Student Learning and Understanding in the CAS Pilot Project

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Twenty-two schools were involved in a pilot scheme to evaluate the use of Computer Algebraic System (CAS) calculators. Teachers underwent professional development on effective ways to use CAS handheld calculators in their year 9 and year 10 mathematics classes and then integrated the calculators into their mathematics classrooms at these year levels. One of the aims of the CAS Pilot Project was to evaluate the effect on student mathematical skills and understanding. Student achievement data was collected using the new PAT[1]:Mathematics tests at the end of year 9 and again at the end of year 10. Achievement data was also collected from a control class in each school. The researchers were able to compare the changes in the levels of mathematical skills and understandings of CAS classes with those of control classes. The researchers were also able to collect qualitative data, both by interviewing teachers and small groups of students and by using questionnaires for teachers and students. The PAT:Mathematics achievement data showed that students in the CAS project progressed at about the same rate as the control class students. There were only minor, educationally insignificant differences between the calculator brands. The researchers' classroom observations and the teachers' responses showed strong changes in teaching style, with a move towards a more constructivist approach rather than a transmission style. Despite no evidence of progress in overall rates of achievement on the PAT:Mathematics results, both teachers and students reported a significant improvement in their understanding of algebra as a result of the CAS approach to teaching and learning.

Background

In 2005, the Ministry of Education and the New Zealand Qualifications Authority began a pilot scheme exploring the effective use of Computer Algebraic System (CAS) calculators in mathematics classrooms. Such use is consistent with an aspect of one of the key competencies in *The New Zealand Curriculum*, using language, symbols, and texts, which states that students should "confidently use ICT" (Ministry of Education, 2007, p. 12).

In 2005, six schools participated in the CAS Pilot Project, which in that year, involved two different brands of calculator. A further 16 schools participated in 2006, when a third brand of calculator was introduced. Each school used just the one brand of calculator. Fourteen schools used the first brand, while four schools each used one of the two other brands. Each brand had its own separate professional development (PD) provider, all with different emphases.

Each school was actively involved in the project for two years. In the first year, two teachers with a year 9 class from each school were given professional development (PD) aimed at using CAS calculators in year 9 algebra and geometry. In the second year, the two original teachers received further PD on using CAS calculators with year 10 classes while two more teachers were given the year 9 PD. The PD involved an introduction on how to operate CAS calculators, but it particularly focused on how to use the CAS calculator effectively for teaching and learning. The PD thus took account of the recommendation that "the use of CAS technology needs to be accompanied by the development of algebraic insight, including the ability to identify the structure and key features of expressions and to link representations" (Anthony & Walshaw, 2007, p. 139). Multiple representations were a feature of the CAS Pilot Project PD.

An evaluation of the 2005 CAS Pilot Project was undertaken by the New Zealand Council for Educational Research (NZCER) and their results published (Neill & Maguire, 2006a, b). The subsequent

[1] Progressive Achievement Tests, NZCER

evaluation, reported on in part in this paper, looked at effects during 2006 and 2007 (Neill & Maguire, 2008).

This paper focuses on just one of the eight research questions of the full evaluation: "How has student learning in mathematics been affected as a result of the pilot scheme?"

Methodology

Twenty-two schools took part in the CAS Pilot Project, with 16 in the North Island and six in the South Island. There was a range of decile ratings among the schools selected, with an overall distribution close to the national distribution of deciles. All of the schools were state schools except for one integrated school. Twelve schools were in major population centres, seven in provincial cities, and three in smaller provincial towns. Seven were single-sex schools (four for girls and three for boys), and 15 were co-educational.

All teachers in the pilot were given a questionnaire, which asked, among other things, for their views on the effect of the CAS Pilot Project on students' mathematics learning and understanding. Teachers were also asked this question in face-to-face interviews.

Students were also asked, in end-of-year questionnaires and in small focus groups, about the effect that their involvement in the CAS Pilot Project might have had on their mathematics learning and understanding.

Student achievement was measured with the PAT:Mathematics scale score (patm). Students were given a baseline test of their mathematical ability at the beginning of year 9. Follow-up achievement tests were also given at the end of year 9 and at the end of year 10, using the relevant version of PAT:Mathematics.

Achievement assessed by different PAT tests can be mapped onto this patm scale score, which is a true interval scale. It measures growth in a linear way so that increases in the patm units are independent of the position of the student on the scale (Darr, Neill, & Stephanou, 2006). This means that an increase in 5 patm units represents the same amount of growth for a highly-able student as it does for a student of average ability. A new version of the test (Test 8) was constructed in conjunction with the PAT development schedule for use in this evaluation. Table 1 shows which PAT:Mathematics test was given to which students and when.

Table 1
Schedule of PAT:Mathematics Achievement Tests

	2006		2007	
	Term 1	Term 4	Term 1	Term 4
2005 cohort	Test 7	Test 8	–	*(NCEA Level 1)*
2006 cohort	Test 6	Test 7	–	Test 8
2007 cohort	–	–	Test 6	Test 7

Student Assessment Results

This section reports on the changes in mathematics achievement as measured by the patm scale score derived from the PAT:Mathematics tests. It is subdivided into results for the 2006 year 9 students, the 2007 year 9 students, and the students who were in year 10 in 2007. The latter group is the group

of students who were in year 9 in 2006. As well as the results shown here, an analysis was also done to look for evidence of growth in achievement in each of the individual CAS and control classes in the study. This analysis showed that some schools and some classes made more significant progress than others, which indicates that teacher effects as well as school effects are likely to be statistically significant. The results below are those aggregated over all classes.

The 2006 Year 9 Students' Growth in Achievement

Control versus CAS class improvement for 2006 year 9 students

In 2006, the average score for students in year 9 control classes went up by 4.83 patm units compared with an increase of 4.03 patm units for year 9 CAS classes. The difference of 0.80 patm units is statistically significant at the 5% level ($p = 0.024$[2]).

These increases indicate that both the CAS and the control class students experienced growth on the patm scale of mathematical understanding that was close to the expected growth of about 5 patm units in a year. A number of different factors may have influenced these results, including the exclusion of all calculators from the PAT:Mathematics tests.

Student achievement and calculator type[3] for 2006 year 9 students

No significant differences were observed when comparing increases in the achievement levels of classes using the different types of CAS calculator. The mean increases were as follows:

Type A: 4.42 patm units; Type B: 3.53 patm units; Type C: 4.24 patm units.

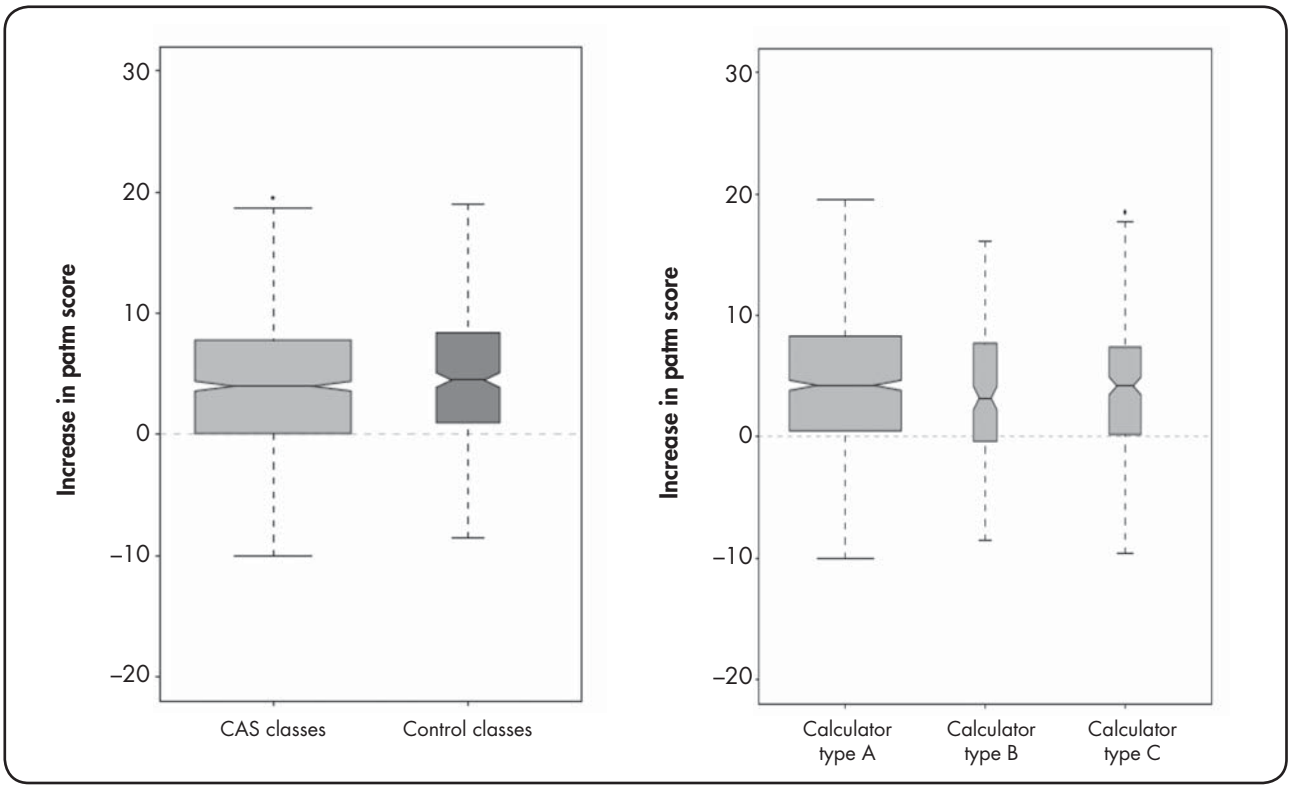

Figure 1. Gain in achievement measured in patm scale points – 2006 year 9 cohort

[2] Probability of 0.024 that the two groups are not actually different, based on a T-test. This means that they are statistically significantly different at the 2.5% level.

[3] "Type" in the context of this paper means one of the three brands of CAS calculators and its associated PD.

The 2007 Year 9 Students' Growth in Achievement

Control versus CAS class improvement for 2007 year 9 students

For the 2007 year 9 cohort, the average score of the control classes went up by 3.91 patm, units, compared with an increase of 4.53 units for CAS classes. The difference of 0.62 patm units is in the opposite direction to that of the 2006 cohort, but the difference is not statistically significant at the 5% level.

These increases indicate that the year 9 students in both the CAS and the control class experienced growth in their mathematical understanding (measured on the patm scale) that was close to the expected growth of about 5 patm units in a year.

Student achievement and calculator type for 2007 year 9 students

Only one small but significant difference was observed between the achievement levels of students in classes using their assigned type of CAS calculator. Classes using type A calculators showed a significantly greater amount of growth than those using type C calculators. This just reached significance at the 5% level ($p = 0.047$) and represents about one-fifth of a year's extra growth, which is small educationally. The mean increases were as follows:

Type A: 4.64 patm units; Type B: 4.17 patm units; Type C: 3.69 patm units.

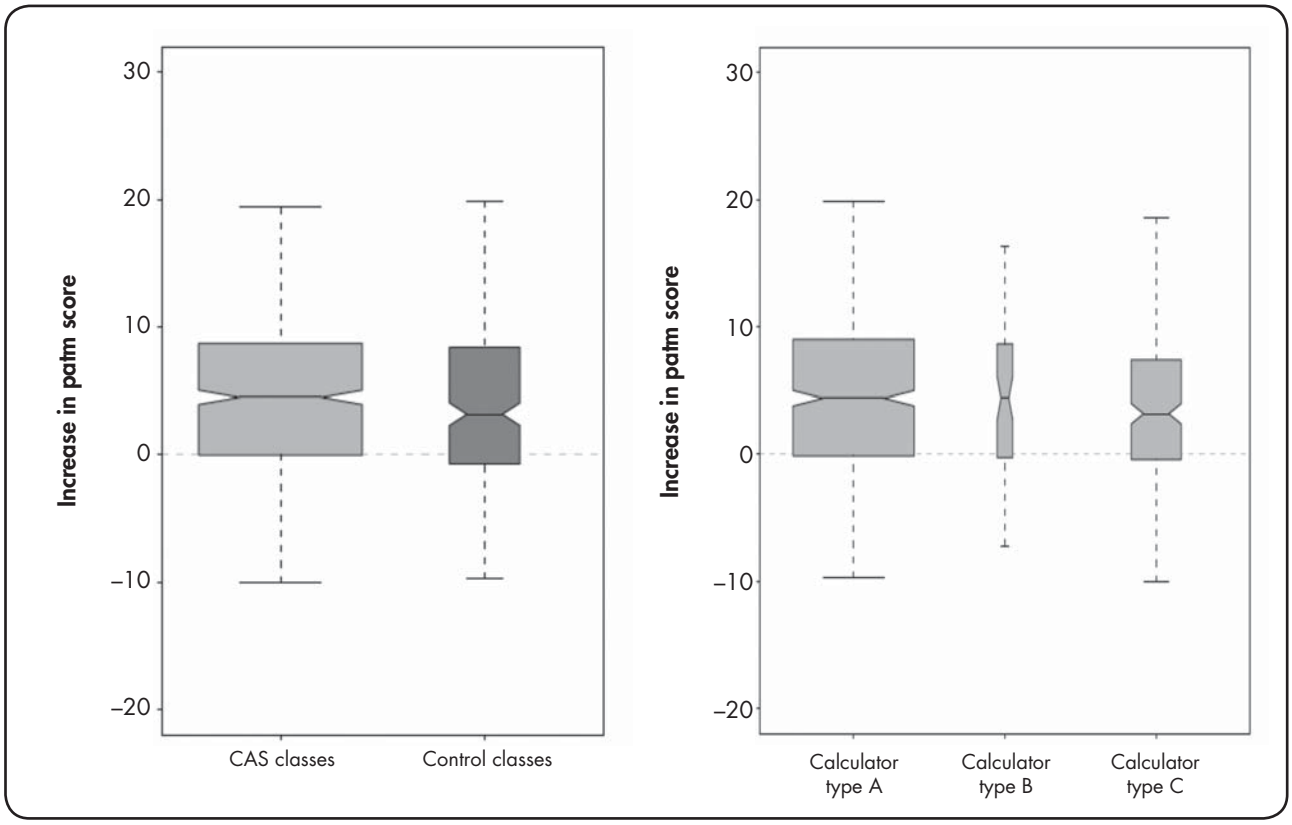

Figure 2. Gain in achievement measured in patm scale points – 2007 year 9 cohort

The 2007 Year 10 Students' Growth in Achievement

The year 10 students showed virtually no growth on the patm scale. This was measured by a new PAT test that has yet to be published. The new test seems to perform as it should psychometrically, and hence it is surprising that it measured no growth because there was uniform growth of about 5 patm units per year on the seven published tests. Further work is being undertaken to ascertain

why this has happened. Notwithstanding this issue, the comparison of the change in mathematical understanding between CAS and control classes, or between the CAS calculator types, is still valid.

Control versus CAS class improvement for 2007 year 10 students

For the 2007 year 10 students, the average score of control classes went down by 0.55 patm units compared with an increase of 0.53 units for CAS classes. The difference of 1.08 patm units is in the opposite direction to that of the same students when they were in year 9 (in 2006). However, neither change is statistically significant at the 5% level, so growth over the two years is virtually identical for CAS and control classes.

Student achievement and calculator type for 2007 year 10 students

Year 10 students in classes using the CAS Type A calculator showed a small but significant growth in patm units, whereas those using CAS Type B or C calculators showed no significant growth.

Type A: 1.59 patm units; Type B: –0.26 patm units; Type C: 0.43 patm units.

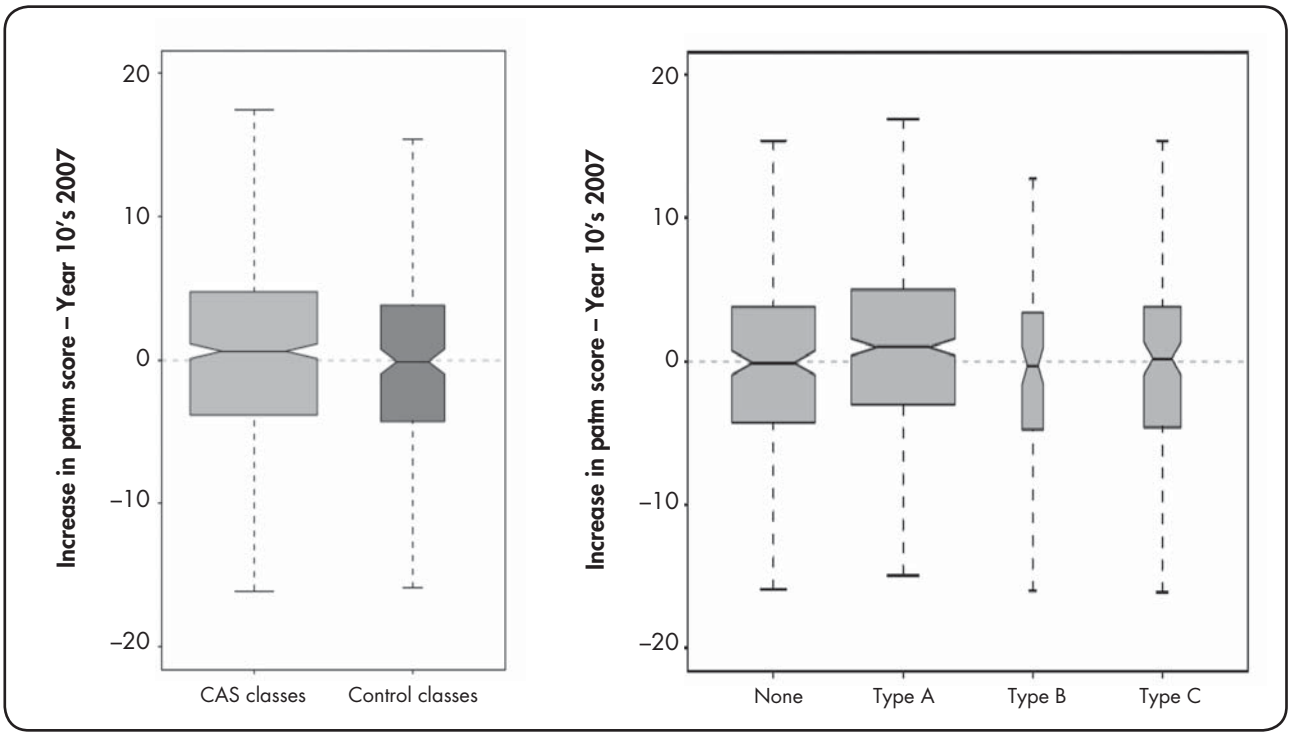

Figure 3. Gain in achievement measured in patm scale points – 2007 year 10 cohort

Overall Effects of the CAS Pilot Project

While there are slight differences between CAS and control-class students, these differences are not consistently in the same direction. The researchers therefore concluded that the two groups showed the same level of growth.

With regard to the CAS calculators, type A calculators show a small but significant advantage over types B and C in year 10 and a slight advantage over type C calculators in the 2007 year 9 cohort. This is about a 1 patm unit, which is barely educationally significant. The brand of calculator fully determined the PD that teachers received, so it is not possible to disentangle the brand effect from the PD effect (that is, they are fully confounded). It may also be that other background factors account for any differences. For these reasons, it would be inadvisable to single out any one calculator–PD mix as resulting in greater growth in mathematical understanding than the other two.

Students' Perception of Their Learning

Questionnaire Responses

All the CAS students were asked in a questionnaire to rate how they thought their own mathematical understanding had changed during the CAS Pilot Project. They were asked this separately in relation to both algebra and geometry. Figures 4 and 5 respectively show the pattern of the students' responses to these questions.

For algebra, in both the 2006 year 9 cohort and the 2007 year 9 cohort, far more students reported an increase rather than a decrease in their understanding as a result of using CAS. This can be seen in Figure 4, in which both distributions in the left-hand graph are positively skewed and virtually identical (chi-sq = 1.75, 4 d.f., N.S.),[4] so the mean response lies about half way between "agree" and "neutral". There was a similar, but less pronounced increase for geometry because the distributions in the left-hand graph of Figure 5 are also positively skewed, but to a lesser extent, and are not significantly different from each other (chi-sq = 6.95, 4 d.f., N.S.).

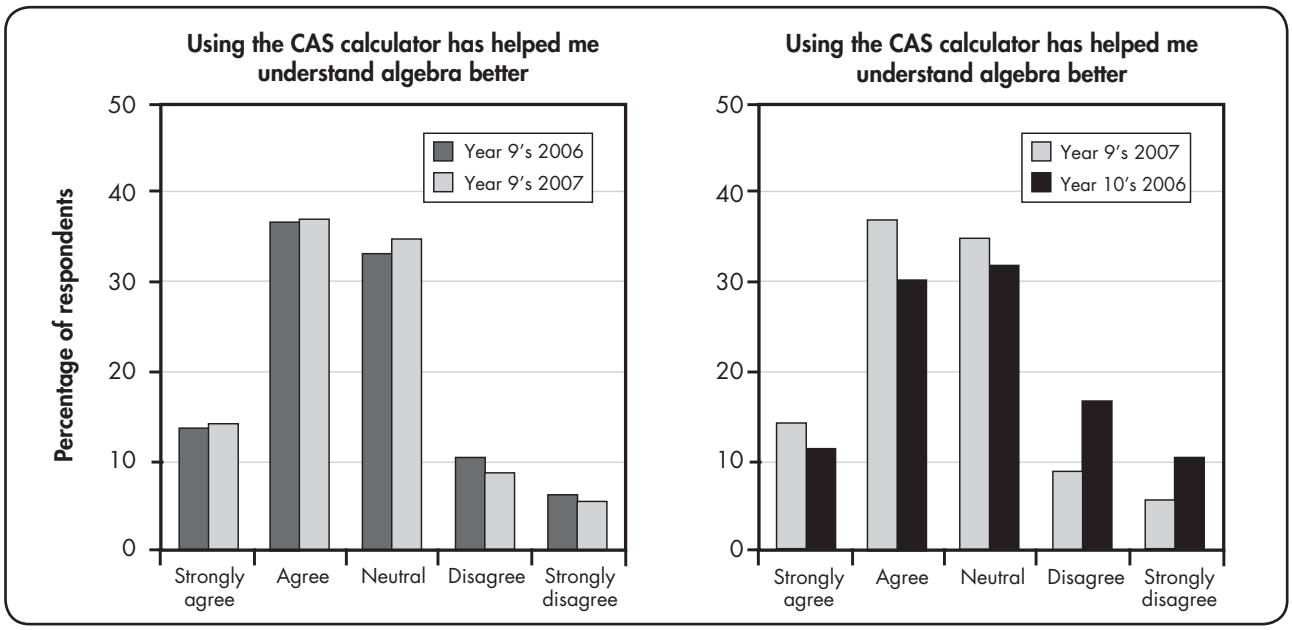

Figure 4. Effect on algebra understanding of CAS calculator use

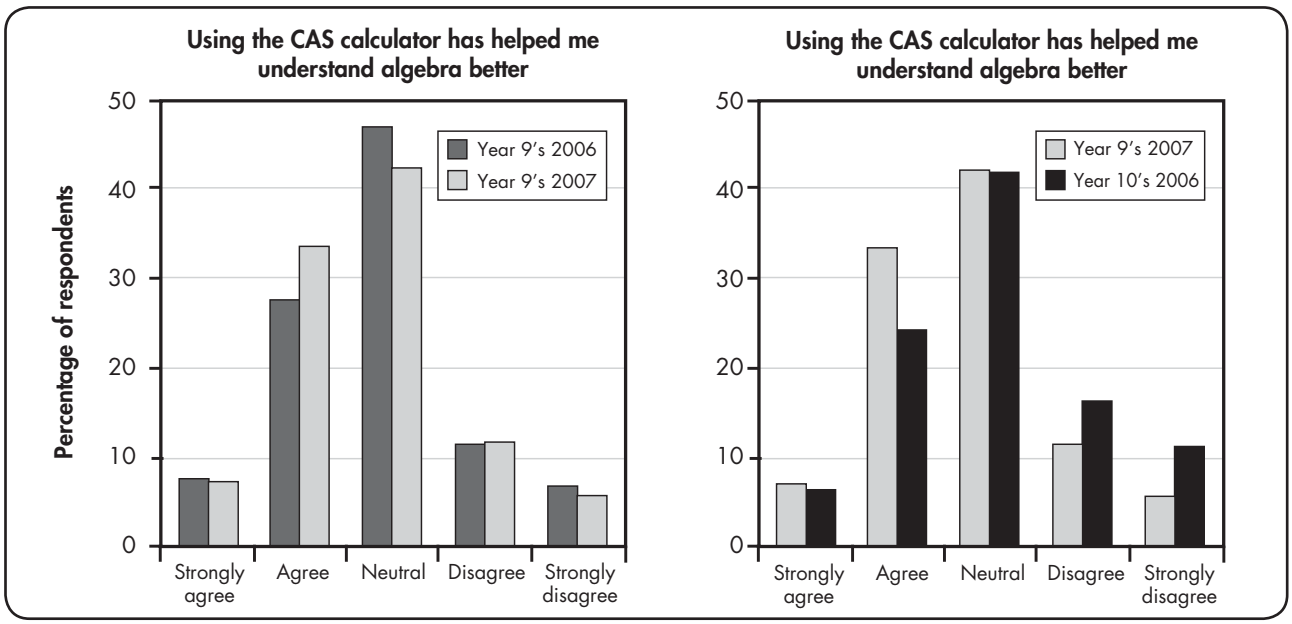

Figure 5. Effect on geometry understanding of CAS calculator use

[4] Based on a chi-squared test, testing whether the two distributions were different, giving a value of 1.75 with 4 degrees of freedom (d.f.), which shows no significant (N.S.) difference between the two at the 5% level.

There is, however, a statistically significant difference between the responses of year 9 and year 10 students.

- For algebra, significantly more year 9 than year 10 students reported improved algebraic understanding following their involvement in the CAS Pilot Project, with fewer students disagreeing or strongly disagreeing that their understanding had improved.

  (Year 9 2006 vs year 10 2007: chi-sq = 29.66, 4 d.f., $p < 0.0001$)

  (Year 9 2007 vs year 10 2007: chi-sq = 34.76, 4 d.f., $p < 0.0001$)

  Overall, however, the year 10 students still reported a slight improvement (testing if the distribution for year 10s in Figure 4 is symmetric, chi-sq = 11.97, 4 d.f., $p = 0.0075$).

- For geometry, year 10 students did not report any significant improvement in their understanding (in testing if the distribution for year 10s in Figure 5 is symmetric, chi-sq = 0.70, 4 d.f., N.S.), even though many year 9 students did report improvement in their understanding.

  (2006 year 9 vs 2007 year 10 chi-sq = 27.30, 4 d.f., $p < 0.0001$)

  (2007 year 9 vs 2007 year 10 chi-sq = 22.27, 4 d.f., $p = 0.0002$)

Responses from Student Focus Groups

Students in the focus groups and individual students in classrooms were also asked if using the CAS calculator had helped them understand mathematics better, especially algebra. Students gave a variety of responses, both positive and negative.

Influence of visual displays

By far the most common reason that students gave for their improved mathematical understanding related to an individual student's learning preference. Several said the visual features of the CAS calculator had been a major positive influence on their understanding. This was stated directly (see the first two quotes below) or implied, as indicated by the latter two quotes, which show that the students did not like a pencil-and-paper approach:

> I can see pictures of graphs rather than seeing it in my head.
>
> [It] helps me understand it because it is visual.
>
> [It has helped on] how to work problems out. [I get] frustrated on paper.
>
> [It] helps do things I couldn't write down.

On the other hand, some students found that writing things down helped their understanding or their recall, as the following quotes show:

> [I'm] better off when writing things down, especially for exams.
>
> [It] helped a bit for understanding, but better to write it down.

This indicates that the technology has the potential to offer visual "advantages" (often referred to as "affordances" in the literature), although some students find these of more help than other students do.

Other positive influences

Students identified some other advantages of the CAS approach:

- Constructivist approach
  The teacher lets us find out how to find out the answer, so [we] get a better understanding.

- Technology feedback loops
  It helps. It showed you the step that was wrong.

- Reinforcement of ideas
  It has changed [for the better] because we go over it more often.

Other negative influences

Students also noted the following negative consequences of the CAS approach:

- Technology as a "black box"[5]
  I haven't really learnt how to do it myself.

  [It is] worse because I didn't have to work it out.

  I don't get how it gets the answer.

- Over-emphasis on technology
  [We are] focusing on what buttons to push rather than why.

- Difficulty with technology
  [Its] more confusing, [for example] how to make a formula.

Teacher Perception of Students' Learning and Understanding

Teachers were asked in both interviews and the questionnaire what they thought the effect of the CAS approach to teaching mathematics was having on student learning, skills, and understanding.

Results from the Follow-up Questionnaire

Figure 6 shows the questionnaire responses of 55 teachers when asked to what extent they agreed with statements about improvement in students' mathematical skills and understanding.

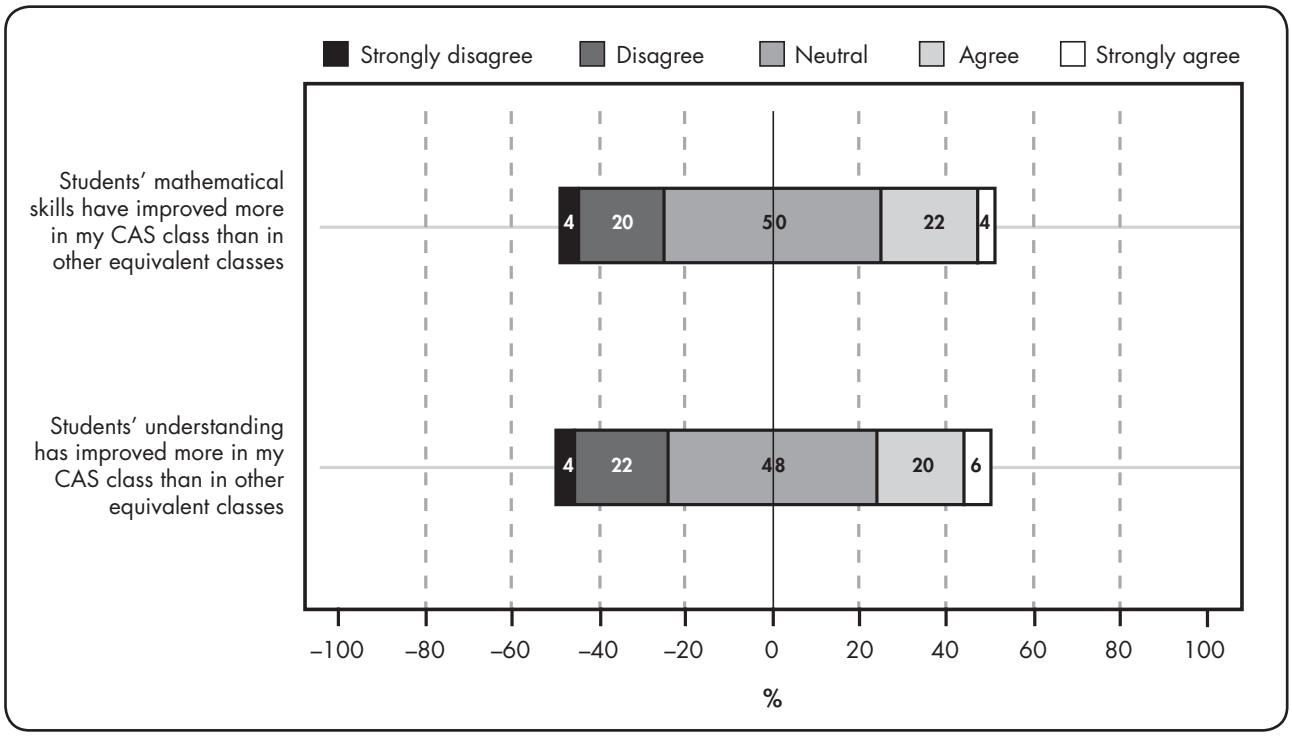

Figure 6. Teachers' views on effect of CAS on mathematical skills and understanding

[5] A "black box" is an object for which a person may not know what's inside but they do need to evaluate what is good or bad for them about not knowing.

Figure 6 clearly shows that, overall, teachers have not perceived any significant change in students' mathematical skills (top bar) or in their understanding. For each point, about half of the teachers gave a neutral response and the others were equally distributed on either side of neutral, with very few strongly agreeing or disagreeing.

Only a small number of teachers elaborated on their response (with a total of 29 comments spread between both questions). The only comments to stand out were that: it was too early to say if either skills or understanding had changed (6 comments); there may be improvements in some areas, especially algebra (5 comments); and students gave up because of poorer attitudes or lack of confidence (4 comments).

Teacher Interview Responses

Teachers offered a range of views on what effect the CAS Pilot Project had on students' mathematics learning and understanding. Most comments from the interviews were either positive or neutral, though some were negative.

Positive effects on algebraic understanding

A number of teachers commented that they thought that there was a positive effect, especially in algebraic understanding:

> It has encouraged their participation. If they are doing more and discussing more, it must have an effect. There is much more discussion in class. The discussions that kids have in algebra make me think that algebra is not so foreign.
>
> Year 9s have picked up algebra better than any other class. Struggling students have benefited and higher-achieving students are inspired to "play" with calculators, exploring and extending themselves.
>
> There is a big difference, especially for students who said "I can't do algebra". [Students] are feeling more encouraged.
>
> It allows students to think. Students are getting understanding … but need reinforcement.

Some teachers thought that, while it was still too early to know, improved understanding and performances might not be observed until students were in the senior school:

> It may give benefits at year 12. It has real potential to do this. Students have the idea of a "variable" better. We don't see the spin-offs yet.
>
> [It] may make a difference at years 12 and 13 for the mathematically able.

Negative effects on algebraic understanding

A number of teachers commented on the potential negative effects that may eventuate from the CAS approach. The first concern was that students might become over-dependent on calculators or might undervalue being able to perform algebraic manipulation manually:

> I don't want students to be over-dependent on CAS calculators.
>
> [I am] concerned the kids will not value being able to do it by hand if the calculator can do it.

The other main concern was that the mathematical content might get lost in the details of how to actually use the calculator:

> With the animations, some were so focused on how to do it that the concept was lost. I found it better when students were watching on the screen.
>
> Kids get lost in the instructions.
>
> [I am] worried that in algebra – will [students] learn the right thing? What are we supposed to be doing? What sort of knowledge do they need? Maths skills versus calculator skills.

Effects on different student groups

Of the teachers who commented on the relative progress of different groups, most thought that the CAS approach had been most useful for students who had moderate or above average mathematical ability:

> I thought it was going nowhere, then it fell into place, especially [for] the better students.

However, some teachers thought that weaker students would not be helped by using the CAS approach:

> [It was] not effective on a weak class. [There were] too many who couldn't get the CAS working for them.

The above quote was from a teacher who was very comfortable and skilled with technology but who taught a class with lower mathematical ability and behavioural issues. That teacher had consequently reverted to using a traditional, less constructivist approach.

Some teachers thought that the CAS approach suited a different type of student:

> It is a different cohort excelling, but the same overall distribution. It requires a different skill set.

However, a comment from another teacher may explain some of the negative comments that previously successful students had made in student focus groups:

> [It is] hard to get students out of their comfort zones and approach maths in a completely different way that requires more problem solving and thinking. Initial hard work is starting to see progress.

Discussion

This paper shows that students who were involved in the CAS Pilot Project achieved about the same levels of increase in mathematical ability as students who did not take part in the pilot. The differing abilities of students are taken into account in this finding because the scale is linear, meaning that the measure of growth is constant at any point on the achievement scale.

This finding is consistent with findings from Heid, Blume, Hollebrands, and Piez (2002), who identified that in eight out of nine studies, students who had used CAS did at least as well as students who had not used CAS, even though the tests did not allow the use of CAS calculators. Heid et al. went on to cite studies that showed that students who used a CAS approach had the same or better conceptual understanding, and also better problem-solving skills, than students who had not used this approach.

The teachers involved in this study generally held the view that the CAS approach had a neutral effect on student mathematical skills and understanding, although some thought that the gains may well become evident at the senior school level. There was some mention that the teaching approach used in the CAS Pilot Project might lay a better foundation for developing mathematical understanding than a more traditional approach would. In their full evaluation, the researchers reported that there were significant shifts in classroom teaching practices as a result of the CAS Pilot project PD (Neill & Maguire, 2008).

On the other hand, many students taking part in this evaluation thought they had made some gains in their mathematical understanding, especially in algebra, with relatively few reporting lower levels of understanding. This did, of course, differ from student to student. It may well be that students who are visual learners will find that the graphic capabilities of the CAS calculators better suit their

learning style. Other researchers have reported similar advantages related to visual displays (see, for example, Thomas, Bosley, Santos, et al., 2007).

The achievement data was collected longitudinally, so there is the potential to continue tracking the growth in student achievement through the three levels of NCEA mathematics results to explore the longer-term effects of the CAS Pilot Project.

Acknowledgments

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