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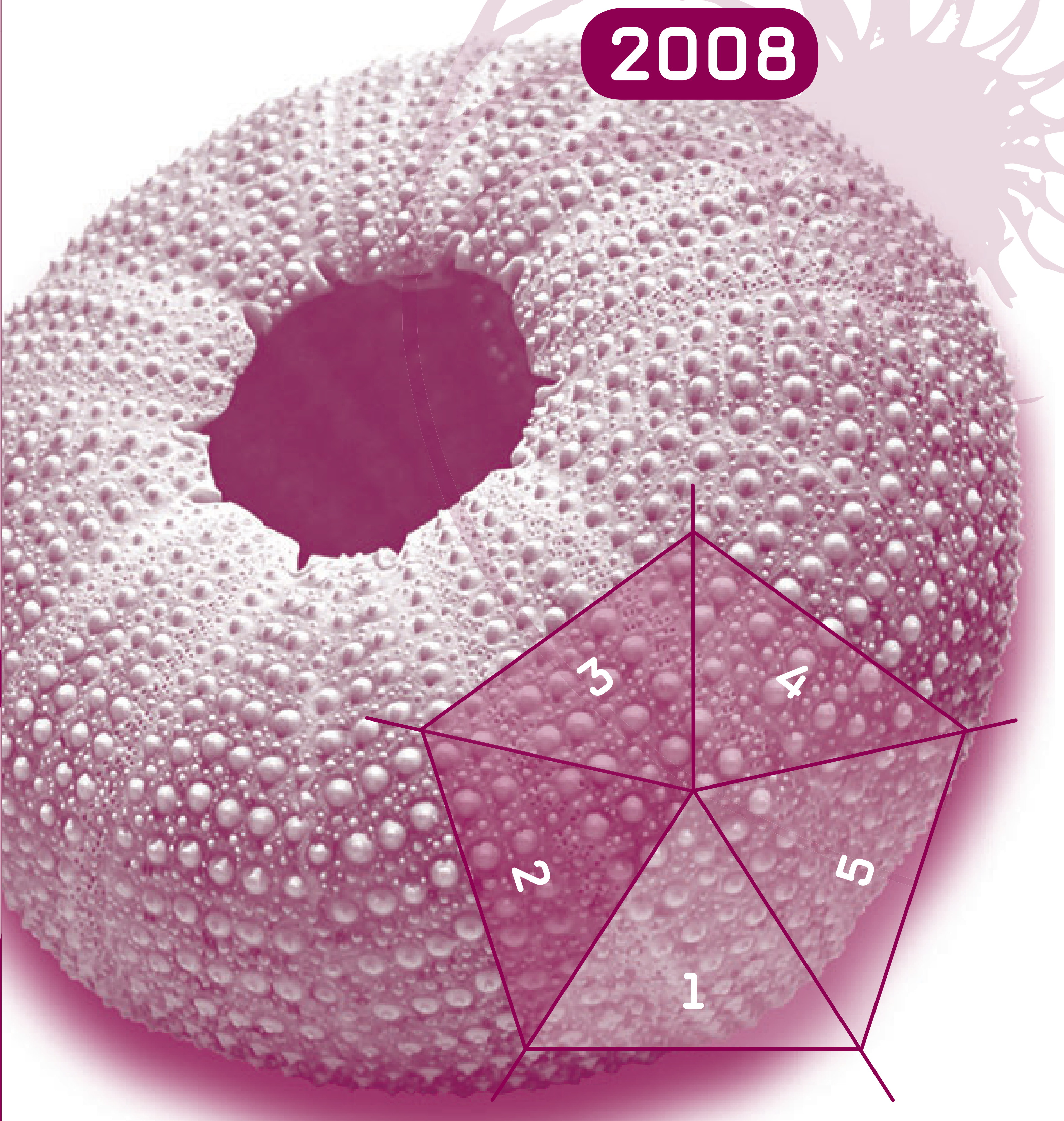
Te Tāhuhu o te Mātauranga

Findings from the New Zealand Secondary Numeracy Project

2008



Findings from the Secondary Numeracy Project 2008

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This resource has been developed to support
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Findings From the Secondary Numeracy Project 2008

Foreword

The Secondary Numeracy Project (SNP) was introduced in 2005 with the aim of helping students develop a deeper understanding of mathematics. This compendium is a collection of papers based on research and evaluation undertaken during the fourth year of the SNP.

As has been the practice in previous years, many researchers have continued to work on themes explored in earlier studies. These include: a fourth year of analysis on attainment data for students in years 9 and 10; results of the further development of a written assessment tool to reliably determine numeracy strategy stages; a further investigation into students' algebraic thinking; the effect of SNP on mathematics teaching practice at senior secondary school level; research into facilitator practices that support effective implementation of teaching and learning in the wharekura Te Poutama Tau project; and a review of the impact of CAS calculators on the learning and understanding of students involved in the CAS Pilot Project.

This compendium shows the benefits of long-term investment by the Ministry of Education in the development of mathematics learning communities. SNP continues to have a positive impact both on student achievement and teacher pedagogy, and there is continuing growth in professional understanding of student learning. Patience is needed for the SNP to show sustained gains.

Student Performance and Progress

Andrew Tagg and Gill Thomas undertake a fourth year of analysis on student data in "Performance of SNP Students on the Number Framework". It is now possible to measure trends in the data from 2005 through to 2008.

A notable difference in the population of students in the SNP occurred in 2008, with a greater proportion of students coming from low-decile schools (36%), compared with a 12% maximum from this sector of schools in previous years. This factor has had an impact on the initial profile of student attainment on entry to year 9, with a smaller percentage of students initially identified as operating in the top two stages of the multiplicative (19% compared with 25% in 2007) and proportional domains (28% compared with 37% in 2007) than in all previous years. Despite this weaker starting position, the 2008 cohort demonstrated similar improvement by the end of year 9 to that in previous years.

The impact of other demographic factors is consistent with previous years: New Zealand European students performed better as a group than Māori or Pasifika students, students from high-decile schools performed better than students from middle- or low-decile schools, and male students attained higher mean stages than female students.

By analysing school data for successive year 9 cohorts, Tagg and Thomas have been able to demonstrate sustained student achievement in numeracy for year 9 students in schools in the second year of the SNP. At the end of 2008, schools that had entered the project in 2007 were able to gather student achievement data for their year 9 students. This was compared with the school's year 9 data for 2007 and any trends noted. While acknowledging that these are two different cohorts of students, the comparisons indicate that the level of student performance in year 9 has been maintained for a second year.

Previous evaluations by Tagg and Thomas have revealed little movement in student attainment during year 10. Of particular concern are those students operating at or below stage 5 on the Number Framework. Tagg and Thomas note: "A comparison of the percentages of students remaining below stage 5 shows that those students rated as 'at risk' at the end of year 9 do not make progress during year 10, with at least as many still in the bottom stages at the end of year 10." These students will by now have effectively disengaged from learning mathematics and appear to have done so for at least a year, if not longer. Some appropriate targeted assistance may be in order.

Written and Oral Assessments of Secondary Students' Number Strategies: Ongoing Development of a Written Assessment Tool

In their 2008 evaluation of a written assessment for use in secondary schools, Gregor Lomas and Peter Hughes continue their work from last year in designing a pen-and-paper assessment tool that can determine a student's numeracy strategy stage as reliably as the oral strategy interview.

Building on what they learned during the first stage of the development of this tool in 2007, Lomas and Hughes repositioned some items, organised each section into strategy and knowledge items, and standardised the number of strategy items in each section. This revised written strategy stage assessment tool (WSSAT) was trialled in the middle of 2008, modified again, and then piloted with two schools at the end of 2008. As with the 2007 model, WSSAT displayed a high level of internal consistency, although some variation existed for stages 7 and 8. It also aligned well with information used for banding students into classes. A comparison with data from the oral assessment was perhaps less conclusive but was more stable than a similar comparison in 2007. There was a strong match between the oral and WSSAT for stage 8 assessments. For lower stages, the data for individual students either matched with both assessments or differed by one stage, with the pen-and-paper assessment rating the student at the higher stage. The researchers point out that this is comparable to the result presented by Thomas and Tagg (2006), who noted that secondary teachers had a tendency to assess student performance "at lower levels based on the teacher's perception of students' needs rather than actual performance on some numeracy assessment tool". On this basis, Lomas and Hughes conclude that they have now developed a significantly improved tool that determines student strategy stages with a reasonable degree of accuracy.

Students' Knowledge and Strategies for Solving Equations

Chris Linsell continues the investigation into students' algebraic thinking that he commenced in 2007. The diagnostic tool developed in 2007 was again used, this time with a further 621 year 7-10 students. The tool identifies strategies that students use to solve linear equations.

In Linsell's 2007 research, the diagnostic tool was able to uncover a range of strategies that students employed to solve linear equations. While some of the strategies were clearly more sophisticated than others, no definitive hierarchy amongst the strategies was suggested. This new research now proposes such a hierarchy. The proposed increasing order of sophistication of strategies used by students is: guess and check; using known basic facts and/or counting techniques; using inverse operations; working backwards, then using guess and check; working backwards, then using known facts; working backwards; and transformations (equation as an object). Linsell suggests that the grouping of students for instruction according to their most sophisticated strategy, as happens in numeracy teaching, is likely to be of benefit for teaching students to solve equations.

The relationship between a student's prerequisite knowledge and skills and the most sophisticated strategy employed to solve equations was investigated further in this research. Students rated at the early strategy stages on the Number Framework employed only primitive solution strategies. As students' understanding of basic facts improved, so did their use of more sophisticated solution strategies. A similar picture emerged for understanding of arithmetic structure, inverse operations, acceptance of a lack of closure, and understanding of equivalence.

It was of interest in the 2007 study that students tended to be able to employ their most sophisticated strategy in both a formal symbolic question and an equivalent question that was described in a context using words or diagrams. This finding is replicated this year. Linsell concludes that this lends weight to a teaching model for the solution of equations comparable with that used for developing an understanding of number: start with a concrete representation of a problem before expecting a visualisation that can then lead to operating on abstract symbolic structures.

Senior Secondary Numeracy Practices in Successful Schools

Roger Harvey and Robin Averill explore the effect of SNP on mathematics teaching practice at senior secondary school level. Questionnaires and semi-structured interviews were used to collect the evidence from six regional professional development facilitators and from mathematics teachers at four low-to-middle-decile schools with effective numeracy practices.

Teaching strategies that were adopted and used by successful numeracy schools in year 11 include: greater emphasis on key ideas; sharing learning intentions at the start of a lesson; an increased focus on student thinking and students explaining their thinking; an increased focus on assessing and developing students' mathematical understanding; and an increased use of real-world contexts.

SNP also appears to be contributing to the development of cohesive mathematics teams, an essential ingredient in a school to ensure continued development and transformation of teacher practice. Ongoing departmental practices fostered through SNP included: giving increased emphasis to professional discussions based around student learning and understanding; carrying out activities for the purpose of discussing the results with colleagues; altering assessment practices; sharing resources; and developing teaching schemes.

The professional conversations between teachers were reported as being richer as a consequence of SNP. It appears that the benefits of the SNP professional development do extend beyond the target classes of years 9 and 10.

Fostering the Growth of Teacher Networks within Professional Development: Kaiako Wharekura Working in Pāngarau

Pania Te Maro, Robin Averill, and Joanna Higgins, joined this year by Brian Tweed, followed up on their evaluative research in 2007, which examined the impact of a pilot project of professional development and support in the wharekura Te Poutama Tau (the Māori-medium version of the SNP) and pāngarau (mathematics) for nine teachers working in wharekura in the Hawke's Bay, Taranaki, Waikato, Wellington, and Whanganui regions. A further cluster of 10 teachers was added to this group in 2008. The 2008 evaluation examined facilitator practices that fostered kaiako (teacher) networks in order to support effective implementation of teaching and learning based on Te Poutama Tau.

New elements were introduced into the modes of delivery of the professional development: a wiki as a store of resources; WiziQ, an Internet-based networking function that replaced video-conferencing; videoing of kaiako working with their classes; and discussions with tumuaki (principals). In addition, the hui were restructured to give kaiako more responsibility for determining their content. Kaiako rated the hui as the most effective mode of delivery, followed by face-to-face visits that also incorporated the use of video. Internet delivery using WiziQ and wiki was deemed the least effective mode, with access to and familiarity with the medium acting as significant inhibitors.

The researchers have continued their exploration of essential characteristics for effective facilitation. Of particular note are the notions of ngāwiritanga and whanaungatanga. Ngāwiritanga captures the idea that a facilitator needs to be flexible, supple, accommodating, non-judgmental, and caring. Such a facilitator "is so knowledgeable about their subject that they are able to give control over to the kaiako, knowing when to participate and when to step back". Whanaungatanga is about facilitating change by engaging as part of a community to foster authentic relationships rather than by acting as a disengaged external expert. Such a relationship could be the foundation for some form of long-term professional development support.

Student Learning and Understanding in the CAS Pilot Project

Alex Neill and Teresa Maguire investigate the progress being made by some of the students from 22 schools involved in the CAS Pilot Programme. Using PAT:Mathematics tests as a measure of mathematical understanding, the researchers collected achievement data to make comparisons between students involved in the pilot and students in a control group, from the same schools, who were not involved.

The data reveals an inconclusive picture. The average score for students in the 2006 year 9 control group was significantly better than the year 9 CAS students and marginally below the 2007 year 9 CAS students, although all groups progressed at near to the expected achievement growth rate in both years. For students in year 10, neither group performed at the expected growth rate, with the control group showing no growth while the CAS group made a small increase that was not statistically significant.

At the end of the two years, no significant difference in performance was detected between the two groups. Qualitative data collected during the research indicated that teachers overall held a neutral position on the impact of CAS on student understanding. On the other hand, many students taking part in this evaluation thought they had made some gains in their mathematical understanding, especially in algebra, with relatively few reporting lower levels of understanding.

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Performance of SNP Students on the Number Framework

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The Secondary Numeracy Project (SNP) has been implemented in New Zealand schools since 2005. This paper analyses the results of students in schools participating in the SNP in 2008, with the aim of quantifying any improvement made in their number knowledge and strategies, and compares their results with those of students from previous years. The findings indicate that the progress made by year 9 students in 2008 schools is comparable with that of year 9 students from 2005 to 2007. The end-of-year performance of year 9 students in second-year schools is very similar to the results of year 9 students from the same schools in 2007. However, the impact of the SNP on year 10 students in second-year schools is small.

Background

The Secondary Numeracy Project (SNP) was first implemented in 2005, with 42 schools participating in a pilot project. Since then, new schools have been added each year. 2008 represents the fourth year of the implementation of the SNP. This paper analyses the results of students in schools participating in the SNP in 2008, with the aim of quantifying any improvement made in their number knowledge and strategies. Comparisons are made with the 2007 results of schools in their second year of involvement. The research questions specifically addressed in this paper are:

- Is the SNP continuing to have an impact on student achievement?
- Do teachers in the second year of the SNP have improved student outcomes?
- Do students make progress in numeracy from year 9 to year 10?
- Does the SNP impact equally on all students?

Method

Procedure

The results reported in this paper were obtained by downloading from the online numeracy database on 27 January 2009 all data from schools participating in the SNP. Schools participating in the SNP for the first time in 2008 were required to enter both initial and final data for year 9 students on the three strategy domains and four knowledge domains of the Number Framework. Schools in their second year of participation were required to enter final data on the seven domains for either year 9 or year 10 students. Students were included in analysis if they had results on all seven of the required domains.

Complete results were available for 2468 year 9 students in 25 schools participating in the SNP for the first time and for 3090 year 9 and 2282 year 10 students in 39 schools in their second year in the SNP. The 2468 year 9 students in the first-year schools represent 76% of the year 9 students in first-year schools for whom complete initial data was entered.

Participants

In first-year schools, 49% of the students were of New Zealand European origin, 26% identified as Māori, and 16% identified as Pasifika. In second-year schools, 64% of students were of New Zealand European origin, 19% identified as Māori, and 6% identified as Pasifika. Nationally, 57% of year

9 and 10 students in 2008 were of New Zealand European origin, 22% were Māori, and 9% were Pasifika. In both first- and second-year schools, there were more female students than male (Ministry of Education, 2009).

Table 1 provides a breakdown by year, gender, and ethnicity of the students included for analysis.

Table 1
Profile of SNP Students by Ethnicity and Gender

	First-year Schools		Second-year Schools			
	Year 9		Year 9		Year 10	
Ethnicity	Male	Female	Male	Female	Male	Female
NZ European	47%	51%	61%	65%	56%	68%
Māori	26%	27%	21%	19%	22%	16%
Pasifika	20%	12%	8%	6%	6%	5%
Asian	5%	6%	6%	4%	8%	7%
Other	3%	5%	5%	5%	8%	5%
Total	1169	1299	1338	1752	862	1420

As shown in Table 2, the decile groups of the participating students were not evenly distributed. About half of all students in both first- and second-year schools came from medium-decile (4–7) schools. Only 13% of students in first-year schools came from high-decile (8–10) schools, with 36% from low-decile (1–3) schools. In second-year schools, a disproportionately low percentage of students came from low-decile schools.

Table 2
Profile of SNP Students by Decile Group

	First-year Schools	Second-year Schools	
	Year 9	Year 9	Year 10
Low decile	36%	21%	17%
Medium decile	51%	56%	47%
High decile	13%	24%	36%
Total	2468	3090	2282

Table 3 presents the decile profile of year 9 students in first-year schools for the four years that the SNP has been implemented. The profile of year 9 students in 2008 is different from that for previous years, with a greater proportion of students coming from low-decile schools. Thirty-six percent of students came from low-decile schools in 2008, compared with a previous maximum of 12% in 2006. Correspondingly, only 13% of year 9 students in 2008 came from high-decile schools, compared with a previous low of 27% in 2006. This reflects the priority given by the Ministry of Education to participation by low-decile secondary schools in 2008.

Table 3
Profile of Year 9 SNP Students in First-year Schools by Decile Group for 2005–2007

	2005	2006	2007	2008
Low decile	11%	12%	8%	36%
Medium decile	52%	61%	54%	51%
High decile	37%	27%	38%	13%
Total	3975	5807	5093	2468

Analysis

The focus for the analysis and discussion in this paper is the students' performance on the multiplicative and proportional strategy domains of the Number Framework. This reflects the curriculum level expectation for students at the end of year 9, which is between stage 7 (advanced multiplicative) and stage 8 (advanced proportional) (Ministry of Education, n.d.). There is also some analysis of student performance on the additive strategy domain and the four knowledge domains of forward number word sequence (FNWS), fractions, place value, and basic facts. The analysis examines the achievement of students in relation to the stages on the Number Framework as well as the progress that students made from the start to the end of the year.

T-tests were used to compare the means of variables with only two categories (gender and year level), and an ANOVA (analysis of variance) was used to compare the means of variables with three categories (decile group and ethnicity). Where overall differences are described between groups, this has been verified to at least the 1% significance level, either by the T-test or by a post-hoc analysis using Tukey's honestly significant difference test. In addition, differences in percentages of students at particular levels of each domain of less than 5% and differences in mean stages of less than 0.2 are not reported because these differences are not considered to be of practical significance.

In all tables, percentages have been rounded. Percentages less than 0.5% are therefore shown as 0%, and where there are no students represented, the cell is left blank. Due to rounding, percentages in some tables may not total to 100.

Effect sizes have been used to quantify the difference between two groups and were calculated by dividing the average difference between two groups by the pooled standard deviation of the two groups. For the purposes of this paper, effect sizes of 0.2 or less are described as small, effect sizes between 0.2 and 0.8 are described as medium, and effect sizes of 0.8 or higher are described as large (Cohen, cited in Coe, 2002).

Findings

The findings of this research explore aspects of the effectiveness of the SNP. They are reported under four headings. The first section addresses the question "Is the SNP continuing to have an impact on student achievement?" Specifically, it looks at how the performance of students in schools participating for the first time in 2008 compares with that of students from first-year schools in previous years. The second section asks "Do teachers in the second year of the SNP have improved student outcomes?" This section compares the performance of year 9 students from schools in their second year in the SNP with that of the year 9 students from the same schools in 2007. The third section addresses the question "Do students make progress in numeracy from year 9 to year 10?" In this section, the results of year 10 students in schools participating for the second year are compared with their year 9 results from

2007. The final section asks "Does the SNP impact equally on all students?" This section compares the impact of the SNP in 2008 on various demographic subgroups of year 9 students.

Appendices A–D (pp. 80–89) provide a detailed breakdown of the percentages of students rated at each stage of the seven domains of the Number Framework.

Is the SNP Continuing to Have an Impact on Student Achievement?

The annual research reports and compendia papers relating to the Numeracy Development Projects (NDP) have consistently shown that students in schools participating in the NDP make greater progress in numeracy as measured on the Number Framework than do students in other schools (for example, Young-Loveridge, 2007). In considering the concept of progression, there are two aspects that need to be addressed: the first is the achievement level of a student at a given point of time; the second is the degree or amount of progress made by a student over a given period of time. Both of these aspects are addressed in this section.

Tables 4 and 5 show the initial and final percentages of year 9 students at each stage of the multiplicative and proportional domains respectively for schools in their first year of SNP for the four years that the SNP has been implemented.

Table 4
Performance of Year 9 Students on the Multiplicative Domain

	Initial				Final			
Stage	2005	2006	2007	2008	2005	2006	2007	2008
0–3: Counting from One	2%	2%	3%	2%	0%	0%	1%	1%
4: Advanced Counting	12%	14%	10%	16%	6%	5%	4%	7%
5: Early Additive	27%	28%	26%	27%	16%	16%	14%	18%
6: Advanced Additive	34%	32%	36%	36%	32%	35%	35%	34%
7: Advanced Multiplicative	20%	18%	20%	16%	30%	29%	31%	31%
8: Advanced Proportional	5%	6%	5%	3%	16%	14%	14%	9%
N =	3975	5807	5093	2468	3975	5807	5093	2468

Table 5
Performance of Year 9 Students on the Proportional Domain

	Initial				Final			
Stage	2005	2006	2007	2008	2005	2006	2007	2008
0–3: Counting from One	1%	1%	1%	2%	1%	0%	0%	1%
4: Advanced Counting	16%	17%	14%	21%	6%	6%	6%	11%
5: Early Additive	29%	31%	30%	35%	23%	24%	22%	27%
6: Advanced Additive	17%	17%	18%	14%	17%	19%	18%	17%
7: Advanced Multiplicative	31%	30%	32%	24%	41%	38%	41%	36%
8: Advanced Proportional	5%	4%	5%	4%	12%	12%	13%	9%
N =	3975	5807	5093	2468	3975	5807	5093	2468

The initial and final achievement profiles of year 9 students in first-year SNP schools in 2008 are different from those of previous years. On both the multiplicative and proportional domains, the proportions of students at the lower stages were higher than in previous years and correspondingly, the proportions of students at the higher stages were lower. For example, at the end of 2008, 38% of year 9 students were still rated at stage 5 or below on the proportional domain, compared with between 28% and 30% in previous years.

Students still rated at or below stage 5 at the end of year 9 are considered to be "at risk" according to curriculum level expectations cited on the nzmaths website (Ministry of Education, n.d.). While the proportions of students that remain in this at risk category on the multiplicative and proportional domains are higher than in previous years, this is largely explained by the fact that the initial proportion of students in this category was also higher than in previous years. For example, at the start of 2008, 58% of year 9 students were rated at stage 5 or below on the proportional domain, compared with between 45% and 49% in previous years. These lower initial and final ratings of students in 2008 are most likely the result of the higher proportion of students from low-decile schools participating in 2008. Previous findings have consistently shown that students from low-decile schools are likely to be at lower stages on all domains (for example, Young-Loveridge, 2007).

While the achievement profile of year 9 students in 2008 was weaker than in previous years, the shifts in the profile between the initial and final assessments were similar. The proportions of 2008 year 9 students rated at the top two stages of the multiplicative domain increased from 19% to 40% from the start to the end of the year, while the percentage in the same two stages on the proportional domain increased from 28% to 45%. Correspondingly, the percentage of students rated at stage 5 or below decreased from 45% to 26% and from 58% to 39% on the multiplicative and proportional domains respectively.

The progress made by students was investigated in more detail. As well as taking account of how students are performing in relation to expected achievement levels, the progress that each student made over a period of time was examined. For example, a student who started the year at stage 5 and ended the year at stage 6 has remained below expected levels of achievement, but in gaining a stage, could be considered to have made substantial progress. Figures 1 and 2 show the percentages of year 9 students gaining stages on the multiplicative and proportional domains respectively in 2005 to 2008, broken down by initial stage. The results from 2008 are very similar to those from previous years, with approximately half of the students not initially rated at the top stage of each domain gaining at least one stage during the course of the SNP. Generally, students with lower initial stages were more likely to make progress, although, consistent with previous years, the proportion of students initially rated at stage 6 on the proportional domain making gains (63%) was slightly higher than that of students initially rated at stage 5 (50%).

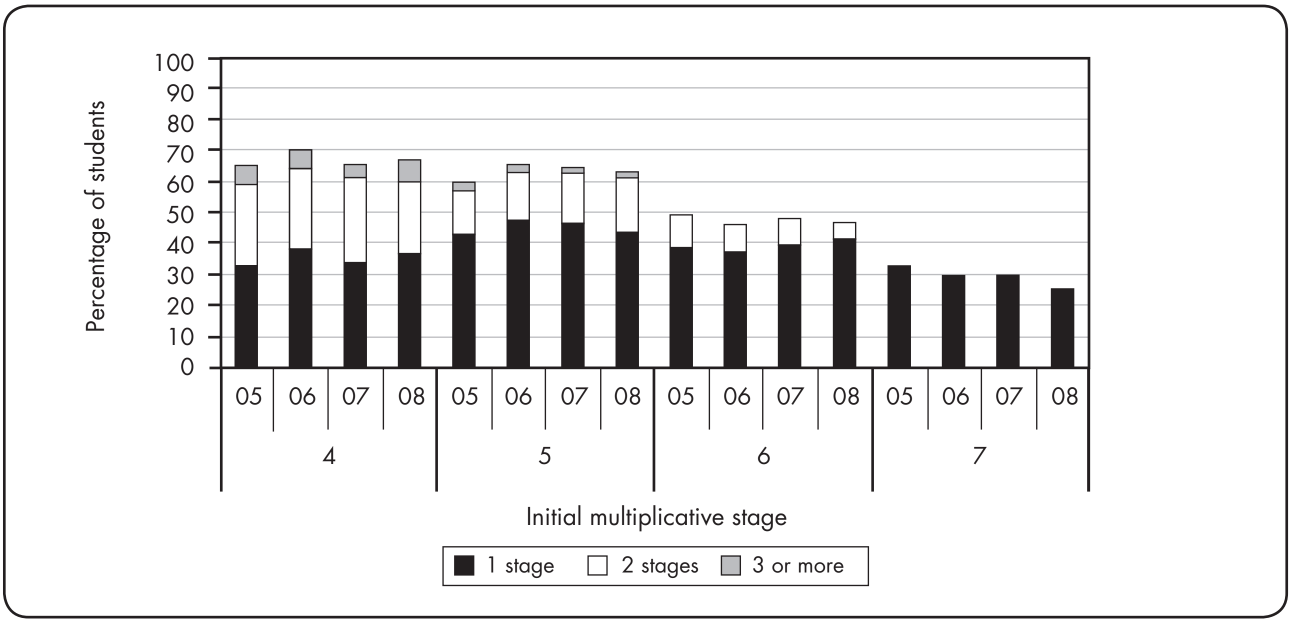

Figure 1. Number of stages gained from initial multiplicative stage for year 9 students in 2005 to 2008

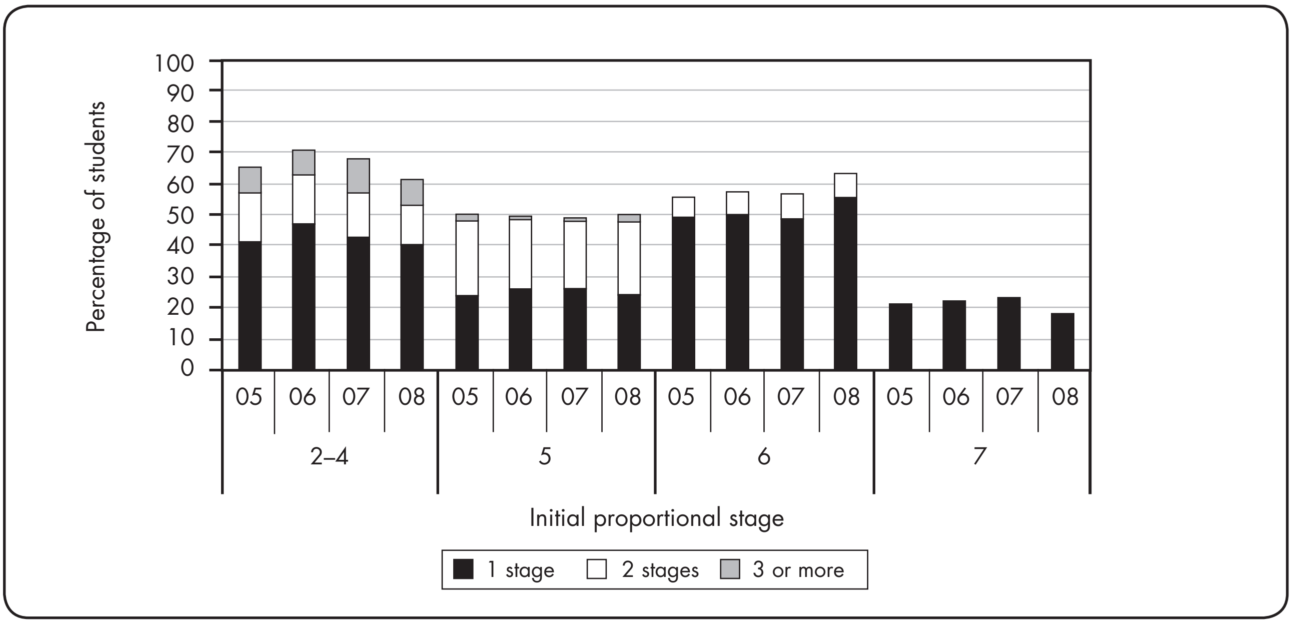

Figure 2. Number of stages gained from initial proportional stage for year 9 students in 2005 to 2008

The mean initial and final stages and effect sizes for the differences for the seven domains are reported in Table 6 to give a measure of the magnitude of the progress made by year 9 students over the course of 2008. Mean gains of at least half a stage were made on all domains, apart from FNWS (0.23) and basic facts (0.33). The apparent lack of progress on the FNWS domain can be explained by a ceiling effect because 49% of all year 9 students rated at the top stage of this domain (stage 6) at the initial assessment. Stages at the top end of the Number Framework are equivalent to levels of the curriculum, so gains of half a stage are in line with expected progress of approximately one level every two years of schooling. On all seven domains, the effect sizes for the gains were medium (between 0.2 and 0.8), with the smallest effect size being for the FNWS domain (0.33) and the largest for the fractions domain (0.56).

Table 6
Effect Sizes for Gains Made by Year 9 SNP Students in 2008

Domain	Mean Initial Stage	Mean Final Stage	Gain	Effect Size
Additive	5.29	5.78	0.50	0.51
Multiplicative	5.56	6.13	0.56	0.50
Proportional	5.22	5.92	0.70	0.46
FNWS	5.37	5.60	0.23	0.33
Fractions	5.23	5.87	0.64	0.56
Place value	5.37	5.95	0.59	0.50
Basic facts	5.63	5.95	0.33	0.34

Do Teachers in the Second Year of the SNP Have Improved Student Outcomes?

Schools are funded to participate in the SNP for two years in the belief that it takes two years of support before teachers and schools are able to effectively implement and sustain SNP teaching practices. This section explores the assumption that teachers in their second year of SNP will have improved outcomes because they have more understanding and experience with SNP practices. This was done by comparing the year 9 results of the 32 second-year SNP schools that entered final results for year 9 in 2008 with the same schools' year 9 results from 2007.

As seen in Table 7, the results of the two groups are very similar, with the most notable difference being the percentage of students reaching the top two stages of the proportional domain. Fifty-two percent of year 9 students in these schools were rated at stage 7 or higher on the proportional domain at the end of 2007, compared with 45% at the end of 2008.

Table 7
Performance on the Multiplicative and Proportional Domains of Year 9 Students in Schools with 2007 and 2008 Data

	Multiplicative		Proportional	
Stage	Year 9 (2007)	Year 9 (2008)	Year 9 (2007)	Year 9 (2008)
0–3: Counting from One	0%	1%	0%	0%
4: Advanced Counting	5%	5%	7%	9%
5: Early Additive	16%	18%	23%	27%
6: Advanced Additive	36%	36%	19%	19%
7: Advanced Multiplicative	31%	30%	38%	32%
8: Advanced Proportional	13%	10%	14%	13%
N =	2771	3090	2771	3090

A T-test was carried out and effect sizes calculated to compare the final scores of year 9 students in second-year SNP schools with the year 9 students from the same schools in 2007. The results are shown in Table 8 below. The shaded cells represent comparisons where the difference was not statistically significant at the 99% confidence level.

Table 8
Effect Sizes for Comparisons of Final Scores of Year 9 SNP Students in Second-year Schools

Domain	2007 Mean Stage	2008 Mean Stage	Difference	Effect Size
Additive	5.95	5.91	–0.04	–0.04
Multiplicative	6.30	6.21	–0.10	–0.09
Proportional	6.21	6.03	–0.18	–0.13
FNWS	5.72	5.69	–0.03	–0.06
Fractions	6.13	6.03	–0.10	–0.09
Place value	6.24	6.14	–0.10	–0.08
Basic facts	6.21	6.24	0.03	0.03

The differences between the final scores of the two groups were not statistically significant on three of the seven domains. While the mean final stages of the students in 2008 on the remaining four domains were statistically lower than in 2007, the effect size for these differences was small (0.2 or less). This indicates, in practical terms, that the performance of year 9 students in SNP schools in the year following the initial implementation of the SNP remains at a similar level to the first year of participation.

Do Students Make Progress in Numeracy from Year 9 to Year 10?

The annual SNP reports have consistently shown that students in year 9 make strong progress in numeracy as measured on the Number Framework (Tagg & Thomas, 2006, 2007, 2008). The challenge is to continue this progress into year 10 so that a greater proportion of students enter year 11 achieving at curriculum level expectations. The online database used to collect numeracy results makes it possible to track the progress of students from one year to the next, as long as schools migrate their students' demographic data at the beginning of the year. It was therefore possible to link 2008 year 10 results for 1510 students in 15 second-year SNP schools to their year 9 results from 2007. Forty-four percent of these students were in high-decile schools, 38% in medium-decile schools, and 18% in low-decile schools.

Table 9 shows the end-of-year results from these 1510 students in 2007 and 2008 on the multiplicative and proportional domains. A comparison of the percentages of students remaining below stage 5 shows that the proportions of students rated as at risk changed little during year 10, with at least as many still in the bottom stages at the end of year 10 as at the end of year 9. A similar comparison at the higher stages shows that the proportions of students in the top two stages increased slightly between the end of year 9 and the end of year 10 on the multiplicative domain but remained fairly similar on the proportional domain.

Table 9
Performance on the Multiplicative and Proportional Domains of the SNP Students with Year 9 and Year 10 Results

	Multiplicative		Proportional	
Stage	Year 9 (2007)	Year 10 (2008)	Year 9 (2007)	Year 10 (2008)
0–3: Counting from One	1%	1%	0%	0%
4: Advanced Counting	4%	6%	6%	7%
5: Early Additive	13%	13%	21%	23%
6: Advanced Additive	39%	31%	17%	16%
7: Advanced Multiplicative	32%	36%	41%	34%
8: Advanced Proportional	11%	13%	15%	20%
N =	1510	1510	1510	1510

While the proportions of students at each stage remain similar, Figure 3 shows that many students' ratings on the multiplicative domain changed between the end of year 9 and the end of year 10. For each starting stage, the numbers of students in each possible category of progress are shown. Those students who either remained at the same stage or moved up at least one stage are shown above zero on the y-axis, while those who were rated at a lower stage at the end of year 10 than they had been at the end of year 9 are shown below zero. Overall, 29% of the students made gains of at least one stage during year 10. Forty-eight percent of students remained at the same stage, and 23% of the students were rated at a lower stage at the end of year 10 than they had been at the end of year 9. The fact that nearly a quarter of students were rated at a lower stage after another year of SNP teaching is both concerning and confusing. Given that the Number Framework represents a progression of learning, it should not be possible for students to regress. One possible explanation is that teachers were more conservative in their judgments at the end of year 10 than at the end of year 9.

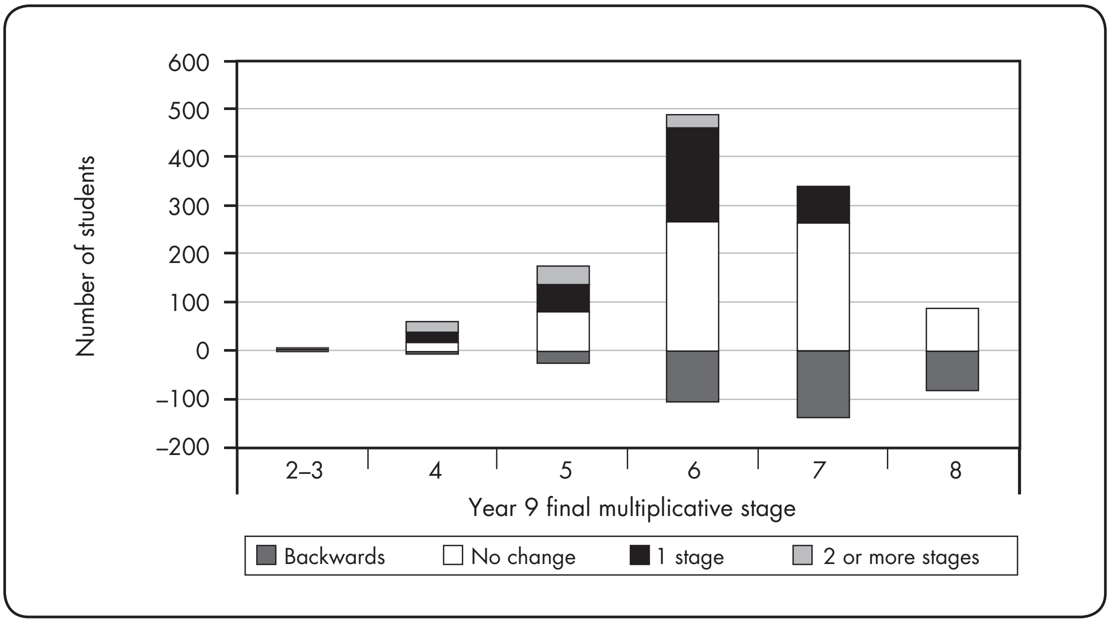

Figure 3. Number of students making gains on the multiplicative domain in year 10 based on year 9 final stage

Table 10 shows the mean initial and final stages of the 1510 students on the seven domains as well as the mean change in stage and the effect size for the impact of the SNP in year 10. The shaded cells represent comparisons where a paired samples T-test indicated that the difference in means was not significant at the 1% level.

Table 10
Effect sizes for Comparisons of Scores of SNP Students with Year 9 and Year 10 Results

Domain	Year 9 Mean Stage	Year 10 Mean Stage	Gain	Effect Size
Additive	5.96	6.14	0.18	0.17
Multiplicative	6.30	6.36	0.06	0.06
Proportional	6.31	6.30	0.00	0.00
FNWS	5.78	5.84	0.06	0.13
Fractions	6.38	6.53	0.15	0.13
Place value	6.49	6.66	0.17	0.14
Basic facts	6.51	6.53	0.01	0.02

As shown by Table 10, the SNP had no measurable impact on these year 10 students in second-year schools for three of the seven domains, while on the remaining four domains (additive, FNWS, fractions, and place value), the effect size of the difference was small.

Does the SNP Impact Equally on All Students?

The results from the SNP from 2005 to 2007 (Tagg & Thomas, 2006, 2007, 2008) indicate that the comparative performances of demographic subgroups of students in the SNP are similar to those found in previous NDP research (Young-Loveridge, 2006). Table 11 shows the mean initial and final stages of demographic subgroups on the multiplicative domain. Mean gains and effect sizes for the differences are included to indicate the magnitude of the impact of the SNP.

Table 11
Effect Sizes for Gains Made on the Multiplicative Domain by Demographic Subgroups of Year 9 Students

	Mean Initial Stage	Mean Final Stage	Gain	Effect Size
Male	5.62	6.17	0.54	0.47
Female	5.51	6.09	0.58	0.54
Low decile	5.21	5.84	0.62	0.53
Medium decile	5.67	6.23	0.55	0.54
High decile	6.11	6.54	0.43	0.40
NZ European	5.83	6.34	0.51	0.48
Māori	5.41	5.92	0.50	0.47
Pasifika	4.95	5.68	0.73	0.66
Total	5.56	6.13	0.56	0.50

The pattern of comparative performance of demographic subgroups on the multiplicative domain is consistent with that found in previous years. A T-test indicated that, at the final assessment, the difference between the mean stages of males and females was not statistically significant. ANOVA tests with post-hoc analysis were carried out to compare the means of students by ethnicity and decile group. The mean stage of New Zealand European students was higher than that of both Māori and Pasifika students ($p < 0.01$). The mean stage of students from high-decile schools was higher than that of students from medium-decile schools ($p < 0.01$), with both being higher than that of students from low-decile schools ($p < 0.01$). The subgroup of year 9 students with the lowest mean final stage was Pasifika students (5.68), while the highest was students from high-decile schools (6.54). In general, the subgroups with the lower mean initial stages tended to make greater mean gains and have a larger effect size, indicating that the SNP is closing the gap between demographic subgroups. This may be partly explained by a ceiling effect for higher-scoring students and by the previously reported finding that students make greater gains at the lower stages of the Number Framework (Tagg & Thomas, 2007; Thomas, Tagg, & Ward, 2003).

Table 12 shows the mean initial and final stages, gains, and effect sizes of demographic subgroups on the proportional domain. The pattern of results is very similar to that shown in Table 11, although females have slightly higher mean stages than males on the proportional domain.

Table 12
Effect sizes for Gains Made on the Proportional Domain by Demographic Subgroups of Year 9 Students

	Mean Initial Stage	Mean Final Stage	Gain	Effect Size
Male	5.17	5.90	0.73	0.46
Female	5.26	5.94	0.68	0.46
Low decile	4.69	5.46	0.77	0.49
Medium decile	5.36	6.07	0.71	0.49
High decile	6.14	6.62	0.48	0.40
NZ European	5.60	6.28	0.68	0.48
Māori	5.00	5.61	0.61	0.40
Pasifika	4.33	5.21	0.88	0.58
Total	5.22	5.92	0.70	0.46

Table 13 summarises the effect sizes for gains made on all seven domains by demographic subgroups. In general, the effect sizes for the impact of the SNP appear to be greater for females than for males and greater for students from low- and medium-decile schools than for students from high-decile schools. Pasifika students appear to have higher effect sizes than New Zealand European or Māori students. An effect size of –0.11 was found for the basic facts scores of high-decile students. Further analysis of the results showed that one school had recorded 68 of the 79 students as having final scores lower than their initial scores on this domain. It is presumed that this represents a data entry error and that final scores were transposed with initial scores.

Table 13
Effect sizes for Gains Made by Demographic Subgroups of Year 9 Students

	Additive	Multiplicative	Proportional	FNWS	Fractions	Place Value	Basic Facts	N =
Male	0.47	0.47	0.46	0.30	0.51	0.45	0.28	1169
Female	0.54	0.54	0.46	0.37	0.61	0.55	0.41	1299
Low	0.54	0.53	0.49	0.36	0.59	0.51	0.46	886
Medium	0.51	0.54	0.49	0.35	0.56	0.55	0.38	1268
High	0.51	0.40	0.40	0.24	0.53	0.37	–0.11	314
NZ European	0.47	0.48	0.48	0.34	0.54	0.52	0.24	1213
Māori	0.47	0.47	0.40	0.28	0.57	0.41	0.42	645
Pasifika	0.64	0.66	0.58	0.45	0.60	0.57	0.43	383
Total	0.51	0.50	0.46	0.33	0.56	0.50	0.34	2468

Key Findings

This section provides a summary of the key findings of the evaluation of student achievement in the SNP in 2008. The SNP continues to be an effective intervention for year 9 students in both first- and second-year SNP schools. However, the results indicate that the SNP has little or no impact on the performance on the Number Framework of year 10 students in second-year SNP schools.

In first-year schools, a comparison of percentages of students at each stage at the initial and final assessments showed that the proportions of students in the higher stages increased while the proportions of students at the lower stages decreased. A comparison of mean initial and final stages showed that students made gains on all seven domains. The effect sizes for these gains were medium (between 0.2 and 0.8) on all domains, indicating that the SNP had a substantial impact on these students.

Teachers in second-year schools appear to have had a similar impact on year 9 student achievement as they had in their first year. Second-year schools were not required to enter initial data, so it is not possible to determine the progress made by these students. However, the end-of-year performance of year 9 students in second-year schools was very similar to that of year 9 students in the same schools in 2007.

Year 10 students in second-year schools made very little progress, with no significant improvement on three of the seven domains. The effect sizes for the gains made on the additive, FNWS, fractions, and place value domains were small. The fact that nearly a quarter (23%) of students were rated at a lower stage at the end of year 10 than at the end of year 9 casts some doubt on the accuracy of teacher ratings.

Demographic factors impacted on the performance of students. Specifically, New Zealand European students had higher mean stages than Māori or Pasifika students, while students from high-decile schools had higher mean stages than students from medium-decile schools, who in turn had higher stages than students from low-decile schools.

References

Coe, R. (2002) *It's the effect size, stupid: what effect size is and why it is important.* Paper presented at the annual conference of the British Educational Research Association, University of Exeter, England, 12–14 September. [Available from www.leeds.ac.uk/educol]

Ministry of Education (n.d.). *Using expectations.* Retrieved 1 February 2009 from: www.nzmaths.co.nz/node/1481

Ministry of Education (2009). *Students by year level 2008.* Retrieved March 2009 from: www.educationcounts.govt.nz/statistics/schooling/julyāschoolārollāreturns/14667/32379

Tagg, A., & Thomas, G. (2006). Performance on the Number Framework. In *Evaluations of the 2005 Secondary Numeracy Pilot Project and the CAS Pilot Project* (pp. 12–35). Wellington: Learning Media.

Tagg, A., & Thomas, G. (2007). Performance of SNP students on the Number Framework, in *Evaluations of the 2006 Secondary Numeracy Project* (pp. 29–39). Wellington: Learning Media.

Tagg, A., & Thomas, G. (2008). Performance of SNP students on the Number Framework. In *Findings from the New Zealand Secondary Numeracy Project 2007* (pp. 5–16). Wellington: Learning Media.

Thomas, G., Tagg, A., & Ward, J. (2003). *An evaluation of the Early Numeracy Project 2002:* Exploring issues in mathematics education. Wellington: Ministry of Education.

Young-Loveridge, J. (2006). Patterns of performance and progress on the Numeracy Development Projects: Looking back from 2005. In *Findings from the New Zealand Numeracy Development Projects 2005* (pp. 6–21). Wellington: Learning Media.

Young-Loveridge, J. (2007). Patterns of performance and progress on the Numeracy Development Projects: Findings from 2006 for years 5–9 students. In *Findings from the New Zealand Numeracy Development Projects 2006* (pp. 16–32). Wellington: Learning Media.

Written and Oral Assessments of Secondary Students' Number Strategies: Ongoing Development of a Written Assessment Tool

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This paper examines the trialling and piloting of a revised Written Strategy Stage Assessment Tool (WSSAT) designed to help identify students' strategy stages and provide formative data for teachers to use in their planning and teaching of numeracy. A year 9 and a 10 cohort of secondary school students trialled the written assessment mid-year, and then another year 9 cohort, from two other schools, piloted its use at the end of the year. Numeracy experts interviewed a sample of the end-of-year students to identify each student's strategy stage, using GloSS (Global Strategy Stage) oral-type protocols. Results from the written assessment gave relatively consistent measures of stages in terms of the criteria set and a relatively close match to national data sets. Comparison of the written and oral assessment results showed the stages identified by the two measures to be generally consistent. The results of this study suggest that the stages established by the WSSAT are likely to be useful for formative and diagnostic purposes.

Background

The extension of the Numeracy Development Projects (NDP) into secondary schools (as the Secondary Numeracy Project [SNP]) has led to a need for a "more efficient" assessment tool to replace the NDP oral assessment tool NumPA (Numeracy Project Assessment) so that an initial assessment of a whole class of students can be carried out. The use of an oral assessment in the primary school area avoids issues of students' reading levels and encourages the teasing out of students' understandings via oral interactions that move beyond the assessment script. NumPA is therefore relatively time consuming. The larger number of students in secondary schools who need assessing per teacher, due to multiple classes, and the expectation of adequate reading levels was the driver for developing the Written Strategy Stage Assessment Tool (WSSAT).

A written assessment tool would be efficient if it:

- reduced the amount of teacher time taken per student;
- provided a record of students' work that largely fulfils the same purposes (formative and summative) as the NumPA oral assessment;
- provided a standardised "marking" schedule.

A standardised marking schedule would enable consistent marking by someone new to the SNP or even by a person with little knowledge of numeracy, such as a parent helper. It would also reduce the individual variability that can occur, for example, when conducting oral assessments, because the written record would be available for re-evaluation and moderation purposes.

Another benefit might be enhanced accuracy of the assessment, particularly in the multiplicative and proportional domains, for which Thomas, Tagg, and Ward's 2006 data set the accuracy of various types of existing numeracy assessments at 76%. Thomas et al. (2006) also found that many teachers "rated students' strategy stages lower than the rating of the researchers, explaining their decisions in terms of consolidating students' understanding at an existing level" (p. 101), that is, they rated students at a lower stage than NumPA would have assigned.

The Nature and Structure of the Written Strategy Stage Assessment Tool

The WSSAT, which is being developed by Peter Hughes at The University of Auckland, aims to identify a global strategy stage (as does the Global Strategy Stage [GloSS] tool) rather than domain-specific strategy stages (as the NumPA does). The approach taken is closely aligned with, and arises from, the NDP strategy and knowledge sections of the Number Framework but consists of short-answer and multi-choice items that draw upon the approaches used in the NDP assessment tools and research findings in related areas (Lomas & Hughes, 2008).

The focus of the WSSAT is primarily strategy stage identification, with seven items at each stage assessing a range of strategies and further items assessing some knowledge aspects (see Table 1). The knowledge items are a mixture of prerequisite knowledge for, and knowledge directly related to, each stage. To achieve a strategy stage, students need to correctly answer four of the strategy items for that stage. As with the NDP oral assessments, the highest strategy stage achieved is taken as that student's stage. For example, if a student meets the criteria for stages 5, 6, and 7, they are classified as being at stage 7, or if they meet the criteria for stage 5 and not that for stage 6, but do meet the criteria for stage 7, they are classified as being at stage 7. Where the students do not meet the criteria for stage 5 or higher, they are assigned to a category that covers stages 1–4.

Table 1
The Number of Items per Strategy Stage and Criteria for Achieving Each Stage

	Strategy Stages				
	1–4	**5**	**6**	**7**	**8**
Total number of items (including seven strategy items)	11	11	12	11	10
Number of knowledge items	4	4	5	4	3
Number of correct items to assign to a strategy stage	< 4	≥ 4	≥ 4	≥ 4	≥ 4

Note: The items used to assess stages 1–4 are the same items as for stage 5.

The WSSAT as a written assessment relies on the written answers without any indication of the process used. This is different from NDP oral assessments, in which students give their answer orally and often talk through the process they used to arrive at their answers. Thus, assigning strategy stages by the oral assessments relates largely to process, whereas assigning strategy stages using the WSSAT is based on outcomes (Lomas & Hughes, 2008).

The WSSAT answer sheet has clear directions for the students to follow and was formatted for ease of marking, giving both a quick indication of a student's strategy stage and more detailed formative data for planning and teaching purposes. The answer sheet has room only for the answers because what was sought was the result of mental processes rather than the process itself or written algorithms.

The range of items selected for each stage attempted to isolate and encapsulate some of the conceptual aspects and elements of strategy relating to that stage as per the domains of the strategy section of the Number Framework. For example, item 28 (see Appendix E [pp. 90–93], Part C), "Work out 5 sixths of 42", and item 8 of the oral assessment (see Appendix F [p. 94], Section C), "What is 3 fifths of 35?", relate to stage 7 of the proportions and ratios domain, in which students are expected to use multiplication strategies to solve problems with fractions.

In addition, the nature of the WSSAT items was designed to reflect elements of students' understanding (process) that might be present in an oral dialogue between teacher and student but would not always be evident in written work (which focuses mainly on outcomes). Thus, a key issue in the WSSAT was to ensure that the items that were used "forced" the student participants to use a particular process and restricted their use of any other approach. That is, the WSSAT minimises the number of items that could be answered procedurally or answered by less sophisticated strategies. For example, some of the written assessment items use combinations of numbers written as words and digits, for example, "Add one-tenth to 4.273" (see Appendix E, Part B, item 17). The aim is to expose the student's in-depth understanding by stating material in a way that limits the use of procedural methods and requires more understanding of number structure.

This approach is also seen as a way of keeping aspects of "oral" language use within a written format, with, for example, "one-tenth" giving a potentially greater access to the underlying meaning than "zero point one" in symbolic form (0.1), while still requiring the students to be able to connect the underlying meanings.

An example of an item that tries to "force" a particular strategy is item 1 (see Appendix E, Part A), "87 + 99 =". Ninety-nine is close to a tidy number, and the most likely solution consists of "one step": making the 99 up to 100 and reducing 87 down to 86 at the same time, resulting in 186 as the answer. The choice of the particular numbers lessens the possibility of students using counting on, which could happen, for example, if 48 + 8 was used, or of them using a variety of different strategies, for example, if 87 + 88 was used. The extent to which this approach has worked will be determined in part by the extent to which the assignment of stages via the written assessment matches the oral assessment.

The sets of items also have the potential to provide a more detailed and standardised diagnostic map of a student's learning needs than an oral assessment. This potential is enhanced by the written format, which allows a student to attempt all the items, thus demonstrating any partial understandings that the student might have (with some items answered correctly) beyond the point where an oral assessment would stop.

2007 Trial Findings

The findings from the trial of the initial version of the WSSAT in 2007 showed that the written assessment, while consistent, did not give stage information that reflected the parallel oral assessment, the national curriculum level expectations, or, more specifically, the low-decile data (Lomas & Hughes, 2008). On this basis, a number of changes were made to the organisation of the items within each part of the WSSAT (reflecting a stage) and the suitability/placement of items in each part.

The organisation of items within each part was changed to create two sections, with the first section consisting of strategy items and the second of knowledge items. The number of strategy items was standardised for each part, as was the number of correct responses required to achieve a stage. The placement of items in each part was reconsidered in terms of each item's suitability for assessing that stage from a theoretical viewpoint (alignment with the strategy stages of the strategy section of the Number Framework) and, in some instances, in terms of the number of correct responses in each item's current placement. For example, "How would you write eleven thousandths as a decimal fraction?", which only 53 students out of 278 answered correctly in the 2007 trial, was shifted from the stage 6 (Part B) to the stage 8 (part D) section, where it was thought that more students would have the understanding required to function at that stage. It was also rewritten in multi-choice format, "Eleven thousandths equals:" with four choices offered: "A. 0.0011 B. 0.011 C. 0.11 D. 11000". This was done for two main reasons: to avoid possible confusion about what "decimal fractions" meant and to avoid the answer "11 over (divided by) 1000", which "side steps" the issue of understanding decimal fraction notation.

Method

The revised WSSAT was trialled mid-year to evaluate the revisions and to identify further aspects for revision on the basis of the data gathered. The further revised WSSAT was then piloted with two schools at the end of 2008, and a parallel oral assessment interview was conducted with a sample from one of the schools. Due to the site-specific nature of the data collected, this research is a form of case study. Thus, the data is unlikely to match the national data sets too closely and care must be taken in generalising any findings.

The participants in the mid-year trial were drawn from the year 9 and 10 cohorts of a large, Auckland, decile 5, girls' secondary school of mixed ethnic composition, with a small number of special needs students being excluded.

The participants in the end-of-year pilot were drawn from two schools: a year 9 cohort of a large, Auckland, decile 3 secondary school of mixed ethnic composition, excluding some special needs students, and the complete year 9 cohort of a medium-sized, Wellington, decile 6 secondary school of mainly New Zealand European students (see Table 2).

Table 2
Percentage Ethnic Make-up of Auckland and Wellington Year 9 Students Included in the Pilot Part of This Study

	NZ European	Māori	Pasifika	Other
Auckland (n = 280) (low decile)	26	18	40	15
Wellington (n = 113) (middle decile)	70	15	7	8

The written assessment was given to both year 9 pilot cohorts, while the oral assessment was given only to a subset (60 students) of the year 9 Auckland cohort. This sample was drawn from several classes from the bands in which the school organised their classes (see Table 3).

Table 3
Auckland School Classes in Bands (High to Low), Showing Student Roll Numbers and the Number of Students Taking the Pilot Written and Oral Assessments

	Pilot Class Name												
	P9A1	P9A2	P9B1	P9B2	P9B3	P9B4	P9C1	P9C2	P9C3	P9C4	P9D1	P9D2	Total
Roll No.	33	32	33	33	33	32	25	20	27	25	28	27	348
Written Ass. No.	26	26	28	24	25	22	25	20	23	16	21	24	280
Oral Ass. No.	10	20	–	–	–	–	–	–	19	11	–	–	60

The interviewer worked around the Auckland school's programme, so the classes accessed were those that least inconvenienced the school. This affected the nature of the sample, which is not representative because it was drawn from upper- and lower-band classes and included no students from the middle-band classes.

The Oral Assessment

The oral assessment research tool (see Appendix F, p. 94) was an expanded form of the GloSS and used some of the GloSS- and NumPA-type items, supplemented by other items that gave increased coverage of higher stages. It also offered the potential for allowing a finer measure of a student's position within a stage, that is, beginning, middle, or later (nearly ready to move to the next stage). As well as the questions being asked orally, a card with the question written on it was placed in front of the student as a reference (as is done with GloSS and NumPA).

Data Collection

For both the trial and the pilot, the WSSAT was generally conducted in each class's usual classroom setting, usually under the supervision of the regular mathematics teacher, in the last few weeks of the second and fourth terms respectively. Standardised instructions were given out explaining how teachers were to conduct the assessments (Lomas & Hughes, 2008), and all the answer sheets were marked by one of the research team to ensure consistency. Copies of the marked answer sheets were returned to the school for potential diagnostic/formative use by the school.

For the Auckland pilot school, a GloSS-type oral assessment was given during the two days after the written assessment to a sample of 60 students, drawn roughly, and at the school's convenience, from across the "ability" bands into which the year 9 cohort was organised (see Table 3). The oral interviews were conducted by external interviewers who had expert knowledge of NDP.

Analysis

The results of the trial and pilot written assessments were first analysed for the internal consistency of the tool in identifying a student's stage, that is, whether a student assigned as being at stage 6 had also been assigned as being at stage 5, and so on. Then they were analysed against three other measures of achievement, one school-based and two based on nationally collected data from the NDP, which give measures of global, rather than domain-specific, strategy stages. The stages that the students achieved were compared with:

- the banding (where applicable) of the class they were in, to see whether this reflected the school's placement of students;
- the national, year 9, low- or middle-decile stage distribution data from the SNP;
- the national, year 9, stage distribution data (the numeracy curriculum expectations).

Issues identified in the mid-year trial were taken into consideration in a further minor revision of the WSSAT prior to this revised version (see Appendix E) being piloted at the end of the year.

The results of the written and oral assessments from the Auckland pilot school were compared to establish a relationship between these two forms of assessment. The oral assessment was assumed to be the more accurate and was taken as the baseline for the comparison due to its alignment with national data collection methods. This assumption was based on two main factors. Firstly, the oral assessment was an extension of the NumPA tool and thus was collecting some of the same data. Secondly, the extra questions were provided by a numeracy expert with an intimate knowledge of the development and use of both GloSS and NumPA. This connection to these existing and "proven" NDP assessment tools provided a sound basis for comparison with the database of student results arising from the WSSAT's use.

Results

The Trial Data

The trial data was collected from both year 9 and year 10 students to meet the school's requirements. Although this was a mid-year rather than an end-of-year assessment, the groups fall either side of the national end-of-year data and so can be related to it, although not directly. The year 9 and the 10 trial data was analysed separately to allow direct comparison of the year 9 data with that of the 2007 trial and of both year 9 and 10 data with the national data.

Internal consistency

Of the 74 year 9 students who could be assigned as being at stage 6 on the basis of the written assessment, only two had not also achieved the criteria (missing by one correct response) for being assigned as being at stage 5. For the 62 students assigned as being at stage 7, only six (almost one-tenth) had not achieved the criteria for being assigned as being at stage 6. Of these six, five had missed the criteria for stage 6 by one correct response and one by two correct responses. For the 56 students who could be assigned as being at stage 8, 18 (almost one-third) had not achieved the criteria for earlier stages. Of the 14 who missed only stage 7, five had missed by one correct response and five by more than one. The other four had not been assigned either stage 6 or stage 7.

Of the 57 year 10 students who could be assigned as being at stage 6 on the basis of the written assessment, only three had not also achieved the criteria (missing by one correct response) for being assigned as being at stage 5. For the 55 students assigned as being at stage 7, only ten (just over one-fifth) had not achieved the criteria for being assigned as being at stage 6. Of these ten, five had missed the criteria for stage 6 by one correct response and five by two correct responses. For the 52 students who could be assigned as being at stage 8, five (almost one-tenth) had not achieved the criteria for earlier stages. Of the four who missed only stage 7, two had missed by one correct response and two missed by two; the fifth student had not been assigned as being at any of stages 5, 6, or 7.

All this data suggests that the WSSAT has a high level of internal consistency in assigning students at stages except at stages 7 and 8, in which a greater level of variation was evident. This indicates that further revision of these stages should be investigated.

Conformity of assigned stages with students banding into classes

The stages assigned by the trial WSSAT generally conformed to the banding of the classes: students in classes in higher bands achieved more of the higher stages, and students in classes in lower bands achieved fewer of the higher stages. Additionally, in line with the internal consistency of the WSSAT, as discussed above, the meeting of the criteria for particular stages also aligned with the banding of the classes, with fewer students from lower-band classes meeting the criteria for each stage. For example, in year 9, class 9A2 had 16 students meeting the stage 8 criteria, 23 the stage 7 criteria, and all 26 the stages 6 and 5 criteria, compared with class 9B2, in which one student met the stage 8 criteria, 10 the stage 7 criteria, 18 the stage 6 criteria, and 24 the stage 5 criteria, while class 9D1 had only one student meet the stage 8 criteria, none the stage 7 criteria, three the stage 6 criteria, and 14 the stage 5 criteria (see Table 4). Similar patterns can be seen in the year 10 data of classes 10A2, 10B2, and 10D1, although more students from these classes achieved higher stages.

Table 4
Classes in Band Order, Showing the Number of Year 9 and 10 Students Taking the Trial WSSAT and the Number of Students Meeting the Criteria for Achieving a Particular Stage

	Trial Classes													
Year 9	**A1**	**A2**	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**	**B7**	**B8**	**B9**	**D1**	**D2**	**Total**
No. of students	26	26	25	24	23	17	20	27	12	16	26	18	15	275
No. meeting stage 5	26	26	23	24	23	17	19	27	10	15	25	14	15	264
No. meeting stage 6	26	26	12	18	15	14	13	16	3	11	17	3	3	177
No. meeting stage 7	21	23	6	10	7	6	4	6	1	3	9	0	4	100
No. meeting stage 8	15	16	0	1	4	2	4	5	3	2	3	1	0	56
Year 10	**A1**	**A2**	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**	**B7**	**B8**	**B9**	**D1**	**D2**	**Total**
No. of students	29	26	21	26	19	25	21	22	–	–	–	13	15	217
No. meeting stage 5	29	26	20	24	19	24	19	21	–	–	–	11	14	207
No. meeting stage 6	29	25	14	17	14	9	16	18	–	–	–	7	2	151
No. meeting stage 7	25	18	9	15	9	5	9	11	–	–	–	1	0	102
No. meeting stage 8	14	15	6	2	2	2	4	7	–	–	–	0	0	52

This data suggests that the banding of classes is reflected in the trial WSSAT measurement of the students' numeracy achievement and the assigning of stages.

Comparison with national numeracy curriculum expectations

The assigning of stages from the written assessment gave rise to a distribution reasonably similar to both the low-decile schools' data and the national expectations (see Table 5) for year 9. While the year 10 percentage data shows a small shift to higher stages (23% at year 9 compared with 25% at year 10 for stage 7 and 20% compared with 24% for stage 8), as expected, the increases are not large. This may well reflect the greater difficulty of the stage 7 and 8 material and the longer time needed to "master" that material.

The area of greatest disparity (around a 50% difference or more) with the middle-decile data for year 9 students at the end of the year is the higher number of middle-decile year 9 trial students assigned as being at stage 8 (20% compared with 10%). The areas of greatest disparity for the year 9 (mid-year) trial students compared with the year 9 national expectations related to the lower number of year 9 trial students assigned as being at stage 7 (23% compared with 40%) and the higher number of year 9 trial students assigned as being at stage 5 (27% compared with 12%). Similar disparities are also evident in the year 10 data. The disparities may partly reflect a difference between students from middle-decile schools and the national expectations, or they may be a feature of the particular school's population.

Table 5
The Percentage of Stages Assigned to Trial Students by the WSSAT and the Middle-decile and National Numeracy Curriculum Expectation Data for Year 9 Students (End of Year)

	Stages				
	1–4	5	6	7	8
Percentage of year 9 students (n = 275)	4	27	27	23	20
Percentage of year 10 students (n = 217)	3	22	27	25	24
Middle-decile[1] (average[2]) percentage end year 9 (Tagg & Thomas, 2008)	6	22	32	30	10
National numeracy curriculum expectation percentage end year 9 (Tagg & Thomas, 2008)	2	12	25	40	21

The patterns evident in this data are considerably different from those found in the 2007 trial, where few students were assigned as being at stage 7 (7%) and none at stage 8, while 46% were assigned as being at stages 1–4, 13% at stage 5, and 34% at stage 6 (Lomas & Hughes, 2008). This suggests that the revision of the 2007 version of the WSSAT has at least partially addressed the major concern identified in the 2007 trial, namely the mismatch between stages assigned by the WSSAT and those evident in the middle-decile and national expectations data, while maintaining high levels of internal consistency.

As a consequence of the higher levels of inconsistency at stages 7 and 8, some minor changes were made to some of the strategy items in parts C and D of the WSSAT (see Appendix E). Only three items in each of parts C and D were changed, while one further item (30) in Part C was reformatted (fraction answers were replaced by pairs of whole numbers) to reduce the operational demand.

In an attempt to enhance consistency, items covering the same strategy but at different levels were created. For example, trial item 35, which deals with a multiplication estimation, became pilot item 24; the new pilot item 35 deals with a division estimation. The pilot item 25, "98 x 5", had easier numbers that replaced the trial numbers "999 x 9", and trial item 37, dealing with fraction addition, was replaced with an item similar to item 25 but with harder numbers, "9998 x 5". Trial item 36, dealing with fractions, was replaced with a harder version of item 28, and trial item 29, dealing with fractions but requiring two answers, was replaced with a multiplication item requiring only one answer.

The Pilot Data

The pilot data was collected from a year 9 cohort in a low-decile school in Auckland and another from a middle-decile school in Wellington. The data for each cohort is analysed separately in this paper to allow direct comparison with the year 9 national data.

Internal consistency

All the Auckland students assigned as being at stage 6 had also been assigned as being at stage 5, while of the 84 students who could be assigned as being at stage 7 based on the written assessment, 21 (one-quarter) had not achieved the criteria for stage 6. Of these 21, 15 had missed the criteria by only one correct response. A further two students who achieved the criteria for being assigned as

1 The middle-decile and national curriculum expectations data in tables 5, 9, and 10 come from Secondary Numeracy Project data collected by researchers. The middle-decile percentages are an average figure derived from the percentage data from the additive, multiplicative, and proportional domains.

2 The average figure (as in i.) gives a more global strategy stage rather than being specific to domains.

being at stage 7 had not achieved the criteria for either stages 5 or 6. For the 46 students assigned as being at stage 8, only ten (over one-quarter) had not achieved the criteria for stage 7, and one further student had not achieved the criteria for either stages 6 or 7. Of the ten not achieving the criteria for stage 7, four had missed by only one correct response.

All the Wellington students assigned as being at stage 6 had also been assigned as being at stage 5, while of the 39 students who could be assigned as being at stage 7 based on the written assessment, eight (almost one-fifth) had not achieved the criteria for stage 6. Of the eight, three had missed the criteria by only one correct response. For the 21 students assigned as being at stage 8, only one had not achieved the criteria for stage 7.

This data suggests that the WSSAT was largely internally consistent in assigning stages except at stages 7 and 8, where a greater level of variation was evident (although less variation was evident in the middle-decile Wellington school data).

Conformity of assigned stages with students banding into classes

The stages assigned by the WSSAT generally conformed to the banding of classes in the Auckland school used in the pilot: classes in higher bands achieved more of the higher stages, and classes in lower bands achieved fewer of the higher stages. Additionally, in line with the internal consistency of the WSSAT, as discussed above, the meeting of the criteria for particular stages also aligned with the banding of the classes, with fewer lower-band students meeting the criteria for each stage. For the Auckland students, for example, class P9A2 had 11 students meet the stage 8 criteria, 20 students the stage 7 criteria, 25 the stage 6 criteria, and 26 the stage 5 criteria, compared with class P9B2, in which six students achieved the stage 8 criteria, 13 the stage 7 criteria, 17 the stage 6 criteria, and 23 the stage 5 criteria, while class P9D1 had only two students meet the stage 7 criteria, five the stage 6 criteria, and 19 the stage 5 criteria (see Table 6).

Table 6
Auckland School Classes in Band Order, Showing the Number of Students Taking the Pilot WSSAT and the Number of Students Meeting the Criteria for Achieving a Particular Stage

	Auckland Pilot Classes												
	P9A1	**P9A2**	**P9B1**	**P9B2**	**P9B3**	**P9B4**	**P9C1**	**P9C2**	**P9C3**	**P9C4**	**P9D1**	**P9D2**	**Total**
No. of students	26	26	28	24	25	22	25	20	23	16	21	24	280
No. meeting stage 5	26	26	28	23	25	22	25	15	22	15	19	22	268
No. meeting stage 6	25	25	21	17	16	16	15	2	15	6	5	4	167
No. meeting stage 7	23	20	11	13	14	5	5	5	11	7	2	4	120
No. meeting stage 8	16	11	2	6	1	4	4	0	1	0	0	1	46

In the case of the Wellington school, the classes were not banded and included the entire cohort (see Table 7). There was no evidence of any particular factors that offered an explanation for the variation of class W9–2, which had higher numbers and proportion of students assigned at stage 8 than the other classes.

Table 7
Wellington Classes, Showing the Number of Students Taking the Pilot WSSAT and the Number of Students Meeting the Criteria for Achieving a Particular Stage

	Wellington Pilot Classes						
	W9–1	**W9–2**	**W9–3**	**W9–4**	**W9–5**	**W9–6**	**Total**
No. of students	17	22	18	19	21	16	113
No. meeting stage 5	17	22	18	19	21	16	113
No. meeting stage 6	12	21	8	11	12	9	73
No. meeting stage 7	7	21	7	8	11	5	59
No. meeting stage 8	3	14	1	2	0	1	21

Comparison between oral and written assessments

The stages determined by the pilot WSATT closely matched the stage determined by the oral assessment of students at stage 8 but less so at other stages (see Table 8). A third of students achieving stage 5 on the oral assessment achieved stage 6 on the WSSAT, and two-thirds of students achieving stage 6 on the oral assessment achieved stage 7 on the WSSAT. However, there were no stage differences of more than one stage, unlike the 2007 trial data (Lomas & Hughes, 2008). This may reflect the more even spread across stages achieved by the 2008 revised WSSAT versions.

Table 8
Comparison of Stages Assigned to Auckland Students by the Pilot WSSAT Compared with the Oral Assessment

	Number of students (n = 60)												
	2	**1**	**–**	**9**	**5**	**–**	**5**	**9**	**–**	**7**	**1**	**3**	**18**
Oral assessment stage	1–4	1–4	5	5	5	6	6	6	7	7	7	8	8
WSSAT stage	1–4	5	1–4	5	6	5	6	7	6	7	8	7	8

Comparison with national numeracy curriculum expectations

The assigning of stages from the WSSAT for the pilot schools gave rise to a distribution reasonably similar to the data for both the middle- and low-decile schools respectively and to the national expectations (see tables 9 and 10).

For the middle-decile year 9 cohort used in the pilot, the areas of greatest disparity with the middle-decile data for end-of-year year 9 students were the higher number of students achieving at stage 8 (19% compared with 10%) and the lower number of students achieving at stages 1–4 (0% compared with 6%) in the trial data. The greatest area of disparity for the middle-decile year 9 students compared with the year 9 national expectations was the higher number of students achieving at stage 5 (28% compared with 12%) in the trial data.

Table 9

The Percentage of Stages Assigned to Wellington Students from the WSSAT and the Middle-decile and National Numeracy Curriculum Expectation Data for Year 9 Students (End of Year)

	Stages				
	1–4	5	6	7	8
WSSAT: percentage of students (n = 113)	0	28	19	34	19
Middle-decile[3] (averaged) percentage at end year 9 (Tagg & Thomas, 2008)	6	22	32	30	10
National numeracy curriculum expectations percentage year 9 (Tagg & Thomas, 2008)	2	12	25	40	21

For the low-decile year 9 cohort, the areas of greatest disparity (around a 50% difference or more) with the low-decile data for end-of-year year 9 students were the higher number of students achieving at stage 8 (16% compared with 5%) and the lower number of students achieving at stages 1–4 (4% compared with 11%) in the trial data (see Table 10). This may partly be explained by the exclusion of a group of lowest-performing year 9 students from the data collection process.

The area of greatest disparity for the low-decile year 9 students with the year 9 national expectations was the lower number of students achieving at stage 5 (28% compared with 12%) in the trial data. This may partly reflect a difference between low-decile students and a national expectation, although a similar disparity was evident in the comparison for the middle-decile data (see above).

Table 10

The Percentage of Stages Assigned to Auckland Students for Each Assessment Tool and the Low-decile and National Numeracy Curriculum Expectation Data for Year 9 Students (End of Year)

	Stages				
	1–4	5	6	7	8
WSSAT: percentage of students (n = 280)	4	28	23	30	16
Oral assessment: percentage of students (n = 60)	5	23	23	13	35
Low-decile[4] (averaged) percentage at end year 9 (Tagg & Thomas, 2008)	11	29	33	22	5
National numeracy curriculum expectations percentage year 9 (Tagg & Thomas, 2008)	2	12	25	40	21

The oral assessment's assigning of stages to students is reasonably close to the national expectation percentages for all stages except those achieving at stages 6 and 7. However, the sample was skewed in that it took half from the upper band and half from the lower band without sampling the middle band, where more students achieved at stages 6 and 7. If the consistency of the stages assigned with the banding of classes (see Table 6) had been taken into account, a less skewed sample would have shown a closer fit overall.

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[3] The middle-decile percentages are average figures derived from the respective percentage data for the additive, multiplicative, and proportional domains.

[4] The low-decile percentages are average figures derived from the respective percentage data for the additive, multiplicative, and proportional domains.

Discussion

A significant factor to consider in comparing the WSSAT and oral assessment results with the national data sets is the degree to which they accurately represent the stages that the students at year 9 have achieved. This is highlighted by Thomas et al.'s (2006) research findings on the accuracy of secondary teachers' assessments (76%) and teachers' tendency to assess at lower levels based on their perception of students' needs rather than actual performance on a numeracy assessment tool. This would suggest an underestimation of student performance overall, but possibly more so at stages 7 and 8, in which the learning demands are greater. This may be evident in the trial data, where there is nearly twice the percentage of students achieving at stage 8 compared to the national middle-decile data, although the percentage of students achieving at stage 7 in the national middle-decile data is higher. The possibility of such a trend is more apparent in both the medium- and low-decile pilot data. For example, compared with the national middle-decile data, there are three times more students achieving at stage 8 and one and a half times more students achieving at stage 7, but only about one-third fewer students achieving at stage 6.

The trial of the revised WSSAT gave a better spread of assigned stages and highlighted a number of issues relating to consistency across stages 7 and 8. With the pilot version, there appears to have been an improvement in consistency across stages 7 and 8 due to the minor revisions to stage 7 and 8 items but less consistency across stages 6 and 7. However, overall, the WSSAT has reasonably high levels of internal consistency for stages 5–8 and can be used to assign students a numeracy (global) strategy stage. In addition, there is a reasonable congruence of the stages assigned with both the low- and middle-decile school data and the national numeracy curriculum expectations and with the oral assessment assignment of NDP stages. Thus, the WSSAT, in its pilot form, determines a student's numeracy strategy stage with a reasonable degree of accuracy (and consistency) for formative and diagnostic purposes.

References

Lomas, G., & Hughes. P. (2008). Written and oral assessment of secondary students' number strategies: Developing a written assessment tool. In *Findings from the New Zealand Secondary Numeracy Project 2007* (pp. 32–38). Wellington: Learning Media.

Tagg, A., & Thomas, G. (2007). Do they continue to improve? Tracking the progress of a cohort of longitudinal students. In *Findings from the New Zealand Numeracy Development Projects 2006* (pp. 8–15). Wellington: Learning Media.

Tagg, A., & Thomas, G. (2008). Performance of SNP students on the Number Framework. In *Findings from the New Zealand Secondary Numeracy Project 2007* (pp. 5–16). Wellington: Learning Media.

Thomas, G., Tagg, A., & Ward, J. (2006). Numeracy assessments: How reliable are teachers' judgments? In *Findings from the New Zealand Numeracy Development Projects 2005* (pp. 91–102). Wellington: Learning Media.

Students’ Knowledge and Strategies for Solving Equations

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This paper presents results from the second year of an investigation into students’ algebraic thinking. The assessment techniques parallel those used for investigating number in the Numeracy Development Projects. In this study of 621 year 7–10 students, oral interviews with supplementary questions were used to investigate the strategies that students used to solve equations. Basic facts, numeracy strategy stage, and knowledge of aspects of algebra were also assessed. Rasch analysis was used to investigate the difficulty of the equations and student ability, and then the strategies associated with each question were examined. The data provides strong evidence that there is a hierarchy of sophistication of strategies. A large number of the students were unable to solve many of the equations because they were restricted to less sophisticated strategies. Clear relationships were found between the most sophisticated strategy a student used and their numeracy stage, basic facts knowledge, and algebraic knowledge.

Introduction

The success of the Numeracy Development Projects (NDP) in raising New Zealand students’ achievement in number (Thomas & Tagg, 2007; Young-Loveridge, 2007) has prompted initiatives to extend the NDP into early algebra. The NDP are centred on the Number Framework (Ministry of Education, 2003), which describes the progression of students’ arithmetical strategies and the knowledge associated with these strategies. The study reported on here examines students’ strategies for solving linear equations and the relationship of these strategies to numeracy and to algebraic knowledge. The goal is to contribute to a research base that may allow the development of an algebra framework.

Background

Many students struggle with introductory algebra, and teachers have little to guide them in designing programmes of learning. Little is known about the effect of students’ numeracy on the learning of early algebra or about the strategies that students use to solve equations. However, a useful summary of strategies used by students is provided by Kieran (1992), who describes the use of known basic facts, counting techniques, guess-and-check, cover-up, working backwards, and formal operations. In this paper, Kieran’s classification of strategies is modified and extended and “transformations” is the term used to describe Kieran’s “formal operations”.

There is a wealth of research on students’ errors and misconceptions in algebra, some of which was summarised in the 2007 findings from the Secondary Numeracy Project (SNP) (Linsell, 2008). These difficulties include understanding of arithmetical structure, inverse operations, algebraic notation and conventions, operating on unknowns, lack of closure, the equals sign, and treating equations as processes rather than objects. It can be argued that much of the research adopts a deficit model approach, detailing what students can’t do. An alternative approach is to investigate what students actually do when presented with equations to solve and to examine the knowledge and skills associated with the various strategies that different students use.

Teaching how to solve equations has traditionally focused on the type of equations presented to students rather than on the strategies that they are using. If a student is successful at solving a given type of equation, the teacher will often present them with harder equations, irrespective of whether the student is solving by, for example, guess-and-check or working backwards. To move to an approach more consistent with the NDP, in which students are taught according to the highest strategy they have available to them, more information is needed about how students solve equations.

The research questions addressed in this study were:

- What is the relative difficulty of equations?
- What strategies do students use, and is there a hierarchy of strategies?
- What is the impact of context on students' solution strategies?
- What prerequisite knowledge is associated with each strategy?
- What stage of numeracy is associated with each strategy?

Method

This study examined relationships between students' strategies for solving equations and their numeracy stage, basic facts knowledge, algebraic knowledge, and whether the equations were symbolic or in context. A structured diagnostic interview was administered to individual students by the researcher or by the students' classroom teacher. The students' responses were coded and then analysed making use of Rasch analysis (Wright & Masters, 1982). Algebraic knowledge was assessed by a written test, while numeracy stage and basic facts knowledge were assessed through routine procedures in place in the schools. Further details of the assessment tools are provided by Linsell (2008).

Subjects

The study took place in two intermediate schools (years 7 and 8), two high schools (years 9 and 10 only) and one college (years 7–9 only). There was no attempt at representative sampling, but instead the aim was to collect data from a wide range of students. The interview was administered to a total of 621 students in year 7 (n = 196), year 8 (n = 43), year 9 (n = 245), and year 10 (n = 137). Clearly, year 8 students are underrepresented, but this is mitigated by the fact that interviews took place throughout the school year, so students at the beginning of year 9 and the end of year 7 were included. In the two schools in which there was streaming, all classes from each year level were included and in all schools, no students were excluded on the basis of ability.

Diagnostic Interview

The diagnostic interview was developed in a previous study (Linsell, McAusland, Bell, et al., 2006) and was guided by the literature on students' strategies for solving equations (Herscovics & Linchevski, 1994; Kieran, 1992). The interview consisted of a series of increasingly complex equations, which the students were asked to solve along with an explanation of their thinking. The series included 12 pairs of parallel questions: ones that were in context (that is, word problems) and ones that were purely symbolic. The questions were presented on cards so that the more difficult questions could be omitted as required without suggesting to the student that they were not coping. Each question was read to the student to minimise the impact of reading difficulties, including difficulties with reading symbolic equations. Calculators and pencil and paper were available for the students to use, but it was stressed to the students that they could use whatever method they chose. The interviewer recorded what the student did and said and then classified the strategy used according to Table 1. Note that there has been a change in terminology since last year's report (Linsell, 2008): strategy i, previously called formal operations/equation as object, is now called transformations/equation as object. This is in order to clarify that the strategy involves transforming an equation into a new equation one or more times and is not simply the following of some given formal procedure.

Table 1
Classification of Strategies for Solving Equations

Code	Strategy
0	Unable to answer question
a	Known basic facts
b	Counting techniques
c	Inverse operation
d	Guess-and-check
e	Cover up
f	Working backwards, then guess-and-check
g	Working backwards, then known fact
h	Working backwards
i	Transformations/equation as object

Knowledge Test

The assessment of algebraic knowledge was administered as a written test because supplementary questions were not required. The areas investigated in this section were: knowledge of conventions and notation, understanding of the equals sign, understanding of arithmetical structure, understanding of inverse operations, and acceptance of lack of closure.

Numeracy Assessment

All the schools in this study were either NDP or SNP schools and therefore routinely collected numeracy data on their students. In instances where this data was not available, stage of numeracy was assessed using a modified GloSS (Global Strategy Stage) and knowledge of basic facts was assessed using a modified section from NumPA (the Numeracy Project Assessment tool).

Data Analysis

The first stage in the analysis was to determine the difficulty of the equations and the ability of the students. In Rasch models, the probability of a specified response (that is, a right/wrong answer) is modelled as a logistic function of the difference between the person and the item parameter. Before applying this model to the data, it was therefore necessary to ascertain that the variable of item difficulty was unidimensional. Factor analysis was initially employed to verify that a one-factor model was an adequate fit to the data from the strategy interview. Following this, Rasch analysis was used to determine item difficulty and student ability. These scores were then related to the strategies that individual students used for each question.

A proposed hierarchy of strategies was then developed by examining the distributions of ability of students, using each strategy on each question. This permitted each student to be classified according to the most sophisticated strategy they used on any question.

From the numeracy assessment and algebraic knowledge test, each student was assigned a score for numeracy, basic facts, conventions and notation, equivalence, arithmetical structure, inverse operations, and lack of closure. Relationships between each of these measures and the most sophisticated strategy used were then examined.

Results

Item Difficulty

For equations that were presented symbolically, there was a huge variation among the students in the number of equations that they were able to solve, with some questions being much harder than others (see Table 2).

Table 2
Item Difficulty (Symbolic Equations)

Equation	Number of Students with Correct Responses	Percentage of Students with Correct Responses	Rasch Score (Item Difficulty)
$n - 3 = 12$	565	91	–3.750
$18 = 3n$	511	82	–2.910
$n + 46 = 113$	523	84	–3.054
$\frac{n}{20} = 5$	206	33	1.111
$4n + 9 = 37$	400	64	–1.405
$3n - 8 = 19$	382	62	–1.153
$26 = 10 + 4n$	362	58	–0.883
$\frac{n}{4} + 12 = 18$	185	30	1.411
$5n + 70 = 150$	283	46	0.185
$2 + \frac{n}{4} = 8$	153	25	1.873
$5n - 2 = 3n + 6$	109	18	2.581
$2n - 3 = \frac{2n + 17}{5}$	24	4	5.064
Rearrange $v = u + at$	5	1	7.283

In general, one-step equations were easier to solve than two-step equations, which in turn were easier to solve than equations with unknowns on both sides. However, it should also be noted that equations involving division were harder to solve than similar equations with other operators. Nevertheless, one-step equations involving division were easier to solve than two-step equations involving division.

Strategies Used

Two strategies (transformations and guess-and-check) could be used to solve any equation, while others (for example, inverse operation for one-step equations, working backwards for two-step equations) could be used for only a limited number of equations. For every equation (except for the final one) there was a range of strategies successfully used by students, but the distribution of strategies varied from question to question. Responses to three questions are shown in Figure 1 to illustrate the ranges of strategies used.

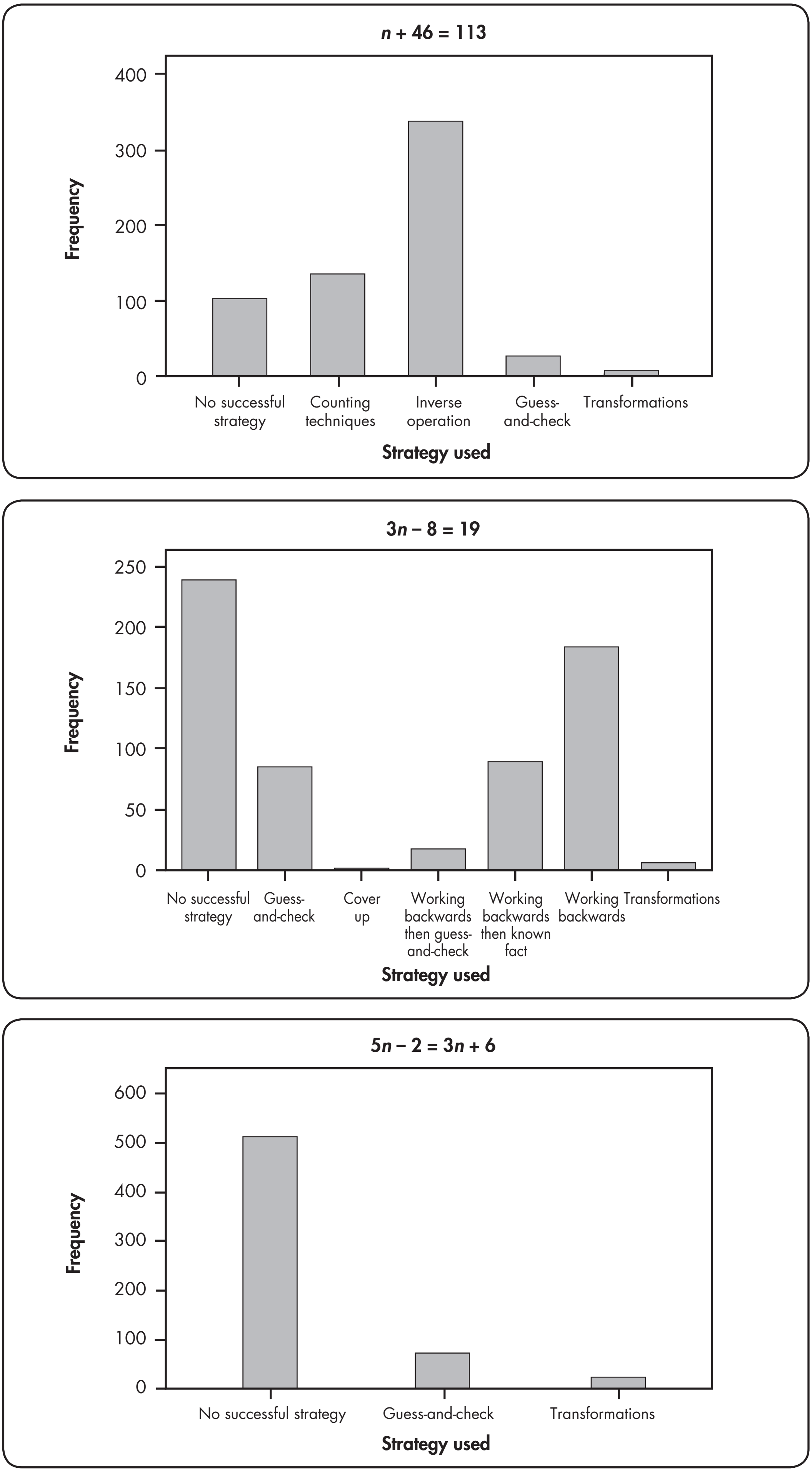
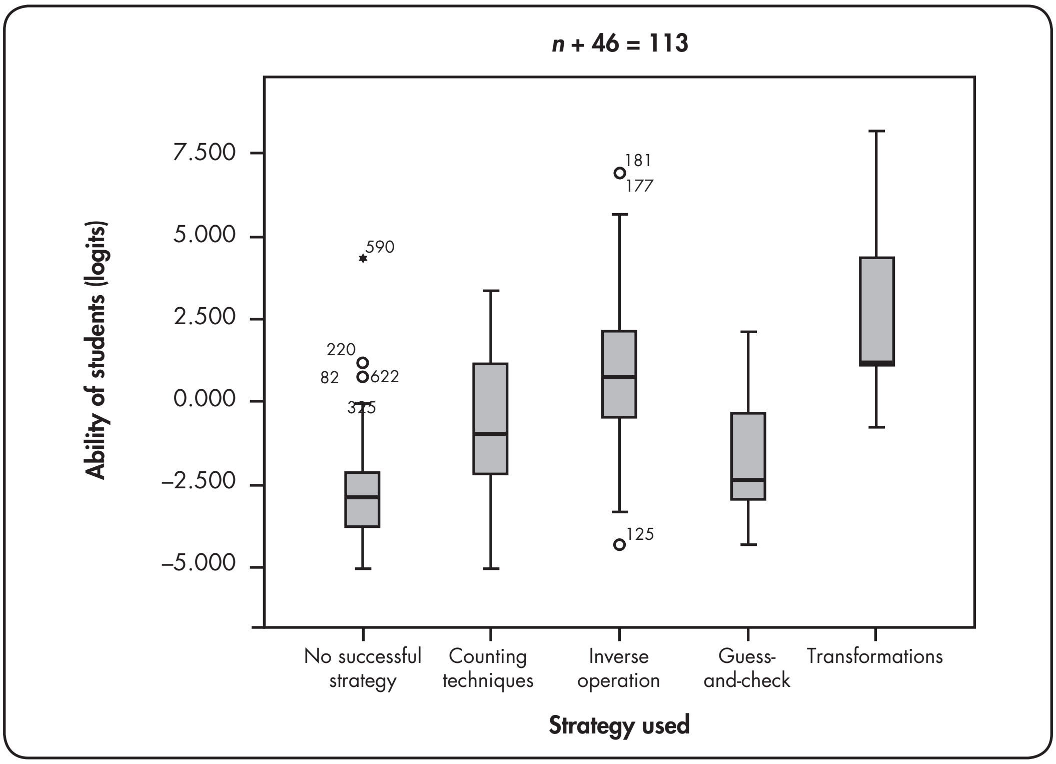


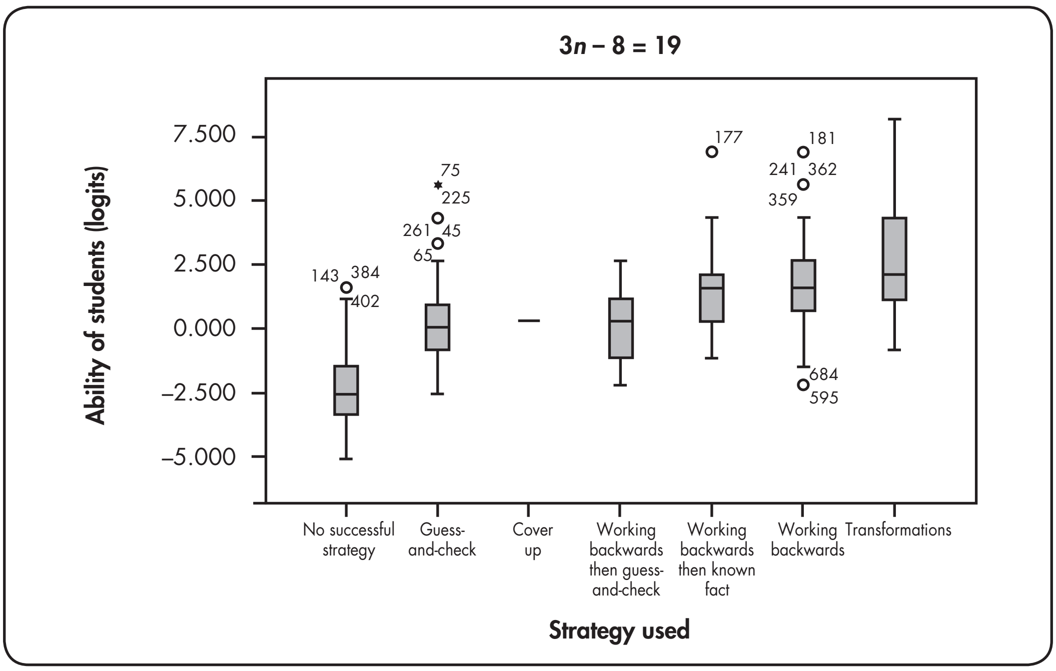
Figure 1. Students’ strategies for three equations

Hierarchy of Strategies

To establish a hierarchy of strategies was not straightforward because the pattern of strategy use varied from equation to equation, with some equations lending themselves to being solved by one strategy rather than by another. Another difficulty was that able students often reverted to guess-and-check for difficult questions, even though they used other strategies for easier equations. Less able students, in contrast, used guess-and-check for easy equations and were unable to solve more difficult equations by any strategy.

Therefore, the approach used was to examine the strategies used on a question-by-question basis. For each question, the ability of students using a particular strategy was investigated. Figure 2 shows the results[1] for the same three equations shown in Figure 1.





[1] Rasch analysis calculates item difficulty and student ability on the same scale.

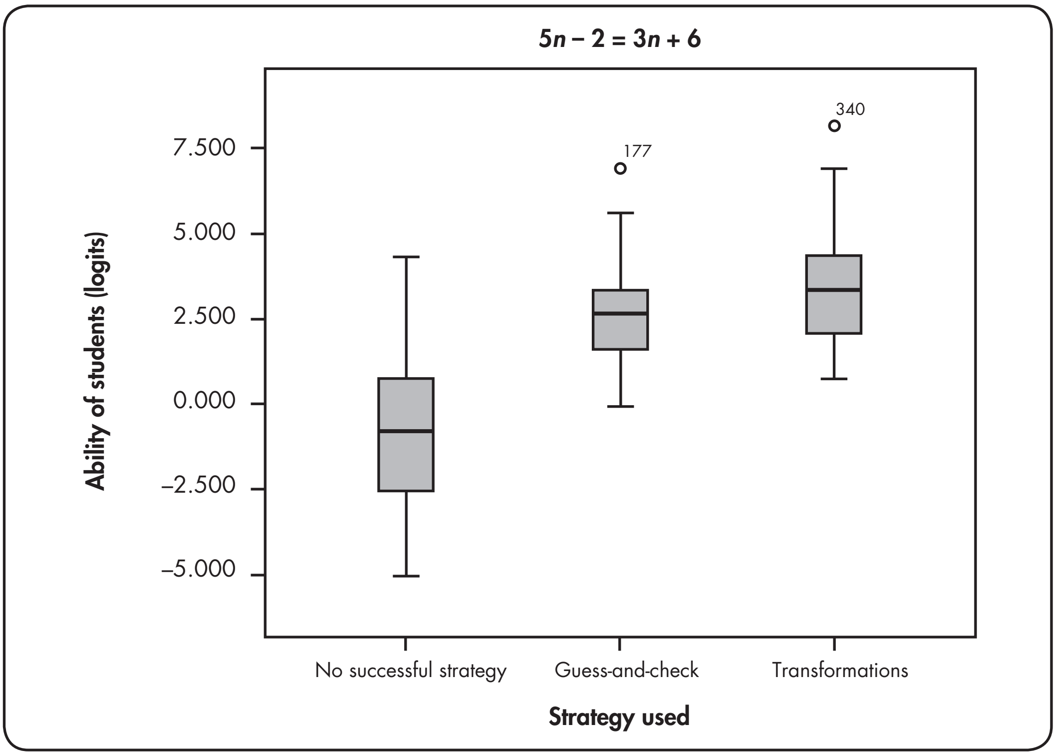


Figure 2. Ability of students using each strategy

For each equation, it was then possible to place the strategies in order according to the mean ability of students using each strategy. For example, the equation $5n - 2 = 3n + 6$ was solved using either guess-and-check or transformations. The mean ability of students using transformations was higher than that of students using guess-and-check, indicating that transformations was the more sophisticated strategy.

The picture that emerged using this approach was fairly consistent, in that the order is the same for nearly all the equations (see Table 3, in conjunction with the explanation of numbers in Table 4).

Table 3
Rank Order of Mean Abilities of Students Using Each Strategy for Each Equation

Strategy	a	b	c	d	e	f	g	h	i
Equation									
$n - 3 = 12$	3		2	4					1
$18 = 3n$	3		2	4					1
$n + 46 = 113$		3	2	4					1
$\frac{n}{20} = 5$	1		3	4					
$4n + 9 = 37$				5	3	6	4	2	1
$3n - 8 = 19$				4		4	3	2	1
$26 = 10 + 4n$	5			6	3	7	4	2	1
$\frac{n + 12}{4} = 18$				5		2	4	3	1
$5n + 70 = 150$				5	6	4	3	2	1
$2 + \frac{n}{4} = 8$				6	5	4	3	2	1
$5n - 2 = 3n + 6$				2					1
$2n - 3 = \frac{2n + 17}{5}$				2					1
$v = u + at$									1

For all equations (except for $\frac{n}{20} = 5$), transformations was the strategy used by the most able students and guess-and-check by the least able. For one-step equations, it was not possible to discern between counting strategies and known basic facts because, although students used a range of strategies over all the equations, for any particular equation this range never included both counting and known basic facts. However, for three of the four one-step equations, inverse operations were used by the more able students rather than by students using either counting strategies or known basic facts. The exception was $\frac{n}{20} = 5$, which was far more difficult than the other one-step equations. For this equation, the most able students solved it using a known basic fact. Cover up was used by such a small number of students that no clear relationship to the other strategies emerged. For five of the six two-step equations, working backwards was used by the more able students rather than by those using working backwards, then known fact, which in turn was used by the more able students rather than by those using working backwards, then guess-and-check. The exception was $\frac{n + 12}{4} = 18$, but the number of students using any strategy other than working backwards was too small to draw any conclusions. The strategies used only on one-step equations clearly could not be compared directly with those used only on two-step equations. However, two-step equations are much harder than one-step, and working backwards involves using inverse operations.

The order of sophistication of strategies indicated by this analysis (see Table 3) is shown in Table 4.

Table 4
Rank Order of Strategies

Rank	Strategy
1	No successful strategy
2	Guess-and-check
3	Counting techniques / Known basic facts
4	Inverse operations
5	Working backwards, then guess-and-check
6	Working backwards, then known fact
7	Working backwards
8	Transformations

The students were then classified according to the most sophisticated strategy they used on any symbolic equation (see Figure 3).

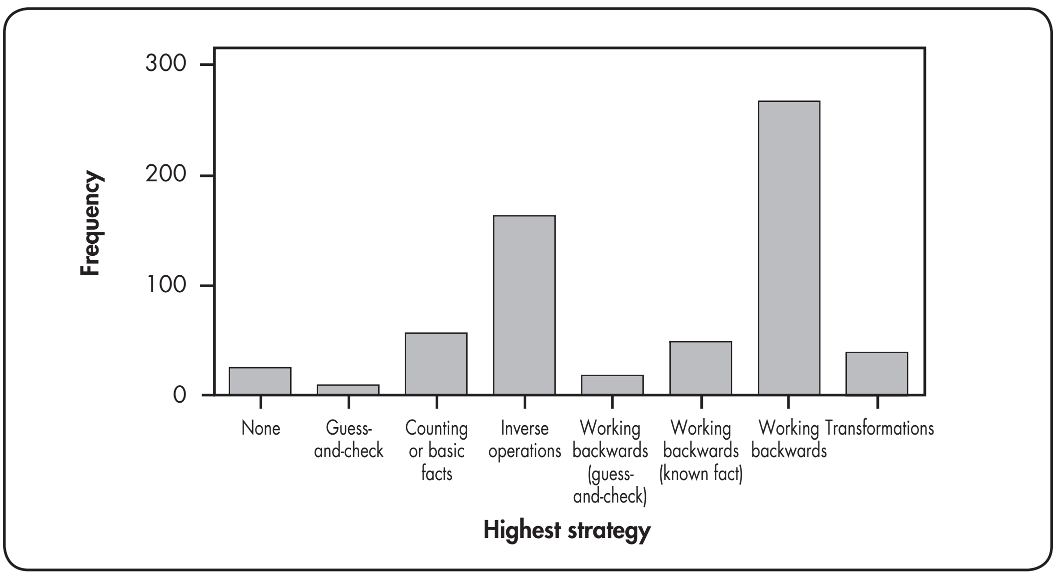


Figure 3. Most sophisticated strategy used on any symbolic equation

The number of students able to solve equations by performing transformations was very low (38). It is also worth noting that a significant number of students (89) were not even able to use inverse operations. Also of note is the number of students (65) who could solve some two-step equations but who were not fully working backwards.

Impact of Context

The picture that emerged in relation to students' solutions of equations that were given to them as word problems in context was very similar to that of symbolic equations. The diagnostic interview included 12 pairs of parallel questions. Table 5 shows the item difficulty of each question. Note that the table shows the structure of the word problems, but the problems read to students did not include these equations. For example, the first problem (shown in the table as $n - 5 = 17$) was "I left home this morning with some money, spent \$5, and have \$17 left. How much did I start with?"

Table 5
Item Difficulty (Rasch Scores)

	Symbolic	In context	Symbolic		In context	
	Equation		% correct	Item difficulty	% correct	Item difficulty
1	$n - 3 = 12$	$n - 5 = 17$	91	–3.75	96	–4.63
2	$18 = 3n$	$24 = 3n$	82	–2.91	67	–1.46
3	$n + 46 = 113$	$n + 24 = 89$	84	–3.05	88	–3.38
4	$\frac{n}{20} = 5$	$\frac{n}{20} = 4$	33	1.11	83	–2.98
5	$4n + 9 = 37$	$4n + 5 = 37$	64	–1.41	75	–2.29
6	$3n - 8 = 19$	$3n - 7 = 17$	62	–1.15	67	–1.55
7	$\frac{n + 12}{4} = 18$	$\frac{n + 11}{4} = 19$	30	1.41	52	–0.31
8	$5n + 70 = 150$	$5n + 70 = 250$	46	0.19	52	–0.34
9	$2 + \frac{n}{4} = 8$	$2 + \frac{n}{4} = 7$	25	1.87	28	1.45
10	$5n - 2 = 3n + 6$	$5n - 3 = 3n + 9$	18	2.58	13	2.89
11	$2n - 3 = \frac{2n + 17}{5}$	$4n - 2 = \frac{5n + 14}{2}$	4	5.06	2	6.26
12	Rearrange $v = u + at$	Rearrange $s = \frac{(v + u)t}{2}$	1	7.28	0	

The harder equations (with unknowns on both sides) were slightly easier when presented symbolically. However, nearly all one-step and two-step equations were easier when presented in context. There was one exception (the second pair of equations) and a few dramatic differences in difficulty that will be commented on when the strategies that students used are examined.

For all of the one-step equations that were in context, compared with their symbolic counterparts, there was a greater use of inverse operations than of counting strategies, known facts, or guess-and-check. This was true even for the second pair of equations, in which the order of difficulty was reversed. The context was "I have 24 CDs. This is three times as many as my brother has. How many CDs does he have?" Many more students than expected got this wrong by multiplying 24 by 3 rather than dividing. It would appear that the words "three times" confused them.

For the fourth, seventh, and ninth pairs of equations, students found the symbolic form much more difficult than one might expect. All these equations involved a division structure. The contexts for the fourth and seventh were: "When I shared a packet of lollies round my class of 20 students, they got 4 each. How many lollies were in the packet?" and "Our kapa haka group is made up of some Māori students and 11 Pākehā students. The whole group is divided into 4 equal-sized groups for practices. Each of the practice groups has 19 students in it. How many Māori students are there in our kapa haka group?" It would appear that students saw these as multiplication rather than division problems and found them much easier than the symbolic equivalents.

For all of the two-step equations, there was a greater proportion of students using the working backwards strategy for equations that were in context compared with those using this strategy for symbolic equations. Conversely, there was a greater proportion of students using less sophisticated strategies for symbolic equations compared with those using these strategies for equations in context. There was also a small number of students who used transformations for two-step symbolic equations.

Prerequisite Skills and Knowledge

The relationships between each student's prerequisite knowledge and skills and the most sophisticated strategy that they were able to employ was then investigated. For these analyses, those students whose most sophisticated strategy was guess-and-check, counting, known basic fact, or no successful strategy were grouped together as primitive strategies. Those students whose most sophisticated strategy was either working backwards, then guess-and-check, or working backwards, then known fact, were grouped together as partially working backwards. The relationship between numeracy strategy stage and highest algebraic strategy is shown in Figure 4.

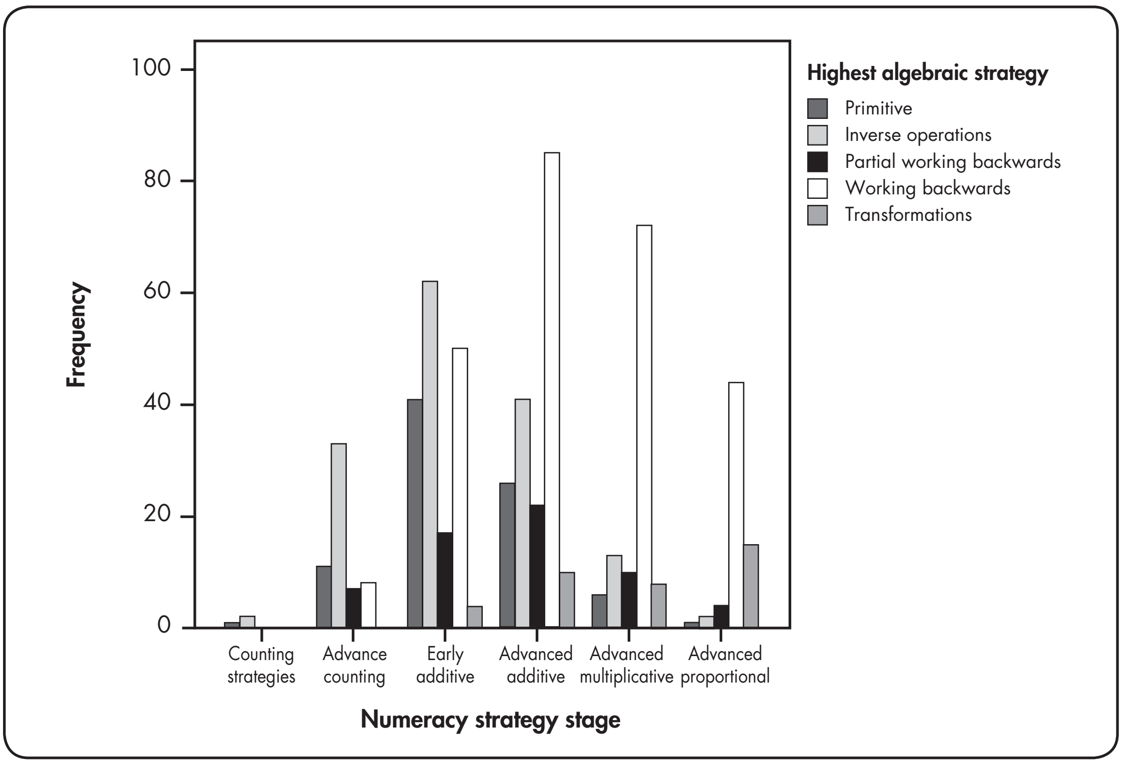


Figure 4. Relationship between students' numeracy and their most sophisticated strategy for solving equations

It is clear that students with poor numeracy skills were largely restricted to less sophisticated strategies. The great majority of students with good numeracy skills were able to use more sophisticated strategies, with very few of them using only less sophisticated strategies. A very similar picture to this emerged for students' knowledge of basic facts. Students with poor knowledge of basic facts were largely restricted to less sophisticated strategies, while most students with good knowledge of basic facts were able to use the more sophisticated strategies.

The relationship between understanding of arithmetical structure and highest algebraic strategy is shown in Figure 5.

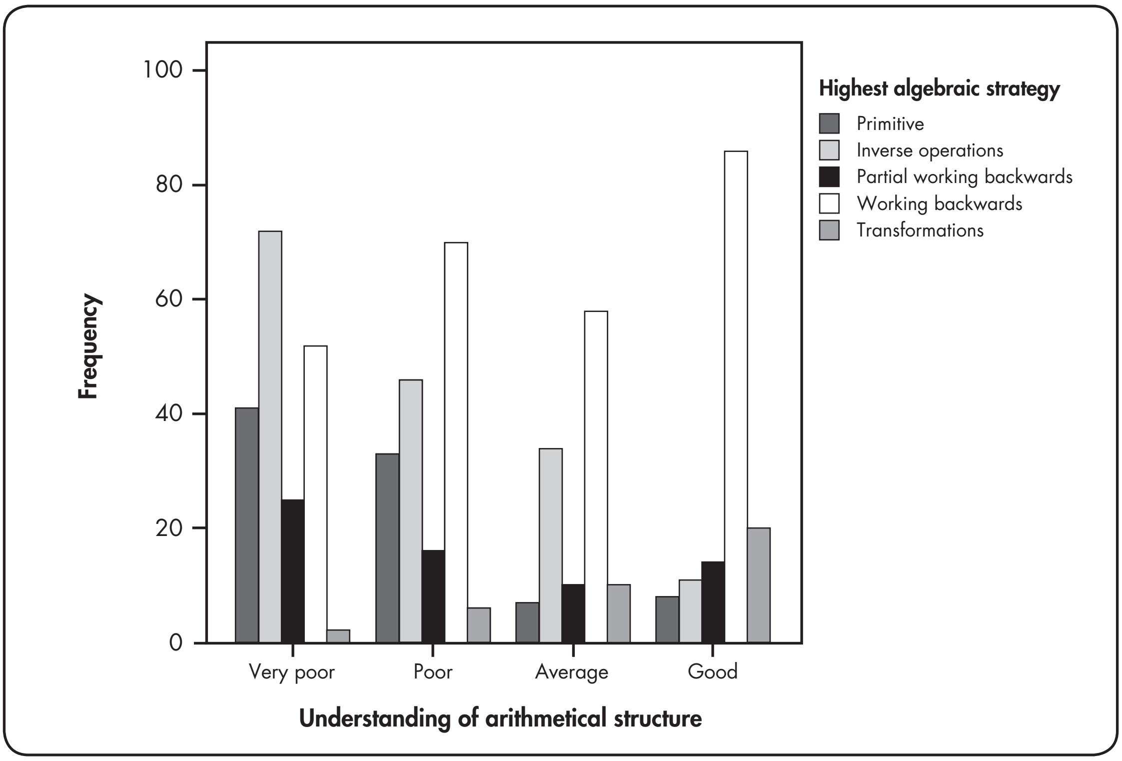


Figure 5. Relationship between students' understanding of arithmetical structure and their most sophisticated strategy for solving equations

Students' understanding of arithmetical structure had a dramatic impact on the most sophisticated strategy they were able to use. As their understanding of arithmetical structure increased, so did their use of more sophisticated algebraic strategies. The pictures for understanding of inverse operations, acceptance of lack of closure, and understanding of equivalence were very similar. The only area of knowledge that did not have such a clear relationship with sophistication of algebraic strategy was algebraic notation and convention. Students with good knowledge in this area used slightly more sophisticated strategies, but the relationship was not so convincing. Of particular note is the impact of students' understanding of equivalence and, to a slightly lesser extent, their acceptance of lack of closure (see Figure 6).

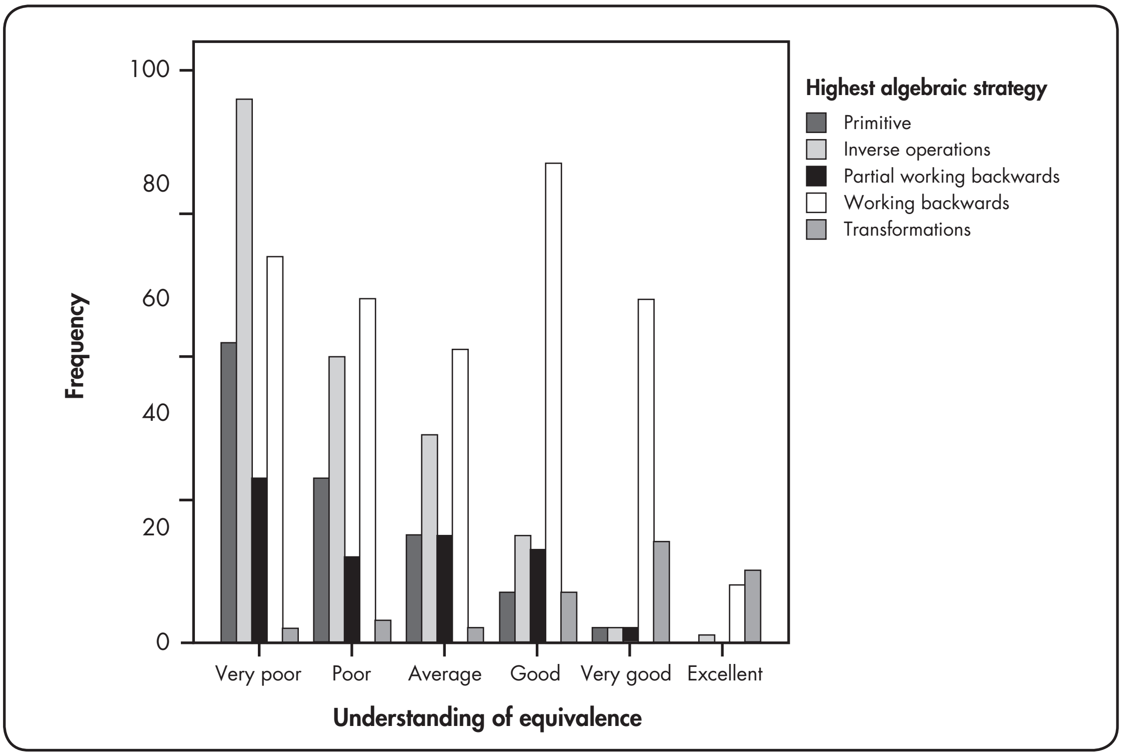


Figure 6. Relationship between students' understanding of equivalence and their most sophisticated strategy for solving equations

As students' understanding of equivalence increased, the ratio of the number of students using transformations to those using working backwards also increased. Of those students with an excellent understanding of equivalence, there was a greater number using transformations than using working backwards. This difference in proportion between those using working backwards and those using transformations was not so marked for the other areas of prerequisite knowledge investigated, although it was also large for acceptance of lack of closure.

Discussion

This study has demonstrated how incredibly difficult some equations are to solve compared with others. It has been known for a long time that students find equations with unknowns on both sides very difficult (Herscovics & Linchevski, 1994), and Sfard (1991) has even suggested that viewing equations as objects may be beyond the grasp of many students. However, this study is the first to compare item difficulty on a Rasch scale. Rasch scores of item difficulty follow an approximately normal distribution, and to find that some items have a difficulty score over two standard deviations above the mean confirms their difficulty. *The New Zealand Curriculum* achievement objectives (Ministry of Education, 2007) give little indication of this difficulty. At level 4, the relevant objective is "Form and solve simple linear equations", and at level 5, it is "Form and solve simple linear and quadratic equations". The level 3 objective of "Record and interpret additive and simple multiplicative strategies, using words, symbols, and diagrams, with an understanding of equality" is useful in focusing on equivalence, but it does not spell out that students need to find unknown values in any position in a statement of equivalence. Educators need to appreciate how huge the range of difficulty is and not trivialise the solving of equations down to a few lessons on specific procedures.

Although equations in context were generally found to be slightly easier than symbolic equations, the general picture for item difficulty was very similar when the two sets of parallel questions were compared. This demonstrates that it is primarily the structure of problems that makes them difficult, not the algebraic symbolism. This finding is backed up by the lack of a strong relationship between students' understanding of algebraic notation and their most sophisticated algebraic strategy. Teaching of algebra should therefore be focusing on the structure of problems. The additional difficulties that students experienced with equations involving division highlight the need for greater emphasis to be placed on a variety of operators within both one-step and two-step equations.

Insights into why students found some equations so difficult to solve were obtained by examining the strategies they employed to solve them. It was clear that students were using a wide variety of strategies to solve all the equations. For one-step equations, many students were getting correct solutions but never used an inverse operation. To attempt to move these students on to two-step equations would be courting disaster. Inverse operations are involved in all the successful strategies for two-step equations other than guess-and-check. Students are therefore likely to be reduced to using guess-and-check or learning a specific procedure that applies only to specific structures. Similarly, many students were obtaining correct solutions to some two-step equations but were only partially using working backwards or were even using guess-and-check. This point, that the strategy of working backwards is less homogeneous than previously reported, is important. Many students are only just grasping the strategy and can use it only when the first step reveals a known basic fact to them for the next step. These students use the strategy of working backwards, then known facts. Other students are prevented from fully using working backwards because of lack of knowledge of multiplication and division facts. These students use the strategy of working backwards, then guess-and-check. To attempt to move these students on to using transformations would almost certainly be premature. When teachers suggest to students that an equation is like a balance pivoted about the equals sign, it is hard to imagine why it would be difficult to do the same thing to both sides of the equation and keep it balanced. However, this study has confirmed just how difficult the strategy of using transformations is for students. Using transformations requires seeing an equation as an object to be acted on (Sfard, 1991), but it is clear that most students see equations as processes.

In order to investigate the relative sophistication of strategies, it was assumed that the more sophisticated strategies would be chosen by the more able students. The findings using this approach were consistent with the fact that there was only one student who was able to perform transformations but could not also work backwards (this low-ability student appeared to be following a procedure of doing the same thing to both sides for simple equations) and that all students who could work backwards could also use inverse operations. This study has shown that the strategy of solving one-step equations by inverse operations is used by the more able students rather than by those who use either known basic facts or counting strategies. These strategies in turn were used by the more able students rather than by those who solved one-step equations using guess-and-check. Similarly, the strategies of solving two-step equations by partially working backwards were used by less able students rather than by those fully working backwards. On any particular equation, students chose a strategy that was sufficient to solve the equation rather than using their most sophisticated strategy. However, it is suggested that the different strategies are not merely a matter of choice but that the most sophisticated strategy that a student ever uses is indicative of conceptual development.

The impact of context on students' ability to solve equivalent problems was very interesting. In general, context problems were found to be easier than symbolic problems until the difficulty level of unknowns on both sides was reached. However, most school programmes focus on teaching skills for solving symbolic equations. Solving word problems is usually regarded as harder and introduced later as an application of these skills. An alternative perspective on contexts is to view them as models of the mathematics. Models are an important feature of Realistic Mathematics Education (RME) (Gravemeijer, 1997). Traditionally, models are derived from formal mathematics, whereas in RME, models are derived from real situations that students have experienced and are chosen to reflect the informal strategies of students. Initially, a model of a situation that is familiar to the students is used. Next, through generalising and formalising, the model becomes an entity in its own right. Finally, it becomes possible to use the model for mathematical reasoning. Gravemeijer describes this as a transition from *model-of* to *model-for*. The nature of a model therefore evolves from being highly context-specific to deriving its meaning from a mathematical framework. In contrast, when pre-existing models are given to students to help them solve problems, the students are expected to use them in prescribed ways that may not be clear to them. The results from this study are consistent with Gravemeijer's perspective and suggest that algebra would be better introduced in context rather than just as symbols.

The impact of context on the strategies that students used may help to explain why students found these problems easier. For one-step equations, there was much higher use of inverse operations than of less sophisticated strategies. It appears likely that contexts allow students to perceive the structure of a problem in more than one way. For example, the problem "When I shared a packet of lollies round my class of 20 students, they got 4 each. How many lollies were in the packet?" has the structure $\frac{n}{20} = 4$, but may be viewed as "The number of lollies is 4 for each of the 20 students", with a structure of $n = 20 \times 4$. The context is therefore naturally leading the student into an inverse operation. If this is the case, then the role of the teacher should be to scaffold the writing of symbolic equations to describe contexts and then to explore and symbolise the solution strategies of the students.

There was a high correspondence between numeracy strategy stage and the most sophisticated strategy a student was able to use to solve equations. Only for students who were at the advanced multiplicative or advanced proportional thinking stages did the majority solve equations by using working backwards or transformations. Students at lower stages of numeracy were largely restricted to less sophisticated strategies. The findings from this study strongly suggest that prerequisite numeracy should be considered when designing teaching programmes for algebra. However, there were students who did not score highly on GloSS but were able to use sophisticated strategies for solving equations. These students were invariably efficient at using algorithms for computations and often came from primary schools that did not promote NDP numeracy. The algebra diagnostic tool may be more useful than GloSS for revealing the thinking of students at the upper end of the Number Framework. This is because GloSS focuses on mental strategies (and does not value the use of algorithms), whereas the algebra tool has a focus on students' understanding of mathematical structure.

There was a very strong relationship between students' knowledge of basic facts and their highest algebraic strategy. Any student who was at stage 6 or below on the Number Framework for basic facts was unlikely to be able to solve equations by working backwards or by transformations. This finding emphasises the critical importance of instant recall of all basic facts, including multiplication and division.

There were also strong associations between students' highest algebraic strategies and their understanding of arithmetical structure, inverse operations, lack of closure, and equivalence. There was not such a strong association between students' highest algebraic strategies and their knowledge of algebraic conventions and notation. The relationship between students' highest algebraic strategy and their understanding of equivalence was particularly interesting. Understanding of equivalence and, to a lesser extent, acceptance of lack of closure had much higher impacts on whether a student could use transformations compared with using the strategy of working backwards than did the other areas of algebraic knowledge. Given the reasonably large number of students who could work backwards and the very small number who could use transformations, these findings may have significant implications for teaching.

Conclusions

Consistent with the perspective of Filloy and Sutherland (1996), it is suggested that the strategies described in this study are not simply alternative approaches to solving equations but represent different stages of conceptual development. Instead of looking at how hard equations are to solve and whether students get them right, it appears to be more useful to look at the strategies that students use. The approach used in this study is very similar to that used in the NDP, with strategy being separated out from the knowledge required for strategy use. This approach allows the classification of the students according to their most sophisticated strategy rather than by the most difficult equation they are able to solve. Within numeracy teaching, students are grouped for instruction according to their most sophisticated strategy. It is suggested that a similar approach to grouping students is likely to be beneficial for teaching students to solve equations.

Also consistent with the NDP, the teaching of prerequisite knowledge needs to be addressed. To solve one-step equations, students need to understand inverse operations and to know their basic facts. To solve two-step equations by working backwards, students also need to understand arithmetical structure. To solve equations by using transformations, students need to understand equivalence and also accept lack of closure.

The third area of findings consistent with the NDP concerns the role of context. Questions that were in context were easier than equivalent symbolic questions. This suggests that a version of the numeracy teaching model should be employed for teaching algebra. We should start with contexts that are meaningful to students, preferably involving concrete materials. Teachers should scaffold students so that they can symbolise the structure of the problems and their solution strategies before expecting them to visualise a concrete representation and finally to operate on abstract symbolic structures.

References

Filloy, E., & Sutherland, R. (1996). Designing curricula for teaching and learning algebra. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds), *International handbook of mathematics education* (Vol. 1, pp. 139–160). Dordrecht: Kluwer.

Gravemeijer, K. (1997). Instructional design for reform in mathematics education. In M. Beishuizen, K. Gravemeijer, & E. Van Lieshout (Eds), *The role of contexts and models in the development of mathematical strategies and procedures* (pp. 13–34). Utrecht: Freudenthal Institute.

Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics*, *27*(1), 59–78.

Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390–419). New York: Macmillan.

Linsell, C. (2008). Solving equations: Students' algebraic thinking. In *Findings from the New Zealand Secondary Numeracy Project 2007* (pp. 39–44). Wellington: Learning Media.

Linsell, C., McAusland, E., Bell, M., Savell, J., & Johnston, N. (2006). *Using action research to learn about learning algebra.* Paper presented at the New Zealand Association for Research in Education, December, 2006.

Ministry of Education (2003). *The Number Framework.* Wellington: Ministry of Education.

Ministry of Education (2007). *The New Zealand Curriculum.* Wellington: Ministry of Education.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as two sides of the same coin. *Educational Studies in Mathematics*, *22*, 1–36.

Thomas, G., & Tagg, A. (2007). Do they continue to improve? Tracking the progress of a cohort of longitudinal students. In *Findings from the New Zealand Numeracy Development Projects 2006* (pp. 8–15). Wellington: Learning Media.

Wright, B. D., & Masters, G. N. (1982). *Rating scale analysis: Rasch measurement.* Chicago: MESA.

Young-Loveridge, J. (2007). Patterns of performance and progress on the Numeracy Development Projects: Findings from 2006 for years 5–9 students. In *Findings from the New Zealand Numeracy Development Projects 2006* (pp. 16–32). Wellington: Learning Media.

Senior Secondary Numeracy Practices in Successful Schools

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The Secondary Numeracy Project (SNP) focuses on developing numeracy teaching practice at years 9 and 10. Previous evaluations showed that teachers involved in the SNP professional development reported growth in their pedagogical content knowledge and mathematics teaching practice. The study reported on in this paper explored effects of the SNP on mathematics teaching at the senior secondary school level. The study sample comprised the mathematics teachers from four schools with effective numeracy teaching practices and six regional professional development facilitators. Data was collected using questionnaires and semi-structured interviews. Results indicate that the SNP has positively influenced teachers' practice in the senior school. Specific examples of changes in practice and departmental contextual factors that appear to have enhanced the impact of the SNP professional development are identified. The study findings indicate that, when certain contextual factors exist, the benefits of the SNP professional development extend beyond the target year levels.

Background

The Secondary Numeracy Project (SNP) is in its fourth year of implementation in New Zealand schools. Its overarching aim is to enhance mathematics teaching practices in secondary school classrooms. While the main focus of the SNP is on mathematics pedagogy for year 9 and 10 classrooms, the project has the potential to alter mathematics teaching at all levels in the secondary school. Schools regarded by regional numeracy facilitators as "successful numeracy schools" are the focus of the research reported in this paper, which aimed is to identify and share practices that the teachers in these schools believe to be transferable from the teaching of their junior classes to their senior classes and advantageous to students' learning.

Schools choosing to join the SNP receive professional development for mathematics department members for two consecutive years. In the first year of participation, teachers focus on developing their year 9 mathematics teaching practices. In the following year, they consolidate their year 9 teaching practices and adapt their teaching of year 10 classes. Schools receive funding to release one member of the mathematics team from 20% of their usual teaching load for the two years of the professional development, to allow them to take on an additional role as the school's in-school facilitator (ISF). The ISFs are inducted at a national hui in the year before their school first participates in the SNP and are mentored and supported within their region by their regional facilitator. Within each school, the ISF has a key role in their department's professional development. The ISF's responsibilities include: conducting workshops to introduce new approaches to teaching particular mathematics topics in years 9 and 10 within their department; supporting individual teachers in their development by observing lessons and providing feedback; and maintaining the impetus of professional development for numeracy teaching within their department. Smaller schools are clustered so that a teacher from one school acts as the ISF for two or more schools.

Previous evaluations of the SNP have shown that the majority of participating teachers have reported growth in their pedagogical content knowledge and mathematics teaching practices in years 9 and 10 (Harvey & Higgins, 2006, 2007). The 2007 investigation (Harvey & Smith, 2008) found that there was considerable variation in the degree to which teachers believed the SNP had impacted on their teaching practice with year 11 classes and that the teachers working with year 11 unit standards[1] classes reported a greater impact on their teaching practice than their colleagues teaching year 11 achievement standards classes.

Analysis of student attainment in two National Certificate of Educational Achievement (NCEA) Level 1 mathematics achievement standards in 2007 alerted researchers to the strong correlation between the decile[2] rating of schools and the performance of students in examinations (Harvey, 2008). Assisting teachers in low- and mid-decile schools to improve students' mathematics achievement is a way to address issues identified in that study.

Features that are generally present in effective mathematics professional development include: a specific pedagogical focus on mathematics teaching, with the development of the teacher's understanding of students' thinking; an extended time frame; and consistency between the development and the working context of the teachers (Timperley, Wilson, Barrar, & Fung, 2007). The SNP professional development extends over two years, has a specific focus on finding out what students do to solve mathematics problems, and involves all teachers from each school's mathematics department in addressing students' teaching and learning in their own school.

Some evidence exists that students involved in the Numeracy Development Projects (NDP) at the primary school level value explaining their own mathematical thinking and knowing others' mathematical strategies (Young-Loveridge, Taylor, & Hāwera, 2005). This paper discusses the ways in which secondary teachers were able to develop their skills in accessing student thinking.

A shared whole-school professional development focus was a noted characteristic of primary NDP schools that were creating strong achievement gains for Māori students (Te Maro, Higgins, & Averill, 2008). The current study explored the impact of whole-school professional development on how mathematics departments worked as teams to foster growth of professional expertise.

The study reported in this paper builds on the previous SNP evaluation studies by investigating the practices of teachers working at year 11 in successful numeracy schools. Consistent with the recommendation regarding decile level (Harvey, 2008), the study sample included only low- and mid-decile schools. The research questions were:

- Which SNP teaching practices are used in the year 11 classrooms of successful numeracy schools?
- How has the SNP impacted on practice in year 11 mathematics classrooms in successful numeracy schools?

Method

The research participants included the year 11 mathematics teachers, the mathematics head of department (HoD), and the ISF of four schools, as well as the regional numeracy facilitators. Participant schools were identified using recommendations by the regional numeracy facilitators and others. Initially, the regional facilitators were asked to suggest schools they deemed to be successful numeracy schools, which were broadly defined as schools where the implementation of the SNP has positively impacted on the teaching practice of the mathematics team. In order to help ensure a geographic

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[1] In year 11, most New Zealand secondary schools offer two different mathematics courses. The course that has a greater focus on abstract mathematics is mainly assessed using achievement standards. The course that has a more practical focus is mainly assessed using unit standards.

[2] Decile is used as a socio-economic indicator of New Zealand schools. A rating of 10 is the highest decile level, and 1 the lowest.

spread, to preserve the schools' anonymity, and to minimise the number of participant schools that had been involved in previous SNP evaluation studies, the researchers and their colleagues around the country compiled another list of successful schools. The combined list was used to select five target schools, which were then approached to take part in the study. Four agreed to participate.

Data was gathered by means of questionnaires for all participants (see Appendix G, pp. 95–98, Questionnaire) and individual semi-structured interviews with the HoDs, the ISFs[3], at least two teachers from each school, and a sample of three regional facilitators (see Appendix H, pp. 99–100, Interview questions). In total, data was gathered from 26 teacher questionnaires, six regional facilitator questionnaires, 15 teacher interviews, and 3 regional facilitator interviews. Teacher interviews were conducted face-to-face and regional facilitator interviews by telephone. All interviews were recorded and transcribed.

Three of the study schools, each in a different city, have between 1200 and 1700 students. The fourth school draws approximately 300 students from a rural town and its surrounding district. Three of the schools are co-educational, and one is a single-sex school. Two of the schools are low-decile, and two are mid-decile. All four had a stable core group of mathematics teaching staff who had been at the school for a number of years. One school commenced its participation in the SNP in 2005, two in 2006, and one in 2007. The three regional facilitators interviewed all had several years' experience working within the SNP.

Results

This section discusses practices that the teachers in the study schools reported using in the classroom. It also considers practices used by the schools' mathematics departments and with the whole staff. In order to detect if the impact of the SNP on year 11 practices differed between achievement standards classes and unit standards classes, the responses for the teachers of these two types of courses are shown separately (see Table 1).

Teachers' Practices

The range of strategies that teachers reported using is diverse, reinforcing the view expressed by many teachers and regional facilitators that there is no single way of teaching mathematics. The responses to questions asking for the frequency of use of specific teaching practices were tabulated, and the strategies that were most commonly cited were classified into broad themes for later discussion.

Teachers were asked to complete a table (see Appendix G) showing how frequently they used specific teaching strategies at year 11. Results from 23 teachers, listed roughly from the most prevalent to the least prevalent practices, showed a wide variation (Table 1).

3 In one school, the ISF was the HoD, and in another school, the ISF was one of the joint HoDs.

Table 1
Teachers' Responses Regarding How Frequently They Use Specific Teaching Approaches with Year 11 Classes

	Course Style	Most lessons	Several times per week	Several times per month	Several times per term	Several times per year	Seldom or never
Students work from textbook or worksheets	Unit standards	7	1				
	Achievement standards	9	6				
Homework set	Unit standards	2	5	1			
	Achievement standards	12	2	1			
Students discuss ideas with other students sitting near them	Unit standards	6				1	1
	Achievement standards	9	5	1			
Students present ideas to the class	Unit standards	5	1		1	1	
	Achievement standards	1	3	6	2	3	
Students are involved in investigations/ problem solving	Unit standards	1	1	4		1	1
	Achievement standards	2	4	5	1	3	
Practical work	Unit standards		1	3	2	1	1
	Achievement standards		1	2	7	2	3
Games	Unit standards		1	1	5		1
	Achievement standards		1		5	6	3
Student work marked by me	Unit standards					1	7
	Achievement standards	2		4	9		
Students use computers	Unit standards					1	7
	Achievement standards					3	12

Note: The table shows data from 23 teachers rather than all 26 because three teachers did not teach a year 11 class in the year of the questionnaire.

Some care should be taken in interpreting the Table 1 results. For example, one teacher responded "Several times per term" to the prompt "Student work marked by me", but added the comment "However, I do check that they have marked the homework". Another teacher indicated that they used textbooks in most lessons but generally for brief periods of time. Comments from other teachers that were not able to be displayed in these results included: teachers would prefer to use certain strategies more frequently than they had indicated in the questionnaire but felt restricted by a lack of emphasis on assessment within mathematical investigations; they didn't know enough suitable games; and there was a lack of access to computers.

The most common practices reported for both unit standards and achievement standards classes were: textbook and worksheet work, setting homework, and students discussing ideas with those around them. Practices used relatively seldom with unit standards and achievement standards classes included using computers and games. Differences in the practices that participant teachers used with unit standards and achievement standards classes included: students in unit standards classes were more likely to learn using games and by presenting their ideas to the class; achievement standards students were more likely to have their work marked by the teacher.

In the interviews, many teachers reported making more opportunities for year 11 students to share their mathematical thinking than had been the case before their involvement in the SNP. Similar to the changes in students' views regarding the value of their own mathematical thinking found by Young-Loveridge, Taylor, & Hāwera (2005), the study teachers reported that they valued students' mathematical discussions more than they had before their involvement in the SNP. Teachers reported having experienced and adopted a range of ways to create access to student thinking (for example, peer teaching, teacher questioning and listening, group and class discussions). It appears that teachers who had used strategies in the junior school that were consistent with the SNP had realised that these provided them with opportunities to assess students' progress and to help individuals learn through being involved in the discussion. Examples included involving students in teaching the class or getting help from another student: "He's just done it, go and ask him to explain it to you right now."

Teachers reported using questioning to generate student contributions to lesson content:

> I basically try to get everything out of them … I don't really want to tell them anything.
>
> [Since my involvement in the SNP], I ask more questions and better questions.

Teachers reported that improving their listening skills had allowed insight into students' mathematics misconceptions:

> I am better at picking up student misconceptions now because I am allowing myself to draw more on individual students' strategies.

One teacher commented on his increased skill at interpreting observations of students:

> Perhaps previously, I might not have noticed or picked up on that … Now I can actually interpret it … [and] make a decision about what I see in the classroom.

Others expressed their own similar development as being better able to "seize the teachable moment".

Several teachers reported increased use of class and group discussion and stated they now interrupt students less often than they used to:

> [I'm now] listening to kids ... There are really rich conversations going on.

A structured approach to eliciting students' thinking is illustrated by one teacher's practice of starting lessons with a true or false statement, such as "A plus A is A squared", for students to debate.

Teachers reported that having better knowledge of student thinking enabled them to be more flexible in their classroom practice:

> I [now] can make [planning] decisions about what I see in the classroom.

Other frequently reported teaching practices that included using exposition, modelling, and encouraging students to practise newly learned skills. Three teachers stated that they now structure their teaching to emphasise understanding of new mathematical ideas.

Explaining changes of practice in year 11 classrooms, one regional facilitator commented:

> I think they tend to explain ideas more; they tend to work with smaller groups more. It's not necessarily pre-planned groups, it's kind of incidental almost, on the day.

Impact of the SNP on Teaching Practices

In the questionnaire, participants were given a list of practices that teachers may have adopted or enhanced as a result of their involvement in the SNP. Teachers were asked to rate, on a five-point scale, the degree to which they had changed their practice since participating in the SNP. A 1 indicated little change in practice and a 5 a major change. In order to compare the impact of SNP-style practices between teachers' practice at the junior and senior secondary school levels, the participants were asked to rate each practice in relation to their teaching of both junior and year 11 classes.

Results are given in tables 2–5 below. Additional tables of results are found in Appendix I, pp. 101–102. Again, the results should be interpreted cautiously. For example, a low rating signifies little change to practice and does not indicate the extent to which the teachers use the practice. Twenty percent of teachers rated the impact of the SNP on their use of a strategy as low but added comments that indicated that the strategy had been an embedded element of their teaching practice before the SNP and remained so.

Consistent with the SNP focus on developing junior secondary school practice, in general the results indicate that the SNP has had a greater impact on teaching at years 9 and 10 than at year 11. The degree to which teachers have adapted their teaching of year 11 classes indicates the extent to which individual teachers have adopted and transferred SNP-consistent pedagogies to their practice outside the junior school. Data from the regional facilitators indicate that they also perceive that the SNP had a greater impact on the teaching of years 9 and 10 than on the teaching of year 11 mathematics.

Table 2

Reported Impact of the SNP (the Teaching Model[4]) on Teacher Practice

Use of the teaching model (materials → imaging → abstraction)	Impact on teaching				
	1	**2**	**3**	**4**	**5**
	Little impact				Major impact
Years 9 and 10	0	1	6	10	6
Year 11 unit standards	0	2	2	2	2
Year 11 achievement standards	1	6	5	2	1

Table 3

Reported Impact of the SNP (Differentiated Teaching[5]) on Teacher Practice

Differentiated teaching	Impact on teaching				
	1	**2**	**3**	**4**	**5**
	Little impact				Major impact
Years 9 and 10	1	4	9	7	2
Year 11 unit standards	0	3	3	1	1
Year 11 achievement standards*	1	5	6	1	1

Note: * denotes 1 non-response

Tables 2 and 3 indicate that the SNP has had a considerable impact on teacher practices of using the teaching model and "differentiated teaching" at the year 9 and 10 levels and some impact on these teacher practices at the year 11 level. Similarly, many teachers reported that they had incorporated increased use of group work and grouping by strategy stage into their years 9 and 10 teaching and, to

[4] Strategies are the mental processes used to solve operational problems with number. NDP *Book 1: The Number Framework* describes a hierarchy of increasingly sophisticated stages.

[5] For example, different work for different students

a lesser extent, into their year 11 teaching (Appendix I). In contrast, responses regarding the impact of the SNP in terms of using real-world examples show similar moderate-impact response patterns for junior, unit standards, and achievement standards class teaching (Table 4).

Table 4
Reported Impact of the SNP (Real-world Examples) on Teacher Practice

Real-world examples used in teaching	Impact on teaching				
	1	2	3	4	5
	Little impact				Major impact
Years 9 and 10*	1	3	8	5	5
Year 11 unit standards*	0	1	3	0	3
Year 11 achievement standards	2	4	3	4	2

Note: * denotes 1 non-response

Perhaps most heartening is that the study teachers reported that the SNP has had a major impact on the degree to which they emphasise understanding of key ideas. In terms of the SNP's impact on their teaching, 70% of teachers of year 9 and 10 students gave ratings of 4 or 5, 39% of year 11 teachers gave ratings of 4 or 5, and 64% of year 11 teachers gave ratings of 3 or higher (Table 5).

Table 5
Reported Impact of the SNP (Greater Emphasis on Understanding of Key Ideas) on Teacher Practice

Greater emphasis on understanding of key ideas	Impact on teaching				
	1	2	3	4	5
	Little impact				Major impact
Years 9 and 10	1	3	3	15	1
Year 11 unit standards	0	2	1	4	1
Year 11 achievement standards*	3	3	4	4	0

Note: * denotes 1 non-response

The results indicate that the SNP-consistent strategies most readily transferred into year 11 practice were: greater emphasis on key ideas, using real-world examples, and sharing learning intentions at the start of the lesson. The practice least often reported as transferred into year 11 classes was use of grouping by strategy stage.

Consistent with teachers' views, changes in year 11 classrooms described by regional facilitators indicate that more emphasis is being placed on making students think as part of the teaching and learning process:

> There is a desire to make mathematics education more accessible at year 11, but time pressure is interfering.
>
> "How did you get that answer?" This question and the consequent sharing are more frequent.
>
> Students are more likely to attempt problems without calculators.

Department and School Contexts

A common theme emerging from the interview data was that the context within which the teachers worked impacted greatly on their ability and inclination to adopt SNP-consistent practices. The research explored departmental and school factors that teachers reported as assisting with the implementation and transfer of SNP practices into year 11.

Department practices

The teachers in the successful numeracy schools participating in this study reported that their departments had continued practices (fostered though the SNP) that helped to enhance the cohesion and professional focus of their mathematics teams. The department practices most commonly described (discussed in turn below) included increased focus on:

- developing and sharing resources;
- giving increased emphasis to professional discussions based around student learning and understanding;
- carrying out activities for the purpose of discussing the results with colleagues;
- developing teaching schemes;
- altering assessment practices.

The teachers from several schools reported that departmental processes such as these assisted in creating an environment in which informal conversations about student learning could naturally occur:

> Good collegial atmosphere where lots of informal sharing goes on.
>
> Opportunities to discuss and share are invaluable.

The availability, sharing, and use of resources featured in many teacher responses. Examples include: support from the HoD in updating and providing resources; resources are easily available; and sharing and pooling resources.

The professional conversations between teachers were reported to be richer as a result of the SNP. The meetings within the mathematics teams were appreciated as times to share and improve practice. At times, these meetings were for the whole mathematics team, and at other times, the meetings involved teachers who were working at a particular level, indicating strategic and focused use of meeting time.

Teachers reported valuing the opportunities that such meetings provided to discuss and solve mathematics problems together, to share ideas, to have new activities demonstrated, and to work together on ways to implement new ideas into their teaching.

In one school, the meetings, which were initially focused on the SNP and led by the ISF, had evolved to focus more generally on mathematics pedagogy and included greater input from a range of staff. As preparation for meetings, some teachers reported carrying out a teaching task with their students, for example, "What do you think '=' means?" This practice was reported as giving a starting point for department discussion grounded in what students really do rather than in what teachers believe students might do.

Another school administered a diagnostic test on a topic and then analysed the results as a team to see "what the results tell us" and decide "how we are going to change our teaching because of the results".

Teachers also described an increase in the richness of informal discussions as a result of their involvement in the SNP. In one school, the creation of a shared departmental space enabled informal discussions to take place more naturally. One teacher noted a shift in collegial discussions towards examining practice, reporting that teachers are now more likely to discuss their own practice, for example:

> I've realised I've been teaching this way, and they (the students) have no idea what I have been talking about, and I've noticed this because I am looking at the data.

Teachers report that as a result of the SNP, their departments have paid more attention to developing their teaching schemes and assessment practices. In one school, the department scheme has been redeveloped to increase the emphasis on mathematical understanding and teaching guides have been developed to focus the teaching of certain topics. Several of the study schools have altered their assessment practices: one school has implemented pre- and post-tests for certain topics, while another has introduced topic assessment throughout the year rather than half-yearly examinations and has included more structure than previously into how they moderate their assessments. Some teachers reported that using asTTLe (Assessment for Teaching and Learning)[6] to determine the skills of students helped them to pitch teaching to their students' needs.

The three regional facilitators who commented on this aspect had seen examples of mathematics teams working together to focus on how best to teach the topic:

> Focused analysis about how to teach each of the components of the course – time consuming but very effective at clarifying ideas about instruction.

School contexts

Consistent with Te Maro, Higgins, and Averill's (2008) findings, synergy between whole-school professional developments (for example, Te Kotahitanga[7]) and the SNP appeared to enhance the uptake of SNP-consistent teaching practices. The Te Kotahitanga project was operating in three of the study schools, and staff reported that the strong reinforcement between the goals and approaches of the two projects contributed to their progress in the SNP. The fourth study school also reported that a whole-school development their school was undertaking, which included focusing on differentiated learning and problem solving, was consistent with their implementation of SNP practices. One aspect of such developments likely to enhance teacher development is the similarities in messages between projects. Another aspect is the environment created through a school culture of focusing on developing effective practice as a team.

Other whole-school practices for SNP implementation included, in one study school, the ISF running the knowledge section of the SNP diagnostic assessment at a whole-school staff meeting. This gave the entire staff a glimpse of the purpose of the SNP and an avenue for finding out more and discussing the possible impact of the SNP on students' work in curriculum areas other than mathematics.

A regional facilitator reported that one of his SNP schools had created kits of mathematics resources to use across the school during whānau[8] time.

Impact on Achievement

Teachers' views of the impact of the SNP on student achievement at year 11 were mixed (Table 6). One-third of the teachers chose not to respond to this question (a higher non-response rate than

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6 www.tki.org.nz/r/asttle/

7 http://edlinked.soe.waikato.ac.nz/departments/index.php?page_id=2639. Te Kotahitanga is a professional development programme that focuses on lifting the achievement of Māori students in mainstream schools.

8 Sometimes called "form time", schools set this time aside for each class to meet with the teacher directly responsible for their pastoral care.

for any other question) and, in several cases, study teachers noted that they did not have sufficient information to tell if there had been an impact on achievement. For teachers of both unit standards and achievement standards classes, the median response was 3 (moderate impact). Table 6 suggests that, in general, the teachers of unit standards classes reported a greater impact than did their colleagues teaching achievement standards classes. However, the difference is not statistically significant for this sample size and is consistent with results from Harvey and Smith's (2008) study. It may be explained in part by the greater emphasis on assessing number work without the use of calculators in key unit standards.

Three of the regional facilitators felt that they did not have enough information to rate the impact of the SNP on achievement at year 11. The other three regional facilitators indicated a modest impact on achievement.

Table 6
Teachers' Views of the Impact on Students' Achievement in Year 11 as a Result of the School's Participation in the SNP

	Little positive impact				Major positive impact	
	1	**2**	**3**	**4**	**5**	**No response**
Unit standards	1		3	1	1	2
Achievement standards	2	1	6	2		5
Total	3	1	9	3	1	7

When asked for their views on the appropriateness of achievement standards for assessing learning supported by the SNP, half of the teachers indicated that they found the current assessments too solution-focused and not useful as a way of measuring the development of strategies. Several of these teachers did, however, indicate that the draft revised standards more appropriately incorporated elements consistent with the SNP.

The regional facilitators were all of the view that many of the current achievement standards assessments are too reliant on skills that do not require understanding and are poorly aligned with the SNP:

> Mostly [the Level 1 achievement standards] do not [appropriately assess learning supported by the SNP]. These are the major reasons for SNP schools not continuing to implement more numeracy-style teaching. Standards are seen as requiring [a] different approach.

The revision of the standards was welcomed by regional facilitators, with two suggesting that specific standards assessing number understanding needed to be included:

> Have a designated number [achievement standard] where the assessment task is focused on gauging understanding.

Issues Impacting on Implementation

The biggest issue that teachers reported regarding implementation of SNP-consistent practices was "time", listed in 60% of responses. The responses from one study school, where junior mathematics was timetabled for three hours per week, showed that the teachers felt that lack of time was a big impediment to implementing SNP-consistent practices. The issue of time appears in many guises, as discussed below. Other issues, each mentioned by a small number of respondents, included: the unsuitability of current texts, the unwillingness of students to try alternative strategies, and staff changes.

Regional facilitators mentioned a range of factors that can impede implementation of the SNP including: lack of buy-in to the SNP by the full school, school-wide assessment and reporting practices that are inconsistent with the SNP, staff turnover, lack of time, and timetabling issues.

Ideas for Further Development and Professional Development

The major themes of ongoing support requested by teachers related to time and more professional development. Requests for time were couched in many different ways: time to think, plan, and prepare; more time needed in the classroom with students because it takes longer to teach this way; and an extended time frame needed for the professional development so that SNP practices could become embedded in both teacher and student practices. Despite the study schools being selected on the basis of successful implementation of the SNP, these teachers still requested more professional development to help them reshape their teaching practices.

The majority of regional facilitators stated that well-funded support for schools should be extended beyond two years. Other themes discussed by regional facilitators included the necessity of aligning national assessment with the SNP and ensuring pre-service teacher education courses include a significant component of material that is consistent with the SNP.

Discussion

The limitations of this study include the fact that only a small sample of schools were used and the study draws on teachers' reported rather than actual practices. However, it is hoped that the results and findings are able to provide a snapshot of effective practice that can be considered by regional facilitators, teachers, HoDs, and ISFs in their work to develop pedagogical and curriculum practice.

Although a study of this type necessarily aggregates results, it was clear that each teacher had their own style and trajectory of change. Some teachers in the study found that there was a high degree of fit between their existing mathematics teaching and that espoused by the SNP and they were still able to incorporate elements from the SNP into their teaching. However, for many teachers, the transition of style was somewhat greater: some teachers had undergone major changes of approach, while others had been more cautious and had adopted relatively few changes.

In the study schools, the SNP has contributed to building more cohesive mathematics teams. The maintenance of these teams is likely to be essential to maximising the ongoing transformation of teacher practice (Timperley et al., 2007). What teachers working in these low- and mid-decile schools have achieved provides a glimpse of how, given appropriate circumstances, professional practice of all secondary mathematics teachers and their departments can be enhanced.

The responses of the teachers in the successful numeracy schools show that many believe the SNP has had a positive impact on their mathematics teaching at year 11. The practices most often transferred into year 11 differed for unit standards classes and achievement standards classes, and this transfer appeared to be enhanced by particular departmental and school practices (for example, a shared professional development focus). The teaching strategies consistent with the SNP that were seemingly most commonly utilised by successful numeracy schools in year 11 included:

- increased focus on student thinking and students explaining their thinking;
- increased focus on developing and assessing students' mathematical understanding;
- increased use of real-world contexts.

This study builds on earlier studies (Harvey& Higgins, 2007: Harvey & Smith, 2008) in finding that the SNP is an effective professional development for improving the quality of mathematics teaching. As such, it is recommended that funding be continued for SNP development. Teachers and facilitators involved in this study suggested that greater funding in the third year of schools' involvement in the SNP had the potential to assist in embedding effective practice. This paper has illustrated some practices that have been used in schools and departments that have enhanced the effectiveness of the SNP and suggests that sharing such practices through the SNP professional development could assist in supporting its development in other schools. In successful numeracy schools, many teachers report increased emphasis on student thinking, focusing on key ideas, and use of real contexts. It would be beneficial if these teaching practices were to be discussed with schools new to the SNP as possible benefits of the project for senior secondary mathematics learners. Cohesion between several professional developments being undertaken within a school is beneficial, with each having the potential to enhance the effectiveness of others.

Recommendations drawn from the findings of this study include:

- continued investment in the SNP professional development across New Zealand secondary schools;
- increased funding for schools in the third year of the SNP to match that given in the second year;
- consideration of department and school contextual factors likely to enhance the implementation of the SNP within the professional development;
- a focus for the senior school on those aspects of the SNP most commonly transferred into successful schools' year 11 practice (student thinking, students explaining their thinking, focusing on key ideas, and use of real contexts);
- regional facilitators, department leaders, and teachers making strategic use of synergies between the SNP and other school-based professional development projects.

Further areas highlighted by this study as important to explore include "successful" teachers' actual classroom practice and junior and senior secondary students' views regarding how teaching approaches consistent with the SNP assist their learning, motivation, and achievement.

References

Harvey, R. (2008). An investigation into the impact of the Secondary Numeracy Project on student performance in two NCEA level 1 mathematics achievement standards. In *Findings from the New Zealand Secondary Numeracy Project 2007* (pp. 17–26). Wellington: Learning Media.

Harvey R., & Higgins J. (2006). Evaluation of the Secondary Numeracy Pilot Project 2005. In *Evaluations of the 2005 Secondary Numeracy Pilot Project and the CAS Pilot Project* (pp. 36–68). Wellington: Learning Media.

Harvey, R., & Higgins J. (2007). Evaluation of the 2006 Secondary Numeracy Project. In *Evaluations of the 2006 Secondary Numeracy Project.* Wellington: Learning Media.

Harvey, R., & Smith , D. (2008). Teachers' views on the impact of the Secondary Numeracy Project on the teaching of year 11 classes. In *Findings from the New Zealand Secondary Numeracy Project 2007* (pp. 27–31). Wellington: Learning Media.

Ministry of Education (2007). *Book 1: The Number Framework* (rev.ed.). Wellington: Learning Media.

Te Maro, P., Higgins, J., & Averill, R. (2008). Creating strong achievement gains for Māori students in English-medium classrooms. In *Findings from the 2007 Numeracy Development Project 2007* (pp. 37–49). Wellington: Learning Media.

Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2007). *Teacher professional learning and development: Best evidence synthesis [BES].* Wellington: Ministry of Education.

Young-Loveridge, J., Taylor, M., & Hāwera, N. (2005). Going public: Students' views about the importance of communicating their mathematical thinking and solution strategies: In *Findings from the Numeracy Development Project 2004* (pp. 97–106). Wellington: Ministry of Education.

Fostering the Growth of Teacher Networks within Professional Development: Kaiako Wharekura Working in Pāngarau

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This 2008 case study follows on from the research carried out in 2007 for the wharekura[1] Te Poutama Tau[2] project with a cluster of kaiako[3] now known as Kupenga. This year's study included an additional cluster known as Waiariki. The study focused on examining how a facilitator fostered and encouraged kaiako networks within professional development in order to support effective implementation of teaching and learning based on Te Poutama Tau. Analysis of data gathered through kaiako and facilitator questionnaires and interviews has revealed that kaiako are becoming increasingly familiar with the professional development components aimed at strengthening content and pedagogical knowledge and developing kaiako capability to sustain further development. Themes from the 2007 study of suitable facilitator characteristics and the importance of te reo pāngarau[4] were reinforced and developed in the 2008 study. It is evident that kaiako need continued facilitator support to enable them to develop fully-functional kaiako networks that can sustain and support their professional growth as kaiako pāngarau[5] at wharekura level.

Background

The wharekura Te Poutama Tau professional development and support project, developed as a result of recommendations from Trinick and Parangi's (2007) report about improving conditions for kaiako wharekura teaching pāngarau, was piloted in 2007 (Te Maro, Averill, & Higgins, 2008). It was designed to cater for the unique working conditions of kaiako wharekura, in which a range of challenges impact on pāngarau delivery. For example, kaiako pāngarau are scattered across the country with little access to peer support and knowledgeable coaches and mentors. Timperley, Wilson, Barrar, and Fung (2007) discuss the successes of professional learning through collaborative planning and execution that occurs in many secondary school departments (p. 208). Similar practice is often not possible in kura[6] because there is usually only one kaiako delivering pāngarau at each kura and this kaiako may also be teaching in other curriculum areas across several year levels. There is little professional development available in te reo Māori[7] for kaiako pāngarau and little designed for kura contexts. The wharekura Te Poutama Tau was designed and delivered to be responsive to kaiako wharekura needs.

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1 Wharekura: Māori-medium secondary school(s)

2 Te Poutama Tau: numeracy development project for Māori-medium educational settings

3 Kaiako: teacher(s)

4 Te reo pāngarau: the language of mathematics in Māori

5 Pāngarau: mathematics (through the medium of Māori language, customs, and cultural lenses)

6 Kura: school(s)

7 Te reo Māori: Māori language

Nine kaiako pāngarau working in wharekura in the Hawke's Bay, Taranaki, Waikato, Wellington, and Whanganui regions (Kupenga) participated in the initial 2007 wharekura Te Poutama Tau. Support was continued in 2008 for the Kupenga cluster (rōpū), and a new cluster of 10 kaiako wharekura from central North Island (Waiariki) was added. The wharekura Te Poutama Tau is specifically designed for kaiako of year 9 and 10 wharekura students (ākonga).

The 2007 study of the wharekura Te Poutama Tau (Te Maro et al., 2008) focused on the usefulness of the programme delivery modes (hui, facilitator in-school visits, and video conferencing) and kaiako and facilitator (kaitakawaenga) perceptions of the impact of this support on teacher knowledge and practice. The findings section of the evaluation paper included discussion of facilitator characteristics identified as essential to the success of the wharekura Te Poutama Tau and of the influence and growth of te reo pāngarau as an important factor in kaiako and ākonga progress with pāngarau. The 2007 kaiako were also starting to informally develop kaiako networks, and the 2008 study was set up in part to explore the facilitator's deliberate attempts to encourage kaiako networks as part of the professional development.

Building Teacher Networks

Teacher networks are useful in supporting the growth of teacher knowledge and practice and have commonly been used in reform initiatives (Cobb & Smith, 2008; Coburn & Russell, 2008; Timperley et al., 2007). Definitions of "teacher network" or "professional community" vary, but, according to Coburn and Russell (2008), they are typically conceptualised to include dimensions such as "shared norms and values, a focus on student learning, social trust, deprivatisation of practice, collective responsibility, and collaboration" (p. 4). Sharing dialogue is also a feature of networks; Timperley et al. (2007) argue that the purpose of dialogue is important in developing teacher knowledge and practice within their networks. For example, dialogue may challenge beliefs and support the efficacy of competing ideas, and by adding outside expertise such as the wharekura Te Poutama Tau facilitator into the networks and dialogue, new perspectives may be introduced to the group that encourage further challenging dialogue. Timperley et al. also suggest that teacher networks can generate collective responsibility for improving student achievement through an analysis of student learning outcomes. They also warn that "... it is possible for teachers to be given generous amounts of time to collaborate and talk together, only to have the status quo reinforced" (p. 201). This underscores the need to evaluate the usefulness of focusing on building kaiako wharekura networks.

The 2008 Professional Development Delivery Model

The delivery model of the 2008 wharekura Te Poutama Tau professional development was adapted from the first year's model, using facilitator reflections. The 2008 pāngarau professional development for both clusters (Kupenga and Waiariki) comprised four new elements and modifications to the 2007 elements. The 2008 development included: four hui[8] (one each term), Internet-based resources and interaction, and at least four in-school visits with each kaiako, arranged according to needs. The same facilitator carried out the professional development in both years. The main change from the 2007 model was to increase the focus on developing kaiako networks while continuing to provide increased opportunities for pedagogical and content knowledge development. New and modified elements included:

- hui (restructured to enhance kaiako ownership). The facilitator began each hui (for Kupenga and, to a lesser extent, Waiariki) by gathering information from the kaiako about what they wanted to focus on and how they wanted the hui to be run. The hui sessions included content-based

[8] Hui: meeting together, usually face-to-face

workshops, learning to use WiziQ and the wiki (see bullet points below), Te Poutama Tau and its connections to NCEA, the pāngarau learning area in *Te Marautanga o Aotearoa* (Ministry of Education, 2008), and, for two hui, visiting a local wharekura for small-group teaching.

- a wiki (new element). All resources from the hui as well as kaiako-selected resources were made available through an online store of materials known as a wiki.
- WiziQ (new element). This Internet-based professional networking function was incorporated to replace the video conferencing used in 2007. WiziQ was intended to provide kaiako with access to virtual hui and workshops that could be run by the facilitator or by kaiako.
- video (new element). The video footage of kaiako working with their classes was useful in allowing individual kaiako to discuss, reflect on, and critique their work with the facilitator.
- discussions with tumuaki[9] (new element) at each wharekura.

Methodology

Research Aims

To build on the 2007 study, the 2008 study focused on the effectiveness of new delivery modes, the growth of kaiako networks, and how kaiako took responsibility for planning and leading their ongoing pāngarau development. In the light of the 2007 findings, the 2008 study focused on the question:

> How does an effective facilitator support kaiako networks to promote numeracy-based teaching and learning of pāngarau?

The study included investigating the facilitator's response to the needs of kaiako and students, namely, by adapting the design of the 2007 professional development programme and putting the revised design into place. One of the facilitator's aims for the modifications was to assist with creating sustainability for Te Poutama Tau by placing more responsibility for the development of the programme with the kaiako groups.

Participants

The research participants for the 2008 wharekura Te Poutama Tau study included the facilitator and 20 kaiako from 15 wharekura. There were 124 year 9 and 10 students in the Kupenga cluster and 50 year 9 and 10 students in the Waiariki cluster. The facilitator carried out the professional development in both 2007 and 2008. The kaiako involved were those carrying on from 2007 in the Kupenga cluster (four kaiako), new kaiako joining the established Kupenga cluster in 2008 (three kaiako), and those in the new Waiariki cluster (nine kaiako). Four kaiako originally involved in the 2007 wharekura Te Poutama Tau did not return in 2008. All participants in both professional development clusters were invited and agreed to take part in this study. As in 2007, all but two of the 2008 kaiako were relatively inexperienced in teaching in wharekura, most had a variety of other responsibilities apart from teaching pāngarau, and all but three were not specialist kaiako pāngarau (taught only pāngarau). One specialist kaiako was secondary-trained in mathematics (in an English-medium setting). Typically, the participating kaiako wharekura worked in isolation from colleagues in the same field because they were the sole teachers of pāngarau in their kura, were geographically distant from kaiako pāngarau in other kura, or were teaching in kura that operated according to unique philosophical underpinnings.

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[9] Tumuaki – principal(s)

Method

Elements consistent with principles of Māori-centred approaches (Cunningham, 1998) and kaupapa Māori approaches (Bishop & Glynn, 1999) were used in this study:

- All aspects of the data gathering were negotiated with the facilitator, who in turn negotiated with the kaiako.
- Participation drew on established relationships (researcher-facilitator and facilitator-wharekura, facilitator-kaiako, researcher-wharekura, researcher-kaiako).
- Steps were taken to establish and maintain relationships (kanohi ki te kanohi[10] meetings before and during data gathering).
- The facilitator was integral to data analysis and report preparation.
- Te reo Māori was an integral part of the interviews, particularly with the Waiariki cluster.

Data Collection

Data collection included:

- written questionnaires, completed by:
  - the facilitator (two questionnaires);
  - kaiako at the initial hui, February 2008 (eight completed questionnaires from the Waiariki cluster);
  - kaiako at the final hui, November 2008 (15 completed questionnaires from both clusters).

  (The kaiako questionnaire is attached as Appendix J, pp. 103–104.)
- facilitator interviews (four in total), two early in the 2008 wharekura Te Poutama Tau and two towards the end of the project, with separate interviews focusing on each cluster. The interview questions were a guide, and the interviews were semi-structured.
- kaiako interviews (three cluster-kaiako interviews in total), which took place at the initial and final Waiariki hui and at the final Kupenga hui. The facilitator was present during parts of the kaiako interviews but did not participate. As in 2007, kaiako chose to be interviewed in their clusters rather than individually. As with the facilitator's interviews, these interviews were semi-structured.
- facilitator-kaiako interviews regarding the nature of pāngarau in the kura with two kaiako (two interviews in total);
- Te Poutama Tau student data at beginning and end of the year. Usually, the kaiako collect the data, which they send to the facilitator to collate. In 2007, the data was analysed collaboratively by the facilitator and the kaiako. In 2008, the data was not yet available to do this collaboratively.

Analysis

As with the 2007 study (Te Maro et al., 2008), the analysis of data collected from kaiako, the facilitator, and students was generative and open. The research team looked for relationships between concepts and ideas that were emerging in relation to the research question. Themes were identified collaboratively by the researchers and then discussed with the facilitator as another means of authentication. The themes were then synthesised to create a story of what occurred for kaiako. Illustrative quotes are included in this paper as evidence to support the discussion. Unless otherwise stated, all quotes are from kaiako questionnaires and interviews.

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[10] Kanohi ki te kanohi: face-to-face

Student data from each of the two groups were sent to the facilitator, who collated and analysed them separately because the groups were at different points in the professional development. One group had not yet finished their final testing, so there was insufficient data from which to draw conclusions for the year 9 cohort of students.

Results

This paper reports on how each of the identified professional development factors (contextual, facilitator characteristics, design and implementation, and te reo Māori) impacted on the professional development of kaiako and then comments on the links to kaiako networks identified by kaiako in their questionnaire responses. Student beginning-and-end-of-year Te Poutama Tau data collected and analysed by the facilitator informed recommendations for future development of the wharekura Te Poutama Tau. The Waiariki group also contributed to the process after discussing the data.

Contextual Factors

The data indicates that a range of contextual factors exist within wharekura that affect kaiako participation in the professional development available and in the development of kaiako networks. Kaiako reported that the support systems in wharekura that facilitated this particular professional development included support from kura managers, boards of trustees, and whānau[11]. These support systems released kaiako to attend hui (seven responses), adjusted timetables, and provided space. Kaiako reported support factors such as: "getting data from kura teina[12] saves us doing interviews" (seven responses); a dedicated support person who assisted with planning programmes (one response); and support from other Te Poutama Tau facilitators who are working with the kura teina and who assist the kaiako wharekura (five responses).

Factors that impacted negatively on the prioritising, delivery, and administration of Te Poutama Tau in participating wharekura were identified by the kaiako. There were five key contextual factors: the range of wharekura activities; the isolation of kaiako; national assessment (NCEA); availability and stability of staffing; and factors relating to the newness of wharekura as a schooling option. Further details about these factors are described below.

Commitment in other areas such as school camps and cultural events was identified as one factor limiting their progress:

> You've got other stuff to do as well as that – like ... sport ... and school – balancing what you as the teacher want to do and the requirements of the school. (Kaiako)

Also identified as problematic was the lack of established networks that could allow kaiako to observe others' practice, engage in professional discussions, and reduce their isolation. Kaiako identified small numbers of people who directly or indirectly supported them in their professional role as kaiako of pāngarau. One comment was:

> ... wanting to observe someone, but it is hard when there are not many around at this level; [not being able to] see it in action; it would probably be good if there were previous teachers to share the knowledge to other teachers. (Kaiako)

The facilitator identified the strong pressure on wharekura to maximise opportunities for students to gain credits towards NCEA, which resulted in students in some wharekura working for pāngarau unit standards in year 9 or earlier. One consequence, he felt, was that the pāngarau programmes can

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11 Whānau: family, extended family, becoming family

12 Kura teina: Levels 1–2 Māori immersion educational settings, equivalent to year 0–6 or year 0–8 schools in the medium of te reo Māori

become assessment-driven and be in conflict with the student-centred, data-driven philosophy of Te Poutama Tau. A kaiako comment on the impact of NCEA was:

> NCEA is the big focus in the school, and so I have put more of my effort into making sure everything is right there and that [the] kura changed the structure so that three times a week the students would do Te Poutama Tau and the other two sessions were "book work" to prepare the students for NCEA. (Kaiako)

Another kaiako comment reinforced the perception that NCEA limited their progress with the Te Poutama Tau professional development:

> There's restructuring in the school ... with me not teaching as much in the junior school as I have previously, with me focusing more on NCEA. (Kaiako)

Kaiako and the facilitator identified three types of staffing changes as having a negative impact. These included: kaiako leaving; kaiako moving out of wharekura and going to kura teina; and kaiako changing roles within the wharekura (for example, changing year levels or subject areas). One comment was:

> We've had a level one maths teacher away for 4 to 5 weeks this year, so I have been taking his class and we've had a reliever in to take my class ... instead of having them five times a week, I'm only having them three times a week. (Kaiako)

Also, most kaiako in wharekura are relatively inexperienced at wharekura level and some are given positions of responsibility early in their careers:

> ... and being HoD, I've got to do that [planning] times three – for the other teachers who are teaching under me – and I have a young inexperienced year-one teacher. (Kaiako)

Wharekura are still relatively new schools with changeable environments in which management structures may be still evolving. One kaiako noted a shift in focus:

> I haven't done as big a focus on Te Poutama Tau this year as I have in previous years. (Kaiako)

Another comment was about the challenge of resourcing, both in terms of staffing and physical material:

> Sharing the class with three other teachers means having to rearrange the classroom after every lesson ... and I am going to get a bigger room next year. (Kaiako)

In addition to the above points, the facilitator reported a range of factors that he felt constrained his actions in fostering the development of kaiako networks. He felt more constrained regarding the amount of time that could be devoted to each cluster due to the need to manage two clusters in 2008 compared with one cluster in 2007. He also stated that he could not always support kura at critical times (for example, with Te Poutama Tau data collection at the beginning and the end of the year) (again, because of a reduction in availability from 2007). He believed that the greater number of kaiako in total presented a greater range of individual issues and increased travel time because the wharekura were spread more widely. The facilitator (like the kaiako themselves) had other commitments, in particular, work in English-medium schools, which further limited the amount of facilitator time and energy available for wharekura.

Facilitator Characteristics

A key focus of this study was to investigate how a facilitator can promote kaiako networks in order to achieve effective pāngarau teaching and learning. The 2007 research reported facilitator characteristics identified by kaiako that made a positive difference to their participation in the wharekura Te Poutama Tau and their success in developing their content knowledge and teaching practice. The facilitator's view of the professional development in 2008 was that kaiako would be helped to build capacity to sustain momentum with Te Poutama Tau through kaiako networks, with support from the facilitator only as required. Reflections from both years' studies indicated that four key facilitator characteristics and behaviours helped develop kaiako networks. These were: first, reflecting ngāwaritanga (see below); secondly, reflecting whanaungatanga (authentic relationships); thirdly, increasing time spent with tumuaki on visits to wharekura; and finally, enhancing kaiako ownership of the professional development through adjustments to its design.

The term ngāwaritanga was coined by the Kupenga cluster in 2007 in describing the facilitator as someone who was ngāwari. Williams (1975) defines ngāwari as soft, supple, moving easily, quick, accommodating, kind, obedient (humble). When the suffix "tanga" is added to the adjective ngāwari, a word to describe a way of being is created. The 2007 descriptions (Te Maro et al., 2008) included facilitator characteristics such as being non-judgmental, meaning that kaiako could take risks without worrying about being wrong or belittled.

> I think he, he also, well for me, he empowers me to actually do what I'm doing, you know, and he doesn't make me feel like oh, you know, kōtiro me mahi koe i tēnei[13]. But actually, you know, ka whakanuia i ngā wā katoa ka kitea a ia[14]... and he likes maths. (Kaiako) (Te Maro et al., 2008, p. 32)

The same study describes a facilitator who is an active listener, making it clear that the kaiako is heard and understood, and who then presents possible pathways. The participants in both years talked about the facilitator as someone who cares about the students and about the wharekura. Being adaptable has also been a trait of the facilitator noted by kaiako, for example, overcoming the lack of relievers by staying and working with a class, being generous in the sharing of resources, and having a vision that is close to the vision of kura. These descriptors indicate that the facilitator was concerned about the students and their safety and development and was proactive rather than negative. Another aspect of ngāwaritanga was identified by a kaiako as pertaining to someone who is so knowledgeable about their subject that they are able to give control over to the kaiako, knowing when to participate and when to step back; they also have the knowledge to ask and answer the right questions. This trait of ngāwaritanga assists in building an environment of trust among the participants, which is necessary for developing networks. Many of these characteristics have also blended in with the next trait of whanaungatanga.

The characteristic of reflecting whanaungatanga is interpreted as setting up relationships as part of the community rather than as an external expert. The facilitator believed that he went into kura to assist with creating a pāngarau culture alongside the wharekura. The facilitator therefore needed to be aware of the individual wharekura cultures, which necessitated fostering authentic relationships (whanaungatanga). In practice, the facilitator needed to get to know the "players" in the kura, their beliefs about and attitudes towards pāngarau, their levels of confidence and competence in pāngarau, and their tolerance to "outsider" interference; therefore the first task was to listen and understand:

> I'm not going to go into a kura to tell them you must do this, do that, and you must do it by this time – this would have an effect of takahi[15] on the rangatiratanga[16] of the kura and individual kaiako. It is about building trust first. (Facilitator)

Increasing the time spent with tumuaki on each wharekura visit was a deliberate strategy by the facilitator, whose intention was to strengthen the connection of the tumuaki to the Te Poutama Tau professional development. The aim was to provide greater consistency of messages about priorities

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[13] Kōtiro me mahi koe i tēnei: girl, you should do this

[14] Ka whakanuia i ngā wā katoa ka kitea a ia: Every time you see him, he uplifts you.

[15] Takahi: tramping or stamping on

[16] Rangatiratanga: inherent right to self-govern and control

for kaiako, so that tumuaki would support what kaiako were doing in terms of resourcing, timing of events, release time for monitoring and assessment purposes, and the fit of NCEA.

Professional Development Design and Implementation

Through altering the professional development design to enhance kaiako ownership, the facilitator wanted to develop the participants' capacity to eventually take charge of their own professional development. The design of delivery was therefore altered to begin the process of transferring responsibility away from the facilitator to the kaiako, as mentioned earlier. The wharekura Te Poutama Tau elements were combined to enable kaiako to work together to evaluate their practice, problem solve issues, and collectively plan for pāngarau teaching, an opportunity not normally available to these kaiako, given their geographical distance from each other. An innovation planned for 2009 was to begin another cluster through the use of seeding strategies rather than the full wharekura Te Poutama Tau programme. In this scenario, kaiako would be involved in a year of preparation before they were expected to fully implement Te Poutama Tau in their classrooms.

The remainder of this section discusses the relative effectiveness of the three modes of delivery: hui; facilitator visits to wharekura; and Internet-based delivery. Kaiako questionnaires indicated that all of the professional development delivery modes were useful for developing content and pedagogical knowledge. Ranked from most useful to least useful, the modes were: hui; kanohi ki te kanohi visits (including video recordings of facilitator and kaiako practice and discussions based on the videos); and lastly, Internet delivery (wiki, then WiziQ). More detailed comments about each mode in terms of the development of kaiako networks follow.

The design, content, and implementation of the hui facilitated development of kaiako networks. Hui gradually moved from being facilitator-driven to being designed collaboratively by a whole group. This change of ownership involved aspects such as setting up group visits for shared pāngarau teaching experiences, kaiako planning the 2009 professional development, and making decisions about the content and timing of the hui.

The facilitator found that in-school visits helped develop kaiako networks through:

- facilitator discussions with the tumuaki to help develop wharekura support for individuals and their understanding of the development as a whole;
- maintaining continuity of the professional development within and across clusters;
- sharing information and news from visits to other wharekura;
- discussions of the videos, enabling kaiako to take personal responsibility for identifying and making changes to their practice;
- ensuring that there was consistency between messages from the facilitator and the shared understandings across the clusters.

The WiziQ and wiki were intended to help develop kaiako networks and facilitate shared ownership of the learning. They included features such as a collaborative "whiteboard" and the uploading of prepared documents and presentations. The intention of the Internet-based elements was to allow the facilitator to communicate with kaiako between hui (usually, to conduct online pāngarau-content workshops) and to allow kaiako to set up WiziQ hui independently, thereby enabling them to gradually take over the organisation of WiziQ hui and further establishing the kaiako networks. WiziQ allowed kaiako to set up or request instant hui as and when the need arose.

Most of the WiziQ sessions were planned during kanohi ki te kanohi hui, with the group deciding when each session would occur and what the focus would be. There were approximately nine sessions.

Attendance at sessions was patchy, with a reliable base of only two to three kaiako attending. All WiziQ sessions were archived, and kaiako unable to attend a WiziQ session were able to access the session; some did so. There was positive kaiako feedback about the usefulness of WiziQ for their professional development and its impact on developing networks. Three kaiako were consistent users of wiki and WiziQ. Some (for example, most of the Waiariki cluster) required assistance and support with the technology. Others were not convinced that it was the best way to engage with the professional development or with each other, or that they would be able to find times that all kaiako would be free or released (in part due to the lack of available relievers for their classes). For some kaiako, the equipment needed was not available. During the year, kaiako and the facilitator contributed lesson plans, schemes, and learning activities to the wiki space. The findings indicate that further work needs to be done to improve kaiako access to, and use of, the wiki and WiziQ tools. About nine WiziQ sessions, usually focused on pāngarau-content topics (for example, fractions, proportional reasoning, and algebra), were set up for each cluster during the year, giving kaiako the opportunity to discuss these topics online. The hope was that all the kaiako involved would set up their own hui eventually, without involvement from the facilitator. This would also lead to increased leadership responsibility within the group and the growth of sustainability through collaboration among them.

The facilitator and kaiako who were videoed saw the videos as a very effective method of effecting change in kaiako practice, with several instances of rapid change and modification of practice observed as a direct result of video observation and associated discussion. In the participants' view, the video component of the professional development provided a useful basis for professional discussion about practice within individual wharekura.

Te Reo Māori

The development of te reo Māori and te reo pāngarau was a focus area in 2008 in all modes of delivery. Kaiako identified their language growth and the need for continued development of both themselves and their students.

> Kia whanake tonu te reo pāngarau ki ngā taumata teitei o te pāngarau; kia whakatairanga ake i taku reo pāngarau, kia maringi noa mai te reo ki ngā tamariki hoki; kia puta ai ngā ākonga e matatau ana ki te reo, me te reo pāngarau – ahakoa tēhea reo, kua mārama[17].

The facilitator identified the need to spend more time on te reo pāngarau in 2009.

Building kaiako networks through te reo pāngarau occurred through kaiako from the kura having others to talk to who speak the same language. Kaiako mentioned that they were the only ones in their kura who talked about Te Poutama Tau and pāngarau at their levels. The hui allowed them to work with other kaiako speaking the same reo and to discuss and grow that reo with each other and the facilitator.

Student Data

The following is a selection of student achievement data from the year 9 and 10 cohort of the Waiariki and Kupenga clusters. There are 85 year 9 and 89 year 10 students represented. The data was collected at the beginning and end of the 2008 year. The facilitator analysed the patterns of achievement in terms of average stage gains. The graphs illustrate results in all domains. The numbers are small and should therefore be viewed with caution.

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17 "... to develop the mathematics register to the highest pinnacles of mathematics; to elevate my own mathematics language, so that it flows for the students as well; so that students leave who are knowledgeable in the mathematical register – no matter which language is used, they understand."

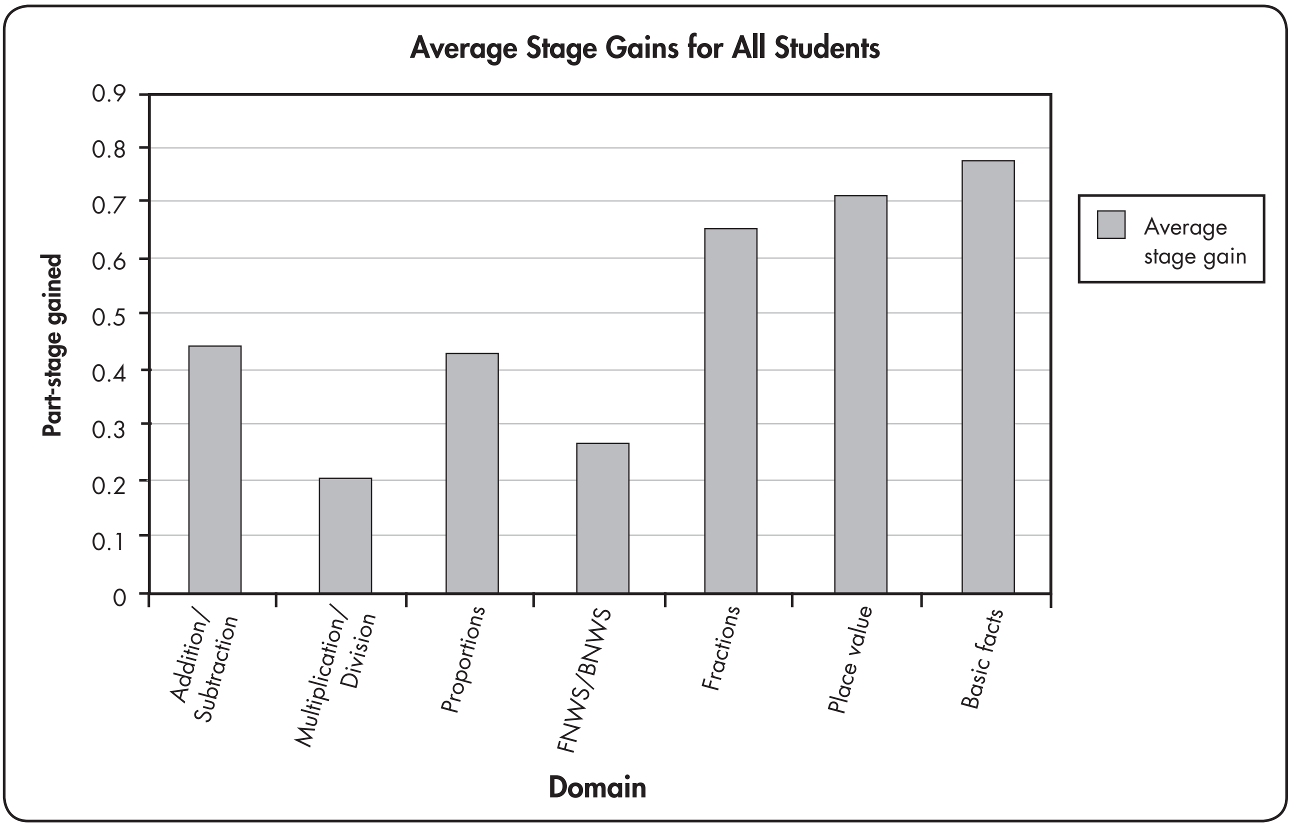

Figure 1. Data for average stage gains of all students

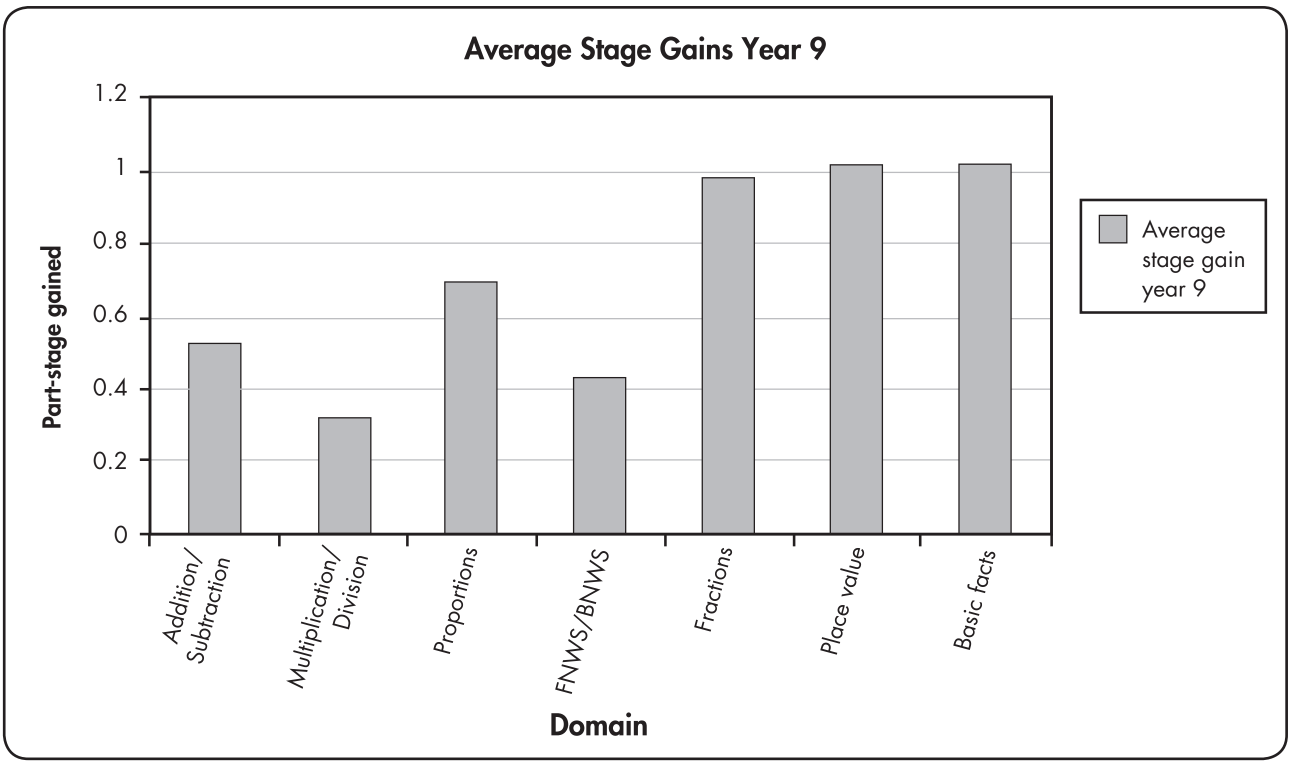

Figure 2. Average stage gains for year 9 students

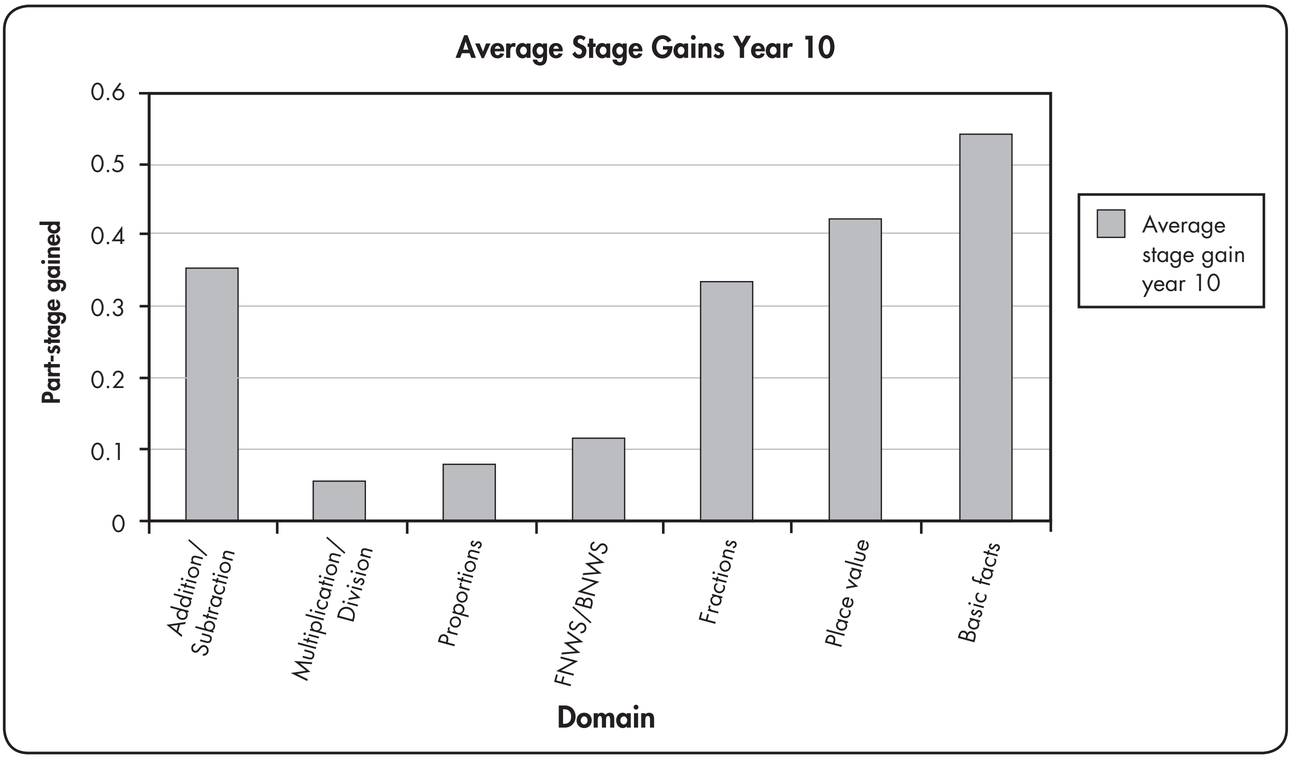

Figure 3. Average stage gains for year 10 students

The first graph (see Figure 1) shows the average stage gains across the two year groups. The stage gains have then been split for year 9 and year 10 to show the difference between the two years. The Waiariki kaiako had the opportunity to collaboratively view the data and make comment.

Waiariki kaiako commented that the range of data presented to them by the facilitator led to opportunities to discuss factors that were inhibiting their focus on Te Poutama Tau. The collaborative analysis of the results led to questioning what their next steps could be in discussion with the facilitator and other kaiako. By identifying the areas where the smallest stage gains were made, kaiako were able to establish future goals and establish priorities for student learning.

This type of collaborative work will continue in 2009 because shared discussion of data among kaiako can enhance planning, teaching, and learning. Undoubtedly, as kaiako better understand Te Poutama Tau, they will be better able to utilise their networks to discuss and collaboratively plan:

> Ko te pātai nui mō tātou hei matapaki ai, he aha ngā take i pērā ai ngā tamariki, he aha kāore ai te nuinga i piki?[18] (Kaiako)

Discussion and Conclusions

This paper reports on factors that have created benefits and constraints for kaiako progress in the wharekura Te Poutama Tau in 2008, including kaiako participation in professional development and in the development of kaiako networks. Supportive structures in kura management include a growing pool of support from kura teina. Kura structures that impact negatively include the transition of kaiako out of wharekura or out of the kura. NCEA was reported as having an impact on the way kura focus on pāngarau and the ways that teaching shifts away from a student-centred approach to an assessment-centred approach.

18 "The big question for us to discuss is what the reasons are for the results looking like this and why most of the students did not make higher gains."

Kaiako again reported that the facilitator continued to play a positive role in their engagement with Te Poutama Tau. The characteristics described by both clusters as positive to their progress remain important. The facilitator reported that having more kaiako and more or less the same hours to deliver meant less coverage, slower collection of data, and less opportunity at the end of the year to collaboratively analyse data in order to plan ahead. Te reo pāngarau and the development of a pāngarau register and common language for sharing pāngarau ideas is still a focus and will continue to be so.

The changes and delivery of the wharekura Te Poutama Tau in 2008 reflected insights gained from 2007, for example, video conferencing was replaced with WiziQ, the facilitator made more opportunities to talk with tumuaki of kura, and the Waiariki cluster took advantage of the proximity of a kura to practice the delivery of lessons and lesson progressions. The preferred mode of delivery remained kanohi ki te kanohi hui, with facilitator visits in individual kura as follow-up. The WiziQ hui had uptake by a small group. Others faced problems with using the technology and having the right tools. Finding a time that suited the kaiako in the cluster remained an issue. Video capturing kaiako practice was useful for those who used it because they were able to critique their teaching with the facilitator to inform future planning.

Alongside the results reported above, other themes emerged over the year in which the study was conducted. These insights are important to include here because they will help shape the next phase of the study. The establishment of a professional development programme dedicated to wharekura pāngarau is in itself a success. This has implications for pre-service training, specifically for kaiako wharekura. Through this initiative, kaiako, in enhancing their knowledge and practice, have also begun to develop collegial processes across wharekura, although undoubtedly there are more challenges ahead in continuing to create effective kaiako networks.

Facilitator and kaiako responses indicate that the wharekura Te Poutama Tau has shown that professional development can be successful within each wharekura as long as it recognises kaupapa Māori; that is, it must be consistent with the aims and aspirations of individual wharekura communities. This means that implementation is likely to vary from kura to kura. From these observations, it can be concluded that professional development for wharekura needs to be adaptive and able to redefine and reinvent itself to meet the needs of each individual kura and each individual kaiako, while still maintaining the integrity of the overall aim. A further conclusion consistent with te ao Māori[19] is that professional development needs to be long term, with the facilitator embedded as part of the whānau of the kura. This challenges notions of sustainability, in which the support for a school would be more likely to be based over a set time, for example, where facilitators are supposed to be in a school for two years with perhaps follow-up visits for sustainability. For wharekura, an alternative, more culturally responsive approach may be that the facilitator is never able to withdraw because, to all intents and purposes, they have become part of the staff and "tangata whenua"[20] in respect to the kura whānau. While the facilitator is part of the professional development, it makes no sense for them to withdraw from the wharekura because they themselves are an integral part of the sustainability of whanaungatanga. Kaiako commented that the facilitator is part of the whānau; whānau never leave – they may go away for a while, but there is a reciprocal relationship that continues in terms of nāu te rourou, nāku te rourou, ka ora te iwi[21]. This results in kura having facilitators who have attained the status of tangata whenua. In practice, this could mean that the facilitator continues to be present, as a colleague, who might not be there as often, but, as a whānau member, is on call when they are needed or wanted.

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19 Te ao Māori: pertaining to the Māori world/Māori world-view

20 Tangata whenua: those who are now part of the particular place because they have been welcomed and share belonging

21 Nāu te rourou, nāku te rourou, ka ora te iwi: with my food basket and your food basket, the iwi can be sustained

Recommendations

Further possibilities for enhancing the effectiveness of the professional development and further developing the kaiako networks include:

- continued facilitator interaction with both existing clusters into 2009;
- supporting kaiako use of Internet systems, and resource sharing and development;
- ensuring that all New Zealand's kura and schools are able to access the Internet by the funding of fibre-optic cabling to all kura and schools, which is a commitment to ensuring that kaiako have access to online kaiako networks quickly and without IT issues.

References

Bishop, R., & Glynn, T. (1999). *Culture counts: Changing power relations in education.* Palmerston North: Dunmore.

Cobb, P., & Smith, T. (2008). *Designing for improvement in mathematics teaching and learning at scale.* Paper presented at the AERA, New York, April.

Coburn, C. E., & Russell, J. L. (2008). District policy and teachers' social networks. *Educational Evaluation and Policy Analysis*, *30*(3), 203–235.

Cunningham, C. (1998). *A framework for addressing Māori knowledge in research, science and technology.* Keynote address to Te Oru Rangahau Māori Research and Development Conference, 7–9 July, Massey University.

Ministry of Education (2008). *Te Marautanga o Aotearoa.* Wellington: Ministry of Education.

Te Maro, P., Averill, R., & Higgins, J. (2008). Evaluation of support for pāngarau teachers working in wharekura. In *Findings of the Secondary Numeracy Project 2007.* Wellington: Learning Media.

Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2007). *Teacher professional learning and development: Best evidence synthesis iteration [BES].* Wellington: Ministry of Education.

Trinick, T., & Parangi, P. (2007). *Te Poutama Tau evaluation report.* Wellington: Ministry of Education.

Williams, H. W. (1975). *A Dictionary of the Māori Language.* Wellington: A. R. Shearer, Government Printer.

Student Learning and Understanding in the CAS Pilot Project

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Twenty-two schools were involved in a pilot scheme to evaluate the use of Computer Algebraic System (CAS) calculators. Teachers underwent professional development on effective ways to use CAS handheld calculators in their year 9 and year 10 mathematics classes and then integrated the calculators into their mathematics classrooms at these year levels. One of the aims of the CAS Pilot Project was to evaluate the effect on student mathematical skills and understanding. Student achievement data was collected using the new PAT[1]:Mathematics tests at the end of year 9 and again at the end of year 10. Achievement data was also collected from a control class in each school. The researchers were able to compare the changes in the levels of mathematical skills and understandings of CAS classes with those of control classes. The researchers were also able to collect qualitative data, both by interviewing teachers and small groups of students and by using questionnaires for teachers and students. The PAT:Mathematics achievement data showed that students in the CAS project progressed at about the same rate as the control class students. There were only minor, educationally insignificant differences between the calculator brands. The researchers' classroom observations and the teachers' responses showed strong changes in teaching style, with a move towards a more constructivist approach rather than a transmission style. Despite no evidence of progress in overall rates of achievement on the PAT:Mathematics results, both teachers and students reported a significant improvement in their understanding of algebra as a result of the CAS approach to teaching and learning.

Background

In 2005, the Ministry of Education and the New Zealand Qualifications Authority began a pilot scheme exploring the effective use of Computer Algebraic System (CAS) calculators in mathematics classrooms. Such use is consistent with an aspect of one of the key competencies in *The New Zealand Curriculum*, using language, symbols, and texts, which states that students should "confidently use ICT" (Ministry of Education, 2007, p. 12).

In 2005, six schools participated in the CAS Pilot Project, which in that year, involved two different brands of calculator. A further 16 schools participated in 2006, when a third brand of calculator was introduced. Each school used just the one brand of calculator. Fourteen schools used the first brand, while four schools each used one of the two other brands. Each brand had its own separate professional development (PD) provider, all with different emphases.

Each school was actively involved in the project for two years. In the first year, two teachers with a year 9 class from each school were given professional development (PD) aimed at using CAS calculators in year 9 algebra and geometry. In the second year, the two original teachers received further PD on using CAS calculators with year 10 classes while two more teachers were given the year 9 PD. The PD involved an introduction on how to operate CAS calculators, but it particularly focused on how to use the CAS calculator effectively for teaching and learning. The PD thus took account of the recommendation that "the use of CAS technology needs to be accompanied by the development of algebraic insight, including the ability to identify the structure and key features of expressions and to link representations" (Anthony & Walshaw, 2007, p. 139). Multiple representations were a feature of the CAS Pilot Project PD.

An evaluation of the 2005 CAS Pilot Project was undertaken by the New Zealand Council for Educational Research (NZCER) and their results published (Neill & Maguire, 2006a, b). The subsequent

1 Progressive Achievement Tests, NZCER

evaluation, reported on in part in this paper, looked at effects during 2006 and 2007 (Neill & Maguire, 2008).

This paper focuses on just one of the eight research questions of the full evaluation: "How has student learning in mathematics been affected as a result of the pilot scheme?"

Methodology

Twenty-two schools took part in the CAS Pilot Project, with 16 in the North Island and six in the South Island. There was a range of decile ratings among the schools selected, with an overall distribution close to the national distribution of deciles. All of the schools were state schools except for one integrated school. Twelve schools were in major population centres, seven in provincial cities, and three in smaller provincial towns. Seven were single-sex schools (four for girls and three for boys), and 15 were co-educational.

All teachers in the pilot were given a questionnaire, which asked, among other things, for their views on the effect of the CAS Pilot Project on students' mathematics learning and understanding. Teachers were also asked this question in face-to-face interviews.

Students were also asked, in end-of-year questionnaires and in small focus groups, about the effect that their involvement in the CAS Pilot Project might have had on their mathematics learning and understanding.

Student achievement was measured with the PAT:Mathematics scale score (patm). Students were given a baseline test of their mathematical ability at the beginning of year 9. Follow-up achievement tests were also given at the end of year 9 and at the end of year 10, using the relevant version of PAT:Mathematics.

Achievement assessed by different PAT tests can be mapped onto this patm scale score, which is a true interval scale. It measures growth in a linear way so that increases in the patm units are independent of the position of the student on the scale (Darr, Neill, & Stephanou, 2006). This means that an increase in 5 patm units represents the same amount of growth for a highly-able student as it does for a student of average ability. A new version of the test (Test 8) was constructed in conjunction with the PAT development schedule for use in this evaluation. Table 1 shows which PAT:Mathematics test was given to which students and when.

Table 1
Schedule of PAT:Mathematics Achievement Tests

	2006		2007	
	Term 1	Term 4	Term 1	Term 4
2005 cohort	Test 7	Test 8	–	*(NCEA Level 1)*
2006 cohort	Test 6	Test 7	–	Test 8
2007 cohort	–	–	Test 6	Test 7

Student Assessment Results

This section reports on the changes in mathematics achievement as measured by the patm scale score derived from the PAT:Mathematics tests. It is subdivided into results for the 2006 year 9 students, the 2007 year 9 students, and the students who were in year 10 in 2007. The latter group is the group of students who were in year 9 in 2006. As well as the results shown here, an analysis was also done to look for evidence of growth in achievement in each of the individual CAS and control classes in the study. This analysis showed that some schools and some classes made more significant progress than others, which indicates that teacher effects as well as school effects are likely to be statistically significant. The results below are those aggregated over all classes.

The 2006 Year 9 Students' Growth in Achievement

Control versus CAS class improvement for 2006 year 9 students

In 2006, the average score for students in year 9 control classes went up by 4.83 patm units compared with an increase of 4.03 patm units for year 9 CAS classes. The difference of 0.80 patm units is statistically significant at the 5% level ($p = 0.024$[2]).

These increases indicate that both the CAS and the control class students experienced growth on the patm scale of mathematical understanding that was close to the expected growth of about 5 patm units in a year. A number of different factors may have influenced these results, including the exclusion of all calculators from the PAT:Mathematics tests.

Student achievement and calculator type[3] for 2006 year 9 students

No significant differences were observed when comparing increases in the achievement levels of classes using the different types of CAS calculator. The mean increases were as follows:

Type A: 4.42 patm units; Type B: 3.53 patm units; Type C: 4.24 patm units.

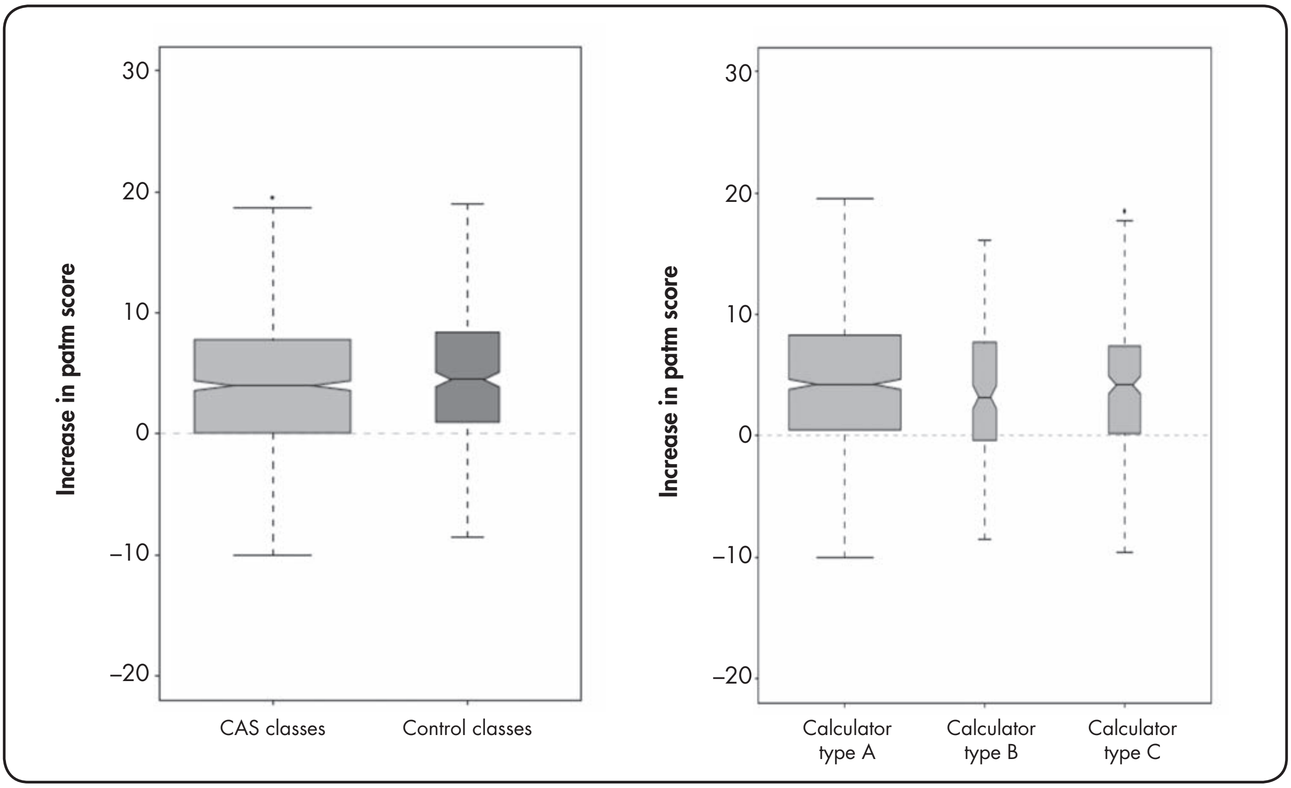


Figure 1. Gain in achievement measured in patm scale points – 2006 year 9 cohort

2 Probability of 0.024 that the two groups are not actually different, based on a T-test. This means that they are statistically significantly different at the 2.5% level.

3 "Type" in the context of this paper means one of the three brands of CAS calculators and its associated PD.

The 2007 Year 9 Students' Growth in Achievement

Control versus CAS class improvement for 2007 year 9 students

For the 2007 year 9 cohort, the average score of the control classes went up by 3.91 patm, units, compared with an increase of 4.53 units for CAS classes. The difference of 0.62 patm units is in the opposite direction to that of the 2006 cohort, but the difference is not statistically significant at the 5% level.

These increases indicate that the year 9 students in both the CAS and the control class experienced growth in their mathematical understanding (measured on the patm scale) that was close to the expected growth of about 5 patm units in a year.

Student achievement and calculator type for 2007 year 9 students

Only one small but significant difference was observed between the achievement levels of students in classes using their assigned type of CAS calculator. Classes using type A calculators showed a significantly greater amount of growth than those using type C calculators. This just reached significance at the 5% level ($p = 0.047$) and represents about one-fifth of a year's extra growth, which is small educationally. The mean increases were as follows:

Type A: 4.64 patm units; Type B: 4.17 patm units; Type C: 3.69 patm units.

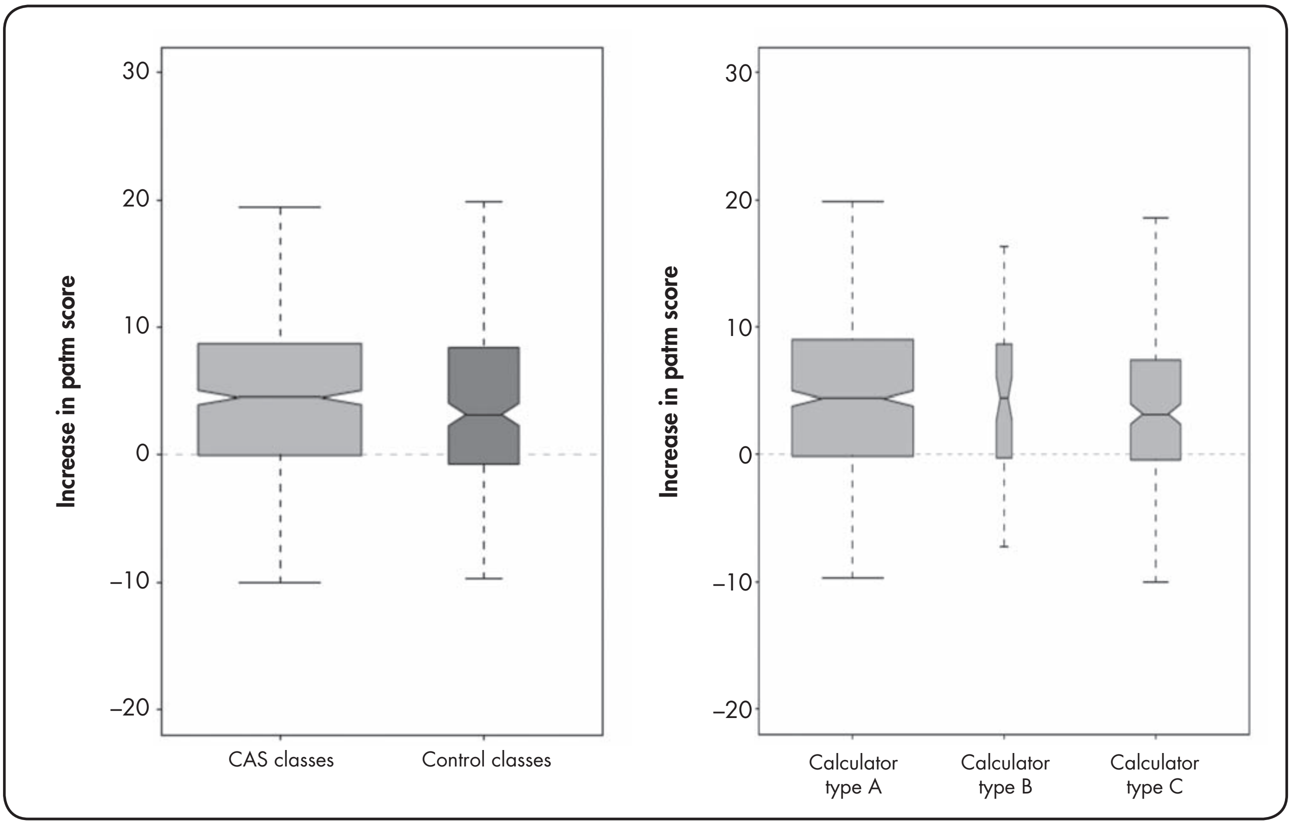


Figure 2. Gain in achievement measured in patm scale points – 2007 year 9 cohort

The 2007 Year 10 Students' Growth in Achievement

The year 10 students showed virtually no growth on the patm scale. This was measured by a new PAT test that has yet to be published. The new test seems to perform as it should psychometrically, and hence it is surprising that it measured no growth because there was uniform growth of about 5 patm units per year on the seven published tests. Further work is being undertaken to ascertain why this has happened. Notwithstanding this issue, the comparison of the change in mathematical understanding between CAS and control classes, or between the CAS calculator types, is still valid.

Control versus CAS class improvement for 2007 year 10 students

For the 2007 year 10 students, the average score of control classes went down by 0.55 patm units compared with an increase of 0.53 units for CAS classes. The difference of 1.08 patm units is in the opposite direction to that of the same students when they were in year 9 (in 2006). However, neither change is statistically significant at the 5% level, so growth over the two years is virtually identical for CAS and control classes.

Student achievement and calculator type for 2007 year 10 students

Year 10 students in classes using the CAS Type A calculator showed a small but significant growth in patm units, whereas those using CAS Type B or C calculators showed no significant growth.

Type A: 1.59 patm units; Type B: –0.26 patm units; Type C: 0.43 patm units.

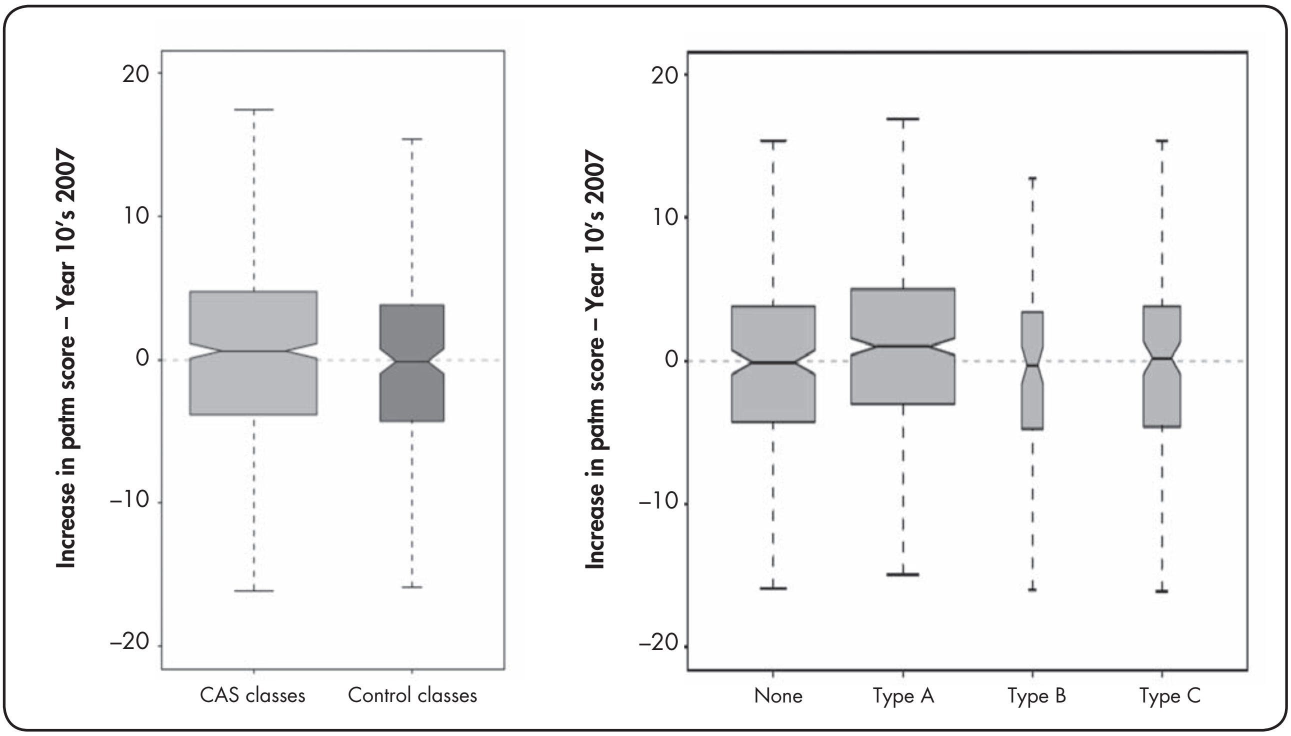


Figure 3. Gain in achievement measured in patm scale points – 2007 year 10 cohort

Overall Effects of the CAS Pilot Project

While there are slight differences between CAS and control-class students, these differences are not consistently in the same direction. The researchers therefore concluded that the two groups showed the same level of growth.

With regard to the CAS calculators, type A calculators show a small but significant advantage over types B and C in year 10 and a slight advantage over type C calculators in the 2007 year 9 cohort. This is about a 1 patm unit, which is barely educationally significant. The brand of calculator fully determined the PD that teachers received, so it is not possible to disentangle the brand effect from the PD effect (that is, they are fully confounded). It may also be that other background factors account for any differences. For these reasons, it would be inadvisable to single out any one calculator–PD mix as resulting in greater growth in mathematical understanding than the other two.

Students' Perception of Their Learning

Questionnaire Responses

All the CAS students were asked in a questionnaire to rate how they thought their own mathematical understanding had changed during the CAS Pilot Project. They were asked this separately in relation to both algebra and geometry. Figures 4 and 5 respectively show the pattern of the students' responses to these questions.

For algebra, in both the 2006 year 9 cohort and the 2007 year 9 cohort, far more students reported an increase rather than a decrease in their understanding as a result of using CAS. This can be seen in Figure 4, in which both distributions in the left-hand graph are positively skewed and virtually identical (chi-sq = 1.75, 4 d.f., N.S.),[4] so the mean response lies about half way between "agree" and "neutral". There was a similar, but less pronounced increase for geometry because the distributions in the left-hand graph of Figure 5 are also positively skewed, but to a lesser extent, and are not significantly different from each other (chi-sq = 6.95, 4 d.f., N.S.).

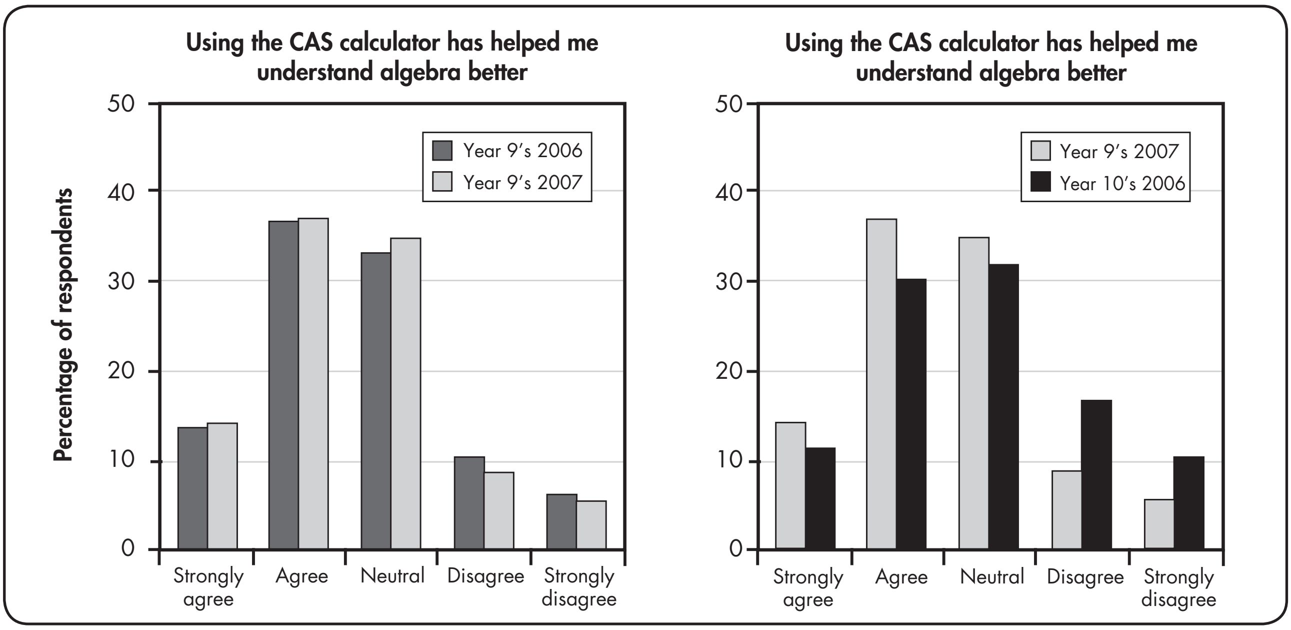


Figure 4. Effect on algebra understanding of CAS calculator use

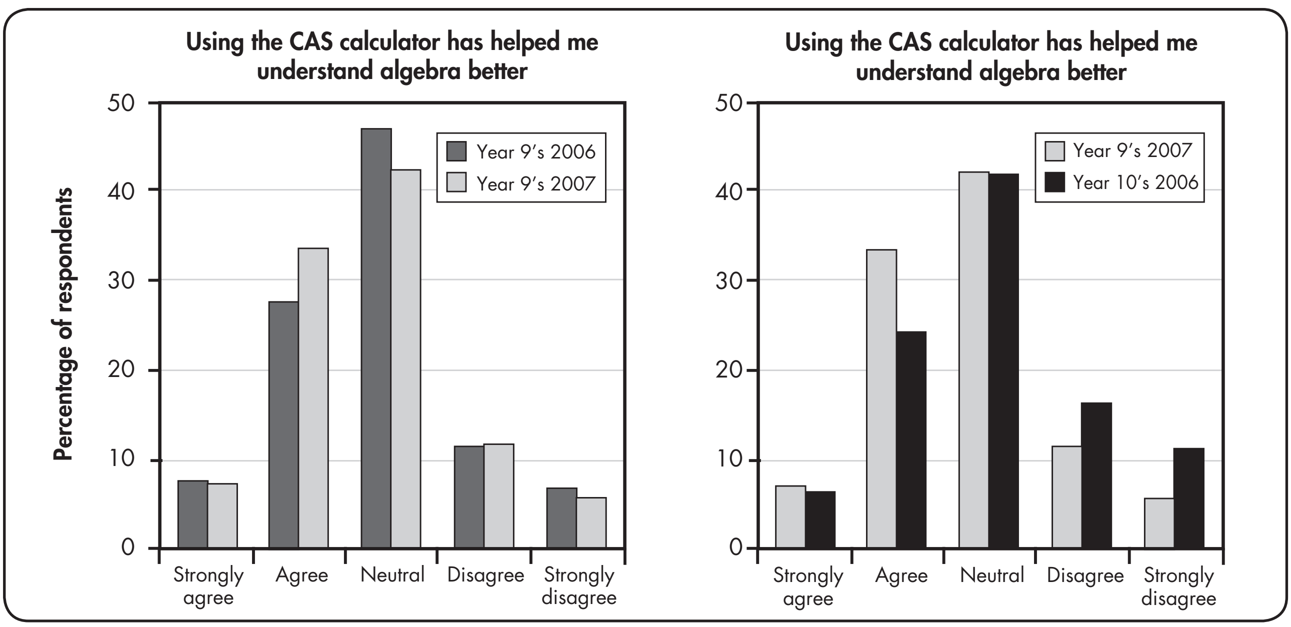


Figure 5. Effect on geometry understanding of CAS calculator use

4 Based on a chi-squared test, testing whether the two distributions were different, giving a value of 1.75 with 4 degrees of freedom (d.f.), which shows no significant (N.S.) difference between the two at the 5% level.

There is, however, a statistically significant difference between the responses of year 9 and year 10 students.

- For algebra, significantly more year 9 than year 10 students reported improved algebraic understanding following their involvement in the CAS Pilot Project, with fewer students disagreeing or strongly disagreeing that their understanding had improved.

  (Year 9 2006 vs year 10 2007: chi-sq = 29.66, 4 d.f., $p < 0.0001$)

  (Year 9 2007 vs year 10 2007: chi-sq = 34.76, 4 d.f., $p < 0.0001$)

  Overall, however, the year 10 students still reported a slight improvement (testing if the distribution for year 10s in Figure 4 is symmetric, chi-sq = 11.97, 4 d.f., $p = 0.0075$).

- For geometry, year 10 students did not report any significant improvement in their understanding (in testing if the distribution for year 10s in Figure 5 is symmetric, chi-sq = 0.70, 4 d.f., N.S.), even though many year 9 students did report improvement in their understanding.

  (2006 year 9 vs 2007 year 10 chi-sq = 27.30, 4 d.f., $p < 0.0001$)

  (2007 year 9 vs 2007 year 10 chi-sq = 22.27, 4 d.f., $p = 0.0002$)

Responses from Student Focus Groups

Students in the focus groups and individual students in classrooms were also asked if using the CAS calculator had helped them understand mathematics better, especially algebra. Students gave a variety of responses, both positive and negative.

Influence of visual displays

By far the most common reason that students gave for their improved mathematical understanding related to an individual student's learning preference. Several said the visual features of the CAS calculator had been a major positive influence on their understanding. This was stated directly (see the first two quotes below) or implied, as indicated by the latter two quotes, which show that the students did not like a pencil-and-paper approach:

> I can see pictures of graphs rather than seeing it in my head.
>
> [It] helps me understand it because it is visual.
>
> [It has helped on] how to work problems out. [I get] frustrated on paper.
>
> [It] helps do things I couldn't write down.

On the other hand, some students found that writing things down helped their understanding or their recall, as the following quotes show:

> [I'm] better off when writing things down, especially for exams.
>
> [It] helped a bit for understanding, but better to write it down.

This indicates that the technology has the potential to offer visual "advantages" (often referred to as "affordances" in the literature), although some students find these of more help than other students do.

Other positive influences

Students identified some other advantages of the CAS approach:

- Constructivist approach
  The teacher lets us find out how to find out the answer, so [we] get a better understanding.

- Technology feedback loops
  It helps. It showed you the step that was wrong.

- Reinforcement of ideas
  It has changed [for the better] because we go over it more often.

Other negative influences

Students also noted the following negative consequences of the CAS approach:

- Technology as a "black box"[5]
  I haven't really learnt how to do it myself.
  [It is] worse because I didn't have to work it out.
  I don't get how it gets the answer.
- Over-emphasis on technology
  [We are] focusing on what buttons to push rather than why.
- Difficulty with technology
  [Its] more confusing, [for example] how to make a formula.

Teacher Perception of Students' Learning and Understanding

Teachers were asked in both interviews and the questionnaire what they thought the effect of the CAS approach to teaching mathematics was having on student learning, skills, and understanding.

Results from the Follow-up Questionnaire

Figure 6 shows the questionnaire responses of 55 teachers when asked to what extent they agreed with statements about improvement in students' mathematical skills and understanding.

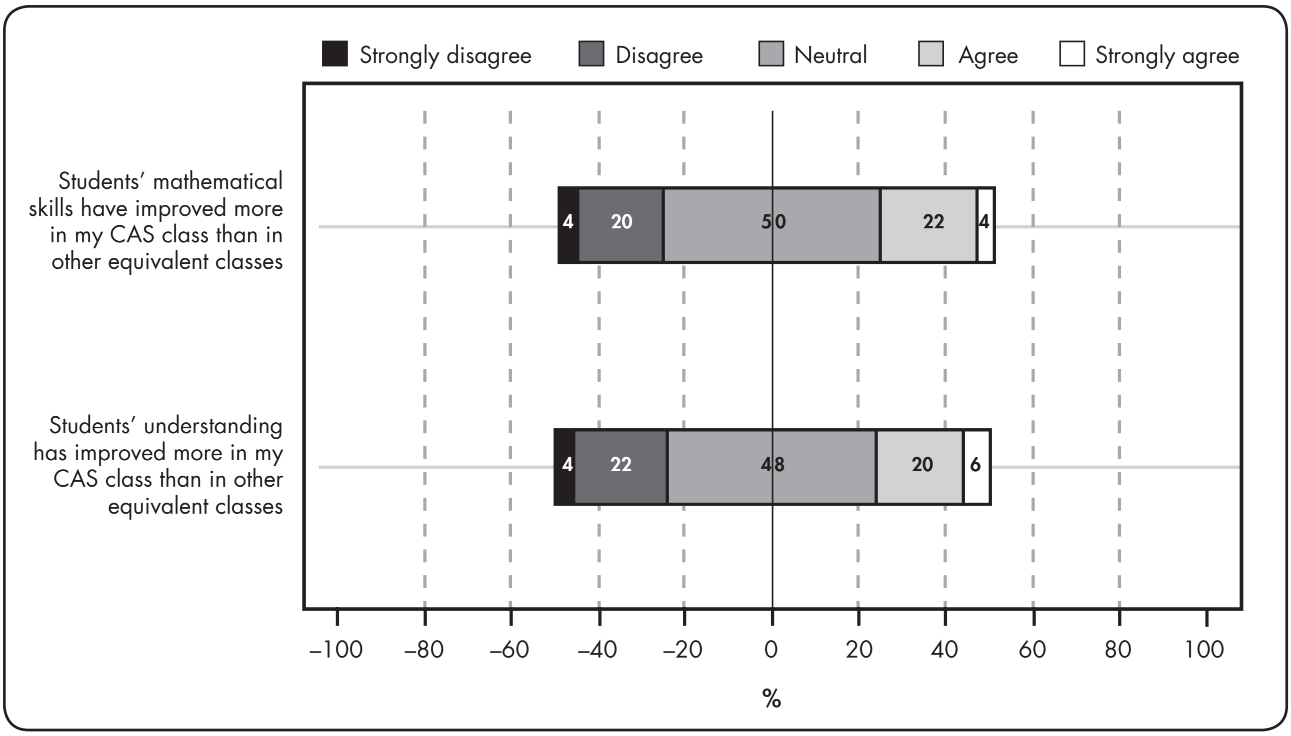


Figure 6. Teachers' views on effect of CAS on mathematical skills and understanding

5 A "black box" is an object for which a person may not know what's inside but they do need to evaluate what is good or bad for them about not knowing.

Figure 6 clearly shows that, overall, teachers have not perceived any significant change in students' mathematical skills (top bar) or in their understanding. For each point, about half of the teachers gave a neutral response and the others were equally distributed on either side of neutral, with very few strongly agreeing or disagreeing.

Only a small number of teachers elaborated on their response (with a total of 29 comments spread between both questions). The only comments to stand out were that: it was too early to say if either skills or understanding had changed (6 comments); there may be improvements in some areas, especially algebra (5 comments); and students gave up because of poorer attitudes or lack of confidence (4 comments).

Teacher Interview Responses

Teachers offered a range of views on what effect the CAS Pilot Project had on students' mathematics learning and understanding. Most comments from the interviews were either positive or neutral, though some were negative.

Positive effects on algebraic understanding

A number of teachers commented that they thought that there was a positive effect, especially in algebraic understanding:

> It has encouraged their participation. If they are doing more and discussing more, it must have an effect. There is much more discussion in class. The discussions that kids have in algebra make me think that algebra is not so foreign.
>
> Year 9s have picked up algebra better than any other class. Struggling students have benefited and higher-achieving students are inspired to "play" with calculators, exploring and extending themselves.
>
> There is a big difference, especially for students who said "I can't do algebra". [Students] are feeling more encouraged.
>
> It allows students to think. Students are getting understanding … but need reinforcement.

Some teachers thought that, while it was still too early to know, improved understanding and performances might not be observed until students were in the senior school:

> It may give benefits at year 12. It has real potential to do this. Students have the idea of a "variable" better. We don't see the spin-offs yet.
>
> [It] may make a difference at years 12 and 13 for the mathematically able.

Negative effects on algebraic understanding

A number of teachers commented on the potential negative effects that may eventuate from the CAS approach. The first concern was that students might become over-dependent on calculators or might undervalue being able to perform algebraic manipulation manually:

> I don't want students to be over-dependent on CAS calculators.
>
> [I am] concerned the kids will not value being able to do it by hand if the calculator can do it.

The other main concern was that the mathematical content might get lost in the details of how to actually use the calculator:

> With the animations, some were so focused on how to do it that the concept was lost. I found it better when students were watching on the screen.
>
> Kids get lost in the instructions.
>
> [I am] worried that in algebra – will [students] learn the right thing? What are we supposed to be doing? What sort of knowledge do they need? Maths skills versus calculator skills.

Effects on different student groups

Of the teachers who commented on the relative progress of different groups, most thought that the CAS approach had been most useful for students who had moderate or above average mathematical ability:

> I thought it was going nowhere, then it fell into place, especially [for] the better students.

However, some teachers thought that weaker students would not be helped by using the CAS approach:

> [It was] not effective on a weak class. [There were] too many who couldn't get the CAS working for them.

The above quote was from a teacher who was very comfortable and skilled with technology but who taught a class with lower mathematical ability and behavioural issues. That teacher had consequently reverted to using a traditional, less constructivist approach.

Some teachers thought that the CAS approach suited a different type of student:

> It is a different cohort excelling, but the same overall distribution. It requires a different skill set.

However, a comment from another teacher may explain some of the negative comments that previously successful students had made in student focus groups:

> [It is] hard to get students out of their comfort zones and approach maths in a completely different way that requires more problem solving and thinking. Initial hard work is starting to see progress.

Discussion

This paper shows that students who were involved in the CAS Pilot Project achieved about the same levels of increase in mathematical ability as students who did not take part in the pilot. The differing abilities of students are taken into account in this finding because the scale is linear, meaning that the measure of growth is constant at any point on the achievement scale.

This finding is consistent with findings from Heid, Blume, Hollebrands, and Piez (2002), who identified that in eight out of nine studies, students who had used CAS did at least as well as students who had not used CAS, even though the tests did not allow the use of CAS calculators. Heid et al. went on to cite studies that showed that students who used a CAS approach had the same or better conceptual understanding, and also better problem-solving skills, than students who had not used this approach.

The teachers involved in this study generally held the view that the CAS approach had a neutral effect on student mathematical skills and understanding, although some thought that the gains may well become evident at the senior school level. There was some mention that the teaching approach used in the CAS Pilot Project might lay a better foundation for developing mathematical understanding than a more traditional approach would. In their full evaluation, the researchers reported that there were significant shifts in classroom teaching practices as a result of the CAS Pilot project PD (Neill & Maguire, 2008).

On the other hand, many students taking part in this evaluation thought they had made some gains in their mathematical understanding, especially in algebra, with relatively few reporting lower levels of understanding. This did, of course, differ from student to student. It may well be that students who are visual learners will find that the graphic capabilities of the CAS calculators better suit their learning style. Other researchers have reported similar advantages related to visual displays (see, for example, Thomas, Bosley, Santos, et al., 2007).

The achievement data was collected longitudinally, so there is the potential to continue tracking the growth in student achievement through the three levels of NCEA mathematics results to explore the longer-term effects of the CAS Pilot Project.

Acknowledgments

The researchers would like to thank the schools, the teachers, and the students involved in this research for making them so welcome and allowing them to observe classroom teaching and conduct interviews and focus-group sessions. (Teresa Maguire has now left NZCER and has taken her expertise, especially in mathematics and early algebraic thinking, back into the classroom.)

The researchers would also like to thank: Hilary Feral for her support in data management, statistical analyses, and production of graphs; Rosemary Hipkins for her guidance during the research and for reviewing this paper; and Jonathan Fisher for his review.

References

Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in mathematics/pāngarau: Best evidence synthesis iteration [BES].* Wellington: Ministry of Education.

Darr, C., Neill, A., & Stephanou, A. (2006). *Progressive achievement test:Mathematics: Teacher manual.* Wellington: New Zealand Council for Educational Research.

Heid, K., Blume, G., Hollebrands, K., & Piez, C. (2002). Computer algebra systems in mathematics instruction: Implications from research. *Mathematics Teacher, 95*(8), 586–591.

Ministry of Education (2007). *The New Zealand Curriculum.* Wellington: Ministry of Education.

Neill, A., & Maguire, T. (2006a). *An evaluation of the CAS pilot project.* Wellington: Ministry of Education. Retrieved 8 April 2008 from:

www.educationcounts.govt.nz/__data/assets/pdf_file/0012/7032/CAS-Pilot-Project.pdf

Neill, A., & Maguire, T. (2006b). An evaluation of the CAS pilot project. *Evaluations of the 2005 Secondary Numeracy Pilot Project and the CAS pilot project* (pp. 69–83). Wellington: Learning Media.

Neill, A., & Maguire, T. (2008). *An evaluation of the CAS pilot project.* Wellington: Ministry of Education. Retrieved on 27 May 2009 from: www.educationcounts.govt.nz/publications/schooling/31083/31084

Thomas, M., Bosley, J., de los Santos, A., Gray, R., Hong, Y., & Loh, J. (2007). *Technology use and the teaching of technology in the secondary classroom.* Wellington: New Zealand Council for Educational Research.

Appendices (Findings From the Secondary Numeracy Project 2008)

Appendix A (Performance of SNP Students on the Number Framework)

Performance of Year 9 Students in First-year SNP Schools on the Strategy Domains

Table 14[1]

Performance of Year 9 Students in First-year SNP Schools on the Additive Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Initial									
0–3: Counting from One	2%	2%	4%	4%	2%	1%	2%	3%	2%
4: Advanced Counting	9%	17%	33%	26%	10%	10%	16%	15%	16%
5: Early Additive	38%	44%	41%	41%	43%	29%	37%	44%	40%
6: Advanced Additive	40%	32%	20%	26%	35%	47%	35%	32%	34%
7: Advanced Multiplicative	10%	5%	2%	4%	9%	13%	9%	6%	7%
8: Advanced Proportional	1%	1%		0%	1%	0%	1%	1%	1%
N =	1213	645	383	886	1268	314	1169	1299	2468
Final									
0–3: Counting from One	1%	1%	1%	1%	0%	0%	1%	1%	1%
4: Advanced Counting	4%	9%	15%	14%	3%	2%	8%	6%	7%
5: Early Additive	26%	33%	37%	32%	30%	25%	28%	32%	30%
6: Advanced Additive	41%	43%	40%	39%	43%	40%	41%	41%	41%
7: Advanced Multiplicative	23%	12%	6%	11%	18%	33%	19%	17%	18%
8: Advanced Proportional	5%	3%	1%	2%	5%	0%	4%	3%	4%
N =	1213	645	383	886	1268	314	1169	1299	2468

Table 15

Performance of Year 9 Students in First-year SNP Schools on the Multiplicative Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Initial									
0–3: Counting from One	1%	2%	6%	4%	1%	0%	2%	2%	2%
4: Advanced Counting	10%	19%	30%	25%	11%	6%	18%	14%	16%
5: Early Additive	24%	30%	31%	29%	27%	25%	23%	31%	27%
6: Advanced Additive	40%	36%	27%	31%	40%	31%	34%	37%	36%
7: Advanced Multiplicative	21%	13%	6%	8%	17%	30%	19%	13%	16%
8: Advanced Proportional	5%	1%	0%	2%	3%	9%	4%	2%	3%
N =	1213	645	383	886	1268	314	1169	1299	2468

[1] In all tables in Appendices A–D, rounded percentages are presented. Percentages less than 0.5% are therefore shown as 0%, and where there are no students represented, the cell is left blank. Due to rounding, percentages in some tables may not total to 100.

Table 15 – *continued*

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Final									
0–3: Counting from One	1%	1%	2%	2%	1%	1%	1%	1%	1%
4: Advanced Counting	4%	11%	14%	12%	5%	3%	8%	7%	7%
5: Early Additive	14%	20%	27%	23%	15%	14%	18%	18%	18%
6: Advanced Additive	33%	37%	33%	31%	38%	28%	31%	37%	34%
7: Advanced Multiplicative	36%	26%	22%	25%	34%	36%	33%	30%	31%
8: Advanced Proportional	12%	5%	3%	6%	8%	19%	11%	7%	9%
N =	1213	645	383	886	1268	314	1169	1299	2468

Table 16
Performance of Year 9 Students in First-year SNP Schools on the Proportional Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Initial									
0–1: Unequal Sharing	1%	2%	4%	3%	1%	0%	2%	2%	2%
2–4: Equal Sharing	14%	24%	40%	32%	17%	7%	24%	19%	21%
5: Early Additive	34%	40%	36%	38%	37%	22%	33%	38%	35%
6: Advanced Additive	16%	13%	8%	10%	15%	21%	13%	15%	14%
7: Advanced Multiplicative	31%	19%	10%	14%	25%	43%	24%	24%	24%
8: Advanced Proportional	5%	2%	1%	2%	4%	7%	4%	3%	4%
N =	1213	645	383	886	1268	314	1169	1299	2468
Final									
0–1: Unequal Sharing	0%	1%	2%	1%	0%	0%	1%	1%	1%
2–4: Equal Sharing	7%	15%	19%	16%	9%	4%	12%	10%	11%
5: Early Additive	21%	32%	38%	35%	24%	13%	27%	26%	27%
6: Advanced Additive	15%	17%	20%	18%	17%	15%	15%	19%	17%
7: Advanced Multiplicative	44%	31%	18%	23%	41%	49%	35%	37%	36%
8: Advanced Proportional	13%	5%	3%	6%	9%	19%	10%	8%	9%
N =	1213	645	383	886	1268	314	1169	1299	2468

Appendix B (Performance of SNP Students on the Number Framework)

Performance of Year 9 Students in First-year SNP Schools on the Knowledge Domains

Table 17

Performance of Year 9 Students in First-year SNP Schools on the FNWS Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Initial									
0–3: To 20	1%	2%	10%	6%	1%	0%	3%	2%	3%
4: To 100	2%	4%	12%	8%	2%	3%	5%	4%	4%
5: To 1000	41%	47%	51%	48%	44%	37%	39%	49%	44%
6: To 1 000 000	57%	47%	26%	38%	53%	60%	53%	44%	49%
N =	1213	645	383	886	1268	314	1169	1299	2468
Final									
0–3: To 20	0%	1%	3%	2%	0%	1%	1%	1%	1%
4: To 100	1%	3%	10%	6%	2%	1%	4%	2%	3%
5: To 1000	25%	36%	38%	35%	27%	26%	26%	33%	30%
6: To 1 000 000	74%	61%	49%	57%	71%	72%	69%	64%	66%
N =	1213	645	383	886	1268	314	1169	1299	2468

Table 18

Performance of Year 9 Students in First-year SNP Schools on the Fractions Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Initial									
0–3: Non-fractions	3%	6%	13%	9%	4%	1%	8%	4%	6%
4: Assigns unit fractions	7%	21%	23%	22%	10%	7%	15%	13%	14%
5: Orders unit fractions	48%	47%	41%	41%	50%	44%	42%	50%	46%
6: Co-ordinates num./denom.	23%	17%	17%	19%	22%	23%	20%	22%	21%
7: Equivalent fractions	14%	7%	5%	8%	12%	18%	13%	9%	11%
8: Orders fractions	4%	1%	0%	1%	2%	8%	3%	2%	2%
N =	1213	645	383	886	1268	314	1169	1299	2468
Final									
0–3: Non-fractions	1%	3%	5%	4%	1%	0%	3%	1%	2%
4: Assigns unit fractions	5%	10%	10%	10%	6%	3%	7%	7%	7%
5: Orders unit fractions	27%	36%	39%	32%	32%	22%	32%	30%	31%
6: Co-ordinates num./denom.	28%	24%	28%	29%	25%	29%	25%	29%	27%
7: Equivalent fractions	31%	23%	16%	21%	29%	35%	26%	28%	27%
8: Orders fractions	8%	4%	1%	4%	6%	12%	7%	5%	6%
N =	1213	645	383	886	1268	314	1169	1299	2468

Table 19
Performance of Year 9 Students in First-year SNP Schools on the Place Value Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Initial									
0–3: Counts in fives and ones	2%	2%	7%	5%	2%	1%	3%	3%	3%
4: 10s to 100, orders to 1000	11%	12%	20%	15%	13%	7%	14%	12%	13%
5: 10s to 1000, orders to 10 000	44%	55%	56%	54%	49%	33%	46%	52%	49%
6: 10s, 100s, 1000s, orders whole numbers	21%	20%	12%	17%	18%	28%	20%	18%	19%
7: Tenths in and orders decimals	14%	8%	4%	5%	13%	18%	12%	10%	11%
8: Tenths, hundredths, and thousandths	8%	2%	1%	2%	5%	13%	6%	5%	5%
N =	1213	645	383	886	1268	314	1169	1299	2468
Final									
0–3: Counts in fives and ones	1%	1%	2%	2%	1%	0%	2%	1%	1%
4: 10s to 100, orders to 1000	4%	7%	12%	9%	5%	5%	7%	6%	6%
5: 10s to 1000, orders to 10 000	29%	44%	45%	40%	36%	23%	35%	36%	36%
6: 10s, 100s, 1000s, orders whole numbers	26%	25%	23%	28%	23%	24%	24%	25%	25%
7: Tenths in and orders decimals	20%	13%	11%	12%	19%	24%	17%	17%	17%
8: Tenths, hundredths, and thousandths	20%	9%	6%	9%	17%	25%	16%	14%	15%
N =	1213	645	383	886	1268	314	1169	1299	2468

Table 20

Performance of Year 9 Students in First-year SNP Schools on the Basic Facts Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Initial									
0–3: Facts to 10	2%	4%	8%	6%	2%	2%	5%	2%	4%
4: Within 10, doubles and teens	7%	7%	12%	11%	7%	2%	9%	7%	8%
5: Addition, multiplication for 2, 5, 10	20%	26%	30%	29%	21%	15%	23%	23%	23%
6: Subtraction and multiplication	55%	57%	44%	47%	60%	50%	51%	57%	54%
7: Division	14%	6%	6%	7%	10%	20%	11%	9%	10%
8: Factors and multiples	3%	0%		0%	0%	11%	2%	1%	2%
N =	1213	645	383	886	1268	314	1169	1299	2468
Final									
0–3: Facts to 10	1%	1%	4%	3%	0%	0%	3%	1%	2%
4: Within 10, doubles and teens	4%	4%	5%	5%	4%	4%	6%	3%	4%
5: Addition, multiplication for 2, 5, 10	14%	19%	26%	23%	14%	13%	20%	14%	17%
6: Subtraction and multiplication	57%	57%	42%	47%	60%	54%	50%	59%	55%
7: Division	21%	17%	21%	20%	20%	26%	20%	21%	21%
8: Factors and multiples	2%	1%	2%	2%	2%	3%	3%	2%	2%
N =	1213	645	383	886	1268	314	1169	1299	2468

Appendix C (Performance of SNP Students on the Number Framework)

Performance of Students in Second-year SNP Schools on the Strategy Domains

Table 21
Performance of Students in Second-year SNP Schools on the Additive Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Year 9									
0–3: Counting from One	0%	0%	1%	0%	1%	0%	1%	0%	0%
4: Advanced Counting	4%	9%	13%	10%	5%	3%	5%	6%	6%
5: Early Additive	25%	36%	48%	39%	27%	26%	28%	30%	29%
6: Advanced Additive	42%	39%	27%	32%	40%	43%	42%	38%	39%
7: Advanced Multiplicative	20%	12%	7%	15%	19%	18%	19%	17%	18%
8: Advanced Proportional	9%	4%	4%	2%	9%	10%	6%	9%	8%
N =	1958	609	215	632	1719	739	1338	1752	3090
Year 10									
0–3: Counting from One	0%	1%	0%	1%	0%	0%	1%	0%	0%
4: Advanced Counting	3%	5%	13%	8%	5%	1%	6%	3%	4%
5: Early Additive	23%	41%	37%	34%	24%	28%	25%	29%	27%
6: Advanced Additive	36%	32%	30%	35%	39%	27%	37%	33%	34%
7: Advanced Multiplicative	22%	14%	15%	18%	22%	19%	19%	21%	20%
8: Advanced Proportional	15%	7%	4%	4%	9%	24%	13%	14%	14%
N =	1452	407	119	393	1061	828	862	1420	2282

Table 22
Performance of Students in Second-year SNP Schools on the Multiplicative Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Year 9									
0–3: Counting from One	1%	1%	1%	0%	1%	0%	1%	1%	1%
4: Advanced Counting	4%	8%	10%	7%	5%	3%	5%	5%	5%
5: Early Additive	16%	24%	27%	25%	16%	16%	18%	18%	18%
6: Advanced Additive	35%	39%	45%	39%	36%	35%	36%	37%	36%
7: Advanced Multiplicative	33%	22%	16%	21%	30%	36%	28%	31%	30%
8: Advanced Proportional	12%	7%	2%	8%	11%	9%	13%	8%	10%
N =	1958	609	215	632	1719	739	1338	1752	3090
Year 10									
0–3: Counting from One	0%	1%	1%	1%	1%	0%	1%	0%	1%
4: Advanced Counting	4%	4%	10%	8%	4%	5%	6%	4%	5%
5: Early Additive	12%	26%	20%	25%	14%	10%	13%	16%	15%
6: Advanced Additive	30%	33%	34%	30%	30%	31%	32%	29%	30%
7: Advanced Multiplicative	36%	25%	30%	25%	32%	42%	30%	37%	34%
8: Advanced Proportional	17%	10%	5%	12%	20%	12%	19%	14%	16%
N =	1452	407	119	393	1061	828	862	1420	2282

Table 23
Performance of Students in Second-year SNP Schools on the Proportional Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Year 9									
0–1: Unequal sharing	0%	0%	1%	0%	1%	0%	0%	0%	0%
2–4: Equal sharing	7%	11%	19%	13%	9%	6%	10%	8%	9%
5: Early Additive	22%	39%	36%	35%	25%	23%	26%	27%	27%
6: Advanced Additive	18%	20%	19%	18%	20%	16%	18%	20%	19%
7: Advanced Multiplicative	36%	23%	18%	26%	32%	38%	33%	31%	32%
8: Advanced Proportional	15%	7%	7%	8%	14%	17%	13%	13%	13%
N =	1958	609	215	632	1719	739	1338	1752	3090
Year 10									
0–1: Unequal sharing	0%	0%	1%	0%	0%	0%	0%	0%	0%
2–4: Equal sharing	5%	10%	14%	10%	8%	3%	9%	5%	7%
5: Early Additive	19%	36%	35%	33%	21%	21%	22%	24%	23%
6: Advanced Additive	14%	17%	20%	20%	16%	13%	15%	16%	16%
7: Advanced Multiplicative	38%	27%	19%	25%	38%	34%	34%	35%	34%
8: Advanced Proportional	23%	10%	10%	12%	17%	28%	20%	20%	20%
N =	1452	407	119	393	1061	828	862	1420	2282

Appendix D (Performance of SNP Students on the Number Framework)

Performance of Students in Second-year SNP schools on the Knowledge Domains

Table 24
Performance of Students in Second-year SNP Schools on the FNWS Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Year 9									
0–3: To 20	0%	1%	2%	1%	0%	0%	1%	0%	0%
4: To 100	1%	3%	4%	3%	2%	1%	2%	2%	2%
5: To 1000	21%	33%	47%	33%	24%	22%	23%	27%	25%
6: To 1 000 000	77%	63%	47%	63%	74%	78%	75%	71%	72%
N =	1958	609	215	632	1719	739	1338	1752	3090
Year 10									
0–3: To 20	0%	1%	0%	1%	0%	0%	0%	0%	0%
4: To 100	1%	3%	3%	4%	1%	0%	2%	1%	1%
5: To 1000	12%	21%	31%	25%	15%	9%	13%	16%	15%
6: To 1 000 000	87%	75%	66%	70%	84%	91%	85%	83%	84%
N =	1452	407	119	393	1061	828	862	1420	2282

Table 25
Performance of Students in Second-year SNP Schools on the Fractions Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Year 9									
0–3: Non-fractions	0%	1%	1%	1%	0%	0%	1%	0%	1%
4: Assigns unit fractions	5%	9%	12%	10%	6%	4%	7%	5%	6%
5: Orders unit fractions	28%	40%	33%	35%	32%	21%	33%	28%	30%
6: Co-ordinates num./denom.	24%	26%	28%	23%	25%	26%	21%	27%	25%
7: Equivalent fractions	31%	21%	21%	25%	27%	37%	28%	29%	29%
8: Orders fractions	11%	4%	5%	5%	10%	12%	10%	9%	9%
N =	1958	609	215	632	1719	739	1338	1752	3090
Year 10									
0–3: Non-fractions	0%	1%	0%	1%	0%	0%	1%	0%	0%
4: Assigns unit fractions	3%	8%	12%	11%	4%	2%	6%	3%	4%
5: Orders unit fractions	18%	35%	29%	33%	24%	10%	23%	20%	21%
6: Co-ordinates num./denom.	21%	24%	29%	27%	22%	21%	22%	23%	22%
7: Equivalent fractions	36%	26%	24%	22%	34%	35%	27%	36%	32%
8: Orders fractions	22%	6%	7%	6%	15%	32%	22%	19%	20%
N =	1452	407	119	393	1061	828	862	1420	2282

Table 26

Performance of Students in Second-year SNP Schools on the Place Value Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Year 9									
0–3: Counts in fives and ones	0%	2%	1%	2%	1%	0%	1%	1%	1%
4: 10s to 100, orders to 1000	4%	8%	10%	8%	5%	2%	5%	5%	5%
5: 10s to 1000, orders to 10 000	25%	35%	49%	42%	27%	22%	30%	28%	29%
6: 10s, 100s, 1000s, orders whole numbers	28%	31%	23%	24%	29%	27%	25%	30%	28%
7: Tenths in and orders decimals	24%	16%	11%	14%	20%	28%	22%	20%	21%
8: Tenths, hundredths, and thousandths	20%	8%	6%	10%	18%	21%	17%	16%	17%
N =	1958	609	215	632	1719	739	1338	1752	3090
Year 10									
0–3: Counts in fives and ones	0%	1%	0%	2%	0%	0%	1%	0%	0%
4: 10s to 100, orders to 1000	2%	7%	7%	9%	1%	2%	3%	3%	3%
5: 10s to 1000, orders to 10 000	13%	25%	27%	31%	16%	9%	19%	14%	16%
6: 10s, 100s, 1000s, orders whole numbers	27%	36%	34%	31%	32%	25%	27%	30%	29%
7: Tenths in and orders decimals	21%	16%	18%	14%	20%	22%	18%	21%	20%
8: Tenths, hundredths, and thousandths	37%	15%	14%	13%	31%	42%	32%	32%	32%
N =	1452	407	119	393	1061	828	862	1420	2282

Table 27

Performance of Students in Second-year SNP schools on the Basic Facts Domain

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Year 9									
0–3: Facts to 10	1%	1%	2%	1%	1%	0%	1%	1%	1%
4: Within 10, doubles and teens	2%	5%	3%	6%	3%	1%	4%	2%	3%
5: Addition, multiplication for 2, 5, 10	13%	18%	16%	17%	14%	9%	15%	12%	14%
6: Subtraction and multiplication	37%	45%	56%	48%	40%	35%	44%	38%	40%
7: Division	43%	30%	22%	25%	37%	52%	32%	43%	39%
8: Factors and multiples	5%	2%	1%	2%	5%	3%	3%	4%	4%
N =	1958	609	215	632	1719	739	1338	1752	3090

Table 27 – *continued*

	Ethnicity			Decile Group			Gender		
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	Total
Year 10									
0–3: Facts to 10	0%	1%	2%	3%	0%	0%	1%	0%	1%
4: Within 10, doubles and teens	1%	3%		2%	2%	0%	2%	1%	1%
5: Addition, multiplication for 2, 5, 10	9%	18%	18%	19%	11%	6%	12%	10%	10%
6: Subtraction and multiplication	29%	40%	50%	50%	35%	18%	39%	27%	32%
7: Division	52%	36%	29%	24%	44%	66%	38%	55%	49%
8: Factors and multiples	9%	2%	2%	1%	7%	10%	8%	7%	7%
N =	1452	407	119	393	1061	828	862	1420	2282

Appendix E (Written and Oral Assessments of Secondary Students' Number Strategies: Ongoing Development of a Written Assessment Tool)

The WSSAT Instrument – Pilot Version

Place Value, Fractions, Decimals, and Percentages

Write only on the answer sheet provided. Do not write on these sheets. In multiple choice questions, write down the correct letter only. **Do not use a calculator.**

Part A

Strategy

1. 87 + 99 =

2. 200 – 99 =

3. 7 tens + 20 =

4. Which number is the biggest?

   a. 70 b. eight tens c. 79 d. 6 tens + 9 ones

5. Subtract one hundred from 3602.

6. Multiply 234 by ten.

7. 167 is nearest to

   a. 160 b. 165 c. 170 d. 175

Knowledge

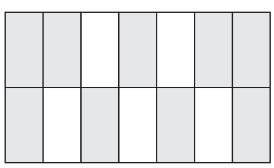
8. In the number 6**3**5 the **3** stands for **3** TENS.

   In the number 6**3** 200 the **3** stands for **3**

9. In the number 545.**8**09 the **8** stands for **8**

10. Write sixty thousand and eighty two as a number.

11. The shading in the diagram shows which fraction?



   a. 9 fifths b. $\frac{14}{9}$ c. $\frac{5}{14}$ d. 9 fourteenths

Part B

Strategy

12. 198 + $\boxed{}$ = 334. What is the number that goes in the box?

13. $\boxed{}$ – 99 = 163. What is the number that goes in the box?

14. 341 – $\boxed{}$ = 299. What is the number that goes in the box?

15. 1 whole – 17 twentieths of a whole equals:

    a. $\frac{20}{3}$ b. $\frac{20}{17}$ c. $\frac{17}{20}$ d. $\frac{3}{20}$

16. What is 20% of 500? $\boxed{}$

17. Add one tenth to 4.273 $\boxed{}$

18. You have $3784 in your savings account that you withdraw in ten dollar notes, and coins. What is the maximum number of ten dollar notes that you can get?

    a. 370 b. 379 c. 378 d. 380

Knowledge

19. What is $\frac{1}{4}$ as a percentage?

20. Thirty percent equals:

    a. 3 halves b. 3 quarters c. 3 fifths d. 3 tenths

21. Write down the correct letter: The largest number is :

    a. 1 third b. 1 quarter c. 1 fifth d. 1 sixth

22. Write down the correct letter: The largest number is:

    a. $\frac{1}{8}$ b. $\frac{1}{7}$ c. $\frac{1}{6}$ d. $\frac{1}{5}$

23. Forty six + fourteen = $\boxed{}$ tens. What is the number that goes in the box?

    a. 5 b. 6 c. 7 d. 8

Part C

Strategy

24. 59.9 x 79.98 is closest to:

    a. 3500 b. 4200 c. 4800 d. 6000

25. What does 98 x 5 equal?

26. Which has the largest answer?

    a. 118 x 117 b. 116 x 118 c. 117 x 119 d. 120 x 101

27. Write down the *two* correct letters:
    Which two numbers multiplied together give an answer closest to the target number 1140?

    a. 13 b. 29 c. 49 d. 39

28. Work out 5 sixths of 42.

29. Which of these is *not* a factor of 336?

    a. 2 b. 5 c. 4 d. 3

30. On a number line, $\frac{48}{7}$ lies between which two numbers?

    a. 5 and 6 b. 6 and 7 c. 7 and 8 d. 8 and 9

Knowledge

31. Round 23.694467 correct to 3 decimal places.

32. Multiply 23.489 by one hundred.

33. Write down any number that is between 777.58 and 778.44 on the number line.

34. The cost of a large bag of rice weighing 12.5 kilograms is $10.95. Which calculation is needed to find the cost of the price of one kilogram of rice?

    a. 10.95 ÷ 12.5 b. 12.5 x 10.95 c. 12.5 ÷ 10.95 d. 12.5 – 10.95

Part D

Strategy

35. 698 ÷ 0.098 is closest to:

a. 70 b. 700 c. 7000 d. 70 000

36. Work out 5 sixtieths of three thousand six hundred.

37. What does 9998 x 5 equal?

38. The percentage profit of $449 999 on sales of $1 999 000 is nearest to:

a. 10% b. 15% c. 20% d. 25%

39. 224% of $4012 is nearest to:

a. $10 000 b. $12 000 c. $14 000 d. $16 000

40. Murray gets a 20% discount on a TV. When he gets home, he sees he paid $200 for it with his cash card. What was the price of the TV before the discount was taken off?

a. $160 b. $220 c. $240 d. $250

41. The share ratio for the two owners of a company, John and Jane, is 240 000 : 360 000. The company's profit is $499 000. John's share of the profit is nearest to:

a. $240 000 b. $200 000 c. $160 000 d. $120 000

Knowledge

42. Write down any number that is between 23.58 and 23.5801 on the number line.

43. Eleven thousandths equals:

a. 0.0011 b. 0.011 c. 0.11 d. 11 000

44. Which of these numbers is the largest?

a. $\frac{30}{39}$ b. $\frac{301}{499}$ c. 68% d. 0.595556

Appendix F (Written and Oral Assessments of Secondary Students' Number Strategies: Ongoing Development of a Written Assessment Tool)

The Oral Assessment Instrument Questions and Criteria

Section A

1.*[5] What is 29 + 7?

2) You have $99, and your grandfather gives you a birthday present of $55. How much money do you have altogether?

3.* You have $47, and you spend $9. How much money do you have left?

Stage: ↓5 5– 5 5+

Section B

4.* There are 63 people on the bus. 39 people get off. How many people are left on the bus?

5. Sandra has collected 399 stamps. She gets another 199 stamps from her brother for her birthday present. How many stamps does she have now?

6. Work out 1 quarter of 40.

Stage: ↓6 6– 6 6+

Section C

7. Work out 97 x 3.

8. What is 3 fifths of 35?

9. Which one of these numbers is the largest?

0.184 14 hundredths 3 tenths

10. What is 20% of $50?

Stage: ↓7 7– 7 7+

Section D

11.* It takes 12 kiwifruit and 8 apples to make a fruit salad. Trevor only has 9 kiwifruit. How many apples does he need to make his salad?

12.* There are 23 boys and 27 girls at a school camp. What percentage are girls?

13.* 12 is 2 thirds of a number. What is the number?

14. 40% of a class eat 20 apples. How many apples does the whole class eat?

Stage: ↓8 8– 8 8+

Overall stage assigned: 1–4 5– 5 5+ 6– 6 6+ 7– 7 7+ 8– 8 8+

Criteria: Previous stage if none correct; x–[6] or beginning of stage if 1 out of 3 or 2 out of 4 correct; x or middle of stage if 2 out of 3 or 3 out of 4 correct, and x+ or later stage if 3 out of 3 or 4 out of 4 correct. For example, 6– indicates someone at the beginning of stage 6.

[5] The asterixed items are drawn directly from NumPA. The other items are all new, except: item 5 is similar to, but numbers are easier than in, the NumPA; item 8 is the same as NumPA except for using "of" rather than x (times) and with changed numbers. Item 6 uses the word "quarter" rather than the symbol form.

[6] x = stage

Appendix G (Senior Secondary Numeracy Practices in Successful Schools)

Questionnaire

Note: Spaces for responses and comments have been reduced in this version.
(The original questionnaire could be completed on paper or electronically.
The format for this was adapted for this version.)

Questionnaire for Management Unit Holders, In-school Facilitators, and Mathematics Teachers

This questionnaire is part of a research project about teaching mathematics in primary and secondary schools. This part of the study focuses specifically on year 11 mathematics teaching and learning.

Your comments are very useful for us. We do realise that you are busy, so we understand you may prefer to provide comments only in places where clarification of your ideas will be most useful for us rather than in every comment box. Please continue on an extra sheet if necessary.

1. I am: (tick all that apply to you)
   - ☐ part of the school senior management team
   - ☐ Head of Mathematics Department
   - ☐ holder of management units in mathematics (but not head of department)
   - ☐ in-school facilitator
   - ☐ teacher of year 11 mathematics

2. Number of years I have taught secondary school mathematics: _________ years

3. Number of years I have been involved in the SNP: _________ years

4. Number of years my school has been involved in the SNP: _________ years

5. (If you teach more than one year 11 class, please select one of the year 11 classes for the purposes of this survey.)
   - ☐ I do not teach a year 11 mathematics class this year.
   - ☐ My year 11 class is mainly assessed using achievement standards.
   - ☐ My year 11 class is mainly assessed using unit standards.

6. Number of students enrolled in the class: _________ students

7. My class has a:
   - ☐ wide range of ability
   - ☐ narrow range of ability

   Add a brief description to clarify (if required).

8. Describe the strategy that you find most effective for teaching year 11 mathematics. Give brief reasons why this strategy is effective.

9. Indicate how frequently you use the following practices with your year 11 mathematics class. (Space was allowed for comment, where appropriate.)

	Most lessons	Several times per week	Several times per month	Several times per term	Several times per year	Seldom or never
Students discuss ideas with other students sitting near them						
Students present ideas to the class						
Students are involved in investigations/problem solving						
Students work from textbook or worksheets						
Games						
Practical work						
Students use computers						
Student work marked by me						
Homework set						

10. Describe any other practices that make your mathematics teaching at year 11 effective.

11. What is your current understanding of the aims of the SNP?

12. How have your understandings of the aims of the SNP changed since you began?

13. Please comment on your use at year 11 of the components of the SNP:

Number Framework
Diagnostic interview
Teaching model
Student data.

14. The following are practices that some teachers have adopted or enhanced as a result of their involvement in the SNP.

Please circle one of the numbers from 1–5 to indicate the degree to which you have *changed your practice* since participating in the SNP professional development.

Under each section, add comments about the impact of the SNP on your teaching of this year 11 class (where appropriate).

1 indicates no significant change of practice.
5 indicates a very significant change of practice.

Increased use of group work

Impact on teaching in years 9 and 10					Impact on teaching in year 11				
1	2	3	4	5	1	2	3	4	5

Students sharing strategies with the class

Impact on teaching in years 9 and 10					Impact on teaching in year 11				
1	2	3	4	5	1	2	3	4	5

Use of the teaching model (materials → imaging → abstraction)

Impact on teaching in years 9 and 10					Impact on teaching in year 11				
1	2	3	4	5	1	2	3	4	5

Differentiated teaching (e.g., different work for different students)

Impact on teaching in years 9 and 10					Impact on teaching in year 11				
1	2	3	4	5	1	2	3	4	5

Use of grouping by strategy stage

Impact on teaching in years 9 and 10					Impact on teaching in year 11				
1	2	3	4	5	1	2	3	4	5

Greater emphasis on understanding of key ideas

Impact on teaching in years 9 and 10					Impact on teaching in year 11				
1	2	3	4	5	1	2	3	4	5

Sharing learning intentions with students at the start of teaching

Impact on teaching in years 9 and 10					Impact on teaching in year 11				
1	2	3	4	5	1	2	3	4	5

Review and reflection at the end of the lesson

Impact on teaching in years 9 and 10					Impact on teaching in year 11				
1	2	3	4	5	1	2	3	4	5

Real-world examples used in teaching

Impact on teaching in years 9 and 10					Impact on teaching in year 11				
1	2	3	4	5	1	2	3	4	5

15. Explain any differences in the ways that you use calculators with your **year 11 students** resulting from your participation in the SNP.

16. What are the most significant changes that you have made to your teaching of **year 11 students** resulting from your participation in the SNP?

17. What additional changes would you like to make to your teaching of year 11 mathematics students?

18. What has been the impact on students' achievement in year 11 as a result of the school's participation in the SNP? (tick appropriate box)

Little positive impact				Very positive impact
1	2	3	4	5

Comment on the impact of the SNP on achievement:

19. Comment on the appropriateness of the mathematics level 1 achievement standards and unit standards for assessing learning supported by the SNP.

20. Describe any departmental practices in your school that have supported your development in teaching year 11 mathematics.

21. What are the biggest challenges in implementing the SNP? Please give reasons.

22. What is necessary to support your implementation of the SNP? Please give reasons.

23. How do you plan to continue developing your year 11 mathematics teaching over the next five years?

24. In what ways has school management or school policy impacted on your ongoing learning in the SNP?

25. Are there any other comments you would like to add, for example, suggestions for improvement?

Thank you very much for your assistance in completing this questionnaire.

Appendix H (Senior Secondary Numeracy Practices in Successful Schools)

Interview Questions

Questions for Mathematics Teachers, HoDs, and ISFs

Describe teaching practices that are effective for enhancing engagement, understanding, and/or achievement with year 11 students.

Describe any practices that you use with your year 11 mathematics class(es) as a result of the SNP professional development.

How effective are they (for enhancing engagement, understanding, and/or achievement)?

Are there any practices that you have experienced as part of the SNP professional development that you do not use with your year 11 class(es)? Why?

Overall, how would your rate the impact of the SNP on your teaching?

What about the impact on your teaching of year 11 classes?

Describe any impact of the SNP on student achievement.

Are there any tensions between the current assessments for mathematics and the SNP professional development?

Describe any mathematics department practices that support the effectiveness of your teaching of the year 11 class.

Describe any school-wide factors that support the effectiveness of teaching and learning in the year 11 mathematics class.

What sort of professional development would be most useful for enhancing your teaching of year 11 mathematics?

Can you answer that question in terms of professional development for all secondary mathematics teachers?

Any other thoughts or comments?

Questions for Regional Facilitators

What changes have you noticed in the way the SNP is used by those more experienced in its use?

What are the challenges for implementation and improvement of practice at year 11?

Describe teaching practices that are most easily/readily adopted with year 11.

What differences in achievement do you know of through schools' involvement in the SNP?

What influences on SNP implementation are there from other professional development that schools may be involved with? In your experience, do other professional development projects (e.g., Te Kotahitanga) sit well alongside the SNP? If so, which elements of the SNP are helped most by teachers' involvement in other professional development?

What differences in student attitude at year 11 have you experienced through their school's involvement in the SNP?

Which SNP practices have been most readily picked up at year 11? Which are less well or less often implemented at year 11?

Describe any impact of the SNP on student achievement.

Are there any tensions between the current assessments for mathematics and the SNP professional development?

Describe any mathematics department practices that support the effective teaching of the year 11 class.

Describe any school-wide factors that support the effectiveness of teaching and learning in the year 11 mathematics class.

What sort of professional development would be most useful for enhancing teaching of year 11 mathematics?

Have you noticed any differences in teacher satisfaction at year 11 for those involved in the SNP?

Have you noticed any impact of the SNP on teacher–student relationships?

Any other thoughts or comments?

Appendix I (Senior Secondary Numeracy Practices in Successful Schools)

Teacher Results Regarding the Impact On their Teaching of SNP-consistent Practices

Table 7

Reported Impact of the SNP (Increased Use of Group Work) on Teacher Practice

Increased use of group work	Impact on teaching				
	1	**2**	**3**	**4**	**5**
	Little impact				Major impact
Years 9 and 10	3	1	7	11	1
Year 11 unit standards	0	4	3	1	0
Year 11 achievement standards	3	6	4	2	0

Table 8

Reported Impact of the SNP (Students Sharing Strategies with the Class) on Teacher Practice

Students sharing strategies with the class	Impact on teaching				
	1	**2**	**3**	**4**	**5**
	Little impact				Major impact
Years 9 and 10	0	2	7	8	6
Year 11 unit standards	0	2	2	2	2
Year 11 achievement standards	1	6	5	3	0

Table 9

Reported Impact of the SNP (Use of Grouping by Strategy Stage) on Teacher Practice

Use of grouping by stretegy stage	Impact on teaching				
	1	**2**	**3**	**4**	**5**
	Little impact				Major impact
Years 9 and 10	3	7	7	6	0
Year 11 unit standards	1	4	2	1	0
Year 11 achievement standards	6	5	4	0	0

Table 10
Reported Impact of the SNP (Sharing Learning Intentions with Students) on Teacher Practice

Sharing learning intentions with students at the start of teaching	Impact on teaching				
	1	**2**	**3**	**4**	**5**
	Little impact				Major impact
Years 9 and 10**	0	1	6	8	6
Year 11 unit standards*	0	0	2	1	4
Year 11 achievement standards*	0	2	4	5	**3**

Note: * denotes 1 non-response, ** denotes two non-responses

Table 11
Reported Impact of the SNP (Review and Reflection at the End of the Lesson) on Teacher Practice

Review and reflection at the end of the lesson	Impact on teaching				
	1	**2**	**3**	**4**	**5**
	Little impact				Major impact
Years 9 and 10*	2	4	6	9	1
Year 11 unit standards*	0	2	3	1	1
Year 11 achievement standards	2	4	3	6	0

Note: * denotes 1 non-response

Appendix J (Fostering the Growth of Teacher Networks within Professional Development: Kaiako Wharekura Working in Pāngarau)

Sample Questionnaire

The Wharekura Project 2008

Questionnaire for Kaiako (October/November)

Note: Spaces for responses have been deleted in this compendium version.

E ngā maunga
E ngā waka
E ngā kārangaranga maha huri noa o te motu
Tēnā rā koutou katoa.
Tēnei te mihi kau atu ana ki a koutou i runga anō i ngā pūtaketanga e here nei i a tātou ki a tātou.
Arā i tēnei wā, ko ngā rangatahi ērā; ko rātou e whai wāhi ana i roto i tēnei ao hurihuri.
Arā ko koutou anō ērā, e whakapau kaha nei ki te poipoi, ki te ārahi i a rātou ko ngā rangatahi kia ngāwari ai te ara e takahi nei rātou hei oranga ake.
Nō reira, tēnā koutou, tēnā koutou, otirā tēnā rā tātou katoa.

1. What is your current understanding of the aims of the wharekura Te Poutama Tau project?
2. How have your understandings of the aims of the project changed since you began?
3. What is your current understanding of how maths should be taught to students in wharekura?
4. What are your long-term goals for teaching maths in wharekura?
5. Please comment on your use of the components of Te Poutama Tau:
   Frameworks
   Diagnostic Interview
   Teaching Model
   Student data.
6. What do you perceive your needs to be:
   (a) for your mathematics content knowledge?
   (b) for your mathematics teaching strategies?
   (c) for your te reo pāngarau?
7. This programme includes hui, wiki, WiziQ, and facilitator visits. What do you think is the most useful aspect of each of these?
   Hui
   Wiki
   WiziQ
   In-class modelling by facilitator
   Discussion of videoed lessons.
8. Please put these (hui, wiki, WiziQ, and modelling/video) in order from least useful to most useful.

   ______ ______ ______ ______ ______

   Least useful **Most useful**

   Please give reasons for your decisions.

9. What are the biggest challenges in implementing this development? Please give reasons.

10. What is necessary to support your implementation? Please give reasons.

11. List all the people who support you to develop your maths teaching. List them by their formal position in the school community, or by their relationship to you (e.g., whānau), and comment on how they support you. Use extra paper as needed.

    Position/relationship **Type of support**

12. How has the wider kura supported you in this development?

13. How do you plan to continue your development over the next five years?

14. In what ways has school management helped or hindered your ongoing learning in this development?

15. What are the most significant changes that you have made to your teaching because of Te Poutama Tau?

16. What other changes would you like to make to your teaching of mathematics?

17. Comment on the appropriateness of the mathematics level 1 Achievement Standards and Unit Standards for assessing learning supported by Te Poutama Tau.

18. Are there any other comments you would like to add, for example suggestions for improvement?

Answer only if you joined the Te Rōpū Kupenga in 2008

How have the existing members of the group helped you to get up to speed with the development?

Answer only if this is your second year in the wharekura project

Describe any difference between this year and last year.

We would like just a little information about you

What was the last school year level of your own maths study? (e.g., Year 12, Form 6)

How many years have you been teaching in wharekura?

How many years have you taught altogether?

Thank you very much for your assistance in completing this questionnaire.