intermediate schools in both metropolitan and provincial regions and of a range of decile ratings (see Table 1). The design of the research involved collection of data that extended across the end of one year and the first portion of the next year. The principal of each school in the study was aware that the study aimed to focus on gifted and talented, Māori, and Pasifika students. Following consultation with principals and lead teachers in the primary and intermediate schools, students from designated year 8 classes were invited to participate in the study. All of the students who agreed to participate were then included in the data collection. Four weeks prior to completion of their year 8, five groups of students (a total of 69) were asked to complete a questionnaire (Appendix O). The questionnaire was based on previous research on transition by Bicknell and Hunter (2009) and Bicknell (2009). All 69 students completed the questionnaire. Focus group interviews (Appendix P) were then completed within each school setting. At the same time, the numeracy lead teachers at each school participated in a semi-structured interview to clarify teaching practices, school policy, and communication practices between the two sectors (Appendix Q). Documents (records of achievement, mathematics programme outlines, and information used in transition) were collected from each of these schools.

The students moved in the following year to different secondary schools. Four of these schools agreed to participate in the next phase of the study. As a consequence, the researchers were then able to track 30 students. Of these 30 students, 12 were Māori, two were Pasifika, 15 were New Zealand European students, and one student classified themselves as "other" ethnicity. This sample included four students who had been identified as gifted and talented by their intermediate schools. The 30 students completed a second phase questionnaire (Appendix R) and participated in focus group interviews (Appendix S). The data collection was completed at each of the schools six to eight weeks after the students had begun their school year. A semi-structured interview also took place with the head of mathematics (HoD) and/or the in-school numeracy facilitators (Appendix T).

Table 1
Summary of Schools in the Study

School	School type	Location	Roll number (approx.)	Decile rating	Number of students in study
A	Intermediate school	urban	370	6	11
B	Intermediate school	provincial	330	2	22
C	Full primary	urban	175	1	7
D	Full primary	urban	510	3	5
E	Full primary	metropolitan	460	3	24
F	Boys' secondary school	urban	1650	9	3
G	Co-educational secondary	urban	750	5	2
H	Co-educational secondary	urban	1370	6	12
I	Co-educational secondary	provincial	670	3	13

Results

The following results are based on Anderson et al.'s (2000) conceptual framework and its three major concepts: preparedness, support, and transitional success or failure. These findings address the following themes:

- students' preparedness and expectations in relation to the transition (these include systemic and academic)
- students' expectations for year 9
- experiences in mathematics at secondary school
- perceived similarities and differences in mathematics between sectors
- numeracy practices
- support for students in the transition.

Preparedness for Transition and Student Expectations

The following section addresses the systemic and academic preparedness for the transition from year 8 to year 9. These results are based on multiple sources of data, which include school documentation, interviews with teachers and students, and student questionnaires.

Systemic preparation

Preparation for student transition across the five schools in the study varied. At school E, a group of year 9 students came to talk to the year 8 students, whereas school C students were visited by representatives from each of the local secondary schools. This was for information and opportunities to ask questions. All of the year 8 students had an opportunity to visit the secondary schools for an orientation visit. One of the secondary schools (school H) arranged student and parent visits for individual interviews to share personal information about student interests. The students concerned commented that this reassured them and orientated them as to what would happen on their first day at secondary school. On enrolment, all students in the five schools were provided with organisational information in a school prospectus.

The primary and intermediate schools completed an information form designed by each of the secondary schools, except for one school (school B), which completed a common form that had been designed collaboratively by the intermediate and secondary schools in their region. The format of the forms varied for every school, but the common elements were current reading age, asTTle data, and Numeracy Development Projects (NDP) data such as the Numeracy Project Assessment (NumPA), Individual Knowledge Assessment for Numeracy (IKAN), and Global Strategy Stage (GloSS). This academic information was supplemented with social and behavioural comments. There was no provision made for assessment data relating to specific learning domains in mathematics such as statistics, measurement, and geometry. Schools requested placement recommendations, in particular for those who were identified as gifted and talented or special needs students. For the special needs students, there was a formal transition process through Ministry of Education, Special Education. The lead teachers and heads of department shared differing views about the value of this shared information. Two schools felt that there was an acceptance that the information they provided would not be used by the secondary schools. The numeracy lead teacher at one school commented:

> We fill in the forms that the high schools give us and we meet with them, and they really only want to talk about the behaviour of the children. They don't really want to talk about learning. (Lead teacher, school C)

One intermediate school had worked with the two secondary schools in its region to improve systems for the sharing of transition data. The secondary schools previously administered an exam to obtain their own data (including mathematics) for student placement in year 9, whereas now the schools had a more collaborative approach:

> They are more trusting of the information we give them. They see our information as valuable; they can see benefits of numeracy data. (Lead teacher, school B)

In the other secondary schools, there was an awareness of a sense of mistrust over the reliability of data provided about the year 8 students' achievement levels in mathematics. They too were trying to address this by working collaboratively on the reliability of data:

> Rather than standardising the information that is coming from the contributing schools, what we are doing is we are standardising it at the time of receipt of the kids. And perhaps the better thing would be to standardise the information that was coming from the contributing schools, but maybe that is too hard. (HoD, school G)

After the completion of the transition data forms by the year 8 schools, members of the secondary senior management team visited the primary and intermediate schools. The purpose of this was to primarily talk about targeted students. The targeted students were viewed as those who were at opposite ends of the learning spectrum and those with behavioural challenges.

Mathematics testing was used by most of the secondary schools for student placement. Students sat the tests at the secondary schools either during their visit in term 4 or at the beginning of the next school year. Four schools used the University of Canterbury's Centre for Evaluation and Monitoring (CEM) entrance test for year 9 students. This mathematics test covers a variety of basic numeracy skills, with emphasis on number, measurement, and simple logic. These schools used this CEM test as a common assessment tool to improve the transition from year 8 to 9. The HoDs explained that they felt that, previously, there were limitations to the usefulness of the data when they had used only Progressive Achievement Tests (PAT) in mathematics and asTTle for placement decisions. There was some retesting by the secondary schools of the number knowledge and strategies using IKAN and GloSS. Justification provided by the secondary schools for reassessing students was to address gaps in the data and to give them confidence in the results. One in-school facilitator commented on the benefits of the year 9 teacher conducting their own diagnostic interviews:

> One of the benefits of the diagnostic testing of course is that the teachers really get an insight and that's what our teachers are positive about … really getting to know the students in that short 15 minute thing and how they are thinking about maths; so that was a real advantage. So if you just had that data coming from another school, that wouldn't give you that insight. (In-school facilitator, school H)

Academic preparation

Four out of five of the phase one schools cross-grouped for mathematics based on the numeracy stages (Ministry of Education, 2007). At the intermediate schools, this meant that classes had a mix of year 7 and 8 students who were working at the same stage. Gifted and talented students in two schools were grouped together so that they were working at an advanced level in mathematics. No special provisions were made in any of the schools for the Māori or Pasifika students, although they were identified on school assessment data systems.

The year 8 teachers outlined in interviews that they focused predominantly on developing students' abilities and confidence in number. Two schools purposefully gave the other strands additional emphasis because of concerns about sending students to secondary school with gaps in their understanding of mathematics.

Year 8 students were given a questionnaire (Appendix O) that required them to rate on a scale (1–4) the importance of various factors that could contribute to their preparation for success at year 9. According to the students' ratings, the most important way in which they had been prepared for success in year 9 was by completing work at a harder level. Six Māori students described how they believed that this was "year 9 and year 10 level mathematics". Two gifted and talented students outlined how their teachers prepared them by having them complete NCEA level 1 tasks.

The focus rated by the students as the next most important for preparation for their secondary transition was on the development of numeracy strategies and mastery of basic facts. In third place was a perception from a number of students that they had received no specific preparation in year 8 mathematics classrooms for year 9. Five students acknowledged that family support also contributed to the transition preparedness. Other aspects of preparation were working at a harder or higher level, a focus on homework, independent skills, and time management.

One teacher was concerned that she did not prepare students well for mathematics at year 9 because of gaps in her own content knowledge. This weakness of teacher content knowledge was also identified by a HoD at a secondary school, who explained that she believed that an intermediate teacher may limit or cap where the students can get to because of their limited mathematical knowledge. According to this HoD, secondary school teachers, on the other hand, are able to make links for the junior students to higher level secondary mathematics.

In their questionnaire (see Appendix O), the year 8 students overwhelmingly recorded that the most important categories (rated by students as "extremely important" or "very important") that supported their mathematics was learning from their mistakes in mathematics (97%), learning from the mistakes of others (88%), being able to ask for help (82%), and knowing their basic facts (80%).

The students expected there to be some differences in the mathematics learning and teaching at their new school, but the majority of them believed that the mathematics content would be the same. For example, they specifically mentioned fractions, basic facts, multiplication, division, decimals, and measurement. However, they believed that the work would be much more challenging and harder to understand. They expected a greater use of textbooks, more bookwork, and increased homework. By being in a streamed class, they expected to be taught as a whole class and therefore "they would advance and excel more quickly". The students expected that the teachers would teach them differently and that there would be less group work. The students also commented on aspects of the physical arrangement of the mathematics classroom, such as seating, where they expected to be in rows or possibly groups.

For all the gifted and talented year 8 students (n = 8), the most important aspects of preparation for success at year 9 included working alone, knowing your basic facts, and learning from your own mistakes and those of others. The next most important aspects, according to most of these students, were working in a group, sharing ideas in a large group, explaining your strategy solutions, and being able to ask for help from the teacher. At least 85% of the year 8 Māori students (n = 29) reported that the most important aspects for preparing for success at secondary school included knowing your basic facts, using a calculator, explaining your strategy solutions, learning from your own mistakes, and being able to ask for help in mathematics.

Year 9 Experiences

Of the 30 year 9 students followed in this study, 90% reported that they predominantly worked alone in their mathematics classes. (The year 9 student questionnaire, Appendix R, which the year 9 students completed six to eight weeks into their year 9 experience, used the same or parallel items as the year 8 questionnaire, Appendix O.) These students indicated that there was a teacher expectation that they work independently.

> We are in a group and we are allowed to talk, but it's pretty much individual work. (Student, school H)

Most of the students (83%) worked predominantly from worksheets or textbooks. Although they felt that they had opportunities to learn from their own mistakes, fewer of them (60%) reported having occasion to learn from the mistakes of others. Most students (73%) felt that they could ask for help from the class teacher but that there was a difference from the interactions they had experienced previously with their year 8 teachers. Instead of the teacher initiating the interactions, five students commented that they felt they had to now raise their hands to receive individual support.

> You probably try not to rely on the teacher teaching you individually as much. (Student, school H)

Most of the students (67%) also confirmed that they were able to share ideas in a large group and to explain their strategy solutions. However, for some (33%), this did not include convincing others about their mathematical thinking.

The majority of students (83%) experienced working in a group with other students only some of the time or occasionally. Basic facts was deemed to be a focus for 60% of the students. Learning using games and activities, writing their own word problems, and taking part in competitions received less attention. Working with the teacher for some or most of the time was reported by only a few students. Most students (87%) had minimal or no use of the calculator in their year 9 programmes at this stage of the year. (One school reported that they refrained from using calculators until year 10.) Homework was a feature of the mathematics programmes for all students in the study. This was not perceived as a negative feature, although it was an aspect that some students felt ill-prepared for as part of transition process.

The gifted and talented students in the study were placed in streamed year 9 classes, based on the secondary schools' assessment data and in one case on the year 8 data. This meant that they were working with like-minded peers. Some of these students felt that the work was harder and included some new topics and the regular use of textbooks. Only one school used a planned approach for acceleration, while the other secondary schools preferred to use enrichment experiences. This practice of not accelerating students was justified on the belief that students should be encouraged to engage fully in school life:

> They get a window of time to get into the cultural and social and sporting aspects of school. (HoD, school H)

Three of the secondary schools provided these students with opportunities to participate in local, national, and international mathematics competitions. The gifted and talented students believed that competitions were an important part of their mathematics programme.

There were no special provisions made for the Māori and Pasifika students to support their transition to the secondary school. However, one school's strategy that was well received by the Māori study students in this school was the use of whānau grouping. This vertical grouping included year 9–13 students. This system allowed students to make connections across year levels and to develop relationships with others. This included a whānau teacher who remained with that group through their time at the school. The HoDs from two schools described the importance of establishing good relationships with students, identifying individual needs, and showing an ethic of care. One HoD explained that "if you can buy their trust, you can get them on board" (school I). The teachers believed that what was good for Māori students could only benefit all students. However, one of the Pasifika students still felt discomfort in speaking in front of the class and felt "a little bit shame".

Similarities and Differences in Mathematics

The year 9 students reported that the main similarities between year 8 and year 9 mathematics were with respect to the content, specifically the number content (the operations), and the use of games. There were, however, several aspects to the mathematics learning and teaching that were different. These were the use of textbooks and worksheets. The classwork was perceived to be easier at times, but the homework was much harder. The students commented on the regular change of topics and that there was a greater variety in the topics that were taught. The students recognised the influence of streaming; this enabled them to work with more students at a similar level. This provided a contrast for the students from regular year 8 (or mixed year 7 and year 8) classes, although some of the students had come from cross-class ability groups or a gifted and talented class. For some students, there was a perception that work from the previous year (or years) was being repeated; as a result, for these students, there was a sense of boredom.

The students recognised that they were being taught by a teacher who was a specialist in mathematics, unlike their year 8 teacher, who taught across multiple subjects. Instead of having primarily one teacher, they now had several teachers, one in each subject. This was perceived as a positive difference because "the secondary teacher explains things better than the intermediate teacher" and "there's more help here; the teacher explains it properly". Contrary to this positive view was a perception that the year 9 teachers engaged less with the students in class than their teachers in year 8. One student's observation was that "the teacher sits at the desk, walks around, then goes back to the desk".

The use of technology was not prevalent in any of the secondary schools for these students at this stage of year 9. The students had expected greater use of technology, in particular the scientific calculator. However, the students found that they were not used in year 9. (The teachers reported that they were deliberately not using these in year 9 because of their focus on mental strategies and the influence of the Secondary Numeracy Project [SNP], part of the NDP.)

Numeracy Practices

All of the secondary schools had had some experience of the SNP. This ranged from one school that was in a pilot study six years ago to a school that was in its first year in the SNP. The in-school numeracy facilitators felt that the SNP had influenced aspects related to transition practices and year 9 students' knowledge and skills base in mathematics. It also influenced the year 9 mathematics programmes (schemes) in the secondary schools.

The secondary schools recognised an urgent need for the standardising of information between schools. In one cluster of schools, the approach was to standardise the sending data, and in another area, they were attempting to standardise at the time of receipt.

> The better thing would be to standardise the information that was coming from the contributing schools, but maybe that's too hard. (HoD, school G)

Numeracy data was provided by the year 8 schools for the secondary schools, but this varied from GloSS results to knowledge stages. Only one of the secondary schools used this numeracy data from the intermediate school. The other schools carried out their own assessments (as previously outlined) but, specifically in numeracy, there was a range of assessment practices. The school new to the SNP carried out full diagnostic interviews with all students. They recognised that this was going to be a one-off situation because of resourcing issues. The other schools selected tools such as the IKAN test and GloSS; in some cases, this was only for selected students, such as those requiring learning support. The information gained from numeracy assessment tools was recognised as providing only one piece of information towards helping the placement of students. The secondary schools recommended that the numeracy assessments be more rigorously applied at year 8 so that results would be perceived to be accurate. The other issue shared by all schools was that accurate data be accessible through an electronic student-record system; this would avoid the physical input of data by the secondary school.

Support for Students

Only a few students acknowledged the support that they had anticipated or received from others, such as friends, siblings, parents, or teachers. There was acknowledgment that it helped to have friends move with you to your new school, but many of the students in this study made the transition with none of their friends from year 8. At their secondary school, they realised that they had little chance of being in the same class as their friends because of school size. One student explained that she caught up with friends by texting during breaks and arranging somewhere to meet. This was an acceptable and allowable practice in the school outside of class time. Generally, the students felt that they were well informed about the secondary school and so additional support was not sought to smooth the transition. However, the students recognised that teachers were an important aspect of the support process. They were aware that they would no longer have the one class teacher to turn to for support, but through school systems, a form teacher or vertical class teacher would provide that support.

Discussion and Conclusions

The transition from year 8 to year 9, when students change school sectors, is a significant event in students' educational lives. Its success involves careful preparation on the part of management staff, classroom teachers, parents, and the students themselves. Although the year 8 students in this study held some reservations about their shift to the secondary school, they generally viewed this next stage in their educational life positively.

There was evidence of a range of orientation practices used to provide support and information for students moving across school sectors. The variety of practices included school visits to the receiving schools and previous students returning to their sending school to share information with the year 8 students. Across the cohort of all students, the process of enrolment appeared to be an important part of orientation. This process involved visits to the secondary schools, and the students were also provided with written documentation about core school organisation, practices, and policies. Anthanasiou and Philippou (2006) suggest transition across school sectors may lead to a loss of student confidence. In this study, the year 8 students did not appear to be overly concerned about the shift. Those students in schools that specifically sought personal contact and sharing of individual information with their new enrolments appeared particularly reassured and reasonably confident about what would happen on their first day at secondary school.

The year 8 students considered that their current teacher's various preparatory strategies, including making them work at a harder level, complete more homework, be more independent, develop specific numeracy strategies, and master basic facts, positively prepared them for the shift. Interestingly, they also identified other attributes that are most commonly associated with a mathematical inquiry community and its discourse (Hunter, 2009), including help seeking and learning from their own mistakes and those of others – attributes that suggest a positive sense of mathematical authority and identity (Esmonde, 2009).

After the move to year 9, the students reported differences in the classroom environment and culture, particularly within classroom interactions. The change in communication patterns as well as physical space, seating arrangements, lack of familiar peers, and the predominance of the use of textbooks or worksheets placed the teacher in a position of mathematical authority. In year 9, the students reported that they had to indicate individual need for help from the teacher and, while they had some opportunities to share their ideas with the class, they were not required to fulfill key inquiry practices such as convincing others. As Pointon (2000) noted previously, these changes in the classroom practices can and do affect student identity as learners. The researchers in this study suggest that because the secondary schools in the study placed little focus on developing a culturally responsive learning environment for their Māori and Pasifika students, these students' cultural sense of identity was at risk, as suggested by Macfarlane and his colleagues (2007). However, four Māori students were in the school that used whānau grouping as a strategy for strengthening students' sense of belonging.

While in year 8, the students anticipated that in year 9 there would be consistency in the mathematical content they studied, and they were correct. At the time of this research, the year 9 students continued to experience a focus on the teaching of basic number content – basic facts, number operations, and place value. Some students noted that they were repeating what they had already been taught earlier. This provides evidence of what is called "a fresh start approach" (Galton & Hargreaves, 2002). In this situation, its use suggests that the year 9 teachers began again with teaching basic content because they did not trust that the students had covered it adequately in their previous school. Bicknell and Hunter (2009) also reported the use of a fresh start approach in their research on transition practices from year 6 to year 7. Other aspects of the evidence supporting fresh start assessment practices will be discussed in the next section.

Evidence is provided in this study of a consistent approach used by the urban secondary schools to develop cohesive systems and structures for transferring relevant data from the year 8 schools. A collaborative approach had been adopted by the provincial year 8 and 9 schools to ensure consistency of data to support students' transition. All the schools' involvement with the NDP and their consistent use of a variety of NDP assessment tools (NumPA, GloSS, and IKAN) had the potential to support successful transition of students from one system to the next. However, as found in the previous New Zealand study on transition (Bicknell & Hunter, 2009), the use of fresh start (Galton & Hargreaves, 2002) interrupted the process unless there were procedures in place to coordinate a collaborative approach. There was evidence of specific collaborative discussions between two intermediate schools and two secondary schools, which focused on the development of a form that used commonly understood assessment data. This was a concerted effort towards a seamless transition. However, despite this, all but one of the secondary schools retested students in mathematics and all but one reported that they had their own testing agenda so that they could place students in streamed classes based on what they perceived to be more reliable and informative data.

All the students identified as gifted and talented in their year 8 schools were placed in top streamed classes in year 9. Only one secondary school used acceleration as a way to meet the needs of their gifted and talented students; the others preferred to use enrichment. However, all those in this specific group of students reported differences in both mathematical content and instruction, and they all reported positively about their opportunities to participate in competitions. Having a specialist mathematics teacher was also valued. Although the Ministry of Education (2000) advocates the use of both acceleration and enrichment for this specific group of students, the schools in this study primarily chose to use one or the other rather than a planned approach to use both strategies.

All in all, as other researchers in New Zealand have suggested (for example, Ministry of Education, 2008; Wylie et al., 2006), the students in this study made the transition across the two sectors without specific problems. However, the researchers recognise that this study is limited by the small sample size and the data was only collected within an early period of the students' secondary experience. Nevertheless, it does provide a picture of what transition might mean for a group of year 8 students, including gifted and talented and Māori and Pasifika students. The researchers reiterate, as they did previously (Bicknell & Hunter, 2009), a need for longitudinal research that follows a larger sample of targeted learners in mathematics across three-tiered transitions, to explore whether, why, and how the students experience the dips in achievement described by Anderson and colleagues (2000). Close examination over a longer period is also needed to see how the collaborative processes enacted in the schools in this study support systemic processes of transition. Further research is also required that focuses specifically on how Māori and Pasifika students can transition and maintain a sense of cultural self as mathematical learners.

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Written and Oral Assessments of Intermediate Students' Number Strategies: Possible Uses for a Written Assessment Tool

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This paper examines the trial with year 7 and 8 intermediate students of a numeracy-focused Written Strategy Stage Assessment Tool (WSSAT) developed for year 9 and 10 secondary students. The trial was carried out in early term 3, 2009. As a comparison measure, a numeracy expert interviewed a sample of the students to identify each student's strategy stage, using GLoSS (Global Strategy Stage) oral-type protocols. Results from the written assessment gave relatively consistent measures of stages. Comparison of the written and oral assessment results showed a greater degree of variation for individuals than in similar assessments in previous years. The overall results of this study suggest that the WSSAT is appropriate for use with year 7 and 8 students for formative and diagnostic purposes. In addition, there is potential for results to assist schools in identifying school-wide trends and possible professional development needs with respect to numeracy teaching.

Background

The Written Strategy Stage Assessment Tool (WSSAT) was developed in response to the extension of the Numeracy Development Projects (NDP) into secondary schools. There was a perceived need for a "more efficient", specifically numeracy-focused assessment tool to replace the NDP oral assessment tool, NumPA, that would allow the initial assessment of a whole class of students at the same time. The WSSAT efficiency "gains" are in reduced teacher time taken per student, the provision of a record of students' work, and the provision of a standardised "marking" schedule (Lomas & Hughes, 2009). However, the WSSAT was always intended to be used alongside teacher supplementary questioning and judgments and not as a replacement for them.

The specific drivers for exploring the use of the WSSAT in years 7 and 8 were whether its potential for capturing diagnostic data could provide useful formative assessment information for intermediate-level teachers and schools to use in planning for students' learning and for school-wide teacher professional development.

The development of a written tool was based on the view that year 9 and 10 (secondary school) students' reading levels would be sufficient to minimise possible language difficulties. Even so, the difficulty level of the language used was carefully monitored. A key issue in its potential usefulness at intermediate level was whether students' reading levels would be sufficient. Thus, trialling the WSSAT in an intermediate-school environment allowed for a check on whether the language used in the questions was accessible to year 7 and 8 students – a prerequisite for any general use with these year groups.

For many students, intermediate-school level is the first time that they meet NDP stage 7 and 8 material on percentages, ratio, and proportion. Those involved in the provision and evaluation of the NDP (Ministry officials, the developers of the Number Framework, numeracy facilitators, and researchers [e.g., Tagg & Thomas (2007b), Thomas, Tagg, & Ward (2003), Young-Loveridge (2008)]) acknowledge that the stages of the strategy domains are not equal in size, with stages 7 and 8 in particular being harder to achieve because of the more challenging content and the time needed to master that content.

The Written Strategy Stage Assessment Tool

The WSSAT (Lomas & Hughes, 2008, 2009) identifies a global strategy stage just as the NDP's Global Strategy Stage tool [GloSS] does, but it also contains knowledge-focused items. The questions used in WSSAT consist of single-number answers and multi-choice items that draw upon the approaches used in the NDP assessment tools and in research findings in related areas (Lomas & Hughes, 2008, 2009).

The focus of the WSSAT is primarily strategy stage identification, with the first seven items at each stage assessing a range of strategies and further items assessing some knowledge aspects (see Table 1). The knowledge items are a mixture of prerequisite knowledge for, and knowledge directly related to, each stage, purely to assist teachers in diagnosing individual students' needs. As with the NDP oral assessments, the highest strategy stage achieved (4 or more of the 7 strategy items correct) is taken as that student's stage. Where the students do not meet the criteria for stage 5 or higher, they are assigned to a category combining the "counting" stages 1–4, even though no evidence on their counting strategies was gathered.

Table 1
The Number of Items per Strategy Stage and Criteria for Achieving Each Stage

	Strategy Stages				
	1–4	5	6	7	8
Total number of items (including seven strategy items)	11	11	12	11	10
Number of knowledge items	4	4	5	4	3
Number of correct items to assign to a strategy stage	< 4	≥ 4	≥ 4	≥ 4	≥ 4

Note: The items for stages 1–4 are the same items as for stage 5.

The WSSAT relies on the written answers when assigning strategy stages, that is, on outcomes alone without knowing the process used. In contrast, the NDP oral assessments rely on both outcomes and students talking through the process when assigning strategy stages (Lomas & Hughes, 2008, 2009).

The range and types of items isolate and encapsulate some of the conceptual aspects and elements of strategy relating to a particular stage as per the domains of the strategy section of the NDP's Number Framework. In addition, the design of the items "force" the students to use a particular targeted strategy or process and attempt to restrict their use of procedural approaches or less sophisticated strategies (Lomas & Hughes, 2009).

The written format allows a student to attempt all the items. Any that the student gets correct demonstrate knowledge that the student might have beyond the point where an oral assessment would stop. Thus, the sets of items provide a more detailed (but standardised) diagnostic map of each student's achievements and learning needs than an oral assessment would do.

Method

Participants

The WSSAT was given to the year 7 and 8 cohorts of a large urban decile 3 intermediate school of mixed ethnic composition (see Table 2). A parallel oral assessment interview was conducted with a sample from the year 7 cohort. Due to the site-specific nature of the data collected, this research is a form of case study. Thus, the data is unlikely to match the national data sets closely.

Table 2
The Percentage Ethnic Make-up of the Year 7 and 8 Cohorts Included in This Study

	NZ European	Māori	Pasifika	Other
Year 7 (n = 406)	6	17	32	45
Year 8 (n = 389)	7	12	35	47

The percentage of "other" is much higher in this study (45% year 7 and 47% year 8) than in other schools where the use of WSSAT has been examined (for example, 15%). Indian students (about 30% of the student population) are the largest group in the "other" category in this study.

The intermediate school in this study was involved with the NDP, which was facilitated on-site over a number of years. In 2006, the focus was on year 8 teachers, and in 2007, on both the year 7 and 8 teachers. In 2008, after a significant staff turnover, there were three focus areas: new teachers, year 7 teachers for their second year, and "expert" teachers with two years' experience of NDP facilitation. This was followed in 2009 by a focus on: new teachers, who attended an off-site programme; lead teacher development (both on- and off-site); teachers upgrading mathematics knowledge; and attempts to meet a group of identified teacher-specific needs.

The classes in the school were not banded in any way, with students being randomly distributed into classes at each year level. These are labelled in this paper using alphanumeric codes and arranged according to data features that became evident. However, due to student numbers, there was one composite year 7–8 class (reported separately by year level as 7A and 8A in Table 3). The school used a standard Assessment Tools for Teaching and Learning (asTTle) assessment across all classes for each year group for mathematics and language. As a consequence, the students were used to written tests and multi-choice question formats; no language or understanding issues with WSSAT were reported to the researchers.

The WSSAT participants were the year 7 and 8 cohorts who were present on the day of the assessment, while the oral participants were a sample (81 students) of the year 7 cohort drawn from three classes nominated by the school (see Table 3).

Table 3
Classes Showing the Number of Students Taking the Written and Oral Assessments

	Class Name														
Year 7	7A	7B	7C	7D	7E	7F	7G	7H	7I	7J	7K	7L	7M	7M	Total
Written ass. no.	15	27	28	29	28	31	33	31	32	32	30	31	27	32	406
Oral ass. no.	–	–	–	–	–	–	28	–	32	–	–	–	–	21	81
Year 8	8A	8B	8C	8D	8E	8F	8G	8H	8I	8J	8K	8L	8M	8N	Total
Written ass. no.	10	29	28	25	33	30	29	32	28	27	30	16	30	30	389

Data Collection

The WSSAT was conducted in each class's usual classroom, under the supervision of their class teacher, early in term 3, 2009. Standardised instructions were distributed to the teachers explaining how they were to conduct the assessments (Lomas & Hughes, 2008). However, the length of time allowed by the teachers varied because the intention was to ensure that every student had as much time as they required to attempt all the items. This may have allowed more procedural (written algorithm) approaches to be used by some students in some classes. All the answer sheets were marked by one of the research team to ensure consistency, and copies of the marked answer sheets were returned to the classroom teachers for potential diagnostic and formative use.

GLoSS-type oral assessment interviews (Lomas & Hughes, 2009) were conducted by an experienced external interviewer, with expert knowledge of NDP, in the days immediately following the written assessment. The oral assessment research tool (Lomas & Hughes, 2009) was an expanded form of the GLoSS. It used some of the GLoSS- and NumPA-type items (and procedures), supplemented by other items that gave increased coverage of higher stages.

Analysis

The focus in this paper is on strategy stage assignments and how these may be used for possible development purposes. The knowledge items, whose primary function is to provide teachers with diagnostic data, are therefore not reported on.

The results of the written assessments were first analysed for the internal consistency of the tool in identifying a student's stage, that is, whether a student assigned as being at stage 7 had also been assigned as being at stage 6, and so on. Secondly, the results of the written and oral assessments were compared to explore the relationship between these two forms of assessment. The oral assessment was taken as the more accurate and used as the baseline for the comparison due to its alignment with national data collection methods and the expertise of the interviewer (Lomas & Hughes, 2009). Thirdly, the data was analysed against the national year 7 and 8 stage distribution data (the Ministry of Education's numeracy expectations), based on nationally collected data from a set of schools involved in the NDP (Thomas & Tagg, 2007a; Thomas & Tagg, 2007b). The expectations give measures of global, rather than domain-specific, strategy stages, just as the WSSAT does. Finally, an analysis of the data by class was conducted for any variations between groups of classes.

Results and Comments

The Data

The data for each of the cohorts (years 7 and 8) was analysed separately, to allow direct comparison with the year 7 and 8 numeracy expectations data, and is presented in year order.

Internal consistency

Of the 57 year 7 students assigned as being at stage 6 based on the written assessment only, one student had also not been assigned as being at stage 5 because they missed achieving the stage 5 criteria by one, that is, they correctly answered only three out of seven strategy items rather than four out of seven, which would have achieved the stage criteria.

Of the 94 year 7 students who could be assigned as being at stage 7, 35 (about one-third) had not achieved the criteria for assigning stage 6. Of these 35, 18 (about half) had missed achieving the stage 6 criteria by one, but all achieved the stage 5 criteria. Of the remaining 17, 14 had achieved the stage 5 criteria, with only three students not achieving the criteria for both stages 6 and 5.

Of the 31 year 7 students assigned as being at stage 8, 17 (about half) had not achieved the criteria for stage 7 and one student who met the stage 8 and 7 criteria had missed achieving the criteria for stage 6 by one. Of the 17 not achieving the criteria for stage 7, two (about one-tenth) had missed doing so by one and these two plus four others achieved the criteria for stage 6. Of the remaining 11, eight had achieved the stage 5 criteria, with only three students not achieving the criteria for both stages 6 and 5.

Of the 70 year 8 students assigned as being at stage 6, only three had not been assigned as being at stage 5. These three had only just achieved the stage 6 criteria (four correct out of the seven strategy questions) and all three missed achieving the stage 5 criteria by two (only two correct out of seven).

Of the 141 year 8 students assigned as being at stage 7, 27 (about one-fifth) had not achieved the criteria for stage 6. Of these 27, 12 (about half) had missed the stage 6 criteria by one, 26 had achieved the stage 5 criteria, and one student missed both the stage 6 and 5 criteria by one.

Of the 30 year 8 students assigned as being at stage 8, nine (about one-third) had not achieved the criteria for stage 7. Of these nine, two (about one-fifth) had not achieved the criteria for stage 7 by one, with one student achieving both the stage 6 and 5 criteria and the other achieving neither the stage 6 or 5 criteria. Of the seven remaining students, two achieved stage 6, four stage 5, and one the stage 1–4 criteria.

The assignment of stages 5 and 6 is highly consistent across both year groups. However, there is less consistency for stages 7 and 8, with the proportion of students missing the previous stage criteria by one being higher than for the other stages and higher for stage 7 than for stage 8 (for example, "about half" to "about one-tenth" for the year 7 cohort and "about half" to "about one-fifth" for the year 8 cohort). In addition, proportionally fewer year 8 students failed to achieve previous stages criteria at both stage 7 and stage 8: for the year 8 stage 8 students, only about one-third had not achieved the stage 7 criteria compared with about a half for year 7, and for the year 8 stage 7 students, only about one-fifth had not achieved the stage 6 criteria compared with about one-third for year 7. Thus the greatest level of inconsistency is in the stage 7 (curriculum level 4) assigned stages for both year groups.

This data is broadly similar to the previous use of the WSSAT in low-decile secondary environments (Lomas & Hughes, 2009). This suggests that the WSSAT is largely internally consistent in assigning stages for intermediate-age students, except at stages 7 and 8, where a greater level of variation was evident. However, the missing of multiple stages, as described here for students assigned stages 7 and 8, was only minimally evident in previous uses of the WSSAT. Also, the proportion of students who missed achieving an earlier stage (but only missing by one) when assigned a later stage was much higher than in previous data sets.

Comment: Given the development of WSSAT and its trialling in low-decile schools that were implementing NDP-type programmes, school environments in which written tests were the norm, and there being no reported language issues with the WSSAT, the cause of these anomalous results is uncertain. The WSSAT gave more consistent results in other apparently similar environments, so there may be particular school or teaching factors that might help explain these results.

To explore this further, an examination of the responses to items for students who had missed stages was conducted. There was no clear pattern to the responses for these students. For example, there were 17 year 7 students who were assigned stage 8 but missed earlier stages. Of the stage 8 strategy items, two dealing with percentages (curriculum level 4 material) were answered correctly by most of the students, three other items by about half the students, and the single item on fractions by only one student. The two percentage items could have been approached using algorithmic methods, while the fraction item, "Work out 5 sixtieths of three thousand six hundred", was less amenable to such an approach. Of the stage 7 strategy items, the item with the most correct responses (ten) was on factors, the next (nine) on multiplication (98 x 5), while the rest had only a few correct responses, with the two fraction items, "Work out 5 sixths of 42" and "On a number line, $\frac{48}{7}$ lies between which two numbers?", having the least number of correct responses. Apart from some items having more correct responses than others, there was no clear pattern as to why this group of students had the same items correct or incorrect. The apparent understanding of percentage strategies alongside a lack of understanding of fraction strategies suggests a possible classroom focus on certain content (curriculum level 4) and approaches (algorithms) rather than a NDP focus of working from what students understand.

Comparison between oral and written assessments

There was considerable variation between the stages assigned to individuals (see Table 4), with the WSSAT-assigned stages varying considerably, by up to three stages in a small number of cases. This variation was greater than in previous studies, where it was limited to single-stage variation (Lomas & Hughes, 2009). For example, half of the 14 students assigned stages 1–4 on the oral assessment were assigned higher stages on the WSSAT: five stage 5s, one stage 6, and one stage 7.

Table 4
Comparison of Stages Assigned to Year 7 Students by the WSSAT and the Oral Assessment

	Oral Assessment Stages				
WSSAT stages	**1–4**	**5**	**6**	**7**	**8**
1–4 (n = 12)	7	5	–	–	–
5 (n = 35)	5	19	8	3	–
6 (n = 9)	1	1	3	4	–
7 (n = 17)	1	6	4	6	–
8 (n = 8)	–	3	2	3	–
Number of students (n = 81)	14	34	17	16	0
Percentage of students	17	42	21	20	0

In addition to the variation in assigning stages, the WSSAT assigned stage 8, while the oral did not.

Comment: The greater level of variation between the oral- and WSSAT-assigned stages evident in this study raises the issue of whether the use of the highest stage criteria achieved to assign a stage with the WSSAT needs to be reconsidered. Where the stage assigned has earlier stage criteria not achieved by more than one, particularly where this is combined with low levels of knowledge, an oral check or the use of other assessment data may be necessary to assign a stage. This may also be necessary when the teacher suspects that the WSSAT-assigned stage does not reflect the student's stage. The format of the answer sheet and mark recording allows an oral check of specific strategies or knowledge to be carried out relatively easily. Thus, for the small number of students in a class likely to be in this situation, an oral checking process should not be overly time-consuming for the teacher.

Comparison with numeracy expectations

The assigning of stages from the WSSAT gave rise to reasonably similar "bimodal" distributions, with relatively higher levels of stage 5s and 7s for both year groups. There is a shift to higher levels evident in the year 8 data as would be expected, although with little change in the percentage of stage 8s (see Table 5). The overall year 8 WSSAT stage distribution is consistent with the year 7 distribution: as students progress from stages 1–4 to stage 5, from 5 to 6, and so on, the percentage of earlier stages reduces and that of higher stages increases. However, because of the greater complexity of both stages 7 and 8, the growth in stage acquisition is usually slower than for earlier stages. Over the course of one year, more students will move into stage 7 than the number who will move up from it. Thus the size of the increase in the stage 7s' percentage from year 7 to 8 is larger than might be expected.

Comment: The NDP philosophy and Number Framework promote a sequential development around both knowledge and strategy acquisition, which implies a more evenly spread distribution rather than a bimodal distribution. Thus the higher percentages of stage 7s compared with stage 6s for both year groups and the increase in percentage of stages 7s from year 7 to 8 (23% to 36%) appears anomalous. This may be a function of the way the WSSAT assigned stages for year 7 and 8 groups, and/or there may be particular school or teaching factors that might help explain these results.

Table 5
The Percentage of Stages Assigned to Year 7 and 8 Students by the WSSAT

	Stages				
	1–4	5	6	7	8
Year 7 cohort stages: Percentage of students (n = 406)	14	41	14	23	7
Year 8 cohort stages: Percentage of students (n = 389)	7	31	18	36	8

For the year 7 cohort, there is a reasonable level of agreement between the overall cohort WSSAT and oral sample WSSAT distributions ($\chi^2 = 10.1$, d.f. = 4, $p = 0.04$), indicating that the sample is fairly representative of the cohort (see Table 6). While the oral stage data distribution for the sample shows a distribution more similar to that of the numeracy expectations, with stages 6 and 7 showing percentages that were similar to although lower than the expectations percentages and with higher percentages in stages 1–4 and 5, there is a highly significant difference between the two ($\chi^2 = 30.9$, d.f. = 4, $p < 0.001$). Such a distribution could well be expected from a lower decile school when compared with national data. There is also a significant difference between orally assigned stages of the oral samples and the WSSAT-assigned stages ($\chi^2 = 13.3$, d.f. = 4, $p < 0.01$), with the main difference occurring for stages 6 and 7, where the percentage of oral stage 6s is greater and that of oral stage 7s is lower than the WSSAT percentages.

Comment: The oral data collection process undertaken in this study was representative of the oral processes used in the NDP assessments that underpin the expectations, and the oral interviewer was an expert in NDP and in conducting NDP oral interviews. Thus, the stages assigned by the oral data are likely to be an accurate representation of the sample's distribution. This suggests that some of the stage 7s and 8s assigned by the WSSAT were assigned incorrectly to students who were actually at earlier stages.

Table 6
The Percentage of Stages Assigned to Year 7 Students for Each Assessment Tool and Numeracy Expectations Data for Year 7 Students (End of Year)

	Stages				
	1–4	5	6	7	8
WSSAT stages for cohort: Percentage of students (n = 406)	14	41	14	23	7
WSSAT stages for oral sample: Percentage of students (n = 81)	15	42	13	21	10
Oral assessment stages: Percentage of students (n = 81)	17	42	21	20	0
Numeracy expectations: Percentage year 7 (Tagg & Thomas, 2007a)	6	26	31	32	5

The school cohort data with its bimodal distribution at both years 7 and 8 (see tables 6 and 7) is significantly different to the numeracy expectations ($\chi^2 = 137.4$, d.f. = 4, $p < 0.001$) and $\chi^2 = 148.7$, d.f. = 4, $p < 0.001$, respectively). At the intermediate level (years 7 and 8), the expectations data indicates low percentages of stage 1–4s and stage 8s, with 60–70% spread roughly equally across stages 6 and 7. The school data shows a different (bimodal) distribution, with higher percentages of stage 5s and 7s for both year groups and with relatively lower stage 6 percentages (see tables 6 and 7).

The percentages for all the year 7 cohort WSSAT stages except stage 8 are quite different (see Table 6). The area of greatest disparity is in stages 5 and 6. At stage 5, the WSSAT percentage (41%) is about one and a half times larger than the expectations percentage (26%), while at stage 6, the WSSAT percentage (14%) is just under half the expectations percentage (31%). The other areas of difference are stages 1–4, where the WSSAT (14%) is over two times larger than the expectations (6%), and stage 7, where the WSSAT (23%) is about two-thirds that of the expectations (32%).

Table 7
The Percentage of Stages Assigned to Year 8 Students by WSSAT and Numeracy Expectations Data for Year 8 Students (End of Year)

	Stages				
	1–4	5	6	7	8
WSSAT cohort stages: Percentage of students (n = 389)	7	31	18	36	8
Numeracy expectations: Percentage year 8 (Tagg & Thomas, 2007a)	6	12	32	37	14

The year 8 percentages of stages 1–4 and 7 are very similar between the two measures, while the WSSAT stage 8s are just over half of the expectations percentage (see Table 6). The area of greatest disparity for the year 8 student data is in stages 5 and 6 (as for year 7). At stage 5, the WSSAT percentage (31%) is about two-and-a-half times larger than the expectations percentage (12%), while at stage 6, the WSSAT percentage (18%) is about half the expectations percentage (32%). This is similar to the differences in the year 7 data for these two stages, with WSSAT stage 5 one-and-a-half times larger and stage 6 about half as large as the expectations percentages.

Comment: The numeracy expectations are averages based on figures covering schools from all decile levels, so it would be reasonable to expect the percentages for this decile 3 school to be higher at the earlier stages and lower at the later stages than the expectations. Although such a trend is clearly evident, there is no evidence in the national data for the overall bimodal distribution seen in the study data.

Patterns in assigned stages by class

In examining the year 7 data by class, there appears to be some variation between the classes (see Table 8), which fall into two main groups.

The data for the first group of four classes (7B to 7E) gives rise to figures more closely aligned with the expectations, allowing for the school-decile level, with reducing percentages meeting later stages and more students achieving only earlier stages.

Table 8
Classes, Showing the Numeracy Expectations Percentages and the Percentage of Year 7 Students Assigned a Particular Stage by the WSSAT for the Non-Composite Classes

		Trial Classes												
Year 7 (n = 406)	**Expectations**	**7B**	**7C**	**7D**	**7E**	**7F**	**7G**	**7H**	**7I**	**7J**	**7K**	**7L**	**7M**	**7N**
Percentage stages 1–4	6	19	14	0	21	23	3	13	34	16	4	19	15	16
Percentage stage 5	26	51	47	41	50	42	55	55	50	40	71	52	48	40
Percentage stage 6	31	11	18	50	11	3	12	0	0	6	0	6	0	13
Percentage stage 7	32	8	14	7	11	22	18	22	3	32	25	17	30	28
Percentage stage 8	5	11	7	2	7	10	12	10	13	6	0	6	7	3

The data for the second group of nine classes (7F to 7N) shows higher percentages of stage 7s than stage 6s. The figures for this group would give rise to the overall year 7 pattern of lower stage 6 and higher stage 7 percentages, which does not match the expectations.

As the students are placed in year 7 classes on a random basis, the differences between the two groups suggest some overarching difference(s) in the class environments that is impacting on student learning.

Comment: Although no data was collected on individual classrooms and the teachers' practices within them, it is possible to make reasonable conjectures as to what might be happening in the groups of classes that could help explain some of the difference. The classes in the first group whose data parallels the expectations distributions may be ones in which the NDP approaches are being more closely enacted, with a particular focus on teaching in relation to the students' current level of number knowledge and strategy development. The classes in the second group, in which there are larger percentages of stage 7s, may be ones where there is a major focus on stage 7 (curriculum level 4) material rather than on NDP approaches and responding to students' individual needs. A teacher/teaching focus on stage 7 material may be partly a response to the school's use of standard written tests that focus on curriculum level 4 mathematics. The impact of assessment on what is taught is widely acknowledged and may well be a factor here.

In examining the year 8 data by class, there appears to be some variation between the classes (see Table 9), as was the case for year 7. The classes fall into two main groups similar to those for the year 7 classes.

The data for the first group of three classes (8B to 8D) gives rise to figures more closely aligned with the expectations, allowing for the school-decile level, with reducing percentages meeting later stages and more achieving earlier stages only.

Table 9
Classes, Showing the Numeracy Expectations Percentages and the Percentage of Year 8 Students Assigned a Particular Stage by the WSSAT for the Non-Composite Classes

	Trial Classes													
Year 8 (n = 389)	**Expectations**	**8B**	**8C**	**8D**	**8E**	**8F**	**8G**	**8H**	**8I**	**8J**	**8K**	**8L**	**8M**	**8N**
Percentage stages 1–4	6	10	7	0	6	13	7	12	11	4	10	10	20	4
Percentage stage 5	12	21	29	36	64	30	45	44	18	48	53	47	7	46
Percentage stage 6	32	35	50	32	0	20	0	0	21	0	14	13	6	0
Percentage stage 7	37	27	11	28	30	24	41	41	46	37	20	27	64	25
Percentage stage 8	14	7	3	4	0	13	7	3	4	11	3	3	3	25

The data for the second group of ten classes (8E to 8N) shows higher percentages of stage 7s than stage 6s. The figures for this group would give rise to the overall year 8 pattern of lower stage 6 percentages and higher stage 7 percentages, which does not match the expectations.

The students are placed in year 8 classes on a random basis (as in year 7), so the differences between the two main groups suggests that there may be some overarching difference(s) in the class environments that is impacting on student learning.

Comment: The conjectures made for the year 7 groups are applicable here because the two patterns in the year 8 class data conform to the two patterns in the year 7 class data. In addition, there are differences between classes within groups at both year levels that are likely to reflect differences between the class environments.

Conclusion

Overall, the WSSAT has reasonable levels of internal consistency for assigning intermediate students a numeracy (global) strategy stage, particularly if used with other numeracy data as intended. Thus, the WSSAT can be used in assigning a student's numeracy strategy stage with a reasonable degree of accuracy and consistency for classroom teachers' formative and diagnostic purposes. However, it is important that teachers use other assessment evidence, such as an oral check, to clarify the assigning of a stage where students are missing earlier stages on the WSSAT.

Examination of the WSSAT data at a school level across year levels and classes has clearly shown groupings of classes with different patterns of achievement. These provide a basis for further exploration within the school to identify underlying factors and possible focuses for school-wide teacher professional development.

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The Role of Leadership in Promoting the Teaching of Pāngarau in Wharekura

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This paper presents the findings of the evaluation of the third year of the wharekura[1] Te Poutama Tau[2] project and examines leadership of pāngarau[3] in a range of contexts. It includes the views of kaiako[4], tumuaki[5], and the kaitakakawaenga[6] (in describing educational success for their context) and whānau[7] influences on leadership. The findings indicate that for pāngarau to be a priority for tumuaki, kaiako, and the kura[8] community, this focus needs to incorporate the growth of te reo Māori[9], te ao Māori[10], and iwi[11] aspirations. Leadership is a critical element in establishing support structures and systems that promote the growth of kaiako knowledge and practice in pāngarau in order to lift student outcomes.

Introduction

Leadership has been identified as one of the major factors influencing the professional growth of kaiako and leading to the successful achievement of students (Robinson, Hohepa, & Lloyd, 2009). Definitions of leadership vary according to the context – the place and the associated focus – and the assigned role. Fay (1987, cited in Robinson et al., 2009) describes leaders as those who "occupy a position which gives them the right to command a course of action or … possess the requisite personal characteristics of leaders, or … who seek an action that is correct or justifiable" (p. 67). Robinson et al. (2009) state that Māori leadership is heavily influenced by concepts such as mana[12]. The community validates the mana of a leader, and leaders in te ao Māori maintain authority as long as their actions promote community objectives. The mana afforded to leaders is derived from either their designated leadership position or a track record of service to the community, achieving success for the group, or serving emancipatory ends. Tumuaki are not simply leaders of education, school curriculum, kaiako, and students; according to Robinson et al., tumuaki have a role in improving the status of Māori in New Zealand society. Beyond this, tumuaki have a role in improving the status of Māori as Māori.

This view of Māori leadership is echoed in the *Ka Hikitia* (Ministry of Education, 2008) vision of Māori students achieving educational success as Māori. It is a foundation of the Tū Rangatira consultation document (Ministry of Education, 2009), where the guiding principles are encapsulated in the concepts of Māori potential, cultural advantage, inherent capability, and mana motuhake[13].

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1 Wharekura: Māori-medium secondary school(s)
2 Te Poutama Tau: Numeracy Development Project for Māori-medium educational settings
3 Pāngarau: mathematics (through the medium of Māori language, customs, and cultural lenses)
4 Kaiako: teacher(s)
5 Tumuaki: principal(s)
6 Kaitakawaenga: facilitator
7 Whānau: family in its widest sense of the word; in this context, it includes all those involved in the education of a student in their community
8 Kura: Māori-medium school(s)
9 Te reo Māori: Māori language
10 Te ao Māori: pertaining to the Māori world/ Māori viewpoint
11 Iwi: tribe
12 Mana: Integrity, respect, and status
13 Mana motuhake: self-determination

Accordingly, tumuaki have responsibility for spreading their influence "into the wider corridors of Māori development" (Durie, 2006, cited in Robinson et al., 2009, p. 70). This can include community goals, such as developing tamariki/mokopuna[14] into adults who can sustain Māori language and kaupapa[15] on the marae. It also includes identifying and assisting the needs of whānau for the ongoing care of our tamariki mokopuna, academically, socially, and culturally. The focus of this paper is on leadership structures and supports in wharekura that promote teacher content knowledge and professional practice as well as student achievement in pāngarau.

Background

Wharekura Te Poutama Tau is a professional development project designed specifically for wharekura kaiako to develop their mathematical content knowledge and teaching strategies (pedagogical content knowledge) to ensure continued gains in wharekura student achievement. The focus on the development of content knowledge and teaching strategies is critical because there is no pre-service training (for example, in mathematics) for subject-specific wharekura kaiako. The majority of wharekura kaiako have primary teacher training that does not include any tertiary level mathematics. For instance, as recently as 2009, only two out of a total of 23 kaiako who had participated in the professional development since 2007 had specialist mathematics qualifications (gained from English-medium institutions) to teach pāngarau in wharekura (Te Maro, Higgins, Averill, & Tweed, 2009).

A catalyst for the first wharekura Te Poutama Tau pilot project was the identification by Trinick and Parangi (2007) of the conditions impacting on wharekura kaiako in planning and delivering quality pāngarau programmes to their students. The conditions included:

- the isolated nature of teaching pāngarau in wharekura
- teachers teaching across multiple curriculum areas
- little professional development support for te reo or pāngarau
- few teaching resources suited for use in wharekura
- outside commitments to marae, whānau, and hapū[16].

Findings from Te Maro, Averill, & Higgins (2008) and Te Maro et al. (2009) are relevant to this evaluation, particularly for reiterating conditions faced by wharekura kaiako, the importance of te reo pāngarau[17] and tikanga[18], and the need to deliver relevant and appropriate professional development that is responsive to the needs of kura and is sustainable over time. Kaiako continue to be vulnerable to these conditions regardless of whether they have specialist mathematical knowledge.

The 2009 Wharekura Te Poutama Tau Delivery Model

Two previous evaluative studies (Te Maro et al., 2008; Te Maro et al., 2009) informed the development of the 2009 wharekura Te Poutama Tau delivery model. Two findings were particularly useful: firstly, the engagement of the kaiako with the project created stronger networks among kaiako, particularly for one of the groups; and secondly, the facilitation was framed in terms of relationships with the community rather than in terms of providing external expertise. In taking a role based on relationships, the facilitator "assisted with creating a pāngarau culture alongside the wharekura [… and] needed to be aware of the individual wharekura cultures, which necessitated fostering authentic relationships

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[14] Tamariki/mokopuna: children/grandchildren

[15] Kaupapa: ways of doing what is right

[16] Hapū: sub-group of an iwi

[17] Te reo pāngarau: mathematics teminology in Māori language

[18] Tikanga: what is deemed to be the correct way to carry out tasks/duties

(whanaungatanga[19])" (Te Maro et al., 2009, p. 40). By taking this approach, the facilitator, from a non-Māori background, could operate in kura and become a member of the whānau of kura. A further finding was the importance of te reo pāngarau and the effect of language on the mathematics teaching and learning of kaiako and students.

Both studies found that the tools chosen for facilitation were instrumental in improving kaiako knowledge and practice and creating achievement gains for students. Specifically, their focus on the usefulness of professional development delivery tools (Te Maro et al., 2008) and on the growth of teacher networks and their influence in supporting wharekura kaiako to assist their students' achievement in pāngarau (Te Maro et al., 2009) informed the 2009 model.

In 2009, the majority of the tools for delivery of the project remained the same as for the two previous years, with slight adaptations to better suit participants' needs (see Te Maro et al., 2008, for a full description of the tools and methods). Adaptations included changing from a video conferencing tool to a WizIq[20] tool to better facilitate interaction amongst participants and seeding a cluster to ready them for a full programme in 2010 rather than starting them as a new full-time cluster. To provide continuity across the initial two years and foster sustainability, the facilitator travelled to each of the original kura to maintain face-to-face delivery as well as online hui.

Methodology

Leadership has many facets, and, for this particular study, it was important that the participants in the professional development defined leadership within their own contexts, particularly with regard to their community's educational priorities and aspirations. This approach aligns with kaupapa Māori[21] research and Smith's (2003) theoretical framework, in which the research is defined by the participants, serves Māori communities, and comes from Māori community needs. These definitions were built into the analysis of data.

Before the evaluation of leadership took place, the research team used a method of backward mapping (Elmore, 1979–80) to create a framework of considerations, goals, and conjectures from the pre-existing data and reports of the two earlier projects. The participant kaiako were consulted before the interviews took place with them and contributed to the framework by critiquing and reframing its elements and adding to the authenticity of the outcomes. The framework then became part of the interview.

Research Aims

To build on both the 2007 and 2008 studies, this evaluation focused on:

- leadership of wharekura Te Poutama Tau
- what leadership means in each wharekura context
- who leads
- how they lead
- what their priorities are
- how those priorities influence Māori educational leadership.

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19 whanaungatanga: relationships similar to those in a family, the act of being familial and creating familial links.

20 WizIq: Internet-based professional networking (see Te Maro et al., 2009, for further information about its use)

21 Kaupapa Māori research – Smith (2003) – for the benefit, conscientisation, and transformation of Māori through research.

The macro view of the issues raised over the last few years in the wharekura project for students in pāngarau is important in dealing with the many influences affecting achievement. The question asked in this study was:

> What leadership structures and supports are in place in wharekura for promoting the teaching of pāngarau?

Participants

Since its inception, the wharekura Te Poutama Tau professional development project has been delivered in two regions (Kupenga and Waiariki) and seeded in another region (Northland/Auckland). The participants of the project have been year 9 and 10 wharekura kaiako. The criteria for the selection of participants for this year's evaluation included the following: those who carried out specific leadership tasks within wharekura and those in wharekura at different stages in the professional development process.

Firstly, the different leadership tasks within wharekura included kaiako teaching pāngarau and tumuaki of wharekura. Kaiako perspectives of leadership are important to provide their perspective of the degree of support afforded to kaiako and students by kura systems and processes. Wharekura kaiako who are teaching pāngarau are generally the only kaiako pāngarau in the kura. As well as being the only kaiako pāngarau, they tend to carry other curriculum areas alone. These kaiako are also likely to play lead organisational and coaching and mentoring roles in areas such as kapa haka, manu kōrero, sporting events, and community events and support systems.

Secondly, the wharekura were selected at different stages in the professional development. They included:

- three tumuaki from a Northland-Auckland seeding cluster, one of whom was also a participant kaiako. The seeding cluster had had two visits: one to introduce the Te Poutama Tau framework and one to follow up and begin planning for 2010 about what Te Poutama Tau was, how it might be introduced to the kura, and whether or not the kura would like to take part in the professional development.
- kaiako in wharekura in their second year of the project (Waiariki: 8 kaiako and three tumuaki. One of the participant kaiako was also a tumuaki). These wharekura, in the second year of Te Poutama Tau, were still gaining insight into the impacts of the project for kaiako and students. They had had regular hui, modelling, and observation (as per the 2007 and 2008 model). Additional reasons that the Waiariki cluster of kaiako was chosen was because they were the only cluster that would be meeting face-to-face in the 2009 year and they had prior experience of, and relationships with, the researcher through their participation in the 2008 project.
- two tumuaki of wharekura from the original cluster group (2007 – Kupenga) where the facilitator felt Te Poutama Tau was embedded into the culture of the kura through the results that kaiako were gaining, the shifts in teaching that kaiako had made, and the positive attitudes that the kaiako had to teaching and learning pāngarau (see Te Maro et al., 2008). Support to the original group aimed to create sustainability by assisting them online (email, text, phone calls) and setting up a one-day hui to plan activities for students from three of the wharekura.

Selection Process

An important part of the selection process was the agreement between tumuaki and kura and the researcher to conduct research with their kaiako, both with them and within the kura. The researcher met face-to-face with all of the tumuaki as part of the consenting process and determined whether, once they had met with the researcher and felt that both the researcher and the research were credible and trustworthy, they were agreeable to the research being carried out. The tumuaki and kaiako from whom this data has been drawn are a small community, and so there is a possibility that they may be easily identifiable. To minimise this possibility, quotes are used in a generic sense and selected to provide examples of what any one of the tumuaki might be expressing.

Interviews

Eight tumuaki were asked a series of open-ended questions during a semi-structured interview that was digitally recorded with their permission. The interviews took place in each of their kura, which was a deliberate choice rather than carrying out telephone interviews because the researcher believed that kanohi ki te kanohi[22] (Smith, 2003) was the most relevant method to use with people who they felt were of higher/rangatira[23] status. As Te Poutama Tau leaders in their kura, the Waiariki cluster of kaiako were interviewed using a semi-structured interview process and their responses were digitally recorded.

Eight kaiako were interviewed as a group, which they decided was their preference, and the interviews were digitally recorded. Themes from the interviews (one at the beginning of the research evaluation and one at the end for both groups) were analysed against the conjectures presented in the backward mapping (Elmore, 1979–80) exercise conducted in the first hui (see Appendix U).

Findings: Leadership Supports for Pāngarau

The study found that the selection of leaders of pāngarau across the wharekura in this study took three forms: an expressed desire to be the kaiako pāngarau; selected by their tumuaki as the most suitable person to fulfil the role (*Ko wai atu? Kei te hiahia kawe, e whakapono ana au ka taea*[24]); or because they displayed aptitude/positive attitude in pāngarau (*"He ngākau nui tōna ki te mahi"*[25]). Tables 1–3 detail the roles and responsibilities for pāngarau kaiako with different levels of experience in order to highlight the complexities of leadership in wharekura. One description of a kaiako is not included because the wharekura focus was undergoing extreme change.

Table 1 lays out the roles and responsibilities of an experienced leader who was both the tumuaki and the pāngarau kaiako. In this example, the leadership focus was fourfold:

- on pāngarau, to develop kaiako as subject specialists
- on kaiako development, through modelling best leadership practice to kaiako as aspiring leaders
- on kura management, through leading the communication and collaboration amongst the wharekura and contributing kura teina[26] of the region, to develop agreement and consistency about what knowledge and skills students need when transitioning to wharekura and how wharekura can best meet the varying needs of those students transitioning to them
- on leadership within the community of kura in the area, through working with kura in the region and providing opportunities for other pāngarau teachers to work together.

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22 Kanohi ki te kanohi: face-to-face

23 Rangatira: person in a position of responsibility or authority

24 Ko wai atu …?: Who else? If you want the responsibility, I believe (have faith) that you can do it.

25 He ngākau nui tōna ki te mahi: They have passion (big heart) for the work.

26 Kura teina: Level 1–2 Māori immersion educational settings, equivalent to year 0–6 or year 0–8 schools in the medium of te reo Māori

Table 1
Roles and Responsibilities of a Tumuaki and Pāngarau Kaiako

Kaiako experience	Roles	Responsibilities
Tumuaki and pāngarau kaiako (1)	Pāngarau leader and kura leader	• Pāngarau (the only subject area) • Kaiako development • Kura management, including within the community whānau • Leadership within the community of kura in the area.

Table 2 lays out the roles and responsibilities of two emerging leaders of pāngarau (both provisionally registered teachers), showing their multifaceted leadership foci. For instance, one beginning kaiako, in their role as pāngarau kaiako, planned the pāngarau programme with the facilitator. They also reported to tumuaki tuarua[27] and negotiated strategic targets for student achievement in Te Poutama Tau.

Table 2
Roles and Responsibilities of Two Emerging Leaders of Pāngarau

Kaiako experience	Roles	Responsibilities
Emerging leaders of kaiako	Pāngarau kaiako and managing wānanga[28] studies for students	*Beginning kaiako 1* • Set up support systems for beginning kaiako • Collaborate in target setting for pāngarau achievement • Planning, monitoring, and assessment for instruction • Planning for kapa haka, manu kōrero, and wānanga studies • Heavy involvement in the strategic planning of the long-term educational strategy for the iwi. *Beginning kaiako 2* • Plan pāngarau delivery through Te Poutama Tau with the tumuaki • Mentor another kaiako in pāngarau • Plan and deliver for NCEA mathematics from year 9 to year 13 • Fostering a Te Poutama Tau culture across the kura • Working with the facilitator to develop kaiako skills and knowledge in pāngarau.

Both beginning kaiako had the following types of responsibilities:

- providing assistance with planning and assessment, while expecting the other kaiako to develop the long-term planning for pāngarau for the wharekura
- encouraging a Te Poutama Tau culture across the entire kura
- working with the kura teina Te Poutama Tau facilitator as well as the wharekura facilitator to develop the content knowledge and skills of kaiako in pāngarau, which meant going to more professional development hui.

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[27] Tumuaki tuarua: deputy or associate principal(s)

[28] Wānanga: Māori tertiary institution

- all planning, monitoring, and assessment
- the work needed for full registration alongside the tumuaki and tutor teacher/s
- managing the wānanga studies that the students were undertaking with other tutors.

Table 3 shows that the roles and responsibilities of experienced leaders of pāngarau kaiako included teaching pāngarau as well as setting up systems to promote pāngarau achievement. Teaching was in years 9–11 (including 1 NCEA level 1 class). The systems set up included establishing a tumuaki tuarua so that each wharekura kaiako was a "department" leader and part of the management team reporting to the tumuaki tuarua. Within this system, decision-making was handed to the team with the expectation that they would problem solve and come to the tumuaki with solutions: "… kaua e oma noa mai mō te pātai, kei a kōrua, mahia."[29] The lead kaiako worked collaboratively with the kura teina to develop consistency across the kura (this has not happened as yet in one kura because Te Poutama Tau is new). The lead kaiako was also responsible for the collection of monitoring and achievement results and reporting results to the tumuaki tuarua as part of the wharekura management team in preparation for reporting the results to the board of trustees. A crucial link with the board was in the strategic planning of the kura for pāngarau. Part of establishing the support system was to have kaiāwhina[30] to assist kaiako in matters such as managing resourcing so that the kaiako was able to attend professional development.

> Karekau i te tino pai te whakahaere o te pāngarau i mua, engari i hanga he tūnga motuhake kia whakarite te tirohanga whānui, mai i ngā pōtiki ki ngā tuākana. Ko te mahi a te rangatira hei rapu huarahi hei whakatikatika hei whakaū i tana kaupapa whai oranga mai ngā tamariki i roto i te ao o te pāngarau. Nō reira i whakaarohia aaa, anei kē.[31]
>
> So the plan is to build expertise in one of the kaiako to act as a rangatira for pāngarau (honohono mai ngā āhuatanga, ngā tāngata katoa mō te pāngarau). Te kura katoa[32]. She becomes the person who plans and takes workshops for the other kaiako, she is the kaiārahi[33], who creates the strategic planning for pāngarau in the school scheme. She becomes the kaiwhakatikatika, the kaipoipoi[34]. My job as a rangatira is to sit with her and to discuss with her what needs to be done.

A second experienced kaiako pāngarau worked alongside the tumuaki as the tumuaki tuarua with responsibility for administration and organisation within the wharekura. The role included assisting kaiako with Te Poutama Tau and managing quality outcomes with the pāngarau facilitator across the kura and kura teina. A third experienced kaiako pāngarau had similar responsibilities for long-term whole-school strategic planning for pāngarau (kura teina and wharekura), for the professional development of the kura kaiako, for reporting progress and identifying the needs of the kura to the tumuaki, and for ensuring the growth of ngāti mathematicians[35].

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[29] … Kaua e oma noa mai … Don't just run to me with questions, you both have the answers, so go ahead. (Also talks about coming with solutions)

[30] Kaiāwhina: teacher aide(s)

[31] The leadership of maths in the kura was not going well before, so a specific position was created to look at the long-term mathematics strategy from the 5 year-olds to the oldest students. The job of the "chief" was to look at how mathematics could be managed so that the students are successful, and so the thought was, aaa, here we already have someone.

[32] Look after mathematics across the entire school (primary and secondary)

[33] Kaiārahi: leader/guide

[34] Kaiwhakatikatika, the kaipoipoi: The person who ensures that mathematics across the school is done correctly and encourages its teaching

[35] Ngati mathematician: a mathematician who works for, through, and about the iwi, rather than as an individual.

Table 3
Roles and Responsibilities of Experienced Leaders of Pāngarau Kaiako

Kaiako experience	Roles	Responsibilities
Experienced pāngarau kaiako (3)	Pāngarau kaiako for their wharekura	• Teaching pāngarau • Setting up support systems for managing the delivery of pāngarau, including setting up tumuaki tuarua • Arranging for kaiāwhina to assist the kaiako • Reporting pāngarau achievement results to the wharekura and the board • Strategic planning focused on raising achievement (alongside the facilitator and kura teina).

The Ngātitanga[36] of Pāngarau – Pāngarau as a Priority for Leaders

It was expressed by tumuaki and kaiako that, while pāngarau was a priority subject in their kura, they had other pressing priorities according to their kura aspirations, such as te reo, upskilling their kaiako, and ngātitanga.

> When we know that our students have te reo, we will know that we have achieved our goals. We can't say that we have reached that pinnacle yet, and that is the greatest challenge for each kaiako of the wharekura – the quality of the reo of the students.

The following priorities were garnered from the data. The numbers to the side are those tumuaki and kaiako in kura who mentioned these priorities:

- In-house professional development, building the reo capability of the kaiako for the sake of the quality te reo of the students (11)
- Developing tools and skills for assessment and monitoring (11)
- Iwi and hapū studies through the wānanga (4)
- Kapa haka (3)
- Develop content knowledge and capability of kaiako in subject areas (11)
- Iwi educational strategic planning (4)
- Ngātitanga (6).

As one of the tumuaki expressed, pāngarau could not usurp kaupapa Māori; it had to find a place to fit and a way to do so that would not diminish the iwi-ness of the students. The tumuaki said that the aspiration was to ensure the survival of te reo Māori, specifically the reo of his iwi, and the iwi way of living and being, "Māku tonu e whakangāti ..."[37]

The vision is that te ao Māori lives again through the kura, through the teachings of the kura. There is no problem with things from the wider world, from science, from maths; however, the tumuaki do not want the specific values of these things to overshadow the overriding values of the kura. It is a matter of selecting what works for the kura (that is, iwi). For example, "Pai noa iho a pāngarau, pai noa iho a science, engari kāore i te pīrangi a 'individuality'."[38] The tumuaki sees their job as being the person who can weave together subjects like pāngarau with the world views of the iwi:

[36] Ngātitanga: collectiveness; whatever the subject, it should be seen to be collective in the way that it is approached and carried out.

[37] It will be me who will determine how pāngarau fits in this iwi.

[38] Pāngarau is fine, science is fine, but we don't want the values of individuality.

> Kāore e paku pīrangi kia kuhu mai ki te takatakahia ngā manaakitanga, ki te takatakahia ngā whānautanga ki konei, te noho a-whānau, te whakakotahitanga.[39] It is about the values inherent in teaching and learning of mathematics. Mā tātou katoa te pāngarau – kia whakatau te pāngarau ki roto i āku uara, i roto i te wairua ake, kia pupū ake te mauri o ngāti ki roto i te kura nei.[40] My desire is that if a student finds their ability in pāngarau, then they have the opportunity to emerge not as a mathematician, BUT as a ngāti mathematician. No problem with the mathematician as long as they are a ngāti mathematician. The vision is that te ao Māori lives again through the kura, through the teachings of the kura.

Concluding Comments

This study asked how leadership supports pāngarau in wharekura. The views of the kaiako, tumuaki, and facilitator add to the emerging shape of leadership within kura, how it is manifested and by whom, at different junctures. An added challenge for kaiako without training at the secondary level is their development as leaders of pāngarau. Tumuaki as leaders have shown that they take cognizance of their responsibilities to support and assist lead kaiako of pāngarau to plan, deliver, monitor, assess, and evaluate pāngarau programmes that will support student achievement. Tumuaki and kaiako work collaboratively in those support structures according to the desire to build, enhance, and increase leadership skills within each kura.

The findings have also indicated that while pāngarau and the development of kaiako and students in pāngarau are a priority for tumuaki, kaiako, and the kura community, it is not the success of pāngarau that is paramount to kura unless it in some way supports the growth of te reo, te ao Māori, and iwi aspirations. One tumuaki expressed it in another way by saying that this was a goal for them, but at the moment not a reality, and that they needed to leave it to others who understand how to do this to explain and assist them because while they want this, they are not there yet. "Waiho mā te tangata e mārama ana ki tērā hei whakamārama mai."[41]

We have not captured the views of students or whānau in this project. It is important that in time those views be captured in order to gain a complete picture of the expectations and aspirations of whānau for the achievement of their tamariki and mokopuna in pāngarau.

Acknowledgment

Nei rā te mihi aroha, te mihi nō te ngākau iti nei i te manawa nui, i te manawa roa, i te pukumahi, i te kaha o ngā tumuaki o ngā kaiako o ngā wharekura i whai wāhi i roto i te rangahau nei. Mei kore koutou, kua ahatia ā mātou nei rangatira, ā mātou nei taonga e whai ana i te ara a kui mā a koro mā i roto i tēnei ake ao? Nā tō koutou kaha ki te tiaki, ki te manaaki i te hunga rangahau nei kua puta mai te whakaaro nui ki te ngātitanga o te pāngarau, ki te pāngarau e whakahaerehia ana ki te reo me ōna tikanga katoa, hei oranga ake mō tātou katoa. E kore rawa e mutu āku nei mihi ki a koutou.

These are greetings of aroha from this smallest of hearts – to the huge hearts, to the hard workers, to the strength of the principals and teachers of wharekura who took part in this research. If not for you, where would our growing chiefs, our treasures, who are following the paths of their ancestors in this contemporary world, be? It is through your strength in protecting and caring for this research and the researcher that ideas about creating leaders who are Māori/iwi mathematicians who operate through the language and the world of Māori thought are able to be recognised as a way for us to flourish in the future – achieving as Māori/iwi. There is no end to the acknowledgment of you all.

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39 Don't want even the tiniest bit for these subjects to trample on the value of care and shared responsibility or the way that we work as whānau, or the way that we work collectively.

40 Pāngarau is for all of us, but it should sit well with my values, in my own spirit, so that the essence of being my iwi is fostered in this kura.

41 Leave it to the people who understand this to explain it to us.

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The Impact of Professional Development Interventions for Numeracy "Pick-ups": Content Knowledge and Teachers' Perceptions of Valued Aspects

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This paper describes some of the effects of in-service interventions for beginning teachers and teachers new to the materials and approaches of the Numeracy Development Projects (NDP). The School Support Services interventions were a contracted delivery of a numeracy "pick-up" programme with a focus on the various aspects of the NDP and on developing an overall understanding of what the NDP required of teachers. The interventions did not address content knowledge as a major focus other than as an integral part of the NDP materials. The 2008 intervention had a facilitator in-class support component, while the 2009 intervention for the group researched did not. Data was collected on the teacher participants' content knowledge before and after the 2009 intervention, and their perceptions of the usefulness of the intervention and the aspects they most valued in their learning for both the 2008 and 2009 interventions were evaluated. The 2008 participants' strategy and content knowledge improved, and both the 2008 and 2009 groups found the interventions to be of value and helpful to their learning.

Introduction

Since the beginning of the Numeracy Development Projects (NDP) in 2000, a major concern has been the issue of teachers new to schools where the NDP had been implemented without themselves having been exposed to the NDP's professional development; the lack of exposure to NDP arising from teacher mobility; and the employment of new, provisionally-registered teachers and overseas teachers new to New Zealand (Lomas, 2009). In some cases, there was a complete staff turnover in a short time period and none of the staff currently at a school might have received NDP professional development, even though the school had been involved previously.

To meet the needs of these types of teachers taking positions in schools that had already received NDP interventions, a different form of intervention, called a "pick-up" programme (Lomas, 2009), was developed and funded. The intervention was provided through School Support Services (SSS) centres around the country and had a local flavour, reflecting the attitudes and experiences of the individual facilitators in each centre.

By about 2008, the initial NDP intervention had reached most primary schools and the NDP moved into a new phase, with a major focus on developing sustainability by providing longer-term, in-depth work with schools (Lomas, 2009). A possible focus for this new work related to improving teachers' content knowledge, which has been acknowledged both nationally (Ward & Thomas, 2007) and internationally (for example, Ball, Thames, & Phelps, 2008) as being weak. It is, however, not yet clear how the strengthening of teachers' content knowledge may best be addressed.

This paper reports on participant teachers' strategy and content knowledge and their evaluations of aspects of the 2009 SSS intervention, compared with a group of teachers who participated in a generally similar SSS intervention in 2008.

Method

Participants and Procedure

Participants for this study were drawn from a group of teachers participating in the Auckland SSS's 2009 semester 2 pick-up induction intervention for beginning teachers, returning teachers, and overseas-trained teachers, none of whom had been exposed to NDP approaches. Of the 30 or so teachers in this SSS intervention, 15 chose to participate in the strategy and content knowledge assessment conducted during both the first and final sessions; of the whole group, ten (of whom only six had their strategy and content knowledge assessed) returned the evaluation forms. The 15 teachers who participated in the assessment consisted of nine provisionally registered, two overseas trained, one returning to work, and three who, while experienced, had not undertaken NDP programmes previously. Details on the evaluation sample composition are not known because, except for the link to having done the Written Strategy Stage Assessment Tool (WSSAT) assessments, this was conducted anonymously.

The evaluations, along with addressed envelopes, were handed out at the last session to all those attending, to be returned at a later date because the intention was to see what the effect had been "over time" as far as possible. Several email reminders were sent to the group near the end of the school year, with the last reminder providing another copy of the evaluation form along with a return address in an attempt to encourage a high return rate. The response rate of around one-third of the 2009 group suggests the need to refine the ways of obtaining feedback in future studies. However, in contrast to the 20% response rate of the 2008 SSS group, the 2009 respondents represent a larger percentage of the teachers in the intervention. Even so, due to the "self-selected" nature of the respondents, it is not clear how representative either the 2008 or 2009 sample is. Another feature of the 2009 sample was the relatively large proportion who did not respond to the in-class support part compared with the 2008 sample. This may have been partly due to there being no formal in-class component in the 2009 intervention, although two of the four who did not respond to the in-class part noted that there was no in-class support in their school. This is of some concern, if it is a correct reading of the situation, as the long-term sustainability of the NDP is largely predicated on the development and provision of localised in-school support in the form of lead teachers of numeracy.

Materials and Analysis

The 2009 SSS intervention participants attended six professional development sessions (over three full days) during the third term. These were conducted by a numeracy facilitator at a central location, in contrast to the 2008 interventions, which were delivered at "local" school sites around Auckland. The sessions focused on the NDP and their implementation in schools and introduced teachers to the NDP "pink books" support material (Ministry of Education, 2006a–f; 2007; 2008) and their use as supplementary resources in developing students' learning. Alongside the sessions, the 2008 participants received in-class visits and/or other support from the SSS facilitator related to the materials delivered in their sessions. In contrast, the 2009 participants did not receive in-class support from the facilitators but may have received in-class support from their school's lead teacher working with them on their classroom practice. Although the lead teachers did not attend the SSS professional development sessions, they were usually involved in ongoing professional development meetings designed to help them support the teachers in their schools.

The focus in the SSS interventions was on professional development, with the sessions being relatively stand-alone and the teachers encouraged to use the materials introduced as they deemed appropriate to the class level they taught.

The 2009 group participants' NDP strategy and content knowledge was assessed using WSSAT (Lomas & Hughes, 2009) early in the first session and then again in the last session. This numeracy assessment tool determines an individual's NDP strategy stage (stages 1–4, 5, 6, 7, or 8) and aspects of numeracy content knowledge related to them.

The items are constructed to "force" the use of mental NDP strategies (Lomas & Hughes, 2009) but rely only on the written answers to determine the stage. However, there is no in-built mechanism to check that algorithmic approaches have not been used to arrive at answers rather than mental NDP strategies.

In this study, many of the assessment items were completed using algorithmic approaches, evidenced by the large amount of written "working out" on both the question and answer papers. This raises questions about the extent to which it was NDP strategies that were measured. However, there was a reduction on the amount of working out from the initial to final assessments, which suggests a growth in understanding of NDP approaches and possible greater confidence with them. In future uses of the WSSAT tool, it may be necessary to place a time limit to restrict the possibility of algorithmic approaches being taken and to state the focus as being on mental work in an effort to gain a clearer picture of purely NDP approaches.

The analysis of the WSSAT scores focuses on individual teachers' initial and final scores and any changes to them over the time period of the sessions. These were considered in terms of both the stage indicated and the number of strategy and knowledge items correct. A paired samples T-test (df = 14) was conducted to test for statistical significance on an assumption of a "normal" distribution of scores.

The participants' perceptions of the sessions and in-class factors were collected via an evaluation form. The original evaluation form (Lomas, 2009) was designed to measure two main components (separated to ensure that each was given equal consideration): the sessions and any in-class support. Each component was rated, using a six-point Likert scale, in terms of the teacher's perception of how valuable and helpful it had been and how worthwhile they perceived their attendance or involvement in meeting their own needs. Self-report data is usually expressed positively in evaluations of professional development programmes (Brown, 2004), so a positively-packed six-point scale (see the Appendix) with four positive and two negative responses was used to allow for the differentiation of varying levels of positive agreement. However, in the analysis of the component rating data, only the three responses that indicated a stronger level of agreement than "agree" were considered as positive.

In addition, a randomised grid of 20 elements that covered the range of practices used in, and alongside, the intervention was created for each component (Lomas, 2009). The teachers were asked to identify, for each component, up to three elements that they saw as being most helpful in learning to work more effectively with the students in their classrooms. Where teachers identified more than three, all were included.

For the 2009 group, the only change to the original evaluation form was to the in-class elements, with the removal of five elements that referred to facilitator involvement outside the sessions (1a, 2a, 2c, 3a, 3d). Element 4a, "Being given resources by the facilitator", was retained because this aspect could arise either during the sessions or as an email follow-up from the facilitator but was not part of the in-class evaluation. A new element (8, "Working with the facilitator outside the session") was added to address any informal contact teachers might have had with the facilitator (see Appendix, Part B). The remaining elements were not relabelled for analysis purposes to ensure that making comparisons with the 2008 group (Lomas, 2009) would be straightforward.

For the session component, the 20 elements were organised into five categories (see Appendix V, Part A). The first two categories contained parallel items that dealt with mathematics content and the learning progression of content (the rest were stand alone):

- personal mathematics knowledge and understanding (6 grid elements: 1a–1f, for example, 1e: "Improving my personal understanding of decimals");
- personal knowledge and understanding of students' learning progressions (6 grid elements: 2a–2f, for example, 2b: "Improving my personal understanding of children's learning progressions in addition/subtraction");
- aspects of development (4 grid elements: 3a–3d, for example, 3a: "Learning more about what students need to be doing next in mathematics");
- self and students as learners (2 grid elements: 4a–4b, for example, 4b: "Gaining a greater understanding of what learning content (e.g., place value) is like for students");
- modelling of practice (2 grid elements: 5a–5b, for example, 5b: "The format and conduct of the sessions (the way the lecturer/ facilitator taught) was a useful model for my classroom").

The 16 in-class component elements for the 2009 group were organised into seven categories (see Appendix V, Part B), like the 2008 group evaluation form arrangement (Lomas, 2009), plus an eighth category of one element dealing with interactions with the facilitator. The first two categories focused on observations, the third on sharing aspects of practice, and the fourth on resources (the rest are self-explanatory):

- being observed with follow-up discussion with a variety of people (3 grid elements: 1b–1c, for example, 1c: "Being observed by, and having follow-up discussions with, another teacher");
- observing a variety of people teaching (3 grid elements: 2b, 2d–2e, for example, 2e: "Observing another classroom teacher teaching their class");
- sharing aspects of practice with a variety of people (4 grid elements: 3b–3c, 3e–3f, for example, 3a: "Sharing planning with a lead teacher");
- receiving resources from a variety of people (3 grid elements: 4a–4c, for example, 4c: "Being given resources by another teacher[s]");
- being able to choose the type of support received (grid element 5);
- discussing their own numeracy issues (grid element 6);
- release time to work with students (grid element 7);
- interaction with the facilitator (grid element 8).

The evaluation was given as late in the year as possible in an attempt to obtain a reflective evaluation and to allow time for the professional development to have had an influence on classroom practice.

The initial 2009 sample data analysis was carried out on the categories of elements. However, as categories had between one and six elements and the teachers were making multiple selections from the 20 or 16 elements, a second level of analysis was then carried out at the individual-element level. The two levels of analysis were then examined to compare the overall ratings by category against the ratings by individual elements. In addition, the evaluation data was compared with the data of a previous 2008 study (Lomas, 2009).

Results

Teachers' NDP Strategy and Content Knowledge Data

The initial assessment assigned the 15 teachers' strategy stages across the full range, with just under half (seven) being assigned stage 7 (see Table 1). This probably reflects the acknowledged weakness in mathematics of primary teachers, particularly around fractions, decimals, and percentages. Of note is the teacher who was assigned only to the stage 1–4 category.

Table 1
Initial and Final Stages assigned to Teachers by WSSAT

	Stages assigned by the WSSAT tool				
WSSAT assessment	1–4	5	6	7	8
Initial	1	–	3	7	4
Final	–	–	1	6	8

The final assessment assigned teachers to stages 6–8, with just over half at stage 8. This suggests that the sessions had an impact on the teachers' knowledge of, and familiarity with, the NDP strategies and content and indicates that the intervention had gone some way to addressing fraction, decimal, and percentage issues for these teachers, with a statistically significant increase in the stages assigned, with $t(14) = 3.67$, $p < 0.005$. However, given the algorithmic approaches used by some teachers in arriving at answers, these stages may be higher than if assigned by NumPA, the NDP oral assessment tool. It also suggests that the impact may be somewhat less positive than first appears.

In terms of individuals' growth, six of the teachers advanced by one stage (three stage 6s to stage 7, three stage 7s to stage 8) and one by at least two stages, from stages 1–4 to stage 6. The stages assigned to the remaining eight teachers did not change.

However, all the teachers improved their overall scores, with most strategy and content items showing increased numbers of correct responses and some showing very large increases. This was particularly evident in stage 7 and 8 items, where the most stage change took place. For example, an item on fractions (stage 7), "What is 3 sixths of 42?", went from four to 14 correct responses, and one on percentage discounts (stage 8) went from three to 12 correct responses. This increase in strategy scores was statistically significant, with $t(14) = 5.05$, $p < 0.0001$. Similarly, the increase in content scores was statistically significant, with $t(14) = 3.55$, $p < 0.005$, although less so than for the strategy scores.

Evaluation data

The teachers' responses for both aspects of the SSS intervention components were similar. The sessions were perceived to be of value and helpful in meeting the teachers' needs, with all ten 2009 teachers offering a "moderately agree" rating or better and 32 out of 34 (94%) doing so for the 2008 sample (see Table 2). Attendance at sessions was similarly perceived as worthwhile, with nine out of ten (90%) offering a "moderately agree" rating or better for the 2009 sample and 32 out of 34 (94%) doing so for the 2008 sample.

Table 2
The Number of Responses Showing Perceived Value and Worth of Sessions and In-class Components of Both the 2009 and 2008 SSS Interventions

		Very strongly agree	Strongly agree	Moderately agree	Agree	Disagree	Strongly disagree	No response
SSS 2009 (n = 10)								
Sessions	Value	5	3	2	–	–	–	–
	Worth attending	5	4	–	1	–		–
In-class	Value	4	–	1	–	1	–	4
	Worth having	4	–	1	–	1	–	4
SSS 2008 (n = 34)								
Sessions	Value	8	17	7	2	–	–	–
	Worth attending	9	16	7	1	–	1	–
In-class	Value	4	13	6	4	1	–	6
	Worth having	5	11	8	3	1	–	6

These patterns are also evident, although less positive, in relation to the in-class support, where the type of support was perceived to be of value, with five out of six (83%) offering a "moderately agree" rating or better for the 2009 group and 23 out of 28 (82%) doing so for the 2008 group. The time involvement with the in-class support was perceived as being worthwhile, with five out of six (83%) offering a "moderately agree" rating or better for the 2009 group and 24 out of 28 (86%) doing so for the 2008 group (see Table 2), which closely matched results for the perceived value. However, there was a small shift towards a less favourable view of the in-class components for both groups.

The categories of session elements identified by the teachers as being of most help in their learning varied between the two groups (see Table 3), but the overall distribution pattern was very similar.

Table 3
The Percentage of Responses to Session Categories of Elements Perceived as Most Helpful for Both the 2009 and 2008 SSS Samples

Category	**SSS 2009** (n = 10)	**SSS 2008** (n = 34)
1[1]: Personal mathematics knowledge	9	6
2: Knowledge of students' learning progressions	16	14
3: Aspects of development	53	60
4: Self and students as learners	16	10
5: Modelling of practice	6	11

The number of responses to individual elements (see Table 4) confirm the pattern shown in Table 3, with the three of the four most highly-rated elements being the same category 3 ("aspects of development") elements for both groups.

[1] The categories are numbered here for reference purposes only, to enable links to be made to grid elements within the text and the appendix.

These align with the overarching purpose of the interventions – introducing teachers to the NDP and its resources as part of developing teachers' professional capability with regard to teaching mathematics. The perceived usefulness of the element (3d) around resources and their use, along with other category 3 elements focusing on how to develop students' mathematical understanding, strongly reflect the purpose of the interventions.

Table 4
Session Category Elements[2] Perceived as the Four (and Fourth Equal) Most Helpful for Both the 2008 and 2009 SSS Samples

SSS 2009 (n = 10)		**SSS 2008** (n = 34)	
3d: Resources and how to use them	70%	3d: Resources and how to use them	56%
3a: What students need to be doing next	50%[3]	3b: What to do next with/and for students	44%
4a: Feeling like a learner again	30%	3a: What students need to be doing next	41%
3b: What to do next with/and for students	30%	4b: What learning context is like for students	26%
		2b: Understanding students' progress in + / –	26%

The presence of elements (category 4) for both groups about what it was like being a learner may reflect an emphasis within the interventions experiential delivery style, with a focus on learning through doing rather than being told.

In general, the ratings of the remaining elements (not shown here) reflect the 2008 group's overall ratings for categories 2, 3, and 4, while those for the 2009 group do not, with two elements of category 5 ranking higher than others.

Similarly to the session data, the categories of in-class elements identified by the teachers as being of most help in their learning (see Table 5) varied between the two groups, but the overall distribution patterns were generally similar. With the small 2009 sample size and even smaller number who responded to the in-class part of the evaluation, the percentage figures need to be treated with caution because a single item response represents 5% of the total.

Also, some of the variation between 2009 and 2008 responses might have arisen from: the reduced level of choice in the 2009 version of the in-class form, in which there were two fewer items in category 2 and 3 and one fewer in category 1, due to the removal of items related to types of interaction with the facilitator; and the addition of a new category (8) of one item dealing with any interactions with the facilitator.

[2] See Appendix V, Part A, for the full text of the grid elements.

[3] The percentages add to more than 100% because each participant was asked to identify up to three elements and some identified more than three.

Table 5
The Percentage of Actual Responses to In-class Categories of Elements Perceived as Most Helpful for Both the 2009 and 2008 SSS Groups

Category	**SSS 2009** (n = 6)	**SSS 2008** (n = 28)
1[4]: Being observed and follow-up discussion	10	7
2: Observing various people teaching	26	35
3: Sharing aspects of teaching practice	16	15
4: Resource acquisition	10	21
5: Choosing the type of support	5	1
6: Discussing own numeracy issues	21	14
7: Release time to work with students	5	7
8: Informal interaction with facilitator	5	NA

The four most highly ranked categories (10% or more for both groups), 2, 3, 4, and 6, were the same for each group, with the "observing various people teaching" category (2) ranking highest for both groups.

The number of responses to individual elements generally confirm the pattern in Table 4, except for the high rating of the single category 6 element concerning discussion focused on the teachers' own issues, which was rated first by the 2009 group and second by the 2008 group (see Table 6). This may be because this category contained only one element and teachers were able to choose multiple elements elsewhere. This lack of an aggregation possibility could reduce the overall rating for category 6.

Table 6
In-class Category Elements[5] Perceived as The Three (and Third Equal) Most Helpful for Both the 2009 and 2008 SSS Groups

SSS 2009 (n = 6)		**SSS 2009** (n = 28)	
6: Discussing own numeracy issues	67%[6]	2c: Observing a facilitator teaching	46%
2d: Observing a lead teacher	50%	6: Discussing own numeracy issues	39%
2e: Observing another teacher	50%	2a: Observing a facilitator teaching my class	25%
4a: Resources from facilitator	25%		

In contrast to the session data, only one of the highest-rated elements (6) for both groups was the same, with each group ranking this element in a different order. In line with the overall category (2) data, "observing others" elements were highly ranked for both groups. However, there were different elements chosen by each group. There was a focus on the facilitator in 2008 (elements 2d and 2e) and on in-school teachers in 2009 (elements 2a and 2c). This difference in elements most likely reflects the non-availability of the facilitator for the 2009 group. However, it does raise issues of how important the availability of an outside expert to be observed might be as a component in maximising the impact of numeracy pick-up professional development.

4 The categories are numbered here for reference purposes only, to enable links to be made to grid elements within the text and the appendix.

5 See Appendix V, Part B, for the full text of the grid elements.

6 The percentages add to more than 100% because each participant could identify up to three elements.

The presence of element 4a in the 2009 group's higher ratings reflects this group's overall category rating for resource acquisition. In general, the ratings of the remaining elements (not shown here) reflect the 2008 group's overall ratings for the categories and the 2009 group's higher overall ratings for categories 2 and 4.

Discussion

There was clear evidence of growth in the 2009 teachers' NDP strategy and content knowledge over the duration of the sessions, with particular evidence of increased understanding of fraction, decimal, and percentage work. The increase in teachers' scores overall was statistically significant in terms of assigned stages and improved strategy and content knowledge. This indicates that this 2009 intervention was effective in dealing with these aspects of NDP. However, the small sample size suggests caution in applying findings generally. Also, issues around the use of algorithmic rather than NDP approaches suggests that the extent of the growth may not be as large as the data indicates. However, it does clearly indicate an increase in knowledge and understanding of the materials presented in the sessions. This would support the continued use of such SSS interventions in assisting teachers new to NDP.

Both groups of participants perceived that their interventions were successful. The evaluation data shows that each of the session and in-class components of the interventions were perceived to be of value and helpful in meeting teachers' needs, as were the time and energy expended in attending the sessions. While the data sets for the two interventions showed differences, the general distribution of responses was very similar, allowing for the small sample size of the 2009 group.

All the categories, with the exception of the in-class "choosing the type of support", were selected by more than one teacher for both groups, indicating a general level of relevance of all categories. While the category ratings varied within each group, the rating of certain categories indicated the greater relevance of some of these to both groups of teachers, namely, the in-sessions category of "aspects of development" and the in-class support category of "observing various people teach". With the latter category, the elements chosen were different for the two groups, with the difference being around the un-availability of the facilitator for support outside the sessions for the 2009 group. This suggests that consideration be given to an ongoing role for facilitators in the area of in-class support as part of their role in delivering pick-up NDP programmes.

These category ratings could be used to refine professional development programmes and guide the ongoing refinement of the categories and items in the evaluation form for future evaluation and research purposes.

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The Impact of a University Course Focusing on PCK and MCK: Does Teachers' Classroom Practice Reflect the Professional Development Experience?

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This paper examines the extent to which primary school teachers incorporated elements of a university course focusing on "reform" practices in the development of mathematics (number) content and pedagogical content knowledge into their classroom teaching practice. Previous studies had shown that, as a result of undertaking this university course, teacher participants' mathematical content knowledge and pedagogical content knowledge for teaching had improved significantly and that teachers valued the course and found it useful for their classroom teaching practice. However, research evidence indicates that even when teachers are knowledgeable about what they need to do to effect change and express the desire to do so, such change does not necessarily happen. To examine the extent to which the university course impacted on the participants' actual teaching practice, data was collected via classroom observations, using the Reformed Teaching Observation Protocol. Although the data showed varying levels of reform-aligned practices in the teachers' classrooms, all the teachers demonstrated elements of reform practices, which suggests either an adoption of, or a shift toward, more reform-oriented practices.

Introduction

New Zealand's Numeracy Development Projects (NDP) were developed to change the way that teachers delivered the number strand of the New Zealand mathematics curriculum (Ministry of Education, 1993) and to promote constructivist-aligned teaching approaches (also commonly referred to as reformed or inquiry-based). Such approaches are perceived by many leading educationalists as being best suited to meeting the objectives of recent educational reforms (for example, Borasi & Fonzi, 2002). Central to this notion of reformed teaching is the assertion that children's learning subject matter "is the product of an interaction between what they are taught and what they bring to any learning situation" (Ball, 1988, p. 1) or, more specifically, the co-construction of knowledge by teacher and students alike. The ultimate goal is that teachers adopt what von Glasersfeld (1995) describes as a "constructivist way of thinking" (p. 178). The rationale is that once teachers are attuned to this way of thinking, they can more readily transfer what they have learnt to their classroom practices.

The American National Council of Teachers of Mathematics (NCTM) has produced a comprehensive approach to implementing reformed teaching using a standards-driven approach. The NCTM Standards are guided by five main goals that encompass the "spirit" of the Standards. Implicit in these goals is that students are expected to:

- become better problem solvers
- learn to reason mathematically
- learn to value mathematics
- become more confident in their mathematical ability
- learn to communicate mathematically. (NCTM, 1989)

These goals are reflected in changes in the United States of America mathematics curricula, assessment, and professional teaching practices (NCTM 1989; 1991; 1995; 2000) and in the sequence of New Zealand curriculums (Ministry of Education, 1993; 2007).

Anderson, Anderson, Varank-Martin, et al. (1994) describe the reforming of teaching and learning as being a movement away from traditional didactic practice towards learning that is compatible with constructivist learning theory. Constructivists (for example, Cobb, Wood, & Yackel, 1991; Steffe & Kieren, 1994; von Glasersfeld, 1995; Steffe & D'Ambrosio, 1995) advocate the use of manipulative materials to assist in developing students' understanding of concepts so that what begins as a manipulative action is internalised, eventually becoming a purely mental activity where students can work with number relationships in the abstract. Reform presupposes that teachers do not emphasise lecturing (using a didactic approach) but rather stress a problem-solving approach and the fostering of active learning (Frantz, Lawrenz, Kushner, & Millar, 1998). In such classrooms, students are grouped according to their learning needs and there is a clear focus on social discourse and the development of thinking skills and, most importantly, on students developing a relational understanding (Skemp, 1976) of the mathematics being learned. Research evidence (for example, Hiebert, 2003) indicates that where reformed teaching methods are implemented consistently, students learn more, and more deeply, than in traditional programmes.

Implicit in this are a number of ramifications for mathematics teachers and the way in which they teach mathematics. Foremost among these is that for a teacher to be able to help a student learn relationally, they must have a relational understanding (Skemp, 1976) of what they themselves are teaching. Ball (2008) stresses that teachers must have the mathematical knowledge for teaching (MKT) that they need in order to "hear their students, to teach content clearly, to be precise, to teach them to reason and to solve and to have the skills they need to do this with every one of their students" (p. 2). In addition, in the New Zealand context, teachers should use practical activities structured to align with the use of the NDP Teaching Model.

A key focus of the Teaching Model is on teachers "noticing" what students are saying and doing (that is, formatively evaluating each student's progress throughout each teaching session) and focusing the students on what they are doing and what is being said and done by others. Ball (1988) holds that students' learning of mathematics subject matter is the product of an interaction between what they are taught and what they bring to any learning situation and that an inquiry-based, reformed, approach to learning and teaching is firmly underpinned by constructivist principles. Ball's work supports the NDP stance that the teacher's role is to listen, promote discussion, and encourage students to explain their mathematical thinking in a climate of co-operation between the teacher and the students.

In addition to noticing, the NDP and Teaching Model promote:

- developing teachers' ability to pose situations (present problems) that will create the necessary perturbation to focus the students' attention on the task in hand;
- the ongoing challenging of students to extend their thinking;
- pacing lessons so that students can accommodate what is being taught;
- encouraging interactive mathematical communication;
- having students present their solutions and record their thinking as it is presented;
- encouraging students to reflect on their solutions (both effective and ineffective);
- making generalisations from their solutions;
- the determining, by reflection, which solution(s) appear to be the most workable or efficient;
- having students write problems for which a particular solution strategy would be the most efficient;
- developing management techniques that will facilitate the type of learning being promoted.

These elements were emphasised throughout the university course, which was specifically designed to support the development of MKT and pedagogical content knowledge (PCK) and promote the use of the Teaching Model in line with the NDP initiative.

The Reformed Teaching Observation Protocol (RTOP)

RTOP is a classroom observation instrument of 25 items (Sawada, Piburn, Falconer, et al., 2000) that embodies the recommendations and standards for the reformed teaching of mathematics (and science) in the United States (NCTM, 1989; 1991; 1995). It gives a quantitative characterisation of the extent to which the mathematics teaching in a particular classroom is in accord with reformed teaching practices, in which the primary focus of reform is on the extent of an "inquiry orientation" within a teacher's classroom practice (Piburn & Sawada, 2000).

RTOP, as a tool to determine how reformed a particular teacher's teaching practice is, has high levels of reliability, at both instrument and sub-scale levels, and validity. It has also been found to be a strong predictor of how much students learn in their classrooms at all levels (Sawada, Piburn, Judson, et al., 2002).

The 25 RTOP scale items are divided into five sub-scales of five items each, with two sets of two sub-scales being grouped together (see Appendix W). The first aspect, *Lesson Design and Implementation,* is assessed by sub-scale 1, which examines the pedagogical setting within which the learning is taking place. The supposition is that a reformed lesson begins by acknowledging and respecting the ideas that the students bring to the classroom. In addition, these items afford an assessment of the level of engagement by the students in exploration in small discussion groups and the effective use of such things as modelling books before attempts are made at explication or generalising. This also involves an assessment of how skilled the teacher is at eliciting ideas from the students and how such information is used to orchestrate the lesson and to extend the students' thinking. For this study, a significant focus with regard to the ranking given was the appropriateness of the materials and examples that were used and the teacher's facility in the use of the NDP Teaching Model.

The second aspect, *Content*, or pedagogical content knowledge (PCK), is assessed by sub-scales 2 and 3, with 2 dealing with the quality of the lesson (propositional pedagogical knowledge) and 3 with the process of inquiry (procedural pedagogical knowledge). They assess the teacher's understanding of the key concept(s) that they are endeavouring to help the students learn, the possible misconceptions that the students might possess, and ways to help them overcome them. It also takes into consideration the teacher's ability to contextualise the lesson by, where appropriate, making connections with other disciplines and real-life situations.

The third aspect, *Classroom Culture* (or climate), is assessed by sub-scales 4 and 5, with 4 dealing with "communicative interactions" and 5 with "student/teacher relationships". They assess the effectiveness of communication (that is, the extent to which it is decentralised), how supportive and non-threatening the learning environment is, and the level of mathematical challenge evident within the classroom.

The RTOP scores can also be examined in terms of three factors: inquiry orientation, content propositional knowledge, and collaboration (see Table 1). Factor 1 includes items from four of the sub-scales (1, 3, 4, and 5); factor 2 includes items only from sub-scale 2; and factor 3 includes items from sub-scales 4 and 5. The first two factors align closely with the face validity of the instrument that focuses on inquiry orientation and content knowledge (Piburn & Sawada, 2000).

Table 1
Factors and Associated Items

Factors	Items
1. Inquiry orientation	1–5, 11–22, 24, and 25
2. Content propositional knowledge	6–10
3. Collaboration	20, 21, and 23

Further examination of the RTOP scores can be carried out in terms of the level of intersection of the three factors, which give rise to five groupings of items (see Table 2) plus two unassigned items. Apart from grouping 2, whose items all came from sub-scale 3 (procedural pedagogical knowledge), each grouping had items from a number of sub-scales.

Table 2
Groupings and Associated Items and the Intersection of Factors

Groupings	Items	Factor(s)
1. Pedagogy of inquiry	3, 4, 11, 12, 13, 14, and 16	1
2. Content base of lesson	6, 7, and 10	2
3. Content pedagogical knowledge	1, 5, 15, and 22	1 and 2
4. Community of learners	2, 18, 20, 21, 24, and 25	1 and 3
5. Reformed teaching	9, 17, and 19	1, 2, and 3

Method

Previous research undertaken with the teachers in this study assessed the effect that the university course had on the teachers' MCK, through a standardised (objective) test, and on their PCK, through a written interpretation of a teaching scenario and what they would do next (Ell, Lomas, Cheeseman, & Nicholas, 2008), and the teachers' positive views (espoused) on the value and usefulness of the course for their classroom practice (Lomas, 2009). This study moves beyond teachers' espoused views on their classroom practice to an external assessment of the teacher participants' actual classroom practice and the extent to which reform practices predominate.

The data for this study was collected by a single classroom observation for each teacher due to time and funding constraints. The observations were carried out by observers trained in the use of RTOP.

Participants

The seven female participants (labelled T1 to T7) for this study were self-selected from the group of 26 primary school teachers approached who had undertaken the university course. Five of the seven had gained high grades where the course was taken for credit, and all had demonstrated high levels of uptake of key ideas in the course sessions or in working with the facilitators (see Table 3). Five of the teachers taught in low decile, multi-cultural schools, while two (T6 and T7) taught in a decile 8 school. The teachers' experience ranged from two years of teaching to over 20 years. One of the teachers (T4) was overseas-trained, with 10 of their 16 years' experience being in South African schools. All of the teachers except T5 had been with their classes from the start of the year when the observations occurred and had well-established classroom routines that allowed teaching and learning to occur. T5 had (1 month prior to the observation) taken over a class that was considered to be "out of control" and was only at the initial stages of establishing such classroom routines.

Table 3
Teacher Participants' Experience, Current Teaching Class Year Level, and Course Grade

Teachers	T1	T2	T3	T4	T5	T6	T7
Years of experience	2	2.5	2	16	5	20+	20+
Teaching-class year level	1	3	8	7	7	3	6
Course grade	na	A–	na	B	B+	A+	A+

Five of the seven (T1 to T5) had attended as part of a local area professional development initiative: two of these as beginning teachers (T1 and T3) and the other three as part of an "expert teacher" of numeracy focus. As part of the initiative, these five teachers received in-class support from NDP facilitators, who worked with them on their classroom practice and assisted them to implement key practices promoted as part of the university course. The other two teachers attended independently and had no direct support in implementing key practices.

Data Collection and Analysis

One observation visit was made to each teacher's classroom, with most observations lasting for just over an hour and the RTOP instrument being used to record key aspects of teacher behaviours. For the first two observations, two observers independently ranked the performance of the teacher participants and the rankings were then compared. The rankings were found to be very similar; points of difference were discussed and an agreed interpretation arrived at for these and further observations. The five remaining sessions were observed by a single observer.

Of the lessons observed, six focused on the development of concepts in the number and algebra strand and the other on the statistics strand. In the latter case, the teacher (T7) wanted to demonstrate how they were incorporating NDP and course principles into the teaching of a strand of the mathematics and statistics learning area other than number and algebra (Ministry of Education, 2007). After each lesson, the observer and the teacher had a discussion that afforded an opportunity to gain clarification, as required, regarding aspects of the lesson and specific teacher actions, including interactions with particular students.

The RTOP data was initially analysed in terms of the overall response, the three aspects, and the five sub-scales and then in terms of the three factors and five-factor intersection groupings, for alignment with reform-oriented teaching.

Each item was scored from 0 to 4 as follows: 0, the behaviour never occurred; 1, the behaviour occurred at least once; 2, the behaviour occurred more than once or very loosely describes the lesson; 3, a frequent behaviour or fairly descriptive of the lesson; 4, pervasive behaviour or extremely descriptive of the lesson. On this basis, a score of 0 or 1 represents a predominance of traditional practices, 2 suggests an awareness and adoption/use of some reform practices, and 3 or 4 represents a predominance of reform practices. Score totals across sets of items (overall, aspects, sub-scales, groupings) can range from zero to four times the number of items being considered, with higher total scores representing a greater predominance of reform practices and lower total scores representing a greater predominance of traditional practices. Means (M) were calculated for each group for comparison purposes, as were standard deviations (SD) for groups of seven or larger.

Findings and Discussion

The mean total of participant scores for all items was 85, with SD = 14.3. These figures show a frequent to pervasive occurrence of reform practices in the teachers' classrooms overall. They represent a stronger reform alignment for this group of teachers than for American university staff who had a commitment to reform, teaching either mathematics content (M = 61.7, SD = 20.9, N = 55) or methods courses (M = 80.2, SD = 10.9, N = 12) (Piburn & Sawada, 2000). They also show a significantly stronger reform alignment than American middle school teachers (M = 48.5, SD = 16.3, N = 28) who were also working with a reform curriculum (Sawada et al., 2002).

The teachers' total scores (see Appendix X) fall into three bands, in which reform-oriented teaching practices were: pervasive (T4, T6, and T7, M = 97.7); frequent to pervasive (T1 and T2, M = 85.5); or occurred more than once to frequent (T3 and T5, with M = 66.0). The means of the two upper bands are higher than all the means for the American university staff or middle school teachers, and the lower-band teachers' mean score is higher than those for the university mathematics content staff and middle school teachers. This shows that all the teacher participants in this study had a stronger reform alignment than many of the people in the American studies. The difference between these American and New Zealand (NZ) teachers may be partly explained by underlying (systemic) differences. However, the stronger alignment overall is indicative of something more than the influence of underlying differences.

The three teachers in the pervasive band are all experienced teachers with more than 16 years of teaching experience, while the teachers in the other bands have fewer years (five or fewer) of experience (see Table 3). This suggests that experience may have been a positive factor in assisting those three pervasive-band teachers to adopt reform-oriented practices. However, T5, with 5 years' experience, had a lower score, which positioned them in the lower band and was lower than others with about two years' experience. T5's score may have been partly caused by the unsettled nature of the classroom in which the teacher found themselves, and this may have overridden any experience effect in comparison with the less experienced teachers (T1, T2, and T3).

The banding of the teachers' scores was reflected in sub-scales 2, 3, and 5 and thus also in the Content aspect (sub-scales 2 and 3 combined). In the other two sub-scales, there were still three bands, but T2 scored as pervasive in sub-scale 1 (the first aspect – Lesson Design and Implementation) and T3 as frequent to pervasive in the fourth sub-scale and therefore as frequent to pervasive for the third aspect – Classroom Culture (sub-scales 4 and 5 combined).

In addition, there was a general consistency for each teacher's individual scores across the items, with teachers such as T4, T6, and T7 scoring mainly 4's, with only two, two, and three 3's respectively; T1 and T2 scoring mainly 3's and 4's; and T3 and T5 scoring mainly 2's and 3's. Thus, for five of the teachers, there was a predominance of reform practices in their classroom teaching for each aspect (and sub-scale), indicating that all of these aspects of reform were generally evident in the teachers' classroom practice. The other two teachers, while not having a predominance of reform practices evidenced in their total score, did score at the frequent (or better) level for about half the items. These scores suggest the adoption of, or move toward, the use of reform practices in their classroom teaching for each aspect. The consistency across the sub-scales and aspects for the teachers in each band indicates a relatively widespread and even use of reform practices rather than the use of a small number focused on only one or two reform practices.

There is, however, a little more variation between the factors, with some of the teachers in higher bands for factors 2 and 3 indicating a small increase in the reform alignment for these two factors. The banding for the overall total scores is reflected for factor 1 (Inquiry orientation), with a mean total of 64.7 (out of 76 possible) and SD = 10.8 with the same teachers in each of the three bands. Thus for the same five teachers (as for the overall total score), there was a predominance of reform practices in their classroom teaching for factor 1. For the other two teachers, the scores suggest the adoption of, or move toward, the use of reform practices for this factor. This is not unexpected, given that factor 1 includes all but 6 of the 25 items and thus is likely to reflect the overall total score, particularly given the considerable degree of consistency in teachers' scores across the items.

For factors 2 (M = 16.7, out of a possible 20) and 3 (M = 11.3, out of a possible 12), there are only two bands evident in each case. For factor 2 (Content propositional knowledge), T3 and T5's total scores indicated that the practices occurred more than once to frequently and for the other five, pervasively. Thus, for the same five teachers (as for the overall total score), there was a predominance of reform practices in their classroom teaching, although for this factor all five scored at the pervasive level.

For factor 3 (Collaboration), T5's total score indicated that the practices occurred frequently and for the other 6, pervasively. Thus for all seven teachers, there was a predominance of reform practices in their classroom teaching. The banding for factor 3 and the number of teachers in the uppermost band indicates a stronger alignment with reform practices for this factor than for overall or for factors 1 and 2.

The extent of variation between the groupings of items is similar to that for factors. Three of the groupings – Pedagogy of inquiry (1), Content base of lessons (2), and Content pedagogical knowledge (3) – reflect the banding of the overall total score with the same teachers in each band. Thus for the same five teachers, there was a predominance of reform practices in their classroom teaching for these groupings. The other two teachers, while not having a predominance of reform practices evidenced in their total score, do have the frequent occurrence for some of the items, which suggests the lesser adoption of reform practices for these groupings.

Grouping 4 of items (Community of learners) also has three bands, but with T5's score indicating a frequent occurrence of reform practices, T3's score indicating a frequent to pervasive occurrence, and the other 5 scores indicating a pervasive occurrence. For this grouping, all seven teachers exhibited a predominance of reform practices in their classroom teaching. This represents a stronger alignment than groupings 1, 2, and 3; overall; and factors 1 and 2, and a slightly weaker alignment than for factor 3. The similarity to factor 3 may be partly explained by the inclusion of two of the three factor 3 items among the six items of grouping 4. Indeed, it would be expected that collaboration would be an intrinsic feature of a community of learners.

Grouping 5 (Reformed teaching) has two only bands, with T1, T2, T3, and T5's scores indicating a frequent occurrence of reform practices and T4, T6, and T7's scores indicating a pervasive occurrence. For this grouping, all seven teachers exhibited a predominance of reform practices in their classroom teaching. This represents a stronger alignment than groupings 1, 2, and 3; overall; and factors 1 and 2, and a weaker alignment than for grouping 4 and factor 3. As this grouping is the strongest indicator of the prevalence of reform practices, it indicates a split between three teachers who can be considered to consistently teach using reform practices and four teachers who frequently use most, or frequently use some, reform practices but cannot yet be considered fully "reform teachers".

All the teachers' RTOP scores show levels of reform practice that are greater than occasional use and indicate a deliberate inclusion of such practices. While the teachers' scores show varying levels of using reform practices, they all show the use of a range of reform practices.

Conclusion

The purpose of this study was to determine the extent to which teachers had incorporated elements of an academic university course, which focuses on the development of mathematics [number] knowledge for teaching (MKT) and pedagogical content knowledge (PCK), into their classroom teaching practice overall. The data shows a clear predominance of reform-aligned practices in five of the teachers' overall classroom practice. These teachers also show a similar predominance for all factors, aspects, and groupings. Where the other teachers' scores indicate a more limited alignment, this may possibly be explained by the teachers having less overall teaching experience (T1, T2, and T3) and, in one case (T5), by the nature of the classroom environment and the relatively short length of time the teacher had been with the particular class. Even for the two teachers who did not have a predominance of reform practices, there is clear evidence of the use of a range of reform practices across all aspects, factors, and groupings. Indeed, for some factors (3) and groupings (4 and 5), these teachers scored more highly, indicating greater use of particular reform practices associated with the items included in them.

Comparison with American teaching staff show a much stronger reform alignment than the American groups demonstrated, indicating that the teachers in this study had developed a significant commitment to reform ideas and practices. The three experienced teachers (T4, T6, and T7) would have experienced the NDP professional development programme in their schools, while the less experienced teachers would have been exposed to the NDP materials and approaches to varying extents within their particular pre-service teacher education programme and within their schools' everyday teaching. However, as the university course was the principle ongoing source of reform ideas and practice, it is likely that it both promoted and maintained their use and underpins much of the teachers' reform-oriented practice.

This study has a number of limitations when trying to draw conclusions and generalise from the data: the sample was small and self-selected; in the only observation in the year after the course, the teachers may have been on their "best" behaviour or the nature of the topic may have limited (or enhanced) the potential for the demonstration of some reform practices'; and there were no before and after observations that would link any growth in reform practices more specifically to the course.

The university course had a positive effect on both MKT acquisition and PCK development, and the teachers expressed positive views on the value and usefulness of the course. These effects, and the findings of this study, suggest that the course may well have had a positive impact on the teachers' classroom practice in encouraging and supporting reform practices. Thus, the university course appears to be an effective professional development mechanism in assisting in changing teachers' classroom practice.

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Appendices (Findings from The New Zealand Numeracy Development Projects 2009)

Appendix A (A Decade of Reform in Mathematics Education: Results for 2009 and Earlier Years)

Percentages[1] of Students at Each Stage on the Number Framework for Each Domain (2009)

Final Stage 2009	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
STRATEGIES									
Number of Students	*85*	*95*	*100*	*95*	*99*	*112*	*336*	*343*	*380*
Additive Domain									
0–3 Counting all	57	23	8	3	2				
4 Counting on	44	56	49	19	11	3	8	4	6
5 Early Additive		19	37	67	43	24	23	22	20
6 Advanced Additive		2	6	11	38	54	49	48	43
7 Advanced Multiplicative					5	19	20	25	31
8 Advanced Proportional						1	0	2	0
Stage 4+	*44*	*77*	*92*	*97*	*98*	*100*	*99*	*100*	*100*
Stage 5+	*0*	*21*	*43*	*78*	*87*	*97*	*92*	*97*	*93*
Stage 6+	*0*	*2*	*6*	*11*	*44*	*73*	*69*	*75*	*74*
Stage 7+	*0*	*0*	*0*	*0*	*5*	*20*	*21*	*27*	*31*
Multiplicative Domain									
Not assessed	87	25	10	2	2				
2–3 Counting all	7	25	13	6	1	2	1		
4 Skip-counting	6	41	46	32	15	5	7	5	6
5 Repeated addition		8	26	48	34	29	17	14	15
6 Early Multiplicative			4	10	40	38	35	34	31
7 Advanced Multiplicative			1	1	7	21	30	30	31
8 Advanced Proportional				1	0	6	10	17	18
Stage 4+	*6*	*50*	*77*	*92*	*97*	*98*	*99*	*100*	*100*
Stage 5+	*0*	*8*	*31*	*60*	*82*	*94*	*92*	*95*	*94*
Stage 6+	*0*	*0*	*5*	*12*	*48*	*65*	*75*	*81*	*80*
Stage 7+	*0*	*0*	*1*	*2*	*7*	*27*	*40*	*47*	*49*
Proportional Domain									
Not assessed	93	40	13	2	2	2			
1 Unequal sharing	0	2	2	1	0	0			
2–4 Equal sharing	7	50	61	55	8	4	9	4	7
5 Early Additive		7	21	24	48	32	28	20	21
6 Advanced Additive		1	2	16	23	30	17	18	16
7 Advanced Multiplicative			1	2	19	30	39	44	43
8 Advanced Proportional						5	6	14	13
Stage 5+	*0*	*7*	*24*	*42*	*90*	*97*	*90*	*96*	*92*
Stage 6+	*0*	*1*	*3*	*18*	*42*	*64*	*63*	*76*	*72*
Stage 7+	*0*	*0*	*1*	*2*	*19*	*35*	*46*	*58*	*56*

[1] Boxes indicate the expected stage at particular year levels in *The New Zealand Curriculum*.

Appendix A – continued

Percentages[1] of Students at Each Stage on the Number Framework for Each Domain (2009)

Final Stage 2009	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
KNOWLEDGE									
Fractions									
Not assessed	87	25	16	2	2				
2–3 Unit fractions not recognised	11	27	9	4	2			1	1
4 Unit fractions recognised	2	41	26	35	10	7	9	4	5
5 Orders unit fractions		6	48	48	49	38	32	28	26
6 Co-ordinates numerator & denominator			1	11	30	22	24	18	22
7 Equivalent fractions					5	19	25	35	28
8 Orders fractions					2	13	9	14	19
Stage 5+	*0*	*6*	*49*	*59*	*86*	*93*	*91*	*96*	*95*
Stage 6+	*0*	*0*	*1*	*11*	*37*	*55*	*59*	*67*	*68*
Stage 7+	*0*	*0*	*0*	*0*	*7*	*32*	*34*	*50*	*47*
Place Value									
0–1 No unit	7	3	1						
2 One as a unit	44	15	3	2	2				
3 Five as a unit	11	14	9	2	2			1	
4 Ten as a counting unit	39	67	64	64	30	14	9	6	3
5 Tens in numbers to 1000/10ths		1	20	25	32	32	24	19	26
6 Hs, Ths in whole numbers/ten 10ths			3	5	25	28	35	29	26
7 10ths in decimals/orders decimals				1	8	18	17	25	15
8 Decimal conversions						8	16	21	29
Stage 5+	*0*	*1*	*23*	*32*	*66*	*86*	*91*	*94*	*97*
Stage 6+	*0*	*0*	*3*	*6*	*33*	*54*	*67*	*75*	*71*
Stage 7+	*0*	*0*	*0*	*1*	*8*	*26*	*33*	*46*	*44*
Basic Facts									
0–1 No addition facts	21	11	3	3	2				
2 Addition facts to 5	35	17	5	1	3	1	1	1	
3 Addition facts to 10	21	25	14	6	1	1	0	1	
4 Addition with 10s & doubles	22	41	53	37	16	4	1	1	2
5 Addition facts		6	21	35	32	27	13	11	13
6 Subtraction & multiplication facts			4	15	27	38	25	36	32
7 Division facts				3	17	27	55	45	47
8 Common factors & multiples					1	3	5	6	6
Stage 5+	*0*	*6*	*25*	*53*	*78*	*95*	*98*	*98*	*98*
Stage 6+	*0*	*0*	*4*	*18*	*46*	*68*	*85*	*87*	*85*
Stage 7+	*0*	*0*	*0*	*3*	*18*	*30*	*60*	*50*	*53*

[1] Boxes indicate the expected stage at particular year levels in *The New Zealand Curriculum*.

Appendix B (A Decade of Reform in Mathematics Education: Results for 2009 and Earlier Years)

Percentages[2] of Students at Each Stage on the Number Framework for Strategy Domains (2003, 2005, 2007)

Final Stage 2003–2007		Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Additive Domain 2003										
Number of Students 2003		*17638*	*18677*	*19942*	*19313*	*18665*	*19338*	*13460*	*11796*	*2223*
0	Emergent	4	1	1				2	2	1
1	One-to-one counting	10	3	1				0	0	0
2	Counting all materials	46	21	7	2	1	1	1	0	0
3	Counting all imaging	21	19	8	3	2	1	1	0	1
4	Counting on	17	43	44	31	22	15	15	10	16
5	Early Additive	2	13	35	50	51	47	43	37	49
6	Advanced Additive		1	5	13	23	36	39	49	33
Stage 4+		*19*	*57*	*84*	*93*	*97*	*98*	*97*	*97*	*98*
Stage 5+		*2*	*13*	*40*	*62*	*74*	*83*	*82*	*86*	*82*
Stage 6+		*0*	*1*	*5*	*13*	*23*	*36*	*39*	*49*	*33*
Additive Domain 2005										
Number of Students 2005		*4737*	*5048*	*5719*	*6966*	*8353*	*8689*	*6348*	*5911*	*4068*
0	Emergent	2								
1	One-to-one counting	11	3	1						
2	Counting all materials	44	16	4	1					
3	Counting all imaging	24	20	7	3	1	1			
4	Counting on	16	45	42	30	21	13	11	8	6
5	Early Additive	2	15	40	49	51	45	41	32	26
6	Advanced Additive		1	5	16	24	33	36	40	45
7	Advanced Multiplicative				1	3	7	11	19	22
Stage 4+		*18*	*61*	*87*	*96*	*98*	*99*	*99*	*100*	*99*
Stage 5+		*2*	*15*	*45*	*66*	*78*	*86*	*88*	*91*	*94*
Stage 6+		*0*	*1*	*5*	*17*	*27*	*41*	*47*	*59*	*67*
Additive Domain 2007										
Number of Students 2007		*2556*	*4222*	*4059*	*4297*	*4457*	*4499*	*7478*	*10291*	*7415*
0	Emergent	2	1							
1	One-to-one counting	8	4	1						
2	Counting all materials	42	20	8	2	1	1			
3	Counting all imaging	25	20	9	3	1	1	1		
4	Counting on	20	41	44	34	21	14	9	6	5
5	Early Additive	2	14	34	45	47	40	33	26	28
6	Advanced Additive			4	14	26	37	41	40	42
7	Advanced Multiplicative				1	3	8	15	25	19
8	Advanced Proportional							1	2	6
Stage 4+		*23*	*56*	*82*	*94*	*97*	*99*	*99*	*99*	*99*
Stage 5+		*2*	*15*	*38*	*60*	*76*	*85*	*90*	*93*	*94*
Stage 6+		*0*	*0*	*4*	*15*	*29*	*45*	*57*	*67*	*67*

[2] Boxes indicate the expected stage at particular year levels in *The New Zealand Curriculum*.

Appendix B – continued

Percentages[2] of Students at Each Stage on the Number Framework for Strategy Domains (2003, 2005, 2007)

Final Stage 2003–2007	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
2003, 2005, 2007 Final Additive									
Number of Students 2003–2007	*24931*	*27947*	*29720*	*30576*	*31475*	*32526*	*27286*	*27998*	*13706*
0 Emergent	3	1					1	1	
1 One-to-one counting	10	3	1				0	0	
2 Counting all materials	46	20	6	2	1	1	0	0	
3 Counting all imaging	22	19	8	3	1	1	1	0	
4 Counting on	17	43	44	31	22	14	12	8	7
5 Early Additive	2	13	36	49	51	46	40	32	31
6 Advanced Additive		1	5	14	23	35	39	44	41
7 Advanced Multiplicative					1	3	7	13	17
8 Advanced Proportional								1	3
Stage 4+	*19*	*57*	*84*	*94*	*97*	*98*	*98*	*98*	*99*
Stage 5+	*2*	*14*	*41*	*63*	*75*	*84*	*86*	*90*	*92*
Stage 6+	*0*	*1*	*5*	*14*	*25*	*38*	*46*	*58*	*61*
Multiplicative Domain 2003									
Number of Students 2003	*17638*	*18677*	*19942*	*19313*	*18665*	*19338*	*13460*	*11796*	*2223*
N/A	89	56	23	9	5	3	4	4	4
2–3 Counting all	5	12	11	6	4	2	2	1	2
4 Skip-counting	6	26	40	36	25	17	14	10	15
5 Repeated addition	1	5	18	28	30	27	24	20	22
6 Early Multiplicative		1	7	18	28	34	35	34	36
7 Advanced Multiplicative			1	3	9	18	21	32	22
Stage 4+	*6*	*32*	*66*	*85*	*92*	*95*	*94*	*95*	*95*
Stage 5+	*1*	*7*	*26*	*49*	*67*	*78*	*80*	*85*	*79*
Stage 6+	*0*	*1*	*8*	*21*	*37*	*52*	*56*	*66*	*58*
Stage 7+		*0*	*1*	*3*	*9*	*18*	*21*	*32*	*22*
Multiplicative Domain 2005									
Number of Students 2005	*2512*	*3625*	*5106*	*6694*	*8210*	*8590*	*6304*	*5852*	*4067*
N/A	82	33	8	3	1	1	1	0	0
2–3 Counting all	8	18	13	6	4	2	1	1	0
4 Skip-counting	8	40	49	38	25	15	12	8	6
5 Repeated addition	1	8	21	30	31	25	23	19	16
6 Early Multiplicative		1	8	19	30	36	38	36	32
7 Advanced Multiplicative			1	4	8	17	20	26	30
8 Advanced Proportional					1	4	6	11	15
Stage 4+	*9*	*49*	*79*	*90*	*95*	*97*	*98*	*99*	*99*
Stage 5+	*1*	*9*	*30*	*52*	*70*	*82*	*87*	*91*	*93*
Stage 6+	*0*	*1*	*9*	*23*	*39*	*57*	*63*	*72*	*78*
Stage 7+	*0*	*0*	*1*	*4*	*9*	*21*	*26*	*36*	*46*

2 Boxes indicate the expected stage at particular year levels in *The New Zealand Curriculum*.

Appendix B – continued

Percentages[2] of Students at Each Stage on the Number Framework for Strategy Domains (2003, 2005, 2007)

Final Stage 2003–2007		Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Multiplicative Domain 2007										
Number of Students 2007		*2956*	*5074*	*4907*	*5014*	*5071*	*5123*	*8385*	*10797*	*8418*
N/A		86	55	25	11	7	5	3	2	0
2–3	Counting all	7	13	12	6	4	2	2	1	1
4	Skip-counting	6	26	40	35	23	14	10	7	6
5	Repeated addition	1	5	18	29	30	27	22	17	16
6	Early Multiplicative		1	5	15	25	32	34	32	34
7	Advanced Multiplicative			1	4	9	17	24	29	31
8	Advanced Proportional					1	3	6	12	12
Stage 4+		*7*	*32*	*64*	*83*	*89*	*92*	*96*	*96*	*99*
Stage 5+		*1*	*6*	*24*	*48*	*66*	*78*	*86*	*90*	*93*
Stage 6+		*0*	*1*	*5*	*19*	*35*	*51*	*64*	*73*	*77*
Stage 7+		*0*	*0*	*1*	*4*	*10*	*20*	*30*	*41*	*43*
Multiplicative 2003–2007										
Number of Students 2003–2007		*23106*	*27376*	*29955*	*31021*	*31946*	*33051*	*28149*	*28445*	*14708*
N/A		88	53	21	8	4	3	3	2	1
2–3	Counting all	6	13	12	6	4	2	2	1	1
4	Skip-counting	6	28	42	36	25	16	13	8	7
5	Repeated addition	1	6	19	29	30	26	23	19	17
6	Early Multiplicative		1	7	18	28	34	35	33	34
7	Advanced Multiplicative			1	3	9	17	22	29	29
8	Advanced Proportional						1	3	7	11
Stage 4+		*6*	*34*	*68*	*86*	*92*	*95*	*96*	*96*	*98*
Stage 5+		*1*	*7*	*26*	*50*	*67*	*79*	*83*	*88*	*91*
Stage 6+		*0*	*1*	*7*	*21*	*37*	*53*	*60*	*70*	*74*
Stage 7+		*0*	*0*	*1*	*3*	*9*	*19*	*25*	*36*	*41*
Proportional Domain 2003										
Number of Students 2003		*17638*	*18677*	*19942*	*19313*	*18665*	*19338*	*13460*	*11796*	*2223*
N/A		89	57	23	9	6	4	4	4	4
0–1	Unequal sharing	3	8	9	6	4	3	2	2	4
2–4	Equal sharing	8	31	48	44	33	23	21	15	20
5	Early Additive		4	16	27	29	27	26	22	23
6	Advanced Additive		1	4	11	19	24	25	26	26
7	Advanced Multiplicative			1	3	8	16	17	21	16
8	Advanced Proportional					1	3	5	11	8
Stage 5+		*0*	*5*	*20*	*40*	*58*	*71*	*73*	*80*	*72*
Stage 6+		*0*	*1*	*4*	*13*	*28*	*43*	*47*	*58*	*49*
Stage 7+		*0*	*0*	*1*	*3*	*9*	*19*	*22*	*32*	*23*

2 Boxes indicate the expected stage at particular year levels in *The New Zealand Curriculum*.

Appendix B – continued

Percentages[2] of Students at Each Stage on the Number Framework for Strategy Domains (2003, 2005, 2007)

Final Stage 2003–2007	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Proportional Domain 2005									
Number of Students 2005	*2522*	*3640*	*5061*	*6667*	*8195*	*8564*	*6277*	*5824*	*4057*
N/A	83	33	8	3	1	1	1	0	0
0–1 Unequal sharing	5	8	7	3	2	1	1	1	0
2–4 Equal sharing	11	53	60	47	32	22	16	12	7
5 Early Additive	1	5	21	31	35	31	29	23	23
6 Advanced Additive			4	12	21	27	28	28	17
7 Advanced Multiplicative			1	3	8	16	21	28	41
8 Advanced Proportional					1	2	4	9	12
Stage 5+	*1*	*5*	*26*	*46*	*65*	*76*	*83*	*88*	*92*
Stage 6+	*0*	*0*	*5*	*15*	*30*	*45*	*53*	*64*	*70*
Stage 7+	*0*	*0*	*1*	*3*	*9*	*18*	*25*	*36*	*53*
Proportional Domain 2007									
Number of Students 2007	*2956*	*5074*	*4907*	*5014*	*5071*	*5123*	*8385*	*10797*	*8418*
N/A	86	55	26	13	10	7	3	3	0
0–1 Unequal sharing	4	6	7	4	2	1	1	1	0
2–4 Equal sharing	10	34	48	41	30	19	15	11	8
5 Early Additive		4	16	27	29	28	25	20	22
6 Advanced Additive		1	3	11	20	26	26	25	18
7 Advanced Multiplicative				3	9	17	25	29	38
8 Advanced Proportional					1	2	5	11	12
Stage 5+	*0*	*5*	*20*	*41*	*58*	*73*	*81*	*86*	*91*
Stage 6+	*0*	*1*	*3*	*14*	*29*	*45*	*56*	*65*	*68*
Stage 7+	*0*	*0*	*0*	*3*	*10*	*18*	*30*	*40*	*50*
Proportional Domain 2003–2007									
Number of Students 2003–2007	*23116*	*27391*	*29910*	*30994*	*31931*	*33025*	*28122*	*28417*	*14698*
N/A	88	53	21	9	5	3	3	3	1
0–1 Unequal sharing	3	7	8	5	3	2	2	1	1
2–4 Equal sharing	9	35	50	44	32	22	18	12	10
5 Early Additive		4	16	28	31	28	26	22	23
6 Advanced Additive		1	4	11	20	25	26	26	19
7 Advanced Multiplicative			1	3	8	16	20	25	35
8 Advanced Proportional					1	3	5	11	12
Stage 5+	*0*	*5*	*21*	*42*	*60*	*72*	*77*	*84*	*88*
Stage 6+	*0*	*1*	*4*	*14*	*29*	*44*	*51*	*62*	*66*
Stage 7+	*0*	*0*	*1*	*3*	*9*	*19*	*25*	*36*	*47*

[2] Boxes indicate the expected stage at particular year levels in *The New Zealand Curriculum*.

Appendix C (A Decade of Reform in Mathematics Education: Results for 2009 and Earlier Years)

Average Initial Stage on the Number Framework for the Additive and Multiplicative Domains (2003, 2005, 2007)

Initial Stage, Additive Domain	Year	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Overall	2003	1.43	2.42	3.37	4.12	4.43	4.67	4.78	4.96	4.78
	2005	1.46	2.52	3.57	4.21	4.55	4.80	4.95	5.14	5.33
	2007	1.45	2.39	3.44	4.15	4.63	4.95	5.17	5.38	5.44
	Average	**1.45**	**2.44**	**3.46**	**4.16**	**4.54**	**4.81**	**4.97**	**5.16**	**5.18**
Boys	2003	1.42	2.44	3.40	4.17	4.49	4.76	4.83	5.01	4.85
	2005	1.44	2.53	3.59	4.27	4.61	4.87	5.00	5.16	5.34
	2007	1.42	2.37	3.46	4.18	4.70	5.01	5.26	5.47	5.56
	Average	**1.43**	**2.45**	**3.48**	**4.20**	**4.60**	**4.88**	**5.03**	**5.21**	**5.25**
Girls	2003	1.45	2.39	3.35	4.07	4.37	4.62	4.72	4.90	4.71
	2005	1.49	2.51	3.55	4.15	4.47	4.70	4.83	5.00	5.14
	2007	1.49	2.41	3.41	4.13	4.56	4.87	5.08	5.29	5.34
	Average	**1.48**	**2.44**	**3.44**	**4.12**	**4.47**	**4.73**	**4.88**	**5.06**	**5.06**
Boys vs Girls	2003	*–0.03*	*0.05*	*0.05*	*0.10*	*0.12*	*0.14*	*0.11*	*0.11*	*0.14*
	2005	*–0.05*	*0.02*	*0.04*	*0.12*	*0.13*	*0.18*	*0.17*	*0.16*	*0.20*
	2007	*–0.07*	*–0.04*	*0.05*	*0.05*	*0.14*	*0.14*	*0.18*	*0.18*	*0.22*
Av = 0.10	**Average**	***–0.05***	***0.01***	***0.05***	***0.09***	***0.13***	***0.15***	***0.15***	***0.15***	***0.19***
Low Decile	2003	1.26	2.18	3.06	3.82	4.16	4.45	4.65	4.82	4.64
	2005	1.12	2.29	3.11	3.88	4.22	4.49	4.66	4.88	4.97
	2007	1.08	2.04	3.05	3.82	4.29	4.59	4.78	5.01	5.16
	Average	**1.15**	**2.17**	**3.07**	**3.84**	**4.22**	**4.51**	**4.70**	**4.90**	**4.92**
Middle Decile	2003	1.42	2.45	3.41	4.16	4.47	4.73	4.83	5.03	4.86
	2005	1.44	2.44	3.49	4.21	4.50	4.78	4.91	5.10	5.19
	2007	1.48	2.35	3.40	4.12	4.62	4.90	5.14	5.37	5.40
	Average	**1.45**	**2.41**	**3.43**	**4.16**	**4.53**	**4.80**	**4.96**	**5.17**	**5.15**
High Decile	2003	1.62	2.64	3.69	4.44	4.69	4.91	5.02	5.23	5.13
	2005	1.63	2.70	3.86	4.40	4.75	4.95	5.05	5.18	5.41
	2007	1.66	2.66	3.76	4.41	4.84	5.21	5.38	5.59	5.61
	Average	**1.64**	**2.66**	**3.77**	**4.42**	**4.76**	**5.02**	**5.15**	**5.33**	**5.38**
High vs Middle	2003	*0.20*	*0.19*	*0.28*	*0.28*	*0.22*	*0.18*	*0.19*	*0.20*	*0.27*
	2005	*0.19*	*0.26*	*0.37*	*0.19*	*0.25*	*0.17*	*0.14*	*0.08*	*0.22*
	2007	*0.18*	*0.31*	*0.36*	*0.29*	*0.22*	*0.30*	*0.24*	*0.22*	*0.21*
Av = 0.23	**Average**	***0.19***	***0.25***	***0.34***	***0.25***	***0.23***	***0.22***	***0.19***	***0.17***	***0.23***
High vs Low	2003	*0.36*	*0.46*	*0.63*	*0.62*	*0.53*	*0.46*	*0.37*	*0.41*	*0.49*
	2005	*0.51*	*0.40*	*0.75*	*0.51*	*0.53*	*0.46*	*0.39*	*0.30*	*0.44*
	2007	*0.58*	*0.62*	*0.71*	*0.59*	*0.56*	*0.62*	*0.61*	*0.58*	*0.45*
Av = 0.52	**Average**	***0.48***	***0.49***	***0.69***	***0.58***	***0.54***	***0.51***	***0.45***	***0.43***	***0.46***

Appendix C – continued

Average Initial Stage on the Number Framework for the Additive and Multiplicative Domains (2003, 2005, 2007)

Initial Stage, Additive Domain	Year	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
European	2003	1.53	2.54	3.53	4.28	4.57	4.80	4.90	5.10	4.98
	2005	1.56	2.56	3.67	4.31	4.64	4.86	5.01	5.18	5.34
	2007	1.60	2.55	3.63	4.36	4.76	5.07	5.30	5.50	5.54
	Average	**1.56**	**2.55**	**3.61**	**4.32**	**4.66**	**4.91**	**5.07**	**5.26**	**5.29**
Māori	2003	1.22	2.16	3.09	3.84	4.18	4.48	4.62	4.80	4.60
	2005	1.19	2.24	3.20	3.93	4.31	4.56	4.76	4.90	4.99
	2007	1.25	2.05	3.09	3.82	4.35	4.73	4.91	5.16	5.24
	Average	**1.22**	**2.15**	**3.13**	**3.86**	**4.28**	**4.59**	**4.76**	**4.95**	**4.94**
Pasifika	2003	1.14	2.08	2.93	3.68	4.00	4.28	4.45	4.56	4.48
	2005	1.15	2.38	3.20	3.83	4.18	4.51	4.49	4.78	4.92
	2007	1.14	2.06	3.02	3.78	4.29	4.59	4.74	4.89	5.08
	Average	**1.15**	**2.18**	**3.05**	**3.76**	**4.16**	**4.46**	**4.56**	**4.74**	**4.83**
European vs Māori	2003	*0.31*	*0.38*	*0.44*	*0.44*	*0.39*	*0.32*	*0.28*	*0.30*	*0.38*
	2005	*0.36*	*0.32*	*0.47*	*0.38*	*0.33*	*0.30*	*0.25*	*0.28*	*0.35*
	2007	*0.35*	*0.50*	*0.54*	*0.54*	*0.40*	*0.34*	*0.39*	*0.34*	*0.30*
Av = 0.37	**Average**	***0.34***	***0.40***	***0.48***	***0.45***	***0.37***	***0.32***	***0.31***	***0.31***	***0.34***
European vs Pasifika	2003	*0.39*	*0.46*	*0.60*	*0.60*	*0.57*	*0.52*	*0.45*	*0.54*	*0.50*
	2005	*0.40*	*0.18*	*0.47*	*0.48*	*0.46*	*0.35*	*0.52*	*0.40*	*0.42*
	2007	*0.45*	*0.49*	*0.61*	*0.58*	*0.46*	*0.48*	*0.56*	*0.61*	*0.46*
Av = 0.48	**Average**	***0.41***	***0.38***	***0.56***	***0.55***	***0.50***	***0.45***	***0.51***	***0.52***	***0.46***
High Decile, European	2003	1.62	2.64	3.69	4.45	4.69	4.90	5.04	5.26	5.18
	2005	1.65	2.68	3.88	4.42	4.78	4.94	5.08	5.21	5.44
	2007	1.71	2.66	3.79	4.45	4.86	5.24	5.42	5.61	5.64
	Average	**1.66**	**2.66**	**3.79**	**4.44**	**4.78**	**5.03**	**5.18**	**5.36**	**5.42**
Low Decile, European	2003	1.42	2.33	3.26	4.04	4.37	4.64	4.85	5.07	4.84
	2005	1.41	2.48	3.27	4.06	4.38	4.61	4.92	5.03	5.10
	2007	1.05	2.35	3.35	4.24	4.49	4.69	4.99	5.21	5.31
	Average	**1.29**	**2.39**	**3.29**	**4.11**	**4.41**	**4.65**	**4.92**	**5.10**	**5.08**
Decile Advantage, European	2003	*0.20*	*0.31*	*0.43*	*0.41*	*0.32*	*0.26*	*0.19*	*0.19*	*0.34*
	2005	*0.24*	*0.20*	*0.61*	*0.36*	*0.40*	*0.33*	*0.16*	*0.18*	*0.34*
	2007	*0.66*	*0.31*	*0.44*	*0.21*	*0.38*	*0.54*	*0.44*	*0.40*	*0.33*
Av = 0.34	**Average**	***0.37***	***0.28***	***0.49***	***0.33***	***0.37***	***0.38***	***0.26***	***0.26***	***0.34***
High Decile, Māori	2003	1.41	2.39	3.31	4.18	4.48	4.66	4.87	5.10	4.92
	2005	1.42	2.46	3.61	4.02	4.55	4.78	4.89	5.09	5.28
	2007	1.52	2.49	3.56	4.17	4.58	5.06	5.16	5.41	5.49
	Average	**1.45**	**2.45**	**3.50**	**4.12**	**4.54**	**4.83**	**4.97**	**5.20**	**5.23**
Low Decile, Māori	2003	1.18	2.11	3.02	3.74	4.12	4.42	4.59	4.75	4.55
	2005	1.00	2.20	2.98	3.81	4.14	4.40	4.66	4.85	4.80
	2007	1.06	1.92	2.99	3.71	4.20	4.59	4.80	5.02	5.06
	Average	**1.08**	**2.08**	**3.00**	**3.75**	**4.15**	**4.47**	**4.68**	**4.87**	**4.80**
Decile Advantage, Māori	*2003*	*0.23*	*0.28*	*0.29*	*0.44*	*0.36*	*0.24*	*0.28*	*0.35*	*0.37*
	2005	*0.43*	*0.26*	*0.64*	*0.21*	*0.41*	*0.38*	*0.23*	*0.24*	*0.49*
	2007	*0.46*	*0.58*	*0.57*	*0.45*	*0.38*	*0.48*	*0.36*	*0.39*	*0.44*
Av = 0.38	**Average**	***0.37***	***0.37***	***0.50***	***0.37***	***0.38***	***0.37***	***0.29***	***0.33***	***0.43***

Appendix C – continued

Average Initial Stage on the Number Framework for the Additive and Multiplicative Domains (2003, 2005, 2007)

Initial Stage, Additive Domain	Year	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
High Decile, Pasifika	2003	1.34	2.41	3.40	4.09	4.42	4.68	4.87	4.67	3.67
	2005	1.54	2.59	3.32	4.22	4.55	4.74	4.94	4.75	5.17
	2007	1.52	2.35	3.47	4.06	4.56	4.98	5.03	5.21	5.32
	Average	**1.47**	**2.45**	**3.40**	**4.12**	**4.51**	**4.80**	**4.95**	**4.88**	**4.72**
Low Decile, Pasifika	2003	1.14	2.06	2.87	3.63	3.93	4.20	4.44	4.56	4.49
	2005	1.03	2.29	3.04	3.73	4.11	4.41	4.42	4.75	4.91
	2007	1.11	1.99	2.90	3.74	4.23	4.52	4.61	4.75	5.04
	Average	**1.09**	**2.11**	**2.94**	**3.70**	**4.09**	**4.38**	**4.49**	**4.69**	**4.81**
Decile Advantage, Pasifika	2003	*0.20*	*0.35*	*0.53*	*0.46*	*0.49*	*0.48*	*0.43*	*0.11*	*–0.82*
	2005	*0.51*	*0.29*	*0.28*	*0.48*	*0.44*	*0.33*	*0.52*	*0.00*	*0.26*
	2007	*0.41*	*0.36*	*0.57*	*0.31*	*0.33*	*0.46*	*0.42*	*0.46*	*0.28*
Av = 0.33	**Average**	***0.37***	***0.34***	***0.46***	***0.42***	***0.42***	***0.42***	***0.46***	***0.19***	***–0.09***

Initial Stage, Multiplicative	Year	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Overall	2003		3.20	3.88	4.74	5.28	5.70	5.89	6.18	6.02
	2005		3.50	3.74	4.13	4.53	4.90	5.14	5.42	5.74
	2007		3.52	3.76	4.17	4.61	5.03	5.29	5.63	5.78
	Average		**3.41**	**3.79**	**4.35**	**4.80**	**5.21**	**5.44**	**5.74**	**5.85**
Boys	2003		3.23	3.93	4.80	5.35	5.77	5.95	6.24	6.15
	2005		3.64	3.82	4.22	4.62	5.03	5.27	5.55	5.89
	2007		3.62	3.84	4.25	4.72	5.11	5.40	5.73	5.90
	Average		**3.50**	**3.86**	**4.43**	**4.90**	**5.30**	**5.54**	**5.84**	**5.98**
Girls	2003		3.17	3.84	4.67	5.20	5.63	5.83	6.11	5.90
	2005		3.32	3.65	4.04	4.43	4.76	5.01	5.27	5.54
	2007		3.40	3.67	4.08	4.48	4.94	5.18	5.52	5.66
	Average		**3.30**	**3.72**	**4.26**	**4.71**	**5.11**	**5.34**	**5.63**	**5.70**
Boys vs Girls	2003		*0.06*	*0.09*	*0.14*	*0.15*	*0.14*	*0.11*	*0.13*	*0.26*
	2005		*0.32*	*0.17*	*0.18*	*0.19*	*0.26*	*0.27*	*0.28*	*0.35*
	2007		*0.22*	*0.17*	*0.17*	*0.24*	*0.17*	*0.22*	*0.21*	*0.24*
Av = 0.20	**Average**		***0.20***	***0.14***	***0.16***	***0.19***	***0.19***	***0.20***	***0.21***	***0.28***
Low Decile	2003		3.13	3.63	4.41	4.93	5.34	5.66	5.94	5.79
	2005		3.33	3.64	3.88	4.24	4.52	4.79	5.03	5.29
	2007		3.24	3.55	3.87	4.20	4.59	4.87	5.17	5.44
	Average		**3.23**	**3.61**	**4.05**	**4.46**	**4.82**	**5.11**	**5.38**	**5.51**
Middle Decile	2003		3.19	3.91	4.78	5.30	5.73	5.97	6.27	6.17
	2005		3.53	3.71	4.10	4.44	4.88	5.13	5.42	5.61
	2007		3.52	3.73	4.16	4.58	5.04	5.25	5.62	5.73
	Average		**3.41**	**3.79**	**4.35**	**4.77**	**5.22**	**5.45**	**5.77**	**5.84**
High Decile	2003		3.27	4.12	5.06	5.66	6.10	6.32	6.65	6.55
	2005		3.52	3.79	4.28	4.76	5.11	5.35	5.59	6.05
	2007		3.59	3.85	4.32	4.82	5.28	5.55	5.88	5.99
	Average		**3.46**	**3.92**	**4.56**	**5.08**	**5.50**	**5.74**	**6.04**	**6.19**

Appendix C – continued

Average Initial Stage on the Number Framework for the Additive and Multiplicative Domains (2003, 2005, 2007)

Initial Stage, Multiplicative	Year	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
High vs Middle	2003		*0.08*	*0.21*	*0.28*	*0.36*	*0.37*	*0.34*	*0.38*	*0.38*
	2005		*–0.01*	*0.08*	*0.18*	*0.31*	*0.23*	*0.22*	*0.16*	*0.44*
	2007		*0.07*	*0.12*	*0.16*	*0.24*	*0.24*	*0.30*	*0.26*	*0.25*
Av = 0.24	**Average**		***0.05***	***0.13***	***0.21***	***0.30***	***0.28***	***0.29***	***0.27***	***0.36***
High vs Low	2003		*0.14*	*0.49*	*0.65*	*0.73*	*0.75*	*0.66*	*0.71*	*0.75*
	2005		*0.18*	*0.15*	*0.41*	*0.52*	*0.59*	*0.56*	*0.56*	*0.76*
	2007		*0.35*	*0.30*	*0.45*	*0.62*	*0.69*	*0.68*	*0.71*	*0.55*
Av = 0.54	**Average**		***0.22***	***0.31***	***0.50***	***0.62***	***0.68***	***0.63***	***0.66***	***0.69***
European	2003		3.23	3.99	4.90	5.46	5.88	6.09	6.40	6.36
	2005		3.49	3.73	4.20	4.63	5.00	5.27	5.57	5.88
	2007		3.59	3.81	4.28	4.74	5.18	5.45	5.79	5.92
	Average		**3.44**	**3.84**	**4.46**	**4.94**	**5.35**	**5.60**	**5.92**	**6.05**
Māori	2003		3.11	3.67	4.42	4.93	5.36	5.63	5.92	5.77
	2005		3.45	3.63	3.90	4.29	4.58	4.90	5.07	5.32
	2007		3.23	3.61	3.93	4.31	4.68	4.95	5.29	5.48
	Average		**3.26**	**3.64**	**4.08**	**4.51**	**4.87**	**5.16**	**5.43**	**5.52**
Pasifika	2003		3.12	3.51	4.20	4.71	5.09	5.38	5.57	5.50
	2005		3.32	3.65	3.80	4.07	4.49	4.61	4.96	5.16
	2007		3.21	3.51	3.84	4.17	4.66	4.80	5.02	5.31
	Average		**3.22**	**3.55**	**3.94**	**4.32**	**4.75**	**4.93**	**5.18**	**5.33**
European vs Māori	2003		*0.12*	*0.32*	*0.48*	*0.53*	*0.52*	*0.46*	*0.48*	*0.59*
	2005		*0.04*	*0.10*	*0.30*	*0.34*	*0.42*	*0.37*	*0.50*	*0.56*
	2007		*0.36*	*0.20*	*0.35*	*0.43*	*0.50*	*0.50*	*0.49*	*0.44*
Av = 0.39	**Average**		***0.18***	***0.21***	***0.38***	***0.43***	***0.48***	***0.44***	***0.49***	***0.53***
European vs Pasifika	2003		*0.11*	*0.49*	*0.70*	*0.75*	*0.79*	*0.70*	*0.83*	*0.86*
	2005		*0.17*	*0.08*	*0.40*	*0.55*	*0.51*	*0.66*	*0.61*	*0.71*
	2007		*0.38*	*0.30*	*0.45*	*0.57*	*0.52*	*0.65*	*0.77*	*0.60*
Av = 0.55	**Average**		***0.22***	***0.29***	***0.52***	***0.62***	***0.60***	***0.67***	***0.74***	***0.73***

Appendix D (A Decade of Reform in Mathematics Education: Results for 2009 and Earlier Years)

Average Gain on the Number Framework for Additive and Multiplicative Domains (2003, 2005, 2007)

Gain in Stage, Additive	Year	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Overall	2003	1.00	0.99	0.81	0.52	0.47	0.47	0.32	0.29	0.31
	2005	1.02	1.02	0.75	0.56	0.50	0.52	0.49	0.54	0.49
	2007	1.16	1.02	0.71	0.52	0.41	0.42	0.45	0.49	0.41
	Average	**1.06**	**1.01**	**0.76**	**0.53**	**0.46**	**0.47**	**0.42**	**0.44**	**0.40**
Boys	2003	1.02	1.02	0.83	0.53	0.48	0.44	0.32	0.27	0.30
	2005	1.02	1.04	0.79	0.58	0.48	0.45	0.40	0.37	0.30
	2007	1.17	1.02	0.72	0.55	0.43	0.40	0.44	0.46	0.36
	Average	**1.07**	**1.03**	**0.78**	**0.55**	**0.46**	**0.43**	**0.39**	**0.37**	**0.32**
Girls	2003	0.98	0.98	0.78	0.50	0.47	0.46	0.33	0.32	0.33
	2005	1.02	1.01	0.71	0.52	0.47	0.47	0.44	0.45	0.40
	2007	1.14	1.02	0.69	0.49	0.39	0.44	0.45	0.51	0.45
	Average	**1.05**	**1.00**	**0.73**	**0.50**	**0.44**	**0.46**	**0.41**	**0.43**	**0.39**
Boys vs Girls	2003	*0.04*	*0.04*	*0.05*	*0.03*	*0.01*	*–0.02*	*–0.01*	*–0.05*	*–0.03*
	2005	*–0.01*	*0.03*	*0.08*	*0.06*	*0.01*	*–0.02*	*–0.04*	*–0.07*	*–0.09*
	2007	*0.03*	*0.01*	*0.03*	*0.06*	*0.04*	*–0.04*	*–0.02*	*–0.06*	*–0.09*
Av = 0.0	**Average**	***0.02***	***0.03***	***0.05***	***0.05***	***0.02***	***–0.03***	***–0.02***	***–0.06***	***–0.07***
Low Decile	2003	0.99	0.94	0.84	0.56	0.51	0.48	0.20	0.18	0.34
	2005	1.07	1.01	0.87	0.56	0.51	0.48	0.46	0.43	0.47
	2007	1.23	0.89	0.72	0.47	0.38	0.37	0.43	0.51	0.43
	Average	**1.10**	**0.95**	**0.81**	**0.53**	**0.47**	**0.44**	**0.36**	**0.37**	**0.41**
Middle Decile	2003	0.98	1.00	0.79	0.51	0.46	0.44	0.39	0.32	0.28
	2005	0.96	1.00	0.77	0.56	0.50	0.44	0.41	0.39	0.36
	2007	1.10	1.03	0.71	0.57	0.41	0.47	0.45	0.48	0.44
	Average	**1.01**	**1.01**	**0.76**	**0.55**	**0.45**	**0.45**	**0.41**	**0.40**	**0.36**
High Decile	2003	1.06	1.05	0.80	0.47	0.46	0.44	0.40	0.37	0.25
	2005	1.07	1.05	0.67	0.53	0.43	0.47	0.42	0.45	0.29
	2007	1.16	1.10	0.70	0.52	0.43	0.42	0.46	0.48	0.39
	Average	**1.09**	**1.07**	**0.72**	**0.51**	**0.44**	**0.44**	**0.43**	**0.43**	**0.31**
High vs Middle	2003	*0.08*	*0.05*	*0.01*	*–0.04*	*0.00*	*0.00*	*0.01*	*0.05*	*–0.03*
	2005	*0.11*	*0.05*	*–0.11*	*–0.03*	*–0.07*	*0.02*	*0.02*	*0.06*	*–0.07*
	2007	*0.06*	*0.07*	*–0.01*	*–0.05*	*0.03*	*–0.05*	*0.01*	*0.00*	*–0.05*
Av = 0.0	**Average**	***0.08***	***0.06***	***–0.04***	***–0.04***	***–0.01***	***–0.01***	***0.01***	***0.04***	***–0.05***
High vs Low	2003	*0.07*	*0.11*	*–0.04*	*–0.09*	*–0.05*	*–0.04*	*0.20*	*0.19*	*–0.09*
	2005	*–0.01*	*0.04*	*–0.21*	*–0.03*	*–0.08*	*–0.01*	*–0.04*	*0.01*	*–0.18*
	2007	*–0.07*	*0.22*	*–0.02*	*0.05*	*0.06*	*0.05*	*0.03*	*–0.02*	*–0.04*
Av = 0.0	**Average**	***0.00***	***0.12***	***–0.09***	***–0.02***	***–0.02***	***0.00***	***0.06***	***0.06***	***–0.10***

Appendix D – continued

Average Gain on the Number Framework for Additive and Multiplicative Domains (2003, 2005, 2007)

Gain in Stage, Additive	Year	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
European	2003	1.01	1.01	0.80	0.49	0.45	0.46	0.39	0.34	0.26
	2005	0.99	1.05	0.74	0.54	0.46	0.45	0.40	0.38	0.32
	2007	1.15	1.07	0.68	0.50	0.41	0.43	0.43	0.47	0.41
	Average	**1.05**	**1.04**	**0.74**	**0.51**	**0.44**	**0.44**	**0.41**	**0.40**	**0.33**
Māori	2003	0.96	0.96	0.84	0.57	0.52	0.47	0.33	0.31	0.32
	2005	1.03	0.99	0.81	0.57	0.49	0.50	0.44	0.46	0.38
	2007	1.15	0.95	0.72	0.55	0.38	0.38	0.44	0.48	0.40
	Average	**1.05**	**0.97**	**0.79**	**0.57**	**0.46**	**0.45**	**0.40**	**0.42**	**0.37**
Pasifika	2003	1.02	0.97	0.81	0.52	0.50	0.46	0.00	0.00	0.39
	2005	1.02	0.92	0.82	0.57	0.49	0.45	0.45	0.44	0.55
	2007	1.24	0.93	0.80	0.50	0.42	0.40	0.51	0.54	0.45
	Average	**1.09**	**0.94**	**0.81**	**0.53**	**0.47**	**0.44**	**0.32**	**0.33**	**0.47**
European vs Māori	2003	*0.05*	*0.05*	*–0.04*	*–0.08*	*–0.07*	*–0.01*	*0.06*	*0.03*	*–0.06*
	2005	*–0.03*	*0.06*	*–0.07*	*–0.03*	*–0.03*	*–0.05*	*–0.04*	*–0.08*	*–0.06*
	2007	*0.00*	*0.12*	*–0.04*	*–0.06*	*0.03*	*0.05*	*–0.01*	*0.00*	*0.01*
Av = –0.01	**Average**	***0.01***	***0.08***	***–0.05***	***–0.06***	***–0.02***	***0.00***	***0.00***	***–0.02***	***–0.04***
European vs Pasifika	2003	*–0.01*	*0.04*	*–0.01*	*–0.03*	*–0.05*	*0.00*	*0.39*	*0.34*	*–0.13*
	2005	*–0.03*	*0.13*	*–0.09*	*–0.03*	*–0.03*	*0.00*	*–0.05*	*–0.06*	*–0.24*
	2007	*–0.09*	*0.14*	*–0.12*	*0.00*	*–0.01*	*0.02*	*–0.08*	*–0.06*	*–0.05*
Av = 0.0	**Average**	***–0.04***	***0.10***	***–0.07***	***–0.02***	***–0.03***	***0.01***	***0.09***	***0.07***	***–0.14***
High Decile, European	2003	1.06	1.06	0.80	0.47	0.46	0.46	0.38	0.32	0.25
	2005	1.05	1.07	0.67	0.53	0.42	0.47	0.41	0.42	0.29
	2007	1.16	1.12	0.71	0.52	0.43	0.42	0.44	0.48	0.40
	Average	**1.09**	**1.08**	**0.73**	**0.51**	**0.43**	**0.45**	**0.41**	**0.41**	**0.31**
Low Decile, European	2003	0.98	0.95	0.85	0.54	0.49	0.47	0.34	0.29	0.31
	2005	0.92	1.02	0.92	0.48	0.54	0.45	0.39	0.38	0.47
	2007	1.50	0.91	0.66	0.28	0.43	0.37	0.34	0.47	0.42
	Average	**1.13**	**0.96**	**0.81**	**0.43**	**0.48**	**0.43**	**0.36**	**0.38**	**0.40**
Decile Advantage, European	2003	*0.08*	*0.11*	*–0.05*	*–0.07*	*–0.03*	*–0.01*	*0.04*	*0.03*	*–0.06*
	2005	*0.13*	*0.05*	*–0.25*	*0.06*	*–0.12*	*0.02*	*0.02*	*0.05*	*–0.18*
	2007	*–0.34*	*0.22*	*0.05*	*0.24*	*0.00*	*0.05*	*0.10*	*0.01*	*–0.03*
Av = 0.0	**Average**	***–0.04***	***0.12***	***–0.09***	***0.07***	***–0.05***	***0.02***	***0.05***	***0.03***	***–0.09***

Appendix D – continued

Average Gain on the Number Framework for Additive and Multiplicative Domains (2003, 2005, 2007)

Gain in Stage, Additive	Year	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
High Decile, Māori	2003	1.09	1.03	0.90	0.54	0.55	0.49	0.36	0.44	0.31
	2005	1.03	1.05	0.60	0.60	0.46	0.48	0.47	0.51	0.26
	2007	1.11	1.06	0.64	0.44	0.44	0.38	0.45	0.43	0.31
	Average	**1.08**	**1.04**	**0.72**	**0.52**	**0.48**	**0.45**	**0.43**	**0.46**	**0.29**
Low Decile, Māori	2003	0.98	0.92	0.85	0.59	0.51	0.47	0.31	0.28	0.31
	2005	1.09	0.98	0.88	0.59	0.53	0.53	0.51	0.51	0.42
	2007	1.20	0.88	0.68	0.53	0.35	0.36	0.39	0.51	0.45
	Average	**1.09**	**0.93**	**0.81**	**0.57**	**0.46**	**0.45**	**0.40**	**0.43**	**0.39**
Decile Advantage, Māori	2003	*0.11*	*0.11*	*0.05*	*–0.05*	*0.04*	*0.02*	*0.05*	*0.16*	*0.00*
	2005	*–0.06*	*0.07*	*–0.28*	*0.01*	*–0.07*	*–0.05*	*–0.04*	*0.00*	*–0.16*
	2007	*–0.09*	*0.18*	*–0.04*	*–0.09*	*0.09*	*0.03*	*0.06*	*–0.09*	*–0.14*
Av = –0.01	**Average**	***–0.01***	***0.12***	***–0.09***	***–0.04***	***0.02***	***0.00***	***0.02***	***0.02***	***–0.10***
High Decile, Pasifika	2003	0.95	1.07	0.82	0.43	0.43	0.46	0.42	0.61	0.44
	2005	0.96	1.00	0.89	0.51	0.41	0.52	0.42	0.58	0.30
	2007	1.13	1.17	0.67	0.52	0.50	0.35	0.47	0.51	0.34
	Average	**1.01**	**1.08**	**0.79**	**0.49**	**0.45**	**0.44**	**0.44**	**0.56**	**0.36**
Low Decile, Pasifika	2003	1.03	0.96	0.82	0.52	0.50	0.49	–0.10	–0.10	0.40
	2005	1.11	0.99	0.81	0.58	0.46	0.43	0.43	0.39	0.62
	2007	1.26	0.84	0.83	0.47	0.38	0.38	0.57	0.56	0.42
	Average	**1.13**	**0.93**	**0.82**	**0.52**	**0.44**	**0.43**	**0.30**	**0.28**	**0.48**
Decile Advantage, Pasifika	2003	*–0.08*	*0.11*	*0.00*	*–0.09*	*–0.07*	*–0.03*	*0.52*	*0.71*	*0.04*
	2005	*–0.15*	*0.01*	*0.08*	*–0.07*	*–0.04*	*0.09*	*–0.01*	*0.19*	*–0.31*
	2007	*–0.13*	*0.34*	*–0.16*	*0.05*	*0.12*	*–0.03*	*–0.09*	*–0.05*	*–0.08*
Av = 0.03	**Average**	***–0.12***	***0.15***	***–0.03***	***–0.04***	***0.00***	***0.01***	***0.14***	***0.28***	***–0.12***

Appendix E (A Decade of Reform in Mathematics Education: Results for 2009 and Earlier Years)

Effect Size for Gains on the Additive and Multiplicative Domains of the Number Framework (2003, 2005, 2007)

Effect Size for Gains, Additive	Year	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Overall	2003	0.99	0.88	0.72	0.52	0.50	0.52	0.31	0.28	0.34
	2005	1.04	0.93	0.71	0.62	0.60	0.65	0.63	0.74	0.66
	2007	1.18	0.93	0.66	0.56	0.46	0.47	0.49	0.52	0.45
	Average	**1.07**	**0.92**	**0.70**	**0.57**	**0.52**	**0.55**	**0.48**	**0.51**	**0.48**
Boys	2003	0.96	0.88	0.71	0.51	0.49	0.48	0.31	0.26	0.31
	2005	1.00	0.91	0.71	0.60	0.55	0.55	0.50	0.50	0.41
	2007	1.16	0.89	0.64	0.55	0.45	0.43	0.47	0.48	0.40
	Average	**1.04**	**0.89**	**0.69**	**0.55**	**0.50**	**0.49**	**0.43**	**0.41**	**0.37**
Girls	2003	1.01	0.91	0.74	0.54	0.53	0.53	0.33	0.32	0.39
	2005	1.09	0.97	0.71	0.62	0.59	0.61	0.58	0.61	0.54
	2007	1.21	0.98	0.70	0.57	0.47	0.52	0.52	0.57	0.50
	Average	**1.10**	**0.95**	**0.72**	**0.58**	**0.53**	**0.55**	**0.48**	**0.50**	**0.48**
Low Decile	2003	1.00	0.86	0.73	0.52	0.51	0.50	0.18	0.16	0.35
	2005	1.10	0.92	0.77	0.58	0.57	0.55	0.50	0.53	0.53
	2007	1.28	0.84	0.67	0.48	0.43	0.41	0.48	0.51	0.43
	Average	**1.12**	**0.87**	**0.72**	**0.53**	**0.50**	**0.49**	**0.39**	**0.40**	**0.44**
Middle Decile	2003	0.98	0.91	0.72	0.54	0.51	0.52	0.44	0.36	0.33
	2005	1.01	0.90	0.73	0.64	0.62	0.56	0.53	0.53	0.47
	2007	1.12	0.95	0.69	0.64	0.47	0.55	0.51	0.53	0.50
	Average	**1.04**	**0.92**	**0.72**	**0.61**	**0.53**	**0.54**	**0.49**	**0.47**	**0.43**
High Decile	2003	1.05	0.96	0.79	0.54	0.55	0.53	0.43	0.42	0.32
	2005	1.10	1.01	0.71	0.63	0.56	0.65	0.60	0.65	0.46
	2007	1.24	1.06	0.72	0.62	0.52	0.52	0.53	0.56	0.45
	Average	**1.13**	**1.01**	**0.74**	**0.60**	**0.54**	**0.57**	**0.52**	**0.55**	**0.41**
European	2003	1.00	0.91	0.76	0.54	0.51	0.54	0.44	0.38	0.31
	2005	1.04	0.99	0.73	0.62	0.58	0.59	0.55	0.53	0.46
	2007	1.21	0.99	0.68	0.58	0.48	0.50	0.49	0.52	0.45
	Average	**1.08**	**0.96**	**0.72**	**0.58**	**0.52**	**0.54**	**0.49**	**0.48**	**0.41**
Māori	2003	1.00	0.88	0.73	0.55	0.54	0.53	0.35	0.31	0.30
	2005	1.06	0.90	0.74	0.64	0.57	0.61	0.57	0.62	0.46
	2007	1.19	0.90	0.66	0.57	0.42	0.42	0.51	0.52	0.43
	Average	**1.08**	**0.89**	**0.71**	**0.58**	**0.51**	**0.52**	**0.47**	**0.48**	**0.40**
Pasifika	2003	1.06	0.93	0.71	0.48	0.48	0.48	0.00	0.00	0.41
	2005	1.08	0.88	0.73	0.59	0.58	0.56	0.52	0.57	0.74
	2007	1.25	0.93	0.74	0.53	0.53	0.46	0.55	0.54	0.53
	Average	**1.13**	**0.91**	**0.73**	**0.53**	**0.53**	**0.50**	**0.36**	**0.37**	**0.56**

Appendix E – continued

Effect Size for Gains on the Additive and Multiplicative Domains of the Number Framework (2003, 2005, 2007)

Effect Size for Gains, Additive	Year	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
High Decile, European	2003	1.05	0.98	0.81	0.56	0.56	0.57	0.43	0.40	0.35
	2005	1.09	1.04	0.72	0.64	0.55	0.66	0.59	0.62	0.47
	2007	1.27	1.08	0.74	0.64	0.51	0.51	0.52	0.56	0.46
	Average	**1.14**	**1.03**	**0.76**	**0.61**	**0.54**	**0.58**	**0.51**	**0.53**	**0.43**
Low Decile, European	2003	0.96	0.85	0.76	0.54	0.52	0.49	0.39	0.32	0.37
	2005	0.91	0.97	0.81	0.48	0.59	0.52	0.44	0.42	0.53
	2007	1.64	0.79	0.63	0.34	0.47	0.40	0.40	0.45	0.41
	Average	**1.17**	**0.87**	**0.73**	**0.45**	**0.53**	**0.47**	**0.41**	**0.40**	**0.44**
High Decile, Māori	2003	1.13	0.95	0.77	0.60	0.65	0.60	0.47	0.60	0.37
	2005	1.03	1.05	0.60	0.67	0.60	0.70	0.59	0.80	0.37
	2007	1.10	1.01	0.69	0.49	0.54	0.50	0.52	0.46	0.35
	Average	**1.09**	**1.01**	**0.69**	**0.59**	**0.60**	**0.60**	**0.53**	**0.62**	**0.36**
Low Decile, Māori	2003	1.02	0.84	0.74	0.55	0.53	0.52	0.32	0.26	0.28
	2005	1.14	0.88	0.79	0.64	0.60	0.57	0.59	0.62	0.47
	2007	1.28	0.84	0.64	0.52	0.38	0.40	0.45	0.56	0.44
	Average	**1.15**	**0.85**	**0.72**	**0.57**	**0.50**	**0.50**	**0.45**	**0.48**	**0.40**
High Decile, Pasifika	2003	0.89	1.09	0.71	0.41	0.49	0.68	0.54	0.57	
	2005	0.89	0.90	0.88	0.61	0.49	0.71	0.62	0.73	0.44
	2007	1.29	1.19	0.58	0.59	0.64	0.43	0.53	0.69	0.44
	Average	**1.02**	**1.06**	**0.73**	**0.53**	**0.54**	**0.61**	**0.57**	**0.66**	**0.44**
Low Decile, Pasifika	2003	1.08	0.92	0.72	0.47	0.46	0.50	–0.07	–0.07	0.42
	2005	1.22	0.89	0.71	0.57	0.55	0.54	0.47	0.52	0.85
	2007	1.21	0.85	0.77	0.49	0.49	0.44	0.62	0.53	0.49
	Average	**1.17**	**0.89**	**0.73**	**0.51**	**0.50**	**0.49**	**0.34**	**0.32**	**0.59**

Appendix E – continued

Effect Size for Gains on the Additive and Multiplicative Domains of the Number Framework (2003, 2005, 2007)

Effect Size for Gains, Multiplicative	Year	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Overall	2003	0.75	0.79	0.64	0.60	0.59	0.47	0.46	0.39
	2005	1.00	0.91	0.78	0.69	0.71	0.62	0.63	0.52
	2007	0.87	0.86	0.74	0.69	0.54	0.55	0.51	0.43
	Average	**0.87**	**0.85**	**0.72**	**0.66**	**0.61**	**0.55**	**0.53**	**0.45**
Boys	2003	0.75	0.77	0.62	0.57	0.57	0.47	0.43	0.36
	2005	0.89	0.90	0.78	0.67	0.69	0.61	0.60	0.50
	2007	0.83	0.84	0.72	0.57	0.50	0.55	0.47	0.39
	Average	**0.82**	**0.84**	**0.71**	**0.60**	**0.59**	**0.54**	**0.50**	**0.42**
Girls	2003	0.76	0.81	0.66	0.63	0.60	0.48	0.50	0.42
	2005	1.22	0.94	0.80	0.72	0.74	0.65	0.68	0.57
	2007	0.97	0.90	0.78	0.64	0.60	0.57	0.56	0.47
	Average	**0.98**	**0.89**	**0.75**	**0.66**	**0.65**	**0.57**	**0.58**	**0.49**
Low Decile	2003	0.65	0.77	0.64	0.61	0.60	0.40	0.37	0.37
	2005	1.15	0.88	0.77	0.69	0.72	0.63	0.69	0.53
	2007	0.60	0.81	0.72	0.55	0.48	0.52	0.48	0.40
	Average	**0.80**	**0.82**	**0.71**	**0.62**	**0.60**	**0.51**	**0.52**	**0.43**
Middle Decile	2003	0.77	0.78	0.64	0.59	0.58	0.53	0.53	0.44
	2005	0.88	0.92	0.79	0.73	0.73	0.63	0.63	0.54
	2007	0.86	0.83	0.77	0.64	0.59	0.59	0.52	0.46
	Average	**0.84**	**0.84**	**0.73**	**0.66**	**0.63**	**0.58**	**0.56**	**0.48**
High Decile	2003	0.89	0.90	0.73	0.68	0.65	0.54	0.52	0.44
	2005	1.09	0.91	0.82	0.68	0.72	0.68	0.70	0.55
	2007	0.99	0.93	0.80	0.64	0.62	0.58	0.56	0.46
	Average	**0.99**	**0.91**	**0.78**	**0.67**	**0.66**	**0.60**	**0.59**	**0.48**
European	2003	0.82	0.84	0.67	0.61	0.62	0.54	0.53	0.40
	2005	1.03	0.97	0.80	0.70	0.73	0.62	0.64	0.51
	2007	0.91	0.90	0.80	0.63	0.57	0.54	0.53	0.43
	Average	**0.92**	**0.90**	**0.76**	**0.65**	**0.64**	**0.57**	**0.57**	**0.45**
Māori	2003	0.66	0.73	0.65	0.64	0.60	0.49	0.48	0.32
	2005	1.05	0.85	0.80	0.69	0.71	0.63	0.64	0.53
	2007	0.92	0.78	0.70	0.53	0.51	0.59	0.50	0.44
	Average	**0.88**	**0.79**	**0.72**	**0.62**	**0.61**	**0.57**	**0.54**	**0.43**
Pasifika	2003	0.61	0.79	0.65	0.59	0.64	0.28	0.29	0.45
	2005	1.18	0.71	0.83	0.76	0.70	0.67	0.66	0.66
	2007	0.71	0.91	0.74	0.65	0.55	0.65	0.51	0.47
	Average	**0.83**	**0.80**	**0.74**	**0.67**	**0.63**	**0.53**	**0.49**	**0.53**

Appendix F (Analysis of the Number Framework)

Table 15
Estimated Difficulty Scores in Logits for Each Stage in Each Domain

Domain	1	2	3	4	5	6	7	8
Additive	–6.13	–3.00	–1.06	–0.53	1.02	3.03	6.30	
Multiplicative				–0.94	1.29	2.60	4.55	7.17
Proportional			–1.48 *		1.47	3.27	4.50	7.45
Place value		–2.38	–0.22	–0.09	2.20	3.98	6.14	8.72
Basic facts		–0.83	0.11	0.23	1.61	3.22	5.14	7.33

* This stage in the proportional domain is labelled "stages 2–4".

Appendix G (Analysis of the Number Framework)

Probability functions for the multiplicative and proportional domains

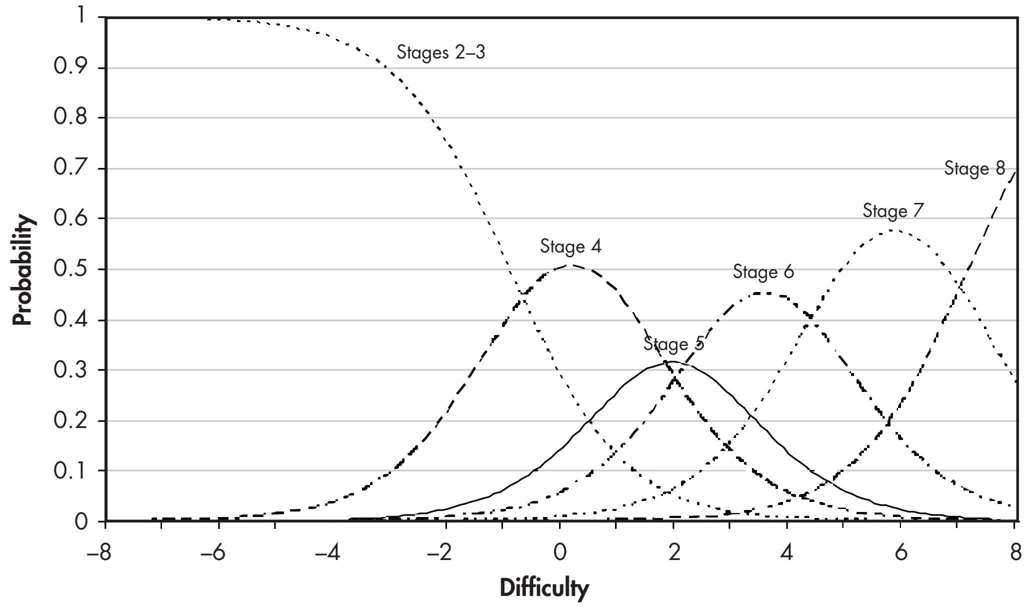


Figure 6. Probability functions for classification at each stage of the multiplicative domain, conditioned on numeracy ability

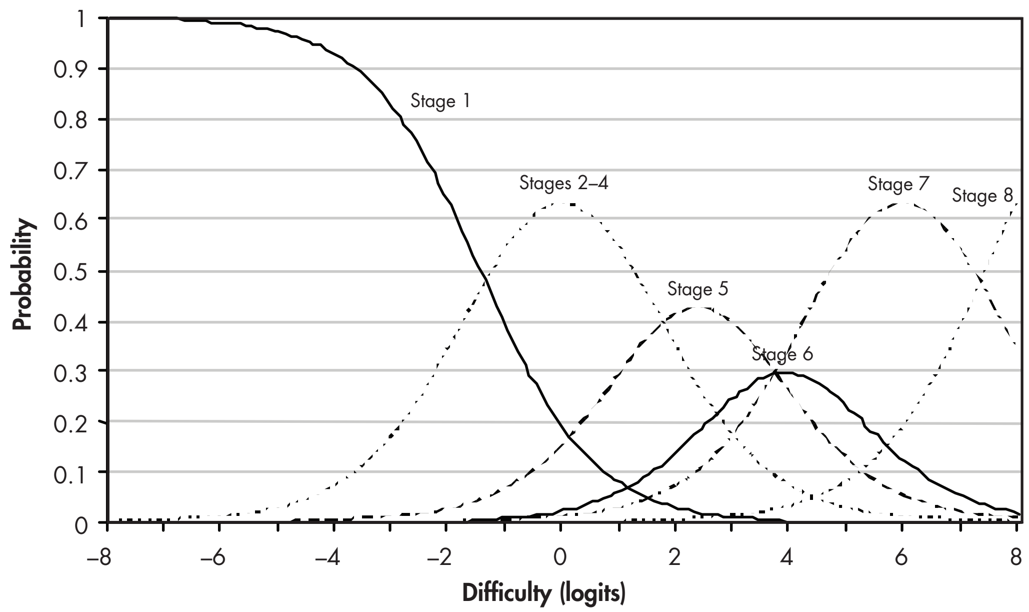


Figure 7. Probability functions for classification at each stage of the proportional domain, conditioned on numeracy ability

Appendix H (Analysis of the Number Framework)

Equi-probable points on the numeracy-ability scale for classification in each of the adjacent stages for the multiplicative, proportional, place value, and basic facts domains, rounded to the nearest 0.05 logit

Table 16
Transition Points on the Multiplicative Domain

Transition between stages				
2→3–4	4–5	5–6	6–7	7–8
–0.70	1.80	2.20	4.35	7.00

Table 17
Transition Points on the Proportional Domain

Transition between stages				
1–2→4	2→4–5	5–6	6–7	7–8
–1.35	1.75	3.70	3.75	7.35

Table 18
Transition Points on the Place Value Domain

Transition between stages					
0→1–2	2–4	4–5	5–6	6–7	7–8
–2.10	–0.20	2.40	3.80	6.05	8.55

Note that stage 3 has been omitted because it is not the most likely stage rating for students at any point on the numeracy ability scale.

Table 19
Transition Points on the Basic Facts Domain

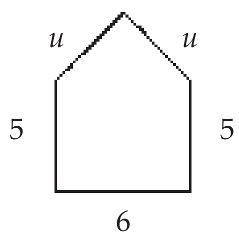
Transition between stages				
0–4	4–5	5–6	6–7	7–8
0.10	1.40	3.05	5.05	7.10

Note that stages 2 and 3 have been omitted because they are not the most likely stage ratings for students at any point on the numeracy ability scale.

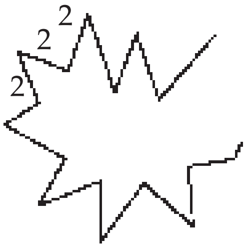
Appendix I (Secondary Numeracy Project Students' Development of Algebraic Knowledge and Strategies)

	Question	Answer	
Q1	If $n = 4$, then what is the value of $3n$?		
Q2	If $h = 4$, then what is the value of $5 + h$?		
Q3	Hine has the following problem to solve: "Find value(s) for x in the expression: $x + x + x = 12$." She answers as follows: a. 6, 3, 3 b. 10, 1, 1 c. 4, 4, 4 Which of her answers is (are) correct? (Note that you may give more than one answer.)		
Q4	Huata has the following problem to solve: "Find value(s) for x and y in the expression: $x + y = 16$." He answers as follows: a. 10, 6 b. 14, 2 c. 8, 8 Which of his answers is (are) correct? (Note that you may give more than one answer.)		
Q5	What is the value of the following? 5 + 6 x 10		
Q6	What is the value of the following? 8 x (7 – 5 + 3)		

Appendix I – continued

	Question	Answer	
Q7	What is the value of the following? $\frac{6 + 12}{3}$		
Q8	What is the value of the following? 18 – 12 ÷ 6		
Q9	Replace the box with +, –, x, or ÷. (1) If $y \div 59 = 7$, then $y = 7 \square 59$		
Q10	Replace the box with +, –, x, or ÷. (1) If $z - 27 = 25$, then $z = 25 \square 27$		
Q11	Replace the box with +, –, x, or ÷. (1) If $\frac{p}{6} = 4$, then $p = 4 \square 6$		
Q12	Replace the box with +, –, x, or ÷. (1) If $5t = 28$, then $t = 28 \square 5$		
Q13	Add 3 to *n*		
Q14	What is the perimeter of this shape? 		

Appendix I – continued

	Question	Answer	
Q15	Take n away from $3n + 1$		
Q16	Part of this shape is not drawn. There are n sides altogether, each of length 2. What is the perimeter?  		
Q17	What number should replace the empty box? $4 + 5 = \square + 3$		
Q18	What number should replace the empty box? $97.23 - \square = 87.23 - 15.84$		
Q19	Given that $13x + 2 = 9$, what should replace the empty box? $13x + \square = 12$		
Q20	Given that $6x + 2 = 9$, what should replace the empty box? $9x + 2 = \square + 9$		

Appendix J (Secondary Numeracy Project Students' Development of Algebraic Knowledge and Strategies)

Solve the following equations. This means finding the value of the letter that makes the equation true.

Note that solutions **might not be whole numbers.**

Please show any working.

	Equation	Solution	Leave blank
Q1	$n - 3 = 12$		
Q2	$3n = 18$		
Q3	$n + 4.6 = 11.3$		
Q4	$29n = 205.9$		
Q5	$\frac{n}{26} = 11.5$		

Appendix J – continued

	Equation	Solution	Leave blank
Q6	$4n + 9 = 37$		
Q7	$3n - 8 = 19$		
Q8	$4(n - 3) = 21$		
Q9	$16n + 78.2 = 147$		
Q10	$\frac{n + 12}{4} = 18$		
Q11	$2.8 + \frac{n}{4} = 8.2$		

Appendix J – continued

	Equation	Solution	Leave blank
Q12	$5n - 2 = 3n + 6$		
Q13	$12n + 2 = 8n + 15$		
Q14	$2n - 3 = \dfrac{2n + 24}{5}$		
Q15	Solve for n, i.e., get into the form $n = \ldots\ldots$ (Solution will be an expression, not a number.) $an - 3p = 5r$		

Appendix K (Performance of SNP Students on the Number Framework)

Table 16

Performance of Year 9 Students in First-year SNP Schools on the Additive Domain

	Ethnicity			Decile group			Gender		Total
	NZE	**Māori**	**Pasifika**	**Low**	**Medium**	**High**	**Male**	**Female**	
Initial									
0–3: Counting from one	0%	1%	3%	2%	1%	1%	1%	1%	1%
4: Advanced counting	6%	16%	17%	20%	8%	6%	8%	10%	9%
5: Early additive	39%	47%	52%	52%	39%	39%	36%	45%	41%
6: Advanced additive	40%	31%	26%	24%	41%	38%	42%	33%	37%
7: Advanced multiplicative	13%	4%	2%	2%	10%	14%	11%	10%	11%
8: Advanced proportional	2%	1%	1%	0%	1%	2%	2%	1%	1%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820
Final									
0–3: Counting from one	0%	0%	1%	0%	0%	0%	0%	0%	0%
4: Advanced counting	3%	7%	3%	7%	4%	3%	3%	4%	4%
5: Early additive	24%	31%	35%	28%	27%	24%	23%	28%	26%
6: Advanced additive	41%	44%	51%	48%	42%	40%	44%	41%	42%
7: Advanced multiplicative	24%	15%	10%	14%	22%	26%	23%	23%	23%
8: Advanced proportional	7%	3%	1%	2%	5%	7%	7%	5%	6%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820

Appendix K – continued

Table 17

Performance of Year 9 Students in First-year SNP Schools on the Multiplicative Domain

	Ethnicity			Decile group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Initial									
0–3: Counting from one	1%	1%	2%	2%	1%	1%	1%	1%	1%
4: Advanced counting	9%	19%	20%	23%	11%	7%	11%	10%	11%
5: Early additive	22%	30%	33%	30%	27%	21%	21%	27%	24%
6: Advanced additive	40%	37%	33%	35%	35%	40%	38%	38%	38%
7: Advanced multiplicative	22%	12%	12%	9%	21%	23%	22%	20%	21%
8: Advanced proportional	6%	1%	1%	1%	5%	7%	7%	4%	6%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820
Final									
0–3: Counting from one	0%	0%	1%	0%	0%	0%	0%	0%	0%
4: Advanced counting	3%	6%	7%	6%	5%	2%	4%	4%	4%
5: Early additive	12%	22%	21%	22%	17%	10%	12%	15%	14%
6: Advanced additive	35%	38%	39%	39%	33%	34%	33%	35%	35%
7: Advanced multiplicative	35%	27%	27%	25%	31%	36%	33%	32%	33%
8: Advanced proportional	15%	7%	6%	7%	14%	17%	17%	13%	15%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820

Table 18

Performance of Year 9 Students in First-year SNP Schools on the Proportional Domain

	Ethnicity			Decile group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Initial									
0–1: Unequal sharing	1%	1%	1%	1%	1%	0%	1%	0%	1%
2–4: Equal sharing	12%	28%	23%	30%	15%	11%	15%	16%	15%
5: Early additive	30%	38%	47%	40%	35%	28%	29%	33%	32%
6: Advanced additive	17%	14%	15%	15%	15%	17%	16%	16%	16%
7: Advanced multiplicative	35%	17%	14%	14%	29%	38%	33%	30%	32%
8: Advanced proportional	5%	2%	1%	0%	5%	6%	6%	4%	5%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820
Final									
0–1: Unequal sharing	0%	0%	0%	0%	0%	0%	0%	0%	0%
2–4: Equal sharing	6%	10%	12%	11%	8%	5%	8%	6%	7%
5: Early additive	21%	37%	35%	36%	25%	20%	21%	26%	24%
6: Advanced additive	17%	20%	18%	22%	16%	17%	17%	17%	17%
7: Advanced multiplicative	41%	29%	31%	28%	36%	42%	38%	38%	38%
8: Advanced proportional	16%	5%	5%	3%	15%	17%	17%	12%	14%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820

Appendix L (Performance of SNP Students on the Number Framework)

Table 19

Performance of Year 9 Students in First-year SNP Schools on the FNWS Domain

	Ethnicity			Decile group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Initial									
0–3: To 20	1%	1%	3%	2%	1%	0%	1%	1%	1%
4: To 100	2%	5%	6%	6%	2%	2%	3%	3%	3%
5: To 1000	34%	36%	51%	31%	39%	34%	29%	40%	35%
6: To 1 000 000	64%	59%	41%	61%	57%	64%	67%	57%	61%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820
Final									
0–3: To 20	0%	1%	1%	1%	1%	0%	0%	0%	0%
4: To 100	1%	2%	5%	3%	1%	1%	1%	1%	1%
5: To 1000	18%	25%	42%	27%	25%	16%	16%	24%	20%
6: To 1 000 000	81%	72%	53%	69%	74%	83%	82%	75%	78%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820

Table 20

Performance of Year 9 Students in First-year SNP Schools on the Fractions Domain

	Ethnicity			Decile group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Initial									
0–3: Non-fractions	1%	3%	4%	4%	2%	1%	2%	1%	2%
4: Assigns unit fractions	9%	12%	16%	16%	11%	8%	11%	9%	10%
5: Orders unit fractions	41%	46%	49%	39%	46%	39%	41%	42%	41%
6: Co-ordinates num./denom.	24%	27%	23%	32%	22%	23%	21%	27%	24%
7: Equivalent fractions	19%	9%	8%	8%	16%	22%	18%	17%	18%
8: Orders fractions	5%	2%	1%	1%	3%	7%	6%	4%	5%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820
Final									
0–3: Non-fractions	1%	1%	1%	1%	1%	0%	1%	0%	1%
4: Assigns unit fractions	3%	8%	9%	10%	5%	3%	5%	4%	4%
5: Orders unit fractions	26%	39%	36%	38%	33%	22%	28%	28%	28%
6: Co-ordinates num./denom.	26%	27%	31%	31%	26%	25%	24%	28%	26%
7: Equivalent fractions	31%	19%	21%	18%	25%	33%	27%	30%	28%
8: Orders fractions	14%	5%	3%	2%	10%	17%	16%	10%	12%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820

Appendix L – continued

Table 21

Performance of Year 9 Students in First-year SNP Schools on the Place Value Domain

	Ethnicity			Decile group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Initial									
0–3: Counts in fives and ones	1%	1%	3%	1%	1%	1%	1%	1%	1%
4: 10s to 100, orders to 1000	9%	11%	12%	13%	11%	8%	9%	10%	10%
5: 10s to 1000, orders to 10 000	38%	47%	51%	43%	43%	39%	39%	43%	41%
6: 10s, 100s, 1000s, orders whole numbers	26%	28%	27%	30%	25%	24%	24%	26%	25%
7: Tenths in and orders decimals	16%	10%	6%	10%	14%	14%	15%	13%	14%
8: Tenths, hundredths, and thousandths	11%	3%	3%	3%	6%	13%	12%	8%	10%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820
Final									
0–3: Counts in fives and ones	0%	0%	1%	0%	1%	0%	0%	0%	0%
4: 10s to 100, orders to 1000	3%	6%	6%	7%	4%	2%	4%	3%	4%
5: 10s to 1000, orders to 10 000	22%	34%	30%	35%	28%	19%	21%	27%	24%
6: 10s, 100s, 1000s, orders whole numbers	28%	35%	41%	39%	28%	27%	28%	30%	29%
7: Tenths in and orders decimals	24%	15%	13%	11%	21%	23%	21%	21%	21%
8: Tenths, hundredths, and thousandths	23%	10%	10%	8%	17%	28%	25%	19%	21%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820

Appendix L – continued

Table 22
Performance of Year 9 Students in First-year SNP Schools on the Basic Facts Domain

	Ethnicity			Decile group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Initial									
0–3: Counts in fives and ones	1%	1%	3%	1%	1%	1%	1%	1%	1%
0–3: Facts to 10	1%	3%	2%	2%	2%	1%	2%	1%	1%
4: Within 10, doubles, and teens	5%	7%	5%	7%	6%	4%	7%	4%	5%
5: Addition, multiplication for 2, 5, 10	18%	24%	26%	24%	24%	15%	20%	19%	19%
6: Subtraction and multiplication	56%	51%	55%	55%	50%	58%	52%	57%	55%
7: Division	18%	16%	13%	12%	17%	20%	18%	17%	18%
8: Factors and multiples	1%	1%		0%	1%	2%	1%	1%	1%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820
Final									
0–3: Facts to 10	0%	1%	1%	1%	1%	0%	1%	0%	1%
4: Within 10, doubles, and teens	2%	3%	3%	3%	3%	2%	3%	2%	2%
5: Addition, multiplication for 2, 5, 10	11%	16%	14%	16%	15%	8%	13%	11%	12%
6: Subtraction and multiplication	42%	47%	38%	46%	43%	41%	40%	44%	42%
7: Division	37%	30%	43%	32%	34%	39%	35%	38%	37%
8: Factors and multiples	7%	3%	3%	2%	3%	10%	8%	5%	6%
Number of students	2482	668	200	570	1185	2065	1666	2154	3820

Appendix M (The Role of the Numeracy Lead Teacher in Promoting the Goals of the Numeracy Development Projects)

Survey content

Leadership Practices in Primary Schools

What is your role in the school? Please tick all that apply.

- ☐ Numeracy lead teacher
- ☐ Principal
- ☐ Deputy principal
- ☐ Assistant principal
- ☐ Associate principal
- ☐ Syndicate leader
- ☐ Teacher (with responsibility for a class)
- ☐ Other – please specify.

Your Role in Leading Numeracy in Your School

The following statements are examples of roles that might be undertaken by the numeracy lead teacher [or principal/senior management team member] in order to develop numeracy instruction and raise student achievement across the school.

For each statement, use a scale of 0–9, where 0 = not at all significant and 9 = extremely significant, to rate the significance of your role.

You will then be asked to list specific actions that you have taken in your numeracy lead teacher [or principal/senior management team member] role to support this.

Numeracy goals

- ☐ Set goals and expectations for numeracy.
- ☐ Communicate numeracy goals and expectations to staff.
- ☐ Maintain a focus on improving student achievement in numeracy.

Numeracy resources

- ☐ Manage classroom numeracy resources.
- ☐ Organise people resources.
- ☐ Ensure teachers have access to numeracy information as it's needed.

Numeracy co-ordination

- ☐ Co-ordinate numeracy curriculum delivery across the school.
- ☐ Develop consistency across numeracy teaching programmes in the school.
- ☐ Monitor and evaluate numeracy teaching and curriculum.

Appendix M – continued

Teacher learning and development

- [ ] Promote teacher learning and development in numeracy.
- [ ] Participate in teacher learning and development in numeracy.
- [ ] Ensure an orderly and supportive school environment.
- [ ] Promote the importance of numeracy.
- [ ] Engage colleagues in constructive problem-talk about teaching and its impact on students' numeracy achievement.
- [ ] Challenge teachers to implement alternative strategies in order to improve student achievement.

Student achievement

- [ ] Decide what school-wide numeracy achievement information will be collected.
- [ ] Establish practices for monitoring and assessing student achievement in numeracy.
- [ ] Analyse and present students' numeracy achievement data.
- [ ] Use students' numeracy achievement data to inform decisions about numeracy.

Numeracy beyond the school

- [ ] Communicate numeracy goals and expectations to the school community.
- [ ] Encourage a closer match between students' experiences of numeracy outside school and the numeracy instruction that they experience.

Developing Numeracy at Your School

- Has numeracy been included as a professional development focus in your school during 2009?
  [Yes/No] Please comment.
- In 2010, is numeracy likely to be a professional development focus in your school?
  [Yes/No] Please comment.
- Was a numeracy goal included as part of teachers' appraisal this year?
  [Yes/No] Please comment.
- Please list the sorts of things that have been the focus of whole-staff numeracy meetings during 2009.
- Please list the sorts of things that have been the focus of team/syndicate discussion about numeracy during 2009.
- Do you believe your actions in the role of numeracy lead teacher [or principal/senior management team member] have made a positive difference to student achievement in numeracy?
  [Yes/No] Please explain.
- Are there any other comments that you would like to make about the role of leadership in developing numeracy in your school?

Appendix M – continued

Background Information

For how many years have you been on the staff at your current school?

____

For approximately how many years have you been in your current leadership role?

____

For approximately how many years in total have you worked in leadership roles in schools?

____

Appendix N (Actions Taken in Two Schools in Which Māori Students Achieve Well in Numeracy)

Sample Interview Questions

The following questions are indicative only:

1. What are the main strategies used by school-based leaders – senior management and numeracy lead teacher/s – to focus teachers' attention on the mathematics achievement of Māori students? (Question for leaders, teachers)
2. How might this have contributed to improving the mathematics achievement of Māori students? (Leaders, teachers)
3. What are the main strategies used by the school to engage parents with the mathematics achievement of Māori students? (Leaders, teachers)
4. In what ways is the school "comfortable" for Māori students and their whānau? (Probe: reporting to parents) (Leaders, teachers)
5. In what ways is the school/would the school be "comfortable" for Māori teachers? (Leaders)
6. What could be done to make the school more comfortable for Māori? (Leaders, teachers)
7. Has your knowledge and understanding of te ao Māori increased since you joined the school community? (Leaders, teachers)
   If so, how? How could your knowledge be further enhanced?
8. Has your knowledge and understanding of Māori learners increased since you joined the school community? (Leaders, teachers)
   If so, how? How could your knowledge be further enhanced?
9. Is teacher practice monitored or appraised using the professional standards? If so, how is this done in relation to the professional standard regarding the Treaty? (Leaders, teachers)
10. In what ways does the school enable Māori students to reflect and develop their Māori cultural identity? (Leaders, teachers)
11. In what ways does the school enable Māori students to reflect and develop their Māori cultural identity in their mathematics learning? (Leaders, teachers)
12. Is any special consideration being given to reporting Māori students' progress to parents/whānau against the mathematics standards? (Principal)

Appendix O (Explorations of Year 8 to Year 9 Transition in Mathematics)

Year 8 Student Questionnaire

Year 8 to Year 9 Transition in Mathematics

Please complete the following questionnaire. The purpose of this questionnaire is to find out about your feelings and ideas about mathematics learning and teaching and the move to year 9.

Thank you for completing the questionnaire. This questionnaire will only be read by the researchers.

Please complete the background information below:

Name: ______________________________

Gender: Female ☐ Male ☐

Ethnicity: ______________________ Number of older brothers/sisters: ☐

Age: ________ Years ________ Months

Date: ________________ Intended school for year 9: ________________

Tick one of the following boxes:

	Very true	True	Somewhat true	Not true
1. Mathematics is easy for me				
2. I like doing mathematics				

Please answer the following questions:

3. What are the key mathematical skills that you have learnt in year 8?
4. Is there anything that you don't enjoy about mathematics? Please explain.
5. What do you think will be the **same** in mathematics in year 9?
6. What do you think will be **different** in mathematics in year 9?
7. How have you been prepared for the move to year 9?
8. Specifically in mathematics, how have you been prepared for year 9?

Appendix O – continued

How important do you think each of the following are in preparing you to do well in mathematics in year 9?

	Extremely important	Very important	Somewhat important	Not important
a. Working in a group with other students				
b. Working alone				
c. Working with the teacher				
d. Sharing your ideas in a large group				
e. Working from a textbook				
f. Working from a worksheet				
g. Learning using games and activities				
h. Knowing your basic facts				
i. Being able to use a calculator				
j. Explaining your strategy solutions				
k. Convincing others about your mathematical thinking				
l. Writing your own word problems				
m. Learning from your mistakes in mathematics				
n. Learning from the mistakes of others				
o. Being able to ask for help in mathematics				
p. Taking part in competitions				

Comments:

Appendix P (Explorations of Year 8 to Year 9 Transition in Mathematics)

Year 8 Student Focus Group Interview Schedule

1. What have been the highlights in your mathematics learning this year?
2. Have there been any less enjoyable or disappointing aspects? Tell me about them.
3. What are your expectations for mathematics next year?
4. Are there any matters that concern you?
5. What other comments do you want to make about the transition to secondary school?

Appendix Q (Explorations of Year 8 to Year 9 Transition in Mathematics)

Lead Teacher (Primary) Interview Schedule

1. What are your school's practices in relation to the Numeracy Project?

2. What are your school's assessment practices in relation to mathematics?

3. What provisions are made for gifted and talented students in mathematics?

4. What provisions are made for Māori students?

5. What provisions are made for Pasifika students?

6. What are your school's transition practices?

   What information is passed on to the secondary school?

   What liaison visits (if any) are made?

   Do you have any placement concerns?

   How do you think the secondary schools use the information?

   How do you think your mathematics teaching practices compare with secondary schools? What do you think are the similarities and what are the differences? Please describe these.

Appendix R (Explorations of Year 8 to Year 9 Transition in Mathematics)

Year 9 Student Questionnaire

Year 8 to Year 9 Transition in Mathematics

Please complete the following questionnaire. The purpose of this questionnaire is to find out about your move to year 9. We want to know about the similarities and differences in your mathematics learning and teaching between year 8 and year 9.

Thank you for completing the questionnaire. This questionnaire will only be read by the researchers.

Please complete the background information below:

Name: ______________________________

Date: ______________________ School: ______________________

How much of the following is happening in your year 9 mathematics programme?

	Most of the time	Some of the time	Occas-sionally	Not at all
a. Working in a group with other students				
b. Working alone				
c. Working with the teacher				
d. Sharing your ideas in a large group				
e. Working from a textbook				
f. Working from a worksheet				
g. Learning using games and activities				
h. Using your basic facts				
i. Using a calculator				
j. Explaining your strategy solutions				
k. Convincing others about your mathematical thinking				
l. Writing your own word problems				
m. Learning from your mistakes in mathematics				
n. Learning from the mistakes of others				
o. Being able to ask for help in mathematics				
p. Taking part in competitions				

Comments:

Appendix R – continued

Please answer the following questions:

1. How often do you have mathematics in a week?

2. How long is a mathematics lesson?

3. What topic are you studying at the moment?

   Have you studied any other topics this year? If so, please list them.

4. Have you completed any maths tests this year? Please describe.

5. What maths resources do you use in class?

6. What maths textbook do you use?

7. What does a typical mathematics lesson consist of?

8. What does your teacher do to help you improve your maths? (Give examples)

9. What aspects of the maths teaching and learning are the same as in year 8?

10. What aspects of the maths teaching and learning are different from year 8?

11. If you were to go back to your previous school to talk with your teacher and the students, what advice would you given them about how best to prepare students for maths in year 9?

Appendix S (Explorations of Year 8 to Year 9 Transition in Mathematics)

Year 9 Student Focus Group Interview Schedule

1. How do you think your shift to secondary school has been?
2. How do you think your shift to secondary school in mathematics has been? Why?
3. Has your mathematics class matched what you expected maths to be like at secondary school?
4. What are some of the differences in mathematics at secondary school compared with intermediate school?
5. What are some of the similarities in mathematics between what you did at intermediate school and secondary school?
6. Do you enjoy mathematics here more, less, or the same as you did at intermediate school? Why?
7. Are you more or less confident in mathematics at secondary school? Why?
8. If you were invited to talk to the year 8 students at your intermediate school, or to the staff, about how to prepare for secondary school mathematics, what would you tell them is most important?

Appendix T (Explorations of Year 8 to Year 9 Transition in Mathematics)

Secondary HoD and/or In-school Facilitator Interview Schedule

1. Grouping/streaming organisation practices for maths at year 9
2. What are your school's practices in relation to the Numeracy Project? SNP involvement?
3. Scheme for year 9: balance between the various strands
4. What are your school's assessment practices in relation to mathematics and what records are kept?
5. What provisions are made for gifted and talented students in mathematics?
6. What provisions are made for Māori students?
7. What provisions are made for Pasifika students?
8. What are your school's transition practices?
9. What data do you receive from the year 8 schools? How do you use it?
10. Is there other information that is important and that you would like to get from the year 8 schools?
11. What data do you collect yourselves about the students?
12. Explain the similarities/differences in the mathematics teaching and learning practices between the intermediate and secondary school.

Appendix U (The Role of Leadership in Promoting the Teaching of Pāngarau in Wharekura)

Backward mapping of pre-existing data and reports

Considerations	What supports do system-level instructional leaders need in order to meet the demands of supporting school-based leaders' development?	What needs to be in place at the system level to support school-based instructional leaders (tumuaki)?	What supports do school-based instructional leaders (tumuaki) need in order to meet the demands of supporting teacher development?	What needs to be in place at the wharekura to support teachers?	What supports do teachers in wharekura need in order to meet the demands of developing high-quality mathematics instruction?	What is the vision for high-quality mathematics instruction in wharekura?
Goals	To improve system-level instructional leaders' knowledge of how to support school-based instructional leaders	To improve instructional leaders' practice in building school-based instructional leaders' knowledge and practice	To improve instructional leaders' (e.g., tumuaki) knowledge of how to support teachers	To improve instructional leaders' (e.g., tumuaki) practice in building teachers' knowledge and practice	To improve teacher knowledge and practice	To improve student achievement and disposition towards mathematics
Conjectures (developed from previous data-sets)	• Facilitators' knowledge of professional development models enables them to provide a contextually responsive approach to the schools with whom they work • Facilitators' understanding of the nuances of different kura cultures.	• Facilitators responding to the context of the school community will be able to provide authentic and appropriate supports for instructional leaders in that school.	• Support in leading the development of the mathematics strategic plan • Support in consulting over the strategic plan with the community (how it is delivered, how whānau contribute to it) • Support in providing the required supports and resources for teachers to improve their knowledge and practice • Knowing how to distribute leadership enables tumuaki to maximise various sources of expertise within the school community.	• Provision of professional development structures enables teachers to improve: – their knowledge of mathematics – their capacity in te reo pāngarau – their instructional practices – and to develop networks with other teachers. • Numeracy prioritised in the schools' strategic plan • Buy-into the strategic direction from school management and whānau.	• Improving te reo pāngarau enables teachers to: – present pāngarau as a cultural construction – make obvious Māori world view in pāngarau – articulate mathematical concepts.	• Student achievement will be improved through: – providing students with more opportunities to talk mathematically through te reo pāngarau – students creating their own representations of mathematical concepts – understanding the link between representations and te reo pāngarau – making links between their everyday culture and a maths culture – providing challenging problems that tauira (students) have to solve in a variety of different ways.

Appendix V (The Impact of Professional Development Interventions for Numeracy "Pick-ups": Content Knowledge and Teachers' Perceptions of Valued Aspects)

Professional Development Programme Evaluation

Part A: Course(s) and/or Sessions

Please respond to the following two statements on the scales provided by *circling* a response.

1. The material of the course/sessions was valuable and helpful in meeting my needs.

   Very strongly agree Strongly agree Moderately agree Agree Disagree Strongly disagree

2. The time and energy involved in attending the course/sessions was worthwhile.

   Very strongly agree Strongly agree Moderately agree Agree Disagree Strongly disagree

3. Now, please *circle up to three elements* in the grid below that were *most helpful* for you in learning to work more effectively with the students in your class.[7]

Please cross out any that do not apply to you.

Learning more about what students need to be doing next in mathematics 3a[8]	Improving my personal understanding of multiplication/division 1c	The teaching practices used by the lecturer/facilitator 5a	Improving my personal understanding of students' learning progressions in fractions 2d	The format and conduct of the sessions (the way the lecturer/facilitator taught) was a useful model for my classroom. 5b
Improving my personal understanding of percentages 1f	Improving my personal understanding of whole numbers 1a	Improving my personal understanding of fractions 1d	Improving my personal understanding of decimals 1e	Learning more about how to organise my classroom effectively 3c
Improving my personal understanding of addition/subtraction 1b	Improving my personal understanding of students' learning progressions in addition/subtraction 2b	Making me feel like a learner again 4a	Gaining a greater understanding of what learning content (for example, place value) is like for students 4b	Improving my personal understanding of students' learning progressions in whole numbers 2a
Learning more about available resources and how to use them 3d	Improving my personal understanding of students' learning progressions in decimals 2e	Improving my personal understanding of students' learning progressions in percentages 2f	Learning more about what I need to be doing next when teaching students 3b	Improving my personal understanding of students' learning progressions in multiplication/division 2c

Any other comments:

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7 The elements are arranged randomly in this table and in the table in Part B, as they were in the study, to allow for the possibility of replicating the study.

8 The elements are numbered in this table and in the table in Part B for reference purposes only, within this paper.

Appendix V – continued

Part B: In-class Support

Please respond to the following two statements on the scales provided by *circling* a response.

4. The in-class support was valuable and helpful in meeting my needs.

   Very strongly agree Strongly agree Moderately agree Agree Disagree Strongly disagree

5. The time and energy involved with in-class support was worthwhile.

   Very strongly agree Strongly agree Moderately agree Agree Disagree Strongly disagree

6. Now, please *circle up to three elements* in the grid below that were *most helpful* for you in working more effectively with the students in your class.

Please cross out any that do not apply to you.

1a	Being observed by, and having follow-up discussions with, a lead teacher 1b	2c	Observing a lead teacher teaching in a class 2d	3a
Discussing **my own** numeracy teaching issues with (an)other teacher(s) 6	Sharing the teaching of my class or a group with a lead teacher 3e	Sharing planning with (an)other teacher(s) 3c	Being given resources by the facilitator 4a	Observing a lead teacher teaching in my class 2b
Sharing the teaching of my class or a group with another teacher 3f	Choosing the type of support I will receive 5	Working with a facilitator outside of the sessions 8	Observing another classroom teacher teaching their class 2e	Sharing planning with a lead teacher 3b
Being given resources by a lead teacher 4b	Being given resources by another teacher(s) 4c	Being observed by, and having follow-up discussions with another teacher 1c	3d	Having release time to work one-to-one with students 7

Any other comments:

Appendix W (The Impact of a University Course Focusing on PCK and MCK: Does Teachers' Classroom Practice Reflect the Professional Development Experience?)

The Reformed Teaching Observation Protocol Items

	LESSON DESIGN and IMPLEMENTATION (Aspect 1)
1	The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.
2	The lesson was designed to engage students as members of a learning community.
3	In this lesson, student exploration preceded formal presentation.
4	This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.
5	The focus and direction of the lesson was often determined by ideas originating with students.
	CONTENT (Aspect 2)
	Propositional Knowledge
6	The lesson involved fundamental concepts of the subject.
7	The lesson promoted strongly coherent conceptual understanding.
8	The teacher had a solid grasp of the subject matter content inherent in the lesson.
9	Elements of abstraction (i.e., symbolic representations, theory building) were encouraged when it was important to do so.
10	Connections with other content disciplines and/or real world phenomena were explored and valued.
	Procedural Knowledge
11	Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent phenomena.
12	Students made predictions, estimations, and/or hypotheses and devised means for testing them.
13	Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.
14	Students were reflective about their learning.
15	Intellectual rigour, constructive criticism, and the challenging of ideas were valued.

Appendix W – continued

	CLASSROOM CULTURE (Aspect 3)
	Communicative Interactions
16	Students were involved in the communication of their ideas to others, using a variety of means and media.
17	The teacher’s questions triggered divergent modes of thinking.
18	There was a high proportion of student talk, and a significant amount of it occurred between and among students.
19	Student questions and comments often determined the focus and direction of classroom discourse.
20	There was a climate of respect for what others had to say.
	Student/Teacher Relationships
21	Active participation of students was encouraged and valued.
22	Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.
23	In general, the teacher was patient with students.
24	The teacher acted as a resource person, working to support and enhance student investigations.
25	The metaphor “teacher as listener” was very characteristic of this classroom.

Appendix X (The Impact of a University Course Focusing on PCK and MCK: Does Teachers' Classroom Practice Reflect the Professional Development Experience?)

RTOP responses for the seven participants

TEACHERS	T1	T2	T3	T4	T5	T6	T7
Item 1	4	3	3	4	2	4	4
Item 2	4	4	3	4	3	4	4
Item 3	3	4	2	3	2	4	4
Item 4	2	4	3	4	2	3	3
Item 5	3	3	3	3	3	4	4
Aspect 1 Total	16	18	13	18	12	19	19
Item 6	4	3	3	4	3	4	4
Item 7	3	4	2	4	2	4	4
Item 8	4	4	2	4	3	4	4
Item 9	4	3	3	4	3	4	4
Item 10	3	4	2	3	1	3	3
Sub-scale 2 total	*18*	*18*	*12*	*19*	*12*	*19*	*19*
Item 11	4	4	2	4	2	4	4
Item 12	3	2	2	4	3	4	4
Item 13	3	4	2	4	2	4	4
Item 14	2	2	3	4	3	4	4
Item 15	4	4	2	4	2	4	4
Sub-scale 3 total	*16*	*16*	*11*	*20*	*12*	*20*	*20*
Aspect 2 Total	34	34	23	39	24	39	39
Item 16	4	4	3	4	3	4	4
Item 17	2	3	3	4	2	4	4
Item 18	3	3	4	4	3	4	4
Item 19	2	2	3	4	3	4	4
Item 20	4	4	4	4	2	4	4
Sub-scale 4 total	*15*	*16*	*17*	*20*	*13*	*20*	*20*
Item 21	4	4	4	4	3	4	4
Item 22	3	3	3	4	2	4	4
Item 23	4	4	3	4	3	4	4
Item 24	4	4	3	4	3	4	4
Item 25	4	4	3	4	3	4	4
Sub-scale 5 total	*19*	*19*	*16*	*20*	*14*	*20*	*20*
Aspect 3 Total	34	35	33	40	27	40	40
OVERALL TOTAL	84	87	69	97	63	98	98