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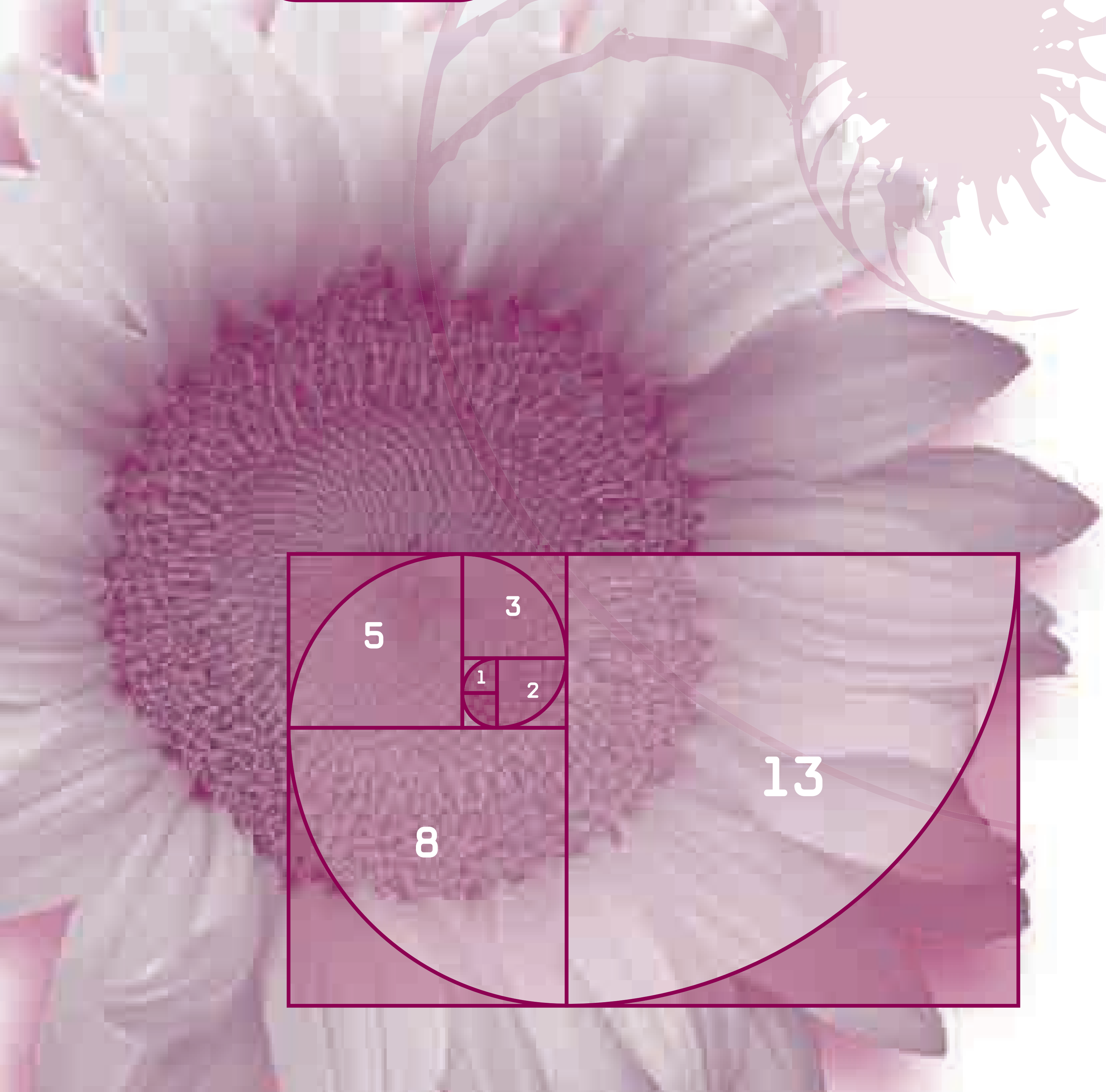
Te Tāhuhu o te Mātauranga

Findings from the New Zealand Numeracy Development Projects

2008



Findings from the New Zealand Numeracy Development Projects 2008

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Foreword

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The New Zealand Curriculum

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Foreword

The Numeracy Development Projects (NDP) began in 2000 with a pilot programme specifically aimed at improving the teaching and learning of number in the early years of schooling in New Zealand primary schools. The NDP have now developed to cover all primary and intermediate years, in both English-medium and Maori-medium (Te Poutama Tau) settings, as well as the first years of secondary school (the Secondary Numeracy Project and the wharekura Te Poutama Tau).

This compendium is a collection of papers based on research in primary and intermediate schools that has been funded by the Ministry of Education. Broadly, these papers cover the following areas: student achievement, professional practice, and evaluations of initiatives. In the rest of this foreword, I want to mention some concerns that the research papers in this volume have highlighted.

A well-known Maori proverb finishes with "Māku e kī atu he tangata, he tangata, he tangata." ("You ask me, what is the most important thing on earth. My reply is, it is people, it is people, it is people.") Paraphrasing this, "What is the most important thing in education? It is teachers, teachers, teachers." This is reinforced in a number of papers in this volume, especially those that emphasise developing and strengthening not only teachers' mathematics content knowledge but also their mathematics pedagogical content knowledge. In other words, it is not sufficient for teachers to know the material that they teach, they also have to know how to teach it and how to respond to individual students' needs during the learning process. Gaining pedagogical content knowledge is an ongoing process for each teacher as their expertise increases and as research extends our overall knowledge of the field. Hence, there is a need for continual investment by the Ministry of Education in both research and professional development. This compendium shows that funding is being made available for both of these important areas.

My main concern in this area is not the professional development of practising teachers but the education of pre-service teachers. I am concerned that a sound grounding in the NDP is not being provided in some teacher education programmes, and I would like to see this aspect of teachers' preparation expanded.

The next two aspects that I want to cover are related to the stages of the Numeracy Framework. The first of these is the different ranges covered at the different levels of the Framework. As a rough generalisation, the early stages cover less mathematics than the later ones and therefore students progress more rapidly through these stages. This gives the false impression that younger students are progressing more rapidly than older ones. This is almost certainly not the case, but when reading some of the papers in this volume, it is important to bear this difference in mind. I would assume that the problem has arisen largely because of our research knowledge of the field. Much more work has been undertaken in the early years of school, and so we know much more about younger students and their development. As research continues world-wide, it will certainly be the case that we will be able to provide a better progression over the whole range of the Number Framework. This raises another issue: the NDP should not be considered as complete and should not be seen as static. As our knowledge grows, the NDP must improve too.

The second aspect relates to student performance and curriculum level. The mathematics and statistics learning area of *The New Zealand Curriculum* is strongly related to the Number Framework. At the moment, the research suggests that a significant percentage of students are not performing on numeracy assessments at expected curriculum levels for their age. It is likely that this was a conscious decision of the curriculum developers in the belief that this positioning would help to raise standards. And their belief may well be justified over the longer haul, so it is too soon to try to make any changes to the curriculum. My concern now, however, is the next step in the education chain – National Standards. These have to be based on the curriculum; this is inevitable. What will happen if students generally cannot be lifted to the curriculum/Standards levels by 2010?

The other matter that is slightly related to the last is the continued poorer achievement of Māori and Pasifika students and students in low-decile schools. Although there is visible improvement for these students as the result of NDP, they do not appear to be catching up with higher-decile New Zealand Europeans. Do we fully understand the extent of the educational and social issues involved here? What has to be done to achieve both equal opportunity and equal achievement?

My last point of concern is the "transfer drop". Why is it that the level of ability of students seems to drop when they go from early childhood education to primary, from primary to intermediate school, and from intermediate to secondary school? What is it about the process of transferring schools that causes a drop in mathematical ability? This phenomenon is not isolated to New Zealand, so it is not something that can be attributed to NDP. However, it ought to worry us sufficiently that we try to overcome it.

Finally, I want to say that there is no doubt that the NDP have been invaluable in improving students' learning and teachers' knowledge and performance, but there are still problems that need to be solved. These problems have to be tackled, and New Zealand has the personnel both in and outside its classrooms to make progress towards their solution.

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The Numeracy Development Projects' Longitudinal Study: How Did the Students Perform in Year 7?

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The Numeracy Development Projects (NDP) have been implemented in schools in New Zealand since the Count Me In Too pilot project in 2000. This paper reports on the year 7 performance of 83 students whose progress in mathematics has been tracked since school entry in 2002. The findings indicate that the proportion of these 83 students achieving at curriculum level expectations over the last seven years has been consistently high. The students' strategy stages correlated positively with their results on a standardised test. Their responses to attitude questions showed a positive correlation between perceived and actual ability in mathematics.

Background

In 2002, as the first schools completed two years of participation in the Numeracy Development Projects (NDP), the Ministry of Education identified the need to address issues of sustainability so that the changes observed in the short term would translate to long-term changes in mathematics teaching and learning. One component of the ongoing research into the effectiveness and sustainability of the NDP has been the collection of information from a sample of schools in the years following their two years of participation in the NDP (Thomas, Tagg, & Ward, 2003; Thomas & Tagg, 2004, 2005, 2006, 2007). While there were a number of replacements of schools participating in this research, every effort was made to keep the demographic profile of the sample consistent. Where new schools were added, they were randomly selected from a list stratified by decile in an effort to keep the sample as representative of the national population as possible.

From 2002 to 2007, the participating schools provided results for their students on the three strategy domains of the Number Framework. These results have been used to inform the Ministry of Education's expectations for student achievement (Ministry of Education, n.d. 2). In 2002 and 2003, qualitative information was collected from teachers and principals in longitudinal study schools to provide insight into the challenges and rewards involved in ongoing implementation of the NDP (Thomas, Tagg, & Ward, 2003; Thomas & Tagg, 2004). Between 2003 and 2006, the study investigated the performance of students from a wider group of schools (Thomas & Tagg, 2004, 2005, 2006, 2007). The focus of the Longitudinal Study in 2007 was on a group of 151 students from the original group who were in year 6 in 2007 and for whom NDP results were available for every year of their schooling (Thomas & Tagg, 2008). The Longitudinal Study in 2008 follows 83 of these 151 students as they move into year 7.

Research has consistently shown that the NDP continue to impact positively on the number strategies of students in NDP-focused schools in the years following their initial implementation (Thomas & Tagg, 2004, 2005, 2006, 2007, 2008). The results of tests relating to mathematics more generally have shown that students in schools that continue to implement NDP practices tend to perform similarly or slightly better on items relating to all areas of mathematics, not just those related to number knowledge and strategies (Thomas & Tagg, 2004, 2005, 2006). The findings from the 2007 study suggested that students who are able to perform at expected levels at any year level have an 80% likelihood of continuing to achieve at expected levels in subsequent years (Thomas & Tagg, 2008).

Method

Materials

A written assessment was developed for the 2008 Longitudinal Study. This assessment included the 41 items from Progressive Achievement Test (PAT): Mathematics Test 4 (NZCER, 2006), five attitude items, and four items designed to assess students' use of number strategies. The four items used to assess number strategy were taken from the NDP Global Strategy Stage (GloSS) forms (Ministry of Education, n.d. 1). These items required the students first to write an answer to a word problem and then to explain their thinking. The problem in Figure 1 was included in the assessment as an example of how to complete the strategy items.

Mani knows that there are 9 teams in the rugby tournament.
Each team has 18 players.
What is the total number of players at the tournament?

Write a number sentence for this problem and then work out the answer.

9 x 18 = 162

Show or explain in a couple of sentences how you worked out the answer to the problem.

I know that 10 x 18 is 180.
I took one lot of 18 away from 180, which gave me 162.

Figure 1. Sample strategy item from longitudinal study assessment

Procedure

The move to year 7 often involves a transition to an intermediate school, so the researchers decided to track the longitudinal study students in consultation with their families rather than through their schools. The schools that the 151 students from the 2007 Longitudinal Study had attended were asked in April 2008 to forward a letter of invitation to the students' families that outlined the purpose and ongoing nature of the study. Informed consent was requested from both the students and their parents for their participation in this research. Eighty-eight students and their parents returned consent forms.

These 88 students were sent the test early in term 4, with a book voucher included as an incentive for them to complete and return the test. One student's tests was returned unopened. 83 of the remaining 87 students returned completed tests. These are the 83 students reported on throughout this paper. An attempt was made to contact the other four families. Two were unable to be contacted, one chose to withdraw from the study, and one returned the assessment several weeks after the analysis had been completed.

Participants

This section compares the demographic characteristics of the full sample of 151 students from 2007 with the 83 students who elected to continue to participate in the study. More of the 83 students are female (61%) than male (39%), and nearly three-quarters (72%) of the students identified as New Zealand European, with 17% identifying as Māori. As illustrated by Table 1, there were larger proportions of Māori and Pasifika students in the 151 students from 2007 than in the 83 students who continued to participate in 2008.

Table 1
Ethnicity and Gender of the 2007 and 2008 Longitudinal Study Students

	2007			2008		
	Male	Female	Total	Male	Female	Total
NZ European	28	53	54%	20	40	72%
Māori	18	19	25%	7	7	17%
Pasifika	9	9	12%	2	2	5%
Asian	8	2	7%	2		2%
Other	2	3	3%	1	2	4%
Total[1]	65	86	151	32	51	83

The 83 students were contacted and tracked through their families rather than through their schools, so no data is available on the schools they attended in 2008. Consequently, the decile data reported in Table 2 for these students is based on the schools they attended in years 1–6. As shown by Table 2, there was a shift in the decile composition of the 2008 participating students, with a smaller proportion having attended decile 1–3 schools in years 1–6 than those who participated in 2007.

Table 2
Decile of the Schools Attended by the 2007 and 2008 Longitudinal Study Students in Years 1–6

	2007	2008
Decile 1–3	18%	11%
Decile 4–7	59%	57%
Decile 8–10	22%	33%
Total	151	83

The self-selecting nature of the 83 students in the 2008 Longitudinal Study has resulted in assessment of a group who had a higher mean strategy stage (6.4) in 2007 than the full 2007 group (6.0).

Analysis

This section describes the analysis of the 83 students' responses to the 2008 assessment. The responses to the 41 items from PAT: Mathematics Test 4 were analysed by NZCER, with comparisons made with national norms for year 8 students. A comparison with year 8 norms rather than with year 7 norms was considered to be more appropriate because the norm data was collected in term 1, whereas the longitudinal study data was collected in term 4. Consequently, the longitudinal study students were generally about 6 months younger than the national reference group.

Each student's GloSS was estimated from their responses to the four strategy items posed within word problems. The minimum evidence required for a rating at each stage is listed in Table 3. Each student was rated at the highest stage for which they met the criteria. Examples of responses at each stage are included in Table 3. Fifty of the students provided sufficiently detailed explanations of their thinking for a strategy stage to be readily assigned. Two researchers independently examined

[1] Percentages in tables may not total to 100 due to rounding.

the remaining 33 students' responses and rated them as either stage 6 (advanced additive), stage 7 (advanced multiplicative), or stage 8 (advanced proportional). Nine students who provided no evidence of strategies at stage 6 or higher were rated as stage 5 or below and were treated as stage 5 for the purpose of calculating means. The two researchers then worked together to agree on the stage ratings for the five students for whom they had initially estimated different stages.

Table 3
Minimum Criteria for Rating Students' GloSS Stage

Stage	Minimum Criteria	Example of Response
Stage 5 or below	No evidence of strategies at stage 6 or higher	143 - 89 = I know 140 - 80 = 60 so if I add 9 + 3 = 12 I will add 60 & 12 together to make 72
Stage 6: Advanced Additive	Uses a partitioning strategy to solve either 143 minus 89 or 28 plus 42	89 + 11 equals 100 so plus 43 equals 143 so in the end 11 + 43 = 54
Stage 7: Advanced Multiplicative	Uses a partitioning strategy to solve either 5.33 minus 2.9 or 154 divided by 7	5.33 − 3.0 + 0.10 = 2.43 Round 2.9 to 3.0 then subtract 3.0 from 5.33 then add the 0.10 on again.
Stage 8: Advanced Proportional	Finds 28 out of 70 as a percentage	there are seventy students. 10% is 7 students. I divided the 28 students by 7 which gave me 4. Then I timed that by 10 so that gave me 40%

Findings

The key research questions addressed in this section include:

- What were the patterns of performance on the Number Framework of the students since school entry?
- How did the students perform on the standardised PAT: Mathematics assessment?
- What is the relationship between the students' strategy stages and their performance on the standardised assessment?
- What were the attitudes of the students to mathematics, and did this relate to their performance?

Patterns of Performance of the Students since School Entry

Curriculum level expectations provide an important set of targets or goalposts for New Zealand schools and teachers (Ministry of Education, n.d. 2). The expectations consist of the stage or stages of the Number Framework that students are expected to be achieving at by the end of each year level of schooling. Also included in the expectations are the categories of "cause for concern" (just below expected levels) and "at risk" (well below expected levels).

While the achievement of students at the end of a single year level provides useful information, more important is the trajectory of their performance over time. Figure 2 shows the performance of the 83 longitudinal study students on the Number Framework over the seven years since school entry in 2002. The students' performance was tracked using an estimated GloSS. A student's GloSS is usually determined by using the GloSS assessment forms, but for the purposes of this study, the year 1–6 GloSS results for each of the 83 students were generated by taking the highest stage reported for that student in that year across the three strategy domains (Thomas & Tagg, 2008). The year 7 GloSS results were determined from the written assessments, as described earlier in this paper.

The shading in Figure 2 indicates the three categories of achievement in relation to curriculum level expectations (Ministry of Education, n.d. 2). Figure 2 shows that a high proportion of the 83 students have achieved at or above expected strategy levels since school entry. Figure 2 also highlights the very low percentage of students who have been considered to be "at risk" in terms of their future achievement in mathematics.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
8 (AP)					6%	13%	15%
7 (AM)			5%	2%	18%	40%	42%
6 (AA)	1%	7%	23%	40%	33%	31%	32%
5 (EA)	10%	29%	45%	45%	31%	12%	11%[2]
4 (AC)	17%	37%	23%	11%	10%	2%	
3 (CAI)	23%	15%	4%	2%	2%	1%	
2 (CA)	36%	8%					
1 (1–1)	8%	4%	1%				
0 (Em)	5%						

Expected level:

Cause for concern:

At risk:

Figure 2. Percentage of the 83 students in the longitudinal study sample at each strategy stage in years 1–7

Table 4 compares the performance of the 83 longitudinal study students with the overall performance of students as reported in the curriculum level expectations on the nzmaths website (Ministry of Education, n.d. 2). As illustrated in Table 4, at least 85% of the 83 students were rated as achieving at or above expectations in every year except year 2. In years 1–3 and year 5, these percentages were very similar to those provided in the expectations. The proportion of year 6 and 7 students achieving at or above expectations was significantly higher for the 2008 longitudinal study students than for the expectations sample.

Table 4
Comparison of Percentages of the 83 Students Performing At or Above Expectations since School Entry in Relation to National Curriculum Level Expectations

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
2008 Longitudinal Study	87%	73%	95%	87%	88%	85%	89%
Curriculum Level Expectations	84%	74%	94%	73%	90%	73%	68%

[2] Includes all students who showed no evidence of strategies at stage 6 or higher

Performance of the Students on a Standardised Mathematics Assessment

Since 2003, the study has investigated the impact of the NDP on student achievement by collecting both numeracy results and information on students' performance in mathematics generally. The items used to assess students' ability in mathematics have come from a variety of sources for which national data is available for New Zealand students. These include the Trends in International Mathematics and Science Study (TIMSS), Achievement Resource Banks (ARBs) and PATs, the National Education Monitoring Project (NEMP), and the Assessment Tools for Teaching and Learning (asTTle). In 2008, the test completed by the 83 longitudinal study students included all 41 items from PAT: Mathematics Test 4.

Figure 3 compares the longitudinal study students' results on PAT: Mathematics Test 4 with those of the PAT year 8 national reference group. The longitudinal study students had a mean PAT scale score of 61.2 (SD = 11.3), compared with a mean of 55.5 (SD = 12.3) for the national reference group, representing a difference in average performance of nearly a stanine. The upper quartile of the longitudinal study results (65.2) was similar to that for the reference group (63.8), while the lower quartile was nearly 10 points higher (56.0 compared with 47.2). This indicates that, while the higher-scoring students in the two groups were performing at similar levels, a smaller proportion of the longitudinal study students received low scores on the test.

While it is important to note that the 2008 longitudinal study students did not represent a randomly selected sample, this favourable comparison with students half a year older indicates that these students were performing well in mathematics.

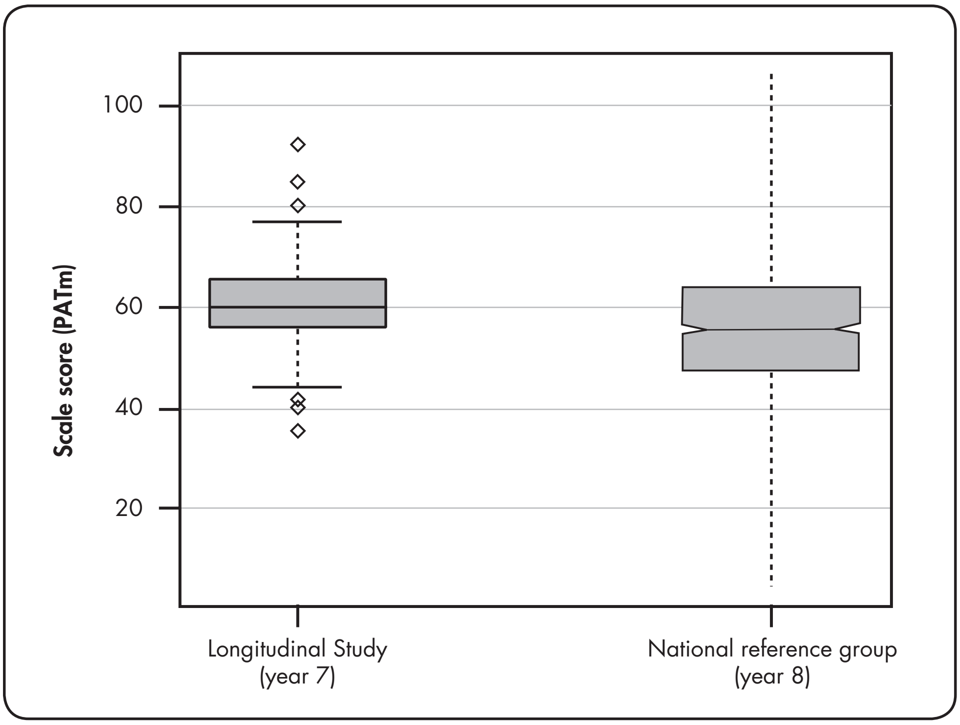

Figure 3. Comparison performance of students on PAT: Mathematics Test 4

The Relationship between the Students' Strategy Stages and their PAT Scores

The NDP focuses on developing students' understanding of numbers and their ability to use numbers to solve mathematical problems. Although the NDP focuses on the Number and Algebra strand of the mathematics and statistics learning area of the curriculum, this is at least partly because an understanding of number also underpins the other strands. The information gathered in the Longitudinal Study in 2008 allows investigation of the relationship between strategy stages and performance on mathematics more generally, as measured by PAT scores.

Table 5 shows the mean scale scores and stanines of the longitudinal study students based on their GloSS stage. Comparing the students' PAT scale scores and GloSS stages yielded a Pearson's correlation coefficient of 0.56 ($p < 0.01$). The strong correlation between strategy stages and PAT scale scores suggests that it is reasonable to use numeracy stages to predict students' overall mathematics ability.

Table 5
The Relationship between the Students' Strategy Stages and their PAT Scores

GloSS	N =	Mean PAT Scale Score	Standard Deviation of PAT Scale Score	Mean PAT Stanine	Standard Deviation of PAT Stanine
≤ 5	9	49.1	9.9	3.8	1.6
6	27	57.2	7.9	5.3	1.4
7	35	62.5	9.0	6.1	1.3
8	12	75.8	9.6	7.8	1.5
All students	83	61.2	11.3	5.9	1.8

The Relationship between Attitudes and Performance in Mathematics

The five attitude questions completed by students took the form of statements for which the students were asked to indicate their level of agreement. Table 6 compares the mean GloSS stage and mean PAT scale score of students based on their responses to the five questions.

Table 6
The Relationship between the Students' Performance and their Responses to Attitude Questions

Statement		Agree a lot	Agree a little	Disagree a little	Disagree a lot
1. I usually do well in mathematics	Mean GloSS	7.0	6.6	6.1	5.0
	Mean scale score	70.4	59.0	56.0	40.0
	N =	22	48	9	1
2. I enjoy learning mathematics	Mean GloSS	6.8	6.6	6.1	6.5
	Mean scale score	62.7	62.4	54.7	59.1
	N =	29	41	10	2
3. I learn things quickly in mathematics	Mean GloSS	7.1	6.6	6.3	6.3
	Mean scale score	69.7	61.5	53.7	56.1
	N =	20	39	19	3
4. I think learning mathematics will help me in my daily life	Mean GloSS	6.5	6.9	6.7	
	Mean scale score	60.4	65.0	53.0	
	N =	54	24	3	
5. I need mathematics to learn other school subjects	Mean GloSS	6.9	6.4	6.7	
	Mean scale score	60.7	61.7	62.9	
	N =	28	41	12	

Two of the statements relate to students' perception of their own ability in mathematics: "I usually do well in mathematics" and "I learn things quickly in mathematics". The students who agreed a lot with these statements had higher mean strategy stages than those who agreed a little, and both groups had higher stages than those who disagreed. An ANOVA (analysis of variance) test indicated that these differences were statistically significant at the 0.01 level for statement 1 and at the 0.05 level for statement 3. The same pattern was true for scale scores, with the differences in means for both statements being statistically significant at the 0.01 level. The differences between means for the other three statements were not statistically significant. This finding is consistent with findings from the 2007 Longitudinal Study, in which the only attitude question for which the differences between mean GloSS stages of responses was significant was "How good do you think you are at maths?"

Concluding Comment

In previous years, this study has examined the impact of the NDP on students' ability to use number strategies in the years following their schools' participation in the NDP professional development programmes. It has also collected information to compare the mathematics performance of students in these schools with the performance of national reference groups. The findings have consistently shown that the NDP continue to impact positively on the strategy performance of students in the years following their initial implementation, and that students in schools that continue to implement NDP practices perform similarly or better on items relating to mathematics more generally.

The findings of the current study support the previous findings, with the 83 students performing better than comparative samples of students nationally on both the strategy stages and on PAT: Mathematics Test 4. It should be noted that, although this sample was not randomly selected, the strategy profile of the group in their first three years of school was very similar to the profile described in the curriculum expectations (Ministry of Education, n.d. 2). The high proportion of students reaching expected strategy stages in year 7 and the strong performance on the PAT indicate the benefits these students gained by remaining until at least year 6 in schools that implement NDP practices.

The responses of students to questions relating to their attitudes to mathematics support previous findings showing that there is a positive correlation between students' perceived ability and their actual performance. This is true of performance on both PAT items and strategy stages.

In tracking the performance of this group of 83 students, the Longitudinal Study continues to find evidence supporting the positive impact of the NDP on students' long-term development of mathematical ability.

Acknowledgments

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Patterns of Performance and Progress of NDP Students in 2008

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The Numeracy Development Projects (NDP) have been implemented in New Zealand schools for almost a decade. The first part of this paper analyses the results of students in schools participating in the NDP for the first time in 2008. Analysis of the students' performance at the end of a year spent on the NDP showed that students in the early school years made substantial progress on the additive domain, with almost all students at stage 4 (advanced counting) or higher by the end of year 3. By the end of years 7 and 8, performance on the multiplicative and proportional domains was higher than on the additive domain. Comparison of students' performance with the curriculum level expectations (Ministry of Education, n.d.) showed that only year 2 students were close to level 1 curriculum expectations. At years 4, 6, and 8, only about half of the students were close to the expectations for curriculum levels 2, 3, and 4 respectively. Analysis showed that students' performance varied as a function of school decile, with students at high-decile schools performing better than those at middle- and low-decile schools. In the second part of the paper, an analysis of students at schools that submitted data in both 2007 and 2008 showed virtually no improvement in the second year of the NDP, except for year 2 students. Analysis of students' average stage at the beginning of the NDP and gain scores showed that in the primary years (0–6), those who started lower on the Number Framework tended to make greater gains, particularly on the additive domain. By the year 7–8 level, those who started higher on the Framework also tended to make greater gains, and this was particularly noticeable on the multiplicative and proportional domains. The net result was that initial differences favouring European students and students at high-decile schools were exacerbated and gaps between groups became wider.

The Numeracy Development Projects (NDP) have been under way for nearly a decade now. Almost all primary and intermediate schools have taken the opportunity to be involved in the NDP. The current focus is on issues of sustainability within schools, and consequently, the amount of data being entered on the nzmaths website by schools in the first or second year of professional development has decreased over recent years.

What Do We Know So Far?

Students whose teachers participated in the NDP made progress on the Number Framework from the beginning of the year their teachers were on the NDP to the end of that year (Young-Loveridge, 2004, 2005, 2006, 2007, 2008). The size of that progress was reasonably substantial, with effect sizes of between a quarter and half of a standard deviation (Young-Loveridge, 2005, 2006, 2007). All students, regardless of ethnicity, gender, age, or socio-economic status (as reflected in school decile) benefited from involvement in the NDP. Students from some groups began at higher stages on the Number Framework and made greater progress than others (Young-Loveridge, 2005, 2006, 2007). However, the size of the differences between groups is quite small compared with group differences on written tests administered to a whole class (Young-Loveridge, 2006). It seems likely that a major reason for smaller group differences on NDP assessments is that students are assessed individually by their own teacher. The questions are presented orally, and this eliminates the complicating effect of reading difficulties. When progress was examined separately for each major ethnic group and each school-decile band (0–3, 4–7, 8–10), it was found that the group of students who made the greatest progress was Pasifika (average effect size [ES] of 0.40), followed by students attending low-decile schools (ES = 0.38), then Māori students (ES = 0.35). This shows that the NDP has had an impact on students' mathematics knowledge and understanding. The focus now needs to shift from *relative* progress to *absolute* progress.

With the imminent introduction of National Standards in literacy and numeracy, it is important to focus on the levels of achievement reached by students at the end of a year in which their teachers focused on the NDP rather than simply looking at the progress made from the level at which students began. Hence, this paper focuses predominantly on data from assessments completed at the end of the year (final data).

How Did NDP Students Perform in 2008?

The analysis reported in this section of the paper was limited to final data sent in by schools participating in the NDP initiative for the first time and includes only students with complete data on all three strategy domains as well as on the knowledge domains of basic facts, place value, and fractions. There were 104 schools who participated in the initiative for the first time in 2008. Complete data was available for students attending 76 of these schools.

Cohort Composition

Table 1 shows the composition of the 2008 NDP cohort (see also Appendix A, p. 169). It is clear that this cohort varied in terms of school decile and ethnicity from one year level to the next. In the initial school years (years 0–3), the cohort was dominated by students from high-decile schools, with more than half of the students coming from the high-decile band (8–10) and the remainder coming almost equally from the low (1–3) and middle (4–7) decile bands. In the middle primary years (years 4–6), the students were more evenly distributed across decile bands. At the intermediate level (years 7–8), more than 50% of the students were from high-decile schools and approximately 10% were from low-decile schools. At the secondary level (year 9), 50% of the students were from the middle-decile band, 37% were from the low-decile band, and only 13% were from the high-decile band. It is important to look at cohort composition when interpreting data on the progress of students between year levels because particularly rapid or slow progress from one year level to another may be explained by the nature of the different cohorts being compared.

NDP cohorts at the year 0–6 level have reduced over the years from more than 10 000 students per year level earlier in the decade to little more than a few hundred in 2008. At the intermediate and secondary levels, the numbers at each year level have reduced from more than 10 000 down to about 3000 in 2008. Because of the reductions in cohort size, some of the fine-grained analyses (for example, the differential impact of the NDP initiative on students from schools differing in decile and/or ethnicity) are no longer possible for more recent data.

Table 1
Composition of the 2008 Cohort (Percentages)[1]

Group	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Number of students	*219*	*225*	*255*	*226*	*221*	*326*	*2989*	*3010*	*2855*
School Decile									
Low (1–3)	23	21	19	19	28	20	11	12	37
Middle (4–7)	20	28	27	47	39	38	35	36	50
High (8–10)	57	51	54	35	34	42	54	52	13
Ethnicity									
NZ European	50	59	53	54	51	66	49	45	48
Māori	21	17	18	19	24	17	11	12	27
Pasifika	10	9	8	9	9	5	11	10	16
Asian	8	4	9	7	5	6	17	18	5
Other	11	11	12	11	11	6	12	15	4

NDP Students' Performance at the End of the First Year

Appendix B (pp. 170–177) shows the percentages of students at each stage on the Number Framework for each year level on each domain. Figure 1 shows the percentages of students who reached particular stages on the Framework by the end of the year that their teachers had first participated in the NDP professional development programme. Only those who had reached at least stage 4 by the end of the year are included in the graph.

It is clear from Figure 1 that there is a lot of progress in the first few years of school, with almost all students at stage 4 or higher by the end of years 3 or 4. In the early school years, performance is better on the additive domain than on the multiplicative or proportional domains. By the end of primary and intermediate years, performance is better on the multiplicative and proportional domains in terms of the percentages of students reaching the higher stages on the Number Framework. While there is a noticeable improvement at each successive year level on all three domains, the performance of year 9 students across the domains is no better than that of year 8 students. The reason for this may be that more than 50% of the intermediate cohort came from high-decile schools, whereas this is the case for only 13% of the year 9 cohort. By the same token, more than 33% of the year 9 cohort came from low-decile schools, whereas this was true for only about 10% of the intermediate cohort.

[1] Totals may be affected by rounding.

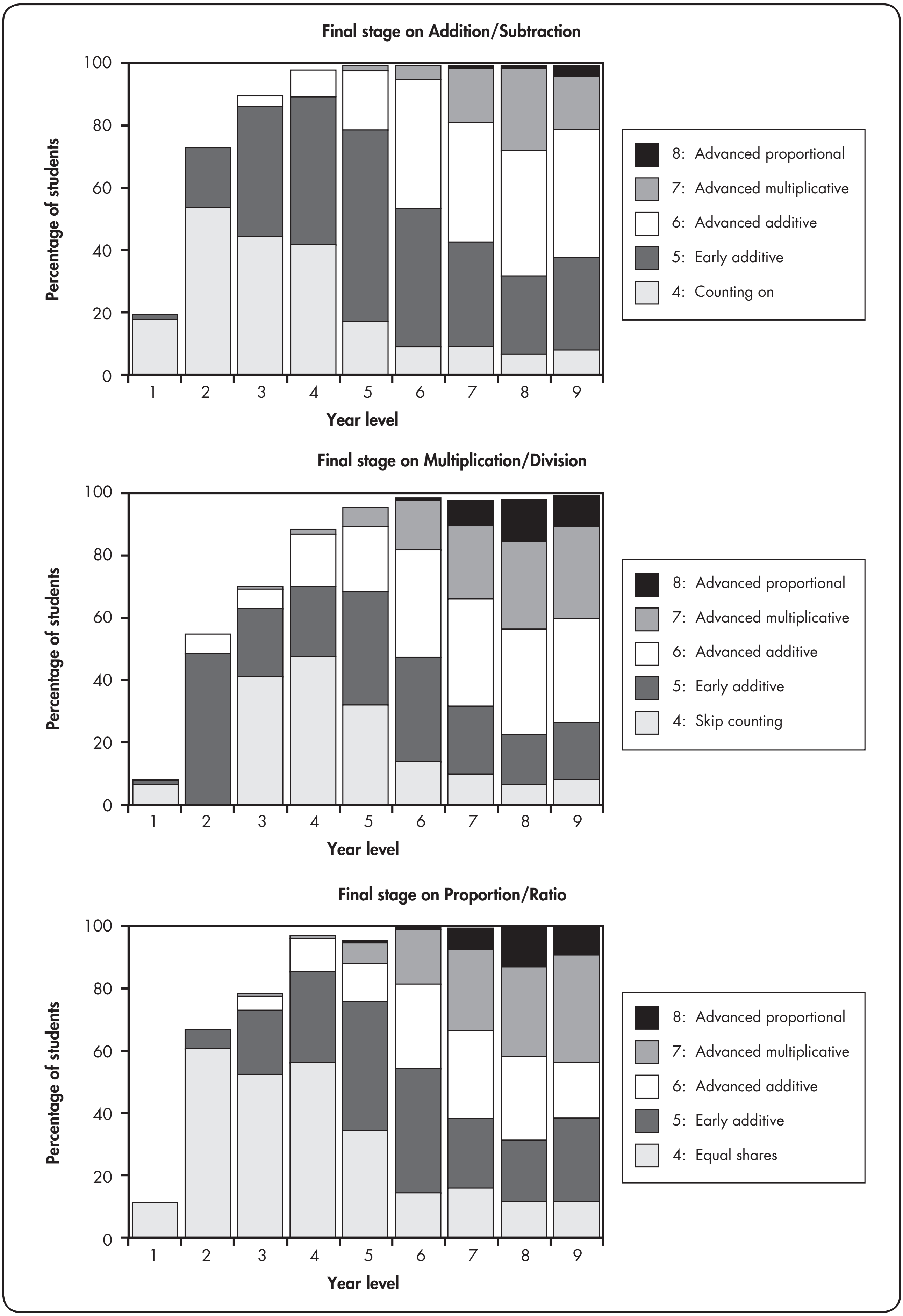

Figure 1. Percentages of students at each stage on the Number Framework on the three strategy domains as a function of year level at the end of the first year

Table 2 presents a summary of percentages of students who were at or above stages 4–7 on the additive, multiplicative, and proportional domains, with boxes indicating the year levels and cumulative stages that are relevant to the Ministry of Education's expectations for students at levels 1–4 of the curriculum. It is clear from Table 2 that, by the end of year 2, 73% of students had reached at least stage 4 (advanced counting) on the additive domain of the Number Framework. This finding is consistent with the expectations for students at level 1 on the curriculum (see Ministry of Education, 2007, n.d.). Fifty-six percent of students had reached at least stage 5 early additive part–whole thinking by the end of year 4 (level 2), and by the end of year 6 (level 3), the proportion of students who had reached stage 6 advanced additive part–whole thinking was 46%. At the end of year 8 (level 4), the proportion who had reached stage 7 advanced multiplicative part–whole thinking was 27% on the additive domain and 42% on the multiplicative and proportional domains. These results for years 4, 6, and 8 differ from the longitudinal percentages published with the curriculum level expectations for students at the end of these year levels.

Table 2
Percentages of Students at Each Stage on the Number Framework by Year Level and Domain at the End of the First Year[2]

Domain/Stage	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Number of students	*219*	*225*	*255*	*226*	*221*	*326*	*2989*	*3010*	*2855*
Additive domain									
Stage 4+	19	$\boxed{73}$	89	98	100	99	100	99	99
Stage 5+	2	19	46	$\boxed{56}$	82	91	90	93	92
Stage 6+	0	0	4	8	21	$\boxed{46}$	57	68	62
Stage 7+	0	0	0	0	2	4	18	$\boxed{27}$	21
Multiplicative domain									
Stage 4+	8	$\boxed{55}$	70	89	97	99	98	98	99
Stage 5+	1	7	29	$\boxed{41}$	65	85	88	92	91
Stage 6+	0	0	7	19	28	$\boxed{51}$	67	75	72
Stage 7+	0	0	0	1	7	16	32	$\boxed{42}$	39
Proportional domain									
Stage 5+	0	6	26	$\boxed{40}$	61	86	84	88	88
Stage 6+	0	0	6	12	20	$\boxed{46}$	61	69	61
Stage 7+	0	0	1	0	7	18	33	$\boxed{42}$	44
Curriculum level		1		2		3		4	

One possible reason for the disappointing results shown at years 4, 6, and 8 (levels 2, 3, and 4) is that the expectation that the majority of year 2 students will be at stage 4 and counting on is not as early as it needs to be to allow for part–whole thinking to be sufficiently developed by the end of year 8. It has been clearly established that the Number Framework is not a linear scale and that progress on the first four stages is much easier than for the upper stages (Young-Loveridge, 2004). By having a large number of "micro-stages" at the lower end of the Number Framework, teachers may feel that

[2] The boxes indicate the percentage of students at a particular stage at the end of each curriculum level according to the national numeracy expectations.

they need to put a much greater emphasis on counting than is desirable in the early years. Counting is just one form of quantifying collections, and it is to be hoped that teachers are also encouraging students to quantify small collections by recognising instantly how many items there are (that is, subitising), rather than laboriously counting each item one by one. If teachers were to expect students to be counting on (stage 4) by the end of year 1 and put a greater emphasis on students knowing the sums of small addends (basic facts) as well as supporting the students' use of subitising from the beginning, then we might see greater progress in terms of the numbers of students reaching stage 5 by the end of year 4.

It is clear that students need much longer to build an understanding of the part–whole relationships among numbers than to progress through the counting stages on the Number Framework, and the sooner they are encouraged to do that, the better. This could have benefits for stages 6 and 7 later on, with many more students reaching those levels earlier than is presently the case.

Impact of School Decile on NDP Students' Performance at the end of the First Year

Figure 2 presents the percentages of students at or above stages 5 and 6 on the three strategy domains as a function of school-decile band (low: 0–3, middle: 4–7, high: 8–10). This figure shows that, in general, students from high-decile schools performed the best, followed by those from middle-decile schools, and then those from low-decile schools. There are likely to be many reasons for these differences.

The decile ranking assigned to a school is based on the income and education levels (from recent census information) of households whose children attend the school. Other factors associated with household income and education, such as the likelihood of students leaving to attend another school (which tends to be higher for lower-decile schools), also affect decile rankings. High levels of school absence or transience make it more difficult for students to maintain satisfactory progress in their learning; these factors are among those associated with lower levels of achievement.

Ritchie (2004) found a tendency for teachers to move towards higher-decile schools, meaning that low-decile schools may have disproportionately more inexperienced teachers than middle- or high-decile schools. Moreover, the expectations of teachers (based on the associations with other variables) may set up self-fulfilling prophecies that contribute to lower levels of achievement than might otherwise have been attained.

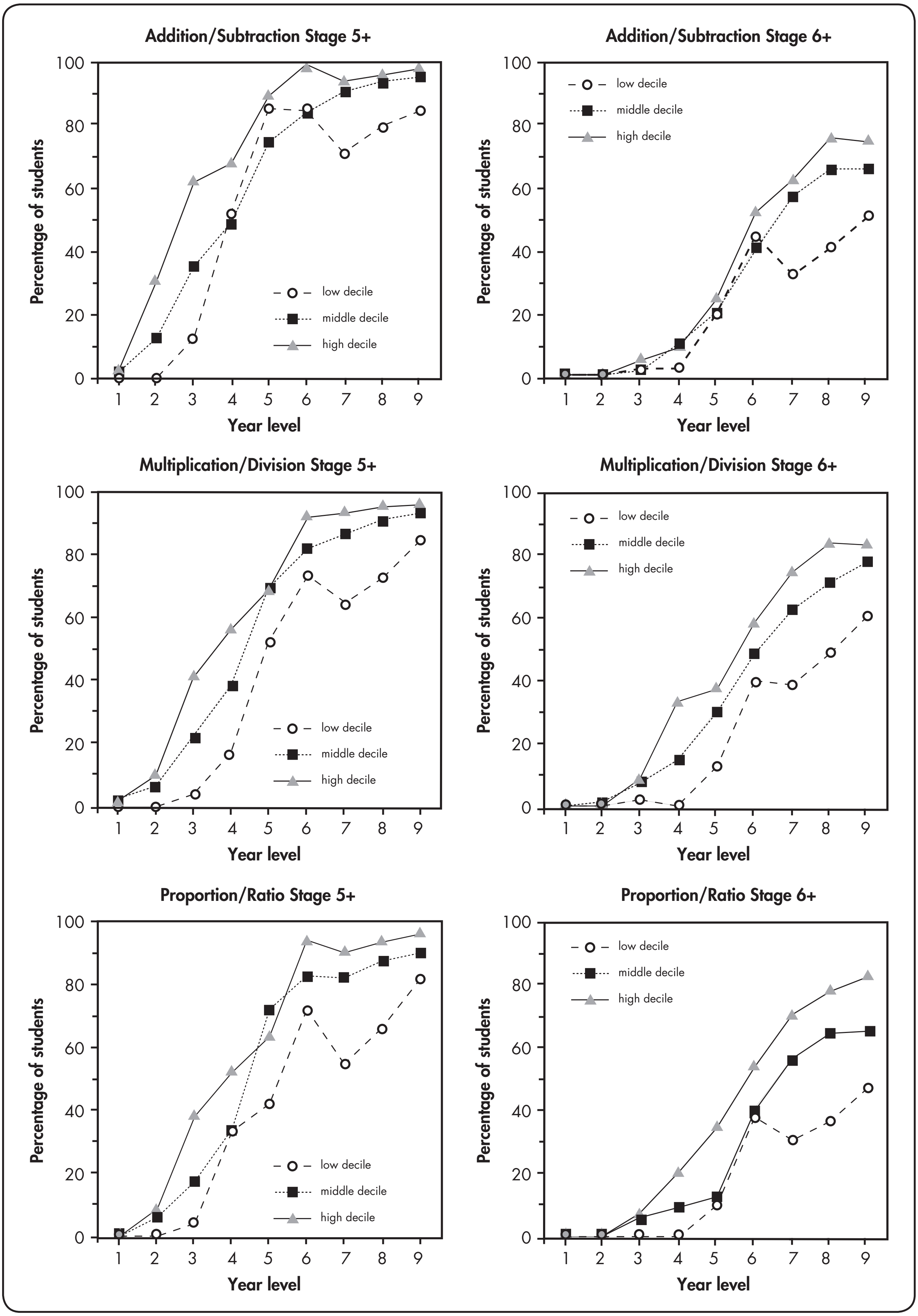

Figure 2. Percentages of students at or above stages 5 and 6 on the three strategy domains as a function of school-decile band at the end of the first year

The pattern shown in Figure 2 for students from low-decile schools indicates a marked drop in performance from year 6 to year 7. One question that has been raised is whether this drop is a reflection of the adjustment needed when students move from year 6 at a primary school to year 7 at an intermediate school. A separate analysis (see second section of this paper) compared the performance of year 7 students attending (low-decile) full primary schools with that of year 7 students attending (low-decile) intermediate schools. Those attending primary schools did slightly better than those attending intermediate schools. However, the results of this analysis were affected by the fact that all of the low-decile intermediate school students studied attended decile 1 schools, whereas some of the students at low-decile full primary schools attended decile 2 and decile 3 schools. Hence any differences could be due to the actual decile rank rather than whether the school was an intermediate or full primary school. Table 3 shows the proportion of students at low-decile schools at each decile ranking.

Table 3
Percentages[3] of Students Attending Low-decile Schools According to Decile Ranking

School Decile	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Number of schools	*51*	*47*	*49*	*42*	*61*	*65*	*336*	*373*	*1068*
Decile 1	8	2	6	7	16	8	90	84	41
Decile 2	26	28	27	24	30	9	5	7	26
Decile 3	67	70	67	69	54	83	5	9	33

As Table 3 shows, a substantial majority of low-decile primary schools (years 0–6) are at decile 3, whereas the vast majority of low-decile schools at the intermediate level (years 7–8) are decile 1 schools. Table 3 makes it clear that the low-decile cohort at year 6 is not comparable with the low-decile cohort at year 7, and this could explain why there appears to be a noticeable drop in student performance between years 6 and 7.

The Impact of Other Variables on Students' Performance and Progress

Appendix C (p. 178) shows the average stages for students at the beginning of the NDP initiative (initial), while Appendix D (p. 179) presents the mean gain from the beginning to the end of the first year of the NDP initiative. It is clear from Appendix C that at the start of the NDP, on average, boys were at a slightly higher stage on the Number Framework than girls, and students from high-decile schools were at a higher stage than students at middle-decile schools, who in turn were at higher stages than those at low-decile schools. On average, New Zealand European and Asian students began the NDP at higher stages on the Number Framework than Māori students, who in turn began higher than Pasifika students.

It is important to interpret the average gain scores presented in Appendix D in the light of students' initial stages on the Number Framework: the lower the students began on the Number Framework, the greater the potential gains they could make, on the additive domain at least. This pattern may be explained in terms of a ceiling effect operating for the additive domain. For example, year 0–1 students from low-decile schools began the NDP with an average stage of 0.82 (compared with corresponding scores of 1.34 and 2.32 for students from middle- and high-decile schools respectively). The students from low-decile schools made an average gain of 1.18 stages (compared with 0.97 and 0.65 for students from middle- and high-decile schools respectively).

[3] Totals may be affected by rounding.

On the additive domain, gain scores in the initial years of school were approximately one stage on the Number Framework (because students could move quickly through several counting stages), whereas gain scores at the upper end of the Number Framework tended to be about half a stage. On the multiplicative and proportional domains, average gain scores were small initially because the younger students were not given the opportunity to do multiplicative or proportional tasks (if assessed using Form A of the Numeracy Project Assessment [NumPA]). However, students gained between about one-half and three-quarters of a stage once they were given the chance to do tasks within the multiplicative and proportional domains. The small numbers of students in some of the groups in years 0–6 means that these figures (mean stages and mean gains) need to be interpreted cautiously.

An examination of year 7 and 8 data on the multiplicative and proportional domains shows that not only did students from high-decile schools start higher on the Number Framework than did students at middle- and low-decile schools, they also made greater gains than either of the other two groups (see figures 3 and 4 and appendices C and D). For example, on the multiplicative domain, year 7 students from high-decile schools scored 5.50 initially (compared with 5.29 and 4.89 for students from middle- and low-decile schools respectively), and gained an average 0.73 stages (compared with 0.58 and 0.45 for students from middle- and low-decile schools respectively). A similar pattern was evident for New Zealand European and Asian students compared with Māori and Pasifika students.

This pattern, which is indicative of a widening gap between groups, suggests that expectancy effects may be operating, with teachers getting what they expect from students according to school decile, ethnicity, and gender. Further research is needed to investigate possible ways of narrowing the gaps between groups. Possible reasons need to be explored as to why this pattern appears to be particularly characteristic of the intermediate level and not of the early and middle years of primary school.

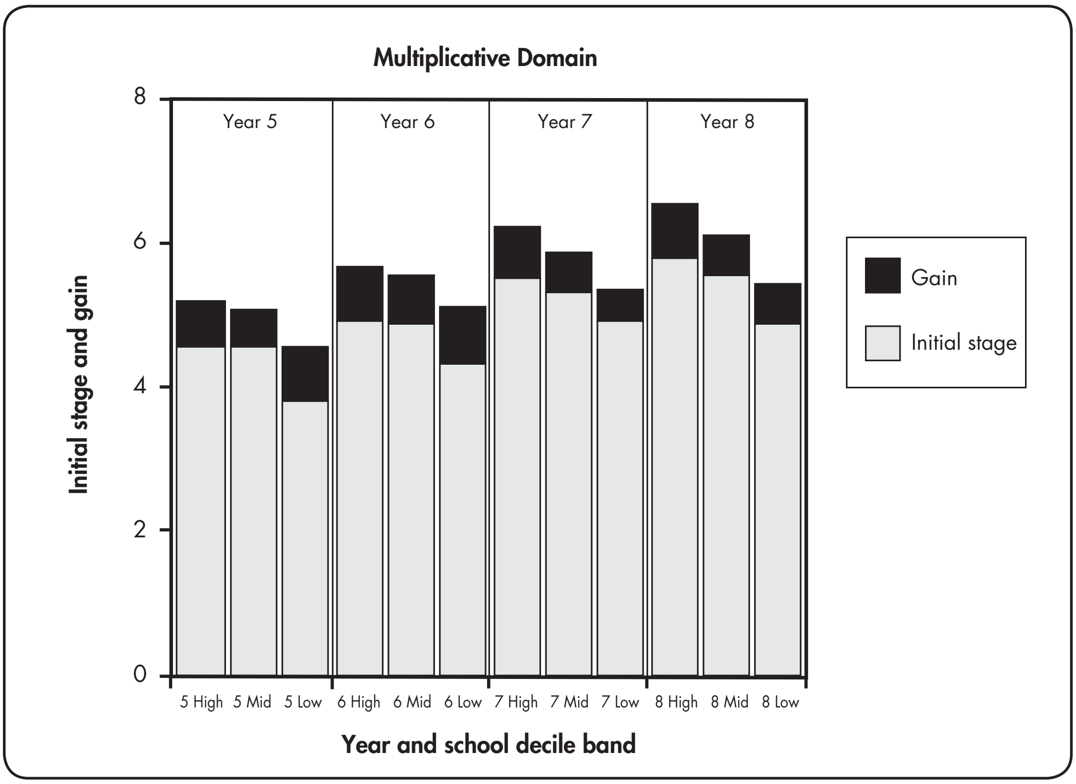

Figure 3. Average initial stage and gain on the multiplicative domain on the Number Framework as a function of year level and school-decile band

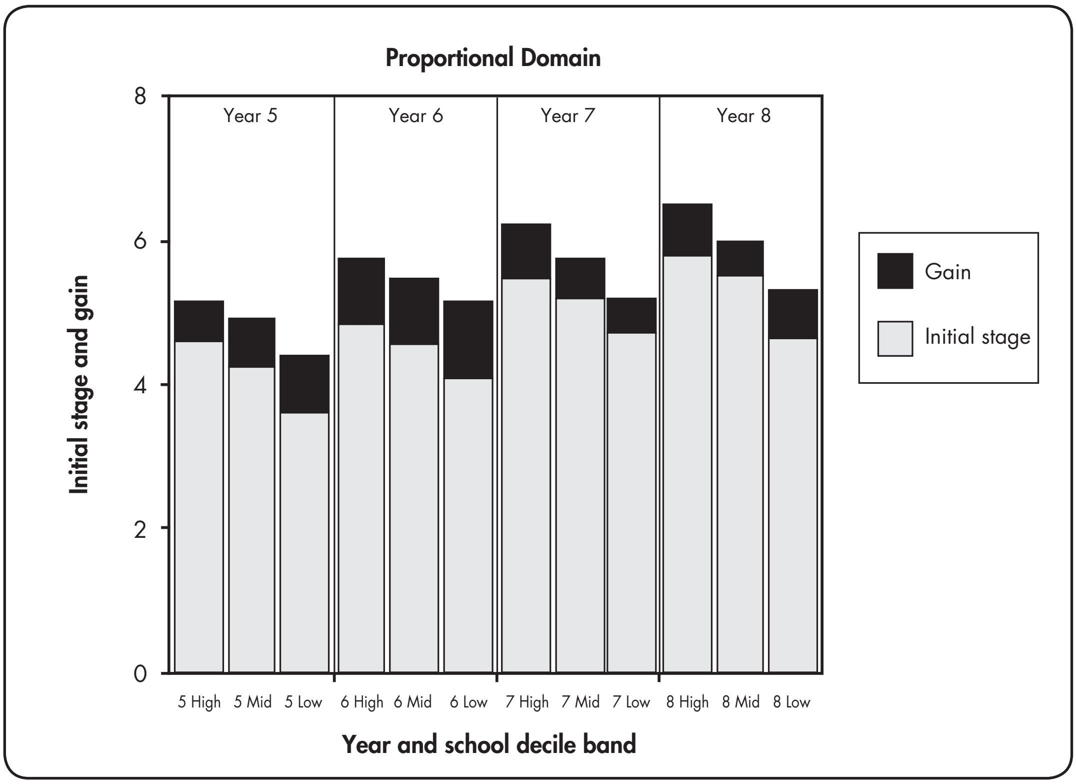

Figure 4. Average initial stage and gain on the proportional domain on the Number Framework as a function of year level and school-decile band

Students' Performance over Two Years of NDP

The Impact on Students' Performance of a Second Year on the NDP

One hundred and four schools participated in the NDP initiative for the first time in 2008, compared with 178 schools in the previous year (2007). Of the 178 schools that participated for the first time in 2007, 92 schools provided complete data for both 2007 and 2008. (Note: 68 schools returned data in 2007 but not in 2008, while three schools returned data in 2008 but not in 2007).

Comparison of the cohorts over successive years shows an increase in the proportion of students from low-decile schools from 2007 to 2008 at all year levels (see Appendix A, p. 169). The pattern for middle- and high-decile schools varies according to year level, with an increase in middle-decile schools at the upper primary level (years 4–6) and a decrease in high-decile schools. Although teachers at these year levels were not required to submit data in the year following their professional development, it is interesting to note that of those who chose to submit data, consistently more were from low-decile schools. This may reflect the higher staff turnover at low-decile schools found in other studies (for example, Ritchie, 2004) and hence the need to up-skill new teachers by involving them in professional development in the following year.

Comparison of the 2007 year 7–8 and year 9 cohorts with the same year-level cohorts in 2008 indicates a slight loss of students from high-decile schools and a relative gain in those from low-decile schools. Such patterns mean that data showing changes in performance and progress must be interpreted carefully because they may reflect changes in the cohort rather than particular aspects of the intervention, such as the teachers' deepening understanding of the Number Framework or the students' development of more strategic approaches to solving problems. Even looking diagonally across (for example, year 7 students in 2007 compared with year 8 students in 2008), it is evident that there is a drop-off in high-decile students in year 8 (24%) compared with year 7 (33%).

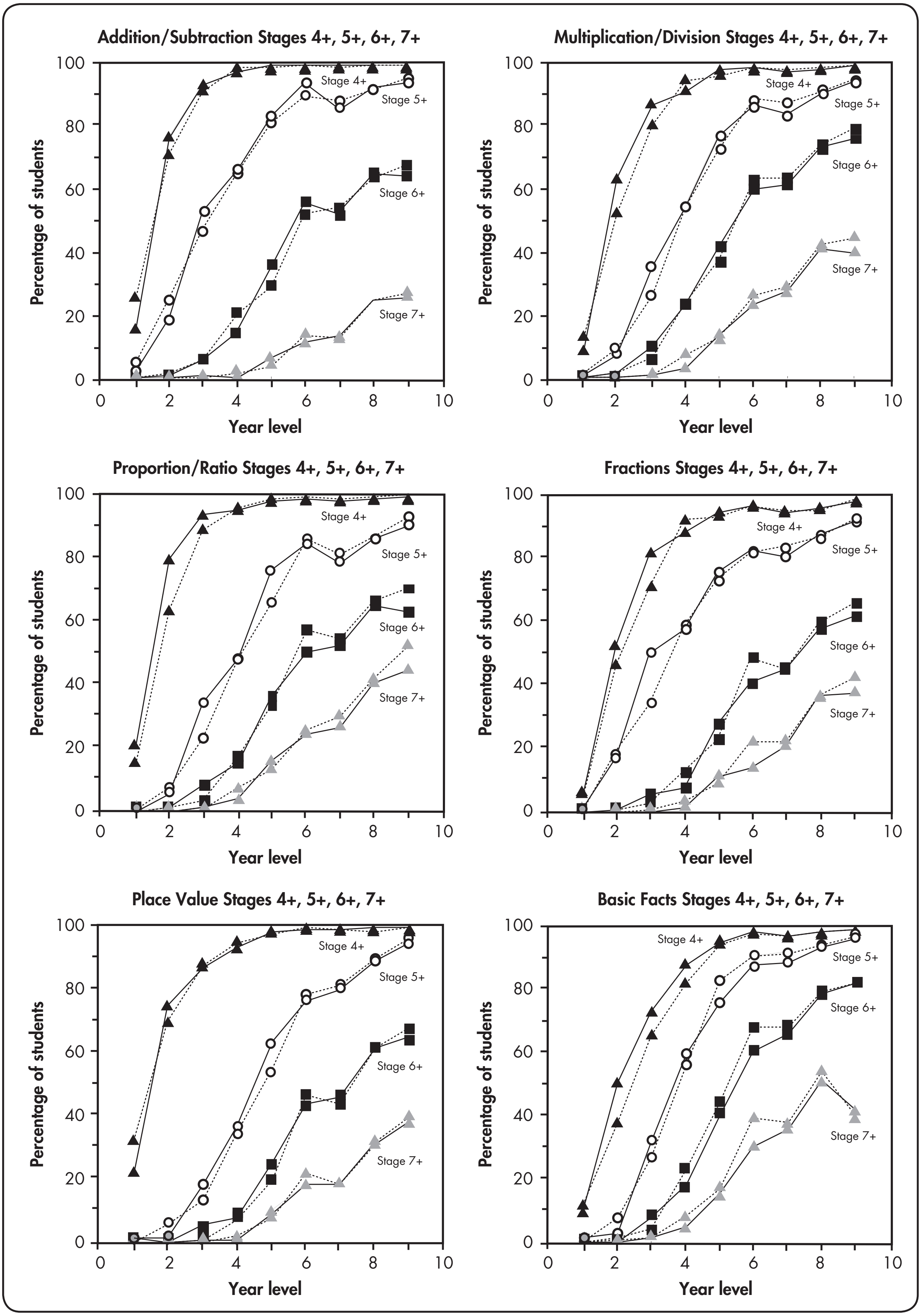

Figure 5. Percentages of students at or above stages 4–7 on strategy and selected knowledge domains as a function of year on NDP (2007, 1st year: broken line; 2008, 2nd year: solid line)

Figure 5 shows the percentages of students at or above stages 4–7 on the strategy and knowledge domains. The graphs show patterns that are fairly similar from one year to the next. Appendix E (pp. 180–184) and Table 4 show percentages for the strategy domains, with boxes indicating the year levels and cumulative stages that are relevant to the Ministry of Education's curriculum level expectations. A summary of the differences between the first year (2007) and the second year (2008) is presented in Table 5.

Table 4

Percentages of Students at or above Stages 4–7 on the Number Framework in the First (2007) and the Second (2008) Year of the NDP as a Function of Year Level and Domain (Final Data)[4]

Domain/Stage	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Additive domain									
First year 2007	*350*	*504*	*573*	*536*	*574*	*470*	*2390*	*2285*	*3327*
Stage 4+	26	71	91	99	99	100	99	100	100
Stage 5+	5	25	47	65	81	90	88	92	95
Stage 6+	0	1	5	21	29	52	54	64	68
Stage 7+	0	0	0	2	4	14	13	25	27
Second year 2008	*274*	*278*	*360*	*449*	*451*	*589*	*2036*	*2447*	*2912*
Stage 4+	16	77	93	97	99	100	99	100	100
Stage 5+	2	19	53	66	83	94	85	92	94
Stage 6+	0	1	6	14	35	56	52	65	64
Stage 7+	0	0	1	0	6	11	14	25	25
Multiplicative domain									
First year 2007	*350*	*504*	*573*	*536*	*574*	*470*	*2390*	*2285*	*3327*
Stage 4+	13	52	80	94	96	99	98	98	99
Stage 5+	1	10	26	54	73	89	88	91	95
Stage 6+	0	1	6	24	37	63	64	74	79
Stage 7+	0	0	1	7	13	26	29	42	45
Second year 2008	*274*	*278*	*360*	*449*	*451*	*589*	*2036*	*2447*	*2912*
Stage 4+	8	63	87	91	98	99	97	98	99
Stage 5+	0	7	35	55	77	87	84	90	94
Stage 6+	0	1	10	23	42	60	61	73	76
Stage 7+	0	0	1	3	14	23	27	41	40
Proportional domain									
First year 2007	*350*	*504*	*573*	*536*	*574*	*470*	*2390*	*2285*	*3327*
Stage 4+	15	64	89	96	99	100	99	100	100
Stage 5+	0	7	23	49	66	86	81	87	93
Stage 6+	0	1	3	17	34	57	55	67	71
Stage 7+	0	0	1	7	13	26	30	42	53
Second year 2008	*274*	*278*	*360*	*449*	*451*	*589*	*2036*	*2447*	*2912*
Stage 4+	21	80	94	95	98	99	98	99	100
Stage 5+	0	5	34	48	76	85	78	87	90
Stage 6+	0	1	8	15	36	51	53	65	63
Stage 7+	0	0	1	4	16	25	27	41	45

[4] The boxes indicate the percentage of students at a particular stage at the end of each curriculum level according to the national numeracy expectations.

Table 5
Changes (between 2007 and 2008) in Percentages of Students at or above Stages 4–7 on the Number Framework in the First and the Second Year of the NDP as a Function of Year Level and Domain

Domain/Stage	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Additive domain									
Stage 4+	–10	6	2	–2	1	0	0	0	0
Stage 5+	–3	–6	6	1	2	4	–3	0	–2
Stage 6+	0	–1	0	–7	7	4	–3	1	–3
Stage 7+	0	0	0	–1	3	–3	1	0	–2
Multiplicative domain									
Stage 4+	–4	11	7	–3	2	0	–1	–1	0
Stage 5+	–1	–3	9	0	4	–2	–4	–1	–1
Stage 6+	0	0	4	–1	4	–3	–2	–1	–3
Stage 7+	0	0	0	–4	1	–3	–2	–1	–5
Proportional domain									
Stage 4+	6	16	5	–1	–1	–1	–1	0	0
Stage 5+	0	–2	11	0	10	–1	–3	0	–3
Stage 6+	0	0	5	–2	3	–7	–2	–2	–8
Stage 7+	0	0	1	–4	3	–1	–3	–1	–8

It is clear from Table 5 that a consistent improvement in performance from one year to the next was evident only at year 2, in relation to the proportion of students who were at or above stage 4 on each domain. The difference was relatively small for the additive domain (6%) but increased for the multiplicative (11%) and proportional (16%) domains. At higher year levels, the difference was small or inconsistent across the domains. It is interesting to note such a difference for year 2 students at or above stage 4. This raises questions about the challenges for teachers of understanding just what is involved in the learning progressions as students move up the Number Framework. It may be that it is much easier for teachers in their second year in the NDP to appreciate just what is involved in moving through the various counting stages (0–4), and that this second year on the NDP enables them to benefit from their involvement the previous year, thus consolidating their learning and increasing the gains for students. Further research is needed to uncover the reasons for these patterns of difference.

It is somewhat surprising to note that the changes between 2007 and 2008 are not in a positive direction for students in years 7–9. Teachers in their schools were funded for two years of professional development, and thus it would be reasonable to expect to find more students reaching higher stages on the Number Framework once their teachers were in the second year of the NDP and had a better understanding of the NDP approach. However, this does not appear to be the case.

It is also interesting to note that a similar analysis carried out independently on year 9 data as part of the analysis for the Secondary Numeracy Project (SNP) showed a very similar outcome (Tagg & Thomas, 2009). One possible reason may be that coming to understand the higher stages on the Number Framework presents an enormous challenge for teachers and it may require considerably more than two years of professional development to have an appreciable impact on teachers' content knowledge and pedagogical content knowledge in mathematics (Ball, Hill, & Bass, 2005; Hill, Schilling, & Ball, 2004; Hill, Rowan, & Ball, 2005; Lamon, 2007). In-depth research in the classrooms of teachers working at the year 7–8 level is consistent with the idea that ways of supporting students' multiplicative thinking, for example, take some years to understand fully (see Young-Loveridge & Mills, this volume).

Discussion

The analysis reported in this paper has shown that students in the early school years make substantial progress on the additive domain, with almost all students working at stage 4 advanced counting or higher by the end of year 3. By the end of years 7 and 8, students' performance on the multiplicative and proportional domains is higher than on the additive domain.

There are issues of comparability across different domains. The biggest discrepancy is students' performance at stage 7 on the additive domain, which is much harder to attain than stage 7 on the multiplicative or proportional domains.

Comparison of students' performance with the curriculum level expectations stated in *The New Zealand Curriculum* (Ministry of Education, 2007) showed that only year 2 students were close to the expectations for level 1. At years 4, 6, and 8, only about half of the students met the expectations for curriculum levels 2, 3, and 4 respectively. This finding has some important implications for teacher knowledge of the complexities of part–whole thinking, both additive and multiplicative. This result is consistent with work elsewhere that shows how important it is for teachers to fully understand the mathematics they are teaching, as well as to have the pedagogical content knowledge in mathematics to anticipate the developmental progressions and difficulties for their learners (for example, Ball et al., 2005; Hill, Schilling, & Ball, 2004; Hill, Rowan, & Ball, 2005).

Analysis showed that students' performance varied as a function of school decile, with students at high-decile schools performing better than those at middle- and low-decile schools. This finding reinforces the need to ensure that teacher quality is maintained at low-decile schools.

An analysis of students at schools that submitted data in both 2007 and 2008 showed virtually no improvement in the second year of the NDP, except for year 2 students. This finding is of particular concern at years 7 and 8, where schools are funded for two years of professional development. Further research is needed to document how teachers' knowledge and understanding of mathematics changes over the course of their professional development in order to gain a better understanding of how teachers experience the professional development that is provided. This issue is of particular significance because the discrepancies between students' performance and the curriculum level expectations are greatest at the end of year 8 (level 4).

Analysis of the students' average stages at the beginning of the NDP and of gain scores showed that in the primary years (years 0–6), those who started lower on the Number Framework tended to make greater gains, particularly on the additive domain. By years 7–8, those who started higher on the Number Framework also tended to make greater gains; this was particularly noticeable on the multiplicative and proportional domains. The end result is that initial differences favouring New Zealand European students and those at high-decile schools were exacerbated and gaps between groups appeared to widen. This finding reinforces the importance of ensuring that low-decile schools maintain high levels of teacher quality.

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Longitudinal Patterns of Performance: Te Poutama Tau

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Te Poutama Tau continues to focus on improving student performance in pāngarau (mathematics) through improving the professional capability of teachers. Te Poutama Tau is based on Te Mahere Tau (Ministry of Education, 2007a), the Number Framework of the Numeracy Development Projects, in which students progress through stages of learning. The considerable corpus of student achievement data collected during Te Poutama Tau provides information on longitudinal patterns of student performance. In general, Te Poutama Tau students' progress is more positive across the knowledge domains of Te Mahere Tau than across the strategy domains. The results show that a student's language proficiency does have some effect on performance, particularly on the use of strategies. The biggest influence on student performance, however, seems to be the teacher.

Background

Te Poutama Tau (the Māori-medium component of the Numeracy Development Projects [NDP]) developed from a 2002 pilot project (Christensen, 2003, 2004) and has evolved considerably over the last six years. The primary catalyst for the development of Te Poutama Tau was the opportunity to develop the teaching of mathematics (pāngarau) in the medium of Māori. Te Poutama Tau continues to focus on improving student performance in pāngarau through improving the professional capability of teachers. Te Mahere Tau (the Number Framework), which provides a clear description of the key concepts and progressions of learning for students, is central to Te Poutama Tau.

Te Poutama Tau data has provided a considerable corpus of data for analysis and investigation. Analyses of student achievement data gathered every year from 2002 has provided a valuable source of information for teachers, schools, and numeracy facilitators involved in Te Poutama Tau.

This paper is in two main parts. Part A reports on the results of the 2008 Te Poutama Tau programme, and Part B reports on longitudinal patterns of student performance. The research focused on the following questions:

- How do patterns of performance and progress compare across the years 2004–2008?
- What are the links between language and achievement?
- Is there a relationship between a student's initial point of entry and progress over time?
- What are the effect sizes between the variables of Te Mahere Tau?
- What are the key factors that affect change in student performance?

Part A: An Evaluation of Te Poutama Tau 2008

Method

Thirty-two schools participated in Te Poutama Tau in 2008, and 22 of these provided data for Part A of this paper. Each year, results for each Te Poutama Tau student, classroom, and school are entered on the national database (www.nzmaths.co.nz). The database shows the progress students have made on Te Mahere Tau between the initial and final diagnostic interviews (Te Uiui Aromatawai, Ministry of Education, 2007b). In this part of the study, the 2008 results were compared with the longitudinal data dating back to 2004. The longitudinal results are discussed in Part B in terms of patterns of performance.

Participants

The following summaries of the data were restricted to those students with both initial and final test results. In 2006, 1153 students completed both the initial and final diagnostic interviews; in 2007, there was complete data for 1323 students; and in 2008, there was complete data for 766 students. Although a few year 9 and 10 students participated in 2006 (see Figure 1), a specific Te Poutama Tau programme was developed in 2007 and 2008 for students in wharekura (Māori-medium secondary), which accounts for the increase in numbers in these years.

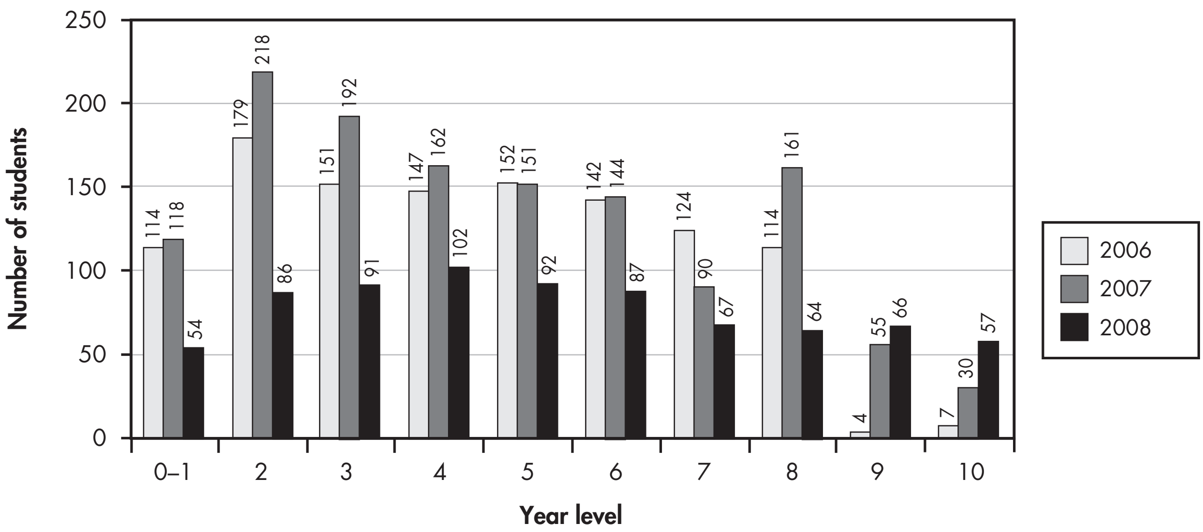

Figure 1. Distribution of Te Poutama Tau students across the year levels 2006–2008

Student Achievement and Year Level 2008

Strategy Domains

The graph in Figure 2 shows the variation in the mean gain for the strategy domains of Te Mahere Tau across the year levels. For example, students at years 0–1 made a mean stage gain of just under 1 for addition and subtraction, while at year 6, the mean gain was just over 0.6. A number of variables need to be considered when interpreting the results, including the increasing complexity of the stages (higher levels are more complex), the ceiling effect, and the number of years that students have been involved in Te Poutama Tau. It is expected that years 0–1 will make more progress in addition and subtraction than in multiplication and division or in proportions. Addition and subtraction is the only strategy domain included in Form A of the diagnostic interview (Uiui A), the interview most year 0–1 students will be assessed on. There is a noteworthy mean stage gain in 2008 in proportional thinking for year 2 (0.9) and year 3 (0.8) in comparison with 2007, where the mean stage gain for year 2 was 0.0 and for year 3 was 0.5 (Trinick & Stevenson, 2008).

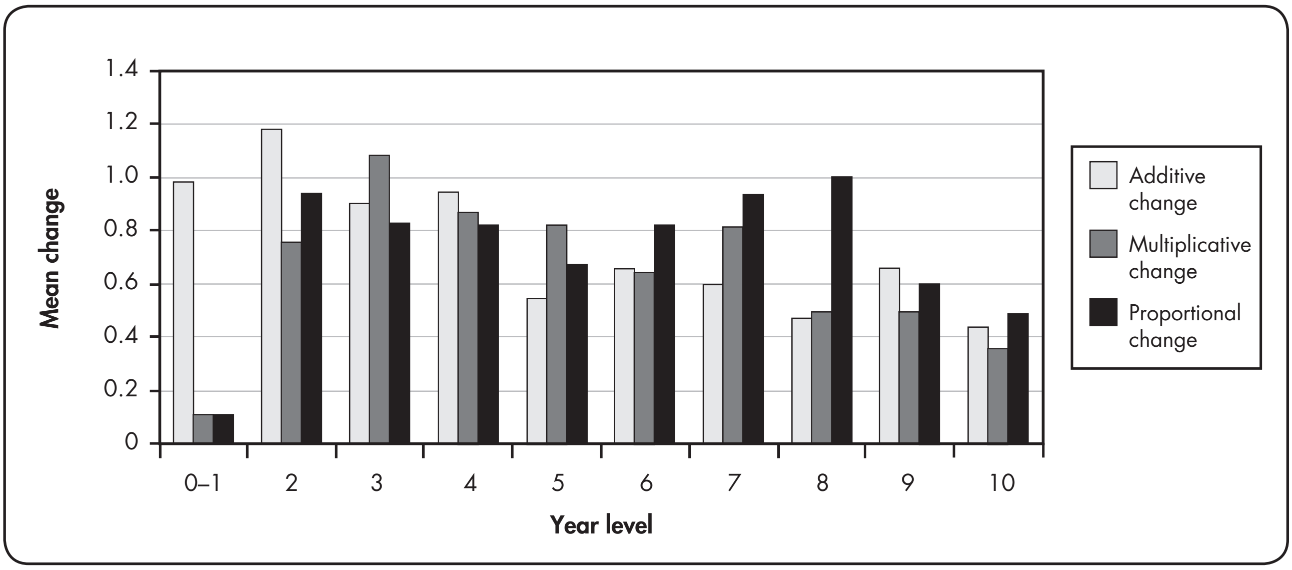

Figure 2. Mean stage gain by year level in 2008 for student achievement in strategy domains

Knowledge Domains

The knowledge domains tend to follow a similar pattern (Trinick & Stevenson, 2007, 2008). There is significant growth in the earlier years in FNWS (forward number word sequence) and BNWS (backward number word sequence), with a similar pattern of regression in later years. The regression can be partly explained by the ceiling effect, that is, a number of students in the older age groups may already be at the upper stages and will therefore not show any progress. It is also important to note that numeral identification (NID) (see Figure 3) as a separate data section is only part of Uiui A. Therefore students who proceed beyond tests A–E or to test U will not register mean stage progress in NID. These results do show that some year 6–8 students were tested on Uiui A.

With fractions, place value, and basic facts, there is growth initially, then a regression around years 5 and 6, and then some growth again. Students tend to start learning fractions later than place value and basic facts, and it is highly likely that the year 0–1 and year 2 students were tested on Uiui A. There is no fractions component in Uiui A, hence the lack of data for years 0–1.

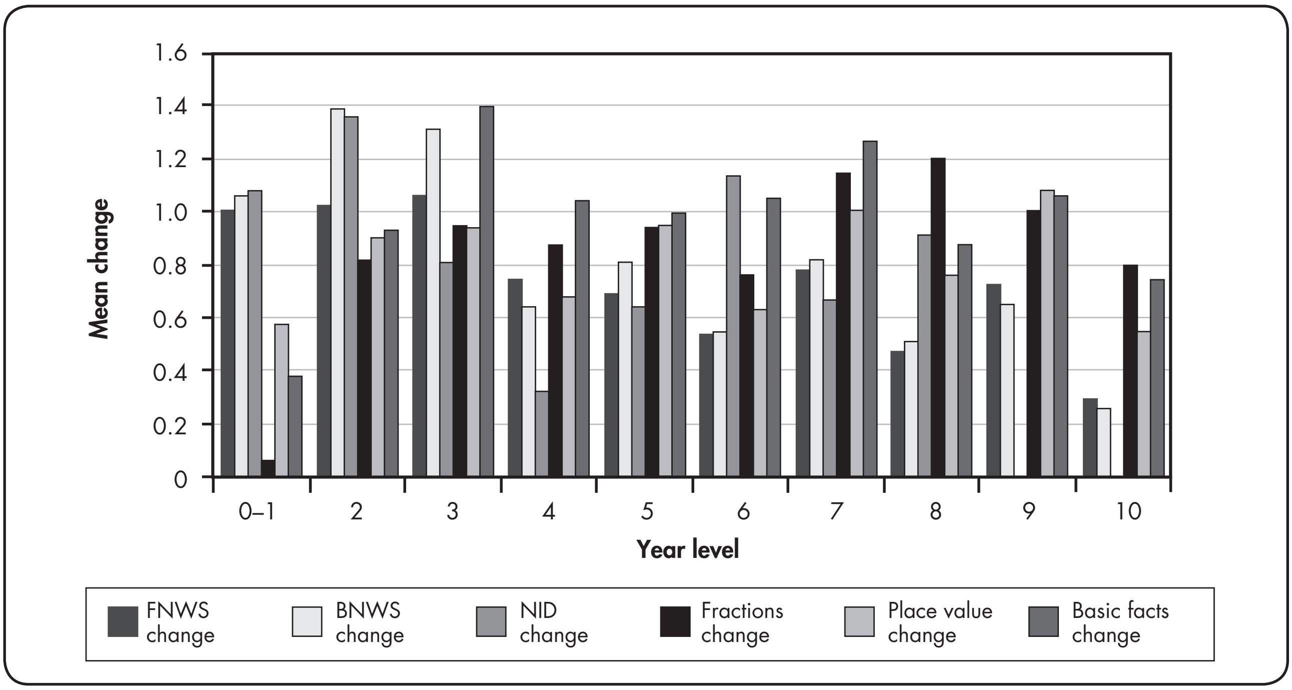

Figure 3. Mean stage gain by year level in 2008 for student achievement in knowledge domains

Language Spoken at Home

One of the key objectives of Te Poutama Tau has been to support the broader aims of Māori-medium schooling in the revitalisation of te reo Māori. The graph in Figure 4 shows that, for 2008, there were few students whose only home language was te reo Māori. This subjective judgment is made by the teacher who enters these results into the national database when they are entering student achievement data. For the majority of students, both English and Māori were spoken equally or English was spoken most of the time.

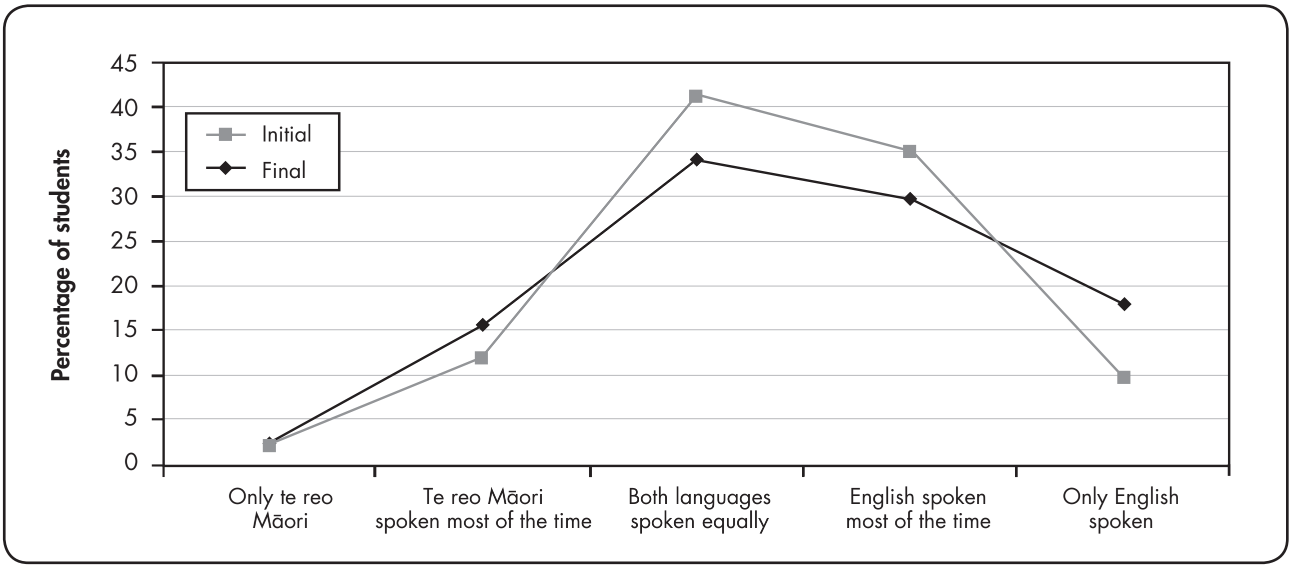

Figure 4. The language spoken at home by the 2008 Te Poutama Tau students

Language Proficiency of the Students

The majority of the students were classified as being either "he matatau" (proficient) and or "he āhua matatau" (reasonably proficient). These ratings rely on the teacher's knowledge of the student's language ability, drawn from a number of indicators, including their oral and written work. As with similar studies (Trinick & Stevenson, 2005, 2006), approximately 80% of the 2008 students are rated as either he matatau or he āhua matatau.

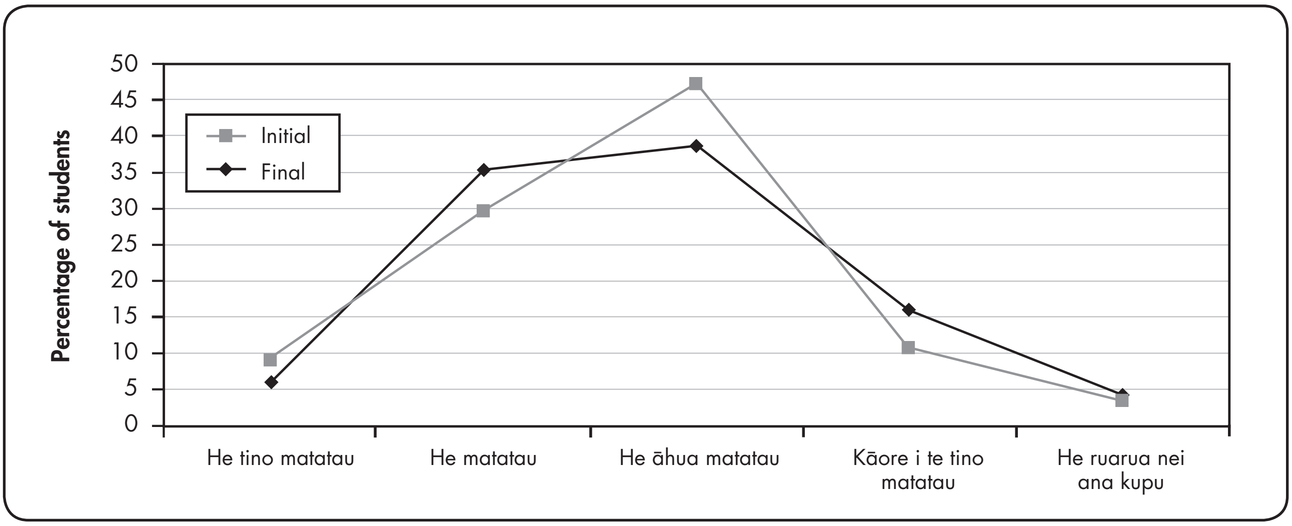

Figure 5. Te reo Māori proficiency of the 2008 students

Home Language by Mean Change

The graph in Figure 6 shows the mean stage gains across the strategy and knowledge domains of Te Mahere Tau in relation to the language(s) predominately spoken at home. The greatest mean stage gain seemed to be made by those students where Māori is spoken most of the time (Māori i te nuinga o te wā). However, the differences between each of the groups is not significant, although it is somewhat surprising that students whose home language was exclusively Māori (Māori anake) did not make similar or greater mean stage gains than those in the other groups. However, as noted previously, the rating is a subjective decision made by the teacher.

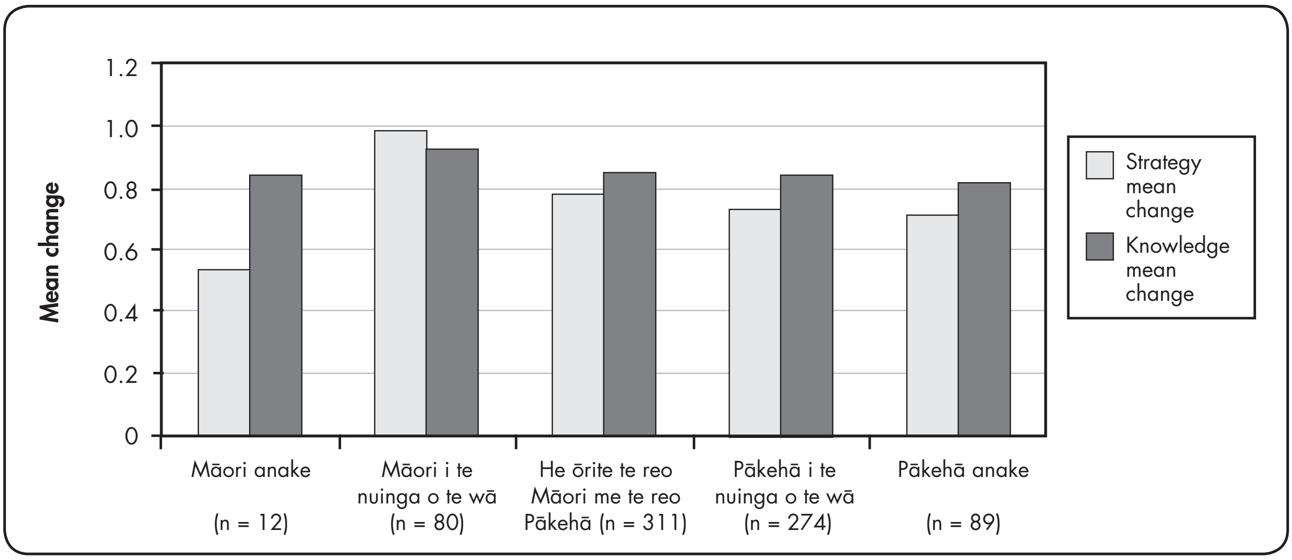

Figure 6. The 2008 mean stage gains across the strategy and knowledge domains in relation to home language

Teacher-rated Te Reo Māori Proficiency by Mean Change

The graph in Figure 7 shows the mean stage gain for the two major components of Te Mahere Tau in relation to students' levels of te reo Māori proficiency. As noted earlier in the discussion on Figure 6, this is a judgment made by the teacher based on the knowledge of their students.

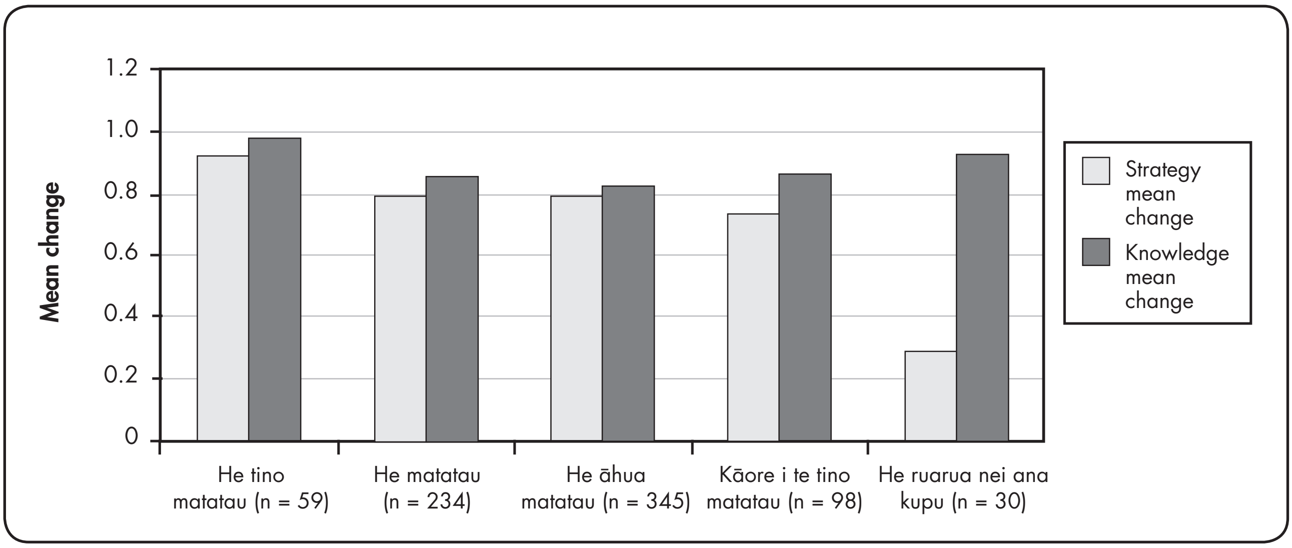

Figure 7. The 2008 mean stage gains across the strategy and knowledge domains in relation to students' language proficiency

This graph shows that the greatest mean stage gains across the two major domains (knowledge and strategies) were made by those students judged very proficient. The lowest mean gains in the strategy domain were from the group identified as having very limited te reo Māori proficiency in the strategy domain. This is not surprising and is consistent with previous studies that have found that students do require a reasonable fluency to articulate their mental strategies (for example, Trinick & Stevenson, 2006).

In summary, the domains of fractions, decimals, percentages, and proportion remain learning challenges for students. These areas will need to remain a focus of the professional development programme that supports Te Poutama Tau. However, there has been positive progress in the area of proportional thinking for students in the early years. The patterns of progress across the various components of the knowledge domains are fairly consistent. There is significant growth in the early years, with a dip, particularly at year 5 (see figures 2 and 3). The language of the home appears to have some effect on student progress in Te Poutama Tau. However, the number of homes identified as speaking only Māori is small in number and the rating is done by the teacher. It is questionable whether the families themselves would provide a similar rating.

Part B: Longitudinal Patterns of Progress and Performance

As noted earlier, a considerable corpus of data has been collected that enables a range of general statements to be made about student achievement in Te Poutama Tau and the factors that do affect progress and performance. This section examines patterns of performance over the four years of the implementation of Te Poutama Tau and only includes the data of students who have participated in Te Poutama Tau for at least two to four years. Te Poutama Tau has predominately focused on the earlier years, hence most of the data comes from the earlier year groups.

Mean Stage Gains across Te Mahere Tau

The graph in Figure 8 shows the mean stage gains across the strategy and knowledge domains of Te Mahere Tau for the years 2005–2008. In general, students' progress is more positive across the knowledge domains of Te Mahere Tau than across the strategy domains. Each year's data has been used to provide a guide and focus for professional learning in subsequent years. For example, as a result of the 2006 and 2007 findings, there has been a continued focus on fractions and proportional thinking. The 2008 results for these two areas show positive stage gains because of this continued attention by facilitators and teachers to these two challenging areas of Te Mahere Tau.

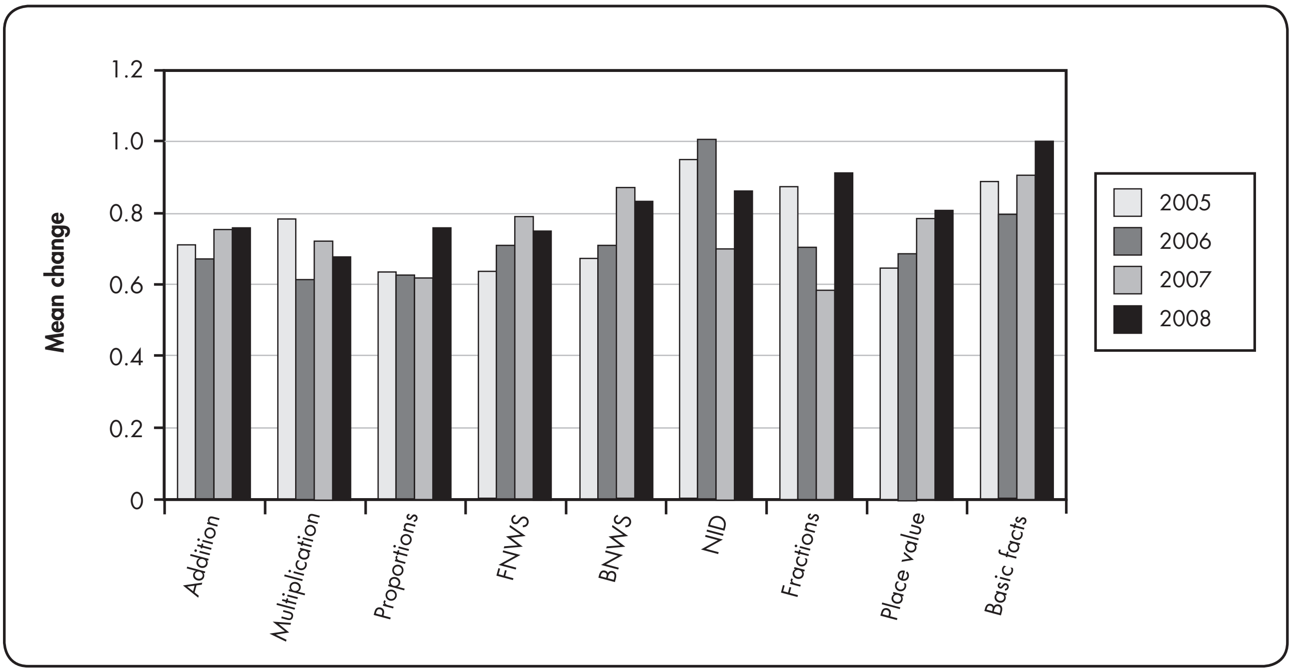

Figure 8. Mean stage gains across Te Mahere Tau

Average of Mean Stage Gains

Figure 9 shows how the average for the final results for all tests varies across the year levels for 2005–2008. This graph reflects the effect on progress through Te Mahere Tau of increasing stages of complexity and the ceiling effect. From year 2 to year 7, the trend is reasonably consistent. In most years, there has consistently been a dip at year 2, with a levelling off or rise in year 6. However, as noted earlier, it is important to interpret cautiously because the stages do not constitute an interval scale.

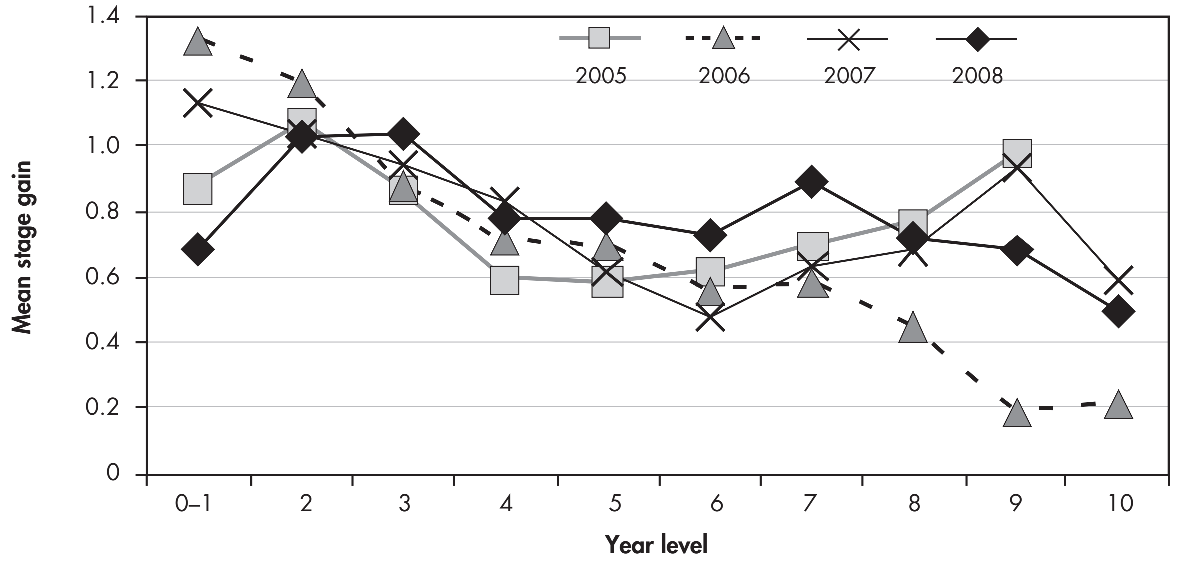

Figure 9. Comparison of students' average mean stage gain across the years 2005–2008

Stage of Entry (Strategy) and Progress over Time on Final Strategy Score

Figure 10 essentially shows that students who initially tested at a higher stage on the strategy tests at year 1 maintained that advantage (albeit a small one) for at least four years. There was only a small amount of data for students beyond year 4 who had been in Te Poutama Tau long enough to model the trends. Outcomes about performance after year 4 can be made when further data is collected or statistical modelling is completed.

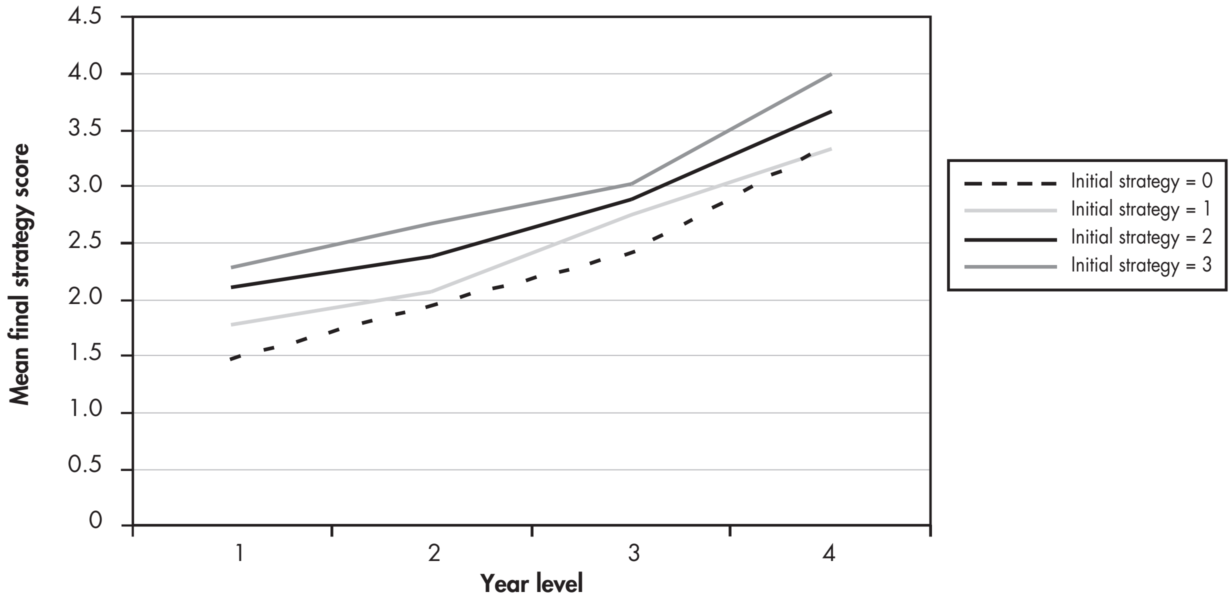

Figure 10. Strategy stage on entry and progress of time on final strategy score

Stage of Entry (Strategy) and Progress over Time on Final Knowledge Score

Figure 11 shows the relationship between the initial strategy stage at entry and students' mean final knowledge score. As with the graph in Figure 10 that shows performance on the tests of strategy, this graph seems to suggest that the higher the student's initial strategy score at year 1, the better their performance in the knowledge domains. This difference may be attributable to student ability as well as to any numerical learning prior to starting school.

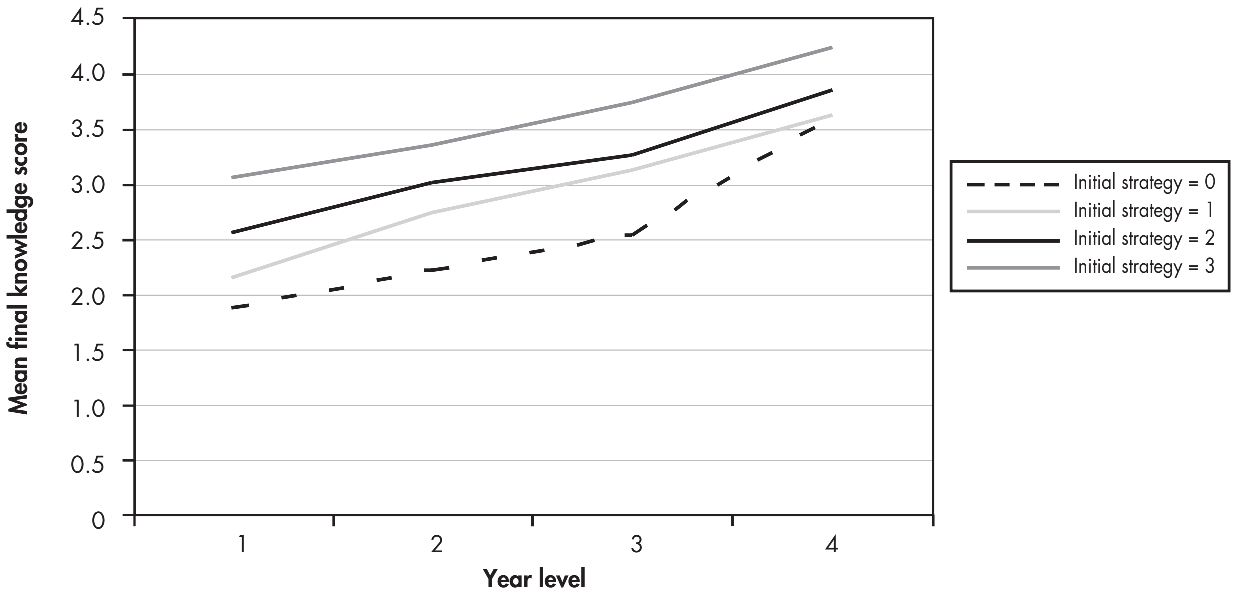

Figure 11. Strategy stage on entry and progress over time on final knowledge score

Home Language of Students

The following graph compares mean change for all domains by home language over the years 2004–2008. It is difficult to know why the mean change where Māori only is spoken at home is so variable across the years 2004–2008. This pattern could be attributed to the small numbers of students from homes that are rated as Māori-only speaking homes.

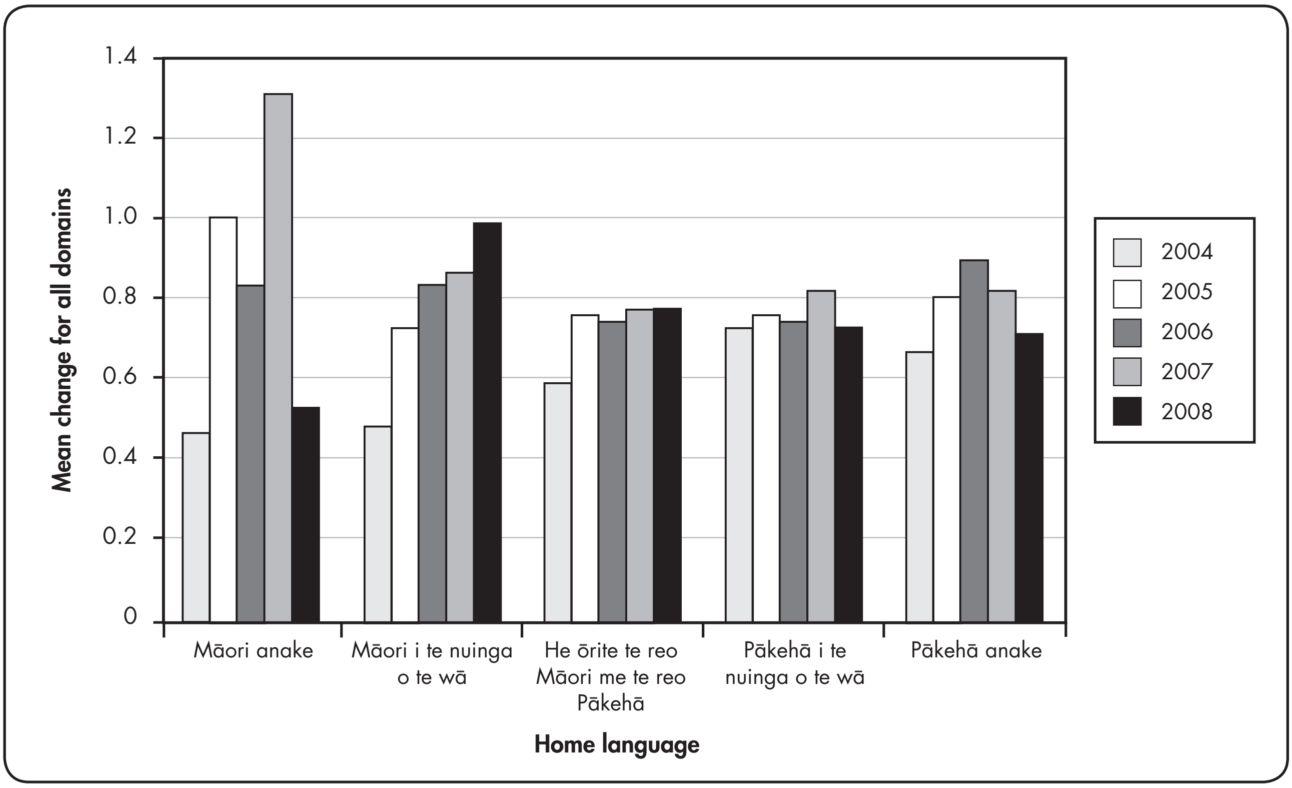

Figure 12. Mean change in relation to language spoken at home

Variables that Impact on Student Performance

One of the aims of Te Poutama Tau has been to identify the variables that impact on student performance and their effect sizes. The results were arrived at using Generalized Estimating Equations (GEE) analysis: students nested within classes, classes within schools, and repeated measurements for all data in the years 2003–2007. The GEE procedure allows the analysis of situations where the observations are correlated, such as repeated measurements and clustered sampling (for example, sampling participants within the same class). Such an analysis was necessary for the Te Poutama Tau data because one cannot assume independence of observations. Results for students will also depend on the class (teacher, class size, and so on) and the school (resourcing, location, bilingual/kura, and so on). The analysis was also conducted over time, with students linked by a common reference number[1]. The table in Appendix F (p. 185) summarises Beta coefficients and significance levels for each final score (on each test, for example, final addition score, final multiplication score) as a dependent variable, with the initial score on all the tests, year, gender, and language as dependents. The model also included all initial scores and language by year as a two-way interaction effect and a class nested within school.

Results

Overall, the analysis found that “class located within school” was statistically significant ($p < 0.001$) for all the dependent variables (the Beta coefficients were several orders larger than any other), with the significance of other terms varying depending on the dependent variable. This result is unsurprising, given that a number of researchers argue that the single largest influence on a student is the teacher (Hattie, 2003).

[1] This was done in Excel by comparing date of birth, school ID, year, and gender to find an appropriate match.

The Relationships between the Various Domains of Te Mahere Tau

The strength of the relationship between the various domains is measured by the correlation coefficient. The correlation coefficient is a measure of the degree of linear relationship between two variables. One cannot draw cause and effect conclusions based on correlation. Correlation refers to the strength of relationship between variables. The variables covered below have been identified as being significant ($p < 0.05$ – see Appendix F).

Addition/Subtraction

Only the class-within-school effect was statistically significant in the domains of addition and subtraction.

Multiplication/Division

A large number of variables were related to success in multiplication. Initial test results for addition, multiplication, and fractions were negatively correlated with multiplication final test results. Why this is so is not clear. Conversely, initial test results for proportions, FNWS, BNWS, NID, and place value were positively correlated with final test scores for multiplication. After accounting for interaction with year, the domains of proportions, FNWS, and place value showed small negative relationships with the final multiplication test result. Addition by year, multiplication by year, and fractions by year all showed small positive correlations.

Proportion

Initial addition scores were negatively related to the final proportions scores. Initial scores on multiplication, proportions, and fractions were positively correlated with final proportions scores.

FNWS and BNWS

Both final scores on FNWS and BNWS were positively correlated with initial scores on FNWS.

NID

Initial scores on proportions were negatively related with the final scores on the NID. The initial scores on the NID were positively related to the final scores.

Fractions

Initial scores on FNWS, fractions, and basic facts were positively related to final scores on fractions. A higher rating of the students’ te reo Māori was positively related to a higher final score. There was also a positive relationship between gender and fractions, where boys tended to do slightly better in fractions than girls did.

Place Value

Initial scores for place value and basic facts were positively related to the place value final scores.

Basic Facts

Initial scores on basic facts were positively related to the students’ final scores on the basic facts test.

Summary of Longitudinal Patterns of Progress

In general, Te Poutama Tau student progress is more positive across the knowledge domains of Te Mahere Tau than across the strategy domains. This may be partly attributed to language proficiency, where students do require a reasonable fluency to articulate their mental strategies. Fractions, decimals, percentages, and proportions remain a learning and teaching challenge for students and teachers. Te Poutama Tau has been valuable in providing opportunities for facilitators and teachers alike to develop the mathematics register to facilitate student learning in domains such as fractions. In teacher professional learning programmes for Māori-medium teachers, it is important to introduce an intellectualised variety of te reo Māori at a high level in order to develop the professional competency of the teachers who will in turn implement an intellectualised variety of te reo Māori in primary and secondary schools. This enhances the development of the school-based Māori-medium mathematics register.

The patterns of progress are very similar across years 2005–2008 for year 2–8 students (see Figure 9), with a consistent dip around the 5–6 year level. The patterns of progress reflect the nature of Te Mahere Tau, that is, the increasing complexity of the stages and the struggle that many students seem to have in transitioning to part–whole thinking. It would seem that where there has been a Te Poutama Tau facilitator and teacher focus on particular areas of Te Mahere Tau, there has been a corresponding improvement in student performance the following year. This suggests that student progress is affected by a number of variables, including teacher competence, quality of time spent learning, and the quality and availability of support resources.

The longitudinal data (see figures 10 and 11) shows that students who initially tested at a higher stage on Te Mahere Tau maintained that advantage to at least year 4. Outcomes about performance after year 4 can be made when further data is collected.

Nationally and internationally, little is known of the impact of a student's language proficiency on their learning of mathematics. The issues of language proficiency and the learning of mathematics are complex areas in their own right. However, the analyses of the data from Te Poutama Tau suggests that the language proficiency of a student does have an effect on their ability to articulate their mental strategies. This is not necessarily a problem if the test is written, but it may impinge on the student's ability to communicate. Many researchers and national policy documents (Pimm, 1987; Ministry of Education, 2008) support the idea of encouraging students to communicate mathematically. Being able to communicate requires students to extract meaning from mathematics statements and to convey that meaning in spoken or written discourse.

As noted, one of the aims of Te Poutama Tau has been to identify the variables that impact on student performance and their effect sizes. Overall, the analysis of the data found that class located within school was statistically significant. This result is unsurprising, given that a number of researchers argue that the single largest influence on a student is the teacher (Hattie, 2003). However, this Te Poutama Tau study does not consider a number of variables that impact on student learning, including the social, cultural, and economic impacts. Mathematics contains many interrelated domains and concepts. This study suggests that students' knowledge of multiplication affects a number of other domains, including fractions, decimals, percentages, and proportions. This has implications for the learning of these challenging areas for students.

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Te Poutama Tau Student Performance in asTTle

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This study examines whether students participating in Te Poutama Tau transfer their knowledge to solving problems that differ in form and context. Additionally, it examines how these students perform in traditional written-type tests, in particular the asTTle test[1], against the national norms for Māori-medium schools. An asTTle test was given to one cohort of year 4 and one of year 7 students who had participated in Te Poutama Tau, and the results were compared with those of a previous study[2]. In this test, both cohorts of students performed above the national norm for Māori-medium schools on number knowledge items. However, across all test items, both the 2007 and 2008 year 4 cohort performed below the national norms for Māori-medium schools. On the other hand, both the 2007 and 2008 year 7 cohorts performed above or close to the national norms for Māori-medium schools, although not noticeably so in algebra.

Background

Initiated as a pilot in 2002, Te Poutama Tau is the Māori-medium component of a key government initiative aimed at raising student achievement by building teacher capability in teaching and learning numeracy in schools (Christensen, 2003). Te Poutama Tau acknowledges professional development as a key to integrating theory and practice for quality outcomes in Māori-medium mathematics (pāngarau) education (Trinick & Stevenson, 2006, 2007). By improving the professional capability of teachers, students' performance in numeracy is also improved (Christensen, 2003). The Number Framework (Te Mahere Tau) is central to Te Poutama Tau. It outlines for teachers the stages of number knowledge and the operational strategies through which students progress in their learning of number (Ministry of Education, 2007a). Students are assessed against the stages of Te Mahere Tau using a diagnostic interview (Te Uiui Aromatawai, Ministry of Education, 2007b), which stresses conceptual understanding and students' internal construction of mathematical meanings (Trinick & Keegan, 2008).

Research to date based on the data from diagnostic interviews indicates that Te Poutama Tau has improved outcomes for students (Trinick & Stevenson, 2005, 2006, 2007, 2008). This study examines whether Te Poutama Tau students transfer their knowledge to solving problems that differ in form and context. Additionally, it examines how these students perform in traditional written-type tests, in particular the asTTle (Assessment Tools for Teaching and Learning [He Pūnaha Aromatawai mō te Whakaako me te Ako]) test, against the national norms for Māori-medium schools. As a result of an earlier Te Poutama Tau/asTTle study, questions arose as to the validity of the asTTle norms for Māori-medium schools and whether the students' results in that study would be consistent with those in future studies (Trinick & Keegan, 2008).

AsTTle is an educational resource for assessing literacy and numeracy (in both English and Māori). It provides teachers, students, and parents with information about a student's level of achievement, relative to curriculum achievement outcomes[3], for levels 2–6 and national norms of performance for

1 asTTle: Assessment Tools for Teaching and Learning (He Pūnaha Aromatawai mō te Whakaako me te Ako)

2 Trinick & Keegan, 2008

3 The asTTle tests used in this study were based on the 1992 *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992). All references in this paper to the curriculum or to curriculum strands are to this 1992 curriculum document.

students in years 4–12. Teachers can use asTTle to create "paper-and-pencil" tests of 40- to 50-minute duration, which means that students must be able to read and write. After the tests are scored, the asTTle tool generates interactive graphic reports that allow teachers to analyse their students' achievement against curriculum levels, curriculum objectives, and population norms (for example, see figures 1 and 2 in this paper).

Aims of the Research

This study examined:

- What aspects of the asTTle test did Te Poutama Tau students perform well in and what were the gaps and areas of weakness?
- How do Te Poutama Tau students' asTTle data compare with the asTTle national norms for Māori-medium schools?
- How do these results compare with the students' performance in the 2007 study?

Method

Participants

Two schools agreed to participate in the 2008 study; one was from a large city and the other was from a small rural town. Both schools had recently participated in Te Poutama Tau. The aim was to replicate the 2007 study as closely as possible, so it was decided to continue focusing on year 4 and year 7 students.

In the 2007 study, year 4 students had been selected because this is the youngest cohort that can be reliably tested using asTTle. Additionally, earlier Te Poutama Tau studies showed a considerable dip in student progress that began in year 3 (Trinick & Stevenson, 2006, 2007). Why this was so is not entirely clear. A number of reasons were considered, including the fact that this is the age group where students are possibly moving towards part–whole thinking. It is also the age group where students may be exposed to a change in teaching pedagogy as they move from years 1–2 to years 3–4 (Trinick & Stevenson, 2007).

The 2007 year 7 cohort had been chosen to provide a comparison with year 4 for showing differences and similarities. Also, schools could use the data when the students were in year 8 to focus on gaps and areas of weakness before the students went on to wharekura (Māori-medium secondary schools) or to English-medium secondary schools.

The Test

An asTTle test focusing on number was generated for each year group in the study, and test scripts were sent out to schools for trialling. The two tests consisted of 32 test items, which were selected to cover number items from the Number and Algebra strands. The aim of the testing was to gain maximum information on students' performance on number and other items relevant to Te Poutama Tau. The nature of asTTle is such that individual test items cannot be selected without losing the capability of the asTTle tool to generate national norms (because norms are not available for individual test items) and associated data. The items in the 2008 test were not identical to those in the 2007 test, but both tests included test items that linked to the same Number and Algebra achievement outcomes in the curriculum. Measurement items were not included in the 2008 test; these were replaced by extra Number and Algebra test items because Te Poutama Tau has tended to focus on these two strands of the curriculum.

The test scripts returned by each participating school were marked, and then a report was compiled for each school. This report included four major reports for teachers, each of which provided different analyses of each year group. These analyses included:

- comparing student performance against a nationally representative Māori-medium sample;
- comparing student performance in relation to curriculum levels and difficulty;
- identifying curriculum outcomes that students had or had not achieved and which of these the students showed strengths in or revealed gaps or areas of weakness;
- allocating each student in a particular curriculum level as being either at the beginning, proficient, or advanced stages.

This report was ideal for assisting teachers to group their students.

AsTTle Tests: Results

All results reported in this section are based on the aggregated results of the 2008 year 4 and year 7 students and are displayed using three types of reports. The results are compared with those from 2007 to identify patterns in achievement.

The Reports

The asTTle reports are primarily aimed at answering the feedback question "How are Te Poutama Tau students doing in comparison with similar students in Māori-medium settings nationally?" AsTTle answers this question by providing comparative or normative information for the group of students in this sample.

Group Performance

Student achievement by year is shown in box-and-whisker plots that display both the national Māori-medium norms and the distribution of the student scores. The reports show the average of the year group and the range of achievement of that group. The box-and-whisker plots are based on five score points (top score, upper quartile, median, lower quartile, and bottom scores) attained by students participating in the test. The white box plot represents the performance of the 2008 Te Poutama Tau students, and the shaded plot represents the performance of the year 4 and year 7 national Māori-medium reference population. Groups that have short ranges within the box and/or the whiskers are more similar in their performance than those with wide ranges. Groups whose median scores are at the top or bottom of the reference group box (the student cohort in this study) probably differ from the national Māori-medium norm by more than chance.

Curriculum Functions

This report shows the aggregated results for each strand of the curriculum that was selected for these particular tests. In the tests generated for this 2008 study, only test items from the Number and Algebra strands were included (as noted earlier).

Learning Pathways Report

These reports were identified by generating learning pathway reports to answer the question "What are the strengths and weaknesses of student performance in regard to the curriculum outcomes?" A percentage is given of the student cohorts that were identified as having achieved/not achieved or as having strengths/gaps in regards to the curriculum outcomes. For this report, "achieved" and "strengths" have been aggregated and are reported under performance highlights. This is where more than 60% of the cohort was identified as having achieved and showed strengths in this outcome. "Not achieved" and "gaps" are aggregated as performance concerns. This is where more than 60% of the cohort was identified as having not achieved and as having gaps in their knowledge.

Comparison of the 2007 and the 2008 Year 4 Students

Results of the Year 4 students

Group performance

The aggregated data of all the 32 test items shows the average of the year group and the range of achievement of the group. Figure 1 shows that the 2008 year 4 Te Poutama Tau students' median in this study was slightly below the norm for students in Māori-medium schools. However, this is an improvement on the 2007 results, which were approximately 200 points below the national Māori-medium norm (Trinick & Keegan, 2008). The results for both these cohorts were not expected. The national Māori-medium norms were established before the implementation of Te Poutama Tau, so it was assumed that, because Te Poutama Tau predominately focuses on Number and, to a lesser degree, Algebra, these Te Poutama Tau students would generally perform better than the national Māori-medium norms in these two strands of the curriculum.

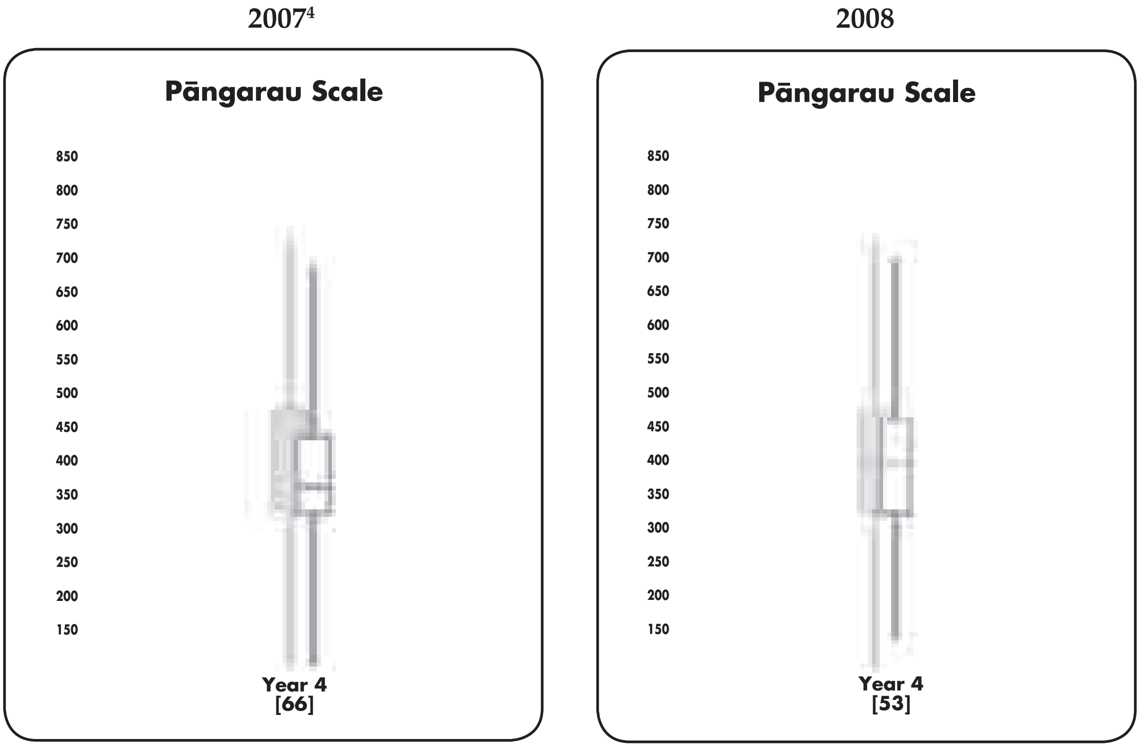

Figure 1. Group performance of year 4 students in 2007 and 2008

Curriculum functions report

Figure 2 shows that the 2008 year 4 students were slightly above the national Māori-medium norm in Number and below for Algebra. Again, this is an improvement on the 2007 results, where students were close to the national Māori-medium norm in Number but were substantially below the national Māori-medium norm in Algebra.

[4] See the explanation on page 41 of the shadings of the box plots.

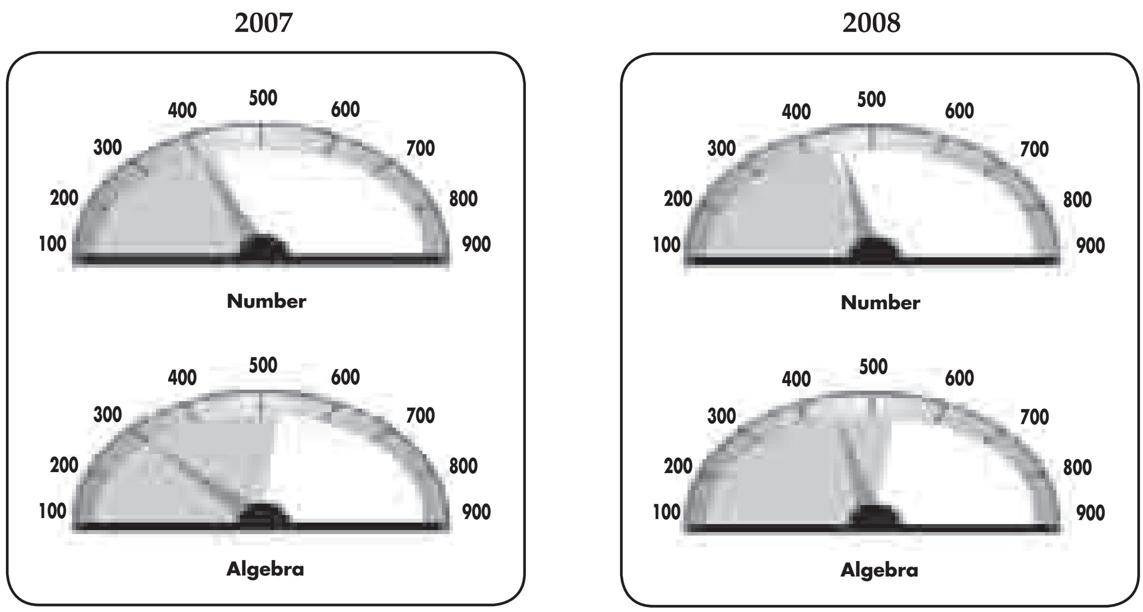

Figure 2. Year 4 student performance in the strands of the curriculum

Learning Pathways Report for Year 4

Performance highlights

Number

The year 4 students in the 2008 study performed positively in the questions that involved ordering whole numbers and decimals. Similarly, student results were positive in questions that required recalling basic facts for addition and subtraction. Number word sequencing and basic facts are both key components of the knowledge domain of Te Mahere Tau in Te Poutama Tau.

Performance concerns

Number

The 2008 year 4 students performed poorly in the questions that involved writing and solving whole- and decimal-number word-story problems with combinations of +, –, x, and ÷. This gap in achievement is consistent with the 2007 results.

Algebra

Both the 2007 and 2008 cohorts of year 4 and year 7 students performed poorly in most of the Algebra questions, including using the mathematical symbols =, <, and >. These also included questions that required entering either the correct symbol or quantity to show a relationship. For example, students were required to enter either <, >, or = in the box to show the appropriate relationship between 80 and 90 (80 $\square$ 90) and the relationship between the multiplication pairs 9 x 2 and 6 x 3 (9 x 2 $\square$ 6 x 3). They also needed to enter the quantity missing in the box in $\square$ + 8 < 10.

Making, describing, and using rules for number and spatial patterns is also an area where a substantial number of 2008 students were below the national Māori-medium norm.

Results of the Year 7 Students

Group performance

Both the 2007 and 2008 year 7 Te Poutama Tau students performed noticeably better than the national Māori-medium norm (Figure 3). The range of performance is much narrower in 2008, suggesting that most of the Te Poutama Tau students in this 2008 study were closer in ability to the national Māori-medium norm.

In the 2007 results, the top scores are off the scale and are much higher than the national Māori-medium norm (Trinick & Keegan, 2008). Notably, in both years there is no long tail of low scores in the Te Poutama Tau cohort.

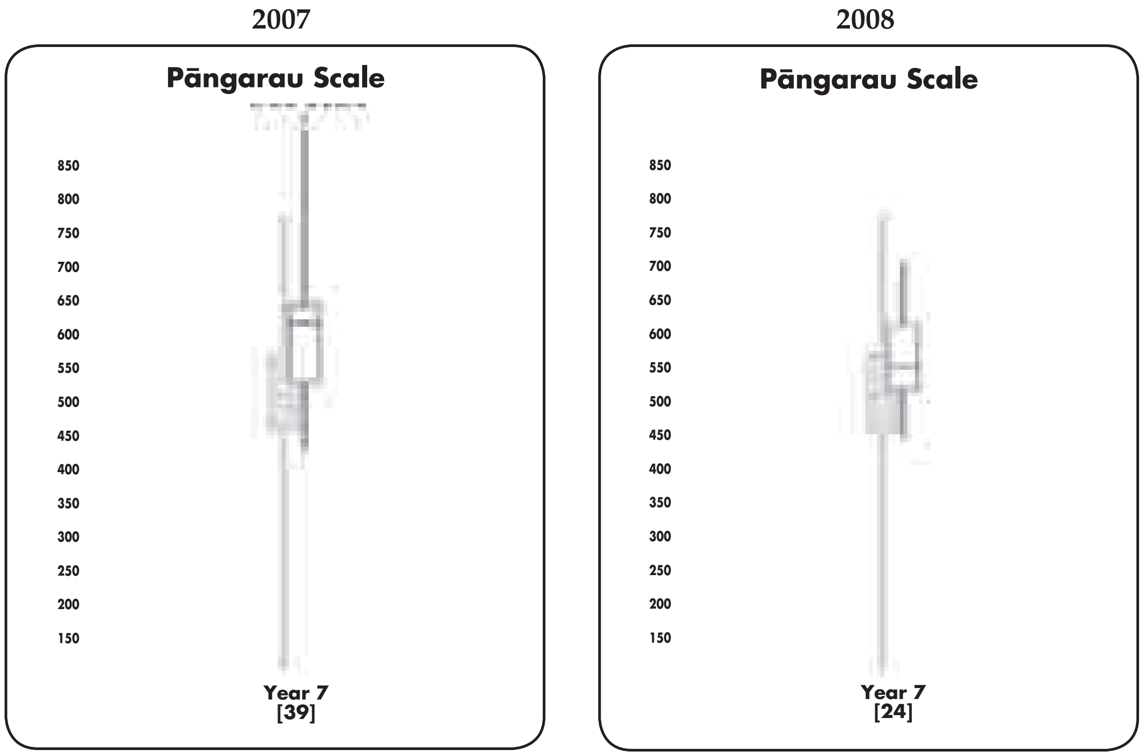

Figure 3. Group performance of year 7 students in 2007 and 2008

Curriculum functions report

In number, both the 2007 and 2008 year 7 cohorts performed well above the national Māori-medium norms. However, performance in algebra was not noticeably different from the national Māori-medium norms for either cohort. As noted in the 2007 study (Trinick & Keegan, 2008), algebra seems to be an area that students find challenging.

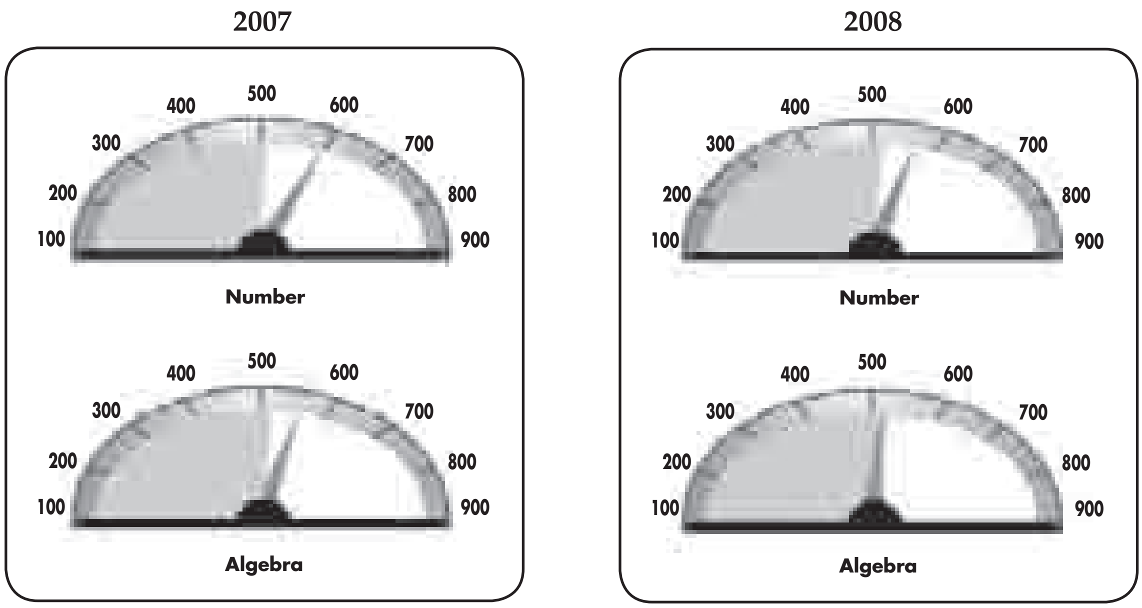

Figure 4. Year 7 student performance in the strands of the curriculum

Learning Pathways Report

Performance highlights

Number

The 2008 year 7 Te Poutama Tau students in this study performed well above the national Māori-medium norms in the questions that involved recalling the basic addition/subtraction and multiplication/division facts. The Te Poutama Tau students also performed particularly well in explaining the meaning of digits in two- to three-digit whole numbers, in expressing quantities as fractions or percentages of a whole, and in finding a fraction or percentage of a quantity. These performance highlights are also consistent with results for the year 7 Te Poutama Tau students in the 2007 study (Trinick & Keegan, 2008). A major focus is given to understanding and developing mental strategies in Te Poutama Tau to solve these types of problems, so this is a very positive outcome.

Algebra

The 2008 cohort of year 7 students performed slightly below the national Māori-medium norm, which is positive considering the year 4 results. The students performed well in questions linked to the learning outcomes, such as continuing sequential patterns.

Performance concerns

Number

The 2008 cohort of year 7 students had some difficulty explaining the meaning of digits in numbers to two or three decimal places, writing and solving problems with decimals in multiplication and division, and using and explaining the meaning of negative numbers. The latter two areas of difficulty are consistent with the 2007 results.

Algebra

About 50% of the 2008 year 7 cohort still had some difficulty with the mathematical symbols =, <, and >. This is discussed in the following section.

Discussion and Concluding Comments

The performance of the year 4 Te Poutama Tau students may be explained partly by fewer years of involvement in Te Poutama Tau. The positive performance highlights that are consistent with Te Poutama Tau include:

- reading and sequencing whole and decimal numbers;
- knowledge of addition and subtraction basic facts.

Some of the areas of concern for students in both the 2007 and 2008 cohorts include the use of the mathematical symbols =, <, and > and being able to describe or make up and use a rule to create a sequential pattern.

The performance of the year 7 Te Poutama Tau students in this study and in the 2007 study is very encouraging. Both cohorts performed above or close to the national Māori-medium norms. The positive results may be due to a range of variables, including teacher effectiveness or participation in other types of interventions such as literacy programmes. Notably, the majority of the year 7 Te Poutama Tau students had participated in Te Poutama Tau for a few years. The positive performance highlights that are consistent with Te Poutama Tau include:

- recalling basic addition, subtraction, and multiplication facts;
- reading and sequencing whole and decimal numbers.

An area of weakness for both the 2007 and 2008 cohorts were test items that involved negative numbers. This can be partly explained by the absence of material in Te Mahere Tau focusing on negative numbers. This is an area for future development. There is a similar issue with solving word problems that involve a variety of operations. Unfortunately, the asTTle test results do not reveal a student's ability to solve problems using mental strategies, which is a feature of Te Poutama Tau.

The year 4 and year 7 groups in both years of the Te Poutama Tau/asTTle study had some difficulty using the mathematical symbols =, <, and >. To learn algebra, students need a conceptual understanding of the use of symbols and the contexts in which they occur (Hiebert, Carpenter, Fennema, et al., 1997). Arcavi (1994, p. 24) introduced the notion of "symbol sense" as a "desired goal for mathematics education". Symbol sense incorporates the ability to appreciate the power of symbols and an ability to manipulate and make sense of symbols in a range of contexts. The concept of equality, for example, is an important idea for developing algebraic concepts among learners of algebra (Carpenter, Franke, & Levi, 2003). This should be an additional area for consideration by the Te Poutama Tau facilitators in 2009 and 2010.

In summary, the 2008 year 4 Te Poutama Tau students performed below the national Māori-medium norms, while the 2008 year 7 Te Poutama Tau students mainly performed above. Why the two age groups performed differently with regard to the asTTle national Māori-medium norms is not entirely clear. However, both the 2007 and 2008 cohorts performed reasonably consistently in a number of areas, particularly in those areas that are a major component of Te Mahere Tau in Te Poutama Tau. These include number knowledge areas such as basic facts. A significant component of Te Poutama Tau is the development of student mental strategies to solve problems. Pencil-and-paper tests such as asTTle are limited in assessing this aspect.

Ko te kōrero whakamutunga, ko te mihi ki ngā ākonga me ngā kura i uru mai ki tēnei rangahau. Nā reira, tēnei te tino mihi atu ki a rātau ko ngā pouako.

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Supporting Multiplicative Thinking: Multi-digit Multiplication Using Array-based Materials

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This paper describes research on the classroom practices of seven teachers who taught a lesson on multi-digit multiplication to their year 7–8 students using array-based materials. The students' understanding of multi-digit multiplication just before the lesson is contrasted with their performance several weeks after the lesson (following a second lesson, on divisibility by nine). Differences among the teachers in the ways they taught the lesson are examined in relation to the students' continued understanding of multiplication. Teachers who emphasised the rectangular structure of multiplication within the context of arrays had students who made greater progress in understanding multiplication. Introduction of traditional algorithms or the "grid" method before using arrays appeared to interfere with the students' understanding of multiplication.

The Numeracy Development Projects (NDP) have now been offered to almost all primary (year 1–6) and intermediate (year 7–8) schools in New Zealand. Analysis of data on the stages of the Number Framework reached by students at the year 8 level shows that fewer than half of these students reach stage 7 (advanced multiplicative part–whole thinking), which, according to the national numeracy expectations, the majority of students at level 4 (year 8) should have reached by the end of that year (Ministry of Education, n.d.). Although the findings from the Longitudinal Study show a greater proportion of year 8 students reaching stage 7, there is evidence to suggest that the students in the Longitudinal Study are more representative of high-decile schools than typical of New Zealand schools overall (Thomas & Tagg, 2008; Young-Loveridge, 2008).

One of the major goals of today's mathematics instruction is to help students understand the structure of mathematics (Lambdin & Walcott, 2007). The greater focus on mathematics structure can be seen in the mathematics and statistics learning area of *The New Zealand Curriculum* (Ministry of Education, 2007d). In contrast to the previous curriculum (Ministry of Education, 1992), where no mention was made of changes in the nature of thinking or of problem solving over year levels, there is a clear progression in the 2007 document (shown in the achievement objectives under number strategies) from simple additive strategies with whole numbers and fractions (level 2), to additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages (level 3), to a range of multiplicative strategies when operating on whole numbers and simple linear proportions, including ordering fractions (level 4), to reasoning with linear proportions (level 5), and to applying direct and inverse relationships with linear proportions (level 6). These progressions are closely aligned with the Number Framework, a key aspect of the NDP (see Bobis, Clarke, Clarke, et al., 2005; Ministry of Education, 2007a).

The work of Mulligan and colleagues supports the idea that students' appreciation of structure and pattern may be at the heart of differences between high and low achievers in mathematics (Mulligan & Mitchelmore, 1997; Bobis, Mulligan, & Lowrie, 2008; Mulligan, Prescott, & Mitchelmore, 2004). Their research shows that low achievers in mathematics do not appear to notice structure and regularity in mathematics, but interventions that draw their attention to structure and pattern can bring about substantial improvement in their mathematics learning.

The literature on multiplicative thinking and reasoning has been growing steadily over the past decade or so. According to Baek (1998), "understanding multiplication is central to knowing mathematics"

(p. 151). The importance of the understanding of multiplication and division for later mathematics has been affirmed in recent writing about the Curriculum Focal Points developed in the United States by the National Council of Teachers of Mathematics (NCTM) (see Beckman & Fuson, 2008; Charles & Duckett, 2008; NCTM, n.d.). Multiplicative reasoning is seen as one of three crucial mathematics themes (along with equivalence and computational fluency) that are interwoven through the Content Standards for the middle grades, forming the foundation for proportional reasoning (NCTM, 2000a, 2000b).

There are several major differences between additive and multiplicative thinking. For example, multiplication and division have proportional structure, whereas addition and subtraction have part–whole structure (Sophian, 2007). This means that multiplicative partitioning must involve equal-sized parts or groups, whereas additive partitioning often involves breaking numbers up into unequal-sized parts. Understanding multiplicative relationships depends on understanding the concept of a unit, and "that is generally developed first in the context of additive reasoning" (Sophian, 2007, p. 103). It is in considering units of quantification other than one that the need for multiplicative relations becomes clear – the unit may be a group (for example, a pair, a trio, or any other composite unit), or it may be a fractional quantity (for example, one-half, one-third, and so on). When students are young, they often don't understand the importance of keeping units constant and have a tendency when doing equal sharing to divide a continuous quantity into a particular number of pieces while ignoring the size of the pieces (Sophian, 2007).

A variety of definitions for multiplicative thinking and reasoning have appeared in the literature. According to support material for New Zealand's NDP (Ministry of Education, 2007c), multiplicative thinking involves:

> constructing and manipulating factors (the numbers being multiplied) in response to a variety of contexts ... [and] deriving [unknown results] from known facts using the properties of multiplication and division [commutative, associative, distributive, inverse]. (P. 3)

Multiplicative reasoning is far more complex than additive thinking and can involve a number of processes, such as: grouping; number-line hopping; folding and layering; branching; making grids or arrays; area, volume, and dimension; steady rise (slope); proportional reasoning; and number-line rotation (for integer multiplication). However, it has been argued that "the most flexible and robust interpretation of multiplication is based on a rectangle" (Davis, 2008, p. 88), thus reinforcing the two-dimensionality of multiplication. An area-based interpretation can be used to show how and why the algorithm for multi-digit whole-number multiplication works and can be extended to multiplication of decimal fractions, common fractions, algebraic expressions, and other continuous values (Davis, 2008; Young-Loveridge, 2005a, 2005b).

In contrast to multiplicative thinking, additive thinking is a linear process, involving a single dimension. Number-line models typically show addition and subtraction as movement either forwards (addition) or backwards (subtraction) along a line. Hence, the use of a repeated-addition strategy to solve a multiplication problem is less advanced than one involving partitioning, manipulating, and recombining quantities (see Ministry of Education, 2006). The inclusion of array diagrams as well as number-line models in the revised edition of *Book 1: the Number Framework* (Ministry of Education, 2007a) helps by showing collections-based (based on partitioning) as well as counting-based (based on skip counting or repeated addition) conceptions of the number system (Yackel, 2001) and provides richer, more flexible models of multiplication and division.

Along with the increased emphasis on multiplicative thinking have come expectations about when students should be able to use multiplicative structure. In New Zealand, there is an expectation that, by the end of year 8, students should be able to reason multiplicatively (Ministry of Education, n.d.).

However, with evidence indicating that only about 33–50% of year 8 students have good control over multiplicative structures (Young-Loveridge, 2007, 2008), it is important to understand more about the challenges for teachers at the year 7–8 level in helping their students become multiplicative thinkers.

This study explores the teaching of multi-digit multiplication using array-based materials in order to understand how different teaching approaches might impact on students' understanding of multiplication.

Method

Participants

Seven female teachers (A–G) working at the year 7–8 level (11- to 13-year-olds) from four schools (one intermediate school, one full primary school, and two middle schools) agreed to participate in the study. The decile ranking[1] of the schools in the study ranged from 2 (low) to 9 (high), reflecting the wide range of socio-economic backgrounds of the students. Teachers varied in years of teaching experience from approximately 1.5 years to 20 years. Likewise, the teachers' experience working with the NDP approach ranged from one to seven years. Each teacher chose a group of students to work with on enhancing multiplicative thinking. A combined total of 46 students took part in lessons and assessments used in the study (two other students, B1and F4, were present for only one of the lessons and are therefore not included in Table 1).

Procedure

Researchers visited each teacher twice. At the first visit, the students were given written assessment tasks to complete before the lesson, with instructions to "explain how you worked out your answer. Where possible, draw a diagram to help show your thinking." The eight tasks included: three about whole-number multiplication; two that involved deriving answers from information given and known number facts (If 4 x 30 = 120, what is 4 x 28? If 5 x 9 = 45, what is 5 x 18?); and one multi-digit multiplication problem (What is 11 x 99?). The teacher then taught a lesson on multi-digit multiplication while the researchers observed. Teachers adapted the lesson according to the knowledge of the students in the group. At some point within the lesson, the problem 23 x 37 was given to the students to solve. The teacher wore a digital audio recorder with lapel microphone to record as much as possible of the dialogue with the students. After the lesson, the researchers talked to the students, and later to the teacher, about their experiences in the lesson, in order to explore their perceptions of the lesson and any confusion that had arisen during the lesson.

After the second lesson (on divisibility by nine), the students and their teacher were interviewed again and the students were given written assessment tasks related to the two lessons. The key task for this study involved students solving the problem 23 x 37 =, showing on a dotty array (provided) how they would solve the problem, and then explaining their answer below the array. In most cases, the interval between the two lessons was between two and three weeks. All teachers taught the same two lessons taken from the support materials for the NDP on teaching multiplication and division (Book 6). This research paper focuses on the first lesson, Cross Products: Multiplication with multi-digit numbers using arrays (Ministry of Education, 2007c, pp. 67–70), and relevant written tasks given before and after the lesson.

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[1] Each school in New Zealand is assigned a decile ranking between 1 (low) and 10 (high), based on the latest census information about the education and income levels of the adults living in the households of students who attend that school.

Results

Students' Prior Knowledge

In the pre-lesson tasks, most (36 out of 46) students were able to find the answer for 4 x 28, and all successfully solved 5 x 18 (see Table 1). Twenty-two students used a rounding and compensation strategy to solve the first problem (4 x 28), deriving their answer by using a combination of the information given and known number facts (for example, 4 x 30 = 120, 4 x 2 = 8, so 4 x 28 = 120 – 8 = 112). One student also used rounding and compensation for the second problem (5 x 18), starting with her knowledge of 5 x 20 = 100, then taking off 10 to get her answer of 90. The majority of students (29) used a doubling and halving strategy to solve 5 x 18 (for example, 5 x 9 = 45, so 5 x 18 = 2 x 45 = 90). Some students (seven on each problem) ignored the information given and instead used standard place value partitioning to work out their answers (4 x 20 = 80 and 4 x 8 = 32, so 80 + 32 = 112; and 5 x 10 = 50, 5 x 8 = 40, so 50 + 40 = 90). Another group (seven on the first problem and nine on the second) used the standard vertical algorithm to work out their answers. Some students made minor errors in calculating their answers to the first problem.

Some students experienced difficulty with the problem 11 x 99, with fewer than half (n = 20) of the students getting the correct answer. Of the correct answers, five students used a rounding and compensation strategy, taking 11 from 1100 to get an answer of 1089, another five students used standard place value partitioning, adding 99 to 990 to work out the answer, and ten students used the traditional vertical algorithm. Four students did not attempt the problem.

Several misconceptions were evident in the students' responses. One notable misconception was to multiply the tens digits and the ones digits but not cross-multiply tens with ones (that is, 10 x 90 = 900, 1 x 9 = 9, 900 + 9 = 909). Four of the six students in teacher E's group used a consistent but incorrect strategy (Note: teacher E was not their usual teacher for mathematics). A different misconception was revealed for the other two students in teacher E's group, both of whom gave the answer as 999. E2 wrote "allways ad *[sic]* 1 more 9", while E4 wrote "x 11 means 1 extra number for the second number that has to be itself." Four students from other groups also gave the answer as 999. C4's response of "11 x 9 = 99, 11 x 99 = 999" suggests that this student may also have been following the "add another 9" rule used by E2 and E4. Four students (A4, C2, D2, G1) attempted the problem using an appropriate strategy but made minor calculation errors on the problem.

The Lesson on Multi-digit Multiplication

There were many commonalities among the seven teachers in the ways they taught the first lesson. For example, six of the seven teachers used a modelling book to record discussions with the students and began by talking about their planned learning intentions for the lesson (see Higgins, 2006). Most discussed the nature of multi-digit numbers and associated issues around place value. Although all of the teachers used the dotty arrays (see Figure 1), some were photocopied onto paper for students to draw on with pencils or marker pens, leaving a permanent record of the processes used by the students. Other arrays were laminated, and the students used a whiteboard pen to record their working, which was erased between problems. Having the paper record to refer back to later was an advantage for both teachers and students. It was also beneficial for the researchers when they were checking back on what had happened during the lesson.

Table 1
Responses of Students on the Multiplication Problems Given Before and After the Lessons

(Correct responses are italicised in bold; RC = rounding and compensation; PVP = place value partitioning; DH = doubling and halving; Alg = algorithm; RA = repeated addition; B = border; P = partitioning; N = numbers shown; S = sum calculated; CPP = cross-product process.)

Student	GloSS	4 x 28 from 4 x 30	5 x 18 from 5 x 9	Before the lessons 11 x 99 =	After the lessons 23 x 37
A1	6	RC 116	***PVP***	***RC***	***BPNS 851***
A2	6	PVP 102	***DH***	Not attempted	BPN
A3	5	***RC***	***DH***	110 – 11 = 99	600 + 30 + 70 = 700
A4		***PVP***	***DH***	PVP 1080	***BPNS 851***
A5	6	***RC***	***PVP***	Not attempted	600 + 90 + 21 + 140 = 841
A6	6	***RC***	***DH***	1100 – 1 = 1999	BPN
A7	5	***RC***	***DH***	Not attempted	***BPNS 851***
B2	7	***RC***	***DH***	***RC***	BPNS 600 + 160 + 90 + 24 = 874
B3	6	***PVP***	***PVP***	***PVP***	***BPNS 851***
B4	7	***PVP***	***PVP***	Alg 1890	***BPNS 851***
B5	6	***PVP***	***Alg***	***Alg***	***BPNS 851***
B6	5	***RC***	***RC***	1100 – 9 = 1091	***BPNS 851***
B7	6	***Alg***	***Alg***	Alg 999	BPN
B8	5/6	***RC***	***DH***	11 x 99 = 999	BPN
C1	5	***RA***	***DH***	1 x 9 = 9,11 x 9 = 99	37 + 23 = 60
C2	6	PVP 104	***DH***	990 + 99 = 1098	600 + 21 = 621
C3	5	RC 118	***DH***	no working 999	20 x 30 = 500 + 21 = 521
C4	5	***RC***	***DH***	11 x 9 = 99,11 x 99 = 999	37 + 23 = 60
C5	5	***RC***	***DH***	Not attempted	600 + 140 + 21 + 9 = 770
C6	5	***RC***	***DH***	Alg 999	2 x 3 = 6 + 10 + 21 = 81
D1	6	***Alg***	***DH***	9 + 9 = 18 + 989 = 1089	***B Alg 851***
D2	5	***Alg***	***Alg***	Alg 1980	6 x 100 + 70 + 70 + 30 + 30 + 30 + 30 = 860
D3	5	***RC***	***DH***	***Alg***	***B Alg 851***
D4	5	***Alg***	***Alg***	***Alg***	***B 851***
D5	7	Alg 102	***Alg***	***Alg***	***BN 851***
D6	6	***Alg***	***Alg***	***Alg***	B 6 x 100 + 2 x 70 + 4 x 30 = 860
D7	5	***Alg***	***Alg***	***Alg***	B 600 + 140 + 80 + 80 = 851 guessed
E1	5	Alg 102	***Alg***	10 x 90 + 1 x 9 = 909	600 + 60 + 140 + 21 = 821
E2	6	PVP152	***DH***	11 x 9 = 99, 11 x 99 = 999 always ad *[sic]* 1 more 9	***B 851: I drew a line 23 dots down & 37 across & worked my way from there***
E3	6	***PVP***	***PVP***	10 x 90 + 1 x 9 = 909	624 because there are 6 full boxes & 2 7s & 4 3s = 624
E4	6	***RC***	***DH***	x 11 means 1 extra number for the 2nd number that has to be itself = 999	111 + 6 x 100 + 70 + 70 = 857
E5	6	***RC***	***PVP***	10 x 90 + 1 x 9 = 909	***600 + 140 = 740 + 90 = 830 + 21 = 851***
E6	6	RC 102	***DH***	10 x 90 + 9 = 909	20 x 30 = 600 + 7 x 3 = 621

Student	GloSS	4 x 28 from 4 x 30	5 x 18 from 5 x 9	Before the lessons 11 x 99 =	After the lessons 23 x 37
F1	8	***PVP***	***PVP***	***PVP***	***B 600 + 21 + 90 + 140 = 851***
F2	8	***RC***	***DH***	***Alg***	***BPNS 600 + 140 + 90 + 21 = 851***
F3	7	***Alg***	***Alg***	Alg = 289 (99 + 190)	***BPNS 600 + 140 + 90 + 21 = 851 Grid***
F5	8	***RC***	***DH***	***Alg***	***BP 600 + 140 + 90 + 21 = 851 Grid***
G1		***RC***	***DH***	RC 1981	BPNS 600 + 90 + 140 + 21 = 751
G2	6	***RC***	***DH***	***Alg***	***BPNS 851***
G3	8	***RC***	***DH***	***RC***	***BPNS Grid 851***
G4	7	***RC***	***DH***	***Alg***	***BPNS 851***
G5	7	RC 118	***DH***	***RC***	***BPNS Grid 851***
G6	8	***RC***	***DH***	***RC***	***BPNS Grid 851***
G7	6	RC 132	***DH***	***PVP***	***BN 851***
G8		***PVP***	***DH***	***PVP***	***BPNS 600 + 140 + 111 = 851***
G9	6	***RC***	***DH***	***PVP***	***BPNS Grid 851***

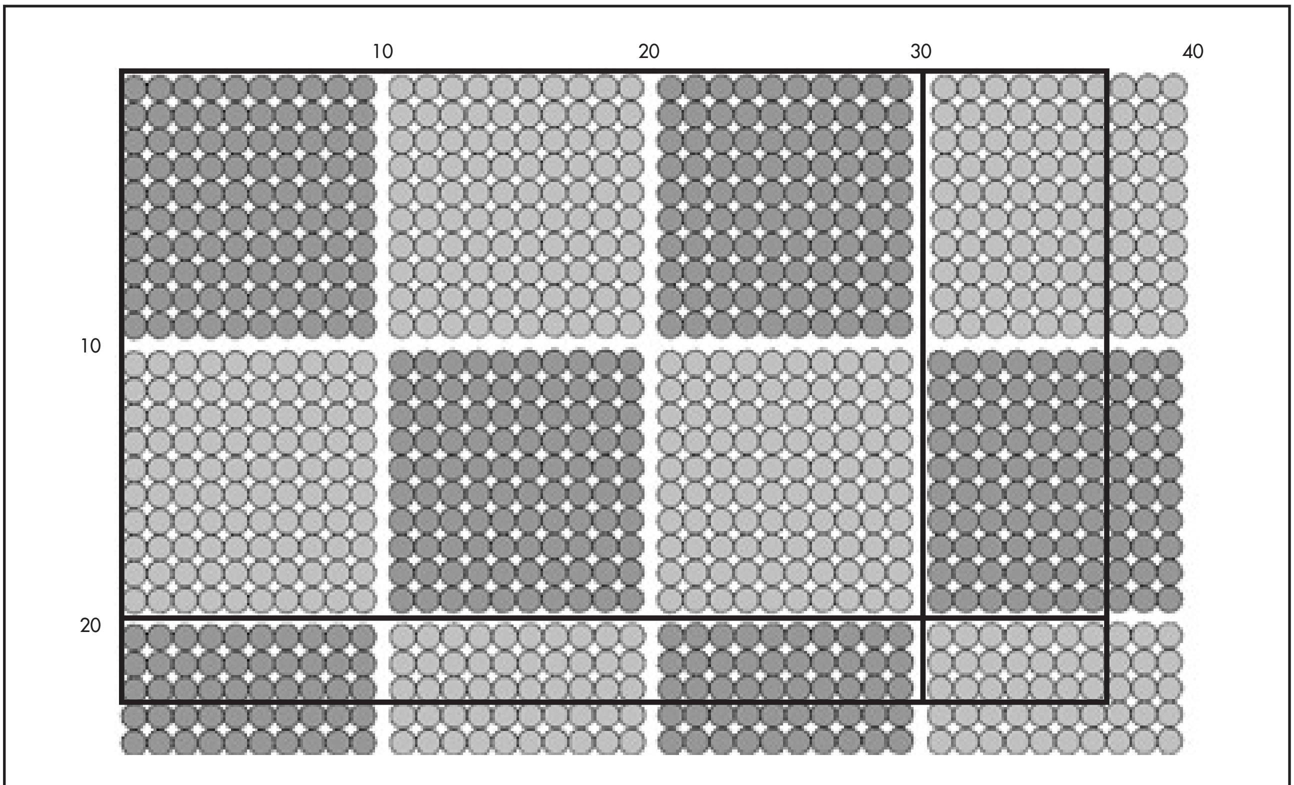

Figure 1. Example of an array showing **23 x 37** as 20 x 30 = 600; 20 x 7 = 140; 3 x 30 = 90; 3 x 7 = 21; 600 + 140 + 90 + 21 = 851

Several teachers began by introducing arrays using single-digit multiplication (for example, 5 x 6), and this appeared to be very helpful for scaffolding the idea of representing multiplication as a rectangle with sides corresponding to each of the factors. Drawing a border around the rectangle formed by the two factors turned out to be important for students' understanding. Although there were some commonalities between the seven lessons, each was very different from all the others.

Students' Understanding of Multiplication After the Lesson

The students in teacher A's group (only one of whom had answered 11 x 99 correctly before the lesson) seemed to have made the greatest progress towards understanding multiplication over the two weeks following the first lesson. Six of the seven in the group successfully used the dotty arrays to work out the partial products for 23 x 37. Three of these students added the partial products to get 851 (A5 miscalculated and got 841). Two students (A2 and A6) did not take the final step of adding the partial products. The only student who did not appear to benefit from the array materials (A3) got partial products of 600, 30, and 70 and added up to get 700. She wrote 3 x 10 = 30 (instead of 3 x 30 = 90), only noticed one group of 70, and completely overlooked the 3 x 7. It was interesting to note that for 11 x 99 before the lesson, this student had written 110 – 11 = 99, failing to notice that her answer was the same as one of the factors. Her strategies on both of these problems may be indicative of carelessness rather than misconceptions about multiplication.

Teacher A started the lesson (after first discussing the learning intention and what multi-digit numbers are) by asking the students what they noticed about the dotty array, drawing their attention to the regular structure of rows and columns of dots and the separation between each group of ten dots so that the 10 by 10 blocks of 100 dots were easy to see. She then asked the students to explain how they would show 6 x 5 = 30 on the array, eventually instructing them to draw a border around it. In the interview, teacher A commented on the importance of drawing a border around the part of the dotty array that represented the problem: "The border is definitely the key word for me – that's why I underline it [on the whiteboard]."

Teacher A made a point of discussing the meaning of key mathematics terms such as "represents" and "partitioning". She asked the students to work in pairs and then to explain each other's work to the group to ensure that they all understood what they needed to do in working with the arrays. When one student showed partitioning without putting a border around the whole problem, teacher A asked the whole group to think about what that student needed to do differently. There was a lot of discussion about partitioning, with students being asked "Has everybody got a partition?" and "Where did you put your partition?" as well as being questioned about "splitting it up".

Teacher A also made sure that her students "numbered the edge" of the array so that the connection between the factors and the array was made clear. She also talked about "factor times factor equals product", linking it to the name of the lesson (Cross Products).

Teacher A did not move to a two-digit by two-digit multiplication problem until more than halfway through the 50-minute lesson. After the students had worked through each of the partial products, teacher A got them to draw up a grid (in preparation for doing the "grid method") in their books (see Figure 2). She then constructed a model of the "cross-product process" by writing the place value partitioned factors on four separate cards (for example, 20 and 3, 30 and 7) and used string stretched between the cards to show the horizontal and diagonal connections between the four parts that were multiplied together to produce the four cross products (see Figure 3).

x	**30**	7	
20	600	140	740
3	90	21	111
	690	161	851

Figure 2. The "grid method" for calculating the answer to the problem: 23 x 37 (a variation on that shown in Ministry of Education, 2007c, p. 68)

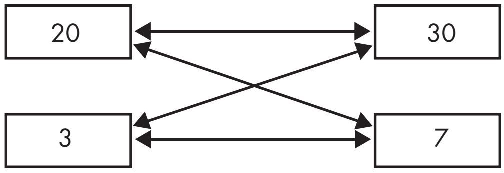

Figure 3. Model showing the "cross-product" process (Ministry of Education, 2007c, p. 69)

In her reflections on the lesson, teacher A rated the students' understanding of double-digit multiplication as "possibly a three ... on a scale of one to ten". In hindsight, it would have been valuable to ascertain how much additional work on multi-digit multiplication teacher A did with the students in further lessons before the researchers' second visit.

Teacher B's students were slightly stronger mathematically than teacher A's. Two of the seven students (B2 and B4) had been assessed as already being at stage 7 (advanced multiplicative part–whole). However, only B2 and B3 used part–whole strategies to solve 11 x 99 before the lesson.

Two weeks after the lesson using dotty arrays, all but one of teacher B's students were able to use dotty arrays to work out 23 x 37, although two students (B7 and B8) did not add their partial products together at the end to find the sum. The only student who did not succeed (B2) had accidentally drawn his array as 23 x 38, resulting in partial products of 160 and 24 instead of 140 and 21. Student B6's response was typical of students who used the dotty array with understanding to solve the problem (see Figure 4).

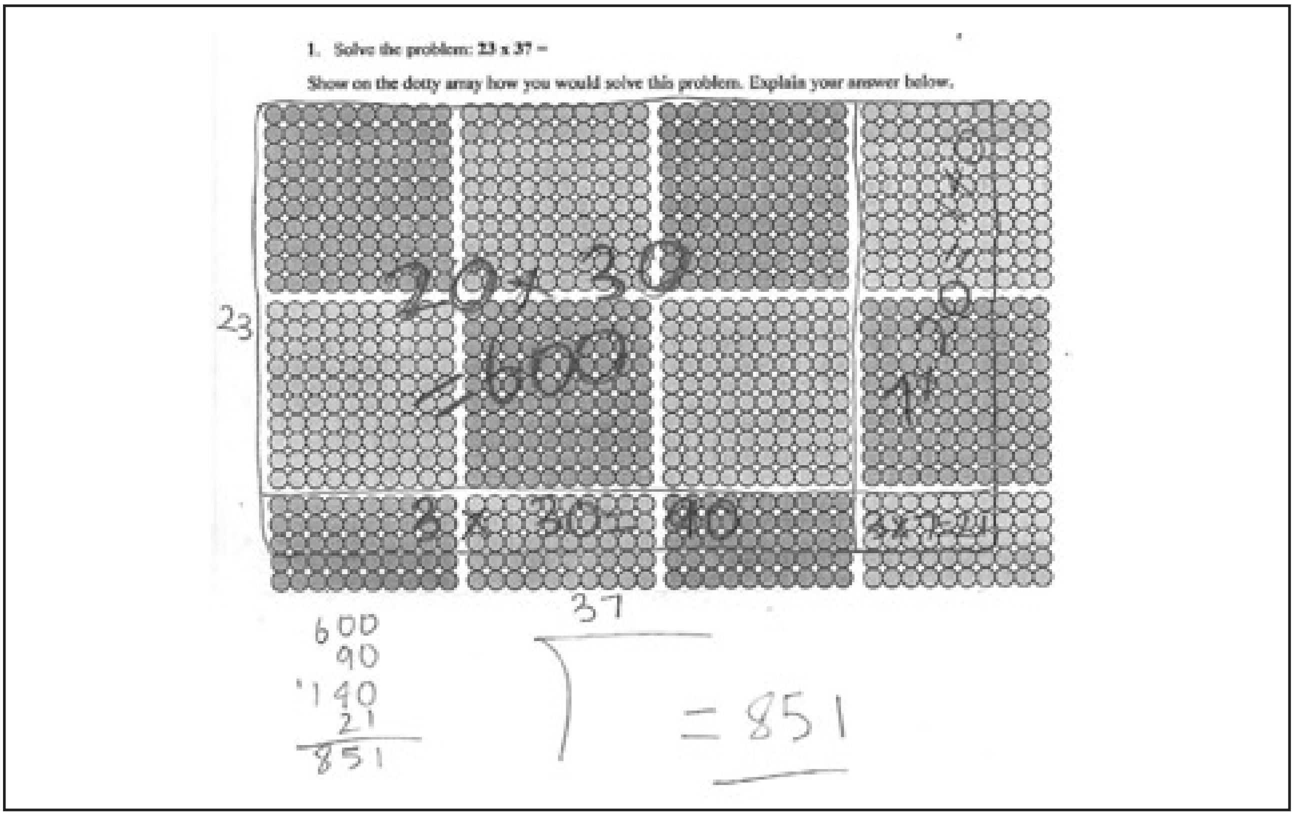

Figure 4. Student B6's record of her solution to 23 x 37 =

In her reflections on the lesson, teacher B, who taught the lesson for the first time on the day of the researchers' first visit, commented that in the future, she would start by stressing the importance of drawing a border around the whole problem. Several of her students progressed from having difficulties with 11 x 99 before the lesson to successfully working out 23 x 37 several weeks after the lesson.

In hindsight, it would have been useful to know what further work teacher B had done with her group on multi-digit multiplication between the two visits. Another interesting area to explore would have been the possible influence of teacher A on teacher B's classroom practice. Given teacher A's many years of experience as a teacher and of working with the NDP approach, together with her leadership role within the school, it seems likely that teacher B benefited from working collaboratively with teacher A.

The Impact of Traditional Algorithms

Teacher D's students were strongly algorithmic in their responses to all of the tasks given. Only one of the seven students derived 4 x 28 from 4 x 30, and two derived 5 x 18 from 5 x 9. Five of the seven used the vertical written algorithm to work out 11 x 99. Although all seven students drew a border around the dots corresponding to the problem 23 x 37, there was little, if any, partitioning (D6 was the exception, partitioning two blocks of 100 and one block of 70 within the array). Two students (D1 and D3) wrote out the vertical written algorithm as part of their explanations. It seems likely that others in the group also used the algorithm mentally to find their answers. In later discussions with teacher D, it became clear that she had focused exclusively on traditional written algorithms in her teaching of multiplication and division. She had undertaken NDP professional development quite recently but, being a second-year teacher, was still finding her feet.

Teacher E appeared to have done considerable work with her students to correct their misconceptions about multi-digit multiplication. However, the students still tended to use the traditional algorithm to work out their answers before drawing a border around the dots representing the problem. Two students (E2 and E5) found the correct answer for 23 x 37 but showed no evidence of partitioning the array – they both simply drew a border around the dots representing the problem. Student E2 wrote "I drew a line 23 dots down and 37 across and worked my way from there." Student E5 wrote "600 + 140 = 740 + 90 = 830 + 21 = 851" underneath her array. Inside her array, she appeared to have numbered the blocks of 100 with "10, 20, 30, 40, 50, 60" and then beneath, numbered across the three rows of 30 and the 3 by 7 section: "21, 42, 63, 84". It is possible that this student worked out the correct answer by applying the traditional algorithm mentally after numbering the array. Student E1 was the only student in this group to partition the array, but she miscalculated 30 x 3 as 60 instead of 90, giving an answer of 821 instead of 851.

Teacher F's students also calculated their answers before drawing borders around unconnected partial products. All four students had been assessed as being at stage 7 or 8 on the Number Framework before the lesson. It became clear during the interview that teacher F did not think that dotty arrays could help her students understand multi-digit multiplication and thought that the vertical written algorithm (which all her students applied correctly) or the grid method (partitioning each factor into tens and ones, then multiplying horizontally and diagonally to create cross products) were the best ways to solve multi-digit multiplication problems. She commented that she didn't think the array "added any value to what they [already] had".

Matching the Lesson to the Learning Needs of the Group

The responses of students in teacher C's group to the 23 x 37 problem revealed considerable confusion about the use of dotty arrays to solve multi-digit multiplication problems, suggesting that perhaps the cross-product lesson was not a good match for the learning needs of the group this teacher had chosen to work with for this lesson. None of the students were able to work out the answer to 23 x 37 two weeks after the lesson. It appeared that most of the students had major issues with place value, confusing tens with hundreds and multiplication with addition. For example, C1 coloured in three blocks of 100 and a row of seven dots (for 37) and, below this, coloured two blocks of 100 and a row of three dots (for 23), then "plused *[sic]* them together" writing the answer as 60 (the sum rather than the product of 23 and 37) (see Figure 5).

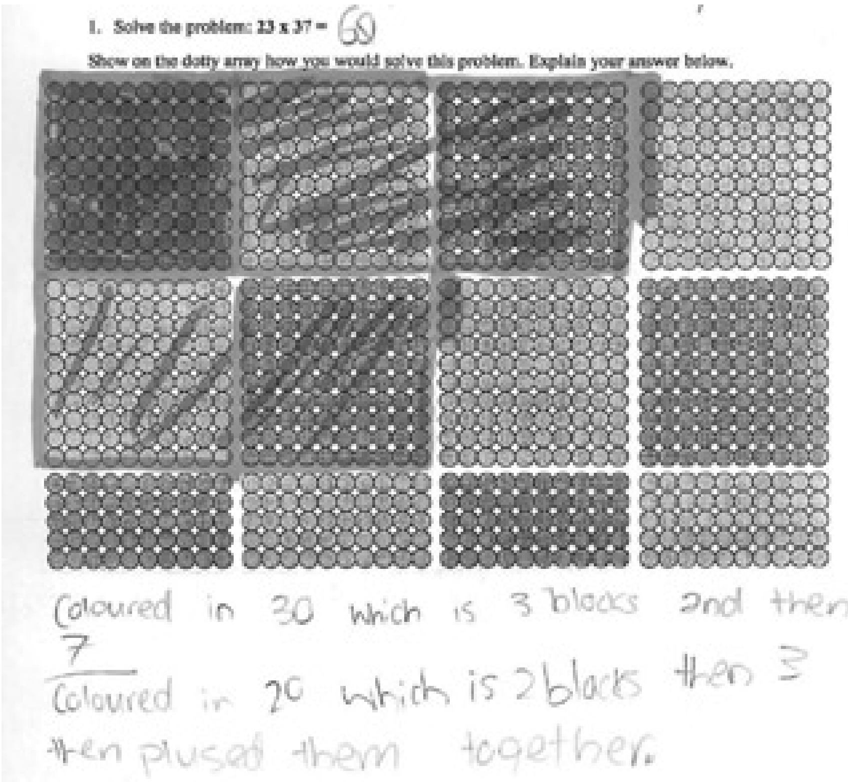

Figure 5. The response of student C1, showing confusion between tens and hundreds and between addition and multiplication

Although C4 drew a border around 37 by 23, like C1, she used a highlighter pen to colour in three blocks of 100 and a row of seven dots (for 37) and, below this, coloured two blocks of 100 and a row of three dots (for 23), giving the answer as 60. C2 drew a border around six blocks of 100 (3 across and 2 down), then a separate border around 21 dots in a 3 x 7 array to the right of the block of 600. He added 600 and 21 to get an answer of 621 (see Figure 6).

C3 drew two almost identical arrays (of 600 and 21) to C2 but cut out one block of 100, writing "20 x 30 = 500", then added the 500 and the 21 from the 3 x 7 array to get an answer of 521. C5 drew a border around an array of 30 by 20 and a border around the 7 x 20 array adjacent to the 600. However, there were also borders around 3 x 7, directly below the 30 x 20, and around a 3 x 3 array (instead of 3 x 30) below the 7 x 20 array, resulting in the sum of cross products being 770.

C6 drew a border around three blocks of 100 and highlighted a column of three dots on the top left corner of the third block (for 23). Underneath, she drew a border around four blocks of 100, highlighting a column of seven dots on the top left corner of the fourth block (for 37). She wrote: "Split the numbers, go back 2 x 3 = 6 and 3 x 7 = 21 then add 10 back onto [drawing an arrow pointing to the 3 in 2 x 3], add them together to equal 81." Her reference to "add 10 back onto" the 3 suggests she was using an "add zero" rule to adjust for the fact that the 2 was really 20 and the 3 was really 30, but she did not recognise that she needed two more zeros (as in 600), not one (as in 60). Evidence of the "add zero" rule was found on some of the work that C2 did during the lesson, where he wrote "37 x 23 = 2 x 3 add 00 = 600". It was interesting to note that before the lesson, C3, C4, and C6 had all given 999 as the answer to 11 x 99 in the initial assessment. Only C2 had correctly partitioned the 99 into 90 and 9 and calculated partial products of 990 and 99 (C2 made a slight error in summing the partial products, getting a final answer of 1098).

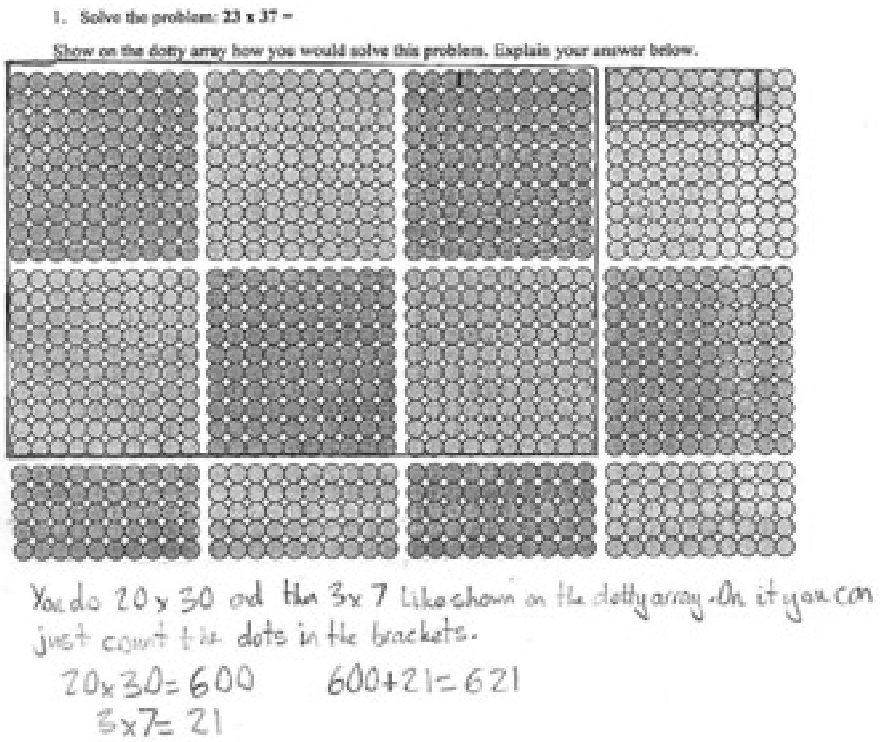

Figure 6. Student C2's response to 23 x 37 =

One reason for the confusion experienced by teacher C's students might have been that this teacher started straight into multi-digit multiplication problems rather than introducing arrays initially with single-digit problems. All but one of the students in this group had been assessed as being at stage 5 early additive part–whole thinking (C2 was assessed as being at stage 6). These students may well have been still trying to get to grips with the complexities of advanced additive part–whole thinking and might not have been ready to deal with the added challenge of working with multi-digit multiplication. It seems likely that they needed a gradual introduction to the use of arrays for single-digit multiplication before moving to work with larger numbers.

Teacher C did not emphasise the importance of first drawing a border around the whole problem. Several of her students tried to work out the partial products first and then put borders around those products, often drawing borders around separate and unconnected cross products. The result was a separate rectangle for each cross product, without coherence or connection to the original factors or the total product. The fact that teacher C accepted her students' strategy of drawing borders around unconnected partial products during the lesson suggested that this teacher did not understand how a rectangular structure can be used to represent multiplication and hence the need for a border around the whole problem before partitioning. Both the teacher and her students seemed confused and a little frustrated during the lesson. Teacher C was aware that her students had found the lesson difficult, later saying "They found it really, really hard."

Teacher C's comment that her students "still struggle a little bit with place value" showed her awareness that this probably contributed to their difficulty using the arrays. Teacher C had seemed somewhat apprehensive about her lesson being observed and audio-recorded, and this may have also played a part in her confusion during the lesson. This was confirmed in the second interview when teacher C commented that she "felt a lot more at ease" during the second lesson. In hindsight, it would have been good to explore with teacher C her thoughts on possible reasons for her students' difficulties with the 23 x 37 problem.

The lesson with dotty arrays may not have been quite such a good match for the group that teacher G chose to work with because they were already quite strong multiplicative thinkers. They had been assessed as already being mostly at stage 7 or above (five out of eight) and started the lesson with a good understanding of multiplication (8 out of 9 had been able to solve 11 x 99 before the lesson). The students tended to use the grid method to work out their answers to 23 x 37. However, all but one student (G7) showed appropriate partitioning inside the border they had drawn around the dots in the array representing the problem. The only student who did not calculate the correct answer (G1) had made a slight calculation error at the last step when adding up the partial products. It was interesting to note that student G1 had also made a slight computation error on 11 x 99, when the student tried to subtract 11 from 1100 and got an answer of 1981. Algorithms were used by only two students (G2 and G4), and then only to solve 11 x 99.

The responses of teacher G's students reflected a strong emphasis on understanding. At the time of the research, teacher G was nearing the end of her first year of NDP professional development and had been focusing on multiplication and division with her class. Although the group she chose to work with almost certainly benefited from the work with the arrays in consolidating their understanding of multi-digit multiplication, it would have been interesting to have observed a group who began the lesson with less prior knowledge and understanding of multiplication.

Teacher Reflections and Comments

The transcription of the teachers' language during the lesson was very useful in helping to identify differences between their approaches that may have been crucial in explaining why some students learned more effectively than others. Teacher A's approach was notable for the way in which she moved very slowly from single-digit multiplication to two-digit multiplication, starting with one-digit by one-digit problems, then moving to one-digit by two-digit problems, before introducing two-digit by two-digit problems. This contrasts with the approach of some other teachers (for example, teacher C) who began the lesson with the 23 x 37 problem. Teacher A also made a particular point of focusing on mathematics language, checking with her students about their understanding of key terms at regular intervals. As well as stressing the importance of the border, she also talked a lot about partitioning, repeatedly asking her students how they had split the numbers up or "made a partition". Teacher A also made a point of getting the students to put the numbers on the sides of the rectangle ("numbering the edge") and to draw a line between each of the digits to highlight the cross products.

Teacher A differed from the other teachers in that she had previously taught the lesson on cross products to another group of students and had clearly refined her technique in response to her previous experience. All of the teachers commented on using the grid method, in which each multi-digit number is broken down according to place value and each part is multiplied with all/both of the parts in the other factor. However, for some students, it seemed to have become just like the traditional written algorithm – a mindless procedure executed without understanding. Only teacher A seemed to appreciate from the outset how the array model enabled students to get a picture of the magnitude of the quantities in the partial products. She was clear about the value of using the array initially to build understanding of the multiplication process. She then followed this with the grid method and the cross-product process (using cards for the place value partitioned factors and string stretched between the parts of one partitioned factor and the parts of the other factor). The formal written algorithm was introduced last as just another way of solving the problem.

It would also have been good to see the lesson go beyond using the dotty arrays on to hand drawing rectangles to represent the problems solved using the dotty arrays.

Discussion

The findings of this study indicate that arrays can be useful for enhancing students' understanding of multi-digit multiplication. Teachers' use of dotty arrays to represent multi-digit multiplication as a rectangle with sides corresponding to the two factors was associated with improved performance on multiplication problems a few weeks after the lesson.

The findings are consistent with the view that "the most flexible and robust interpretation of multiplication is based on a rectangle" (Davis, 2008, p. 88). An advantage of dotty arrays is that they help students to appreciate differences in the magnitude of partial products and the impact of place value on the size of the sections (that is, partial products) within an array. For example, the six blocks of 100 dots representing 20 x 30 (the "tens") was substantially larger than the 21 dots in the 3 x 7 (the "ones") array.

The results reported here support the views of Mulligan and colleagues (Mulligan & Mitchelmore, 1997; Bobis et al., 2008; Mulligan et al., 2004) that coming to understand the underlying structure of the mathematics is vitally important for effective mathematics learning. Some students who were only using vertical written algorithms or the grid method appeared to have difficulty understanding how arrays could be useful for solving multiplication problems. An emphasis on procedural knowledge and rules, as reflected in the use of algorithmic approaches to multiplication, may undermine conceptual understanding. As Pesek and Kirshner (2000) have shown, once students have been taught to use standard written algorithms, it can be extremely difficult to then try to help them develop relational understanding.

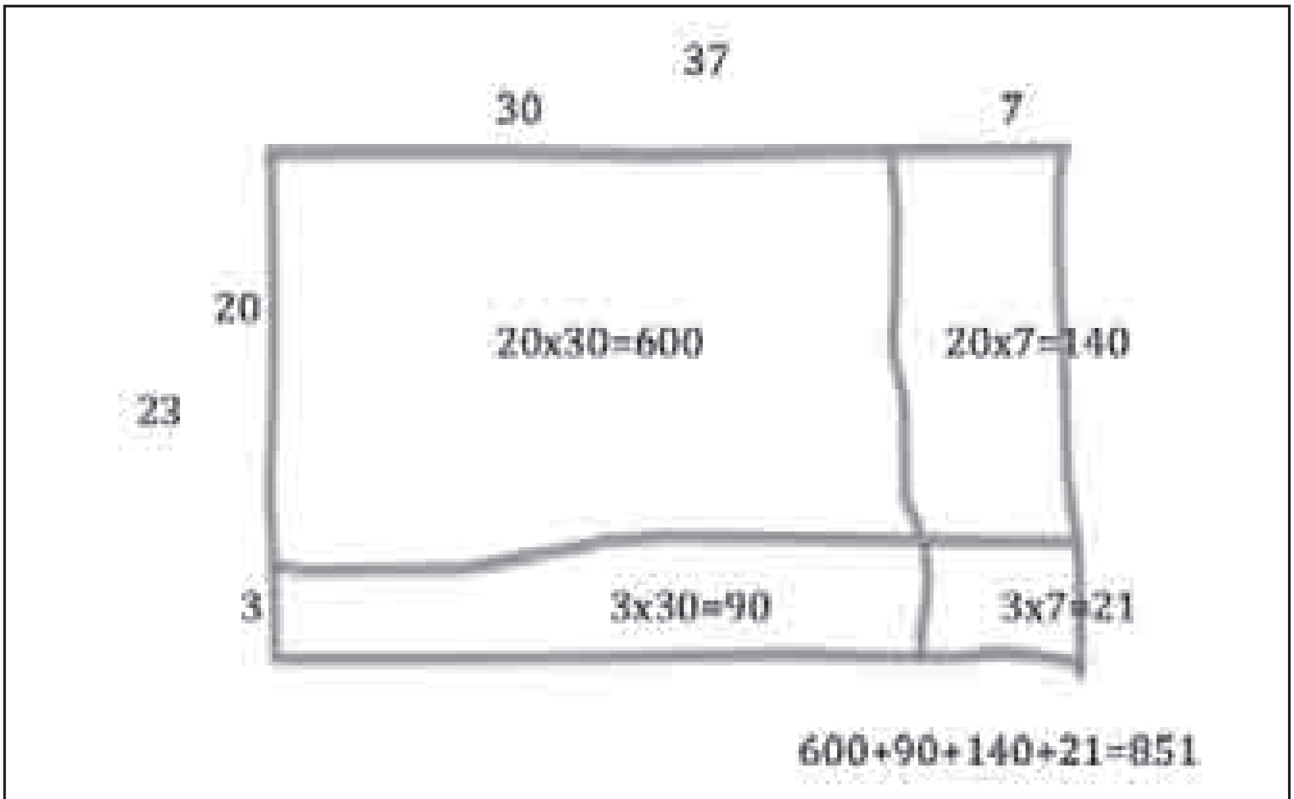

Figure 7. An example of a hand-drawn rectangle showing an array-based solution strategy for the problem: 23 x 37

The researchers have concluded that it is rather unfortunate that in the NDP support materials (Book 6), the lesson on traditional written algorithms for multiplication (entitled Paper Power) comes before rather than after the lesson on using dotty arrays to make sense of multi-digit multiplication. The order of the lessons in the planning sheets also has the traditional written algorithms for multiplication coming before instead of after the lesson on dotty arrays (see www.nzmaths.co.nz). The introduction of hand-drawn rectangles as a means of representing the array would provide a useful intermediary step between the dotty arrays and the grid method (see Figure 7). An example of hand-drawn arrays in Book 6 would show teachers that pictures and diagrams can provide a useful means of recording the problem solving in multi-digit multiplication. The digital learning objects on the nzmaths website allow students to play around with different ways of partitioning arrays to make multiplication easier and to reinforce the two-dimensional structure of multiplication problems (see The Multiplier, nzmaths, n.d.).

It is interesting to note that none of the teachers referred to the distributive property, either in their work with the students or in their discussions with the researchers. It is the use of the distributive property (whether or not students refer to it by name) that distinguishes those who are advanced multiplicative thinkers and can therefore construct and manipulate factors in response to a variety of contexts and derive the answers to unknown problems from known facts, using the properties of multiplication and division (that is, commutative, associative, distributive, inverse) (Ministry of Education, 2007c). It would be good to see further information about these number properties, particularly in Book 3, where the teaching model is described (Ministry of Education, 2007b).

Some teachers may not be aware that the number properties that are part of the teaching model are the same number properties that they may have been taught about when they were at school in the so-called "New Maths" era. Understanding how numbers can be added or multiplied in any order (the commutative property) or grouped in any way (the associative property, as reflected in halving and doubling, tripling and "thirding", or quadrupling and quartering) underpins the part–whole strategies that are characteristic of students' thinking at stages 5 and above on the additive and multiplicative domains. Unique to multiplication is the distributive property, which allows factors to be partitioned additively so that students can derive answers to unknown multiplication problems by partitioning factors in such a way that known facts can be used to solve parts of the problem (that is, partial products) and the parts can then be joined together to create the final product, for example, $23 \times 37 = (20 + 3) \times (30 + 7) = (20 \times 30) + (20 \times 7) + (3 \times 30) + (3 \times 7) = 600 + 140 + 90 + 21 = 851$.

If we are to support teachers in helping their students to become advanced multiplicative, then it is important to provide the teachers with the principles that underpin the processes we are encouraging them to use. While number properties (commutative, distributive, associative, inverse) are mentioned in Book 6, examples given, and suggested vocabulary included, no definitions are provided to help teachers appreciate that the essence of the associative property is about changing the *grouping* and the essence of the distributive property is about *partitioning* factors into convenient chunks. It is likely that many teachers do realise that the commutative property is about changing the *order* of factors ("turn-abouts"), but it would be interesting to know how many of them have also made links with the number properties mentioned in the teaching model.

The inverse property refers to reversing and doing and undoing, but it should also show how multiplication of the product by a unit fraction that is the reciprocal of one factor results in the other factor, in just the same way that division reverses multiplication (for example, $7 \times 4 = 28$, so $28 \div 4 = 7$, and $28 \div 7 = 4$; also $\frac{1}{4}$ of $28 = 7$, and $\frac{1}{7}$ of $28 = 4$).

Book 1 can also be used to make more links with the number properties now that the revised version shows how array models can be used to model multiplication as well as number-line models (Ministry of Education, 2007a).

The findings of this study have some important implications for the grid method that seems to have become a popular method for teaching multiplication (see Figure 2). The results indicate that the grid method is possibly being taught as an alternative to the traditional written algorithm and may suffer from the same problem of being applied in a mindless way (as rules and procedures without meaning) that has contributed to warnings and cautions for the vertical written algorithm.

As with the related cross-product process (see Figure 3), the understanding developed when students draw arrows (both horizontal and diagonal) between the place value partitioned factors and then add up the partial products that result from these multiplications is not much greater than the understanding they develop from using traditional vertical written algorithms.

What is important for students to appreciate is that, when the two "tens parts" are multiplied together, the partial product is extremely large, whereas the multiplication of the two "ones parts" yields a relatively small partial product by comparison. They need to see where the "tens" by "ones" and the "ones" by "tens" partial products come from in the rectangular structure that represents the problem as a whole (see Davis, 2008). It was very striking to see the number of students in teacher C's group whose answer to 23 x 37 was less than 100, even though they had apparently done quite a bit of work with the grid method (see Figure 2).

The differences between teacher A's approach and the approaches of the other teachers highlights the importance of mathematics language and the need for teachers to help their students become familiar with the terms used to describe the processes that are involved in solving problems by applying their understanding of number properties and using part–whole strategies. The challenges for teachers of coming to understand that language themselves and developing suitable ways for helping their students learn about it have probably been underestimated.

The findings of this study are consistent with the idea that teachers' knowledge and understanding of mathematics has a great impact on their teaching (see Ball, Hill, & Bass, 2005; Hill, Rowan, & Ball, 2005; Hill, Schilling & Ball, 2004). It is clear that multiplicative reasoning is complex and multifaceted. There are many challenges for teachers in fully understanding the many aspects of multiplicative thinking so that they can then decide on the best ways to support their students in acquiring that conceptual understanding. The findings of this study suggest that teachers need considerable support in coming to understand what multiplicative thinking involves and how they can use NDP resources, such as the booklets (for example, Ministry of Education, 2007a, b, c), to help their students. For teachers working at the upper primary level, the two years of professional development provided by the NDP was not enough for most of them to become familiar with superficial aspects of the Number Framework and the assessment tools, let alone build an in-depth understanding of the NDP approach overall.

The two years' of professional development that most of these intermediate (year 7–8) teachers had been given does not appear to have been sufficient for them to fully understand the complexities of multiplicative thinking. Teacher A, on the other hand, had more than 20 years of teaching experience, including four years' experience with the NDP, as well as considerable ongoing professional development provided because of her role as a lead teacher in her school and as a numeracy coach for teachers in other schools.

The NDP has provided teachers with a great start on the journey but, on average, the distance travelled so far is small compared with the distance to the final destination. This destination is the point at which all students will be taught by teachers who have the necessary levels of content knowledge and pedagogical content knowledge in mathematics to ensure that the majority of their students do reach the curriculum level expectations.

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Some Strategies Used in Mathematics by Māori-medium Students

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Ko te manu kai i te miro, nōna te ngahere.
Ko te manu kai i te mātauranga, nōna te ao!

This study set out to explore the perspectives of students attending Māori-medium schools. Sixty-one year 5–8 students in four schools were interviewed individually to identify their mathematics strategies for subtraction. The 44 year 7–8 students were also asked to solve a multiplication problem. The findings show that there was a range of mental strategies displayed by the students.

Background

Best & Hongi (2002) state that pre-European Māori had a "very good system of numeration and doubtless quite elaborate enough for their purposes" (p. 1). Such a system enabled them to count items singly, in pairs, in twenties, and in hundreds.

With the advent of European-style schooling, Māori students were assimiliated into a Western system of education in which little was done to explore connections with a Māori education paradigm. This contributed to low achievement generally for Māori in the European school system, and mathematics was no exception. This situation remained constant for a long period of time (Knight, 1994; Forbes, 2002).

In recent times, this negative trend has begun to reverse for Māori students in mainstream schools (Caygill & Kirkham, 2008). There have been significant increases in the mean mathematics achievement of these Māori students as measured by international benchmarks (Ministry of Education, 2006). For example, the National Education Monitoring Project, report 38, (Crooks & Flockton, 2006) states that students in Māori-medium schools performed well in tasks that involved recall of basic facts and geometric reasoning.

These gains may be a reflection of curricular changes in the 1992 *Mathematics in the New Zealand Curriculum* document, which explicitly stated that many Māori students' mathematics needs had not been catered for. A greater emphasis was therefore placed on schools to provide more appropriate mathematics teaching and assessment tools, taking cognisance of the background and experiences of Māori students (Ministry of Education, 1992, 1996; Christensen, 2002). Encouraging the confidence and greater participation of Māori students when they were learning mathematics became a key obligation for schools (Garden, 1997; Ministry of Education, 1992; 1996).

Māori initiatives such as kōhanga reo and Māori-medium schools have placed emphasis on improving Māori achievement in education (Smith, 1991). These developments contributed to a demand for appropriate resources for use in Māori-medium settings. A later emphasis on numeracy in New Zealand resulted in Te Poutama Tau, a professional development project based on the Numeracy Development Projects (NDP) (Christensen, 2002). Te Poutama Tau focuses on teacher development to enhance students' learning of numeracy in Māori-medium schools.

In an effort to support the improvement of mathematics learning, there has been an increasing expectation that mathematics education needs to shift from learning and remembering procedures involving number to solving problems within a range of meaningful contexts (Ministry of Education, 1992, 1996). The mathematics learning area in *Te Marautanga o Aotearoa* emphasises that students should be making connections within and across different mathematics ideas (Ministry of Education, 2008). The ability to make such connections is perceived to be critical for the development of number sense (Anghileri, 2006).

Students who have developed a rich sense of number display flexibility of thinking and an awareness of the links and relationships between number ideas. These students are able to make generalisations about number and choose the most efficient and appropriate operations to solve problems in ways that make sense to them. Such processes can aid problem solving in alternative situations (Angliheri, 2006; Dowker, 2005).

Part of number sense involves being able to utilise mental computation strategies in a variety of situations. In order to develop such strategies, students need to learn and readily access a wide base of number facts. Knowledge of number facts can provide a platform for helping students to develop further thinking about number. The more known facts students can access mentally, the greater their potential for constructing strategies to solve mathematics problems (Dowker, 2005; Thompson, 1999a).

In recent years, the development of robust methods of mental computation has become a focus generally in mathematics education in the western world. In Britain, for example, the National Numeracy Strategy advocates exposing students to a variety of mental strategies. From such a range of strategies, students are encouraged to explore and choose the most appropriate for the situation or problem they are engaged in. This process assists them to develop confidence in their ability to problem solve (Suggate, Davis, & Goulding, 2006). These precepts are also embedded in *Te Marautanga o Aotearoa* (Ministry of Education, 2008).

Being able to articulate a mental computation strategy is deemed beneficial for students' learning in mathematics (Zevenbergen, Dole, & Wright, 2004; Ittigson, 2002). The ability to do so must be learned (Anthony & Walshaw, 2007). All recent curriculum documents and Te Poutama Tau reflect the importance of learners developing proficiency in the articulation of their mathematics thinking. Furthermore, Moschkovich (2002) reminds us that mathematics discourse is more than learning mathematics terminology. Students also have to learn to participate in valued mathematics discourse practices.

Māori-medium settings support learners to articulate their mathematics thinking in Māori. These settings have necessitated the development of appropriate vocabulary in Māori for learning mathematics. Mathematics discourse that is interwoven with the conceptual development of mathematics ideas continues to be a challenge for learners, including Māori (Barton, 2008; Christensen, 2004).

It is helpful for students to become familiar with written ways of recording their mathematics thinking. This enables them to record their mathematics thinking systematically and then to communicate those ideas. Written methods should closely align with their mental processes of calculation (Anghileri, 2006; Beishuizen & Anghileri, 1998; Suggate et al., 2006). Traditional western algorithmic procedures for calculating implemented in schools have seldom mirrored mental computation processes and have contributed instead to "cognitive passivity"(Thompson, 1999a, p. 173). It is argued that mathematics instruction should therefore include ways of helping all students to integrate their mathematics thinking and recording so that they are not using procedures by rote in meaningless ways (Gilmore & Bryant, 2008).

In order to align students' mental and written computation strategies, close examination of their mathematics thinking for solving mathematics problems is necessary. For example, when subtracting, students can exhibit a range of mental computation strategies. The recording of these strategies should reflect their mathematics thinking. This is important when students have to deal with more complex calculations (Anghileri, 2006).

Several subtraction strategies used by students have been identified. One example is the counting-back strategy (CB) that can be employed when subtracting one, two, or three items. However, this strategy is deemed to be inefficient and problematic when larger amounts need to be taken away (Zevenbergen et al., 2004). Another subtraction strategy is one where students count up (CUP) from the lower number to the higher one and thereby find the difference (Jordan, Hanich, & Uberti, 2003). A bridging-through-ten strategy (BTT) is noted by Thompson (1999b). This process involves partitioning the subtrahend (the amount subtracted) into two chunks. Subtracting one chunk from the minuend (the starting amount) takes the total to a decade number from which the remaining chunk is subtracted. For example, 37 – 9 becomes (37 – 7) – 2 = 30 – 2 = 28. Subtraction by compensation (COM) involves working from known facts. The subtrahend may be changed to a more convenient number that is then subtracted from the minuend. The change made to the original subtrahend must then be compensated for (Anghileri, 2006). For example, 37 – 9 becomes 37 – (9 + 1) = 37 – 10 = 27. So 37 – 9 = 27 + 1 (the extra 1 that was subtracted with the 9) = 27 + 1 = 28.

For students to develop efficient subtraction strategies, they need to be presented with numerous opportunities to practise the automatic retrieval of number bonds such as complements of or to ten. Immediate retrieval of this type of number knowledge will aid the development of mental flexibility with number (Beishuizen & Angliheri, 1998).

Being aware of how numbers can be manipulated is also important for the development of multiplication strategies. Siemon (2005) suggests that multiplicative thinking demonstrates an individual's "capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in various contexts" (p. 1). The simplest form of a multiplicative situation that students will meet is one where there is a one-to-many correspondence between two sets, for example, 1 car, 4 wheels. The most basic strategy students use to solve this type of problem is when they add the multiplier the number of times indicated by the multiplicand. So for 17 cars each with 4 wheels, 17 x 4 becomes 4 + 4 = 8 + 4 = 12 + 4 ... (Nunes & Bryant, 1996).

Another strategy that students might use is to double numbers to create "clumps" that are added together to find a total. These can include a double-double strategy (DD), where 17 x 4 becomes (17 + 17) + (17 + 17) = 34 + 34 = 68, or a times-doubling strategy (TD), in which 17 x 4 becomes (17 x 2) x 2 = 34 x 2 = 68. The doubling strategy may be seen as less efficient when larger numbers are involved. A standard place value partitioning strategy (SPVP), in which the multiplicand is partitioned and then each part multiplied by the multiplier and both results combined to find the solution, is deemed to be more efficient (Ambrose, Baek, and Carpenter, 2003; Baek, 2006). 17 x 4 becomes 17 partitioned into 10 and 7, 10 x 4 = 40, and 7 x 4 = 28, 40 + 28 = 68. Baek (2006) argues that compensating is a very sophisticated strategy that requires flexible thinking and a fluent understanding of both the numbers and the process of multiplication. For example, 17 x 4 becomes (17 + 3) x 4 = 80; 17 x 4 = 80 – 3 x 4 (the extra amount added on to the 17 at the beginning] = 80 – 12 = 68.

When multiplying, some students use traditionally-taught written procedures (ALG). Difficulties arise if they do not have a clear understanding of place value and try to perform calculations by following poorly understood rules (Lawton, 2005).

Method

Participants

This study focuses on the responses of 61 year 5–8 Māori students in four schools. Two schools were kura kaupapa Māori catering for students from years 0–8. Another kura catered for students from years 1–15, and the wharekura catered for students from years 0–13. Three of the kura had participated in Te Poutama Tau, the Māori-medium equivalent of the NDP, for some years prior to this study. Twenty of the students were from a decile 1 kura, 21 from decile 2 kura, and 20 from a decile 5 kura. Thirty-eight students were female, and 23 were male.

Table 1
Composition of the Students by Kura and Year Level

Kura	Year 5	Year 6	Year 7	Year 8	Total
1*	2	2	0	6	10
2*	2	3	3	2	10
3*	4	4	8	4	20
4	0	0	12	9	21
Number of students	8	9	23	21	61

* Te Poutama Tau participants

Procedure

Schools were asked to nominate year 5–8 students from across a range of mathematics levels. These students were interviewed individually for about 30 minutes in te reo Māori or English (their choice) in a quiet place away from the classroom. They were told that the interviewer was interested in finding out about their thoughts regarding their learning of pāngarau/mathematics.

The questions this paper focuses on are part of a larger collection of questions that the students were asked to respond to. Other questions have been previously analysed and discussed elsewhere (Hāwera & Taylor, 2008; Hāwera, Taylor, Young-Loveridge, & Sharma, 2007). The subtraction and multiplication questions were selected on the basis that the mathematics involved should be accessible to the students. The questions were designed to elicit the use of important and relevant strategies and knowledge that this age group could be expected to employ. The questions analysed here are:

1. E $37 i roto i te pēke o Te Āwhina. I pau i a ia te $9 ki te toa. E hia ana moni kei te toe?

   Te Āwhina had $37 in her bag. She spent $9 at the shop. How much money did she have left?

2. E hanga motokā ana te kamupene o Hera. E 4 ngā wīra mō ia motokā. E hia katoa ngā wīra mō te 17 motokā?

   Hera has a car manufacturing company. She needs 4 wheels for each car. How many wheels does she need for 17 cars?

Audiotapes of the interviews were transcribed by a person fluent in te reo Māori. Transcripts were subjected to a content analysis to identify common strategies in the students' responses. The students' responses have been coded[1] to maintain confidentiality and to be consistent with the reporting of other data from the larger study.

[1] See explanation of code on page 70.

Results

Subtraction Strategies

All 61 students in the study were asked the subtraction question, and all provided a solution. Fifty-three solved the subtraction problem correctly, and all of these students could articulate the strategy they used. As might be expected, there was a range of responses to this question. The responses have been grouped according to the following strategies:

BTT: bridging through ten, for example, $37 - 9 = (37 - 7) - 2 = 30 - 2 = 28$

COM: compensating, for example, $37 - 9 = 37 - (9 + 1) + 1 = 37 - 10 + 1 = 27 + 1 = 28$

CB: counting back by one, in this case, from 37

CUP: counting up by one, in this case, from 9 to 37

ALG: using a traditionally-taught written procedure

Nexp: no explanation of the strategy used.

Table 2
Strategies used for the Subtraction Task

Kura	Number of Students in Study	BTT	COM	CB	CUP	ALG	Nexp
1*	10	5	1	4 (2W)	0	0	0
2*	10	5 (1W)	3	1	0	0	1 (1W)
3*	20	7	9	2 (1W)	0	0	2 (1W)
4	21	7	2	3 (1W)	2 (1W)	2	5
Total	61	24	15	10	2	2	8

*Te Poutama Tau participants
(*n*W) indicates the number of incorrect solutions

The two most commonly used strategies were bridging through ten and compensation. These strategies require efficient and flexible mathematics thinking (Beishuizen & Anghileri, 1998). Of the 61 students interviewed, 24 used the bridging-through-ten strategy for solving this problem. The compensating strategy was used by 15 of the students. It is noted that a significant proportion of students from each of the schools demonstrated the use of these strategies.

The less efficient counting-back strategy was used by 10 of the students. These students came from each of the kura. Four of them gave an incorrect solution: two used the counting-up-by-one strategy, and the other two used an algorithm. Of the 12 students who used a counting-up-by-one strategy, almost half reached an incorrect solution. Eight of the students did not divulge their strategy for this problem, although most of these students calculated a correct solution. Of the group who did not disclose their strategy, most were from kura 4.

Six of the students were able to offer a second strategy for finding a correct solution (see Table 3). (Four of these students were from kura 3 and two from kura 4.) This indicates a desired flexibility of thinking and an ability to employ number sense in a pressured situation.

Table 3
Subtraction: Two Strategies (as stated by the students)

Names	Kura	Strategy 1	Strategy 2
K37f7[2]	3	30 – 9 = 21 + 7 = 28	9 – 7 = 2, 30 – 2 = 28
K39m6	3	37 – 10 = 27 + 1 = 28	40 – 9 = 31 – 3 = 28
K46m7	3	39 – 11 = 28	37 + 2 = 39 – 9 = 30 – 2 = 28
K38f7	3	37 – 10 = 27 + 1 = 28	37 – 9 = 28
K61m7	4	37 – 7 = 30 – 2 = 28	* 37 on the top and then you take 9 away from 7 but you cannot do that so you slash the 3 and put 1 on the 7, [that] makes it 17, and then you take 9 away from 17, which is 8, and then you write a 2, and then it's 28.
K51m7	4	40 – 10 = 30 – 3 = 27 + 1 = 28	37 – 7 = 30 – 2 = 28

* asked for paper to record on.

One such example of this is articulated by K37f7.

Her first strategy was:

> I tango au te iwa mai ..., mai i te toru tekau ... āe, arā ko tērā te rua tekau mā tahi.
>
> Tāpiri ki te whitu.
>
> (I took the 9 from 30 and got 21. I added that to the 7.)

Her second strategy was:

> ... ka tango te whitu mai i te iwa, ā, ko te toru tekau, ka tango te rua mai i te toru tekau.
>
> (I took 7 from 9, made 30, took 2 from 30.)

Multiplication Strategies

Only the 44 year 7–8 students in the study were asked to complete the following multiplication task:

> E hanga motokā ana te kamupene o Hera. E 4 ngā wīra mō ia motokā. E hia katoa ngā wīra mō te 17 motokā?
>
> Hera has a car manufacturing company. She needs 4 wheels for each car. How many wheels does she need for 17 cars?

Thirty-six out of 44 students attempted to solve the multiplication problem. Of these, 29 were able to do so correctly. As with the subtraction problem, the students used a range of strategies to solve this problem. These responses have been categorised into the following strategies:

SPVP: standard place value partitioning, for example, 4 x 17 = (4 x 10) + (4 x 7) = 40 + 28 = 68

DF: derived fact, for example, 4 x 17 = (4 x 10) + (4 x 5) + (4 x 2) = 40 + 20 + 8 = 68.

TT: times twice, for example, 4 x 17 = (2 x 17) x 2 = 34 x 2 = 68

[2] K = kura, 37 = the 3rd group out of the 61 students and the 7th student in that group, f = female, and 7 = year level

DD: double double, for example, 4 x 17 = (17 + 17) + (17 + 17) = 34 + 34 = 68

TD: times doubling, for example, 4 x 17 = (2 x 17) + (2 x 17) = 34 + 34 = 68

C4: counting up in fours, for example, (4, 8, 12, 16 ... 68)

ALG: a traditionally taught written procedure

NA: no attempt made or no strategy offered.

Table 4
Strategies Used for the Multiplication Task

Kura	Number of Year 7–8 Students	SPVP	DF	TT	DD	TD	C4	ALG	NA
1*	6	0	0	0	0	1	0	1 (1W)	4
2*	5	1	1	1	1	1	0	0	0
3*	12	6	0	0	0	1	0	2	3
4	21	4 (1W)	4 (1W)	0	0	1	5 (1W)	6 (3W)	1
Total	44	11	5	1	1	4	5	9	8

* Te Poutama Tau participants
(*n*W) indicates the number of incorrect solutions

11 of the students used the standard place value partitioning strategy. An example of this is:

> I took the 7 away, and I just did 10 times 4 equals 40, then I done 4 times 7, which equals 28. Then I added those 2 answers together and got 68. (K68m8)

The derived-fact strategy was used by five of the students to reach a solution. Further analysis indicates that this group of students was able to make use of known facts that they were instantly able to recall. For example:

> Whā whakarau tekau ka puta whā tekau, ā, whā whakarau rima ka puta rua tekau, ā, whā whakarau rua, ka puta waru, ā, ka tāpiri ērā mea kia ono tekau mā waru. (K25f7)
>
> (4 times 10 makes 40, and 4 times 5 makes 20, and 4 times 2 makes 8, and you add those to make 68.)

Another student, who worked from a fact that she knew, said:

> ...um, I went 4 times 12 which is 48, and then I just went 4 times 5 is 20 and then added the 20 to the 48. (K65f7)

Variations of the doubling strategy (TT, DD, TD) were used by six of the students. For example:

> I rounded the 17 down to um 15. I times'd the 15 times 4, times'd 15 times 4, which equals 60.
>
> [How'd you know that equals 60?]
>
> Two 15s equals 30, then two 30s equal 60.
>
> [60 and the 2 that you took off?]
>
> You times it by 4, which equals 8, and 60 plus 8 equals 68. (K51m7)

Counting up in fours was used by five of the students, all of whom came from kura 4. The traditionally-taught written procedure was used by nine of the 44 students, most of whom came from kura 4. Just over half of the students who used the algorithmic strategy were able provide the correct solution.

Eight of the students indicated that they did not know how to do the multiplication task and made no attempt to do so. Of the 36 students who did attempt the problem, seven of these provided an incorrect solution.

For the multiplication task, nine of the students shared more than one strategy for finding the solution. All of these solutions were correct. This group included three of the students who had also offered a second strategy for the subtraction problem.

Table 5
Multiplication: Two Strategies (as stated by the students)

Names	Kura	Strategy 1	Strategy 2
K27f8	2	2 x 17 = 34 + 34 = 68	4 x 10 = 40, 4 x 7 = 28, 40 + 28 = 68
K25f7	2	4 x 10 = 40, 4 x 5 = 20, 4 x 2 = 8, 40 + 20 + 8 = 68	4 x 20 = 80, 4 x 3 = 12, 80 –12 = 68
K36m7	3	17 x 2 = 34 + 34 = 68	4 + 1 = 5, 5 + 5 = 10, 10 x 17 = 170 ÷ 2 = 85 – 17 = 68
K37f7	3	10 x 4 = 40, 7 x 4 = 28, 40 + 28 = 68	17 x 2 = 34 x 2 = 68
K46m7	3	10 x 4 = 40, 7 x 4 = 28, 40 + 28 = 68	* 4 x 7 = 28, 4 x 1 = 4 + 2 = 68
K38f7	3	4 x 10 = 40, 4 x 7 = 28, 40 + 28 = 68	17 + 17 + 17 + 17 = 68
K610f7	4	4, 8, 12, 13, 14, 15, ... 68	* 4 times 7 is 28, and then you stick the 2 there, and then … 4 times 1 is 4, plus the 2 is 6.
K64f8	4	4 x 7 = 28, 4 x 10 = 40 28 + 40 = 68	17 x 2 = 34 x 2 = 68
K57f7	4	4, 8, 12 … 68	* 4 times 7 is 28. Put the 8 down here and the 2 up there. 2 and 4 is 6.

* asked for paper to record on

For example, K36m7's first strategy was:

> Um, tekau mā whitu whakarau rua, ka toru tekau mā whā, toru tekau mā whā tāpiri toru tekau mā whā, ka ono tekau mā waru.
>
> (17 times 2 equals 34. 34 plus 34 equals 68)

His second strategy was:

> Ka taea ki te tāpiri te kotahi i runga i te whā ka rima, kātahi huri te rima ki te tekau; tekau whakarau tekau mā whitu ka tahi rau whitu tekau, kātahi me hāwhe te tahi rau whitu tekau ka waru tekau mā rima, kātahi tango tekau mā whitu ka ono tekau mā waru.
>
> (You can add 1 to the 4 to make 5, then change the 5 to 10; 10 times 17 makes 170, then half the 170 makes 85, then take away 17 equals 68.)

Discussion

It is heartening to see that many of these students seemed to readily make sense of these two word problems, given that they had only just met the interviewer, whose Māori language usage they may not have been accustomed to. The students were also isolated from their normal classroom and were expected to adjust quickly to an unfamiliar situation.

Of the students interviewed, most were able to successfully complete the problems. For the subtraction task, almost two-thirds of the students used the bridging-through-ten (Thompson, 1999b) and compensation strategies (Anghileri, 2006). The strategies were used by some students from each of the kura. Many students were able to demonstrate the ability to reason from what they knew and understood about basic facts and the ways that numbers can be manipulated. This flexibility with number is an important aspect of developing rich number sense and thereby increases the facility to utilise number in different contexts (Anghileri, 2006; Dowker, 2005).

While most of the students could solve the problems presented, some used less efficient strategies that proved to be cumbersome and time consuming for them. For example, some students from kura 4 displayed the use of the less efficient counting-up-in-fours strategy when multiplying, while others were able to draw on the more efficient standard place value partitioning and derived-fact strategies (Dowker, 2005; Baek, 2006). More consideration of and emphasis on pedagogical practices to help some students become more efficient thinkers and manipulators of number may be required. For example, when appropriate, students should be encouraged to multiply rather than to continue with repeated addition. Such connections need to be made more explicit for some learners (Anghileri, 2006). This ability to think multiplicatively has implications for developing and appreciating algebraic relationships and should therefore not be underestimated (Lamon, 2007; Watson, 2008).

It is interesting to note that some students were able to share more than one strategy to solve the subtraction and multiplication problems. This indicates the flexibility of being able to use number knowledge in different ways and awareness that there can be more than one pathway to a solution (Young-Loveridge, 2006). Zevenbergen et al. (2004) maintain that expecting students to explain, listen to, and reflect on a range of strategies helps them make better sense of the mathematics they engage with. Such thinking is illustrative of current expectations of students' learning in mathematics education (Ministry of Education, 2007, 2008).

Of the 12 students who used a counting-up-in-one or a counting-back-in-one strategy for subtraction, almost one-half calculated an incorrect solution. This indicates that the cognitive demand of these strategies predisposes students to make errors as they try to keep track of their calculations. More students made errors when subtracting using this process than those who used alternative methods. This confirms Zevenbergen et al.'s (2004) view that when subtracting more than three items, a more efficient and effective strategy should be employed.

An emphasis on communication is reflected in recent curriculum documents and support resources in New Zealand (Ministry of Education, 1992, 1996, 2007, 2008; Christensen, 2002). It is noteworthy that 41 of the 61 students chose to be interviewed in te reo Māori and were able to express their mathematics reasoning clearly and succinctly in that language. This indicates a confidence in their knowledge and use of appropriate mathematics vocabulary and discourse. Barton (2008) argues that mathematics development is affected by language development and vice versa. Parallel advancement in both of these aspects has implications for students' ability to learn about and share mathematics ideas. Students who did not share their strategies in this study may not have been able to make the necessary connections between their mathematics thinking and the language required to express it.

Analysis of the multiplication results from this study indicates that 15 of the 44 year 7–8 students were not able to either access the problem or solve it correctly. Of the seven students who provided an incorrect solution, four used an algorithmic strategy. These students did not appear to fully understand the process of manipulating numbers when using the algorithmic procedure (Gilmore & Bryant, 2008; Thompson, 1999a).

Eight of these year 7 and 8 students did not seem to have a strategy to solve or even begin the multiplication problem. Given that these students are likely to have been exposed to the process of multiplication for at least three years (Ministry of Education, 1992), it is a concern that they appeared unable to access and use a mathematics strategy to solve the problem. Learning about multiplication is a complex process, and research indicates that the development of multiplicative thinking has proved to be a challenge for many students (Lamon, 2007).

Although data has been collected from four Māori-medium schools, a limitation of this paper could be that these findings pertain to just 61 students in total, only 44 of whom were asked to respond to the multiplication problem. This makes it difficult to generalise, but it does give some insights about students learning mathematics in Māori-medium settings.

There seemed to be no significant trends emerging from any one school because students from all the schools shared a range of mathematics strategies. As expected, those students involved in Te Poutama Tau were more inclined to use strategies promoted in that project. It should be noted though, that use of those strategies was also apparent in the kura not involved in Te Poutama Tau. This illustrates that students may well be able to generate a variety of mathematics strategies without formal instruction. However, overt teaching that promotes efficient strategy development and number knowledge appears to be beneficial for increasing students' facility with number.

Conclusion

This study indicates that the majority of students interviewed in Māori-medium settings were able to solve the two problems offered to them and articulate their mathematics thinking. They displayed a diverse range of strategies for mental computation for subtraction and multiplication, some of which were more efficient than others. The process of multiplication appeared to be a challenge for about 15 of the year 7–8 students. This is a concern and warrants further investigation.

Many of these students in Māori-medium settings demonstrated that they are developing mathematics competency. Most were able to communicate clearly their ideas in te reo Māori. Further facilitation of their mathematics learning is required to support the use of more efficient strategies so that they can continue to develop more sophisticated ideas in the realm of numeracy. This is essential if we are serious about Māori gaining more equitable access to all the opportunities that the New Zealand societal landscape has to offer.

Concluding Comments

It has been a privilege to listen to the mathematics ideas presented by these Māori students. These results indicate that the majority of these students have experienced success in demonstrating a way to solve these problems and articulate their thinking. What are the challenges for those who have not been able to do this? Further research into identifying the barriers preventing the development of multiplicative thinking in Māori-medium settings should be explored and acted upon to ensure success for all students.

Furthermore, because we expected students to be able to access and complete these problems, it would be enlightening to explore the ways in which year 7–8 students approach more complex mathematics problems that encompass a broader range of ideas. This can only provide further insights into students' thinking and learning of mathematics in Māori medium and contribute to the paucity of the knowledge base in this area.

Hei Mihi

Hei kapi ake, ka haere tonu ngā mihi ki te iti me te rahi me ā rātou tautoko, mō tā rātou ūtanga mai ki te kaupapa. Mei kore rātou, e kore e pēnei rawa te puta o ngā māramatanga me ngā momo kōrero hei tautoko i te kaupapa nei. Mauri ora ki a tātou!

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Linking Teacher Knowledge and Student Outcomes

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This paper reports on the ongoing analysis of the validity of a teacher-knowledge assessment as a predictor of students' progress. The assessment aims to identify the teachers who have the weakest knowledge for the teaching of fractions in order to enable the provision of targeted professional development support. The relationship between teachers' scores on the assessment and student achievement gains as measured against the Number Framework was investigated. Rasch analysis was employed to obtain a measure of student gain that could be compared irrespective of starting stage. Results provide evidence of the validity of the content measures in the assessment, with students of teachers who scored highly in the content measures making significantly greater gains than students of teachers who scored poorly in the content measures. Findings for the measures based on pedagogical content knowledge were unexpected, with students of teachers scoring highly in these measures making significantly smaller mean gains than the students of teachers scoring poorly.

Background

It is well established that teachers of mathematics require a broad range of knowledge in order to be effective (Wayne & Youngs, 2003) and that professional development programmes have an important role to play in supporting the development of teacher knowledge (Putnam & Borko, 1997). Recent studies have indicated that programmes focused on the development of teacher knowledge are successful in changing teacher practice and improving student performance (Borko, 2004). The most effective professional development programmes build on teachers' existing knowledge, so an initial assessment of teachers' understandings may provide valuable information that can be used to provide targeted professional development support (Guskey, 2000).

This paper reports on the ongoing analysis of the validity of an assessment (see Appendix G, pp. 186–188) that focuses on teachers' knowledge for the teaching of fractions as a predictor of students' progress (Ward & Thomas, 2007, 2008). The assessment was developed for use in professional development programmes within the Numeracy Development Projects (NDP). The assessment is based on previous research in which measures of teachers' content knowledge for teaching were aligned with student achievement information; this research found that students with teachers in the lowest third of the knowledge distribution had significantly lower achievement (Hill, Rowan, & Ball, 2005). Based on this work, the assessment aims to identify those teachers who have the least knowledge for the teaching of fractions in order to enable the provision of targeted professional development support.

Previous analysis investigating the relationship between teacher scores on the assessment and student achievement information was limited by the non-linear scale of the stages of the Number Framework (Ward & Thomas, 2008). Students progress more quickly through the lower stages of the Framework than through the upper stages (Irwin, 2003; Irwin & Niederer, 2002; Ward & Thomas, 2002), so a simple comparison of stage gains in relation to teacher scores is not possible. The current study employs Rasch analysis to obtain a linear measure of student gain that can be compared across students, irrespective of starting stage. Similar work has been carried out previously (Irwin, 2003; Irwin & Niederer, 2002), but there was a need to update this analysis to take account of changes to the Number Framework.

The current analysis investigates the relationship between teacher scores on the assessment and Rasch measures of student achievement gain. If teachers who score poorly in the assessment have students who make significantly lower gains, this is evidence that the tool is fit for purpose; it is able to identify those teachers with the greatest need for professional development.

Method

Materials

Information on teacher knowledge was collected using the teaching of fractions assessment. The assessment is designed to collect information on teachers' content knowledge (CK) and pedagogical content knowledge (PCK) in the area of fractions. Teacher responses were evaluated using a marking criteria developed alongside the assessment. The assessment was marked out of a total of 17 points: the CK component had a total possible score of 7 points and the PCK component was worth 10 points. Further information about the development, nature, and trial of the assessment can be found in previous reports (Ward & Thomas, 2007, 2008).

Student achievement data was collected by teachers using the Numeracy Project Assessment (NumPA). The NumPA is an individual interview that provides information on students' knowledge and strategy stages aligned to the Number Framework.

Method

Participating schools received teacher assessments by post, along with a set of detailed instructions for their completion. Assessments were completed at the school and returned to the researcher for marking.

Two sets of student achievement information were collected by teachers: initial data before instruction began and final data near the end of the 2007 school year. Student data was entered into the online numeracy database.

Further information about data collection methods can be found in previous reports (Ward & Thomas, 2007, 2008).

Participants

The final sample consisted of 17 schools: one low-decile school (1–3), 10 medium-decile schools (4–7), and 6 high-decile schools (8–10). Results from 88 teachers were included in the sample. Table 1 provides a summary of the year levels taught by these teachers. Participating teachers had between 6 months and 30 years teaching experience, with the majority of teachers' experience being at the year level of the students they were currently teaching.

Table 1
Participating Teachers

Year levels	Number of Teachers	Percentage of Teachers
1–3	27	31
4–6	18	20
7–8	32	36
9	11	13
Total	88	100

On average, each participating teacher provided complete results for 22 students, with between 2 and 49 sets of student results per teacher. Table 2 shows the distribution of participating students across year levels.

Table 2
Participating Students

Year levels	Number of Students	Percentage of Students
1	168	9
2	188	10
3	172	9
4	119	6
5	140	7
6	122	6
7	371	19
8	445	23
9	221	11
Total	1946	100

Analysis

A Rasch analysis was used to describe student gain in a three-step process. Initially, item difficulty estimates for each of the stages within the three strategy and the six knowledge domains within the Number Framework were calculated using a maximum log likelihood procedure and software developed for the purpose using R (a statistical programming language). The difficulties of all stages of all nine domains were estimated on the same logistic (Rasch) scale. The estimates were calculated using the initial results of 4000 students randomly selected from the 47 005 entries in the 2007 national numeracy results database.

Initial and final student ability estimates for the students in the sample were calculated using the item difficulty estimates and the students' initial and final stage ratings on each of the nine domains. Student gain scores were then calculated for each student by subtracting the initial ability estimate from the final ability estimate. These gain scores provided a measure of student progress in logits, which could be compared across all students irrespective of starting stage.

The relationship between student gain scores and teacher scores on the assessment was investigated. For comparison purposes, teachers were split into low-scoring and high-scoring groups. These groups were based on the distribution of scores, with one-third of the teachers placed in the low-scoring group and one-third of the teachers placed in the high-scoring group for each of the three score types: CK, PCK, and total. Table 3 shows the distribution of scores for each of these groups.

Table 3
Scoring Range of Low- and High-Scoring Comparison Groups

	Low Scoring		High Scoring	
Score	Score	Number of teachers	Score	Number of teachers
Content knowledge (CK)	≤ 4	25	7	26
Pedagogical content knowledge (PCK)	≤ 2	25	≥ 6	23
Total (CK and PCK)	≤ 7	27	≥ 12	26

T-tests were used to compare the mean gain score of students with low-scoring teachers to the mean gain score of students with high-scoring teachers for each of the three groups. For the purposes of this paper, effect sizes of 0.2 or less are described as small, effect sizes between 0.2 and 0.8 are described as medium, and effect sizes of 0.8 or higher are described as large (Cohen, cited in Coe, 2002).

Findings and Discussion

This section reports on the findings of this research under three headings. Firstly, the achievement gains of the students whose teachers attained low or high scores on the assessment are compared. This is followed by a breakdown of gains by year level. Finally, the effect of CK scores is investigated for groups in which PCK is low, and similarly, the effect of varying PCK scores is described for groups in which CK is high.

Comparison of Student Achievement Gains for Low- and High-scoring Teachers

Significant differences were found between the mean gains of students with low- and high-scoring CK teachers, with a medium effect size of 0.24. No significant differences between the mean gains of low- and high-scoring groups were found using PCK or total scores. Table 4 shows these results.

Table 4
Comparison of Mean Student Gain for Low- and High-scoring Teacher Groups

	Mean Gain (logits)				
Score	Students of low-scoring teachers (n)	Students of high-scoring teachers (n)	Difference in means	Effect size	Significance level
Total	0.95 (583)	1.03 (546)	0.08	0.11	Not significant
CK	0.96 (540)	1.15 (582)	0.19	0.24	<0.01
PCK	1.00 (554)	1.03 (449)	0.03	0.05	Not significant

Note: n = number of students

The differences between the mean gains for low- and high-scoring CK groups seem small, but they indicate a substantial difference in gains between the groups. Students with a teacher in the high-scoring-CK group gained, on average, an additional 0.19 logits. This is 20% more than the average gain made by students of low-scoring-CK teachers. Over the course of a school year, this effectively equates to an additional eight weeks of instruction for the students of high-scoring teachers.

The lack of significant differences in the mean gains of students with teachers in the low- and high-scoring-PCK groups was unexpected, as was the lack of significant differences in the mean gains of students with teachers in the low- or high-scoring total groups. Further analysis was therefore carried out by year level.

Comparison of Student Achievement Gains by Year Level

Because the assessment was developed to assess knowledge for teaching at stages 7 and 8 of the Number Framework, it might be expected that the relationship between student gain scores and teacher scores on the assessment would be strongest at higher year levels. Comparisons were made between the student gains of low- and high-scoring teachers for all three score types (CK, PCK, and total) by year level. Significant differences were found between the means of low- and high-scoring-CK groups at years 7–8 and 9. Table 5 displays the results for CK scores. No significant differences where found between the mean gains of low- and high-scoring groups for PCK or total scores.

Table 5
Mean Gain Comparison: Low- and High-scoring Groups by Year Level

	Mean Gain (logits)				
Year	Students of low-scoring teachers (n)	Students of high-scoring teachers (n)	Difference in means	Effect size	Significance level
1–3	0.99 (206)	0.94 (117)	0.05	0.13	Not significant
4–6	0.89 (116)	0.87 (141	0.02	0.03	Not significant
7–8	1.00 (205)	1.42 (254)	0.42	0.42	<0.01
9	0.43 (13)	1.13 (70)	0.69	0.91	<0.01

Note: n = number of students

The differences in the student gains of low- and high-scoring-CK teachers in years 7–8 and 9 were substantial, with effect sizes of 0.42 in years 7–8 and 0.91 in year 9. At years 7–8, students of high-scoring-CK teachers made on average 42% more progress than students of low-scoring-CK teachers. At year 9, students of high-scoring-CK teachers made on average 160% more progress than students of low-scoring-CK teachers, although the small sample size for students of low-scoring teachers at this level indicates a need to be cautious when interpreting these results.

While the analysis by year level shows a clear relationship between measures of student gain and teachers' CK scores, teachers' PCK scores were found to be unrelated to student gain scores at any level. This may be explained, in part, by the relationship between CK and PCK scores, which makes a comparison of high and low scores problematic. A group of teachers with low CK scores will necessarily have low PCK scores, whereas a group of teachers with high CK scores may have a range of PCK scores (Ward & Thomas, 2008). In other words, those teachers who are unfamiliar with specific content are unlikely to know effective ways to teach that content, while teachers who are familiar with the content may or may not know how to teach it effectively. Conversely, a group of teachers with high PCK scores will necessarily have high CK scores, whereas a group of teachers with low PCK scores may have a range of CK scores. Figure 1 shows a plot of teachers' CK scores versus their PCK scores to illustrate this relationship. A line of best fit is included, and the numerals in brackets give the number of teachers at each of the data points.

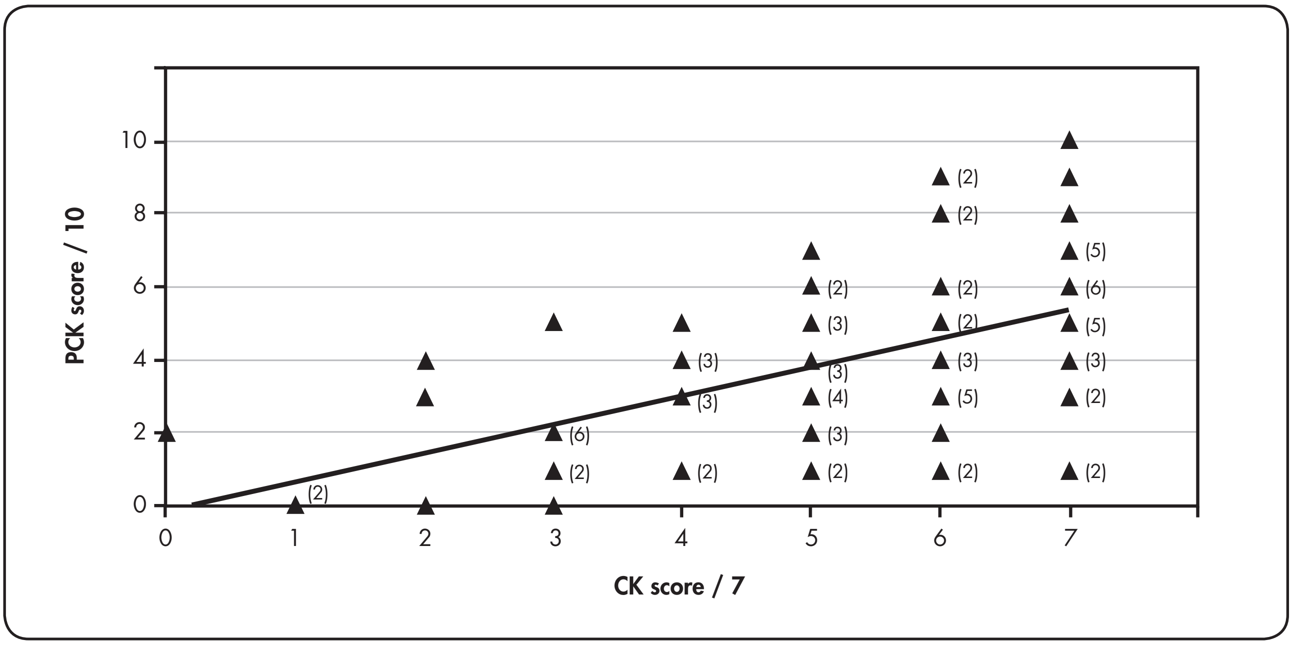

Figure 1. Teachers' CK and PCK scores

Comparisons of Individual Effects of CK and PCK

As described in the previous section, the relationship between CK and PCK scores complicates comparisons of low- and high-scoring groups. To mitigate this effect to some extent, two further analyses were carried out: firstly, an investigation of the student gains of low- and high-scoring-CK teachers whose PCK was low, and secondly, an investigation of the student gains of low- and high-scoring-PCK teachers whose CK was high. The analysis was carried out using students from years 7–8 and 9 for whom a relationship between student gain and teacher CK scores had already been established.

Three groups of year 7–9 teachers were established to enable these comparisons. Group 1 consisted of teachers who were low scoring in both CK and PCK, group 2 was made up of teachers who were low scoring in PCK and high scoring in CK, and group 3 teachers were high scoring in both CK and PCK. Figure 2 illustrates these groups.

	Low CK	High CK
High PCK		Group Three: 7 teachers 140 students
Low PCK	Group One: 6 teachers 140 students	Group Two: 2 teachers 50 students

Figure 2. CK and PCK comparison groups, years 7–9

Comparing groups 1 and 2 enables us to look at the effect of CK scores where PCK is low. Teachers in the high-scoring-CK group were found to have a significantly higher mean student gain than teachers in the low-scoring-CK group ($p < 0.01$). Figure 3 illustrates the mean gain for groups 1 and 2, using error bars with a 95% confidence interval for the mean.

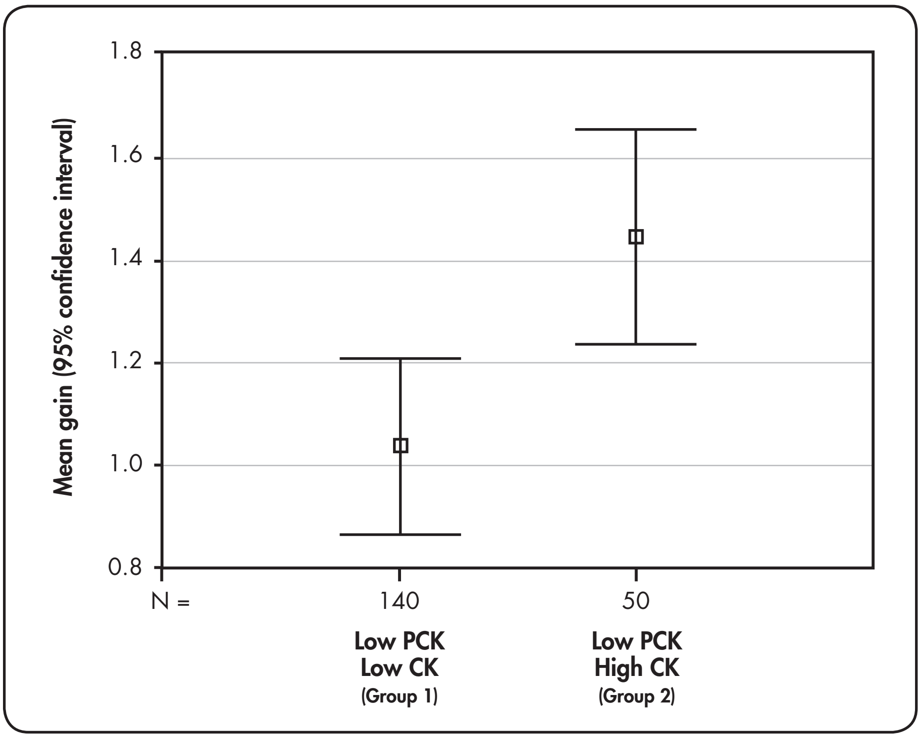

Figure 3. Effect of CK scores when PCK is low

A T-test was also carried out using student gain scores from the two groups. Students with low-scoring-CK teachers had a mean gain of 1.04 logits, while students with high-scoring-CK teachers had a mean gain of 1.44 logits. This difference was found to be significant ($p < 0.01$), with a medium effect size of 0.42. These results are in accordance with the relationship between student gain scores and CK measures already identified.

A comparison of groups 2 and 3 enables a look at the effect of PCK scores where CK is high. Unexpectedly, teachers in the low-scoring-PCK group were found to have a significantly higher mean student gain than teachers in the high-scoring-PCK group. Figure 4 illustrates the mean gain for groups 2 and 3, using error bars with a 95% confidence interval for the mean.

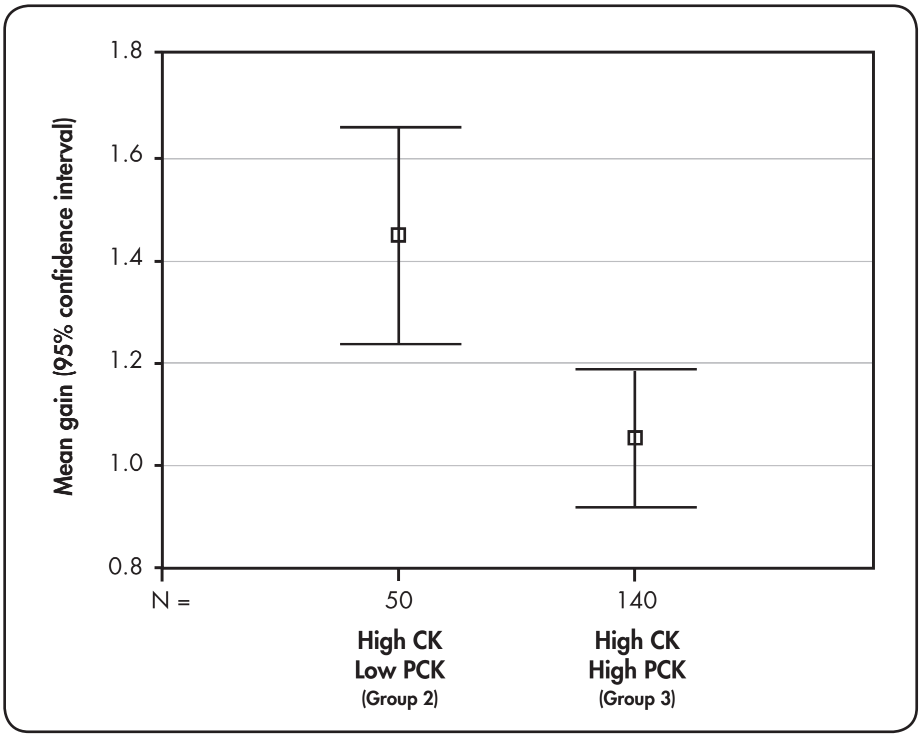

Figure 4. Effect of PCK scores when CK is high

A further T-test on the student gains of these two groups showed that students with teachers in the low-scoring-PCK group had a mean gain of 1.44 logits, while students of teachers in the high-scoring-PCK group had a mean gain of 1.05 logits. This is a difference in the means of 0.40 logits, which is significant ($p < 0.01$), with a medium effect size of –0.51. This result was unforseen and surprising; students with teachers in the high-scoring-PCK group had significantly smaller mean gains than students with teachers in the low-scoring-PCK group.

A review of the literature provides some explanation for the result in which higher PCK scores are related to diminished student gain. While there have been several attempts to measure teachers' PCK in mathematics (see, for example, Carpenter, Fennema, Peterson, & Carey, 1988 and Hill, Schilling, & Ball, 2004), attempts to align teachers' scores on these measures with student achievement data have been unsuccessful. This is in contrast to measures of teachers' content knowledge that have successfully been aligned with student achievement data (see, for example, Hill, Rowan, & Ball, 2005). The lack of alignment between measures of PCK and student achievement data has been attributed to the multidimensional nature of PCK (Alonzo, 2007; Hill, Ball, Blunk, et al., 2007; Hill, Ball, & Schilling, 2008). As a construct, PCK is problematic to measure because it consists of several elements. These include: knowledge of the development of student thinking in particular domains, knowledge of common student misunderstandings, and knowledge of effective teaching representations. The broad nature of PCK results in measurement difficulties.

Concluding Comment

Fractions are a complex and crucial area of mathematics instruction. They are recognised as both one of the most important areas in the primary school mathematics (Behr, Lesh, Post, & Silver, 1983; Lamon, 2007) and one of the most challenging to teach and learn effectively (Smith, 2002; Lamon, 2007). In addition, they represent the initial development of proportional reasoning, an area of the NDP in which both students' and teachers' understanding is of concern (Young-Loveridge, 2006, 2007; 2008; Young-Loveridge, Taylor, Hāwera, & Sharma, 2007; Ward & Thomas, 2007).

When establishing the validity of any assessment, the purpose for which items were developed and the context in which they are to be used are paramount (Kane, 2007; Lawrenz & Toal, 2007). On this basis, the question must be asked: are teachers' scores in the assessment useful for identifying those teachers whose students are making the least progress? The CK measures are valid in this regard at years 7–9, in which the content of instructional programmes is directly aligned with the content of the assessment. At these years, students of high-scoring-CK teachers make significantly greater gains than students of low-scoring-CK teachers. Contrastingly, the PCK measures cannot be regarded as valid. The unexplained result for years 7–9, in which the students of high-scoring-PCK teachers made significantly less gains than the students of low-scoring-PCK teachers, is puzzling.

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Mathematics from Early Childhood to School: Investigation into Transition

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There is current interest in how the mathematics content, understanding, and practices of the new entrant classroom connects with the learning that children had experienced within early childhood settings. It is acknowledged that children start school with a wealth of mathematical knowledge and experiences and that recognition of this rich resource by the new entrant teacher may facilitate the smooth transition of a child into school. This paper reports on research investigating the existing transition practices between four early childhood education services and four new entrant classes with regard to mathematics learning and teaching. This study found tenuous links between these sectors with regard to transition practices in mathematics.

Background

There has been a recent surge of interest in the development of mathematics in early childhood as a direct result of research into early mathematics learning. Issues raised by Clements and Sarama (2007) in their synthesis of research into early childhood mathematics in the USA are also evident in New Zealand. The upsurge in the number of American children in early childhood education (ECE) is mirrored in New Zealand, where it is actively and financially encouraged by the Ministry of Education (2002). Clements and Sarama also indicate that increased recognition of the importance of mathematics brings with it concerns in relation to students' mathematical achievements. Researchers have reviewed their position that young children have little or no knowledge of mathematics and now recognise the "mathematical power" (Perry & Dockett, 2005, p. 36) that young children have on entry to formal schooling. The understanding that a child's competence in mathematics at the end of the first year of schooling is a strong predictor of later success in mathematics has raised the awareness of the need for an early focus on mathematics. Clements and Sarama conclude that gaps that develop in a child's knowledge in their early years of schooling are due to the lack of connectedness between their formal and intuitive knowledge and school mathematics.

Kagan and Neuman (1998) suggest that there are high costs in not ensuring continuity between sectors; these costs relate to lower success rate at school, difficulties in making friends, and vulnerability to adjustment problems. Impediments to smooth transitions may develop through different visions and cultures, structural divisions, and communication (Broström, 2002; Kienig, 2002). Margetts (2007) asserts that the complexity of transition, particularly in making sense of the differences and discontinuities between the sectors, must be addressed because, she argues, "starting school is not a simple process" (p. 106). There are changes in the roles, activities, and interpersonal relationships between the teacher, the parent, and the child as the child moves on to school (Bronfenbrenner, 1979). Successful transition to the school setting has been described as an ecological transition between two "microsystems" (Bronfenbrenner). Tensions arise as a result of change from a learning environment based on socio-cultural and co-constructivist ideas of learning (Bronfenbrenner) to more structured activities and formal instruction. Past work on transition has been dominated by concepts of children's readiness and the need for the child to adjust to school and be the one who must change, be resilient, and cope with the difference (Dunlop, 2007). Tensions now exist between this "child-ready" approach and the "school-ready" dialogue; there is a growing expectation that the school setting should be flexible in order to help the child through changes.

According to the New Zealand early childhood curriculum, *Te Whāriki* (Ministry of Education, 1996), teaching in an early childhood setting involves "reciprocal and responsive interaction with others", building on the "child's current needs, strengths, and interests by allowing children choices and by encouraging them to take responsibility for their learning" (p. 20). The child is viewed as a competent learner and communicator, and their dispositions toward learning are an important outcome for early childhood education. *Te Whāriki* suggests: "Dispositions to learn develop when children are immersed in an environment that is characterised by well-being and trust, belonging and purposeful activity, contributing and collaboration, communicating and representing, and exploring and guided participation" (Ministry of Education, 1996, p. 45). The child's dispositions towards learning are reflected in the nature of assessment within early childhood settings (Carr, 2001). Assessment focuses on the child as a learner in specific contexts rather than on achievement objectives and skills. Narratives of incidences of a child's learning are often in the form of a "learning story" (Carr, 2001) and focus on dispositions such as curiosity, trust, perseverance, confidence, and responsibility.

The Numeracy Development Projects (NDP) are a strong influence on mathematics teaching and learning in New Zealand schools (Ministry of Education, 2001). The key focus of the NDP since 2001 has been to improve student performance in mathematics through improving the professional capability of teachers. The NDP provides a variety of assessment tools for ascertaining a student's current number knowledge and strategy use. This identification of the student's learning stages allows the teacher to provide more focused and relevant learning experiences for that student as an individual. This approach is similar to the approaches in early childhood settings; that is, progressing the student to further their understanding. The student's progress is supported by the teacher's increased pedagogical knowledge, provision of support materials, and specific assessment.

Another key element of the NDP in 2001 was to "ensure continuity between early childhood education and schools and at various transition points during schooling" (Ministry of Education, 2001, p. 5). Ensuring continuity is further supported in the 10-year strategy for early childhood, which aims "to promote collaborative relationships" (Ministry of Education, 2002, p. 2). Transition is a vital tool in this process to merge schools, with their focus on teaching curriculum content, and early childhood programmes, with their focus on learning dispositions (Peters, 2004).

The New Zealand Curriculum (Ministry of Education, 2007) acknowledges and encourages the development of dispositions in the form of "key competencies" (p. 12) that "young people need for growing, working, and participation in their communities and society" (p. 38). Although in its infancy, the implementation of this document heralds, within a formal curriculum, a focus on children's competencies in developing capabilities for living and life-long learning. The alignment of dispositions and key competencies may also develop a continuity of the learning environments across the sectors (Carr, 2006).

This research investigated the contention that what happens at the transition from early childhood education to primary school directly influences a child's ability in and dispositions towards mathematics and that a successful transition will ensure the continuity of a child's physical, social, and philosophical experiences. The research reported here was centred on one key question: "What early childhood education and new entrant practices facilitate positive transitions in mathematics between early childhood settings and primary schools?"

Methodology

Procedure

The research investigated the existing transition practices, particularly with regard to mathematics learning and teaching, between ECE services and primary schools in a small town in New Zealand. The year-long study began in the ECE settings in June and moved to the new entrant classes in the following February. A case-study approach allowed the researchers to focus on interactions between specific instances or situations (Bell, 2002) within focused time frames. Evidence was systematically collected, enabling the relationships between variables to be studied (Cohen, Manion, & Morrison, 2007). Data collection involved: observations in both sectors; questionnaires to ECE parents (6–8 weeks before the child started school); questionnaires to new entrant parents (6–8 weeks after the child started school), regardless of whether their child had attended a focus (that is, one used in the research) ECE service; interviews with those parents whose child had attended both a focus ECE service and a focus new entrant class; teacher interviews; documentation, including examples of children's work, teacher planning, and assessment information; policies relevant to teaching programmes and transition; copies of newsletters; information about schools that was available to ECE parents; copies of assessments; and photographs of mathematics in action.

Participants

Initial attention was centred on four ECE services: two early education and care centres and two kindergartens. The two early education and care centres (referred to hereafter as centres) are privately owned and open from 7.30 a.m. to 5.30 p.m. on weekdays. In the "over two" section of these centres, three teachers/care givers provide full day care to 12–15 children, a teacher:child ratio of 1:4 or 1:5. The two kindergartens are funded by the Ministry of Education and employ only registered early childhood teachers. They provide early childhood education for under-five-year-olds within set hours and have a teacher:child ratio of 1:15. They are not full day-care facilities.

Early in the following year, attention moved to the two primary schools that many of the ECE children in the study now attended. For research purposes, the schools were labelled Nikau School (decile 4, with 480 pupils) and Punga School (decile 3, with 440 pupils). Nikau School had children continually feeding into either one of two new entrant classes. Punga School had one new entrant class that had been full (25 children) from the beginning of the year and a second new entrant class that was continuing to fill. All four teachers had been involved in professional development through the NDP.

Analysis

This study used the theoretical framework in Bronfenbrenner's (1979) analogy of the child's learning environment as "interconnected systems". Here, the learner and their engagement within the immediate environment (or microsystem) is situated at the first level of learning, and the second level (or mesosystem) extends to the relationships between the immediate learning environments. Successful transition to the school setting has been described as an ecological transition between two microsystems (Bronfenbrenner).

This paper considers five key aspects of the research: structural provisions for mathematics; the assessments that are made with regard to children's mathematical understanding; how information is conveyed between sectors; processes and provisions for transition; and parental perceptions and expectations. The examples are chosen to illustrate the range of transition practices relating to children's mathematical experiences and understandings within this case study. Results of this small study are relevant to these study sites and may not be able to be generalised.

Findings and Discussion

Structural Provisions

Early childhood education services

The approach to learning in ECE is holistic in nature, based on Bronfenbrenner's (1979) idea of the child engaging with the learning environment. In ECE settings, children are immersed in rich learning across a range of subject curriculum areas; a strong focus on the child's interest is often embedded in play. ECE teachers reflect this approach in their philosophy to mathematical learning:

> Children need to have autonomy of their learning and to be able to make some choices for themselves. (Kindergarten teacher)

If a child shows a particular interest, the teacher may build on and nurture that interest, illustrating Bronfenbenner's first level of learning as a "responsive and reciprocal relationship":

> It happens throughout the whole engaged curriculum. It doesn't stand as a solitary stand-alone exercise unless it is extending a child's interest. So it is based around a child's interest and we can seize an opportunity and teachable moment and extend it. (Kindergarten teacher)

The provision of opportunities for mathematical learning and language development arose in areas such as the sand pit, block corner, and family corner. Water play, play with toy animals or cars, carpentry, and computer games were commonplace in all ECE settings:

> The water trough, tipping, measuring, pouring, full. You know, that measurement stuff. Colours as well in the water trough. Sandpit where digging holes. (Centre teacher)

Within the learning environment, children worked alone, in solitary play or in parallel play, or played together. The teachers observed, interacted, challenged, scaffolded, co-constructed, or were not present. More formal activities, such as shape matching, bead threading, puzzles, and jigsaws, also provided for mathematical learning. At times, a teacher would remain at the activity, encouraging and extending the learning through conversations and challenges. Mat time or whānau [group] time often provided an opportunity for a mathematics focus as a result of child observations or a teacher-initiated focus:

> I was working with a little boy who was going 1, 2, 3, and I thought, there is a whole heap of stuff there ... So that is why I thought of bringing in [at mat time] the actual 1, 2, 3 ... It's not giving him the knowledge, it's like developing an awareness. (Kindergarten teacher)

The two centres provided a planned "focus on four-year-olds" programme, with a strong "preparation for school" approach. The weekly sessions had either a strong subject focus (literacy or numeracy) or allowed these children to engage with extra resources less suitable for younger children. The numeracy emphasis was on children developing an awareness of numbers. The children were also encouraged to concentrate, complete simple tasks, and develop social skills:

> The focus-on-four programme is mostly a preparation for school. For instance, just learning their numbers for a start. Learning to recognise 1 to 10, maybe 1 to 20, learning to count it, and learning to write them. That's what I had hoped in my programme. (Centre teacher)

Margetts (2007) suggests that a component of successful transition is the provision of culturally important academic and social understanding and skills before the child starts school. However, she cautions that care must be taken to ensure that developmentally appropriate early childhood programmes are not changed to be more like a school class. This care was reflected by both kindergartens, where no allowances were made for those children about to move to a new entrants' class:

> The whole *Te Whāriki* is more of a holistic approach to children's learning. They will learn all those things in due course through stages of development, through their own interest-driven activities. (Kindergarten teacher)

School new entrant classes

The approach to learning in a school setting may be viewed as a change in focus from personal, social, and emotional development in the ECE setting to the formal beginning of specific subjects and prescribed content (Stephenson & Parsons, 2007). Having lessons is a big contrast to the socio-cultural experiences emphasised in the ECE setting. In Bronfenbrenner's (1979) framework, this move is towards the second level of learning. The children are being affected by what is happening outside their own microsystem.

Mathematics learning in the four new entrant classrooms was teacher-initiated, with focused learning intentions. The teachers had fixed ideas as to the particular needs of new entrant children and planned and directed the children's learning accordingly. Similarly to a New Zealand study of five new entrant classes (Sherley, Clark, & Higgins, 2008), the teachers were in control of the learning environment, providing activities to "plug the gaps" in children's knowledge:

> I guess you are really quite restricted, but you have your planning and guidelines for numeracy project, so usually that really controls most of what you do. (Punga teacher)

In all four classes, children were placed in achievement groups from their first day at school and experienced formal whole-class mat time followed by group rotations. Opportunities for less structured learning were provided during times without teacher contact, when groups of similar-ability children were provided with specifically focused games or equipment designed to stimulate self-generated activities. While teachers expressed a belief in the importance of play and in learning through play, they did not reflect this in practice. There was a strong belief that games or activities from the NDP replicated the children's earlier experiences of learning through play:

> I suppose that helps them transition. I suppose we just expect them to start participating in the games. (Nikau teacher)

When children were in the non-contact group, they had some control over their learning in their choice of equipment, but they had little opportunity to interact with the teacher. The teacher was unable to scaffold or respond interactively to children's initiations because the teacher was engaged elsewhere in instruction or classroom management:

> I think there is an expectation of when they come [pause] well how they behave when they are at school, and numeracy time is a set time. So we cater to those children by doing games. (Nikau teacher)

One classroom teacher provided practical experiences and opportunities for "free" play so that children could experience confidence and success:

> It is a bit of both, really. That is where I have developmental-type activities – so they have a little bit of structure on the mat. Then they have freedom of other activities [and] at the same time they are learning that rotation process. (Punga teacher)

It has been demonstrated that children in classes in which teachers have used more developmentally appropriate practices exhibit less stressed behaviours (Margetts, 2007). Stephenson and Parsons (2007) suggest that children become impassive and disempowered with more formal approaches to teaching, which may lead to anxiety and low self-esteem. Perhaps more teacher professional development is needed to develop a comprehensive understanding of the pedagogy appropriate for children in transition. In another small New Zealand study, Belcher (2006) suggests that teacher beliefs and lack of understanding of the NDP may limit the experiences they provide. Although it has been suggested (Margetts, 2007) that school teachers should be responsive and reflective in the early weeks of schooling to the diversity of backgrounds, little evidence was found in this study of authentic social contexts for learning.

Assessment

Early childhood education services

Narrative assessments were the most common form of documentation in the ECE services. These tended to document, in written and photographic form, the dispositions exhibited by the child rather than focusing on a specific subject. The foreground of these narratives described the whole experience to ensure that the complexity of the learning was preserved (Carr, 2001). Within the background, there was evidence of specific mathematical concepts being developed, practised, or achieved. For example, one kindergarten teacher wrote for a child:

> You enjoyed your time in the water, filling bottles using jugs and small containers. You had really good concentration and showed awesome control when pouring the water into the bottles. You lined the new cylinders up from smallest to largest and filled these too. You were not only developing your fine motor control but discovering all about volume. (Learning Story)

Te Whāriki suggests a very clear purpose of assessment is to "feedback to children on their learning and development [and it] should enhance their sense of themselves as capable people and competent learners" (Ministry of Education, 1996, p. 30). Peters (2004, p. 8) suggests that this focus on learning dispositions "provides a potential link between sectors, and is consistent with the definition of numeracy underpinning the Numeracy Strategy" which is "to be numerate is to have the ability and inclination to use mathematics effectively in our lives, at home, at work, and in the community" (Ministry of Education, 2001, p. 1).

School new entrant classes

The assessment practices undertaken at school are very different from those in the ECE services. In the study by Sherley et al. (2008), the five teachers whom they interviewed did not attend to the knowledge and skills that the children already had on entry to school. This was confirmed in this study:

> A huge jump for children who didn't know anything [about numeracy] when they started. (Nikau teacher)

Teachers indicated their use of either the Individual Knowledge Assessment for Numeracy (IKAN) checklists (Ministry of Education, 2005) or the Numeracy Project Assessment (NumPA) tool (Ministry of Education, 2006) to assess children, in some cases, in the first few days of arrival at school. Concerns have been raised regarding the use at a new entrant level of such tools (Peters, 2004), with their focus on narrowly defined goals and checklists meaning that little attention is paid to the situated nature of learning experienced by children before they start school:

> We do observation assessment for the first six weeks and then in the sixth week, we do the NumPA Form A ... and after that we carry on with a tick chart, one from the numeracy project stage that they are at. (Nikau teacher)
>
> So they come out at 0 [Stage 0 of Number Framework], so they don't know any of the things [that the NDP assesses]. (Nikau teacher)

The reporting practices on children's progress were consistent across both schools in this study. Each school completed an oral and written report at six weeks as to how the child had settled in and the progress they were making. Contrary to findings from Belcher (2006), parents spoke confidently about having or being able to receive information easily on their children's progress in numeracy:

> We had an interview after the six weeks and they went through how she was and how she was sort of ordering numbers and they have been introducing numbers out of sequence so that they could place them in the order. (Parent, Nikau)

Parental Perceptions

Positive relationships between sectors are important because they develop continuity between home, ECE settings, and the new entrant classroom (Bronfenbrenner, 1979). The researchers in this study wanted to unpack the parent/caregiver's view of the transition partnership between teachers and parents of the child. A questionnaire given to ECE parents provided information on the parents' perceptions of transition and on aspects of children's learning of mathematics within the ECE environment. A similar questionnaire focusing on new entrant classes was given to the school parents in the second phase of the research.

Parents (n1[1] = 114, n2[2] = 55) believed that in ECE services, mathematics happens "often" as children play with puzzles and games (43%), during mat time (45%), at block construction (30%), in water play (29%), and on the computer (32%). Parents think the ECE teachers interact with children on things mathematical "all the time" through conversations (33%) and "often" during mat time (49%) and with inside equipment (43%). ECE parents agreed on how mathematical learning occurs through play in the early childhood setting and believed school numeracy to be the more formal learning of numbers. However, they anticipated that school mathematics would still be simple and incorporated into everyday situations, fun, challenging, and engaging children's interests. All ECE parents expected to see changes from structured play-based activities to more formal teaching and learning in school (Stephenson & Parsons, 2007):

> Very basic. I would expect the transition between formal learning and learning by play to be fairly slow at first to help the child settle into school. (Parent, centre)

The parents' perceptions of "mathematical learning" changed once the child had started school (n1[1] = 59, n2[2] = 28). Nine (33%) of the parents whose children were in the new entrant classroom responded consistently across all options that they "did not know" where mathematics occurred within a new entrant classroom. The rest of the parents believed that mathematics occurred "all the time" in the maths corner and in structured mathematics lessons and "often" in puzzles and games. When asked how the teacher helped their child learn mathematics, 46% (13 parents) "did not know", 21% (6 parents) responded that teachers used a formal mathematics lesson "all the time", while others said that teachers "often" used conversation (25%, 7 parents), mat time (25%, 7 parents), and formal lessons (18%, 5 parents). Findings supported those of Belcher (2006) that parents "were unaware of the extent of their children's shift" (p. 114) as they moved from early childhood to the school environment.

Parents were able to articulate quite clearly their perception of their child's mathematical knowledge both before and after starting school. Parents indicated that their children showed competence in: the use and understanding of positional language, recognising shapes, comparing lengths and heights, saying the number names in order (to 20), and accurately counting a group of objects (to 10).

Formal Transition Process

Within the process of transition (involving the child and parent/s as the child moves from an ECE service to school), it was difficult to isolate any formal process pertaining to mathematics. The researchers assumed that successful transition of the child would have a direct impact on the child's mathematical learning. Successful transition is said to be a result of a process designed to familiarise participants with the school environment and the challenges and demands associated with starting school (Broström, 2002; Margetts, 2007). Within Bronfenbrenner's (1979) theoretical framework of "levels of learning", the child is affected by situations in which they participate and also by decisions

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1 Number of questionnaires distributed

2 Number of questionnaires returned

and events outside the child's environment of which they have no knowledge or control. Hence, Margetts (2007) recommends that transition programmes should create links between and actively involve children, parents, families, and teachers, and, furthermore, the "voice" of all participants should be valued and information shared.

The new entrant teachers in this study did not visit the ECE service before a particular child started school. For one kindergarten, visits from Punga School (two to three a year) were more a roll-gathering exercise. The second kindergarten had a closer relationship with Nikau School, and alternating visits were arranged once a term for teachers to share common interests:

> Kiwi [a kindergarten] is our main feed into school, so we try and go and see them once a term. It is pretty informal, and we sit there with our lists and say "What about this child?" and "Who would be good with this one?" (Nikau teacher)

> It is a pity because I think as a new entrant teacher it would be great to go out and visit and see what's happening. (Punga teacher)

In the four ECE services in the study, school visits before the child started school were the ultimate responsibility of the parent or caregiver. Material was available in all four ECE services for parents, providing them with information about all the local schools, together with information on the enrolment and transition process.

The two schools offered different processes for the child who was about to start school. At Nikau School, parent/s formally enrolled the child and then toured the school with the principal and met the child's class teacher. At this stage, future class visits were discussed. The number of visits depended on the request from the parent, with the parent's attendance optional:

> Usually, it's for one-and-a-half hours ... They are allowed as many as they like or as few as they like, we don't mind. ... Some people don't come for any, and other people come for 10. The average is probably five visits. I leave it up to the parents now, I don't say anything to them. (Nikau teacher)

As part of the transition process, Punga School offered school sessions called "Ready-Go". The child and a parent/caregiver were encouraged to attend up to 10 weekly sessions from 1.30 to 3.00 p.m. in a spare teaching space. Sessions could have between 4 and 17 children, who were from nine local ECE services or who had no ECE experience. The assistant principal had developed a regular programme, which for the children started with mat time and focused on social skills and literacy, while the parents had a "guest" speaker, such as the school principal or an administration person who outlined school policy or practices. The children then completed a range of activities with parental and teacher support. Children also had up to three class visits immediately before starting school. Parents, new entrant teachers, and ECE teachers spoke very positively about the "Ready-Go" programme:

> I think it was great that there was the Ready-Go programme just so the parents as well as the children were familiar with the environment of the school and who was who. (Parent, Punga)

> It is to give the children an easier start to school. Where they are learning basic things like sitting on the mat, putting their hand up, and also learning some early language skills. (Punga teacher)

> It was wonderful to see the children [those preparing to start school] coming up to her to say hello. Because they know her because of the Ready-Go programme. How cool that is for those children and for her for when they do go to school; it has broken down so many of those barriers. (Kindergarten teacher)

Parents from both schools spoke positively of how the class visits helped settle and meet the social needs of their children:

> Yes, I think so, because then she knew her classroom, knew her teacher, knew some of the children that were already in the classroom. (Parent, Nikau)

The two new entrant classes at Nikau School combined to provide an "exploration" time for the children, usually one day a fortnight for 80 minutes, where children experienced self-selected activities. Teachers saw this time as an important link back to early childhood for new entrant children. They also encouraged children on school visits to attend the exploration sessions:

> Normally hooks all our kids into coming into the school. Because they go, "Oh that's like kindy – I quite like it here now." (Nikau teacher)

However, it was also apparent that the transition process did not necessarily successfully meet the needs of all children:

> But for some children, starting anywhere new is going to be traumatic because of their personality you know. ... I guess we just manage on a case-by-case basis. (Nikau teacher)

Concerns were also raised regarding those children who did not attend any ECE service:

> They are the ones who are going to take longer to settle, longer to learn. ... No pre-school, and they are taking a long time to get underway. (Punga teacher)

Information Sharing

There were no specific policies between any of the ECE services or schools determining "what and how" should be shared on transition. Lead teachers from the junior school of both schools visited the kindergartens – but not the centres – a few times a year, for various reasons. These relationships indicate the "professionalism and collegial development" of the third level of learning described by Bronfenbrenner (1979) as the "exosystem".

ECE teachers considered that the portfolios of narrative assessments contained sufficient information for the new entrant teacher to use as a starting point in getting to know the child. However, they were unsure whether the new entrant teacher would use the portfolios. It was left to parents to decide if they would take their child's portfolio to school:

> I put this in the child's profile book with a link about the learning involved, and I thought, wouldn't this be great if I could hand it on to the teachers so they had a knowledge of where they were at. But I don't know, maybe they have their own assessment. (Centre teacher)

New entrant teachers had little knowledge of the learning focus and placed little value on the child's portfolio:

> What kind of things the children have been doing. What kind of things they are interested in. That sort of thing. Because they don't do any kind of assessment at all. It would be kind of useful to know what kinds of things they do know. (Nikau teacher)

Parents' expectation of what information the new entrant teacher would seek about the child's mathematical understanding was gathered from the questionnaire given 6–8 weeks before the child started school (n1 = 114, n2 = 55). ECE parents overwhelmingly expected discussions with the parent or caregiver (89%, 49 parents), discussions with the child (85%, 47 parents), a written report from the ECE teacher (80%, 44 parents), and, to a slightly lesser extent, the child's portfolio (70%, 38 parents) to be the prime information sources. However, in the follow-up questionnaire with the new entrant parents (n1 = 58, n2 = 28) regarding information that the new entrant teacher sought, only 50% (14 parents) said the teacher had had discussions with the parent and 46% (13) with the child. Only 25% (7) of the parents could say that the new entrant teacher had acknowledged their child's previous mathematical learning. New entrant teachers appeared to want little information from the ECE experience:

> We don't actually get anything from the early childhood setting as far as records go to do with the child's academic achievement. (Punga teacher)

> I think that is something that is really lacking in New Zealand across the board. ... A lack of contact between kindergarten and school. It is almost like you go to kindy, right that's over. Now you go to school, and there is no flow, and I haven't seen anywhere there is. (Nikau teacher)

Teachers have little understanding of each other's educational settings. By sharing information, teachers may better understand the changes the child will experience during transition (Belcher, 2006). The key competencies outlined in *The New Zealand Curriculum* (Ministry of Education, 2007) provide an avenue for closer links between sectors (Belcher, 2006; Carr, 2006; Peters, 2005) because the attention of the primary school sector is drawn to developing life-long competencies. The focus on dispositions and key competencies could initiate closer links between the sectors and help ease the process of transition.

Conclusions

The richness of mathematical learning experiences that children bring with them to school has been well researched (Aubrey, 1993; Perry & Dockett, 2004; Young-Loveridge, 1989). Perry and Dockett (2005) analysed the many mathematical experiences that children have in early childhood that demonstrate "immense knowledge ... including mathematics" and the "mathematical power of young children's skills in mathematising, making connections and argumentation" (p. 36). There was limited recognition by the new entrant teachers in this study of their new entrants' mathematical power and the need to nurture it by providing learning experiences that make connections to the children's existing mathematical understanding.

Within this small study of four ECE services, there was a diverse range of mathematical experiences available to the children within each of the ECE settings. Assessments were very holistic in nature, focusing on dispositions to learning. Subject curriculum areas were not emphasised, but they were evident in the background of the assessment narratives.

Involvement in the NDP dominated the teaching of mathematics in the four new entrant classes. Children experienced structured numeracy lessons involving whole-class mat time followed by achievement-group rotations. The use of NDP activities and games during group rotation were believed to replicate the ECE learning approach. NDP assessment tools, which were the main methods of assessment, did not allow for assessment of the nature of children's previous mathematical learning.

Parents were able to articulate clearly how their children learn mathematics in ECE services and what their expectations were for how connections should be made to this learning in the new entrant classroom. There was a high expectation among parents that some information sharing would take place regarding their child during the move to school. However, very limited dialogue took place. ECE teachers expected the portfolios of narrative assessments would provide adequate information, but this source was not used by the new entrant teachers.

There was considerable variation in the extent to which early childhood and new entrant teachers were prepared to adapt learning approaches for children moving to school. The changes anticipated by the researchers in the roles, activities, and interpersonal relationships between the teacher, the parent, and the child as the child moves to school (Bronfenbrenner, 1979) were confirmed.

It was evident that positive transition practices in mathematics between the early childhood setting and the new entrant classroom were tenuous. Further effort is needed to make the recommendation in *The New Zealand Curriculum* (Ministry of Education, 2007), "this new stage [the transition from the ECE setting to school] in children's learning builds upon and makes connections with early childhood learning and experiences" (p. 41), become a reality. A future focus on dispositions and key competencies could well initiate closer links. From this small case study, it is clear that the need for a reform of transition practices warrants further investigation to ensure that "schools can design their curriculum so that students find the transitions positive and have a clear sense of continuity and direction" (Ministry of Education, 2007, p. 41).

Recommendations

- Discourse should be promoted between the sectors to develop further understanding.
- The use of subject-specific assessment in the narrative assessments from early childhood settings should be promoted.
- A more "appropriate" method for assessing new entrant children should be developed.
- In the NDP booklets, teaching approaches and activities provided in pre- and early-counting stages should reflect more closely learning approaches from early childhood settings.
- Key competencies and dispositions should provide an avenue for closer links and less obvious differences in learning and assessment approaches between sectors.
- Transition programmes that include a focus on mathematical learning should be promoted.

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Explorations of Year 6 to Year 7 Transition in Numeracy

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This paper reports on school transition systems and practices in numeracy when students move from one type of school to another between years 6 and 7. The study used multiple case studies across six primary schools, three intermediate schools, and one middle school in two different geographic regions. Qualitative data (from interviews and questionnaires) was collected from students, teachers, lead teachers in numeracy, and parents. The data was analysed using a conceptual framework for examining transition. The first phase included 67 year 6 students; from this group, six students were selected for phase two in year 7 on the basis of being gifted and talented and ten based on Māori and Pasifika ethnicity. The study was concerned with these two particular groups of students because they do not always make the transition easily. Findings showed that the transition process from year 6 to year 7 was not problematic for most of the students. The gifted and talented students had all completed a successful move. The move had not been so smooth for some of the Māori and Pasifika students, who were still, at this early stage, making adjustments to their new class setting. In numeracy, there was evidence of "fresh-start" practices that included numeracy reassessments, a lack of curriculum continuity, and mistrust between sectors.

Introduction

The move from one type of school in the education system to another type of school is often described as transition. For most New Zealand students, an example of this is the move from year 6 (primary school) to year 7 (which may be to an intermediate[1], a year 7–13, or a middle school[2]). The move from one school to another is a very important occasion in a student's schooling and, at the ages in this study, often coincides with the onset of adolescence. The transition from primary to intermediate[3] school is viewed almost as a rite of passage. It is an interim move that may challenge students personally, socially, and academically, and also structurally in relation to the way their new school is organised. The intermediate school provides a model that combines aspects of primary school along with some of the subject specialisation, streaming, and timetabling practised in secondary school.

A transition may present many challenges for students. These include: difficulties with sustaining commitments to learning and in understanding the continuities in learning (Anderson, Jacobs, Schramm, & Splittgerber, 2000; Demetriou, Goalen, & Rudduck, 2000); lack of continuity of teaching style; and changes in teacher expectations, friendships, and subject matter. There is also the practice of schools adopting a "fresh start" policy (Galton & Hargreaves, 2002), in which those schools receiving new students fail to consider the academic information passed on by the students' previous schools (in this study, the primary schools). The justification for this practice is the belief by schools at the next level that they are better equipped to make judgments about students' abilities in subject areas such as mathematics because they have more academically-specific knowledge. This means that written records are mistrusted, although, according to Fabian (2007), communication systems contribute an important role in effective management to "make a transition meaningful to everyone" (p. 10).

Consistent research findings from both the United Kingdom and the United States provide evidence of dips in student progress at each point in transition, such as primary to middle school or middle to junior high (Anderson, et al., 2000; Galton & Morrison, 2000). Noyes (2006) showed that, in mathematics, school transition acted like a prism diffracting the social and academic trajectories

[1] Year 7 and 8 classes only
[2] Years 7–10
[3] Intermediate, in this paper, includes the middle school concept.

of students as they passed through it. Galton, Morrison, and Pell (2000) explained that, although students often showed signs of anxiety and excitement at the prospect of moving to a new school (often a much larger school), for the majority of students, any fears had largely disappeared after the first term. Typically, what remained was a lack of continuity across the curriculum. Other studies showed that students were often presented with revision of material previously encountered and a lower level of task demand that led to boredom, lack of motivation (Anderman & Maeher, 1994), disengagement from school (Anderson et al., 2000), and dips in progress (Catterall, 1998). The research on transition does not at this stage indicate whether such dips in progress are cumulative, so there is a need for more longitudinal studies. Recent New Zealand research (Cox & Kennedy, 2008) on transition from primary to secondary schooling found that by the end of the first year at secondary school, most students made good achievement gains in mathematics, although there was a marked drop in positive attitudes towards this subject (including in high achievers).

Students may be supported in their school transitions in a variety of ways. This support may be systemic (for example, school visits and written material) or provided by people such as friends and peers, parents, and teachers. Several studies have shown that friends influence adolescents' adjustments to a new school (Berndt & Keefe, 1995; Whitton & Perry, 2005) and that friendship, peer acceptance, and group membership have an established link with students' academic achievements (Wentzel & Caldwell, 1997). Parental involvement can also play an important part in the transition process (Dauber, Alexander, & Entwistle, 1996; Mizelle, 2005), although levels of involvement drop off as a student progresses through the school system. Teachers may help prepare students for this move by teaching coping skills such as self and time management, decision making, and conflict resolution (Schumacker & Sayler, 1995). Hawk and Hill's (2001) study found that "so many teachers are so focused on curriculum coverage that they do not take the time to incorporate these [for example, self and time management and study skills] into the programme" (p. 31). Successful transitions usually occur when the new schools involve students, parents, and teachers in the process (Smith, 2001) so that students experience a sense of community and belonging.

This study is concerned with two particular groups of students who do not always make this transition easily. The first group under consideration are Māori and Pasifika students. Whilst not suggesting that these students are a homogeneous group, they do have many social and cultural factors in common. Although the literature does not directly address transition, it (for example, Hawk, Cowley, Hill, & Sutherland, 2001; Hunter, 2007; Higgins, Makoare, Wilson, & Klaracich, 2005; Hill & Hawk, 2000; Macfarlane, 2004, 2007) draws attention to the need for teachers to ensure what Anthony and Walshaw (2007) term "social nurturing" (p. 60) exists in order for Māori and Pasifika students to be successful in classroom situations. To provide social nurturing, teachers need to be culturally responsive and able to build collaboratively on what Māori and Pasifika students bring to the classroom. Hunter's recent (2007) New Zealand study illustrated that when teachers attend to concepts of whānau [family], they develop classroom cultures that enact reciprocity – mutual respect that empowers all members of the community. In her study, the teachers carefully crafted care and support for student talk. Group norms provided the students with "the gift of confidence, the sharing of risks in the presentation of new ideas, constructive criticism, and the creation of a safety zone" (Mahn & John-Steiner, 2002, p. 52).

For transition to be successful for gifted and talented mathematics students, it is most important that they are grouped in their new school in a way that enables them to work with like-minded peers (Assouline & Lupkowski-Shoplik, 2003; Robinson, 2004). The mathematics programme should be designed to be challenging (Diezmann & Watters, 2004) and the curriculum differentiated[4] so that

[4] Qualitatively different (see Ministry of Education, 2000).

they are provided with a learning environment that not only maintains but strengthens their interests and achievements in mathematics. Mathematically gifted and talented students should also have continued opportunities to participate in competitions (Bicknell, 2008; Riley & Karnes, 2007).

The transition process for the two groups of students targeted in this study was examined using a framework developed by Anderson and colleagues (2000). They proposed three major concepts for understanding and improving school transition. These concepts are: preparedness, support, and transitional success or failure. Preparedness is multidimensional and includes academic preparedness, independence and industriousness, conformity to adult standards, and coping mechanisms. The second concept, support, which may be informational, tangible (resources), emotional, or social, facilitates successful transition. This support may come from peers, parents, or teachers. Transitional success or failure can be judged by factors such as results, appropriateness of a student's post-transition behaviour, social relationships with peers, and academic orientation. These indicators are what are commonly commented on in students' school reports (namely, achievement, conduct, and effort). This conceptual framework provided the basis for the data analysis and subsequent findings.

Research Design

The research reported in this study was designed to examine the transition process in mathematics for two groups of students moving from year 6 to year 7. The aim of the study was to understand the systems and structures in place in mathematics for students in the final year of primary school and the first year of intermediate school. These included assessment practices, achievement records, grouping practices, and the more specific provisions made at year 7 for gifted and talented students and for Māori and Pasifika students. The researchers wanted to know the ways in which practices between the two sectors, particularly those relating to the Numeracy Development Projects (NDP), coincided. They also wanted to examine the communication between the sectors, the depth of information provided about students, and the extent to which the intermediate schools drew on this information. How the students viewed their transition across school sectors, their perceptions about learning mathematics in the next sector, their preparedness for it, how successfully they had managed the transition process, what changes they had experienced, and how well they had coped with these changes were also considered.

The design of the research involved a collection of data that covered the end of one year and the early portion of the next year (see appendices H–I, pp. 189–194, for questionnaires and interview questions). Six weeks before the completion of year 6, all students and their parents from one class in each of six primary schools (within a decile range of 3–7) from two different geographical regions in New Zealand were asked to complete questionnaires. Sixty-seven of 141 questionnaires were returned (48%). Within each school setting, focus group interviews, in groups of about 10 students, were then completed with the 67 students who had returned questionnaires. At the same time, the corresponding year 6 classroom teachers completed a questionnaire. Numeracy lead teachers in each primary school participated in a semi-structured interview to clarify school policy and practices. Documentation (records of achievement, records used in transition, and teacher plans) were collected.

The students moved in the following year to three large intermediate schools and one year 7–10 school. Ten Māori and Pasifika students (one Māori student was also gifted and talented) were selected for the second phase of interviews. This group included all of the Māori and Pasifika students from one of the geographical areas who were available for interview. Half of these students were of both Māori and Pasifika ethnicity. Six other students who were identified in year 6 as gifted and talented in mathematics were also selected for the second phase of the study. Semi-structured focus-group interviews were held at each school six weeks after the students had begun their school year. A semi-structured interview also took place with the four lead teachers in the intermediate and middle schools. The parents of the 16 students interviewed in this phase were all asked to complete a questionnaire. However, only three responses were received from parents of the gifted and talented students and one from a parent of a Pasifika student (this response was discounted because it was felt that one parent's view was not likely to be representative of the ten Māori and Pasifika students).

Results

The results presented are based on Anderson et al.'s (2000) conceptual framework and its three major concepts: preparedness, support, and transitional success or failure. These concepts have been reconsidered and the findings reported under the following subheadings: systemic, academic support, and transitional success and failure. The findings are also supported by the voices of the key stakeholders: the students, the teachers, and the parents.

Systemic

All students from the six schools were prepared for transition by making a visit to their new school, where they were shown around and given organisational information. Students were also provided with written material (a prospectus) about the school's structure and practices. One school in the study provided an additional visit for gifted and talented students (nominated by the year 6 teacher and/or parents). The purpose of this visit was for students to complete group problem-solving activities in mathematics. These results contributed to the school's identification of gifted and talented students.

A senior-management representative from each intermediate-level school visited the primary schools to talk with the year 6 teachers. The aim of this visit was for the year 6 teachers to have an opportunity to talk about specific students, such as gifted and talented students or those with specific learning or behavioural needs. These visits were deemed valuable because teachers shared information about students that they were reluctant to put in writing on the placement forms. In one case, a senior-management staff member attended Individual Education Plan (IEP) meetings for specific year 6 students with needs who were going to move on to their school.

Every year 6 classroom teacher in this study prepared written or electronic Teaching-Assessment-Planning (eTAP) records to pass on to the intermediate school. The format varied for every school, but the common elements were: current reading age or Supplementary Test of Achievement in Reading (STAR) results; Progressive Achievement Test (PAT) results for reading and mathematics; Assessment Tools for Teaching and Learning (asTTle); NDP data (Numeracy Project Assessment [NumPA], Individual Knowledge Assessment for Numeracy [IKAN], and Global Strategy Stage [GloSS]); and social and behavioural factors (recorded on a scale). None of the placement forms made provision for the dating of assessment data.

> I don't know if this was done at the beginning of year 6, the end of year 6, or at any other point of time. (Year 7 teacher)

Apart from NDP-related data and PAT results, limited provision was made for assessment data relating to other aspects of mathematics (such as geometry, measurement, and statistics). The forms allowed for a teacher to record additional notes. The section pertaining to work habits and social behaviours was recognised by one teacher as the most useful information received from the primary school.

Ten students in the original sample of 67 students were identified in year 6 as gifted and talented in mathematics, but only three of these were confirmed as such in year 7 and placed in special classes or programmes. One student had clearly been misidentified by the primary school, and six more were described by their new schools as hard workers but not gifted and talented. This was based on their new assessments. This confirmed the practice of fresh start. All of the schools believed that gifted and talented students should be performing on at least stage 8 of the NDP Number Framework or achieve stanine 9 on the PAT.

All the intermediate schools in term 1 conducted their own numeracy reassessments. These assessments included PAT, asTTle, and the NDP's NumPA, GloSS, and IKAN. Two schools used exactly the same GloSS interview as had been used in year 6 by the primary school. The students verified this practice and expressed concern that they felt they had cheated because they had completed the same problems in year 6. Justification provided by the intermediate schools for reassessing students rather than using the primary school data was an expressed mistrust in the stated student numeracy stages. One intermediate school had developed a joint cluster project with their three primary schools that aimed to develop consistency in the transition data. However, despite this initiative, they continued to reassess all their year 7 students on entry:

> One thing we have noticed is we cannot believe the data, but we have our own tests too ... We start again to see what they can do and go from there. (Lead teacher, intermediate school)

This message was confirmed by all intermediate schools in this study. The lead teachers also reported that their retesting for numeracy placed many students on lower strategy levels than those reported in the primary school data. They believed (without full analysis of the assessment data) that the overall students' results for NDP stages were one or two stages lower than that recorded by the year 6 teacher. This generalised view further justified their reasons for this practice of "fresh start":

> We will just take the fresh start rather than looking at the old data, but I guess there would be a way perhaps of not having to do that ... if we learnt to trust that the year 6 teachers had done it properly. But we always feel that we need to start fresh, and so we do. (Lead teacher, intermediate school)

Reassessment of all students was also justified by the lead teachers on the grounds that the high number of schools contributing to the year 7 cohort (for example, up to seven for one intermediate) led to variation in assessment practices and cast doubt on the validity of their received data.

Grouping practices in year 6 were commonly based on students' numeracy strategy levels. One teacher grouped her students based on numeracy knowledge levels, and the other teacher used mixed-ability grouping rather than strategy grouping. At year 7, the approach to grouping across all the schools consistently drew on NDP strategy levels, with additional information provided from PAT and asTTle. Specific grouping practices at year 7 within a range of organisational strategies for mathematics were used to provide for the gifted and talented students in three of the four case-study schools. These included a full-time gifted and talented class, a withdrawal class for gifted and talented students, and a practice of spreading the gifted and talented mathematics students evenly across four year 7 classes. One school used no organisational strategy to group gifted and talented students for mathematics. Two of the four schools (one in each region) specifically considered grouping strategies in mathematics for Māori and Pasifika students. One school offered full-time bilingual classes, and another used a withdrawal group for its Pasifika students, placing them together for mathematics.

Academic

The year 6 teachers prepared the students academically for success in numeracy by focusing on some key mathematical skills and concepts. These included basic facts knowledge, problem-solving strategies, numeracy strategies, and, in some cases, the written algorithm with renaming. Other foci were place value knowledge, experience with textbooks, making mathematics relevant to the students' lives, and self-awareness of levels and experience in mathematics. The students described how key players (teachers and parents) in their transition had worked towards developing their thinking skills, risk-taking ability, confidence, and independence. One student explained:

> Our teachers challenge us and give us different work nearly every day, and we either get a maths book, a textbook, or just a sheet, and we work off those. Each time, there are different levels and challenging levels for your group. (Year 6 student)

Another student described the increased expectations of his parents:

> My parents have been more strict on me that way. I learn discipline and I don't muck around because they want me to get to a good college and they want me to push myself to the limit. (Year 6 student)

The students expected there to be some differences in the mathematics learning and teaching at their new school. Specifically in numeracy, there was an expectation that they would have to solve problems using new strategies. Other expected differences mentioned by the year 6 students included: mathematics would be harder and more complex, different, more work to do, less fun, more confusing, bigger numbers, and there would be less teacher time and greater use of textbooks. They also thought that the teacher would have higher expectations and use a different teaching style. The students described some of the desirable characteristics of their new teacher. These included: one who explains and encourages questions, one who spends several days on a strategy, and one who listens, gives regular homework, and caters for all ability levels. They anticipated that there would be some changes in grouping and seating arrangements. For the gifted and talented and the Māori and Pasifika case-study students, the major perceived difference was that the work would be a lot harder and more complex. The gifted and talented students saw this as being a challenge and also thought that there would be less time to complete the work. The Māori and Pasifika students believed that in year 7 they would have to solve problems using a greater range of strategies and that there would be higher teacher expectations.

The parents indicated that some of the important factors that would enable a smooth transition in mathematics from year 6 to year 7 would be if their children were mathematically confident (especially with basic facts), had no gaps in their mathematics knowledge, and had good work habits, especially in being more responsible for themselves. There was some concern from parents from one school that there were indications in the student profile that their children had not been taught some mathematical topics, such as fractions and percentages.

Several parents commented on the part that homework might play and that they anticipated more homework in the following year. The parents of the students identified in year 6 as gifted and talented in mathematics were particularly interested in the placements of their children; they were aware that their students were in the "top set" at primary school but showed an awareness that their children might be "little fish in a big pond" at their new schools. They wanted the mathematics to match their children's abilities, so that, for example, "he is not bored; when bored, he plays up for the teachers":

> I hope that there will be adequate and appropriate testing at the end of year 6 to assess where the child is at to make sure that the teaching level at year 7 suits the child's needs without overwhelming or boring them. (Parent of a gifted and talented student)

Support

Support for students was provided in tangible form by the systems described above, but support also came from others. These were peers, friends, siblings, and parents. For most of the students (gifted and talented, and Māori and Pasifika) moving to a new school with a friend assisted a smooth transition. Siblings and other whānau members also helped the process by sharing information about school uniform, organisational practices, and teachers:

> My brother has told me all about what the school does and everything that goes on and how the teachers are, so I am not as scared as I was before. (Year 6 Māori student)

> My sister tells me a lot about the teachers and all the different opportunities we can use, but I am still a bit nervous about the first day where you just go into the hall and they say what class you are in, but now that I know [about] the teachers it makes it a bit easier. (Year 6 student)

Many of the parents showed a real sense of commitment towards supporting their children. One parent commented:

> I know he has the ability to grasp new concepts easily, although he hasn't pushed himself in year 6, is quite capable of doing harder work. I feel he will either "sink or swim" depending on how he starts the year [year 7]. I am hopeful he will do well and am preparing him and myself to get into the new year's studies as I think he might need help initially to settle into a work routine. (Parent of a gifted and talented student)

Transitional Success or Failure

The gifted and talented students in special classes or programmes expected and appreciated the opportunity to work with like-minded peers in numeracy. They found the mathematics more challenging, at a more advanced level than in year 6, but still enjoyable. They were invariably working approximately one year or more in advance of their same-age peers. They found themselves with a larger group of students working at a similar level. As one student said:

> It's good to be with lots of people who are good at maths rather than just two working at the same level. Now it's about a third of the CWSA [Children with Special Abilities] class maybe. (Gifted and talented student)

These students were learning to cope as a small fish in a big pond. The two teachers from the gifted and talented class and withdrawal gifted class at the intermediate schools described key aspects of their programme that specifically catered for these students. These were summed up by one teacher:

> We are faster paced and we go further through the work, and so we can cover more ground, and we try to bring in that balance with the creative things as well ... You have to like maths, and if they see you like maths and you value maths, then they start to take that on board and they start to go that way too. (Year 7 teacher of gifted and talented)

The gifted and talented students in these two classes were also expected to participate in national and international competitions. This opportunity was not extended to other students in the school but reserved for those who had been identified for these special classes.

The six students who had previously been identified as gifted and talented students but were now classified as "hard workers" were placed in regular classes. They also talked positively about their current experiences in mathematics. They felt that the numeracy knowledge and strategies that they brought with them from primary school gave them a sound basis to continue with success in year 7. They were also learning new strategies and continued to feel confident about their achievements. These students had used textbooks as part of their year 6 numeracy programme and continued with greater use of them in year 7. Most of the year 6 teachers had deliberately made greater use of textbooks later in year 6 as part of the transition preparation.

Post-transition, three of the parents of gifted students responded to a questionnaire, and they all reported smooth transitions for their children. Two students had been placed in special classes (one was a full-time gifted class) at the beginning of year 7, and the third was promoted to a higher class (cross-class grouped for mathematics) after the school had completed its own reassessments. The positive comments from parents addressed their children's attitudes and the level of challenge of the mathematics. Two parents mentioned good communications from their children and teachers about the mathematics programme, whereas the third parent was waiting for parent interviews to gain insight into how well her child was achieving in year 7. The parents all recognised the part that homework played in helping to keep them informed about the mathematics programme, and one parent commented that “we haven’t had this for a long time”. They did recognise that “intermediate is more ‘hands off’ for parents than primary”.

Parental support in mathematics takes a variety of forms at primary school but usually lessens as children progress through the school system. This is usually due to age factors and parents’ awareness that their understanding of mathematics is challenged as the level of mathematics increases. The parents of the gifted and talented mathematics students who responded in this study showed an interest in the transition process. They had no issues and no concerns about placement.

The Māori and Pasifika students, when talking about their transition experiences, focused on different aspects of numeracy learning and teaching in their new classes. The most common themes they identified were associated with basic facts, number operations (specifically addition and subtraction), numeracy strategies, group work, written work, pace, time, and level of challenge. The students recognised the importance of knowing their basic facts, but many still felt that they did not have mastery. They were surprised at the continued emphasis on whole-number work, specifically “pluses and minuses”. There was an expectation that the work would be a lot harder and that they would be learning more about fractions. Several students were surprised that they had to remember and were expected to use previously learnt strategies (from year 6):

> Well, I do find it hard because I have forgotten the strategies I had to use last year and then there are the new ones this year. So it is all confusing, and I am still trying to remember them. (Year 7 Pasifika student)

The Māori and Pasifika students’ collective voices made explicit links between basic facts and strategies as important knowledge to be learnt in numeracy lessons. However, rather than viewing strategies as a tool for solving number problems, they appeared to view strategies as a further collection of facts to be learnt and practised. They had expected that this practice, along with learning basic facts, would provide the main focus for their homework. No one appeared to have issues about year 7 homework.

These students spoke positively about the opportunities for working in groups. They liked being able to explain and talk about their strategies to others and felt that they learnt more in this situation compared with working from worksheets and textbooks. However, collectively they expressed some concerns about the quantity (too much) of written work and the fast pace of the lesson content, which led to limited time for students to understand and practise their old and new strategies. This also led to less time for question asking and answering and a fear of getting the wrong answer:

> I do not like it [year 7 mathematics] because you cannot ask questions because the teacher thinks you are not listening and other kids look at you like you are weird. (Year 7 Pasifika student)

There were contradictory student views about the appropriateness of the level of challenge for the students in numeracy. On one hand, the Māori and Pasifika students believed that the level of challenge was good for them because it was at a higher level than they had encountered at primary school. On the other hand, this level of challenge undermined their confidence:

> It’s been very challenging for me, and I have not been working my best so far. I have found it really hard to concentrate in mathematics because I have had to step up more than I did in primary school. (Year 7 Pasifika student)

> I like year 6 maths better because when you make mistakes you could learn from them, but now it is too challenging. (Year 7 Māori student)

One school identified differences in their numeracy results between Māori and Pasifika students. The Māori students were out-performing the Pasifika students, and so the school, in discussion with parents, was implementing a remedial programme. This withdrawal remedial programme was designed to build student confidence and competence in numeracy. This new initiative had not yet been evaluated. Interestingly, a case-study Pasifika student selected for this programme voiced her lack of understanding of its intent:

> What I have found challenging [post-transition] is the maths because we have to go to another room and that's a big step up for me. We do have maths in our class, but I think it is to get us used to maths in another class. (Year 7 Pasifika student)

Discussion and Conclusions

The transition from year 6 to year 7 when students change schools is an important event in students' educational lives. Its success involves considerable preparation on the part of schools and their management staff, classroom teachers, specialist teachers and support workers, parents, and the students themselves. While still in year 6, the students in this study considered their transition to their new school as a necessary rite of passage, which, with some reservations, they generally viewed positively. They recognised that the larger school held wider curricula choice:

> Well, looking at the curriculum activities there is a lot more there; like, you will get a better education. (Year 6 Māori gifted and talented student)

The students described how visits to the new school, information from their family and friends, and their current teacher's various preparatory strategies that emphasised specific concepts and skills positively prepared them for the shift. After the move to year 7, the key positive factors that the students collectively reported as being important for their successful transition were peer support, friendships, and group membership. Researchers (for example, Berndt & Keefe, 1995; Wentzel & Caldwell, 1997; Whitton & Perry, 2005) have identified such support as important for students' academic success. Consistent with the findings of Galton, Morrison, and Pell (2000), the anxiety the students described before the transition had generally dissipated after the event.

Before the move to year 7, both the gifted and talented and the Māori and Pasifika students voiced a hope for further mathematics challenge with teachers who "were keen on it [mathematics], and you actually do like [the teacher], because if you do not you are likely to have doubts about listening and [about] learning strategies" (Māori gifted and talented student). All the students anticipated a change in teacher expectations relating to the type of work, the volume and pace of the work, and the amount of time they would be given to complete the work. Māori and Pasifika students also expressed anxiety at the expected increase in challenge, in particular, the challenge that related to their current knowledge of basic facts and numeracy strategies and what mathematics knowledge they would be required to have in order to succeed in their new mathematics learning environment. Many of these concerns remained relevant in year 7 for these students as they grappled with the discontinuities they encountered with changes in the mathematics programme and teacher expectations. These findings are consistent with those previously described by Anderson et al. (2000). It was evident that some of these case-study Māori and Pasifika students were encountering difficulties in year 7 sustaining commitment to their mathematics learning.

Evidence is provided in this study of the serious approach all the schools took towards developing cohesive systems and structures to support the students in making the transition in numeracy. Careful attention was given to the transfer of relevant data both in oral and written form. The consistent use of a variety of NDP assessment tools (NumPA, GloSS, and IKAN) and the schools' teaching and learning practices grounded in the NDP had the potential to support a seamless and successful transition of students from one mathematics system to the next. However, the use of fresh start assessment (Galton & Hargreaves, 2002) impeded this process. The mistrust of the transition numeracy data caused many schools to retest, although the anecdotal evidence given by the year 7 lead teachers about students' lower levels could be accounted for by both the early timing of testing and a natural hiatus post-transition and post-holiday period. A few of the teachers recognised these contributing factors. The outcome of reassessment was a delay in grouping and in numeracy teaching and learning and a possible cause of too much mathematics challenge, as described by some students. Alternatively, it was the cause of an initial lack of challenge experienced by a gifted and talented Māori student who commented:

> "I knew that we would not learn new stuff right away. We have to learn all this stuff even if we did it at primary because that's how schools work. So you do the easy stuff first." (Māori gifted and talented student)

What this student was noting was what Hawk and Hill (2001) reported, that curriculum coverage appeared to be the key focus.

The potential limitation of some of the assessment tools used by both the primary and intermediate schools needs to be considered. The data from the primary schools invariably included PAT and asTTle scores and/or numeracy stages, for example, from NumPA. The intermediate school data also included newly completed PAT, GloSS, or IKAN results and used this information to form numeracy groups at the start of the year. Using PAT and asTTle tests in a summative form limits their potential. Likewise, IKAN is limited to numeracy knowledge stages, while only a few GloSS questions assess strategy stages, which can be quite different from knowledge stages. These practices may have led to the misidentification of a set of students identified as gifted and talented at year 6 but not at year 7. For specific groups of students, full diagnostic information would be beneficial. However, the use of NumPA to identify gifted and talented students also poses problems with its potential ceiling effect. This means that it cannot be used effectively to differentiate within the subset of gifted and talented students in mathematics. The need for differentiation was supported by the teacher of the full-time intermediate gifted class, who recognised noticeable levels of difference among her gifted and talented students.

Three of the four intermediate schools directly addressed ways to meet the needs of their gifted and talented students, and, as a result, this group of students reported differences in both mathematics content and instruction. Similar to the findings of Diezmann and Watters (2004), the results of this study showed enhanced levels of engagement for these students because they had increased opportunities to be mathematically challenged and work with like-minded peers. Two of the four schools made available bilingual or withdrawal groups for their Māori and Pasifika year 7 students. For one gifted and talented Māori student, this posed a challenge in that he had to decide between the special provisions offered in a gifted class and those in a bilingual class. As a group, the intermediate-school experiences of many of the Māori and Pasifika students appeared to be less than positive at this early stage in the school year. While they described the importance to their mathematics learning of talking with friends in small groups, the larger mathematics classroom situation posed many risks compared with their year 6 experiences. These findings illustrate the importance of teachers attending to classroom culture and explicitly affirming what the students bring to the classroom in what Macfarlane (2004) describes as "culturally responsive" (p. 27) ways so that learning competence is engendered.

Of interest in this study was the fact that no students referred to the use of technology (such as calculators and computers) or mathematics equipment. Perhaps, at year 7, the students expected a reduced emphasis on the use of manipulatives to support their mathematics learning. In reference specifically to the gifted and talented students, one of the lead teachers stated:

> All these children are preparing for high school maths, so we don't use a lot of equipment as you might have done at the lower levels. But we still have it here. There is the odd group of children who may need it. (Lead teacher, intermediate)

However, the use of concrete materials at all numeracy levels is a key component of the NDP. Many of the students believed that the strategies they learnt in year 6 would be replaced by a new set of strategies in year 7. This belief seemed to suggest that they considered numeracy strategies not as problem-solving tools but rather as knowledge to be learnt and mastered in a similar way to basic facts.

Although this study is limited by the small sample size, it does provide a picture of what transition might mean for students who are gifted and talented and/or Māori and Pasifika students. Further research is needed to follow longitudinally a larger sample of targeted mathematics learners across transitions to ascertain the effect of dips in achievement described by Anderson et al. (2000), and when, why, and what factors mitigate these dips.

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Embedding the Numeracy Development Projects in Two Schools

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The exploratory case study described in this paper aimed to identify the nature of school structures and practices that support schools to embed the Numeracy Development Projects (NDP). Forty-one teachers, numeracy lead teachers, and senior management members at two urban primary schools completed a survey, part of which focused on the school structures and practices that help them embed the NDP initiative in their school. Fifteen staff later participated in individual interviews to expand on their survey responses. Staff at both schools had recently completed a renewed, whole-school focus on numeracy. The schools appeared to be at different stages within the embedding phase of this reform, with both recognising that they needed to modify some of their organisational structures in order to continue focusing on improving students' achievement with greater independence from outside facilitators. A focus on students' achievement was emerging at the first school, and at the second, students' achievement had an established role in driving decision-making about numeracy.

Background

The initial phase of implementing the Numeracy Development Projects (NDP) in New Zealand primary schools was drawing to a close in 2008. By that point, most primary schools had completed an initial teacher-focused, classroom-based introduction to the NDP, led by an external facilitator. Starkey, Yates, Meyer, et al. (2009) describe the initial implementation stage of a reform as being "designed nationally and delivered regionally with an emphasis on consistency and quality" (p. 181). The consolidation phase of the NDP is now underway and focuses on individual schools taking responsibility for embedding the initiative in their particular contexts.

The notion of sustainability appears frequently in the literature but has been labelled as ill-defined (Knight, 2005) and as an article of faith (Timperley, Wilson, Barrar, & Fung, 2007). Knight (2005) describes the notion of sustainability as "a traditional fixation of keeping something going over time with continued support from external providers" (p. 467). A theme of adaptation to specific contexts by the adopters of new practices and knowledge also appears in discussions about sustainability (Knight). Knight points out that the definition of sustainability outlined above ignores the way an intervention might be adapted to its context in order to institutionalise it (p. 467). A second theme associates sustainability with improved student outcomes (Knight, 2008; Timperley et al., 2007). Timperley et al. define sustainability as being "in terms of continual or improved student outcomes once the support provided during the earlier phases of professional development has been largely or totally withdrawn". Robinson (2008) similarly discusses the links between distributed leadership and sustaining improved student outcomes. Both Timperley et al. (2007) and Robinson (2008) use literacy as the project context, whereas Knight (2005) situates her discussion of sustainability in the context of the NDP.

Another slant on sustainability is linked to the idea of "scaling up" or taking an initiative across many sites (Coburn, 2003; Cobb & Smith, 2008; McNaughton & Lai, 2009). Coburn argues for a multidimensional definition: implementing a reform on a larger scale that specifically incorporates issues of depth, sustainability, spread, and shift in reform ownership. Cobb and Smith (2008) suggest "in educational contexts, improvement at scale refers to the process of taking an instructional innovation that has proved effective in supporting students' learning in a small number of classrooms and reproducing that success in a large number of classrooms" (p. 1). In considering implementing new practices on a larger scale, McNaughton and Lai (2009) talk about sustainability as being a generalisation of the effects of an intervention across time as well as across new cohorts of learners and teachers. They link this to treatment integrity or fidelity and the idea of scaling up. A fourth idea related to sustainability is to do with the stage of implementation of a project. For instance, Starkey et al. (2009) suggest that "teacher professional development during the embedding stage of a national or large scale reform may differ fundamentally from other forms of teacher education and professional development" (p. 181). In this paper, embedding the NDP in their second phase is conceptualised as an individual school not only sustaining the initiative but also making necessary adjustments to the school's structures and practices to ensure that the NDP become deeply embedded in everyday practice.

The current study builds on previous investigations around the topic of sustainability of the NDP. Earlier studies have addressed such aspects as the impact on teachers and their classroom numeracy programmes and on students' achievement (for example, Thomas, Tagg, & Ward, 2003). Teachers' perspectives on student achievement and their evaluation of various components of the professional development programme have also been examined (Thomas & Tagg, 2004). A number of studies have investigated factors that contribute to sustaining the NDP (Ell & Irwin, 2006; Thomas & Tagg, 2004; Thomas, Tagg, & Ward, 2003; Thomas & Ward, 2006; Ward, Thomas, & Tagg, 2007), as did Higgins, Bonne, and Fraser (2004), who described "components that fuel the dynamic of sustainability" (p. 59). Other investigations of aspects of sustainability in the New Zealand numeracy context include: Pritchard and McDiarmid (2006), who identified enabling and constraining factors that contribute to sustainability; Anthony and Walshaw (2006), who explore factors associated with sustained changes to teacher practice; and Ell (2007), in her longitudinal study of sustainability in a rural school.

Methodology

This exploratory case study[1] aimed to define questions and conjectures for a subsequent study. The focus on the structures and practices that help schools to embed the NDP was one component of a larger study[2] that also investigated school-based instructional leadership, networks of support and influence, and the diagnostic interview as a smart tool.

This paper aims to answer the research question:

- What is the nature of the school structures and practices that support schools to embed the numeracy initiative?

Participants

The two Wellington-region schools participating in the study had originally completed numeracy professional development in the NDP's early years of implementation and, more recently, had undertaken a renewed focus on numeracy. The schools continued to call on outside facilitators to support the continued improvement of numeracy instruction. Both these urban full primary schools were selected because they reported recent improvements in their students' achievement; for example, data from school B showed that the achievement of a target year group had improved. Also, both schools were geographically convenient to the researchers.

School A was a medium-sized, high-decile, state primary school. The school had one numeracy lead teacher – a classroom teacher and syndicate leader – who had been in the lead teacher role for approximately eight years and who currently worked with two colleagues towards embedding the NDP in their school setting.

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[1] See Yin (2003) for a discussion of case-study methodology.

[2] See other papers by Higgins & Bonne in this volume.

School B was a large, high-decile, state primary school. The lead teacher responsibilities at school B were shared by a classroom-based lead teacher (lead teacher 1) and a lead teacher who was a "walking" member of senior management (lead teacher 2). Lead teacher 1 had been a numeracy lead teacher at her previous school and had been in her current role at this school for three years. Lead teacher 2 had taken up the second lead teacher role when a colleague had left the school the year before and had also worked in the role several years previously.

The two schools can be thought of as being at different stages of implementation: school A was still undergoing the organisational redesign needed to support full implementation of the NDP; school B was embedding the structural changes they had already put in place.

Procedures

All teachers, numeracy lead teachers, and senior management (including principals) were invited to participate in the study. Initially, all teachers and senior management members were asked to complete a survey (see Appendix J, pp. 195–197) at each school's numeracy-focused staff meeting, at which the researchers made field notes. Several teachers at both schools took up the option of completing the survey after the meeting for later collection. Survey questions were generated using Cobb and Smith's (2008) frame of leadership priorities and were designed to elicit responses about leadership, networks of support and influence, formative assessment, and the roles these play in sustaining the NDP in their school[3]. At both schools, almost 90% of staff returned completed surveys.

Audio-taped interviews were subsequently carried out with lead teachers of numeracy, all members of senior management, and a representative sampling of teachers from both schools. The six people interviewed at school A were: the numeracy lead teacher (who also taught year 3–4 students), the principal, the deputy principal (who also taught new entrants), the assistant principal (who also taught year 5–6 students), and two other teachers who were part of the numeracy development team (one taught year 7–8 students; the other taught year 5–6 students).

The nine people interviewed at school B were: numeracy lead teacher 1 (who also taught new entrants), numeracy lead teacher 2 (who was also the deputy principal), the principal, the assistant principal, a teacher with special responsibility for curriculum, and one classroom teacher from each of the following year groups: years 3–4, years 5–6, and years 7–8. An additional teacher (of year 5–6 students), who had undergone NDP development elsewhere and who was identified as having a particular strength in teaching numeracy, was interviewed at the principal's recommendation. All school and numeracy leaders were interviewed with the questions shown in Appendix J. Teachers' interview questions are also shown in Appendix J. In the case of dual roles, leadership roles took precedence over teaching roles; for example, the numeracy lead teacher at school A, who also taught year 3–4 students, was interviewed using the questions for school leaders.

Lead teachers were also asked to provide copies of school documentation that supported the development of high-quality numeracy instruction and student achievement data for the current and previous year. In summary, the school's dataset comprised surveys, interview transcripts, school documentation, and student achievement data.

Analysis

This paper is focused on the school as a unit of analysis rather than on the individual teacher and draws on the work of Anthony and Walshaw (2006) and Starkey et al. (2009) to unpack the extent to

3 See other papers by Higgins and Bonne in this volume.

which planned, purposeful, whole-school adaptations were shaped by the stage of implementation for individual schools. The data analysis therefore included electronic searches of interview transcripts and a compilation of survey responses for key words, such as “whole”, “plan”, and “achievement”. These searches led to additional themes, including appraisal and the timetabling of instruction, which were then used in searches of the documents. Other school documentation was read by the researchers to identify links.

Limitations

The exploratory case study focused on staff at two schools in order to develop conjectures for wider investigation during the embedding phase of the NDP. Data relating to the sustainability of the NDP at these schools is particular to their contexts, so the findings from this initial study cannot reasonably be generalised to other schools.

Findings

Planned, Purposeful, Whole-school Adaptation

What emerged from the data from the two schools was similar to what was described by Anthony and Walshaw (2006):

> Schools have adapted the Numeracy Development Project in a wide variety of ways to meet their individual needs. For some schools, the adaptation has been school wide and purposeful and supported through various layers of leadership. In other schools, adaptation is unplanned, convenient, and individualised at the syndicate or classroom level. (p. 26)

In the study reported here, lead teachers and senior management at both schools described similar school-wide changes in relation to NDP practices as either “becoming embedded” or “intended adaptations”. Both schools had undertaken their renewed focus on numeracy as whole schools.

The numeracy lead teacher and senior management at school A were planning to make some organisational changes in order to embed practices, such as the provision of extra support where needed and the monitoring of student achievement, and talked about the need to be “... making sure that things continue to happen, that you are in a constant state of continuous improvement” (senior management member, school A, interview). For example, the numeracy lead teacher said she “would like to see some remedial maths development for extra support for the slow learners in maths” (lead teacher, school A, interview).

A focus on students’ achievement was becoming established, with:

> ... term monitoring for students, and onto that term monitoring would go your basic facts and your GloSS [Global Strategy Stage] level or your NDP level ... From that monitoring, students are identified as being extension or having difficulties. (Lead teacher, school A, interview)

Their students’ achievement data was being used to identify areas of need that could be targeted in teacher professional development:

> We looked at ... the national benchmarks and where we were coming, and then from there identifying that we needed perhaps to work on multiplication or that we needed to work on fractions, so looking at where our students were ... and identifying weaknesses. (Lead teacher, school A, interview)

As a school, they were exploring a range of mathematics assessments and had yet to settle on the combination that best met the needs of their students and community:

> We did the PAT[4] maths for our interest but also to report to parents through our blue portfolio system in the first term, but it's a bit complicated. So we don't think we'll do those again. We like the "I can"[5] sheets and so do the parents because ... it's an example of their thinking and strategy that they're using, and the parents have found that really useful. We have used the actual number results against the norms and reported to the board against those as well and [the lead teacher] has done an analysis of those that have been useful for the board but also very useful for us about where we stand nationally. (Senior management member, school A, interview)

Adaptations that were planned included aligning numeracy targets with the appraisal system and planning for lead teacher work in teachers' classrooms. Numeracy goals were not a mandatory component of the appraisal process and were included only if a teacher elected this as an area for appraisal:

> Occasionally there are people that would identify it as a need, like [a member of senior management] when she first went back into a class. (Senior management member, school A, interview)

There were not yet structures in place to allow the lead teacher to regularly work with individual teachers in their classrooms, modelling and observing numeracy instruction. This was planned to happen over the 2009 school year.

Staff at school B had deliberately made changes to a number of structures and practices in order to help embed their adaptation of the NDP into numeracy instruction, school-wide. Elements of their school organisation that they chose to redesign included: shifting from having a mathematics committee to having two lead teachers; deliberately having a classroom teacher as one of the lead teachers and a member of senior management as the other; timetabling numeracy teaching across the school; and making regular provision for release time for the numeracy lead teachers to support teachers in their classrooms. Their practices around the use of student achievement data also supported the embedding of the NDP.

Both lead teachers agreed that replacing the more traditional mathematics committee with two numeracy lead teachers had been a positive shift. One of them commented:

> I think having two strong numeracy leaders this year has made a huge difference. We are very focused, we know what we want to happen, but we're not quite sure we're doing all the right things to get it there, but we are working very hard with teachers to get there. And we've improved the resources with the resource boxes, that kind of thing, and I think ... we're at the stage we need to sit down with somebody to give us some guidance about what we need to do next year ... because we're really determined to continue, but it's all very well being determined ... it's about knowing exactly the needs. (Lead teacher 2, school B, interview)

A decision had been made by the numeracy lead teachers and senior management that numeracy would be taught across the school at the same time: "Maths is quarter past 9 to quarter past 10 in every classroom every day" (lead teacher 2, school B, interview) in order to allow for complementary timetabling, that is, withdrawal of groups of students who need extra support and of those who need extending.

The lead teachers at this school had established school structures and routines for working one-to-one with teachers and their students. This typically involved the lead teacher modelling an aspect of a mathematics lesson for the teacher to observe, then discussing what they observed. Later, the lead teacher would return to observe the teacher's instruction, after which they would meet to discuss the

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4 *Progressive Achievement Test: Mathematics Years 4–10*, New Zealand Council for Educational Research (2006). Wellington: NZCER.

5 "I can" assessment sheets are a resource available from www.nzmaths.co.nz.

lead teacher's feedback. In their survey responses to a question that asked them to describe the three most important factors that have contributed to sustaining their learning and development, there were 22 comments from 28 respondents relating to the value of the modelling, observation, and feedback processes. A teacher of year 7–8 students commented that an important factor was:

> Leaders modelling strategy and knowledge warm-ups for class, as well as watching you do the same and giving feedback. Important to see it being used with your own class as you can see how it is meant to be used properly. (Teacher, school B, survey)

Another teacher valued:

> modelling and feedback – teaching practice can change through teacher modelling and feedback, rather than dialogue alone. (Teacher of year 0–1 students, school B, survey)

The challenge of changes in staff was mentioned by principals in both schools. While this may have been an issue in the past, school B had recognised the importance of consistency of practice across all teachers, so they had put in place several strategies to cater for teachers arriving at the school. In their implementation plan for the mathematics and statistics learning area, professional development for new teachers was described as including "mentoring, modelling, observing, and other practical ways of helping teachers develop their understanding and confidence" (Implementation Plan, school B). This was the responsibility of the two lead teachers, and because this was a large school, they were sometimes able to group new staff for such development. In relation to the beginning of the school year, one lead teacher observed:

> We have got quite a few new teachers coming in next year, so we will be starting back supporting new teachers again. (Lead teacher 2, school B, interview)

Also in the implementation plan, consistency of practice was clearly stated as a goal and was supported by expectations for such aspects as programme planning and assessment:

> We're calling it an implementation plan. So we've got aims, we've got achievement expectations, we've got professional development, reporting to parents and the board, special needs and special abilities, assessment practices, our goals, we've got classroom expectations ..." (Lead teacher 2, school B, interview)

Students' numeracy achievement data drove decision-making about numeracy at school B. This included decisions about setting the school's goals for students' achievement and developing programmes to meet the learning needs of targeted groups of students: "... there is both remedial and extension. I had forgotten about that, but we just do that as a matter of course" (senior management member, school B, interview). Student results were also used to identify staff's professional development needs and to help individual teachers identify how they needed to shape their classroom programme to cater for their students' needs.

> Student data is initially used by a classroom teacher as part of formative assessment and the where to next. Very important, and then obviously school-wide data is analysed by the team for reporting to parents as a whole and then ... assessment for each classroom, as far as each classroom teacher is concerned, is used for reporting to parents. (Senior management member, school B, interview)

At school B, appraisal was also used as a means of maintaining teachers' focus on improving their numeracy instruction:

> As part of our appraisal system, we have targeted areas that we go in to observe ... Numeracy has been a targeted area, and that comes and goes depending on our focus for professional development. So it has been quite a focus over the last few years, and it's around modelling and feedback. I was going to say positive feedback – constructive feedback is a better way of putting it. [We've got] past the fluffy stuff. (Senior management member, school B, interview)

Appraisal visits with a numeracy focus were an established norm in the school, as one teacher described:

> and also, I think the appraisal system as well, because they come in and it's not threatening, it's just you know, they just come in and watch and say "Look, it's going like this, have you thought about this?" Yeah, it's fine. (Year 3–4 teacher, school B, interview)

Discussion and Conclusion

McNaughton and Lai (2009) talk about sustainability as being about generalisation across time of the effects of the intervention as well as across new cohorts of teachers and learners. The process of generalisation is likely to be associated with embedding practices in the school structure so that they can be sustained. School A appeared to be at an earlier stage of embedding the NDP than school B: school A had fewer structures and practices in place to support embedding the initiative into their context, but continued organisational redesign should support this. At school B, their adaptation of the NDP "has been school wide and purposeful and supported through various layers of leadership" (Anthony & Walshaw, 2006, p. 26). Particularly evident at school B was a combination of leadership enactments, all focused on the goal of raising students' numeracy achievement, with which they had already had some success. This may be related to the way leadership is distributed in their school. As Robinson (2008) suggests:

> Schools with stronger distributed leadership will, it is argued, have more staff who are knowledgeable about and take responsibility for the improvement of educational outcomes. (p. 242)

Starkey et al. (2009) suggest that:

> teacher professional development during the embedding stage of a national or large scale reform may differ fundamentally from other forms of teacher education and professional development. (p. 181)

The description in this paper of how two schools were working to embed the NDP in their respective contexts illustrates that, within this important phase, schools may be at different stages. Timperley et al.'s (2007) point that "sustainability was not neglected in the literature, but it was treated as an article of faith rather than a condition subject to empirical verification" presents a challenge to both researchers and implementers in determining the focus appropriate for the stage of the implementation (Starkey et al., 2009). In the current phase of NDP implementation, it may be appropriate for facilitators to shift their focus from developing individual teachers to considering ways in which they can help schools to ensure their organisational structures and practices support the adaptation of the NDP to their context in order for the NDP to endure.

For Further Research

Further investigation of leadership functions and practices is needed in order to understand what is important in embedding the NDP. Identification of the hallmarks of a school that can be said to have successfully embedded the NDP should be included in such an investigation.

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Networks of Support and Influence in the Numeracy Development Projects: A Case Study of Two Schools

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This paper reports on an exploratory case study of the networks that support and influence teachers' numeracy instruction. Forty-one teachers, numeracy lead teachers, and senior management in two schools that had recently undergone a renewed, two-year focus on the Numeracy Development Projects completed a survey that included creating a diagram of the network of people who support and influence their numeracy instruction. Fifteen staff then participated in an interview that included discussion of their network diagram. Characteristics of networks shown in participants' diagrams included strong within-school networks and elements of ongoing support from facilitators. A smaller role was played by networks that extended beyond the school site, such as lead teacher networks. The provision of release time was an organisational and resourcing consideration for schools that appeared to support teachers to engage with colleagues from their networks, with the shared purpose of improving their instruction.

Background

In New Zealand professional development projects, collaborative work focused on improving instructional practices in schools is often reported in studies of literacy (McNaughton & Lai, 2009; Timperley & Phillips, 2003) and, to a lesser degree, numeracy (Anthony & Walshaw, 2007). Numerous forms of school organisation (syndicate groupings, whole-school staff meetings, and senior management team meetings) provide opportunities for collaborative efforts in setting visions, planning, assessment practices, and professional learning. A variety of descriptors of this shared work in use in New Zealand primary and intermediate schools include communities of practice, communities of learning, and networks of practice. How teachers and school leaders see these various networks as supporting and influencing instruction in numeracy is the focus of the current study.

In their work on situating teachers' instructional practices in the institutional setting of the school and school district, Cobb and McClain (2006) used Wenger's (1998) three interrelated dimensions of a community of practice, "a joint enterprise, mutual relationships, and a well-honed repertoire of ways of reasoning with tools and artifacts" (p. 86), to analyse communities and networks of practice. A joint enterprise "entailed the teachers developing a relatively deep understanding of the mathematical intent of instructional activities so that they could achieve their instructional agendas by capitalizing on students' reasoning" (Cobb & McClain, 2006, p. 86). The dimensions of mutual relationships are the norms of participation, such as the sharing of instructional activities. Cobb and McClain suggest that through the three interrelated dimensions, "members of each community therefore afford and constrain the practices developed by members of other communities" (p. 81).

Coburn and Russell (2008), in reviewing definitions of professional community found in the literature, conclude that "they generally conceptualise it as including such dimensions as shared norms and values, a focus on student learning, social trust, deprivatisation of practice, collective responsibility, and collaboration" (p. 204). They caution that, while the field of study of professional community highlights the importance of teachers' social relations in improving instruction, there are several conceptual and methodological limitations to do with identifying which features of social relations assist teachers to improve their instructional practices. They suggest that at least four dimensions of teachers' social networks are important, including "structure of ties, trust, access to expertise, and the content of interaction" (p. 204). These dimensions are similar to those that appear in Timperley, Wilson, Barrar, and Fung (2007). In summary, it is the depth and focus of the teacher interactions in a school that are important (Cobb & Smith, 2008).

This paper uses the term "networks of support and influence" because the notion of networks is relevant to up-scaling professional development (Cobb & McClain, 2006, p. 87) and sustaining new instructional practices within and across schools. The role of school-based networks of support and influence is critical to supporting ongoing learning and development in numeracy. Dimensions that may constrain and support the development and sustaining of networks include school size, school-based forms of reporting, school-based and project-based management structures (for example, senior management team, numeracy lead teacher), and the stage of implementation of the Numeracy Development Projects (NDP). Factors such as people's interpretation of roles, such as the role of the numeracy lead teacher, may also be important.

Methodology

This exploratory case study[1] aimed to define questions and hypotheses for a subsequent study. The focus on networks of support and influence was one component of a study[2] that also investigated instructional leadership, embedding the NDP, and the diagnostic interview as a smart tool. This paper examines the role of networks in supporting teachers' ongoing learning and development in numeracy and aims to answer the research question:

- What are the characteristics of structures of influence and support on which teachers draw for the ongoing development of their teaching practice, and how are they supported or constrained by the school organisation?

The study builds on the work of Coburn and Russell (2008) and Cobb and Smith (2008) to examine how school organisation "can support or constrain the development of productive social interaction between teachers that enables them to make positive instructional change" (Coburn & Russell, 2008, p. 48). Drawing on Cobb and Smith, 2008 (p. 14), the teacher and senior management team interviews incorporated questions to find out:

- who influences how teachers teach mathematics;
- each person's understanding of the school's policies for mathematics instruction (or their vision for instruction);
- the person's informal professional networks;
- official sources of assistance each person can draw on.

Participants

The two Wellington-region schools participating in the study had originally completed numeracy professional development in the NDP's early years of implementation and, more recently, had undertaken a renewed focus on numeracy. The schools continued to call on outside facilitators to support the continued improvement of numeracy instruction. Both these urban full primary schools were selected because they reported recent improvements in their students' achievement; for example, data from school B showed that the achievement of a target year group had improved. Also, both schools were geographically convenient to the researchers.

School A was a medium-sized, high-decile, state primary school. The school had one numeracy lead teacher – a classroom teacher and syndicate leader – who had been in the lead teacher role for

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1 See Yin (2003) for a discussion of case-study methodology.

2 See other papers by Higgins and Bonne in this volume.

approximately eight years and who currently worked with two colleagues towards embedding the NDP in their school setting.

School B was a large, high-decile, state primary school. The lead teacher responsibilities at school B were shared by a classroom-based lead teacher (lead teacher 1) and a lead teacher who was a "walking" member of senior management (lead teacher 2). Lead teacher 1 had been a numeracy lead teacher at her previous school and had been in her current role at school B for three years. Lead teacher 2 had taken up the second lead teacher role when a colleague had left the school the year before and had also worked in the role several years previously.

The two schools can be thought of as being at different stages of implementation: school A was still undergoing the organisational redesign needed to support full implementation of the NDP; school B was embedding the structural changes they had already put in place.

Procedures

All teachers, numeracy lead teachers, and senior management (including principals) were invited to participate in the study. Initially, all teachers and senior management members were asked to complete a survey (see Appendix J, pp. 195–197) at each school's numeracy-focused staff meeting, at which the researchers made field notes. Several teachers at both schools took up the option of completing the survey after the meeting for later collection. Survey questions were generated using Cobb and Smith's (2008) frame of leadership priorities and were designed to elicit responses about leadership, networks of support and influence, formative assessment, and the roles these play in sustaining the NDP in their school[3]. At both schools, almost 90% of staff returned completed surveys.

As part of the survey, each staff member was asked to create a diagram of the network of people who support and influence their numeracy instruction. Of the total number of respondents, 11 from school A and 25 from school B created diagrams. The researchers were aware that respondents' interpretations of this task would vary and were testing its usefulness in preparation for future application on a wider scale.

Networks of support and influence was also a focus of one section of audio-taped interviews, which were subsequently carried out with lead teachers of numeracy, all members of senior management, and a representative sampling of teachers at both schools. At school A, there were six interviews, held with: the numeracy lead teacher (who also taught year 3–4 students), the principal, the deputy principal (who also taught new entrants), the assistant principal (who also taught year 5–6 students), and two other teachers who were part of the NDP team (one taught year 7–8 students; the other taught year 5–6 students).

At school B, there were nine interviews, held with: numeracy lead teacher 1 (who also taught new entrants), numeracy lead teacher 2 (who was also the deputy principal), the principal, the assistant principal, a teacher with special responsibility for curriculum, and one classroom teacher from each of the following year groups: years 3–4, years 5–6, years 7–8. An additional teacher of year 5–6 students, who had undergone NDP development elsewhere and was identified as having a particular strength in teaching numeracy, was interviewed at the principal's recommendation. All school and numeracy leaders were interviewed with the questions shown in Appendix J, which also includes teachers' interview questions. In the case of dual roles, leadership roles took precedence over teaching roles; for example, the numeracy lead teacher at school A, who also taught year 3–4 students, was interviewed using the questions for school leaders.

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[3] See other papers by Higgins and Bonne in this volume.

Lead teachers were also asked to provide copies of school documentation that supported the development of high-quality numeracy instruction and student achievement data for the current and previous year. In summary, the school's dataset comprised surveys, interview transcripts, school documentation, and student achievement data.

Analysis

A qualitative approach was taken to the diagram analysis, using three major patterns referred to as "spoke", "chain", and "net" structures by Kinchin, Hay, and Adams (2000, p. 43). These structures were used by Kinchin, Hay, and Adams in relation to students' depth of conceptual development. Adapted for the current study, the structures provided a helpful analytical lens through which to identify the complexity of networks with which teachers associated themselves. The three structures resemble the following patterns:

- Spoke structure: all members of the network are connected directly to the teacher but are not linked to one another;
- Chain structure: members of the network are linked in a linear order, perhaps implying hierarchy;
- Net structure: interconnections between various members of the network are indicated.

Characteristics of networks of support and influence were drawn from the diagrams, a sample of which is reproduced here. Data from interviews was used to identify how teachers' engagement with networks was supported or constrained by the school organisation.

Findings

Part 1: Characteristics of Networks of Practice

There were a number of challenges involved in analysing the diagrams: not all staff completed a diagram; some made what seem to be very cursory diagrams; the diagrams were collected across two sites; the diagrams capture one point in time in the professional development process; and because the diagrams were completed by individuals in the school, they do not necessarily reflect the stage of the embedding process at the school level. More research is needed to establish the depth and focus of the interactions between the people in the networks. In some cases, using the diagrams as a prompt in the interviews exposed the possibility of changes to the overall structure, so that, for instance, spoke structures might become net structures by the addition of interconnections.

Network diagrams are used here to illustrate the study's findings. For each school, diagrams are presented here from the lead teacher/s, a classroom teacher, and a senior management member.

The first three diagrams (see figures 1–3) are from staff at school A, which is still implementing the organisational redesign needed to support further development of their numeracy instruction. The lead teacher at this school is also a classroom teacher, and the net structure she created (Figure 1) includes supports and influences from people who are outside the immediate school context: a family member, a friend, and mathematics advisors. Also noted in the diagram is the typical focus of the lead teacher's interactions with members of her network.

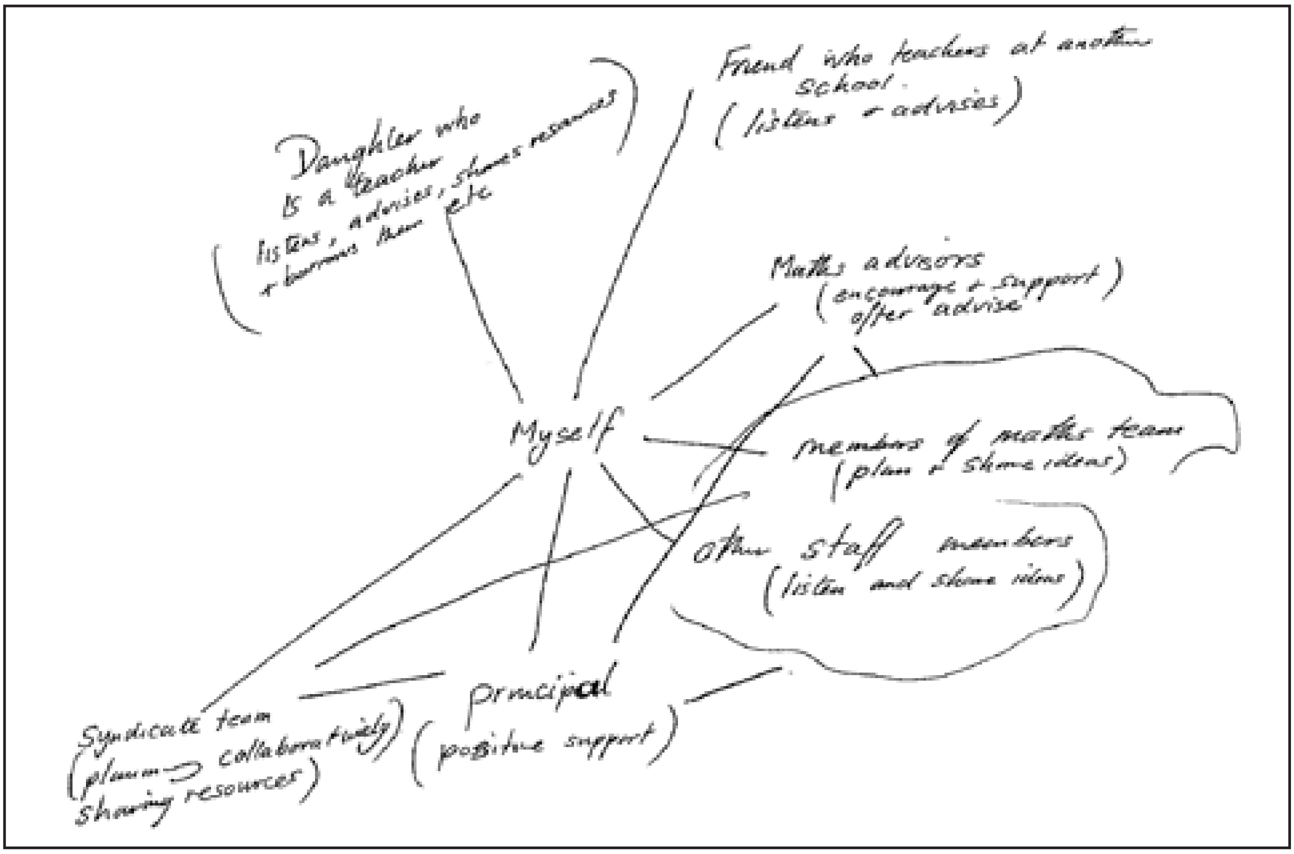

Figure 1. Classroom-based lead teacher's diagram of the network of people who support and influence their numeracy instruction (school A)

Figure 2, from another teacher, is a spoke structure that also includes as a support and influence a family member who is a teacher.

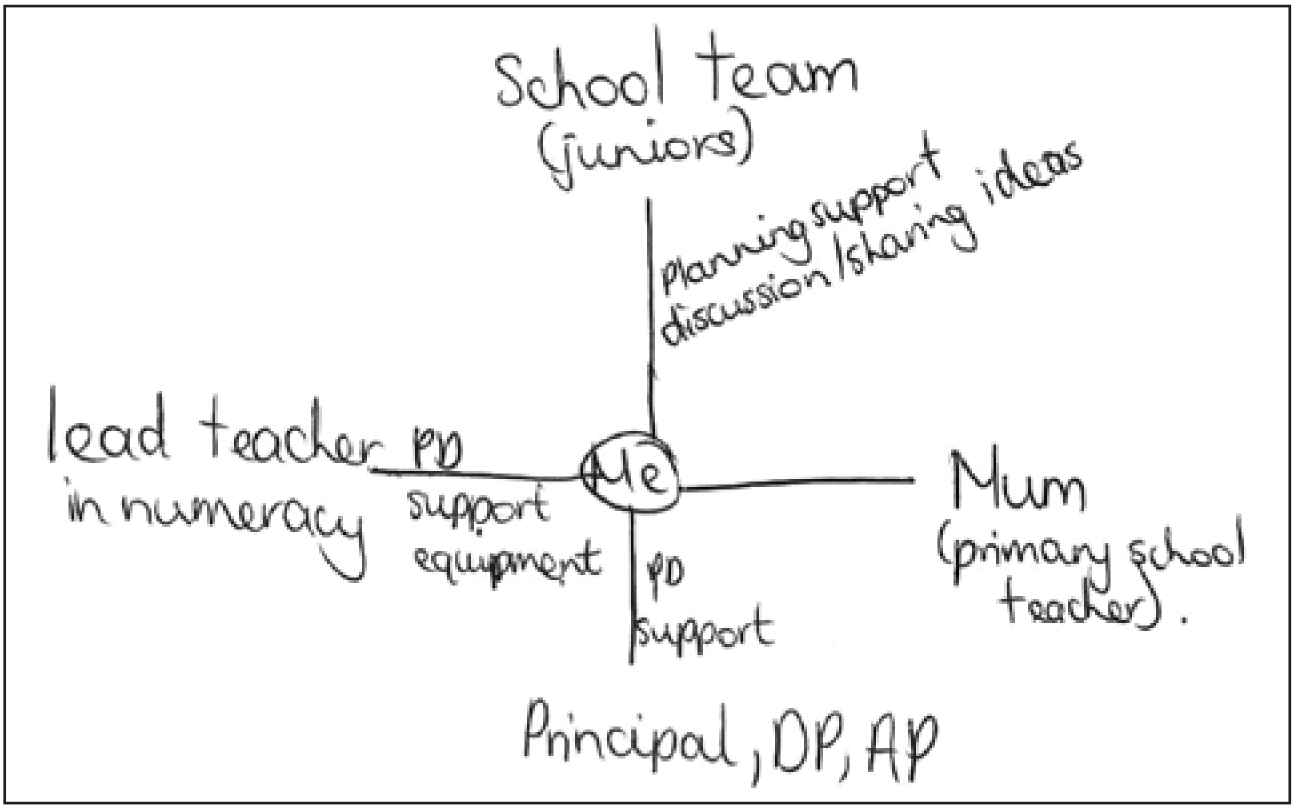

Figure 2. Teacher of year 1–2 students' diagram of the network of people who support and influence their numeracy instruction (school A)

Figure 3 from school A is from a member of the senior management team and reflects a strong emphasis on pedagogy and student learning. This is another example of a net structure, showing interconnections between network members.

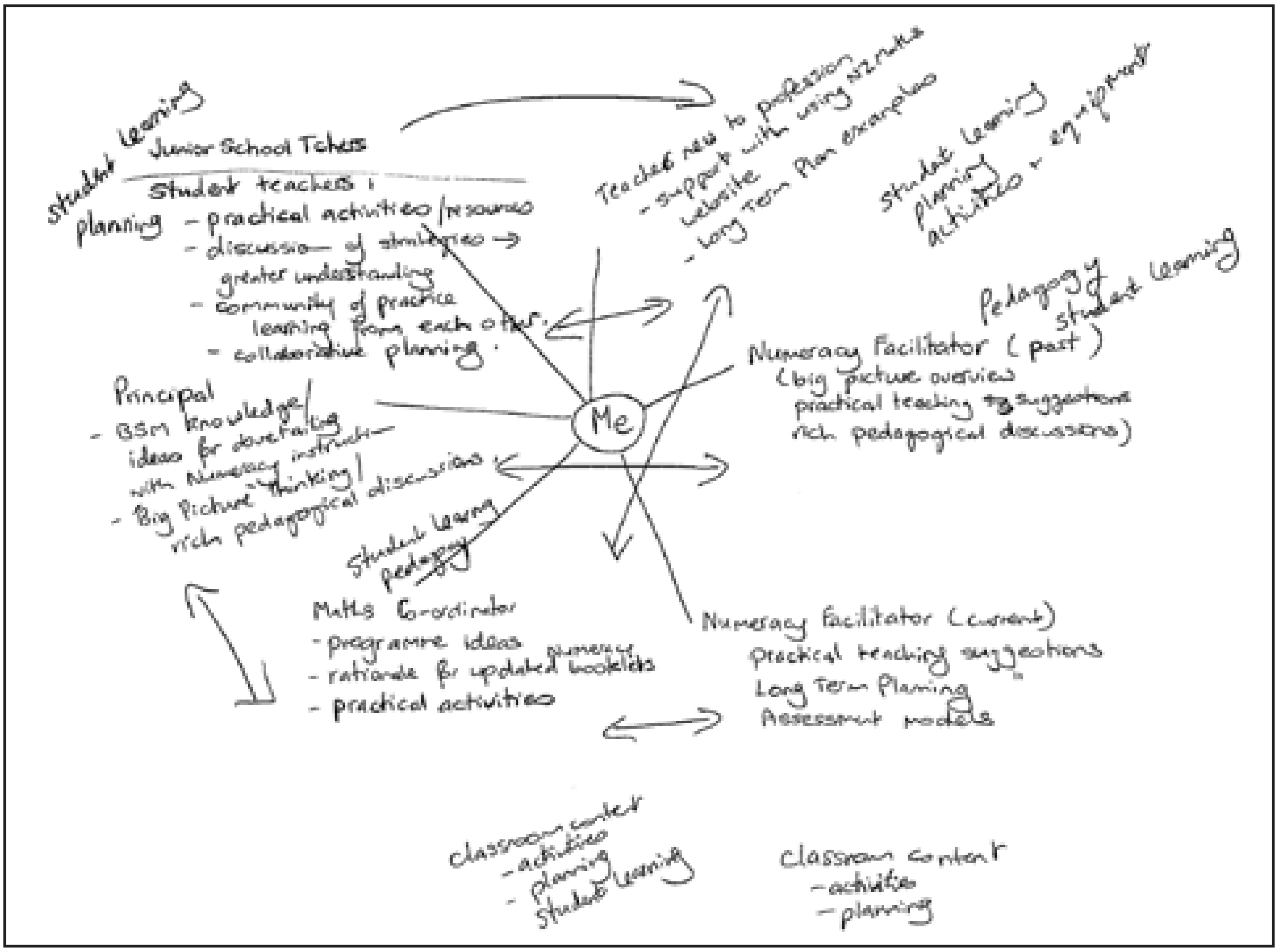

Figure 3. Senior management member's diagram of the network of people who support and influence their numeracy instruction (school A)

School B had two lead teachers of numeracy. Figure 4, a net structure, was drawn by a lead teacher who was also responsible for teaching a class.

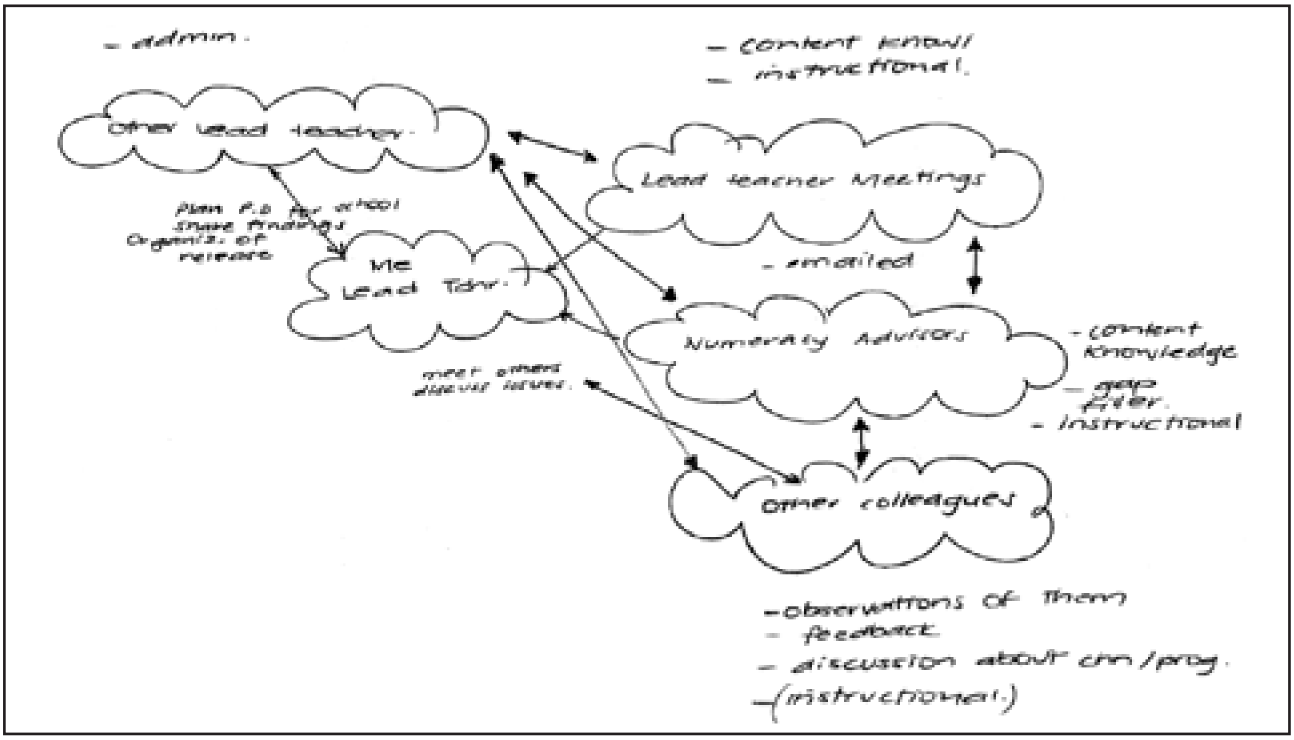

Figure 4. Lead teacher 1's diagram of the network of people who support and influence their numeracy instruction (school B)

The diagram in Figure 5, by a member of senior management who was also the second lead teacher of numeracy at school B, is closer to what Kinchin et al. (2000) describe as a chain structure. This person's interpretation of the task may have been affected by the fact that they did only a small amount of numeracy teaching; the diagram tends to reflect the senior manager's/lead teacher's role in supporting and influencing others.

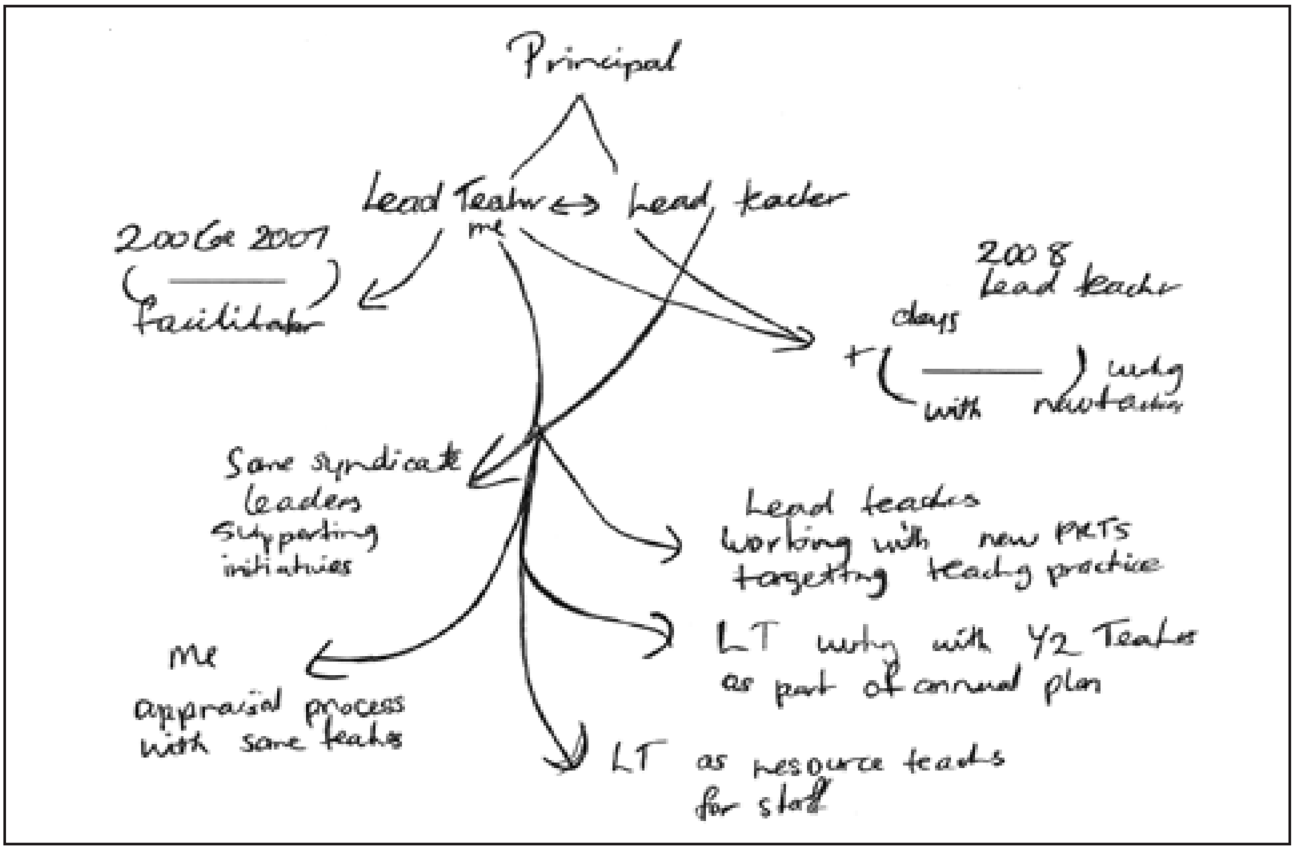

Figure 5. Lead teacher 2's diagram of the network of people who support and influence their numeracy instruction (school B)

The final diagram – another net structure – was created by a teacher of year 5–6 students.

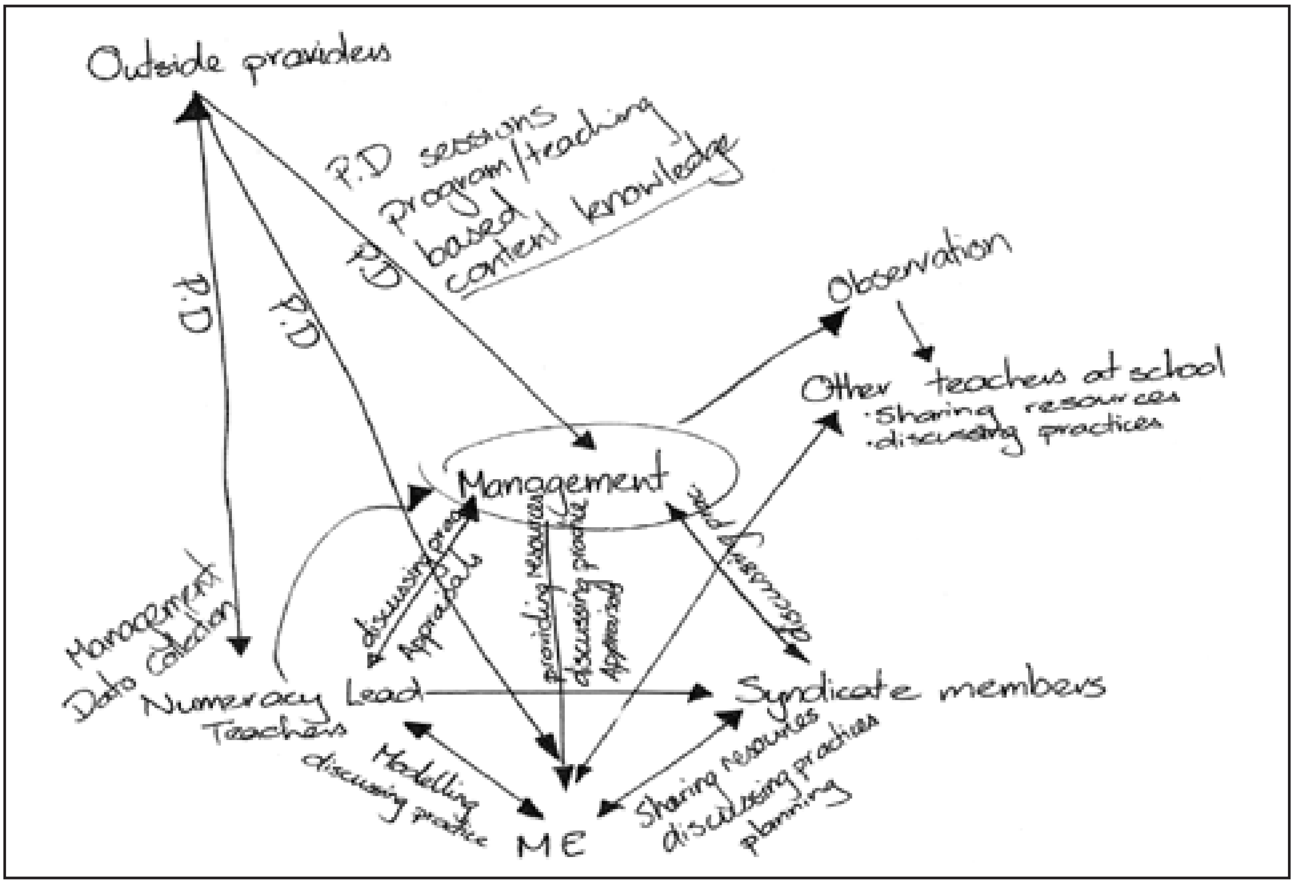

Figure 6. Teacher of year 5–6 students' diagram of the network of people who support and influence their numeracy instruction (school B)

From the three diagrams from school B reproduced here, as well as from those drawn by others at the same school, it seems that the teachers at school B tend to draw support from within their own school rather than seeking it beyond their immediate context. It was interesting to note that there was little evidence of professional, collegial relationships with teachers at other local schools. One likely reason for this is that this data was collected in a large school, in which there were many opportunities for observing colleagues who teach a similar year level. Structures and practices that support the continued improvement of numeracy instruction, such as the lead teacher being released to model for colleagues and then observe them and give critical feedback, were by now embedded in the fabric of the school. The apparent self-sufficiency of the staff might also be influenced by the whole-school professional development model; the more typical model of delivery begins by drawing together teachers of year 1–3 students from a cluster of neighbouring schools, before working with teachers from years 4–6, and then years 7–8. It may be that this more common model provides opportunities for teachers to build networks with colleagues at other schools.

Looking across the diagrams completed by staff from the two schools, the structures most often used are spokes (19 diagrams), with net structures the second most common (10), and chains the least common (7).

Part 2: Supports and Constraints of School Organisation

A variety of structures were in place at school B to provide teachers with opportunities to develop their numeracy instruction by drawing support from, and being influenced by, colleagues within their own school. These structures included: a buddy-teacher system (teachers were paired with a colleague who taught a different year level and who often had a different degree of expertise); syndicate groupings; neighbouring teachers (typically teaching the same year group); lead teachers being released to model and then to observe colleagues and give feedback; and numeracy goals being part of every teacher's appraisal.

The provision of release time for lead teachers allowed them to attend lead teacher professional development meetings at the local university. Both lead teachers at school B referred to lead teacher days or lead teacher meetings in their diagrams (see figures 4 and 5), although both commented during their interviews that they had not attended these consistently and were unconvinced of the usefulness of attending these sessions, which sometimes required employing a relief teacher. One of the lead teachers did, however, remark on the potential helpfulness of such a network:

> It would be nice to be in contact with other lead teachers in other schools, particularly those who you might get along with in some way and you felt you could ring that person and ask "Hey, what are you doing about this or what are you doing about that?" Cos sometimes, you know, I do feel like sometimes we're in an island in a very big sea. (Lead teacher 1, school B, interview)

Where release time was not made available, teachers attended after-school courses. An advantage of attending courses outside school hours was that a whole syndicate was able to attend: "and that is so much better than one person, that's so good to actually ... really be involved" (senior management member, school B, interview).

Release time was a necessary component of the lead teachers' classroom modelling, observation, and feedback process and was highly valued by teachers as a means of collaborating with members of their networks of support and influence to develop their numeracy instruction. At school B, this practice was reported by 22 of 28 staff as being one of the three most important factors that had contributed to sustaining the development of numeracy instruction in their school, with comments such as:

> Modelling and feedback – teaching practice can change through teacher modelling and feedback, rather than dialogue alone. (Year 0–1 teacher, school B, survey)

Release time was also needed for liaison visits to early childhood centres. These visits occasionally included a focus on numeracy.

Email was used to maintain communication between various members of networks of practice, as needed:

> I've just emailed [the facilitator] with a couple of questions, and she said "I'll just [call in] and answer one of those" ... (Lead teacher 1, school B, interview)
>
> [the facilitator] and I still email over various things over mathematics. (Senior management member, school A, interview)
>
> I provide the feedback form and I email that through to [the teacher] plus our AP ... whoever's in charge of that area, and if it was a provisionally-registered teacher, a copy also goes to their tutor teacher. (Lead teacher 1, school B, interview)

During the interviews conducted as part of this study, lead teachers and those in management roles were described as lynchpins and hubs, signalling their pivotal role in connecting people in networks in order to develop numeracy instruction:

> So even if it's not a professional development focus, [the lead teacher] is really our lynchpin in that she goes to the workshops and she brings back into the school the new thinking. (Senior management member, school A, interview)
>
> And within school management, well yeah, because I'm part of middle management, so I guess you've got, we're bridging that gap between our scale A teachers and our senior management and sharing what's going on between the two ... (Year 7–8 teacher, school B, interview)

> And it's having that relationship with teachers that they come and say "Oh I just can't get so and so", and then we're able to think on the spot and say "Oh right, okay, I'd bet ... [the lead teacher] would know", or "[another teacher with expertise in mathematics teaching] has been using ..." So if you know what everyone's doing ... you're a hub to sort of say, "Oh okay, go and talk to this person" or "Let's have a look at the data, what does your number test say?" or "What does your GloSS [Global Strategy Stage] say?" (Senior management member, school A, interview)

One lead teacher described networks of practice as supporting teachers to become confident numeracy teachers:

> I think that everybody is confident, or if they're not confident, [they] know where they can find the help they need, that they've got the guidelines and the processes [so] that they can become confident teachers. So they've got the support networks in place for them. (Lead teacher, school A, interview)

Discussion and Conclusions

Characteristics of the networks of support and influence at these two schools are particular to their contexts, so the findings from this initial study cannot reasonably be generalised to other settings. The study showed that, in the two participating schools, a greater number of functional, within-school networks supported and influenced teachers' numeracy instruction than networks that reach beyond the school. While the main involvement of the facilitators was two years prior to the diagrams being completed, many participants included the facilitator in their network diagram. What is not known is the extent to which the scope of the networks was shaped by the stage of embedding the professional development or by the fact that the teachers had undertaken their most recent numeracy professional development as a whole school led by an external facilitator. The provision of release time enabled numeracy lead teachers, senior management members, and teachers to interact with others in their networks in order to collaborate with the goal of improving instruction. Both schools indicated their intention to visit other schools in the coming year to observe numeracy instruction. What remains to be seen is whether doing this will impact on the teachers' networks in terms of the depth and focus of the interaction, how this might in turn influence their numeracy instruction, and, ultimately, how this might be linked to students' achievement in numeracy.

For Further Research

- "There is abundant evidence that the mere presence of collegial support is not by itself sufficient: both the focus and the depth of teachers' interactions matter" (Cobb & Smith, 2008, p. 5). Further research is needed to examine these aspects of teachers' interactions within their networks of support and influence.
- What are the effects of teachers having no networks that extend beyond their own school? How is a teacher being part of a wider network associated with the achievement of their students?
- In what ways are networks of support and influence of more experienced teachers different from those of less experienced teachers?

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The Role of the Diagnostic Interview in the Numeracy Development Projects

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Formative assessment practices in two case study schools are described in this paper. The two high-decile, urban primary schools that participated in this exploratory study were at different stages of embedding the Numeracy Development Projects (NDP). Teachers, numeracy lead teachers, and members of the senior management teams completed surveys and then a cross-section of staff was interviewed. Students' numeracy achievement data, field notes from one school's staff meeting that focused on the diagnostic interview, and other school documentation completed the data sets. The findings describe how a school's use of the diagnostic interview to generate formative assessment data can contribute to the development of school-based conditions for the sustainability of a professional development project. In particular, for the NDP, consistent, school-wide use of the diagnostic interview and its resultant achievement data appeared to strengthen a school's emphasis on improving students' achievement.

Background

Professional development that gets to the core of educational practice is important in projects aimed at system-wide change, such as the Numeracy Development Projects (NDP). Elmore (1996) defines the core of practice as "how teachers understand the nature of knowledge and the student's role in learning, and how these ideas about knowledge and learning are manifested in teaching and classwork" (p. 2). This paper examines the role of the diagnostic interview in getting to the core of practice so that both individual teachers and the school as a collective experience powerful professional learning. Such a tool provides teachers and schools with a means of establishing core ideas of quality instruction and enables teachers to develop more complex practice rather than merely making superficial changes (Higgins & Parsons, in press).

The diagnostic interview[1] is one of three pedagogical tools of the NDP that ensure powerful professional learning in a school setting, the others being the Number Framework and the strategy teaching model (Higgins & Parsons, forthcoming). Higgins and Parsons have identified the following three embedded design elements of the diagnostic interview for teacher professional learning:

> Firstly, it is designed as a model for the types of questions that teachers might use in teaching students strategies; secondly, teachers deepen their understanding through the items in the diagnostic interview illustrating the different stages of the Number Framework; and thirdly, the information gained through the interview enables teachers to develop more specific expectations of student learning. The strategy and knowledge components of the interview build teachers' knowledge of the interconnectedness of mathematical ideas. (Higgins & Parsons, forthcoming)

Such design elements suggest that the diagnostic interview has some of the characteristics of what Robinson and Timperley (2007) term, a "smart tool". They reserve the term "smart tool" for tools that meet the criterion of "following two characteristics: (a) they incorporate a valid theory of the task for which they were designed and (b) the tools themselves are well designed" (p. 256). Specifically, Robinson and Timperley argue for the importance of a smart tool having embedded theories about quality instruction and acting as a mechanism for collecting evidence of the quality of teacher performance and related student outcomes. The diagnostic interview, like the literacy "wedge

[1] For the purposes of this discussion, the term "diagnostic interview" refers to the following formative assessment tools for numeracy: the diagnostic interview (NumPA) (Ministry of Education, 2008), IKAN (Individual Knowledge Assessment for Numeracy), and GloSS (Global Strategy Stage assessment) (all available online at www.nzmaths.co.nz).

graph" used in Robinson and Timperley's example of a smart tool (p. 256), is a well-designed tool that incorporates a theory of teacher professional learning, the task for which it was designed. The diagnostic interview provides teachers with a model of types of strategy questions, illustrations of Number Framework stages, specific expectations, and a means of gathering evidence of student learning at different levels. The importance of teachers understanding the interconnectedness of mathematical ideas is illustrated through the strategy and knowledge sections of the diagnostic interview.

Methodology

This exploratory case study[2] aimed to define questions and conjectures for a subsequent study. The focus on formative assessment was one component of a study[3] that also sought to elicit teachers' and school leaders' thinking around the themes of instructional leadership, networks of practice, and embedding the NDP.

The following two research questions were investigated in this case study:

- In what ways are students' numeracy achievement information collected?
- In what ways does the data contribute to embedding the NDP in a school?

Participants

The two participating schools in the Wellington region had originally completed numeracy professional development in the NDP's early years of implementation and, more recently, had undertaken a renewed, two-year focus on numeracy. The schools continued to call on outside facilitators to support the continued improvement of numeracy instruction. Both of these urban, full primary schools were selected because they reported recent improvements in their students' achievement; for example, data from one school showed that the achievement of a target year group had improved. Also, both schools were geographically convenient to the researchers.

School A was a medium-sized, high-decile, state primary school. The school had one numeracy lead teacher – a classroom teacher and syndicate leader – who had been in the lead teacher role for approximately eight years and who currently worked with two colleagues towards consolidating the NDP in their school setting.

School B was a large, high-decile, state primary school. The lead teacher responsibilities at this school were shared; one classroom-based lead teacher had been a numeracy lead teacher at her previous school and had had three years in her current role at this school. The second lead teacher was a "walking" member of senior management who had taken up the second lead teacher role when a colleague had left the school the year before. She had also worked in the role several years previously.

The two schools can be thought of as being at different stages of implementation: school A was still undergoing the organisational redesign needed to support full implementation of the NDP; school B was embedding the structural changes it had already put in place.

Procedures

All teachers, numeracy leader teachers, and senior management (including principals) were invited to participate in this study. Initially, all teachers and senior management members were asked to complete a survey (see Appendix J, pp. 195–197) at each school's numeracy-focused staff meeting,

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2 See Yin (2003) for a discussion of case-study methodology.

3 See other papers by Higgins & Bonne, in this volume.

at which the researchers made field notes. Several teachers at both schools took up the option of completing the survey after the meeting for later collection. Survey questions were generated using Cobb and Smith's (2008) frame of leadership priorities and were designed to elicit responses about leadership, networks of practice, formative assessment, and the roles these play in sustaining the NDP in their school[4]. At both schools, almost 90% of staff returned completed surveys.

Audio-taped interviews were subsequently carried out with lead teachers of numeracy, all members of senior management, and a representative sampling of teachers at both schools. At school A, a total of six interviews were held with: the numeracy lead teacher (who also taught year 3–4 students), the principal, the deputy principal (who also taught new entrants), the assistant principal (who also taught year 5–6 students), and two other teachers who were part of the NDP team (one taught year 7–8 students; the other taught year 5–6 students). Responses to other questions around the themes of leadership, networks of practice, and embedding the NDP also elicited comments relating to the diagnostic interview and the use of students' numeracy achievement data.

At school B, nine interviews were held with: numeracy lead teacher 1 (who also taught new entrants), numeracy lead teacher 2 (who was also the deputy principal), the principal, the assistant principal, a teacher with special responsibility for curriculum, and one classroom teacher from each of the following year groups: years 3–4, years 5–6, and years 7–8. An additional teacher of year 5–6 students, who had undergone NDP development elsewhere and who was identified as having a particular strength in teaching numeracy, was interviewed at the principal's recommendation. All school and numeracy leaders were interviewed with the questions shown in Appendix J. Teachers' interview questions are also shown in Appendix J. In the case of dual roles, leadership roles took precedence over teaching roles; for example, the numeracy lead teacher at school A, who also taught year 3–4 students, was interviewed using the questions for school leaders.

Lead teachers were also asked to provide copies of school documentation that supported the development of high-quality numeracy instruction and student achievement data for the current and previous year. In summary, the schools' data sets comprised surveys, interview transcripts, school documentation, and student achievement data.

Analysis

It is important to lay the foundations for the development of school-based conditions for the sustainability of a professional development project (Coburn & Russell, 2008; Higgins with Bonne & Fraser, 2004; Timperley, Wilson, Barrar, & Fung, 2007). Drawing on the school-level foundational conditions identified by Timperley et al. (2007) and the leadership dimensions described by Robinson, Hohepa, and Lloyd (in press), the data was analysed using the following seven dimensions:

- consistency of interpretation of the NDP at the school level;
- setting direction and school targets for student achievement;
- consistency in grouping students for instruction;
- evaluation of the impact of an initiative at the school level;
- the formation of focused school-wide networks of practice;
- evidence-based and school-wide consistency in reporting to parents;
- information for school-based instructional leaders on individual teachers and groups of teachers.

The formative assessment tools of the NDP were analysed against these seven dimensions.

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[4] See other papers by Higgins & Bonne in this volume.

Findings

The evidence that follows demonstrates how two schools at different stages of establishing numeracy practices used achievement data to consolidate the changes in their school. While school B had quite well-established structures and practices in place for numeracy assessment, school A was still in the process of making decisions about the combination of assessment tools that best met the needs of their students, teachers, and community. Their numeracy lead teacher explained:

> We were doing the whole [diagnostic interview], but then that was getting too much, so then we were doing GloSS[5] because ... when you compared it to the national norms, we were up and above, so we then went back to just doing GloSS ... (Lead teacher, school A, interview)

The full diagnostic interview was still used occasionally with students:

> ... new to the school, coming into the school, we do a full thing when they come in if we can, if we have time. And any children that we've got concerns with, we might do one ... (Lead teacher, school A, interview)

Teachers at school A were no longer using the diagnostic interview school-wide:

> This year, we're trialling doing a full one for the year fours and the year sevens. (Lead teacher, school A, interview)

They had explored using the revised PAT: Mathematics[6] but were unsure that it was a good fit with their other mathematics assessments. Also:

> We have made a decision not to go down the asTTle[7] track, we're just waiting and seeing how that all pans out, so we are still playing around with stuff ... (Senior management member, school A, interview)
>
> In the meantime, we have term monitoring for students and on to that term monitoring would go your basic facts and your GloSS levels ... (Lead teacher, school A, interview)

At school B, the lead teachers had established appropriate school structures and resources for the administration of the diagnostic interview and supported their colleagues to become confident in using the interview. This was reported by teachers as "taking staff meetings, i.e., ... Numeracy Diagnostic" (year 0–1 teacher, school B, survey) and working with teachers on a one-to-one basis: "Tomorrow I am going to work with [a lead teacher] – administering a diagnostic test to a pupil" (year 3–4 teacher, school B, survey). Another comment was "The school expectations for interviewing every student were supported by good systems – diagnostic tests" (year 7–8 teacher, school B, survey).

Consistency of Interpretation of the Diagnostic Interview

Although school A was a smaller school than school B, the staff were also aware of the need for consistency in administration of the diagnostic interview. Their strategy for addressing this was to have one teacher do all the interviews for their two targeted year groups:

> What we've decided is that in year 4 and year 7, we are going to actually do at the end of each year a full number survey and release people like [teacher's name] to do that testing. Then we can actually be sure that our GloSSing is up to scratch as well as getting consistency across. (Senior management member, school A, interview)

School B was a large school, and the lead teachers and senior management strove for a degree of consistency of teachers' practices. As the end-of-year, school-wide diagnostic interviewing drew near,

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5 GloSS: Global Strategy Stage, an assessment tool that can be used to determine which global strategy a student uses, available online at www.nzmaths.co.nz

6 PAT: *Progressive Achievement Test: Mathematics,* New Zealand Council for Educational Research

7 Assessment Tools for Teaching and Learning (asTTle) is an educational resource for assessing literacy and numeracy developed for the Ministry of Education by The University of Auckland.

the two lead teachers at school B led a staff meeting that emphasised the importance of consistency of administration as they worked to establish norms around how the interview was used:

> This is the way we do [the diagnostic interview] at [school] ... It's important we're consistent here at [school], otherwise we won't get good data. (Lead teacher 1, school B, speaking at staff meeting)

Teachers were aware of the importance of "making sure that I am doing it right and that I understand exactly where their level should be" (year 5–6 teacher, school B, interview), and were supported by the lead teachers to administer the diagnostic interview in a consistent manner, as is evidenced by this survey response:

> New staff have a diagnostic test modelled for them, and then they have an opportunity to observe them in return, offering feedback and further support. (Year 7–8 teacher, school B, survey)

The facilitator with whom the staff had worked had also stressed the importance of consistent practice when training the lead teachers in the use of the diagnostic interview:

> When we started, he trained [a former lead teacher] and I to do the numeracy diagnostic test and then we trained the other teachers until he was satisfied with the way we were doing it. (Lead teacher 2, school B, interview)

Opportunities were provided by the numeracy lead teachers for teachers to revisit this:

> There are times when they'll say "Look, we're having this meeting, and we're going to show you all a video about how to do the diagnostic." [A lead teacher] says "I'm going to do some, come into my room and watch me do a diagnostic ..." (Year 3–4 teacher, school B, interview)

Setting Direction and School Targets for Student Achievement

School-wide data at school A was used to identify groups of students whose learning needs might demand special programmes:

> We're looking for pockets of excellence where we say "Mmm, where are we going to take these kids?" And we're looking for pockets of "Here's a group of kids who we need to support." (Senior management member, school A, interview)

The degree to which their students' numeracy data was aligned with students around New Zealand was also a focus:

> For mathematics, we want to know that our number testing is aligning with the national picture, in the same proportion. (Senior management member, school A, interview)

School B had a very clear school-wide focus on improving students' numeracy achievement, and this was supported by a focus on data. In their implementation plan, it was stated:

> The diagnostic interview results undertaken in term 4 will inform the subsequent year's focus for goal setting and development of programmes to determine student learning. (School B's Implementation Plan)

The school's implementation plan also includes a copy of the end-of-year curriculum level expectations for students' numeracy stages, taken from www.nzmaths.co.nz, which provides this and other NDP information for teachers and schools. School B's 2007 end-of-year data for year 1 students had shown that 86% of students had been assessed as operating at stages 2–4 of the Number Framework, with the remaining 14% below the curriculum level expectation for this year group. Although this compares favourably with around 16% being below the expectation[8], it caused concern:

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[8] See www.nzmaths.co.nz

> In a school of this decile, we were shocked at [this result] because kids come in with good maths knowledge ... (Lead teacher 2, school B, interview)

This data had been analysed and shared with staff at the start of 2008 as this cohort of students moved into year 2:

> When we got the numeracy diagnostic results, we were a bit shocked to find where our year 2 children were, and I think it sort of galvanised [the other lead teacher] and I into wanting to sustain the project in the school. (Lead teacher 2, school B, interview)

> As a result of the diagnostic interviews last year, our year 2s are quite low ... well below as far as looking at the national benchmarks ... so we highlighted the need to work with teachers of year 2 children. (Lead teacher 1, school B, interview)

The senior management team drew on this data to set this strategic goal for the 2008 school year: "70% of the target group of children at [the school] will be moving into stage 5 by the beginning of year 3" (school B's annual plan, 2008). It was the lead teachers' responsibility to set up a withdrawal programme in order to achieve this goal. This was informed by a presentation at a lead teacher development day:

> We went over to the very first lead teacher meeting, and there was a woman speaking about the programme she'd put into place ... And so we came back here all fired up ready to do something similar and so we ... worked with teacher aides ... (Lead teacher 1, school B, interview)

The 2008 end-of-year data showed that "46% of the targeted year 2 children are working at stage 5" and "43% of the targeted year 2 children are working in stage 4 in readiness to transition into stage 5" (school B's annual plan, 2008).

Consistency in Grouping Students for Instruction

At school A, the "I can"[9] assessment sheets were used for ongoing monitoring:

> The "I can" sheets are also part of what syndicate leaders might look at [at] different times too across the syndicate to see if there are any areas of weaknesses or strengths that they might need to note for individual children, in particular gifted and talented kids ... (Senior management member, school A, interview)

At school B:

> Student data is initially used by a classroom teacher as part of formative assessment and the where to next. Very important. (Senior management member, school B, interview)

There was emphasis on administering the diagnostic interview in a consistent manner across the school because they used their data to inform key decisions about numeracy. At the start of the school year, end-of-year data from the previous year was used to determine groupings for instruction, based on students' strategy stages:

> Diagnostic testing is used for the following year by the new teacher for groupings, and following on from that are the "I can" sheets done on a regular basis to see what stage they are at and confirm that stage before they move on to the next stage. (Senior management member, school B, interview)

This teacher talked about the accuracy of the information received about the strategy stages of their students, who were drawn from several of the previous year's class groups:

> I went on last year's [diagnostic interview data]. I did "I can" sheets at the start of the year, and it pretty much confirmed everything that was in those things, so I just took it, that was the benchmark that I went from ... (Year 3–4 teacher, school B, interview)

9 "I can" assessment sheets can be found at www.nzmaths.co.nz/node/1491

Evaluation of the Impact of the Diagnostic Interview

At school B, students' achievement results were treated as a key indicator of the impact of the NDP, as this member of the senior management team reported:

> It's school-wide data to inform how we are doing as a school, moving student achievement up to the target areas ... (Senior management member, school B, interview)

Numeracy data was discussed as a whole staff:

> Now and again, we'll do testing and they'll throw it up on a big screen and just say "Look at this, we've got a real trend here where we're dropping in this, we need to get onto this." And everyone's more than keen. I mean we're looking after our kids, so we do want to hear it. (Year 3–4 teacher, school B, interview)

The lead teachers evaluated progress towards each year's strategic goal for numeracy. Most recently, the improvement in achievement of the year 2 students, described earlier, had been seen as a positive result of the programme established by the lead teachers working closely with the teachers in that part of the school, as well as with a team of teacher aides. One of the lead teachers talked about anecdotal evidence of positive change related to the diagnostic interviewing process:

> So then we see much more confident teachers, and we see children achieving at a much better rate, and I haven't got all the diagnostics, they are still doing their diagnostics, but the talk at the moment is "Wow, my kids have done so well in fractions, not so good in ad-sub." I thought they might have done better, but their fractions are fantastic, so that's all really exciting. (Lead teacher 2, school B, interview)

The Formation of Focused School-wide Networks of Practice

During the study, the researchers attended a staff meeting at school B where the focus was on the diagnostic interview. This official network was used to set norms around how teachers were to administer the interview, with the group discussing questions about what counts as a correct response from a student and how many attempts they should be allowed for each question. There was some debate over whether or not teachers should ask a student to explain their strategy when they gave an incorrect answer.

Other well-established within-school networks that provided opportunities for teachers to discuss the diagnostic interview and its data included buddy teachers, a large group of provisionally registered teachers and their tutor teachers, and neighbouring teachers. Syndicate groupings were another important network for discussing and reflecting on practice. One teacher talked about how the focus on numeracy was included by a syndicate leader, who:

> always makes sure there's part of our meetings ... that are just open to any, like she always says "Is everyone ok, how are you going with this, what do you need?" (Year 3–4 teacher, school B, interview)

The same teacher alluded to the opportunities afforded by the school's virtual network:

> I find it confusing how big stage 5 is. There are some kids who are just on it who aren't that good at maths and there are some who are fantastic, but they get the same grade, and so we had a huge discussion. There was an email that went right around the school straight after it, you know, it was real fast and, you know, good effective communication. (Year 3–4 teacher, school B, interview)

A related study[10] describes how within-school networks appeared to have greater influence on numeracy instruction in both schools than networks that reach beyond the school. At school B, the provision of release time enabled numeracy lead teachers to support teachers – individually, in groups,

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[10] See Higgins & Bonne, Networks of support and influence in the Numeracy Development Projects: A case study of two schools, in this volume.

and as a whole staff – in order to develop consistent practices around the use of the diagnostic interview and the resultant data, with the ultimate goal of improving students' numeracy achievement.

Evidence-based and School-wide Consistency in Reporting to Parents

At school A, a teacher described the value of the diagnostic interview data for reporting to parents of 5-year-olds:

> We have a six-week conference with children where we share our number data with parents ... and talk about where their kids are at and where we're going to take them. (Senior management member, school A, interview)

The same teacher reported that information from the diagnostic interview completed with students on school entry meant that:

> ... my teaching of number knowledge is sharper – the NumPA testing gives me a clear picture of what I need to target, e.g., counting forwards and backwards, teen numbers, and I use teaching techniques and resources that I know work ... (Senior management member, school A, survey)

The diagnostic interview was not being used across the whole school. Instead, GloSS data was informing reporting to parents. At school B:

> School-wide data is analysed by the team for reporting to parents as a whole, and then assessment for each classroom, as far as each classroom teacher is concerned, is used for reporting to parents. (Senior management member, school B, interview)

Descriptors of students' strategy stages, in particular, seemed to raise questions for teachers when reporting to parents. With reference to the broad range of understandings encompassed by a single strategy stage, one teacher asked:

> How do we communicate that through to the parents? And we're just talking about what we can put in the reports. You know, just to tell the parents "Look, they've made great progress but are still within this, they are close to transition, but it's not quite there." (Year 3–4 teacher, school B, interview)

Information for School-based Instructional leaders on Individual Teachers and Groups of Teachers

The focus on students' achievement data at school B included using it as a means of identifying possible development needs for teachers:

> I guess it can identify for the numeracy leaders who might be having some issues, like the teacher, and therefore maybe just through that we can then put the support in that we actually need. So it's used in a constructive supportive manner, it's not saying they haven't done well. (Senior management member, school B, interview)

A colleague commented:

> It also shows areas for PD [professional development] that we need to focus in on ... Obviously, for the numeracy leaders, it helps them to work out maybe teachers that they, or classes in particular, that they feel need extra support with either teacher knowledge or following the programme itself. (Senior management member, school B, interview)

So not only might it be possible to diagnose a need for development across the whole staff, but the data might also give an indication of support needed by individual teachers. Supposedly, it also has the potential to identify areas of strength of individuals and of the whole staff, although this was not reported in these two schools.

Limitations

This exploratory study focused on two urban, high-decile schools. Only a small number of survey and interview questions specifically focused on formative assessment in numeracy, although responses to a number of other questions included references to assessment. Further research will be needed to determine whether other types of schools that are working to embed the NDP share any characteristics of practice relating to formative assessment.

Discussion and Conclusion

The schools demonstrated the laying of all seven dimensions[11] of the foundations of school-based conditions helpful in sustaining the NDP. Tracking the laying of these conditions through the use of the diagnostic interview in the school enabled a view of how both schools got to the core of practice, enhancing the likelihood of deep-level changes both to individual teachers as well as to school-wide collective practices. This viewpoint also demonstrates the power of the diagnostic interview as a pedagogical tool for professional development. The evidence demonstrates that the diagnostic interview, as well as the data generated from its use, contributed to embedding the NDP in the schools studied.

The study suggests that professional development designers and implementers should pay attention to the power of well-designed assessment tools that embody underlying theories of professional development initiatives.

For Further Research

- Can the NDP be embedded in a school using forms of assessment other than the diagnostic interview?
- What is the relative importance of the actual choice of assessment tool and having systems in place to ensure its consistent, school-wide use?

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[11] See description under Analysis.

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The Impact of Two Professional Development Programmes for Numeracy "Pick Ups": Teachers' Perceptions of Valued Aspects

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This paper describes some of the effects of two in-service interventions for beginning teachers and teachers new to the materials and approaches of the Numeracy Development Projects (NDP). One was a university course focusing on the mathematics knowledge necessary for teaching, with associated in-class work, and the other was a School Support Services contracted delivery of a numeracy "pick-up" programme. The main focus of the first was on further developing the personal mathematics of participating teachers in conjunction with an introduction to the NDP, while the second focused on the various aspects of the NDP and developing an overall understanding of what it required of teachers. While both the university course and the pick-up programme used particular teaching approaches, there were a number of common delivery elements, one of which was in-class support. The researchers evaluated participants' perceptions of the usefulness of the intervention and the aspects they most valued in their learning. The participants found both programmes to be of value and helpful to their learning. However, there were differences in the specific aspects valued in each programme, and this has implications for the nature of future pick-up programmes.

Introduction

Since their beginnings in 2000, a key aspect of the Numeracy Development Projects (NDP) has been the development of teachers' professional capability with regard to teaching mathematics through a programme of professional development. This was initially provided to teachers when their schools were involved in the first phase of the implementation, which aimed to deliver the NDP to all New Zealand primary schools. By 2008, most primary schools had been involved in this professional development. In the early years of the NDP, the professional development was delivered initially to all the junior-syndicate teachers in a primary school and then, when that was completed, to all the senior-syndicate teachers. From then on, it was delivered to all teachers at a primary school at the same time.

As the initial phase neared completion in 2008, the NDP moved into a new phase, with a major focus on developing sustainability by providing longer-term, in-depth work with schools. In this new phase, all schools are being "revisited" by SSS to provide a three-year ongoing professional development programme based around school needs in numeracy that have been identified by both the school and SSS. A suggested focus for this new work relates to improving teachers' content knowledge, which has been acknowledged both nationally (Ward & Thomas, 2007) and internationally (for example, Ball, Thames, & Phelps, 2008) as being weak. However, it is not yet clear how this particular focus may best be addressed.

However, throughout the initial phase, teacher mobility and the employment of new, provisionally-registered teachers meant that some teachers at schools where the NDP had been implemented had not been exposed to the NDP's professional development. In cases where there was a complete staff turnover over a short period, none of the staff currently at a school might have received NDP professional development, even though the school had been involved previously.

To meet the needs of these teachers on an ongoing basis, a different form of intervention from the whole-school model used generally within the NDP was required, and so "pick-up" programmes were developed. These were provided through School Support Services (SSS) centres around the country and had a local flavour, reflecting the attitudes and experiences of the individual facilitators at each centre. Ongoing funding for pick-up programmes has become a stable feature of the Ministry of Education's NDP funding in recognition of the fact that, each year, a significant number of "new" teachers will require induction into the NDP.

In South Auckland, the Manurewa Enhancement Initiative (MEI) school cluster, funded by the Ministry of Education, developed their own pick-up intervention, combining a university course that had a content component with an enhanced pick-up programme that used SSS facilitators. This intervention built upon the previous involvement of The University of Auckland with MEI, in which the university provided professional development support for the MEI programme, whose goal was the development of "expert teachers" in mathematics.

This paper reports on teachers' evaluations of aspects of both the MEI and the SSS interventions.

Method

Participants

Participants for this study were drawn from two groups of teachers. As part of the professional development being offered through the MEI (a large-scale project designed to raise student achievement in schools in the Manukau area), one group of teachers was attending a course at The University of Auckland entitled "Understanding and Extending Mathematical Thinking". This group comprised beginning teachers (who were attending as part of their induction programme), returning teachers (those with experience who were returning to teaching), and overseas-trained teachers, all of whom had not been exposed to NDP approaches. Of the 30 teachers attending, 26 completed the evaluation forms in their last course session near the end of 2008.

The other group of participants was drawn from eight of the 15 groups of teachers participating in the Auckland SSS's induction programme for beginning teachers, returning teachers, and overseas-trained teachers, all of whom had not been exposed to NDP approaches. Of the 260 teachers (approximately) in this programme, 169 chose to participate in other aspects of the research during the sessions; of these, 34 completed the evaluation forms, which were emailed to all the teachers near the end of the year. The evaluations were emailed because the SSS programme sessions had all been completed at various times earlier in the year. The low response rate (20% of the participating group) suggests the need to use other ways of obtaining feedback in future studies. In contrast to the MEI group, 34 respondents represents a small subset of the total number of teachers in the SSS pick-up intervention, and due to the "self-selected" nature of the respondents, it is not clear how representative this sample is.

Materials and Procedure

The university course participants attended a year-long 12-session course in Understanding and Extending Mathematical Thinking, taught by a School of Science, Mathematics and Technology Education lecturer at a school in their area. The course had both academic and professional development content, which focused mainly on the mathematics of place value, fractions, and decimals as seen through the lens of young students as learners. The focus was therefore on how students construct mathematical knowledge, the conceptual pitfalls inherent in seemingly basic concepts, and ways in which mathematics could be presented to students so they could develop robust ways of thinking about fundamental concepts. There was a strong relationship to classroom practice, with an integral part of the course being "homework", which consisted of trialling course material with students and reporting back in the next session on what they had found. As well, the participants were introduced to the NDP and also received in-class support from SSS numeracy facilitators, who worked with the teachers in developing their classroom practice and helping them implement the NDP. To ensure a close alignment between the two components of the intervention, these facilitators also sat in on the course so that they could be aware of what the teachers were being presented with.

The SSS programme participants attended six professional development sessions, usually over two terms, conducted by numeracy facilitators at a school in their area. The sessions focused on the NDP and their implementation in schools and introduced teachers to the NDP "pink books" support material (Ministry of Education, 2006a–f; 2007; 2008) and their use as supplementary resources in developing students' learning. In addition, the participants received in-class support, with the facilitator and/or their school's lead teacher taking the sessions and working with them on their classroom practice. Although the lead teachers did not attend the SSS professional development sessions, they were generally involved in ongoing professional development meetings designed to help them support the teachers in their schools.

While the two interventions had similarities, there were significant differences. These differences included: a focus on academic and professional development for the MEI initiative versus a focus on professional development in the SSS programme; the degree to which the in-class support connected with the session work; the homework of the university course required the teachers to use the course material in their classrooms and feed back into later sessions, while the SSS sessions were more stand-alone, with teachers free to use material but with no integrated feedback mechanism; the teachers in the SSS programme had a variety of facilitators working with them, whereas the MEI group of teachers had a single lecturer and one of two facilitators working with them on an individual basis.

The evaluation form (see Appendix K, pp. 198–199: Professional Development Programme Evaluation) was designed to measure two main components: the sessions and the accompanying in-class support. These two components were separated to ensure that each was given equal consideration. They were rated, using a six-point Likert scale, in terms of the teacher's perception of how valuable and helpful each component had been and how worthwhile they perceived their attendance or involvement in meeting their own needs. This allowed the researchers to determine which of the components, if any, was more or less valued/helpful or worthwhile. Self-report data is usually expressed positively in evaluations of professional development programmes (Brown, 2004), so the researchers used a positively-packed six-point scale to allow for the differentiation of varying levels of positive agreement. In the analysis of the data, only the responses that indicated a stronger level of agreement than "agree" were considered as positive.

After consultation with all the lecturers and facilitators involved in the interventions, a set of 20 elements that covered the range of practices used in both interventions was identified for each component. The teachers were asked to identify, from a randomised grid arrangement for each set of 20 elements, up to three elements that they saw as being most helpful in learning to work more effectively with the students in their classrooms. For the session component, the elements were organised into five categories (see Appendix K, Part A). The first two categories contained parallel items that dealt with mathematics content and the learning progression of content (the rest were stand alone):

- personal mathematics knowledge and understanding (6 grid elements: 1a–1f, for example, 1d: "Improving my personal understanding of fractions");
- personal knowledge and understanding of students' learning progressions (6 grid elements: 2a–2f, for example, 2d: "Improving my personal understanding of students' learning progressions in fractions");

- aspects of development (4 grid elements: 3a–3d, for example, 3b: "Learning more about what I need to be doing next when teaching students");
- self and students as learners (2 grid elements: 4a–4b, for example, 4a: "Making me feel like a learner again");
- modelling of practice (2 grid elements: 5a–5b, for example, 5a: "The teaching practices used by the lecturer/facilitator").

The in-class component elements were organised into seven categories (see Appendix K, Part B). The first two categories focused on observations, the third on sharing aspects of practice, and the fourth on resources (the rest are self-explanatory):

- being observed with follow-up discussion with a variety of people (3 grid elements: 1a–1c, for example, 1a: "Being observed by, and having follow-up discussions with, a facilitator");
- observing a variety of people teaching (5 grid elements: 2a–2e, for example, 2d: "Observing a lead teacher teaching in a class");
- sharing aspects of practice with a variety of people (6 grid elements: 3a–3f, for example, 3a: "Sharing planning with the facilitator");
- receiving resources from a variety of people (3 grid elements: 4a–4c, for example, 4b: "Being given resources by a lead teacher");
- being able to choose the type of support received (grid element 5);
- discussing their own numeracy issues (grid element 6);
- release time to work with students (grid element 7).

The evaluation was given as late in the year as possible in an attempt to obtain a reflective evaluation and to allow time for the professional development to have had an influence on classroom practice.

The initial data analysis was carried out on the categories that grouped elements in some cases. However, as categories had between one and seven elements and the teachers were making multiple selections from the 20 elements, a second level of analysis was then carried out at the individual-element level. The two levels of analysis were then examined to compare the overall ratings by category against the ratings by individual elements.

Results

The teachers' responses for both aspects of the MEI and SSS intervention components were similar. The sessions were perceived to be of value and helpful in meeting the teachers' needs, with 23 out of 26 (88%) teachers offering a "moderately agree" rating or better for the MEI group and 32 out of 34 (94%) doing so for the SSS group (see Table 1). Attendance at sessions was similarly perceived as worthwhile, with 23 out of 26 (88%) offering a "moderately agree" rating or better for the MEI group and 32 out of 34 (94%) doing so for the SSS group.

Table 1
The Number of Responses Showing Perceived Value and Worth of Sessions and In-class Components of Both the MEI and the SSS Interventions

		Very strongly agree	Strongly agree	Moderately agree	Agree	Disagree	Strongly disagree	No response
MEI (n = 26)								
Sessions	Value	8	9	6	3	–	–	–
	Worth attending	5	15	3	3	–		–
In-class	Value	4	10	5	5	–	–	2
	Worth having	5	9	4	5	1	–	2
SSS (n = 34)								
Sessions	Value	8	17	7	2	–	–	–
	Worth attending	9	16	7	1	–	1	–
In-class	Value	4	13	6	4	1	–	6
	Worth having	5	11	8	3	1	–	6

These patterns are also evident, although less positive, in relation to the in-class support, where the type of support was perceived to be of value, with 19 out of 26 (73%) offering a "moderately agree" rating or better for the MEI group and 23 out of 34 (68%) doing so for the SSS group. The time involvement with the in-class support was perceived as being worthwhile, with 18 out of 26 (69%) offering a "moderately agree" rating or better for the MEI group and 24 out of 34 (71%) doing so for the SSS group (see Table 1), which closely matched results for the perceived value. There was, however, a small shift towards a less favourable view of the in-class components for both groups, which is evident in the overall distribution of responses.

The categories of session elements identified by the teachers as being of most help in their learning varied between the two groups (see Table 2). The largest variation was evident in the "personal mathematics knowledge" elements (MEI 25% and SSS 6%) and the "aspects of development" elements (MEI 33% and SSS 60%). This variation is most likely a natural consequence of the different foci of the interventions, with the MEI group focusing on both academic (mathematic content for teaching) and professional development (introduction to the NDP), while the SSS group focused primarily on professional development.

Table 2
The Percentage of Responses to Session Categories of Elements Perceived as Most Helpful for Both the MEI and the SSS Groups

Category	MEI (n = 26)	SSS (n = 34)
1[1]: Personal mathematics knowledge	25	6
2: Knowledge of students' learning progressions	16	14
3: Aspects of development	33	60
4: Self and students as learners	16	10
5: Modelling of practice	10	11

[1] The categories are numbered here for reference purposes only, to enable links to be made to grid elements within the text and the appendices.

The other three categories did not differ as much, with two of them ("knowledge of students' learning progressions" and "modelling of practice") being rated nearly the same by the two groups.

The number of responses to individual elements (see Table 3) confirm the pattern shown in Table 2, with the three most highly-rated elements being in category 3 ("aspects of development") for the SSS group, while the higher rated elements for the MEI group were primarily a mixture of the elements in categories 3 and 1 ("personal mathematics knowledge").

Four of the five elements that received the highest rating from both groups were the same (Table 3: 3a, 3b, 3d, 4b), although rated differently and in a different order for each group. These align with the overarching purpose of both interventions – developing teachers' professional capability with regard to teaching mathematics. The two elements rated at over 50% by the MEI group focus on students and their learning, while the one element rated at over 50% by the SSR group focuses on the teachers' knowledge and use of resources. The fifth of the five top elements focused on personal understanding of fraction content (1d) for the MEI group and on students' learning progressions in addition and subtraction (2b) for the SSS group.

A feature of the remaining responses is the higher response level for more elements (4) by the MEI group compared with the SSS group (1), with the elements selected by the MEI group relating primarily to content areas and learning progressions, which align with the focus of the course component of that intervention. This content focus of the MEI course is evident in the number of content-related elements identified overall and their more advanced level (multiplication and division, fractions, and decimals), compared with the one content-related element (addition and subtraction) identified by the SSS group.

Table 3

Session Category Elements[2] Perceived as Most Helpful for Both the MEI and SSR Groups (arranged in descending levels of percentage response [where the percentage response level is equal to or greater than 15%])

MEI (n = 26)		**SSS** (n = 34)	
		3d: Resources and how to use them	56%
3a: What students need to be doing next	54%[3]		
4b: What learning content is like for students	54%		
		3b: What to do next with/and for students	44%
		3a: What students need to be doing next	41%
3b: What to do next with/and for students	38%		
1d: Understanding of fractions	31%		
3d: Resources and how to use them	31%		
1c: Understanding of multiplication and division	27%		
5b: Modelling of teaching practices	27%		
		4b: What learning context is like for students	26%
		2b: Understanding students' progress in addition and subtraction	26%
1e: Understanding of decimals	23%		
2c: Understanding students' progress in multiplication and division	23%		
		5a: Teaching approaches as models	21%
2e: Understanding students' progress in decimals	15%		

2 See Appendix K, Part A (p. 198) for the full text of the grid elements.

3 The percentages add to more than 100% because each participant was permitted to identify up to three elements.

As for the session data, the categories of in-class elements identified by the teachers as being of most help in their learning (see Table 4) varied between the two groups. The categories, one of which was rated above 20% by one group, showed the greatest variation between groups. The largest variation between the two groups (roughly three times as great) was evident in the "being observed and follow-up discussion" category (MEI 23%, SSS 7%) and the "resource acquisition" category (MEI 6%, SSS 21%). The MEI higher rating in regard to observation and related discussion may have been influenced by this being a common professional development practice in the MEI schools in addition to its place within the MEI intervention. The SSS higher rating of resources possibly reflects the single focus of the SSS programme compared with the dual focus of the MEI intervention, but it may also reflect the SSS teachers' focus on resource acquisition in comparison with the MEI teachers' focus on pedagogical issues.

Table 4
The Percentage of Responses to In-class Categories of Elements Perceived as Most Helpful for Both the MEI and the SSS Groups

Category	MEI (n = 26)	SSS (n = 34)
1[4]: Being observed and follow-up discussion	23	7
2: Observing various people teaching	28	35
3: Sharing aspects of teaching practice	23	14
4: Resource acquisition	6	21
5: Choosing the type of support	0	1
6: Discussing own numeracy issues	10	11
7: Release time to work with students	3	6

There was a smaller variation between two other categories: the "observing various people teaching" category (MEI 28%; SSS a quarter higher at 35%) and the "sharing aspects of teaching practice" (SSS 14%; MEI more than a half higher at 23%).

The remaining three categories, containing more limited numbers of elements, were not rated differently to any extent, with two of them being rated nearly the same. However, one, "discussing own numeracy issues", was rated more highly than the other low categories (MEI 10%, SSS 11%), while "choosing the type of support" barely rated at all (MEI 0%, SSS 1%).

The number of responses to individual elements generally confirm the pattern in Table 4, except for the high rating of the single category 6 element concerning discussion focused on the teachers' own issues, which was rated first by the MEI group and second by the SSS group (see Table 5). This may be because this category contained only one element and teachers were able to choose multiple elements elsewhere. This lack of an aggregation possibility would reduce the overall rating for category 6.

[4] The categories are numbered here for reference purposes only, to enable links to be made to grid elements within the text and the appendices.

Table 5

In-class Category Elements[5] Perceived as Most Helpful for Both the MEI and SSR Groups (arranged in descending levels of percentage response [where the percentage response level is equal to or greater than 15%])

MEI (n = 26)		**SSS** (n = 34)	
6: Discussing own numeracy issues	38%[6]	2c: Observing a facilitator teaching	38%
1a: Observed by facilitator	35%		
		6: Discussing own numeracy issues	32%
		2a: Observing a facilitator teaching my class	21%
		4a: Resources from facilitator	21%
2e: Observing another teacher	19%		
		7: Release time for working one-to-one with students	18%
2c: Observing a facilitator teaching	15%		
3d: Sharing the teaching with a facilitator	15%		

In contrast to the session data, only two of the five highest-rated elements for both groups were the same (2c and 6), but each group also ranked the elements in a different order. Element 2c, "observing a facilitator teaching", was rated highest by the SSS group and fourth by the MEI group. However, both groups rated two category 2 elements in the top five, reflecting the overall rating for this category (MEI 28%, SSS 35%).

The presence of elements 1a, 2c, and 2e for the MEI group reflect this group's overall ratings for observation and being observed respectively. Similarly, the presence of element 4a in the SSS group's ratings reflects this group's overall category rating for resource acquisition. The SSS group also rated the single category 7 element (relating to release time) fifth, which was higher than for any of the observation elements.

The ratings of the remaining elements (not shown here) reflect the MEI group's higher overall ratings for categories 1, 2, and 3 and the SSS group's higher overall ratings for categories 2, 3, and 4.

Discussion

The data shows that each of the session and in-class components of both interventions were perceived to be of value and helpful in meeting teachers' needs, as were the time and energy expended in attending the meetings. All categories except the in-class "choosing the type of support" were selected by more than one teacher, and although ratings varied, they indicate the general relevance of most of the categories to both groups of teachers.

The data sets for the two interventions showed differences that reflected the groups' different foci: the much higher rating of the session content-related category and higher-level elements (for example, fractions and decimals) by the MEI group, and the SSS group's higher rating of the session "aspects of development" category. This was also evident in the in-class categories, with "being observed and discussing" rating more highly for the MEI group and "resource acquisition" rating more highly for the SSS group.

5 See Appendix K, Part B (p. 199) for the full text of the grid elements.

6 The percentages add to more than 100% as each participant could identify up to three elements.

Both groups of participants felt that their interventions were successful, with the MEI group having had a greater exposure to mathematics content issues. In terms of the suggested focus on content in phase two of the NDP, this raises questions about which intervention might be most appropriate and effective for pick-up teachers. The university course, which is a fundamental part of the MEI intervention, had a statistically significant positive effect on teachers' mathematical content knowledge and professional content knowledge (Ell, Lomas, Cheeseman, & Nicholas, 2008) and so could potentially play a useful part in the SSS pick-up programme and in the implementation of phase two more generally.

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Numeracy Development Projects' Patterns of Performance and Progress: National Pick-up Programme 2008

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The national pick-up programme provides professional development for teachers who are unfamiliar with the Numeracy Development Projects (NDP) and who are working in schools that have completed their initial involvement in the professional development programme. This paper reports on the performance and progress of year 0–6 students in the classes of teachers who participated in the NDP pick-up programme in 2008. The findings indicate that pick-up students in all year levels make good progress in numeracy, as measured against the Number Framework and in comparison with students in schools participating in the full-school NDP professional development.

Background

The New Zealand Numeracy Development Projects (NDP) were first implemented in 2000 with a pilot project, and each year, research reports and compendia papers have been written describing the performance and progress of students (for example, Thomas, Tagg, & Ward, 2003; Thomas & Tagg, 2004, 2005, 2006, 2007, 2008; Higgins, 2003; Higgins, Bonne, & Fraser, 2004; Young-Loveridge, 2004, 2005, 2006, 2007, 2008).

The NDP have now been implemented in nearly all New Zealand primary, intermediate, and composite schools. Many schools have had changes to staffing in the years since they participated in the professional development programme. These staff changes include new graduates, teachers returning from either overseas or from a break in their teaching careers, and teachers who have moved between schools within New Zealand. The national pick-up programme is targeted towards these new teachers in schools that have completed their initial involvement in the NDP. The teachers were clustered with new teachers from other schools wherever possible and provided with a workshop programme and at least one individual in-class modelling and discussion session. Provisionally-registered beginning teachers were provided with workshops that built on their preservice education.

This paper describes the performance and progress of year 0–6 students in the classes of teachers involved in the national pick-up programme in 2008.

Method

Participants

The results reported in this paper were obtained from the online numeracy database by downloading the data for all year 0–6 students in the accounts allocated to the national pick-up programme. Year 0–3 students were included for analysis if there was initial and final stage data for them for the additive, forward number word sequence (FNWS), place value, and basic facts domains. For the year 4–6 students to be included for analysis, they also needed to have initial and final stage data for the multiplicative, proportional, and fractions domains.

Tables 1 and 2 show the demographic profile of the 1818 students who met the criteria described above. These students are referred to as pick-up (PU) students throughout this paper.

Table 1
Demographic Profile of Year 0–3 Students

	Years 0–1		Year 2		Year 3	
Ethnicity	Male	Female	Male	Female	Male	Female
NZ European	54%	46%	48%	48%	60%	60%
Māori	13%	23%	15%	15%	18%	16%
Pasifika	13%	14%	19%	18%	8%	10%
Asian	12%	12%	6%	12%	6%	9%
Other	8%	5%	12%	7%	8%	6%
Total	234	196	241	257	180	145

Table 2
Demographic Profile of Year 4–6 Students

	Year 4		Year 5		Year 6	
Ethnicity	Male	Female	Male	Female	Male	Female
NZ European	52%	62%	58%	62%	40%	56%
Māori	16%	11%	16%	12%	14%	15%
Pasifika	13%	14%	14%	10%	22%	15%
Asian	7%	2%	7%	7%	9%	5%
Other	12%	10%	5%	8%	16%	9%
Total	121	125	108	98	58	55

Analysis

The performance of students was measured by comparing the achievement of students on the Number Framework at the initial and final assessments. Four main methods have been used to analyse and report findings from the NDP. Changes in percentages of students at each stage have been reported for all domains of the Number Framework. Progress made by students from each initial stage has been compared. Effect sizes have been used both to assess the impact of the NDP and to compare demographic subgroups (ethnicity, decile, and gender). The research has also compared the final results of one year level with the initial results of the next year level.

All the tables in this paper (including the tables in Appendix L, pp. 200–221) show rounded percentages. Percentages less than 0.5% are therefore shown as 0%, and where there are no students represented, the cell is left blank. Due to rounding, percentages in some tables may not total to 100.

Effect sizes, where used, have been calculated by dividing the average difference between the mean stages of two groups by the pooled standard deviation of the two groups. For the purpose of this paper, effect sizes of 0.2 or less are described as small, effect sizes between 0.2 and 0.8 are described as medium, and effect sizes of 0.8 or more are described as large (Cohen, cited in Coe, 2002).

Findings

The findings discussed in this paper are divided into four sections. The first two sections describe the performance and progress on the strategy domains of students in year 0–3 and 4–6 respectively. For year 0–3 students, the additive domain is used to compare results, while for year 4–6 students, the multiplicative domain is used. The performance of each year level and the progress they made is discussed. The effect sizes for the mean stage gains made by demographic subgroups (ethnicity and gender) are reported. The third section reports on the performance of students on the knowledge domains. For year 0–3 students, results are reported for the forward number word sequence (FNWS), place value, and basic facts domains. For years 4–6, the results for the fractions domain are reported instead of those for the FNWS domain.

Curriculum level expectations cited in the NDP information section of the nzmaths website describe criteria for rating students as "at risk", "cause for concern", "achieving at or above expectations", or "high achievers" (Ministry of Education, n.d.). Students rated as at risk are described as "those who are sufficiently below expectations that their future learning in mathematics is in jeopardy", while those rated as cause for concern are below expectations, but "it is reasonable to expect classroom teachers to be able to move them to the expected stage". In each table, the cells representing those students who are performing at or above NDP expectations at the final assessment are shaded.

The progress of the PU students is compared with complete national results from 2008 as downloaded from the online database on January 27 2009. These national-results students are referred to as NDP students throughout this paper.

Performance and Progress of Year 0–3 Students on the Additive Domain

Performance

Table 3 shows the impact of the NDP on the performance on the additive domain of year 0–3 students. The percentages of students at the lower stages of the Number Framework decreased, and the percentages of students at the higher stages of the Framework increased.

Table 3
Performance of Year 0–3 Students on the Additive Domain

	Years 0–1		Year 2		Year 3	
	Initial	Final	Initial	Final	Initial	Final
0: Emergent	14%	1%	1%	0%		
1: One-to-one counting	23%	5%	10%	1%	6%	1%
2: Counting from one on materials	48%	41%	44%	17%	17%	2%
3: Counting from one by imaging	11%	30%	23%	20%	20%	7%
4: Advanced counting	3%	21%	17%	46%	38%	47%
5: Early additive part–whole	0%	2%	5%	14%	17%	36%
6: Advanced additive part–whole				2%	2%	6%
7: Advanced multiplicative part–whole						2%
N =	430	430	498	498	325	325

Table 3 shows that the percentage of year 0–1 students still rated at the emergent and one-to-one counting stages and below end-of-year curriculum level expectations decreased from 37% to 6%. The percentage of students who were able to at least count on or count back (advanced counting) to solve addition and subtraction problems increased from 3% to 21%. The percentage of year 2 students who were at least advanced counting (stage 4) and therefore meeting end-of-year expectations increased from 22% to 62%. The percentage of year 3 students who were at least advanced counting (stage 4) and therefore meeting end-of-year expectations increased from 57% to 91%.

The final results of the year 0–1 students show that the improvement is greater than that expected over time alone, with 53% of year 0–1 students achieving at or above stage 3 at the end of the year compared with the 45% of year 2 students who were initially at these stages. There was also a lower percentage (6%) of year 0–1 students remaining at the emergent and one-to-one counting stages compared with the 11% of year 2 students who were initially at these stages. This trend is also observed in the final year 2 student results, with 62% of students rated as at least stage 4, compared with 57% of year 3 students at the start of the year. In previous research, similar results have been found when comparing final results of one year level with the initial results of the next year level (Young-Loveridge, 2004, 2005).

Progress

Figures 1–3 show the percentage of year 0–3 students gaining stages on the additive domain, linked to their initial stage. The lower stages are smaller than the higher stages, so it is easier to make progress through the lower stages than the higher stages. The findings show that, in each year level, the percentage of students who made progress tended to be greater at the lower initial stages. These results are consistent with findings from the NDP, which have shown that the students assessed at lower initial stages made greater stage gains than students who were initially at higher Number Framework stages (Thomas & Ward, 2002; Thomas, Tagg, & Ward, 2003; Thomas & Tagg, 2004).

Figure 1 compares the progress of year 0–1 PU students on the additive domain with that of NDP students. It shows that over 90% of year 0–1 students initially rated at stage 0 or 1 moved up at least one stage, while over half of students initially rated at stages 2 or 3 moved up at least one stage. From all starting stages, the percentages of PU students making gains are similar to those of year 0–1 NDP students.

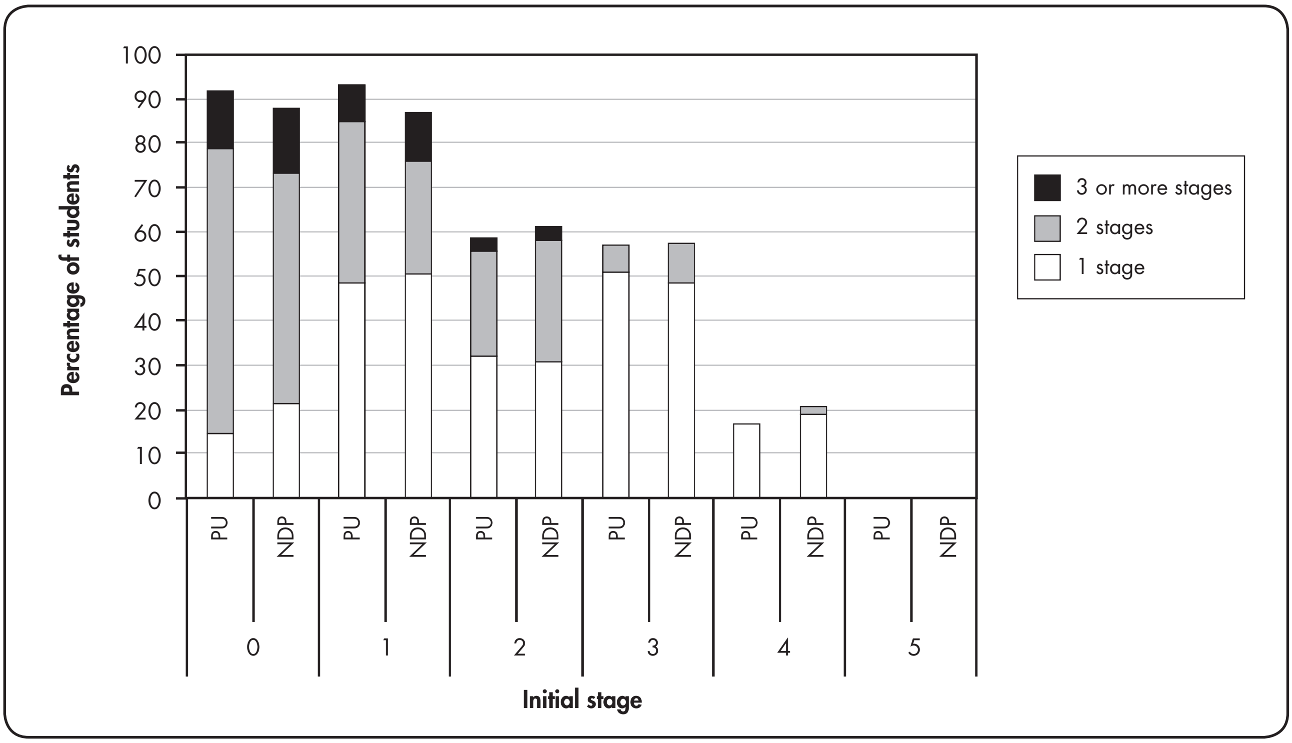

Figure 1. Number of stages gained from initial additive stage for year 0–1 students

The general trend of a larger percentage of students making progress from lower initial stages than from higher stages continues with the year 2 students (Figure 2). Although the numbers of stages gained fluctuates over the first three stages, the trend of lower total proportions of students gaining at least one stage from higher initial stages is clear. The percentages of students making gains are again similar to those of NDP students, although a lower proportion of year 2 PU students initially rated at stage 3 made progress (68%) compared with NDP students (81%).

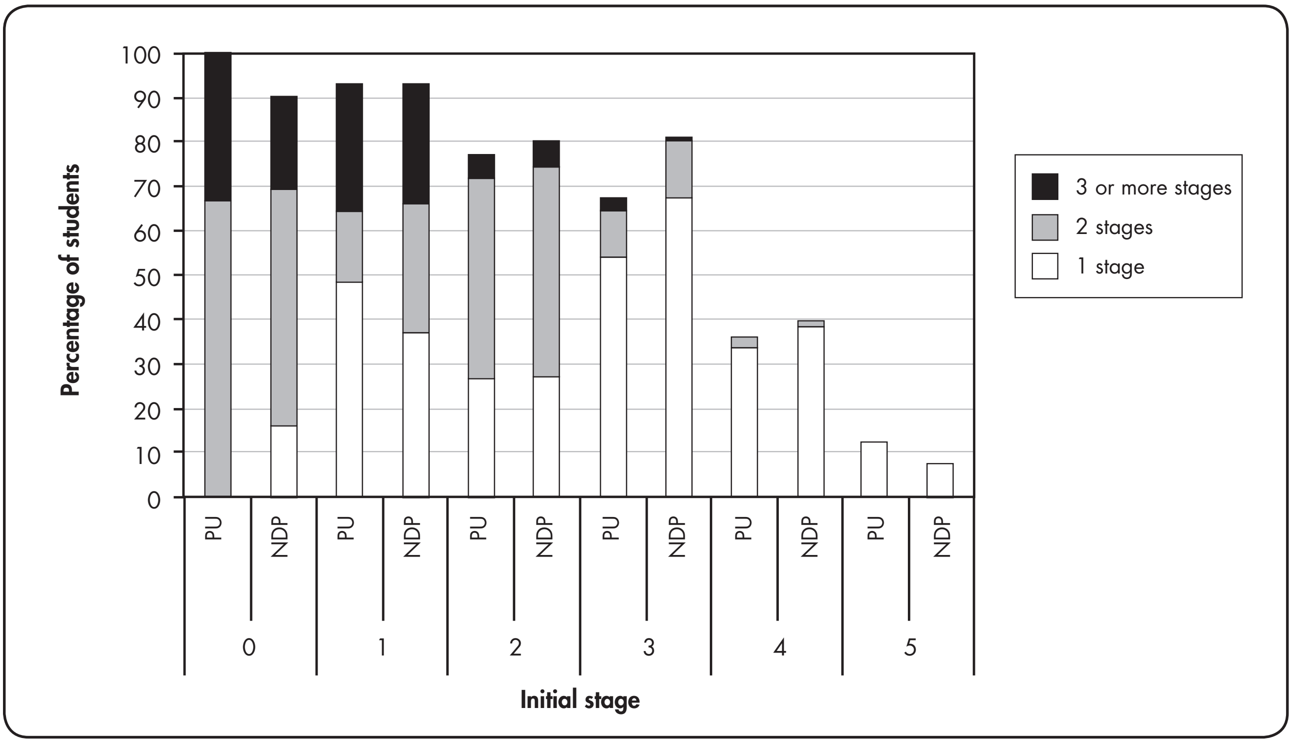

Figure 2. Number of stages gained from initial additive stage for year 2 students

As shown in Figure 3, over 80% of year 3 students initially at stages 1, 2, or 3 moved up at least one stage, with 83% of students initially at stage 1 and 65% at stage 2 moving two or more stages. Again, a larger percentage of students initially at these lower stages made stage gains than did students initially at stages 4 and 5. There is little difference in progress made between the results of PU students and NDP students, with the differences in percentages of students making gains from each stage less than 10%.

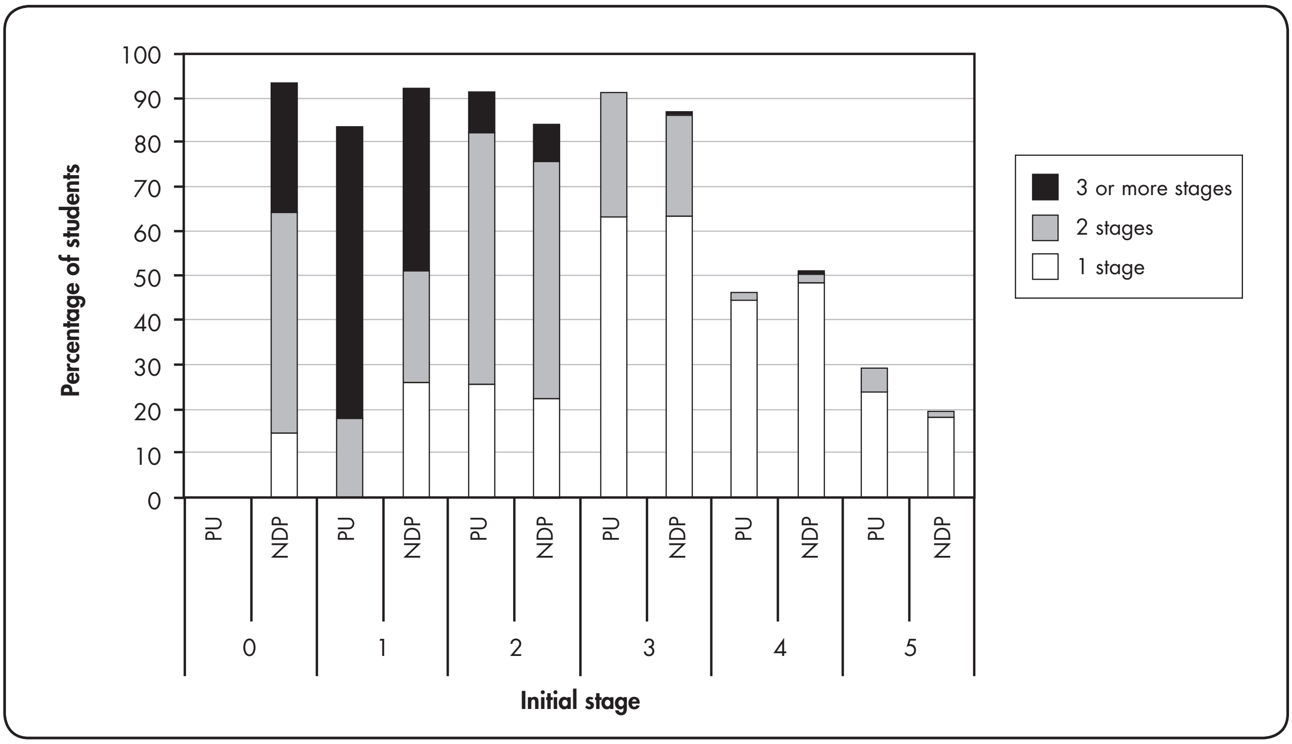

Figure 3. Number of stages gained from initial additive stage for year 3 students

Effect sizes for mean gains made on the additive domain by demographic subgroups of year 0–3 students

The tables in this section show the mean initial and final stages on the additive domain for each demographic subgroup of students in years 0–3. Mean gains and effect sizes for the impact of the NDP are also shown. The effect sizes for the impact of the NDP on all the demographic subgroups of year 0–1 students are described as large (Table 4). There was very little difference between the ethnic groups and genders. The largest difference is between the Māori students (0.97) compared with the New Zealand European students (1.14). The effect size of 1.14 for the New Zealand European subgroup is the largest size recorded. This suggests that the NDP had the largest impact on this demographic group.

Table 4
Effect Sizes for Gains Made on the Additive Domain by Subgroups of Year 0–1 Students

	Mean Initial Stage	Mean Final Stage	Mean Gain	Effect Size
NZ European	1.77	2.83	1.06	1.14
Māori	1.61	2.56	0.95	0.97
Pasifika	1.33	2.28	0.95	0.99
Male	1.64	2.70	1.06	1.08
Female	1.70	2.74	1.04	1.09
All students	1.67	2.72	1.05	1.08

Table 5 shows that the effect sizes for the impact of the NDP on all the demographic subgroups of the year 2 students are again described as large, although they are generally not as large as those for the year 0–1 students. There is again little difference between the ethnic groups and genders. The Pasifika subgroup had the highest effect size (1.01), with the lowest being for Māori (0.83).

Table 5
Effect sizes for Gains Made on the Additive Domain by Subgroups of Year 2 Students

	Mean Initial Stage	Mean Final Stage	Mean Gain	Effect Size
NZ European	2.71	3.64	0.93	0.93
Māori	2.50	3.41	0.91	0.83
Pasifika	2.49	3.47	0.98	1.01
Male	2.63	3.64	1.01	0.94
Female	2.60	3.56	0.96	0.95
All students	2.61	3.60	0.99	0.95

Table 6 shows large effect sizes for the impact of the NDP on all the demographic subgroups of year 3 students. The demographic subgroups all had similar effect sizes, with the exception of the Pasifika students, whose effect size of 1.11 was 0.26 higher than the next highest rating ethnicity, New Zealand European students.

Table 6
Effect Sizes for Gains Made on the Additive Domain by Subgroups of Year 3 Students

	Mean Initial Stage	Mean Final Stage	Mean Gain	Effect Size
NZ European	3.52	4.42	0.90	0.85
Māori	3.31	4.13	0.82	0.80
Pasifika	3.52	4.38	0.86	1.11
Male	3.51	4.45	0.94	0.86
Female	3.52	4.34	0.82	0.82
All students	3.51	4.40	0.89	0.84

Performance and Progress of Year 4–6 Students on the Multiplicative Domain

Performance

Table 7 shows the impact of the NDP on the performance of year 4–6 students on the multiplicative domain. As expected, the percentages of students at the lower stages of the Number Framework decreased and the percentages of students at the higher stages of the Framework increased.

Table 7
Performance of Year 4–6 Students on the Multiplicative Domain

	Year 4		Year 5		Year 6	
	Initial	Final	Initial	Final	Initial	Final
Not rated	15%	6%	6%	0%	1%	0%
2–3: Counting from one	14%	4%	5%	2%	4%	0%
4: Advanced counting	44%	34%	37%	19%	17%	6%
5: Early additive part–whole	19%	35%	31%	34%	35%	23%
6: Advanced additive part–whole	7%	19%	17%	33%	34%	42%
7: Advanced multiplicative part–whole	1%	3%	3%	10%	10%	27%
8: Advanced proportional part–whole		0%	0%	1%	0%	1%
N =	246	246	206	206	113	113

Table 7 shows that the percentage of year 4 students still rated at the advanced counting stage or lower and below the end-of-year curriculum level expectations decreased from 73% to 44%. The percentage of year 5 students who were at least early additive (stage 5), and therefore meeting end-of-year expectations, increased from 51% to 78%. The percentage of year 6 students who were at least advanced additive (stage 6), and therefore meeting end-of-year expectations, increased from 44% to 70%.

The final results of the year 4–6 students show that the improvement is again greater than that expected over time alone. The percentage of year 4 students achieving at or above the stage 5 at the end of the year is 57%, compared with the 51% of year 5 students who were initially at these stages. This trend is less evident when the final year 5 student results are compared with the initial year 6 results, with the two sets of results being very similar.

Progress

Figures 4–6 show the percentage of year 4–6 students gaining stages on the multiplicative domain of the Number Framework linked to their initial stage. The findings show that, in each year level, the percentage of students who made progress tended to be greater at the lower initial stages. The exception to this is the fact that fewer of the year 4 students initially not rated made progress than of those students initially rated at stages 2–3. Figure 4 shows that at least 50% of year 4 students with an initial stage below 6 moved up at least one stage. The percentages of PU students who made gains were similar to those of NDP students. (Note: There were only two PU students initially rated at stage 7, so the 50% who made a gain represents only one student.)

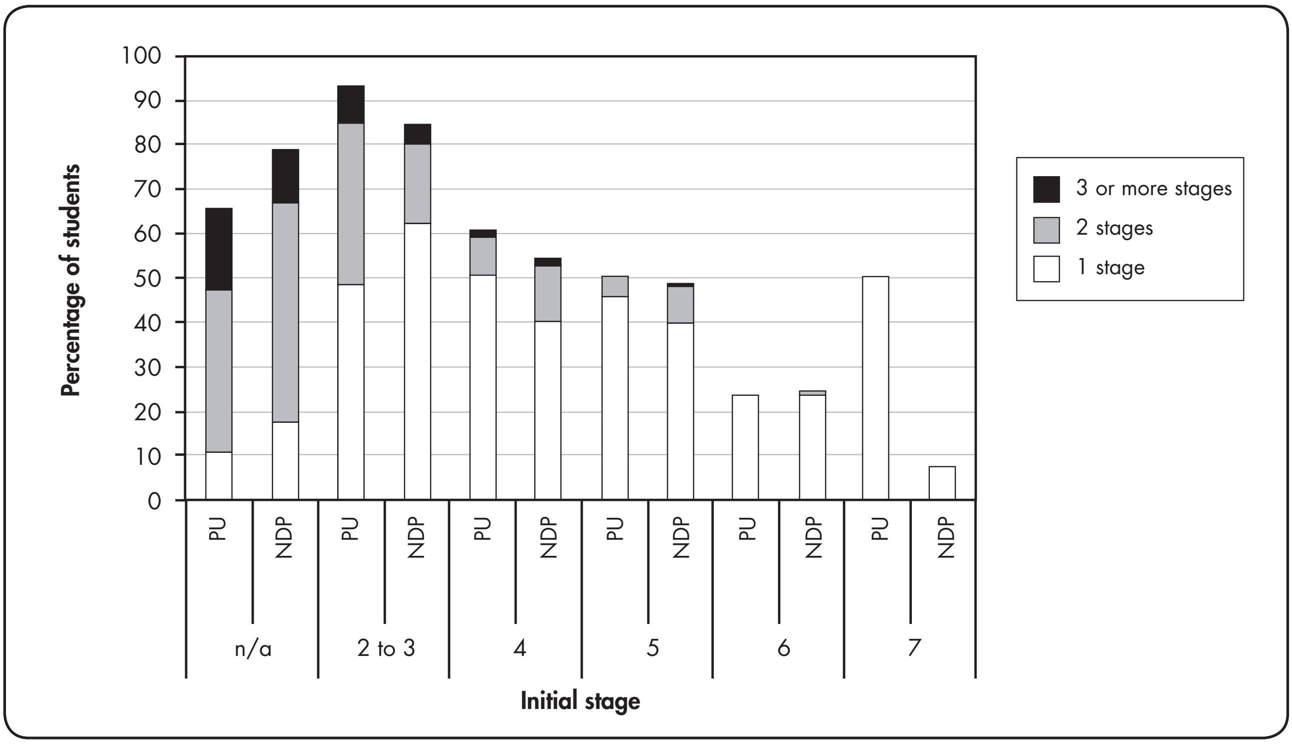

Figure 4. Number of stages gained from initial multiplicative stage for year 4 students

Figure 5 shows that over half of the year 5 PU students initially rated below stage 6 made gains of at least one stage. Seventy percent of PU students initially rated at stages 2–3 made progress, compared with 82% of NDP students. However, higher proportions of PU students initially not rated or rated at stages 5 or 6 made gains (92%, 62%, and 42%, compared with 81%, 54%, and 29% respectively).

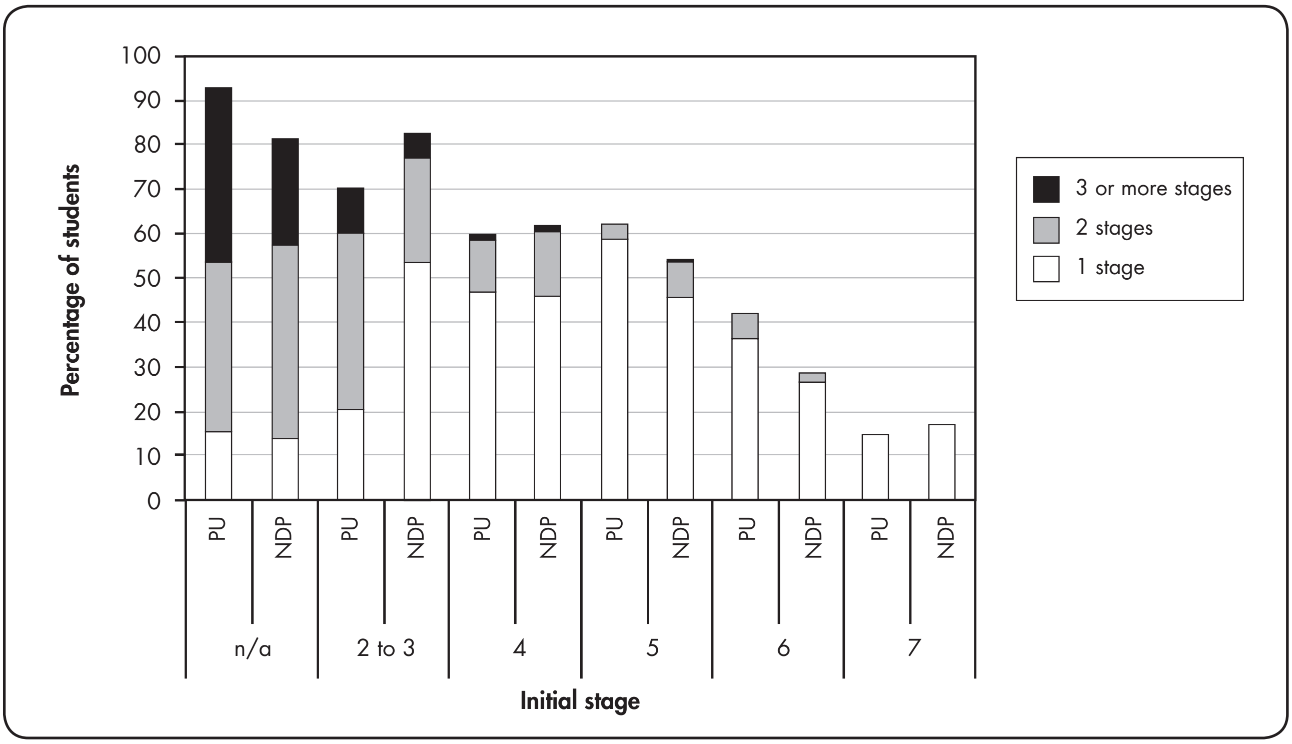

Figure 5. Number of stages gained from initial multiplicative stage for year 5 students

Figure 6 shows that over 60% of year 6 PU students initially rated below stage 6 made gains of at least one stage, with all students initially rated below stage 4 making gains. From all starting stages apart from stage 7, at least as high a proportion of PU students made gains as of NDP students. Only one of the 11 year 6 PU students initially rated at stage 7 moved to stage 8 by the end of the year.

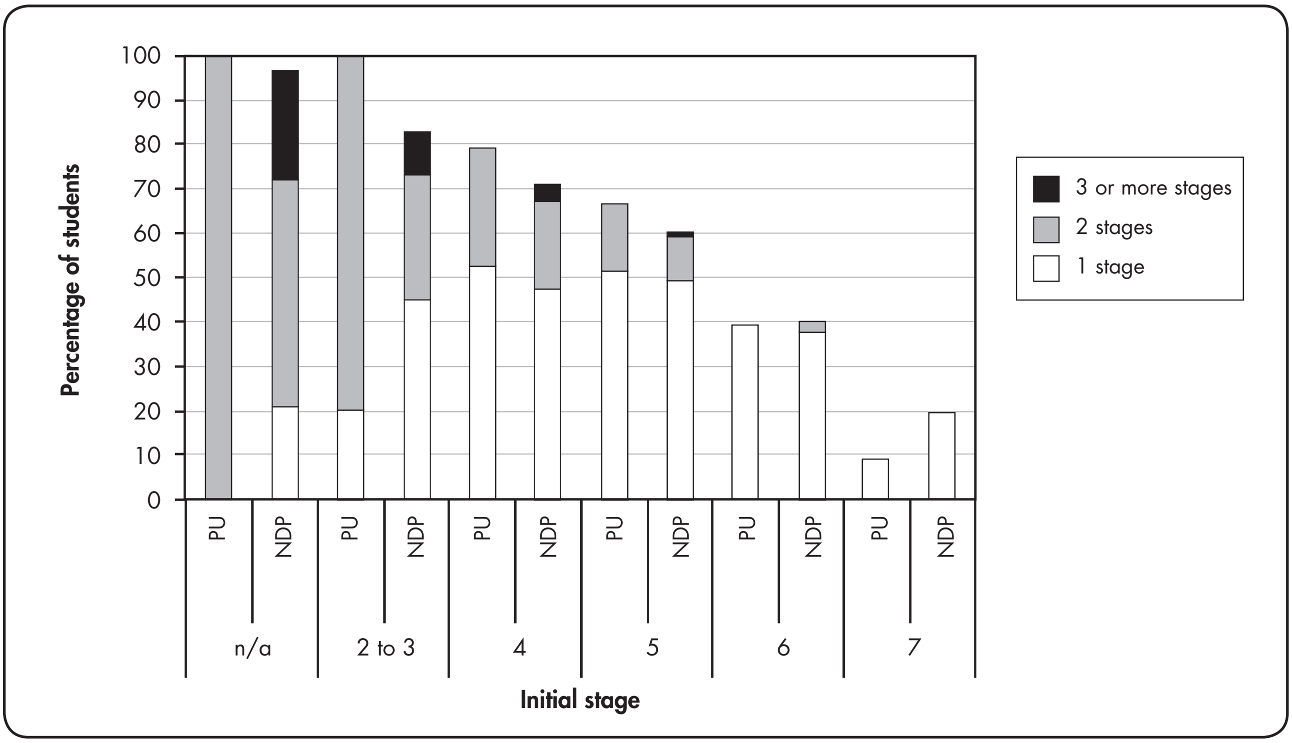

Figure 6. Number of stages gained from initial multiplicative stage for year 6 students

Effect sizes for mean gains made on the multiplicative domain by demographic subgroups of year 4–6 students

The tables in this section show the mean initial and final stages on the multiplicative domain for each demographic subgroup of students in years 4–6. Mean gains and effect sizes for the impact of the NDP are also shown. The effect sizes for the impact of the NDP on all the demographic subgroups of year 4 students are described as medium (Table 8). There was very little difference between the ethnic groups and genders. The largest effect size (0.69) and mean gain (1.26) were recorded by Pasifika students, although this is likely to be due at least in part to their low mean initial stage.

Table 8
Effect sizes for Gains Made on the Multiplicative Domain by Subgroups of Year 4 Students

	Mean Initial Stage	Mean Final Stage	Mean Gain	Effect Size
NZ European	3.77	4.78	1.00	0.66
Māori	3.06	4.11	1.05	0.65
Pasifika	2.64	3.89	1.26	0.69
Male	3.63	4.70	1.07	0.61
Female	3.42	4.40	0.98	0.67
All students	3.52	4.55	1.03	0.63

Table 9 shows that the effect sizes for the impact of the NDP on all the demographic subgroups of the year 5 students are again described as medium, with the exception of Māori students, for whom the effect size was 0.82, which is described as a large effect. The Pasifika subgroup had the highest gain in mean stage size (1.12), with the lowest being for New Zealand European students (0.83).

Table 9
Effect sizes for Gains Made on the Multiplicative Domain by Subgroups of Year 5 Students

	Mean Initial Stage	Mean Final Stage	Mean Gain	Effect Size
NZ European	4.46	5.29	0.83	0.63
Māori	3.84	4.91	1.07	0.82
Pasifika	4.24	5.36	1.12	0.75
Male	4.63	5.48	0.85	0.70
Female	4.22	5.09	0.87	0.62
All students	4.43	5.29	0.86	0.65

Table 10 shows a large effect size (1.00) for the impact of the NDP on year 6 Māori students. The effect size for females was also large (0.80), while the effect sizes for all other students were medium. In a result contrasting with all other year levels, Pasifika students had the lowest effect size (0.24) and mean gain (0.26).

Table 10

Effect sizes for Gains Made on the Multiplicative Domain by Subgroups of Year 6 Students

	Mean Initial Stage	Mean Final Stage	Mean Gain	Effect Size
NZ European	5.30	5.96	0.67	0.71
Māori	4.84	5.81	0.97	1.00
Pasifika	5.50	5.76	0.26	0.24
Male	5.47	6.09	0.62	0.64
Female	4.94	5.78	0.85	0.80
All students	5.21	5.94	0.73	0.70

Performance of Students on the Knowledge Domains

This section describes the performance of the students on the knowledge domains of the Number Framework. Results are reported for all students on the place value and basic facts domains because these are key areas of knowledge for students at all levels. Results for the forward number word sequence (FNWS) domain are only provided for year 0–3 students because by the end of year 3, the majority of students are in the top two stages of this domain, limiting the usefulness of further analysis. Similarly, results on the fractions domain are only provided for year 4–6 students because the majority of year 0–3 students are not assessed on this domain.

Performance of year 0–3 students on the forward number word sequence domain

Table 11 shows the initial and final results for year 0–3 students on the FNWS domain. Within each year level, the percentage of students at the lower stages decreased and the percentage at the higher stages increased. The percentage of students able to identify the number after a given number up to at least 100 (stage 4) increased from 11% to 43% for year 0–1 students, from 45% to 76% for year 2 students, and from 80% to 96% for year 3 students. The final results of year 3 students show that 70% of students were able to at least identify the number after any number up to 1000 (stage 5) and 16% were able to identify the number after any number up to 1 000 000 (stage 6).

Table 11

Performance of Year 0–3 Students on the FNWS Domain

	Year 0–1		Year 2		Year 3	
	Initial	Final	Initial	Final	Initial	Final
0: Emergent	7%	1%	1%			
1: Initial to 10	22%	4%	6%	0%	1%	0%
2: To 10	35%	16%	16%	4%	5%	0%
3: To 20	25%	35%	32%	19%	14%	4%
4: To 100	10%	39%	35%	46%	42%	26%
5: To 1000	1%	4%	10%	27%	33%	54%
6: To 1 000 000		0%		3%	5%	16%
N =	430	430	498	498	325	325

Performance of year 4–6 students on the fractions domain

Table 12 shows the initial and final results for year 4–6 students on the fractions domain. Again, within each year level the percentage of students at the lower stages decreased and the percentage at the higher stages increased. The percentage of students able to at least order unit fractions (stage 5) increased from 21% to 58% for year 4 students, from 44% to 78% for year 5 students, and from 58% to 83% for year 3 students. The final results of year 6 students show that 34% of students were able to at least coordinate numerators and denominators (stage 6) and 14% were able identify equivalent fractions (stage 7).

Table 12
Performance of Year 4–6 Students on the Fractions Domain

	Year 4		Year 5		Year 6	
	Initial	Final	Initial	Final	Initial	Final
Not assessed	16%	6%	6%	0%	1%	0%
2–3: Non-fractions	39%	11%	21%	4%	13%	1%
4: Assigns unit fractions	22%	26%	28%	17%	27%	16%
5: Orders unit fractions	19%	48%	35%	54%	42%	49%
6: Co-ordinates numerators/denominators	2%	7%	5%	14%	12%	20%
7: Equivalent fractions	0%	2%	2%	7%	4%	12%
8: Orders fractions	0%	1%	2%	3%	0%	2%
N =	246	246	206	206	113	113

Performance of year 0–6 students on the place value domain

Tables 13 and 14 show the place value domain results for year 0–3 and year 4–6 students respectively. The results show a similar pattern to other domains, with the percentages of students at the lower stages decreasing between the initial and final results within each year level and the percentages at the higher stages increasing.

Table 13
Performance of Year 0–3 Students on the Place Value Domain

	Years 0–1		Year 2		Year 3	
	Initial	Final	Initial	Final	Initial	Final
0–1: Counts by ones	45%	12%	12%	1%	3%	1%
2: Counts in ones	51%	55%	55%	29%	25%	6%
3: Counts in fives and ones	1%	12%	7%	12%	10%	10%
4: 10s to 100, orders to 1000	4%	20%	26%	51%	52%	55%
5: 10s to 1000, orders to 10 000	0%	1%	0%	5%	9%	24%
6: 10s, 100s, 1000s, orders whole numbers				1%	0%	5%
7: Tenths in and orders decimals						0%
8: Tenths, hundredths, and thousandths						
N =	430	430	498	498	325	325

Table 14
Performance of Year 4–6 Students on the Place Value Domain

	Year 4		Year 5		Year 6	
	Initial	Final	Initial	Final	Initial	Final
0–1: Counts by ones	1%		0%	0%	0%	0%
2: Counts in ones	9%	2%	3%	1%	0%	0%
3: Counts in fives and ones	10%	4%	4%	2%	2%	0%
4: 10s to 100, orders to 1000	63%	52%	56%	29%	43%	19%
5: 10s to 1000, orders to 10 000	15%	32%	26%	50%	34%	42%
6: 10s, 100s, 1000s, orders whole numbers	0%	9%	8%	11%	18%	29%
7: Tenths in and orders decimals	0%	0%	1%	6%	4%	8%
8: Tenths, hundredths, and thousandths	1%	1%	1%	1%	0%	3%
N =	246	246	206	206	113	113

Performance of year 0–6 students on the basic facts domain

Tables 15 and 16 show the results on the basic facts domain for year 0–3 and year 4–6 students respectively. As for all other domains, the percentages of students at the lower stages decreased between the initial and final results within each year level and the percentages at the higher stages increased.

Table 15
Performance of Year 0–3 Students on the Basic Facts Domain

	Years 0–1		Year 2		Year 3	
	Initial	Final	Initial	Final	Initial	Final
0–1: Non-grouping	84%	48%	48%	13%	18%	2%
2: Facts to 5	11%	27%	28%	21%	18%	5%
3: Facts to 10	3%	15%	9%	20%	18%	11%
4: Within 10, doubles and teens	2%	8%	14%	39%	34%	42%
5: Addition, multiplication for 2, 5, 10	0%	1%	1%	4%	7%	31%
6: Subtraction and multiplication		0%	0%	2%	3%	5%
7: Division				1%	2%	3%
8: Common factors, multiples						
N =	430	430	498	498	325	325

Table 16
Performance of Year 4–6 Students on the Basic Facts Domain

	Year 4		Year 5		Year 6	
	Initial	Final	Initial	Final	Initial	Final
0–1: Non-grouping	3%	2%	1%	1%	0%	0%
2: Facts to 5	10%	7%	3%	1%	1%	0%
3: Facts to 10	10%	23%	6%	2%	0%	1%
4: Within 10, doubles and teens	41%	43%	27%	13%	9%	4%
5: Addition, multiplication for 2, 5, 10	25%	21%	35%	42%	44%	30%
6: Subtraction and multiplication	9%	3%	19%	26%	29%	34%
7: Division	2%	1%	6%	13%	16%	29%
8: Common factors, multiples		2%	2%	2%	1%	3%
N =	246	246	206	206	113	113

Concluding Comment and Key Findings

The results from the 2008 national pick-ups programme indicate that year 0–6 PU students are making good progress as measured against the Number Framework and in comparison with NDP students. The findings indicate that, at each year level, the percentage of students rated at the lower stages of each domain decreased between the initial and final assessments and the percentage of students at the higher stages increased. This pattern was observed on the additive, forward number word sequence, place value, and basic facts domains for year 0–3 students, and on the multiplicative, fractions, place value and basic facts domains for year 4–6 students.

Consistent with previous NDP findings, in each year level, the percentage of students making progress on the Number Framework was greater at the lower stages than at the higher stages.

Effect sizes were used to measure the impact of the NDP on the demographic subgroups. At each year level except year 6, the effect sizes for the impact of the NDP on the various demographic subgroups were similar (within a range of no more than 0.2). In year 6, the effect size for Pasifika students was only 0.24, compared with 1.00 for Māori students. The effect sizes for the impact of the NDP on year 0–3 students on the additive domain are considered large. The effect sizes for the impact of the NDP on year 4–6 students on the multiplicative domain are in almost all instances considered medium.

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Do Groupies Do Better? Perceptions of Practising Teachers Undertaking Tertiary Study in Mathematics

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In 2005, the Ministry of Education began investigating ways in which the teachers who had participated in the teacher development programme could be supported in maintaining and extending their knowledge and expertise. This paper describes the perceptions of a group of teachers whose tertiary studies were partially funded through a Ministry initiative designed to help sustain the gains of the NDP. The study found that group support, unexpectedly, was not identified by the participating teachers as important for success. Despite finding the papers valuable in meeting many of their immediate and long-term needs, many teachers identified issues such as the lack of time available for concentrated study and variable levels of support given by their schools as impediments to their study.

Introduction

Since the introduction of the Numeracy Development Projects (NDP) in 2000, almost all teachers in primary and intermediate schools have undertaken the extended school-based development in teaching mathematics that forms the basis of the NDP. These facilitated workshops and in-class support sessions focus on improving student performance in mathematics by improving the professional capability of teachers.

Evaluations have consistently shown that teachers' confidence and professional capability have improved substantially as a result of their involvement in the NDP (for example, Young-Loveridge, 2004). However, most teachers spent only two years in the professional development programme, and it soon became evident that for many teachers this was simply not enough time to fully understand the NDP and implement it with all their students (Cheeseman, 2006).

By 2005, the issue of sustainability had become increasingly important:

> The NDP is moving into a phase in which the emphasis is not only on improving the teaching and learning of mathematics in New Zealand schools but also on enhancing the capacity of schools to sustain and build on that learning. (Ministry of Education, 2005, p. 4)

A response to this move was an initiative developed by the Ministry of Education to provide funding support for selected primary and intermediate school teachers to undertake approved tertiary study as a strategy for sustaining high-quality mathematics teaching. The relationship of the scheme to the NDP was clearly explained in the Ministry's announcement on the nzmaths website in November 2006:

> The scheme is part of the Ministry of Education's efforts to further develop teachers' pedagogical content knowledge and understanding of mathematics education. This scheme builds on from the work started with the Numeracy Development Projects by supporting schools efforts to raise the achievement of all their students through increasing the expertise and knowledge of their teachers. (Retrieved November 2006 from www.nzmaths.co.nz)

The initiative provided for approximately 600 teachers each year to be supported to study a mathematics education paper at graduate or post-graduate level. Tuition fees were to be paid for, half by the Ministry of Education and half by the teacher's school. The scheme did not provide for release from teaching duties.

In 2007, there were 108 applicants, all of whom were granted funding. The low number of applicants meant that the criteria for selection outlined on the website did not need to be applied. The geographical spread was wide and roughly proportionate to that of all New Zealand teachers. Seventy percent of the applicants were lead teachers of numeracy within their schools. Almost half applied with others from their school (in groups of between two and five teachers). Class levels taught covered the full range of primary and intermediate school classes, and teacher applicants had a wide range of teaching experience (with 20+ years being the most represented) and covered the full range of experience in the NDP (with 50% starting in 2004 or 2005).

Faculties and colleges of Education from six New Zealand universities provided approved papers. Many of these were closely related to the NDP work already undertaken by the teachers in their schools, and two universities provided opportunities for teachers to study at a local school site rather than on the university campus. A number of papers were available through distance education.

The Study

This paper is the initial part of a wider study that seeks to investigate how teachers undertaking tertiary study in mathematics can best be supported. It reports on the results of a questionnaire sent to teachers who undertook tertiary study funded through the Ministry scheme in 2007 while continuing to teach. The questionnaire sought information relating to sources of support for these teachers, asking them to identify the extent to which they felt each of a range of factors had provided support for their studies and to make general comments relating to that support. It was anticipated that the information from the questionnaires would provide direction for follow-up, in-depth interviews. The overall study can thus be described as a mixed-mode design, drawing on both quantitative and qualitative approaches (Kervin, Vialle, Herrington, & Okely, 2006).

The questionnaires were sent to 98 teachers, and 32 responses were received. Response rates for each of the universities ranged from 20% to 50%. All but one of the respondents indicated that they had passed the papers in which they had enrolled.

Results and Discussion

Many research studies have identified a number of general principles that encourage teachers to adopt innovative practices (for example, Hill, Hawk, & Taylor, 2002; Corcoran, 1995). Most of these included suggestions that:

- effective professional development should take place over an extended period of time and as close as possible to the teacher's own working environment;
- groups of teachers should be involved, rather than individuals from a school;
- the teachers should be fully supported by the school administration;
- schools should use the services of a consultant.

The in-school teacher development associated with the NDP was carefully designed to include all of these principles (Bobis, Clarke, Clarke, et al., 2005), and succeeded in developing the "communities of learners" described by Birman, Desimone, Porter, & Garet (2000).

However, planning for the sustainability of the NDP provided an opportunity to consider other aspects that were relevant to teachers' learning. Issues such as differing levels of experience (National Council of Teachers of Mathematics, 1991, p. 161–162), variations in the readiness of the teachers to change their teaching practice, their personal knowledge of content and pedagogy, and their commitment to on-going study were difficult to accommodate in whole-school development programmes. The provision (and funding) of tertiary courses to complement the in-service work seemed to enable these issues to be dealt with, as McLaughlin & Talbert (2006), suggest:

> High quality offsite professional development affords teachers unique opportunities to access knowledge of content, to rethink their practice and to experience learning in a community of peers. (P. 65)

Selection of Papers

It was envisaged that the act of enrolling in tertiary study would involve a conscious commitment on the part of the teacher, thus ensuring a feeling of personal involvement. Papers could be selected to address issues of concern recognised by teachers themselves and would provide ongoing opportunities for reflection and feedback.

However, while this conscious commitment and individual selection was clearly true in some cases, almost half of the teachers enrolled in tertiary papers that were closely related to their in-school numeracy development work. These papers were seen as a natural extension of learning in the NDP, and most of those enrolling in them were lead teachers acting on the recommendation of their facilitators and/or principals of their schools.

Most teachers provided one or two reasons for selecting specific papers. Twenty of the respondents commented on the relevance of the paper to their teaching and/or their role as a lead teacher in their school. Eight teachers chose a paper to continue or complete degrees or diplomas that they had already begun, five because the cost was subsidised, and two based their choices on interest. Two teachers commented on the convenience of the venue. Although almost half of the respondents reported enrolling with others from their school, only one included the perceived advantage of studying with colleagues as a reason for choosing a specific paper.

Relevance of Personal Experience

Teachers were asked about the extent to which aspects of their personal experience influenced their performance in the papers they studied. The broad groupings in Table 1 show that a significant number of respondents felt that their personal experience and knowledge had some impact on their performance.

Table 1
Extent to Which Personal Experience Influenced Performance

	Personal Mathematical Knowledge	Previous Tertiary Study	Teaching Experience	NDP Experience
Greatly	31%	44%	47%	37%
Somewhat	60%	53%	53%	53%
Not at all	9%	3%	0%	6%

This was especially evident in the group who chose to study extramurally. The years of teaching experience in this group was high, ranging from 16–30 years, with an average of over 20 years. Their responses rated their personal mathematical knowledge, previous tertiary study, and teaching experience as important factors in their success. Most of the teachers in this group were continuing or completing qualifications that they had already begun, and their decision to enrol in the papers was a “conscious commitment” to further study. They did not see their NDP experience as being a major factor in their success.

Teachers who participated in "face-to-face" papers that were very specifically related to the NDP were a much less experienced group, with an average "teaching life" of just over 8 years. Assessment in these papers required teachers to use their current NDP involvement to achieve success.

Support from the School

Teachers in the survey were asked to consider ways in which they felt their schools supported their tertiary studies. The view of the Ministry of Education is clear:

> Principals and numeracy lead teachers are central to sustaining and developing effective practices. They need to be enthusiastic, supportive, and involved. (Ministry of Education, 2005, p. 4)

Perceptions relating to these issues are shown in Table 2.

Table 2
Sources of School-based Support

	Support from Principal *	Support from Lead Teacher **	Support from Colleagues	Time Allowance / Reduced Responsibilities
Greatly	32%	20%	28%	3%
Somewhat	36%	40%	12%	3%
Not received	32%	40%	60%	94%

*/** Participants included a principal and 12 lead teachers, who are excluded from these categories.

There are several factors that may have contributed to the perceived lack of support. The impending introduction of *The New Zealand Curriculum* (Ministry of Education, 2007) became a necessary focus for schools, with the result that almost half of the respondents were working in schools where numeracy was no longer a professional development focus at the time of their study. The links between numeracy development in the schools and some of the tertiary papers offered was very clear, with the papers being fully supported by the facilitators who had provided teachers in the school with the initial NDP programme. However, particularly in the case of the distance-education papers, other numeracy papers may not have been seen to have the same relevance to the school-wide NDP programme. Several lead teachers described being torn between supporting others and attending to their own studies.

Where support was offered by the principal and or senior management, it was clearly appreciated:

> I had a very supportive principal who was interested in my study and the benefits of the study both to myself and to the school. He provided feedback on my assignments, and we discussed aspects from my study in relation to our school practices and performance. (Lead teacher, large medium-decile school)

Teachers who enrolled as part of a group that included a member of the school's senior management team felt advantaged in this regard:

> Have doubts whether enough support would have been given from my school had there not been a colleague from the school management team doing the paper with me. (Class teacher, large high-decile school)

However, several respondents commented on the difficulties of continuing to teach full time and maintain a wide range of school responsibilities while studying, and those who were not classroom teachers had difficulty in completing the on-going practical teaching requirement of some papers.

The Papers

Table 3
Extent to Which Aspects of the Papers Helped Teachers to Succeed

	Feedback	Fit with Current Teaching	Meeting Long-term Needs	Support of Others In the Class
Greatly	72%	66%	66%	28%
Somewhat	28%	34%	31%	53%
Not at all	0%	0%	3%	19%

Most of the teachers felt well supported by the lecturers and facilitators who led the papers in which they enrolled. Several commented favourably on the organisation of the papers and the enthusiasm and professionalism with which they were presented. Most of the teachers felt that the papers they took met both their immediate and long-term needs. The supportive nature of the feedback they received was appreciated by most:

> Feedback from the lecturer, while seeming critical to begin with, did develop my understanding of the teaching of maths. (Lead teacher, large primary school)

Concluding Comments

Many of the issues raised by the teachers in this study relate to the difficulties of studying at a tertiary level while dealing with the on-going demands of a teaching job. While individuals named different aspects that were supportive of their studies, two common themes appeared in their comments. The lack of time available for concentrated study was, as expected, an issue for almost all of the participants. Although it was anticipated that teachers would find the support of a group important to their success, this was not the case. Rather, teachers commented on the importance of a single supporter or "critical friend" as the most helpful area of support. This finding, together with the discrepancies evident in the support given to teachers by their schools, would seem worthy of further investigation.

The Ministry of Education initiative that provided funding for the teachers in this study aimed to build on the NDP school-based teacher development in numeracy by further developing teachers' pedagogical content knowledge and understanding of mathematics education. Despite the range of difficulties experienced by the teachers, their comments suggest that, for many, this aim had been achieved:

> Learning from doing the paper has certainly made me a more confident teacher of maths and has also increased my confidence in my own maths ability. It has also given me the confidence to take more of a leadership role in my school regarding the maths curriculum. (Teacher, large integrated school)

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Appendices (Findings From the New Zealand Numeracy Development Projects 2008)

Appendix A[1] (Patterns of Performance and Progress of NDP Students in 2008)

Cohort Composition of Students in their First Year of NDP in 2008 and of Students whose First Year Was 2007 and Second Year Was 2008

Group	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
First year 2008	*219*	*225*	*255*	*226*	*221*	*326*	*2989*	*3010*	*2855*
School decile									
Low (1–3)	23	21	19	19	28	20	11	12	37
Middle (4–7)	20	28	27	47	39	38	35	36	50
High (8–10)	57	51	54	35	34	42	54	52	13
Ethnicity									
NZE	50	59	53	54	51	66	49	45	48
Māori	21	17	18	19	24	17	11	12	27
Pasifika	10	9	8	9	9	5	11	10	16
Asian	8	4	9	7	5	6	17	18	5
First year 2007	*348*	*503*	*573*	*534*	*571*	*468*	*2386*	*2284*	*3327*
School decile									
Low (1–3)	6	4	7	3	4	4	7	7	3
Middle (4–7)	38	37	43	26	12	11	60	55	69
High (8–10)	56	59	51	71	85	85	33	38	28
Ethnicity									
NZE	67	62	66	63	62	59	57	59	63
Māori	20	22	21	13	13	15	23	20	20
Pasifika	5	7	5	4	5	6	5	6	7
Asian	5	5	3	5	5	8	6	6	4
Second year 2008	*274*	*278*	*360*	*449*	*451*	*589*	*2036*	*2447*	*2912*
School decile									
Low (1–3)	27	19	22	21	12	12	22	17	22
Middle (4–7)	6	22	24	35	35	30	47	60	53
High (8–10)	68	59	55	44	53	58	31	24	25
Ethnicity									
NZE	60	67	63	60	62	66	55	55	63
Māori	26	19	19	26	22	20	20	21	20
Pasifika	4	3	7	5	8	4	11	11	7
Asian	6	4	6	5	4	5	8	7	5

[1] For appendices A–E:
- totals may be affected by rounding
- NZE = New Zealand European
- in appendices A and B, except for year group totals, figures are percentages
- in Appendix A, ethnicity percentages for "other" are not shown
- percentages less than 0.5% are shown as 0%
- where there is no data entered for a student, the cell is left blank.

Appendix B (Patterns of Performance and Progress of NDP Students in 2008)

Percentages of Students at Year Level at Particular Stages on the Number Framework for Various Domains (Strategy and Knowledge; Final 2008)

Additive Domain	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Overall	*219*	*225*	*255*	*226*	*221*	*326*	*2989*	*3010*	*2855*
0–3 Counting all	81	27	11	2	1	1	1	1	1
4 Counting on	17	54	44	42	17	9	10	6	8
5 Early Additive	2	19	42	48	61	45	33	25	30
6 Advanced Additive			3	8	20	42	39	41	41
7 Advanced Multiplicative			0		2	4	18	26	17
8 Advanced Proportional							0	1	3
Stage 4+	*19*	*73*	*89*	*98*	*100*	*99*	*100*	*99*	*99*
Stage 5+	*2*	*19*	*46*	*56*	*82*	*91*	*90*	*93*	*92*
Stage 6+	*0*	*0*	*4*	*8*	*21*	*46*	*57*	*68*	*62*
Stage 7+	*0*	*0*	*0*	*0*	*2*	*4*	*18*	*27*	*21*
Low decile	*51*	*47*	*49*	*42*	*61*	*65*	*336*	*372*	*1068*
0–3 Counting all	94	53	24	2	0	0	2	3	1
4 Counting on	6	47	63	45	15	15	27	18	14
5 Early Additive			10	50	66	40	39	39	33
6 Advanced Additive			0	2	20	45	19	32	38
7 Advanced Multiplicative			2				13	8	11
8 Advanced Proportional							1	1	2
Stage 4+	*6*	*47*	*76*	*98*	*100*	*100*	*98*	*97*	*99*
Stage 5+	*0*	*0*	*12*	*52*	*85*	*85*	*71*	*80*	*85*
Stage 6+	*0*	*0*	*2*	*2*	*20*	*45*	*32*	*41*	*51*
Stage 7+	*0*	*0*	*2*	*0*	*0*	*0*	*14*	*9*	*13*
Middle decile	*44*	*64*	*68*	*106*	*86*	*125*	*1046*	*1084*	*1421*
0–3 Counting all	91	33	12	3	0	2	1	0	1
4 Counting on	7	55	53	48	26	14	9	6	4
5 Early Additive	2	13	34	39	55	44	33	28	30
6 Advanced Additive			2	10	20	35	41	45	44
7 Advanced Multiplicative						5	16	19	18
8 Advanced Proportional							0	2	5
Stage 4+	*9*	*67*	*88*	*97*	*100*	*98*	*99*	*100*	*100*
Stage 5+	*2*	*13*	*35*	*49*	*75*	*84*	*90*	*94*	*96*
Stage 6+	*0*	*0*	*2*	*10*	*20*	*40*	*57*	*66*	*66*
Stage 7+	*0*	*0*	*0*	*0*	*0*	*5*	*17*	*21*	*22*

Appendix B – continued

Percentages of Students at Year Level at Particular Stages on the Number Framework for Various Domains (Strategy and Knowledge; Final 2008)

Additive Domain	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
High decile	*124*	*114*	*138*	*78*	*74*	*136*	*1607*	*1554*	*366*
0–3 Counting all	72	13	5	1	1	0	0	0	0
4 Counting on	26	56	33	31	10	1	6	4	2
5 Early Additive	2	31	57	59	65	47	31	20	23
6 Advanced Additive			5	9	19	47	42	40	39
7 Advanced Multiplicative				5	5	20	36	34	8
8 Advanced Proportional							0	1	1
Stage 4+	*28*	*87*	*95*	*99*	*99*	*100*	*100*	*100*	*100*
Stage 5+	*2*	*31*	*62*	*68*	*89*	*99*	*94*	*96*	*98*
Stage 6+	*0*	*0*	*5*	*9*	*24*	*52*	*63*	*76*	*75*
Stage 7+	*0*	*0*	*0*	*0*	*5*	*5*	*21*	*36*	*35*

Multiplicative Domain	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Overall	*219*	*225*	*255*	*226*	*221*	*326*	*2989*	*3010*	*2855*
0–3 Counting all/NA	92	45	30	11	3	1	2	2	1
4 Skip counting	6	48	41	48	32	14	10	7	8
5 Repeated addition	1	6	22	22	37	34	22	16	19
6 Early Multiplicative		0	7	17	21	35	35	34	34
7 Advanced Multiplicative			0	1	6	16	24	28	30
8 Advanced Proportional					1	0	8	14	9
Stage 4+	*8*	*55*	*70*	*89*	*97*	*99*	*98*	*98*	*99*
Stage 5+	*1*	*7*	*29*	*41*	*65*	*85*	*88*	*92*	*91*
Stage 6+	*0*	*0*	*7*	*19*	*28*	*51*	*67*	*75*	*72*
Stage 7+	*0*	*0*	*0*	*1*	*7*	*16*	*32*	*42*	*39*
Low decile	*51*	*47*	*49*	*42*	*61*	*65*	*336*	*372*	*1068*
0–3 Counting all/NA	98	77	69	17	3	0	5	7	2
4 Skip counting	2	23	27	67	44	26	30	20	13
5 Repeated addition			2	17	39	34	26	25	24
6 Early Multiplicative			2		12	32	24	30	31
7 Advanced Multiplicative					2	8	13	16	24
8 Advanced Proportional							2	2	6
Stage 4+	*2*	*23*	*31*	*83*	*97*	*100*	*95*	*93*	*98*
Stage 5+	*0*	*0*	*4*	*17*	*52*	*74*	*65*	*73*	*85*
Stage 6+	*0*	*0*	*2*	*0*	*13*	*40*	*39*	*49*	*61*
Stage 7+	*0*	*0*	*0*	*0*	*2*	*8*	*15*	*18*	*30*

Appendix B – continued

Percentages of Students at Year Level at Particular Stages on the Number Framework for Various Domains (Strategy and Knowledge; Final 2008)

Multiplicative Domain	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Middle decile	*44*	*64*	*68*	*106*	*86*	*125*	*1046*	*1084*	*1421*
0–3 Counting all/NA	96	47	29	8	4	2	2	1	1
4 Skip counting	2	47	49	54	27	16	11	7	5
5 Repeated addition	2	5	15	24	40	34	24	20	16
6 Early Multiplicative		2	6	14	21	31	35	37	37
7 Advanced Multiplicative			2	1	9	18	23	25	33
8 Advanced Proportional							6	10	8
Stage 4+	*5*	*53*	*71*	*93*	*96*	*98*	*98*	*99*	*99*
Stage 5+	*2*	*6*	*22*	*39*	*70*	*82*	*87*	*92*	*94*
Stage 6+	*0*	*2*	*7*	*15*	*30*	*49*	*63*	*72*	*78*
Stage 7+	*0*	*0*	*2*	*1*	*9*	*18*	*28*	*35*	*41*
High decile	*124*	*114*	*138*	*78*	*74*	*136*	*1607*	*1554*	*366*
0–3 Counting all/NA	89	31	16	13	3	2	1	1	1
4 Skip counting	10	60	43	31	28	6	6	4	3
5 Repeated addition	2	10	33	23	31	34	19	12	13
6 Early Multiplicative			9	31	30	40	37	33	29
7 Advanced Multiplicative				3	5	18	27	33	36
8 Advanced Proportional					3	1	11	19	19
Stage 4+	*11*	*69*	*84*	*87*	*97*	*99*	*99*	*100*	*99*
Stage 5+	*2*	*10*	*41*	*57*	*69*	*93*	*94*	*96*	*97*
Stage 6+	*0*	*0*	*9*	*33*	*38*	*59*	*75*	*84*	*84*
Stage 7+	*0*	*0*	*0*	*3*	*8*	*19*	*38*	*52*	*55*

Proportional Domain	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Overall	*219*	*225*	*255*	*226*	*221*	*326*	*2989*	*3010*	*2855*
0–1 Unequal sharing/NA	89	34	22	4	5	0	1	0	1
2–4 Equal sharing	11	60	52	56	34	14	16	11	11
5 Early Additive		6	21	29	42	40	22	20	27
6 Advanced Additive			5	11	13	27	29	27	18
7 Advanced Multiplicative			1	0	6	18	26	29	35
8 Advanced Proportional					1	1	7	13	9
Stage 5+	*0*	*6*	*26*	*40*	*61*	*86*	*84*	*88*	*88*
Stage 6+	*0*	*0*	*6*	*12*	*20*	*46*	*61*	*69*	*61*
Stage 7+	*0*	*0*	*1*	*0*	*7*	*18*	*33*	*42*	*44*

Appendix B – continued

Percentages of Students at Year Level at Particular Stages on the Number Framework for Various Domains (Strategy and Knowledge; Final 2008)

Proportional Domain	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Low decile	*51*	*47*	*49*	*42*	*61*	*65*	*336*	*372*	*1068*
0–1 Unequal sharing/NA	96	66	55	7	10	2	5	1	1
2–4 Equal sharing	4	34	41	60	48	26	40	33	16
5 Early Additive			4	33	33	34	24	29	35
6 Advanced Additive					8	25	19	22	19
7 Advanced Multiplicative					2	12	11	13	23
8 Advanced Proportional						2	1	3	6
Stage 5+	*0*	*0*	*4*	*33*	*43*	*72*	*55*	*66*	*82*
Stage 6+	*0*	*0*	*0*	*0*	*10*	*38*	*31*	*37*	*47*
Stage 7+	*0*	*0*	*0*	*0*	*2*	*14*	*12*	*16*	*29*
Middle decile	*44*	*64*	*68*	*106*	*86*	*125*	*1046*	*1084*	*1421*
0–1 Unequal sharing/NA	96	28	12	4	1	0	1	0	0
2–4 Equal sharing	5	66	71	62	27	17	17	12	10
5 Early Additive		6	12	25	59	43	26	23	24
6 Advanced Additive			6	9	9	24	29	30	18
7 Advanced Multiplicative					4	16	23	27	40
8 Advanced Proportional							4	7	8
Stage 5+	*0*	*6*	*18*	*34*	*72*	*83*	*82*	*88*	*90*
Stage 6+	*0*	*0*	*6*	*9*	*13*	*40*	*57*	*65*	*66*
Stage 7+	*0*	*0*	*0*	*0*	*4*	*16*	*27*	*34*	*48*
High decile	*124*	*114*	*138*	*78*	*74*	*136*	*1607*	*1554*	*366*
0–1 Unequal sharing/NA	84	24	15	1	6	0	0	0	0
2–4 Equal sharing	16	68	46	46	31	6	9	6	4
5 Early Additive		9	31	32	28	40	20	15	13
6 Advanced Additive			6	19	20	32	30	26	15
7 Advanced Multiplicative			1	1	14	22	31	34	49
8 Advanced Proportional					1	1	10	19	19
Stage 5+	*0*	*9*	*38*	*53*	*64*	*94*	*91*	*94*	*97*
Stage 6+	*0*	*0*	*7*	*21*	*35*	*54*	*71*	*79*	*83*
Stage 7+	*0*	*0*	*1*	*1*	*15*	*23*	*41*	*53*	*68*

Appendix B – continued

Percentages of Students at Year Level at Particular Stages on the Number Framework for Various Domains (Strategy and Knowledge; Final 2008)

Fractions	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Overall	*219*	*225*	*255*	*226*	*221*	*326*	*2989*	*3010*	*2855*
0–3 Unit fractions not recognised/NA	95	65	41	11	11	3	3	2	2
4 Unit fractions recognised	4	30	27	27	25	14	10	9	8
5 Orders unit fractions	2	5	30	55	51	52	34	27	31
6 Co-ordinates numerator & denominator		0	3	7	9	20	25	23	26
7 Equivalent fractions				0	1	6	18	21	26
8 Orders fractions					2	4	11	18	6
Stage 5+	*2*	*6*	*33*	*62*	*64*	*82*	*87*	*89*	*90*
Stage 6+	*0*	*0*	*3*	*7*	*13*	*30*	*53*	*62*	*58*
Stage 7+	*0*	*0*	*0*	*0*	*4*	*10*	*28*	*40*	*32*
Low decile	*51*	*47*	*49*	*42*	*61*	*65*	*336*	*372*	*1068*
0–3 Unit fractions not recognised/NA	96	89	69	21	12	6	13	9	4
4 Unit fractions recognised	2	9	20	33	33	22	22	26	12
5 Orders unit fractions	2	2	10	43	49	59	38	34	33
6 Co-ordinates numerator & denominator				2	7	9	18	14	28
7 Equivalent fractions						5	7	13	20
8 Orders fractions							3	4	4
Stage 5+	*2*	*2*	*10*	*45*	*56*	*72*	*65*	*66*	*84*
Stage 6+	*0*	*0*	*0*	*2*	*7*	*14*	*27*	*31*	*51*
Stage 7+	*0*	*0*	*0*	*0*	*0*	*5*	*10*	*17*	*23*
Middle decile	*44*	*64*	*68*	*106*	*86*	*125*	*1046*	*1084*	*1421*
0–3 Unit fractions not recognised/NA	96	64	43	10	9	3	2	2	1
4 Unit fractions recognised	0	31	34	24	28	16	13	9	7
5 Orders unit fractions	2	3	21	56	52	47	38	35	33
6 Co-ordinates numerator & denominator		2	3	11	8	22	20	25	25
7 Equivalent fractions					1	10	19	18	29
8 Orders fractions					1	2	8	11	6
Stage 5+	*2*	*5*	*24*	*67*	*63*	*81*	*85*	*89*	*92*
Stage 6+	*0*	*2*	*3*	*11*	*11*	*34*	*47*	*54*	*59*
Stage 7+	*0*	*0*	*0*	*0*	*2*	*11*	*27*	*30*	*34*

Appendix B – continued

Percentages of Students at Year Level at Particular Stages on the Number Framework for Various Domains (Strategy and Knowledge; Final 2008)

Fractions	**Y0–1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Y8**	**Y9**
High decile	*124*	*114*	*138*	*78*	*74*	*136*	*1607*	*1554*	*366*
0–3 Unit fractions not recognised/NA	93	54	30	8	12	2	2	1	0
4 Unit fractions recognised	6	38	25	27	16	10	5	4	3
5 Orders unit fractions	2	8	41	62	51	53	31	20	21
6 Co-ordinates numerator & denominator			4	4	12	24	29	23	27
7 Equivalent fractions					3	4	19	26	35
8 Orders fractions					5	7	14	26	15
Stage 5+	*2*	*8*	*45*	*65*	*72*	*88*	*94*	*95*	*97*
Stage 6+	*0*	*0*	*4*	*4*	*20*	*35*	*63*	*75*	*77*
Stage 7+	*0*	*0*	*0*	*0*	*8*	*12*	*33*	*52*	*50*

Place Value	**Y0–1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Y8**	**Y9**
Overall	*219*	*225*	*255*	*226*	*221*	*326*	*2989*	*3010*	*2855*
0–2 One as a unit/NA	66	24	13	3	1	0	0	0	0
3 Five as a unit	3	7	5	4	6	4	0	1	1
4 Ten as a counting unit	30	65	71	62	55	29	13	9	7
5 Tens in numbers to 1000/10ths	1	4	9	27	28	31	32	28	37
6 Hs, Ths in whole numbers/ten 10ths			2	4	5	27	28	28	25
7 10ths in decimals/orders decimals					3	6	15	18	17
8 Decimal conversions					2	2	10	17	14
Stage 5+	*1*	*4*	*11*	*31*	*38*	*67*	*86*	*90*	*92*
Stage 6+	*0*	*0*	*2*	*4*	*10*	*36*	*54*	*63*	*56*
Stage 7+	*0*	*0*	*0*	*0*	*5*	*8*	*25*	*35*	*31*
Low decile	*51*	*47*	*49*	*42*	*61*	*65*	*336*	*372*	*1068*
0–2 One as a unit/NA	96	53	35	10	0	0	1	1	1
3 Five as a unit	2	15	8	14	18	5	2	4	1
4 Ten as a counting unit	2	32	57	50	72	52	35	23	10
5 Tens in numbers to 1000/10ths				24	10	29	30	40	42
6 Hs, Ths in whole numbers/ten 10ths				2		14	24	23	27
7 10ths in decimals/orders decimals							5	7	11
8 Decimal conversions							4	2	8
Stage 5+	*0*	*0*	*0*	*26*	*10*	*43*	*63*	*72*	*88*
Stage 6+	*0*	*0*	*0*	*2*	*0*	*14*	*33*	*32*	*46*
Stage 7+	*0*	*0*	*0*	*0*	*0*	*0*	*8*	*9*	*19*

Appendix B – continued

Percentages of Students at Year Level at Particular Stages on the Number Framework for Various Domains (Strategy and Knowledge; Final 2008)

Place value	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Middle decile	*44*	*64*	*68*	*106*	*86*	*125*	*1046*	*1084*	*1421*
0–2 One as a unit/NA	77	20	16	2	0	0	0	0	0
3 Five as a unit	5	6	7	1	2	5	0	1	1
4 Ten as a counting unit	18	67	66	71	50	31	15	11	5
5 Tens in numbers to 1000/10ths		6	9	23	37	34	35	31	37
6 Hs, Ths in whole numbers/ten 10ths			2	4	9	26	25	29	23
7 10ths in decimals/orders decimals					1	5	15	18	19
8 Decimal conversions							9	11	16
Stage 5+	*0*	*6*	*10*	*26*	*48*	*64*	*84*	*89*	*94*
Stage 6+	*0*	*0*	*2*	*4*	*11*	*30*	*49*	*58*	*58*
Stage 7+	*0*	*0*	*0*	*0*	*1*	*5*	*24*	*29*	*35*
High decile	*124*	*114*	*138*	*78*	*74*	*136*	*1607*	*1554*	*366*
0–2 One as a unit/NA	55	15	4	0	3	0	0	0	0
3 Five as a unit	4	4	2	3	1	2	0	0	0
4 Ten as a counting unit	40	78	78	56	46	17	8	4	4
5 Tens in numbers to 1000/10ths	1	4	13	36	32	30	31	22	22
6 Hs, Ths in whole numbers/ten 10ths			2	5	4	35	31	28	24
7 10ths in decimals/orders decimals					8	10	17	21	24
8 Decimal conversions					5	5	13	25	26
Stage 5+	*1*	*4*	*15*	*41*	*50*	*81*	*92*	*96*	*96*
Stage 6+	*0*	*0*	*2*	*5*	*18*	*51*	*61*	*74*	*74*
Stage 7+	*0*	*0*	*0*	*0*	*14*	*15*	*29*	*46*	*50*

Basic Facts	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Overall	*219*	*225*	*255*	*226*	*221*	*326*	*2989*	*3010*	*2855*
0–1 NA	51	12	3	3	0	0	0	0	0
2 Addition facts to 5	25	25	11	7	3	0	0	0	0
3 Addition facts to 10	10	20	12	7	3	2	1	1	1
4 Addition with 10s & doubles	13	36	41	31	22	12	6	5	5
5 Addition facts	1	7	23	39	40	28	19	15	17
6 Subtraction & multiplication facts			9	12	21	32	34	31	54
7 Division facts			1	2	9	23	30	30	21
8 Common factors & multiples					2	2	10	18	2
Stage 5+	*1*	*7*	*33*	*53*	*72*	*86*	*92*	*94*	*94*
Stage 6+	*0*	*0*	*1*	*2*	*32*	*57*	*74*	*79*	*77*
Stage 7+	*0*	*0*	*0*	*0*	*11*	*25*	*40*	*48*	*23*

Appendix B – continued

Percentages of Students at Year Level at Particular Stages on the Number Framework for Various Domains (Strategy and Knowledge; Final 2008)

Basic facts	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Low decile	*51*	*47*	*49*	*42*	*61*	*65*	*336*	*372*	*1068*
0–1 NA	57	15	6	14	0	0	0	1	0
2 Addition facts to 5	20	38	20	14	3	2	2	1	1
3 Addition facts to 10	8	32	12	12	7	3	3	4	2
4 Addition with 10s & doubles	16	15	55	38	36	35	17	14	5
5 Addition facts			4	21	48	39	31	25	23
6 Subtraction & multiplication facts			2		5	14	30	34	46
7 Division facts					2	6	14	17	20
8 Common factors & multiples						2	3	5	2
Stage 5+	*0*	*0*	*6*	*21*	*54*	*60*	*78*	*81*	*91*
Stage 6+	*0*	*0*	*2*	*0*	*7*	*22*	*48*	*56*	*68*
Stage 7+	*0*	*0*	*0*	*0*	*2*	*8*	*17*	*22*	*22*
Middle decile	*44*	*64*	*68*	*106*	*86*	*125*	*1046*	*1084*	*1421*
0–1 NA	64	13	4	0	0	0	0	0	0
2 Addition facts to 5	25	19	10	2	2	0	0	0	0
3 Addition facts to 10	7	27	13	8	0	3	1	0	0
4 Addition with 10s & doubles	5	39	46	26	14	7	6	5	4
5 Addition facts		3	19	47	48	26	19	15	14
6 Subtraction & multiplication facts			6	17	27	34	37	37	60
7 Division facts			2	1	8	27	28	28	19
8 Common factors & multiples					1	2	10	14	2
Stage 5+	*0*	*3*	*27*	*65*	*84*	*90*	*93*	*95*	*95*
Stage 6+	*0*	*0*	*7*	*18*	*36*	*63*	*74*	*79*	*82*
Stage 7+	*0*	*0*	*2*	*1*	*9*	*29*	*38*	*42*	*22*
High decile	*124*	*114*	*138*	*78*	*74*	*136*	*1607*	*1554*	*366*
0–1 NA	44	11	1	0	0	0	0	0	0
2 Addition facts to 5	27	23	8	9	3	0	0	0	0
3 Addition facts to 10	12	12	11	3	3	0	1	0	0
4 Addition with 10s & doubles	15	42	33	35	20	6	4	3	4
5 Addition facts	2	11	32	37	26	25	16	12	12
6 Subtraction & multiplication facts			14	13	27	38	33	27	49
7 Division facts			1	4	16	28	34	34	31
8 Common factors & multiples					5	3	11	24	4
Stage 5+	*2*	*11*	*46*	*54*	*74*	*94*	*95*	*97*	*96*
Stage 6+	*0*	*0*	*15*	*17*	*49*	*69*	*79*	*85*	*84*
Stage 7+	*0*	*0*	*1*	*4*	*22*	*31*	*46*	*58*	*35*

Appendix C (Patterns of Performance and Progress of NDP Students in 2008)

Average Stage on the Number Framework at the Beginning of the NDP Initiative (Initial) as a Function of School Decile, Ethnicity, and Gender

Domain/Group	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Additive domain									
Overall	1.80	2.85	3.86	4.31	4.62	4.94	5.24	5.40	5.28
School decile									
Low decile	0.82	2.03	2.87	4.08	4.50	4.74	4.88	4.85	4.98
Middle decile	1.34	2.50	3.61	4.31	4.63	4.87	5.24	5.36	5.42
High decile	2.32	3.34	4.28	4.44	4.71	5.05	5.29	5.26	5.57
Ethnicity									
NZE	2.01	2.99	3.99	4.39	4.60	4.90	5.28	5.47	5.49
Māori	1.14	2.45	3.38	4.33	4.61	4.90	4.90	5.09	5.19
Pasifika	1.06	2.27	3.54	3.95	4.44	4.71	4.93	4.83	4.81
Asian	2.17	2.60	4.43	4.64	5.30	5.50	5.47	5.68	5.27
Gender									
Boys	1.88	2.94	3.99	4.41	4.77	4.95	5.34	5.53	5.35
Girls	1.73	2.77	3.72	4.22	4.47	4.92	5.12	5.26	5.22
Multiplicative domain									
Overall	2.08	2.59	3.31	3.83	4.34	4.79	5.37	5.60	5.56
School decile									
Low decile	2.00	2.12	2.42	3.41	3.80	4.30	4.89	4.86	5.23
Middle decile	2.05	2.31	3.04	3.70	4.52	4.85	5.29	5.54	5.68
High decile	2.12	2.91	3.70	4.25	4.57	4.90	5.50	5.78	6.01
Ethnicity									
NZE	2.12	2.70	3.45	3.97	4.51	4.82	5.41	5.70	5.82
Māori	2.00	2.33	2.76	3.62	4.11	4.58	4.92	5.20	5.40
Pasifika	2.00	2.18	3.15	3.26	4.17	4.21	5.01	4.85	4.98
Asian	2.00	2.40	3.86	4.21	5.30	5.25	5.66	5.94	5.66
Gender									
Boys	2.10	2.65	3.40	4.00	4.58	4.81	5.47	5.77	5.64
Girls	2.06	2.54	3.20	3.67	4.10	4.77	5.27	5.42	5.49
Proportional domain									
Overall	2.08	2.57	3.32	3.73	4.16	4.58	5.26	5.55	5.46
School decile									
Low decile	2.00	2.15	2.58	3.33	3.59	4.07	4.69	4.63	5.08
Middle decile	2.03	2.39	3.02	3.58	4.20	4.53	5.15	5.47	5.56
High decile	2.13	2.83	3.69	4.18	4.56	4.79	5.43	5.77	6.13
Ethnicity									
NZE	2.11	2.68	3.49	3.91	4.27	4.63	5.34	5.67	5.76
Māori	2.00	2.24	2.86	3.43	3.87	4.21	4.87	5.05	5.27
Pasifika	2.00	2.36	2.92	3.37	3.83	4.14	4.79	4.71	4.82
Asian	2.00	2.40	3.71	4.00	5.30	5.30	5.47	5.94	5.57
Gender									
Boys	2.08	2.62	3.37	3.85	4.25	4.56	5.37	5.65	5.46
Girls	2.08	2.54	3.27	3.62	4.06	4.61	5.15	5.44	5.47

Appendix D (Patterns of Performance and Progress of NDP Students in 2008)

Average Gain on the Number Framework from the Beginning of the NDP Initiative to the End of the First Year as a Function of School Decile, Ethnicity, and Gender

Domain/Group	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Additive domain gain									
Overall	0.83	0.9	0.51	0.31	0.43	0.48	0.49	0.54	0.5
School decile									
Low decile	1.18	1.12	0.97	0.41	0.52	0.56	0.45	0.51	0.53
Middle decile	0.97	1.03	0.57	0.25	0.30	0.41	0.43	0.47	0.48
High decile	0.65	0.73	0.34	0.34	0.50	0.52	0.53	0.59	0.51
Ethnicity									
NZE	0.72	0.89	0.48	0.29	0.50	0.55	0.50	0.53	0.47
Māori	0.97	0.97	0.76	0.19	0.31	0.33	0.45	0.47	0.43
Pasifika	1.25	1.00	0.77	0.42	0.33	0.21	0.41	0.51	0.57
Asian	0.75	1.00	0.29	0.29	0.80	0.40	0.53	0.57	0.75
Gender									
Boys	0.89	0.84	0.62	0.27	0.46	0.46	0.52	0.59	0.53
Girls	0.76	0.95	0.42	0.35	0.39	0.51	0.45	0.49	0.47
Multiplicative domain gain									
Overall	0.17	0.64	0.60	0.67	0.63	0.75	0.65	0.66	0.57
School decile									
Low decile	0.09	0.39	0.39	0.56	0.77	0.81	0.45	0.55	0.60
Middle decile	0.11	0.76	0.79	0.75	0.56	0.69	0.58	0.55	0.55
High decile	0.23	0.66	0.58	0.62	0.61	0.78	0.73	0.76	0.53
Ethnicity									
NZE	0.21	0.67	0.58	0.57	0.55	0.72	0.71	0.69	0.53
Māori	0.00	0.67	0.86	0.81	0.74	0.79	0.67	0.59	0.49
Pasifika	0.13	0.27	0.31	0.68	0.33	0.57	0.32	0.50	0.72
Asian	0.42	0.60	0.48	0.71	0.80	1.10	0.71	0.67	0.83
Gender									
Boys	0.11	0.58	0.61	0.69	0.73	0.77	0.66	0.74	0.59
Girls	0.24	0.71	0.59	0.65	0.54	0.74	0.65	0.60	0.54
Proportional domain gain									
Overall	0.17	0.83	0.63	0.76	0.67	0.94	0.68	0.62	0.58
School decile									
Low decile	0.12	0.48	0.24	0.85	0.79	1.07	0.47	0.64	0.56
Middle decile	0.11	1.11	0.96	0.81	0.68	0.91	0.59	0.48	0.60
High decile	0.21	0.80	0.59	0.63	0.57	0.92	0.76	0.72	0.53
Ethnicity									
NZE	0.20	0.89	0.59	0.67	0.73	0.97	0.72	0.64	0.60
Māori	0.00	0.85	0.76	0.86	0.57	0.94	0.56	0.56	0.48
Pasifika	0.13	0.18	0.46	0.74	0.67	0.79	0.43	0.51	0.62
Asian	0.42	0.80	0.67	0.79	0.70	0.85	0.77	0.65	0.66
Gender									
Boys	0.09	0.74	0.58	0.79	0.60	0.94	0.72	0.65	0.57
Girls	0.24	0.93	0.67	0.73	0.74	0.93	0.64	0.59	0.58

Appendix E (Patterns of Performance and Progress of NDP Students in 2008)

Percentages of Students at Particular Stages on the Number Framework as a Function of Year on the Project (First Year 2007 or Second Year 2008)

Additive Domain	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
First year 2007	*350*	*504*	*573*	*536*	*574*	*470*	*2390*	*2285*	*3327*
0–3 Counting all	75	29	9	1	1	0	1	1	0
4 Counting on	21	46	44	34	17	10	11	8	4
5 Early Additive	5	24	42	44	53	37	34	29	28
6 Advanced Additive	0	1	5	19	25	39	41	39	41
7 Advanced Multiplicative		0	0	2	4	13	13	23	20
8 Advanced Proportional						0	0	1	7
Stage 4+	*26*	*71*	*91*	*99*	*99*	*100*	*99*	*100*	*100*
Stage 5+	*5*	*25*	*47*	*65*	*81*	*90*	*88*	*92*	*95*
Stage 6+	*0*	*1*	*5*	*21*	*29*	*52*	*54*	*64*	*68*
Stage 7+	*0*	*0*	*0*	*2*	*4*	*14*	*13*	*25*	*27*
Second year 2008	*274*	*278*	*360*	*449*	*451*	*589*	*2036*	*2447*	*2912*
0–3 Counting all	84	23	7	3	1	0	1	0	1
4 Counting on	14	58	40	31	16	6	14	7	6
5 Early Additive	2	18	48	53	48	38	34	28	30
6 Advanced Additive		1	5	14	29	45	38	40	39
7 Advanced Multiplicative			1	0	6	11	13	24	17
8 Advanced Proportional						0	1	1	8
Stage 4+	*16*	*77*	*93*	*97*	*99*	*100*	*99*	*100*	*100*
Stage 5+	*2*	*19*	*53*	*66*	*83*	*94*	*85*	*92*	*94*
Stage 6+	*0*	*1*	*6*	*14*	*35*	*56*	*52*	*65*	*64*
Stage 7+	*0*	*0*	*1*	*0*	*6*	*11*	*14*	*25*	*25*
Stage 4+ Change 07/08	***–10***	***6***	***2***	***–2***	***1***	***0***	***0***	***0***	***0***
Stage 5+ Change 07/08	***–3***	***–6***	***6***	***1***	***2***	***4***	***–3***	***0***	***–2***
Stage 6+ Change 07/08	***0***	***–1***	***0***	***–7***	***7***	***4***	***–2***	***1***	***–3***
Stage 7+ Change 07/08	***0***	***0***	***0***	***–1***	***3***	***–3***	***1***	***0***	***–2***

Multiplicative Domain	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
First year 2007	*350*	*504*	*573*	*536*	*574*	*470*	*2390*	*2285*	*3327*
0–3 Counting all	87	48	20	6	4	1	2	2	1
4 Skip counting	11	43	54	40	23	10	10	7	5
5 Repeated addition	1	8	20	31	36	25	24	17	15
6 Early Multiplicative	0	1	5	16	25	37	35	32	35
7 Advanced Multiplicative			1	7	12	21	24	30	31
8 Advanced Proportional					1	5	6	12	14
Stage 4+	*13*	*52*	*80*	*94*	*96*	*99*	*98*	*98*	*99*
Stage 5+	*1*	*10*	*26*	*54*	*73*	*89*	*88*	*91*	*95*
Stage 6+	*0*	*1*	*6*	*24*	*37*	*63*	*64*	*74*	*79*
Stage 7+	*0*	*0*	*1*	*7*	*13*	*26*	*29*	*42*	*45*

Appendix E – continued

Percentages of Students at Particular Stages on the Number Framework as a Function of Year on the Project (First Year 2007 or Second Year 2008)

Multiplicative Domain	**Y0–1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Y8**	**Y9**
Second year 2008	*274*	*278*	*360*	*449*	*451*	*589*	*2036*	*2447*	*2912*
0–3 Counting all	92	37	13	9	2	1	3	2	1
4 Skip counting	8	57	52	37	21	12	14	8	5
5 Repeated addition	0	6	25	31	36	27	22	17	18
6 Early Multiplicative		1	9	20	28	37	34	32	36
7 Advanced Multiplicative			1	3	13	21	22	30	30
8 Advanced Proportional					1	2	5	12	9
Stage 4+	*8*	*63*	*87*	*91*	*98*	*99*	*97*	*98*	*99*
Stage 5+	*0*	*7*	*35*	*55*	*77*	*87*	*84*	*90*	*94*
Stage 6+	*0*	*1*	*10*	*23*	*42*	*60*	*61*	*73*	*76*
Stage 7+	*0*	*0*	*1*	*3*	*14*	*23*	*27*	*41*	*40*
Stage 4+ Change 07/08	***–4***	***11***	***7***	***–3***	***2***	***0***	***–1***	***–1***	***0***
Stage 5+ Change 07/08	***–1***	***–3***	***9***	***0***	***4***	***–2***	***–4***	***–1***	***–1***
Stage 6+ Change 07/08	***0***	***0***	***4***	***–1***	***4***	***–3***	***–2***	***–1***	***–3***
Stage 7+ Change 07/08	***0***	***0***	***0***	***–4***	***1***	***–3***	***–2***	***–1***	***–5***

Proportional Domain	**Y0–1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Y8**	**Y9**
First year 2007	*350*	*504*	*573*	*536*	*574*	*470*	*2390*	*2285*	*3327*
0–1 Unequal sharing	85	37	11	4	1	0	1	1	0
2–4 Equal sharing	15	56	66	47	33	14	18	13	7
5 Early Additive	0	6	20	31	32	29	27	20	22
6 Advanced Additive		1	3	10	20	32	25	25	18
7 Advanced Multiplicative			1	7	12	22	26	32	39
8 Advanced Proportional					1	3	4	11	14
Stage 4+	*15*	*64*	*89*	*96*	*99*	*100*	*99*	*100*	*100*
Stage 5+	*0*	*7*	*23*	*49*	*66*	*86*	*81*	*87*	*93*
Stage 6+	*0*	*1*	*3*	*17*	*34*	*57*	*55*	*67*	*71*
Stage 7+	*0*	*0*	*1*	*7*	*13*	*26*	*30*	*42*	*53*
Second year 2008	*274*	*278*	*360*	*449*	*451*	*589*	*2036*	*2447*	*2912*
0–1 Unequal sharing	79	21	6	5	2	1	2	1	0
2–4 Equal sharing	21	75	60	47	22	14	20	13	9
5 Early Additive		4	26	33	40	34	26	22	27
6 Advanced Additive		1	7	11	21	26	26	24	18
7 Advanced Multiplicative			1	4	15	23	22	31	32
8 Advanced Proportional					1	2	4	10	13

Appendix E – continued

Percentages of Students at Particular Stages on the Number Framework as a Function of Year on the Project (First Year 2007 or Second Year 2008)

Proportional Domain	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
Stage 4+	*21*	*80*	*94*	*95*	*98*	*99*	*98*	*99*	*100*
Stage 5+	*0*	*5*	*34*	*48*	*76*	*85*	*78*	*87*	*90*
Stage 6+	*0*	*1*	*8*	*15*	*36*	*51*	*53*	*65*	*63*
Stage 7+	*0*	*0*	*1*	*4*	*16*	*25*	*27*	*41*	*45*
Stage 4+ Change 07/08	***6***	***16***	***5***	***–1***	***–1***	***–1***	***–1***	***0***	***0***
Stage 5+ Change 07/08	***0***	***–2***	***11***	***0***	***10***	***–1***	***–3***	***0***	***–3***
Stage 6+ Change 07/08	***0***	***0***	***5***	***–2***	***3***	***–7***	***–2***	***–2***	***–8***
Stage 7+ Change 07/08	***0***	***0***	***1***	***–4***	***3***	***–1***	***–3***	***–1***	***–8***

Fractions	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
First year 2007	*350*	*504*	*573*	*536*	*574*	*470*	*2390*	*2285*	*3327*
Overall	81	33	7	1	1	0	0	0	0
0–3 Unit fractions not recognised/NA	14	20	22	6	5	2	3	3	1
4 Unit fractions recognised	4	28	37	34	20	15	12	9	6
5 Orders unit fractions	0	19	32	47	51	34	39	28	26
6 Co-ordinates numerator & denominator	0	1	2	9	14	27	23	23	24
7 Equivalent fractions	0		0	3	6	14	14	20	32
8 Orders fractions	0		1	1	4	9	8	17	11
Stage 4+	*5*	*47*	*72*	*93*	*94*	*98*	*96*	*97*	*100*
Stage 5+	*1*	*19*	*35*	*60*	*74*	*83*	*85*	*88*	*94*
Stage 6+	*0*	*1*	*2*	*13*	*23*	*50*	*46*	*60*	*67*
Stage 7+	*0*	*0*	*1*	*4*	*9*	*23*	*22*	*37*	*43*
Second year 2008	*274*	*278*	*360*	*449*	*451*	*589*	*2036*	*2447*	*2912*
Overall	81	12	3	4	0	0	1	0	0
0–3 Unit fractions not recognised/NA	13	35	14	7	4	2	4	3	1
4 Unit fractions recognised	4	36	32	31	19	14	14	9	7
5 Orders unit fractions	2	17	45	51	49	42	37	30	30
6 Co-ordinates numerator & denominator		0	5	6	17	27	24	21	25
7 Equivalent fractions			0	1	8	7	13	21	29
8 Orders fractions			0	0	3	8	8	17	10
Stage 4+	*6*	*53*	*83*	*90*	*96*	*98*	*96*	*97*	*99*
Stage 5+	*2*	*17*	*51*	*59*	*77*	*84*	*82*	*88*	*93*
Stage 6+	*0*	*0*	*6*	*8*	*28*	*41*	*45*	*59*	*63*
Stage 7+	*0*	*0*	*0*	*2*	*11*	*14*	*21*	*37*	*38*
Stage 4+ Change 07/08	***1***	***6***	***11***	***–4***	***1***	***0***	***–1***	***1***	***0***
Stage 5+ Change 07/08	***1***	***–2***	***16***	***–1***	***3***	***0***	***–3***	***1***	***–1***
Stage 6+ Change 07/08	***0***	***0***	***3***	***–5***	***5***	***–8***	***0***	***–2***	***–4***
Stage 7+ Change 07/08	***0***	***0***	***0***	***–2***	***2***	***–8***	***–1***	***0***	***–5***

Appendix E – continued

Percentages of Students at Particular Stages on the Number Framework as a Function of Year on the Project (First Year 2007 or Second Year 2008)

Place Value	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
First year 2007	*350*	*504*	*573*	*536*	*574*	*470*	*2390*	*2285*	*3327*
0–2 One as a unit/NA	61	23	7	1	1	0	1	1	0
3 Five as a unit	7	8	5	4	2	1	1	1	0
4 Ten as a counting unit	32	64	74	60	42	21	17	9	4
5 Tens in numbers to 1000/10ths	0	6	12	27	35	32	37	28	29
6 Hs, Ths in whole numbers/ten 10ths			1	5	12	24	26	29	28
7 10ths in decimals/orders decimals			0	2	6	16	13	18	20
8 Decimal conversions				0	2	6	6	14	20
Stage 4+	*32*	*70*	*88*	*95*	*97*	*100*	*98*	*98*	*100*
Stage 5+	*0*	*6*	*14*	*35*	*55*	*79*	*81*	*89*	*96*
Stage 6+	*0*	*0*	*1*	*7*	*20*	*47*	*44*	*61*	*68*
Stage 7+	*0*	*0*	*0*	*2*	*8*	*22*	*18*	*32*	*39*
Second year 2008	*274*	*278*	*360*	*449*	*451*	*589*	*2036*	*2447*	*2912*
0–2 One as a unit/NA	72	22	6	3	0	0	1	0	0
3 Five as a unit	7	4	7	4	2	1	1	1	1
4 Ten as a counting unit	20	73	68	56	35	23	19	10	5
5 Tens in numbers to 1000/10ths	0	2	13	29	38	32	34	28	29
6 Hs, Ths in whole numbers/ten 10ths	2		5	7	16	25	28	30	27
7 10ths in decimals/orders decimals			1	1	8	14	12	18	21
8 Decimal conversions					2	5	6	14	17
Stage 4+	*22*	*75*	*87*	*93*	*98*	*99*	*99*	*99*	*99*
Stage 5+	*2*	*2*	*19*	*37*	*63*	*76*	*79*	*89*	*94*
Stage 6+	*2*	*0*	*6*	*8*	*25*	*44*	*46*	*61*	*65*
Stage 7+	*0*	*0*	*1*	*1*	*10*	*19*	*18*	*31*	*38*
Stage 4+ Change 07/08	***–10***	***5***	***–1***	***–2***	***1***	***–1***	***0***	***1***	***0***
Stage 5+ Change 07/08	***2***	***–4***	***5***	***2***	***8***	***–3***	***–2***	***0***	***–2***
Stage 6+ Change 07/08	***2***	***0***	***4***	***1***	***5***	***–3***	***2***	***0***	***–2***
Stage 7+ Change 07/08	***0***	***0***	***1***	***–1***	***2***	***–4***	***0***	***–1***	***–2***

Basic Facts	Y0–1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
First year 2007	350	504	573	536	574	470	2390	2285	3327
0–1 NA	38	16	6	3	0	0	0	0	0
2 Addition facts to 5	32	25	10	5	1	0	1	1	0
3 Addition facts to 10	19	21	19	9	4	2	2	1	0
4 Addition with 10s & doubles	11	30	39	26	12	7	5	4	3
5 Addition facts		7	23	33	39	22	24	15	15
6 Subtraction & multiplication facts		1	3	15	27	29	30	25	43
7 Division facts		0	1	8	15	30	31	36	35
8 Common factors & multiples				1	2	9	6	18	5

Appendix E – continued

Percentages of Students at Particular Stages on the Number Framework as a Function of Year on the Project (First Year 2007 or Second Year 2008)

Basic Facts	**Y0–1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Y8**	**Y9**
Stage 4+	11	38	66	82	95	98	97	98	99
Stage 5+	0	8	27	57	83	91	92	94	97
Stage 6+	0	1	4	23	44	68	68	79	82
Stage 7+	0	0	1	8	17	40	38	54	39
Second year 2008	*274*	*278*	*360*	*449*	*451*	*589*	*2036*	*2447*	*2912*
0–1 NA	53	11	2	2	0	0	0	0	0
2 Addition facts to 5	27	21	9	2	1	1	1	0	0
3 Addition facts to 10	11	17	16	8	4	1	2	1	1
4 Addition with 10s & doubles	8	48	41	28	19	11	8	5	3
5 Addition facts	2	3	24	41	36	28	23	15	14
6 Subtraction & multiplication facts			6	14	25	30	30	27	41
7 Division facts			1	5	13	26	28	35	38
8 Common factors & multiples			0		2	4	8	16	4
Stage 4+	*10*	*51*	*73*	*88*	*95*	*99*	*97*	*99*	*99*
Stage 5+	*2*	*3*	*32*	*60*	*76*	*88*	*89*	*94*	*96*
Stage 6+	*0*	*0*	*8*	*19*	*40*	*60*	*66*	*79*	*83*
Stage 7+	*0*	*0*	*1*	*5*	*15*	*31*	*36*	*51*	*42*
Stage 4+ Change 07/08	***–2***	***13***	***7***	***6***	***1***	***1***	***0***	***0***	***0***
Stage 5+ Change 07/08	***2***	***–5***	***5***	***3***	***–7***	***–2***	***–3***	***0***	***–1***
Stage 6+ Change 07/08	***0***	***–1***	***3***	***–5***	***–4***	***–8***	***–2***	***–1***	***1***
Stage 7+ Change 07/08	***0***	***0***	***0***	***–4***	***–2***	***–9***	***–2***	***–3***	***3***

Appendix F (Longitudinal Patterns of Performance: Te Poutama Tau)

Table 1

GEE Analysis: Table Showing Beta and Significance for the Independent Variables for Each Dependent Variable

	Dependent Variable								
Independent Variable	Addition final	Multiplication final	Proportions final	FNWS final	BNWS final	NID final	Fractions final	Place value final	Basic facts final
N =	**1816**	**1447**	**1468**	**1815**	**1815**	**1736**	**1466**	**1816**	**1816**
Year	0.000	–0.256	–0.130	0.010	–0.044	0.089	–0.019	0.021	0.041
Additive initial	0.205	**–0.865 *****	**–0.186 ***	–0.065	–0.046	–0.048	0.030	0.066	0.161
Multiplicative initial	0.064	**–0.594 *****	**0.173 ***	–0.059	–0.047	0.109	0.050	0.091	0.095
Proportional initial	–0.093	**10.401 *****	**0.389 *****	–0.093	–0.196	**–0.195 ***	0.038	–0.055	–0.057
FNWS initial	0.156	**0.433 *****	0.032	**0.272 ***	**0.272 ***	0.041	**0.197 ****	0.069	0.088
BNWS initial	0.012	**0.311 ***	0.035	0.097	0.166	–0.026	–0.047	0.035	0.104
NID initial	0.004	**0.148 ***	0.039	0.046	0.021	**0.191 ***	0.029	–0.00003	–0.110
Fractions initial	0.001	**–10.184 *****	**0.291 *****	0.021	0.159	–0.048	**0.415 *****	0.082	0.049
Place Value initial	–0.033	**0.752 *****	–0.040	0.040	0.016	0.013	–0.005	**0.198 ****	0.075
Basic Facts initial	0.126	–0.070	0.080	0.130	0.088	0.082	**0.102 ***	**0.146 ***	**0.280 *****
Home Language	0.013	**–0.361 *****	0.022	0.074	0.018	–0.034	–0.137	–0.023	0.030
Māori Language	0.054	**0.444 ****	–0.005	0.012	–0.025	0.194	**0.223 ***	0.161	0.014
Class ID (institute ID)	***	***	***	***	***	***	***	***	***
Gender	00.032	–0.020	–0.022	0.096	0.054	0.013	**–0.093 ****	0.028	–0.024
Year by Additive initial	0.022	**0.132 *****	**0.064 *****	0.039	0.041	0.005	0.020	0.017	0.008
Year by Multiplicative initial	0.005	**0.142 *****	–0.007	0.021	0.020	–0.027	0.002	0.000	0.002
Year by Proportional initial	0.029	**–0.213 *****	–0.022	0.019	0.035	**0.041 ***	0.010	0.023	0.024
Year by FNWS initial	–0.016	**–0.094 *****	0.000	–0.014	–0.035	–0.009	–0.031	0.002	–0.002
Year by BNWS initial	–0.014	0.010	–0.006	–0.024	–0.008	–0.002	0.019	–0.016	–0.026
Year by NID initial	0.018	0.032	–0.006	0.010	0.013	–0.025	–0.011	0.011	0.022
Year by Fractions initial	–0.002	**0.192 *****	**–0.038 ***	–0.003	–0.029	0.015	–0.019	–0.002	–0.009
Year by Place Value initial	0.007	–0.102 ***	0.020	–0.010	–0.005	–0.010	0.009	–0.005	–0.005
Year by Basic Facts initial	–0.012	0.023	–0.007	–0.016	–0.010	–0.015	–0.013	–0.016	–0.003
Year by Home Language	0.003	**0.043 ***	–0.006	–0.008	0.003	0.010	0.032	0.008	–0.008
Year by Māori Language	–0.047	**–0.068 ***	0.003	–0.049	–0.041	**–0.058 ***	–0.034	–0.056	–0.032

Note: *** p < 0.001; ** p < 0.01; * p < 0.05

Appendix G (Linking Teacher Knowledge and Student Outcomes)

Teaching of Fractions Assessment

Note: The first part of each question (a) probes the teacher's CK. Remaining questions investigate the teacher's PCK.

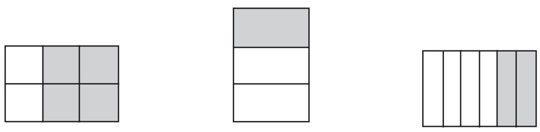
Teaching Fractions

Each of the questions below describes a teaching scenario and asks you to briefly outline the student's reasoning, the key understandings required to solve the problem, or the feedback and actions you would take as a teacher. Please be as specific as possible in your responses. You may like to mention what you would say to the student, how you would use any materials with the student, and any further examples you would use to help the student.

If you are unsure about the most appropriate response for the given situation, tick the box at the end of the question to indicate this.

Question One ☐ I am unsure of an appropriate response to Question One.

Which shape has $\frac{2}{3}$ of its area shaded?



Mark insists that none of the shapes have $\frac{2}{3}$ of their area shaded.

a) Do any of the shapes have $\frac{2}{3}$ of their area shaded? If yes, circle the correct answer to the problem.

b) What action, if any, do you take?

Question Two ☐ I am unsure of an appropriate response to Question Two.

You observe the following equation in Sally's work:

$$\frac{3}{5} + \frac{2}{3} = \frac{5}{8}$$

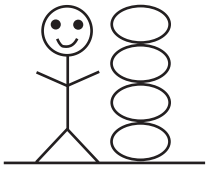
a) Is Sally correct?

You question Sally about her understanding, and she explains: I scored 3 goals out of 5 in the first half and 2 goals out of 3 in the second half. Overall, I scored 5 out of 8 goals.

b) What, if any, is the key understanding she needs to develop to solve this problem?

Question Three ☐ I am unsure of an appropriate response to Question Three.

This is Mr Short.



The height of Mr Short is 4 large buttons.
The height of Mr Tall is 6 large buttons.

When paper clips are used to measure Mr Short and Mr Tall, the height of Mr Short is 6 paper clips. What is the height of Mr Tall in paper clips?

Jim answers 8 paper clips.
Steve answers 9 paper clips.

a) Who is correct?

b) What is the possible reasoning behind **each** of their answers?

Question Four ☐ I am unsure of an appropriate response to Question Four.

Cameron is working on the following problem:

> There is $\frac{3}{4}$ of a pizza left after the party. $\frac{1}{3}$ of the leftovers are given to Sarah to take home. What fraction of a pizza does Sarah take home?

You hear Cameron say "One-third of three-quarters; that's the same as one-third times three-quarters."

a) What is the answer to the problem?

b) What action, if any, do you take?

Question Five ☐ I am unsure of an appropriate response to Question Five.

Sam is working on this problem and having difficulty:

> Order these fractions from the smallest to largest: $\frac{3}{5}, \frac{1}{3}, \frac{4}{8}.$

You question him about his thinking, and he explains, "I am trying to put them all over the same thing, but 8 doesn't go into 15."

a) What is the correct order for the fractions?

b) What feedback would you give Sam?

c) What action, if any, do you take?

Question Six ☐ I am unsure of an appropriate response to Question Six.

Julia is trying to answer the following problem but is having trouble getting started:

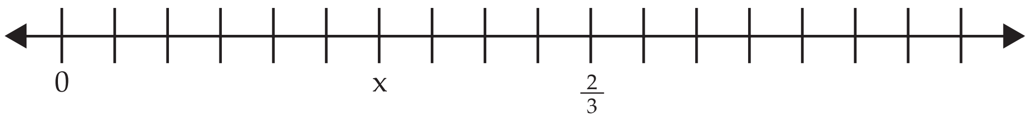
> Rebecca and Simon have the same number of rugby cards in their collections. 12 of Simon's cards are forwards, and this is $\frac{2}{3}$ of his collection. $\frac{5}{9}$ of Rebecca's collection are forwards, how many cards is this?

a) What is the answer to the problem?

b) What is the key understanding Julia needs to develop to get started on this problem?

Question Seven ☐ I am unsure of an appropriate response to Question Seven.

Grace is working with fractional number lines and looking at the following problem:



> If $\frac{2}{3}$ is at the point marked, what fraction should be at x?

Grace has recorded her answer to this problem as $\frac{6}{10}$.

a) What fraction should be at point x?

b) What is the possible reasoning behind Grace's answer?

c) What action, if any, do you take?

Appendix H (Explorations of Year 6 to Year 7 Transition in Numeracy)

Year 6 to Year 7 Transition in Numeracy, Phase One

Student Questionnaire

(Spaces for responses have been deleted in this version.)

Please complete the following questionnaire. The purpose of this questionnaire is to find out about your feelings and ideas about mathematics learning and teaching and the move to year 7.

Thank you for completing the questionnaire. Your answers to this questionnaire will only be read by the researchers. Please complete the background information below:

Name: ______________________________

Gender: Female ☐ Male ☐

Ethnicity: ____________________ Number of older brothers/sisters: ____________

Age: Years __________ Months __________

Date: ____________________

Tick one of the following boxes:

	Very true	True	Somewhat true	Not true
1. Mathematics is easy for me				
2. I like doing mathematics				

Please answer the following questions:

1. What do you enjoy the most about learning mathematics in year 6?
2. Is there anything that you don't enjoy about mathematics? Please explain.
3. How do you think year 7 mathematics will be different from year 6 mathematics?

4. How important do you think each of the following are in preparing you to do well in mathematics in year 7?

		Extremely important	Very important	Somewhat important	Not important
a.	Working in a group with other students				
b.	Working alone				
c.	Working with the teacher				
d.	Sharing your ideas in a large group				
e.	Working from a textbook				
f.	Working from a worksheet				
g.	Learning using games and activities				
h.	Knowing your basic facts				
i.	Being able to use a calculator				
j.	Explaining your strategy solutions				
k.	Convincing others about your mathematics thinking				
l.	Writing your own word problems				
m.	Learning from your mistakes in mathematics				
n.	Learning from the mistakes of others				
o.	Being able to ask for help in mathematics				
p.	Taking part in competitions.				

5. Choose from the list above, (a) to (p), which activities usually happen in your mathematics class in year 6.

6. What do you think are the most important things to do to be prepared for mathematics in year 7?

7. What have you been told about what will happen in mathematics in year 7?

Comments:

Parent Questionnaire

1. What do you think are the important factors that will enable a smooth transition from year 6 to year 7?

2. In what ways do you think your child needs to be prepared so that he/she will succeed in numeracy/mathematics in year 7?

3. Explain how you think your child has been prepared to succeed in numeracy/mathematics in year 7.

Student Focus-Group Interview

1. Who decided on your new school?
2. What things did you/your parents think about when choosing your new school?
3. In what ways have you been prepared for the move to intermediate school?
4. What do you think will be the main differences in maths at your new school?
5. What are you hoping maths will be like at your new school?
6. What do you hope your new maths teacher will be like?
7. Have you any expectations/hopes/concerns you'd like to talk about with the move to your new school?

Teacher Interview

1. Why do you think the transition is easier now that the intermediates are involved in the NDP?
2. Tell me more about the records that are passed on. Did the different intermediates your students "feed" to have different requirements regarding the numeracy records?
3. Tell me more about the liaison visits. Who came? With whom did they talk? What was on their agenda?
4. Did you have any placement concerns? How do you think the intermediate will address these?
5. Tell me more about your grouping and results of the students in relation to numeracy stages.
6. (Researcher to follow up on the expectations concerning gifted and talented students and Māori and Pasifika students.)

Teacher Questionnaire

Number of years' teaching experience:

Number of years teaching year 6 students:

Number of years involved in the NDP:

Comments:

	All of the time	Most of the time	Some of the time	Never
1. I am confident at teaching mathematics				
2. I believe that mathematics is one of the most important curriculum areas				
3. I emphasise the numeracy component of my mathematics				

4. The weighting I give numeracy in my mathematics programme is approx ______________%.

5. Outline how you prepare students in mathematics for the transition to year 7.

6. Do you use grouping? If so, how are your students grouped for mathematics?

7. Briefly outline a typical mathematics lesson or attach a mathematics lesson plan. You may want to consider the following:
   - Grouping (independent, pair, small group, whole class)
   - Student problem solving in small groups
   - Students explaining their thinking in small groups/large groups/whole class
   - Use of materials/equipment
   - Use of resource material such as numeracy material
   - Learning and teaching approach
   - Role of questioning

   Briefly describe the assessment practices you usually employ in numeracy.

9. What school-wide forms of record keeping are used?

10. Outline how mathematics records are transferred from year 6 to year 7.

11. Describe what is included in the transfer records in relation to numeracy (e.g., numeracy stage, achievement level, and mathematics disposition).

Any other comments?

Appendix I (Explorations of Year 6 to Year 7 Transition in Numeracy)

Year 6 to Year 7 Transition in Numeracy, Phase Two

Student Interview

(Spaces for responses have been deleted in this version.)

1. How do you think your shift to intermediate school has been?
2. How do you think your shift to intermediate school mathematics has been? Why?
3. What are some of the differences in mathematics at intermediate school? What are some of the similarities in mathematics to what you did at primary school?
4. Do you enjoy mathematics here more, less, or the same, as you did at primary school? Why?
5. Are you more or less confident in mathematics at intermediate school? Why?
6. If you were invited to talk to the year 6 students at your primary school about how to prepare for intermediate school in mathematics, what would you tell them is most important?

Lead Teacher/Class Teacher Interview

1. How long have you been teaching?
2. How confident are you teaching mathematics?
3. Can you please describe your daily mathematics lesson?
4. What grouping practices do you use? Does this differ for different students (i.e., for gifted and talented, Māori and Pasifika, special needs)?
5. What balance do you give to numeracy and the other mathematics strands? Does this differ for different students (i.e., for gifted and talented, Māori and Pasifika, special needs)?
6. What assessment practices do you use in the classroom and school-wide?
7. What mathematics data do you receive from primary schools and how do you use it?
8. What other information from primary schools do you think is important or you would ask for if you could?
9. What other programmes or other provisions are made for the gifted and talented, Māori and Pasifika, and special needs students?

Parent Questionnaire

Introduction:

Thank you for agreeing to this second phase of the research. This questionnaire contributes to this next phase that focuses on the post-transition experiences for students and school systems in relation to mathematics/numeracy. We would be most grateful if you would complete this questionnaire and return it in the pre-paid envelope. Thank you very much for taking the time to contribute to this research. If you would like a copy of the final report, please add your address.

Name:

Address:

1. How well do you think your child has made the move to their new school?
2. Have there been any issues (social/academic) that you have had to address since your child began at this new school?
3. What do you know about your child's current mathematics programme? How have you gained this information?
4. How well do you think your child is achieving in their current mathematics programme?
5. What have been the significant changes that you've noticed in terms of the mathematics programme?
6. What role does homework play?
7. What opportunities are you aware of for your child to take part in maths competitions?

Any other comments?

Appendix J (Embedding the Numeracy Development Projects in Two Schools)

Survey and Interview Questions

Teacher and School Leader Survey

(Spaces for responses have been deleted in this version.)

Name: .... *Year level taught: ....*

This survey is in three sections: sustaining the Numeracy Development Projects, networks of support and influence, and leadership.

Sustaining the Numeracy Development Projects

1. From your point of view, what are the three most important factors that have contributed to sustaining teachers' learning and development in numeracy in your school since your original involvement in the numeracy professional development? Briefly explain why these factors have been so important.
2. In what ways do teachers in your school work together to develop numeracy practice?
3. What evidence do you have that your practice is improving? Please be specific.
4. Briefly describe the numeracy professional development activities in which you have engaged this year.

Networks of Support and Influence

5. To illustrate the network of people who support and influence your numeracy instruction, please create a diagram on an A4 or A3 sheet of paper. Please label people by their roles (e.g., lead teacher of numeracy or year 3 teacher in neighbouring school) rather than by their names. This will allow for patterns to be identified between diagrams, as well as maintaining a degree of anonymity. Please also label the links between people to describe the ways in which they influence and/or support you (e.g., share resources, or plan collaboratively with me). You should include both formal and informal sources of support (e.g., lead teacher of numeracy, sister-in-law who's a teacher, respectively) over the last couple of years. For people who are outside your own school, please indicate how you came to know them (e.g., met at regional numeracy professional development day or taught together at a previous school).

Leadership

6. In what ways have instructional leaders (school management, numeracy lead teachers, etc.) been effective in supporting your ongoing learning and development in numeracy?
7. From your point of view, what are the most important things leaders can do to support your ongoing learning and development in numeracy?

Thank you for completing this survey.

Interview Questions for Teachers

This interview is divided into two sections. In the first section, your diagram will provide a focus for developing your views about professional networks. The second part of the interview will focus on leadership and will overlap with and build on some of the questions from the survey.

First, let's return to your diagram. (Use a different colour pen for info added during the interview and take 2 photocopies; 1 as a back-up and 1 to make notes on during the interview.)

1. Would you please label the strongest 3 supports 1 to 3, with 1 the strongest support.
2. Next, please indicate which of these supports is formal (by recording F) and which are informal (by recording I).
3. Now, please note next to each one its timing and duration, e.g., was it a workshop that occurred once and lasted half a day, or is it something that happens every month for an hour?
4. Please tell me whether these are one-off, isolated supports, or if they're connected – if so, how?
5. What was the focus of each one – instructional or managerial/administrative or personal content knowledge, etc.?

Thinking about professional networks, both official and unofficial:

6. What are the official sources of support you can call on to assist your teaching of numeracy? Which of these do you use and why? What form does the support take? Which do you <u>not</u> use and why?
7. What are the unofficial sources of support you call on? Why do you call on these people? What form does the support take? If these people are outside this school, how did you come to know them? (e.g., met at regional numeracy PD day or short course, taught at the same school, worked on some other PD together in the past.)
8. Within these different groups, or communities of practice, what is your role?
9. How does your school organisation support your engagement in these networks?
10. What else could a school do to further enable engagement in these networks?
11. School structures and practices are generally intended to support you in your role as ______________________________. In what ways can you be constrained by school structures?

Thinking now about leadership in numeracy:

12. In what ways does your school's leadership contribute to building more effective numeracy instruction?
13. How do you see your own role regarding the leadership of ongoing numeracy learning and support in relation to the school's formal numeracy leaders?
14. Thinking about the notion/concept of leadership, rather than thinking about the actual people in leadership roles in your school, what do you see is the role of leadership in relation to the continued development of effective numeracy instruction? (Cobb, McClain, de Silva, & Dean, 2003, p. 20)
15. How is your vision for numeracy teaching and learning the same as/different from your leadership team's vision? (Cobb et al., 2003, Cobb & Smith, 2008)

Interview Questions for School Leaders

The first series of questions relates to the role of school leadership in supporting teachers' ongoing learning and development in numeracy.

1. What's your vision for numeracy instruction in your school?
   Why? How do you plan to do this?
2. What can you tell me about your school's strategic goals for teaching and learning in numeracy? How does this influence your role?
3. What is your role in supporting teachers' sustained professional learning and development in numeracy?
4. What are some of the opportunities and challenges of this role?
5. From your point of view, what are the most important things leaders can do to support teachers' ongoing learning and development in numeracy?
6. What actions have you taken to promote professional learning opportunities for your teachers?
7. What actions have you taken to promote the implementation of new practices in classrooms?
8. How do you go about creating the conditions for developing and distributing leadership in the school?
9. How do you see the role of your teachers regarding the leadership of ongoing numeracy learning and support in relation to the school's formal numeracy leaders?
10. To what degree have you actively participated in the numeracy learning and development in which your teachers have engaged? Why is this?

The following questions are to do with how numeracy is being sustained in the school.

11. What school structures and practices support the sustainability of the numeracy initiative?
12. What actions have sustained the increased levels of student achievement? Who took those actions?
13. In what ways is your student data used, and by whom?
14. Is there any evidence of teachers working together to keep the project going? Have you, for example, noticed more conversations about numeracy in the staffroom or numeracy as the topic of staff meetings?
15. Has the focus of teachers' performance appraisal been influenced by the numeracy professional development?
16. Finally, thinking about communities or networks of practice, are you involved in any groups within the school or beyond, which you haven't already mentioned, whose agendas involve numeracy instruction? What role do they play in relation to numeracy instruction? Within these different groups, or communities of practice, what is your role?

Appendix K (The Impact of Two Professional Development Programmes for Numeracy "Pick Ups": Teachers' Perceptions of Valued Aspects)

Professional Development Programme Evaluation

Part A: Course(s) and/or Sessions

Please respond to the following two statements on the scales provided by circling a response.

1. The material of the course/sessions was valuable and helpful in meeting my needs.

   Very strongly agree　Strongly agree　Moderately agree　Agree　Disagree　Strongly disagree

2. The time and energy involved in attending the course/sessions was worthwhile.

   Very strongly agree　Strongly agree　Moderately agree　Agree　Disagree　Strongly disagree

3. Now, please *circle up to three elements* in the grid below that were *most helpful* for you in learning to work more effectively with the students in your class[7].

Please cross out any that do not apply to you.

Learning more about what students need to be doing next in mathematics 3a[8]	Improving my personal understanding of multiplication/division 1c	The teaching practices used by the lecturer/facilitator 5a	Improving my personal understanding of students' learning progressions in fractions 2d	The format and conduct of the sessions (the way the lecturer/facilitator taught) was a useful model for my classroom. 5b
Improving my personal understanding of percentages 1f	Improving my personal understanding of whole numbers 1a	Improving my personal understanding of fractions 1d	Improving my personal understanding of decimals 1e	Learning more about how to organise my classroom effectively 3c
Improving my personal understanding of addition/subtraction 1b	Improving my personal understanding of students' learning progressions in addition/subtraction 2b	Making me feel like a learner again 4a	Gaining a greater understanding of what learning content (for example, place value) is like for students 4b	Improving my personal understanding of students' learning progressions in whole numbers 2a
Learning more about available resources and how to use them 3d	Improving my personal understanding of students' learning progressions in decimals 2e	Improving my personal understanding of students' learning progressions in percentages 2f	Learning more about what I need to be doing next when teaching students 3b	Improving my personal understanding of students' learning progressions in multiplication/division 2c

Any other comments:

[7] The elements are arranged randomly in this table and in the table in Part B, as they were in the study, to allow for the possibility of replicating the study.

[8] The elements are numbered in this table and in the table in Part B for reference purposes only, within this paper.

Part B: In-class Support

Please respond to the following two statements on the scales provided by circling a response.

4. The in-class support was valuable and helpful in meeting my needs.

   Very strongly agree   Strongly agree   Moderately agree   Agree   Disagree   Strongly disagree

5. The time and energy involved with in-class support was worthwhile.

   Very strongly agree   Strongly agree   Moderately agree   Agree   Disagree   Strongly disagree

6. Now, please *circle up to three elements* in the grid below that were *most helpful* for you in working more effectively with the students in your class.

Please cross out any that do not apply to you.

Being observed by, and having follow-up discussions with, a facilitator 1a	Being observed by, and having follow-up discussions with, a lead teacher 1b	Observing a facilitator teaching in a class 2c	Observing a lead teacher teaching in a class 2d	Sharing planning with the facilitator 3a
Discussing my own numeracy teaching issues with (an)other teacher(s) 6	Sharing the teaching of my class or a group with a lead teacher 3e	Sharing planning with (an)other teacher(s) 3c	Being given resources by the facilitator 4a	Observing a lead teacher teaching in my class 2b
Sharing the teaching of my class or a group with another teacher 3f	Choosing the type of support I will receive 5	Observing a facilitator teaching in my class 2a	Observing another classroom teacher teaching their class 2e	Sharing planning with a lead teacher 3b
Being given resources by a lead teacher 4b	Being given resources by another teacher(s) 4c	Being observed by, and having follow-up discussions with another teacher 1c	Sharing the teaching of my class or a group with the facilitator 3d	Having release time to work one-to-one with students 7

Any other comments:

Appendix L (Numeracy Development Projects' Patterns of Performance and Progress: National Pick-up Programme 2008)

Table 17[1]

Performance of Year 0–1 Students on the Additive Domain

	Ethnicity			Gender		
	NZE	Māori	Pasifika	Male	Female	Total
Initial						
0: Emergent	9%	17%	26%	16%	13%	14%
1: One-to-one counting	25%	24%	22%	22%	24%	23%
2: Counting from one on materials	50%	43%	47%	49%	47%	48%
3: Counting from one by imaging	12%	13%	3%	11%	12%	11%
4: Advanced counting	3%	1%	2%	3%	3%	3%
5: Early additive part–whole	0%	1%		0%	1%	0%
6: Advanced additive part–whole						
7: Advanced multiplicative part–whole						
N =	218	75	58	234	196	430
Final						
0: Emergent	0%	1%	5%	2%	1%	1%
1: One-to-one counting	4%	4%	7%	6%	4%	5%
2: Counting from one on materials	37%	49%	55%	39%	43%	41%
3: Counting from one by imaging	33%	29%	21%	31%	29%	30%
4: Advanced counting	24%	15%	12%	20%	22%	21%
5: Early additive part–whole	2%	1%		3%	2%	2%
6: Advanced additive part–whole						
7: Advanced multiplicative part–whole						
N =	218	75	58	234	196	430

1 All the tables in Appendix L show rounded percentages. Percentages less than 0.5% are therefore shown as 0% and, where there are no students represented, the cell is left blank. Due to rounding, percentages in some tables may not total to 100.

Table 18
Performance of Year 0–1 Students on the FNWS Domain

	Ethnicity			Gender		
	NZE	Māori	Pasifika	Male	Female	Total
Initial						
0: Emergent	4%	11%	17%	6%	9%	7%
1: Initial to 10	18%	31%	31%	21%	22%	22%
2: To 10	39%	35%	24%	35%	36%	35%
3: To 20	28%	17%	22%	26%	24%	25%
4: To 100	10%	7%	5%	12%	9%	10%
5: To 1000	1%			1%		1%
6: To 1 000 000						
N =	218	75	58	234	196	430
Final						
0: Emergent			3%	1%	1%	1%
1: Initial to 10	2%	8%	9%	4%	5%	4%
2: To 10	15%	25%	16%	17%	16%	16%
3: To 20	37%	33%	43%	32%	38%	35%
4: To 100	39%	32%	28%	40%	37%	39%
5: To 1000	7%	1%		5%	4%	4%
6: To 1 000 000			2%	1%		0%
N =	218	75	58	234	196	430

Table 19
Performance of Year 0–1 Students on the Place Value Domain

	Ethnicity			Gender		
	NZE	Māori	Pasifika	Male	Female	Total
Initial						
0–1: Counts by ones	41%	48%	55%	48%	40%	45%
2: Counts in ones	54%	49%	45%	48%	54%	51%
3: Counts in fives and ones	0%			0%	1%	1%
4: 10s to 100, orders to 1000	4%	3%		3%	5%	4%
5: 10s to 1000, orders to 10 000	0%			0%		0%
6: 10s, 100s, 1000s, orders whole numbers						
N =	218	75	58	234	196	430
Final						
0–1: Counts by ones	10%	12%	19%	15%	9%	12%
2: Counts in ones	54%	57%	64%	50%	60%	55%
3: Counts in fives and ones	13%	16%	7%	11%	13%	12%
4: 10s to 100, orders to 1000	22%	15%	10%	23%	17%	20%
5: 10s to 1000, orders to 10 000	1%			1%	1%	1%
6: 10s, 100s, 1000s, orders whole numbers						
N =	218	75	58	234	196	430

Table 20
Performance of Year 0–1 Students on the Basic Facts Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Non-grouping	83%	96%	93%	84%	85%	84%
2: Facts to 5	13%	1%	3%	10%	12%	11%
3: Facts to 10	3%	1%	3%	3%	3%	3%
4: Within 10, doubles and teens	1%	1%		3%	1%	2%
5: Addition, multiplication for 2, 5, 10	0%			0%		0%
6: Subtraction and multiplication						
7: Division						
N =	218	75	58	234	196	430
Final						
0–1: Non-grouping	44%	57%	64%	49%	47%	48%
2: Facts to 5	32%	23%	21%	23%	33%	27%
3: Facts to 10	14%	15%	10%	16%	13%	15%
4: Within 10, doubles and teens	8%	4%	5%	10%	6%	8%
5: Addition, multiplication for 2, 5, 10	1%	1%		1%	1%	1%
6: Subtraction and multiplication				0%		0%
7: Division						
N =	218	75	58	234	196	430

Table 21
Performance of Year 2 Students on the Additive Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0: Emergent			2%		1%	1%
1: One-to-one counting	8%	15%	9%	12%	8%	10%
2: Counting from one on materials	45%	46%	45%	43%	45%	44%
3: Counting from one by imaging	22%	19%	26%	20%	26%	23%
4: Advanced counting	20%	15%	16%	19%	16%	17%
5: Early additive part–whole	6%	5%	1%	6%	4%	5%
6: Advanced additive part–whole						
7: Advanced multiplicative part–whole						
N =	239	74	91	241	257	498

Table 21 – *continued*

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Final						
0: Emergent					0%	0%
1: One-to-one counting	1%	1%		1%	1%	1%
2: Counting from one on materials	14%	26%	22%	16%	18%	17%
3: Counting from one by imaging	21%	20%	19%	20%	20%	20%
4: Advanced counting	48%	38%	49%	45%	48%	46%
5: Early additive part–whole	15%	14%	10%	16%	12%	14%
6: Advanced additive part–whole	0%	1%		2%	2%	2%
7: Advanced multiplicative part–whole						
N =	239	74	91	241	257	498

Table 22
Performance of Year 2 Students on the FNWS Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0: Emergent			3%	0%	1%	1%
1: Initial to 10	4%	5%	4%	7%	5%	6%
2: To 10	17%	26%	14%	15%	18%	16%
3: To 20	31%	34%	31%	29%	34%	32%
4: To 100	37%	27%	41%	34%	37%	35%
5: To 1000	12%	8%	7%	15%	6%	10%
6: To 1 000 000						
N =	239	74	91	241	257	498
Final						
0: Emergent						
1: Initial to 10	1%			0%	0%	0%
2: To 10	3%	3%	7%	4%	4%	4%
3: To 20	15%	28%	24%	19%	20%	19%
4: To 100	49%	38%	48%	42%	51%	46%
5: To 1000	31%	28%	18%	31%	24%	27%
6: To 1 000 000	1%	3%	3%	4%	2%	3%
N =	239	74	91	241	257	498

Table 23
Performance of Year 2 Students on the Place Value Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Counts by ones	10%	7%	16%	14%	10%	12%
2: Counts in ones	52%	65%	57%	51%	60%	55%
3: Counts in fives and ones	7%	7%	9%	5%	8%	7%
4: 10s to 100, orders to 1000	30%	22%	18%	29%	23%	26%
5: 10s to 1000, orders to 10 000	0%			1%		0%
6: 10s, 100s, 1000s, orders whole numbers						
N =	239	74	91	241	257	498
Final						
0–1: Counts by ones	3%			1%	2%	1%
2: Counts in ones	25%	42%	40%	30%	28%	29%
3: Counts in fives and ones	10%	15%	13%	8%	16%	12%
4: 10s to 100, orders to 1000	58%	38%	43%	52%	51%	51%
5: 10s to 1000, orders to 10 000	5%	4%	2%	7%	4%	5%
6: 10s, 100s, 1000s, orders whole numbers	0%	1%	2%	1%	1%	1%
N =	239	74	91	241	257	498

Table 24
Performance of Year 2 Students on the Basic Facts Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Non-grouping	47%	47%	45%	46%	49%	48%
2: Facts to 5	29%	27%	24%	28%	28%	28%
3: Facts to 10	5%	8%	22%	10%	8%	9%
4: Within 10, doubles and teens	18%	12%	9%	14%	14%	14%
5: Addition, multiplication for 2, 5, 10	0%	4%		2%	1%	1%
6: Subtraction and multiplication		1%			0%	0%
7: Division						
N =	239	74	91	241	257	498

Table 24 – *continued*

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Final						
0–1: Non-grouping	11%	16%	20%	12%	15%	13%
2: Facts to 5	18%	30%	26%	21%	21%	21%
3: Facts to 10	23%	15%	15%	17%	22%	20%
4: Within 10, doubles and teens	43%	34%	35%	41%	37%	39%
5: Addition, multiplication for 2, 5, 10	4%	3%	2%	5%	4%	4%
6: Subtraction and multiplication	1%	1%	1%	2%	1%	2%
7: Division		1%		1%		1%
N =	239	74	91	241	257	498

Table 25
Performance of Year 3 Students on the Additive Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0: Emergent						
1: One-to-one counting	5%	9%	3%	5%	6%	6%
2: Counting from one on materials	17%	20%	10%	20%	13%	17%
3: Counting from one by imaging	21%	18%	21%	21%	19%	20%
4: Advanced counting	37%	36%	62%	32%	46%	38%
5: Early additive part–whole	18%	16%	3%	19%	14%	17%
6: Advanced additive part–whole	2%			3%	1%	2%
7: Advanced multiplicative part–whole						
N =	195	55	29	180	145	325
Final						
0: Emergent						
1: One-to-one counting	1%			1%	1%	1%
2: Counting from one on materials	2%	4%		2%	1%	2%
3: Counting from one by imaging	7%	13%	7%	6%	8%	7%
4: Advanced counting	47%	51%	52%	47%	47%	47%
5: Early additive part–whole	35%	33%	38%	34%	37%	36%
6: Advanced additive part–whole	8%		3%	7%	5%	6%
7: Advanced multiplicative part–whole	2%			3%	1%	2%
N =	195	55	29	180	145	325

Table 26
Performance of Year 3 Students on the FNWS Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0: Emergent						
1: Initial to 10		4%		1%	1%	1%
2: To 10	6%	2%	4%	6%	5%	
3: To 20	13%	18%	10%	12%	16%	14%
4: To 100	38%	47%	59%	39%	47%	42%
5: To 1000	34%	29%	31%	37%	29%	33%
6: To 1 000 000	8%			8%	1%	5%
N =	195	55	29	180	145	325
Final						
0: Emergent						
1: Initial to 10	1%			1%		0%
2: To 10					1%	0%
3: To 20	3%	11%		4%	4%	4%
4: To 100	26%	27%	31%	22%	31%	26%
5: To 1000	54%	51%	55%	56%	51%	54%
6: To 1 000 000	16%	11%	14%	18%	13%	16%
N =	195	55	29	180	145	325

Table 27
Performance of Year 3 Students on the Place Value Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Counts by ones	4%	2%		3%	4%	3%
2: Counts in ones	21%	40%	31%	23%	28%	25%
3: Counts in fives and ones	8%	15%	14%	9%	11%	10%
4: 10s to 100, orders to 1000	56%	42%	52%	54%	49%	52%
5: 10s to 1000, orders to 10 000	10%	2%	3%	11%	8%	9%
6: 10s, 100s, 1000s, orders whole numbers	1%				1%	0%
7: Tenths in and orders decimals						
N =	195	55	29	180	145	325

Table 27 – *continued*

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Final						
0–1: Counts by ones	1%			1%	1%	1%
2: Counts in ones	6%	7%	3%	5%	6%	6%
3: Counts in fives and ones	11%	13%	7%	8%	12%	10%
4: 10s to 100, orders to 1000	53%	65%	62%	60%	48%	55%
5: 10s to 1000, orders to 10 000	25%	13%	24%	21%	28%	24%
6: 10s, 100s, 1000s, orders whole numbers	4%	2%	3%	5%	4%	5%
7: Tenths in and orders decimals	1%				1%	0%
N =	195	55	29	180	145	325

Table 28
Performance of Year 3 Students on the Basic Facts Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Non-grouping	18%	24%	14%	17%	20%	18%
2: Facts to 5	18%	18%	14%	17%	19%	18%
3: Facts to 10	16%	25%	28%	19%	17%	18%
4: Within 10, doubles and teens	35%	29%	41%	33%	35%	34%
5: Addition, multiplication for 2, 5, 10	8%	4%	3%	8%	6%	7%
6: Subtraction and multiplication	3%			4%	1%	3%
7: Division	2%			2%	2%	2%
N =	195	55	29	180	145	325
Final						
0–1: Non-grouping	4%	2%		1%	4%	2%
2: Facts to 5	6%	7%		3%	8%	5%
3: Facts to 10	10%	11%	14%	11%	11%	11%
4: Within 10, doubles and teens	42%	55%	45%	44%	41%	42%
5: Addition, multiplication for 2, 5, 10	30%	24%	41%	32%	30%	31%
6: Subtraction and multiplication	5%	2%		5%	5%	5%
7: Division	3%			4%	2%	3%
N =	195	55	29	180	145	325

Table 29
Performance of Year 4 Students on the Additive Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0: Emergent						
1: One-to-one counting	1%			1%	1%	1%
2: Counting from one on materials	7%	9%		8%	4%	6%
3: Counting from one by imaging	4%	9%	15%	7%	6%	6%
4: Advanced counting	40%	48%	61%	34%	52%	43%
5: Early additive part–whole	38%	33%	24%	39%	34%	37%
6: Advanced additive part–whole	9%			11%	3%	7%
7: Advanced multiplicative part–whole				1%		0%
N =	141	33	33	121	125	246
Final						
0: Emergent						
1: One-to-one counting						
2: Counting from one on materials				1%		0%
3: Counting from one by imaging	3%	3%	3%	3%	2%	3%
4: Advanced counting	24%	42%	55%	26%	34%	30%
5: Early additive part–whole	50%	39%	42%	42%	55%	49%
6: Advanced additive part–whole	21%	15%		26%	8%	17%
7: Advanced multiplicative part–whole	1%			2%	1%	1%
N =	141	33	33	121	125	246

Table 30
Performance of Year 4 Students on the Multiplicative Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
Not rated	11%	21%	30%	17%	14%	15%
2–3: Counting from one	13%	18%	18%	12%	16%	14%
4: Advanced counting	43%	45%	39%	39%	50%	44%
5: Early additive part–whole	24%	12%	12%	21%	16%	19%
6: Advanced additive part–whole	8%	3%		10%	4%	7%
7: Advanced multiplicative part–whole	1%			2%		1%
8: Advanced proportional part–whole						
N =	141	33	33	121	125	246

Table 30 – *continued*

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Final						
Not rated	4%	6%	12%	7%	5%	6%
2–3: Counting from one	2%	9%	9%	3%	4%	4%
4: Advanced counting	28%	48%	39%	30%	38%	34%
5: Early additive part–whole	40%	24%	27%	26%	42%	35%
6: Advanced additive part–whole	21%	12%	12%	27%	10%	19%
7: Advanced multiplicative part–whole	6%			6%	1%	3%
8: Advanced proportional part–whole				1%		0%
N =	141	33	33	121	125	246

Table 31

Performance of Year 4 Students on the Proportional Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
Not rated	18%	24%	24%	21%	17%	19%
1: Unequal sharing	6%	15%	21%	10%	9%	9%
2–4: Equal sharing	54%	48%	45%	45%	60%	52%
5: Early additive part–whole	18%	12%	9%	18%	11%	15%
6: Advanced additive part–whole	3%			5%	3%	4%
7: Advanced multiplicative part–whole	1%			2%		1%
8: Advanced proportional part–whole						
N =	141	33	33	121	125	246
Final						
Not rated	9%	9%	12%	10%	7%	9%
1: Unequal sharing	1%	3%	9%	3%	2%	2%
2–4: Equal sharing	27%	58%	45%	33%	40%	37%
5: Early additive part–whole	47%	27%	21%	36%	37%	37%
6: Advanced additive part–whole	13%	3%	12%	12%	13%	13%
7: Advanced multiplicative part–whole	4%			4%	2%	3%
8: Advanced proportional part–whole				1%		0%
N =	141	33	33	121	125	246

Table 32
Performance of Year 4 Students on the Fractions Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
Not assessed	13%	24%	27%	17%	15%	16%
2–3: Non-fractions	38%	45%	52%	39%	40%	39%
4: Assigns unit fractions	23%	27%	18%	18%	26%	22%
5: Orders unit fractions	24%	3%	3%	21%	18%	19%
6: Co-ordinates numerators/denominators	2%			3%	1%	2%
7: Equivalent fractions	1%			1%		0%
8: Orders fractions				1%		0%
N =	141	33	33	121	125	246
Final						
Not assessed	4%	9%	12%	7%	6%	6%
2–3: Non-fractions	9%	27%	12%	12%	10%	11%
4: Assigns unit fractions	23%	24%	33%	24%	27%	26%
5: Orders unit fractions	53%	33%	42%	47%	50%	48%
6: Co-ordinates numerators/denominators	7%	6%		6%	7%	7%
7: Equivalent fractions	2%			2%	1%	2%
8: Orders fractions	1%			2%		1%
N =	141	33	33	121	125	246

Table 33
Performance of Year 4 Students on the Place Value Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Counts by ones	1%			2%	1%	1%
2: Counts in ones	9%	15%	15%	7%	11%	9%
3: Counts in fives and ones	9%	9%	18%	7%	12%	10%
4: 10s to 100, orders to 1000	65%	64%	55%	64%	62%	63%
5: 10s to 1000, orders to 10 000	14%	12%	12%	16%	14%	15%
6: 10s, 100s, 1000s, orders whole numbers	1%			1%		0%
7: Tenths in and orders decimals				1%		0%
8: Tenths, hundredths, and thousandths	1%			2%		1%
N =	141	33	33	121	125	246

Table 33 – *continued*

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Final						
0–1: Counts by ones						
2: Counts in ones	2%		3%	1%	3%	2%
3: Counts in fives and ones	4%	9%	6%	6%	3%	4%
4: 10s to 100, orders to 1000	50%	52%	76%	45%	58%	52%
5: 10s to 1000, orders to 10 000	31%	36%	12%	33%	30%	32%
6: 10s, 100s, 1000s, orders whole numbers	11%	3%	3%	12%	5%	9%
7: Tenths in and orders decimals	1%			1%		0%
8: Tenths, hundredths, and thousandths	1%			2%		1%
N =	141	33	33	121	125	246

Table 34

Performance of Year 4 Students on the Basic Facts Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Non-grouping	5%	3%		2%	5%	3%
2: Facts to 5	9%	15%	15%	9%	10%	10%
3: Facts to 10	8%	12%	21%	13%	6%	10%
4: Within 10, doubles and teens	38%	52%	42%	32%	50%	41%
5: Addition, multiplication for 2, 5, 10	28%	18%	18%	28%	22%	25%
6: Subtraction and multiplication	11%		3%	13%	6%	9%
7: Division	1%			2%	1%	2%
8: Common factors, multiples						
N =	141	33	33	121	125	246
Final						
0–1: Non-grouping						
2: Facts to 5	2%	6%		2%	2%	2%
3: Facts to 10	6%	15%	9%	10%	5%	7%
4: Within 10, doubles and teens	24%	21%	36%	15%	30%	23%
5: Addition, multiplication for 2, 5, 10	40%	39%	48%	40%	45%	43%
6: Subtraction and multiplication	23%	18%	6%	27%	15%	21%
7: Division	4%			3%	2%	3%
8: Common factors, multiples	1%			2%		1%
N =	141	33	33	121	125	246

Table 35
Performance of Year 5 Students on the Additive Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0: Emergent	0%	0%	0%	0%	0%	0%
1: One-to-one counting	0%	0%	0%	0%	0%	0%
2: Counting from one on materials	2%	0%	4%	2%	2%	2%
3: Counting from one by imaging	2%	0%	0%	0%	2%	1%
4: Advanced counting	27%	52%	40%	30%	35%	32%
5: Early additive part–whole	49%	41%	52%	48%	48%	48%
6: Advanced additive part–whole	19%	7%	4%	19%	13%	16%
7: Advanced multiplicative part–whole	1%	0%	0%	2%	0%	1%
N =	124	29	25	108	98	206
Final						
0: Emergent	0%	0%	0%	0%	0%	0%
1: One-to-one counting	0%	0%	0%	0%	0%	0%
2: Counting from one on materials	0%	0%	0%	0%	0%	0%
3: Counting from one by imaging	1%	0%	0%	1%	0%	0%
4: Advanced counting	10%	17%	12%	8%	15%	12%
5: Early additive part–whole	49%	69%	84%	50%	61%	55%
6: Advanced additive part–whole	36%	10%	0%	33%	20%	27%
7: Advanced multiplicative part–whole	2%	3%	4%	6%	3%	5%
N =	124	29	25	108	98	206

Table 36
Performance of Year 5 Students on the Multiplicative Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
Not rated	6%	10%	12%	4%	9%	6%
2–3: Counting from one	3%	10%	8%	5%	5%	5%
4: Advanced counting	40%	45%	32%	35%	40%	37%
5: Early additive part–whole	34%	28%	16%	32%	29%	31%
6: Advanced additive part–whole	14%	7%	28%	20%	14%	17%
7: Advanced multiplicative part–whole	4%	0%	4%	4%	3%	3%
8: Advanced proportional part–whole	0%	0%	0%	0%	0%	0%
N =	124	29	25	108	98	206

Table 36 – *continued*

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Final						
Not rated	1%	0%	0%	0%	1%	0%
2–3: Counting from one	3%	3%	0%	3%	2%	2%
4: Advanced counting	18%	31%	12%	15%	23%	19%
5: Early additive part–whole	33%	34%	44%	31%	38%	34%
6: Advanced additive part–whole	33%	31%	40%	36%	30%	33%
7: Advanced multiplicative part–whole	10%	0%	4%	14%	5%	10%
8: Advanced proportional part–whole	2%	0%	0%	2%	1%	1%
N =	124	29	25	108	98	206

Table 37
Performance of Year 5 Students on the Proportional Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
Not rated	8%	14%	12%	7%	9%	8%
1: Unequal sharing	1%	14%	8%	5%	3%	4%
2–4: Equal sharing	53%	52%	36%	47%	50%	49%
5: Early additive part–whole	23%	21%	24%	24%	24%	24%
6: Advanced additive part–whole	8%	0%	20%	8%	12%	10%
7: Advanced multiplicative part–whole	6%	0%	0%	7%	1%	4%
8: Advanced proportional part–whole	1%	0%	0%	1%	0%	0%
N =	124	29	25	108	98	206
Final						
Not rated	3%	3%	0%	4%	1%	2%
1: Unequal sharing	1%	7%	0%	2%	1%	1%
2–4: Equal sharing	21%	24%	32%	23%	20%	22%
5: Early additive part–whole	40%	48%	40%	32%	50%	41%
6: Advanced additive part–whole	26%	17%	24%	25%	22%	24%
7: Advanced multiplicative part–whole	7%	0%	4%	12%	5%	9%
8: Advanced proportional part–whole	2%	0%	0%	2%	0%	1%
N =	124	29	25	108	98	206

Table 38
Performance of Year 5 Students on the Fractions Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
Not assessed	6%	14%	8%	3%	10%	6%
2–3: Non-fractions	18%	38%	32%	27%	15%	21%
4: Assigns unit fractions	25%	34%	40%	29%	28%	28%
5: Orders unit fractions	43%	14%	16%	31%	40%	35%
6: Co-ordinates numerators/denominators	5%	0%	4%	5%	5%	5%
7: Equivalent fractions	2%	0%	0%	3%	2%	2%
8: Orders fractions	2%	0%	0%	4%	0%	2%
N =	124	29	25	108	98	206
Final						
Not assessed	1%	0%	0%	0%	1%	0%
2–3: Non-fractions	2%	7%	8%	5%	3%	4%
4: Assigns unit fractions	9%	38%	32%	19%	15%	17%
5: Orders unit fractions	60%	52%	48%	50%	58%	54%
6: Co-ordinates numerators/denominators	16%	0%	12%	11%	17%	14%
7: Equivalent fractions	10%	3%	0%	9%	5%	7%
8: Orders fractions	2%	0%	0%	6%	0%	3%
N =	124	29	25	108	98	206

Table 39
Performance of Year 5 Students on the Place Value Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Counts by ones	0%	0%	0%	0%	0%	0%
2: Counts in ones	2%	7%	4%	4%	2%	3%
3: Counts in fives and ones	3%	3%	8%	3%	6%	4%
4: 10s to 100, orders to 1000	56%	69%	68%	53%	60%	56%
5: 10s to 1000, orders to 10 000	24%	21%	20%	25%	27%	26%
6: 10s, 100s, 1000s, orders whole numbers	11%	0%	0%	12%	4%	8%
7: Tenths in and orders decimals	1%	0%	0%	2%	1%	1%
8: Tenths, hundredths, and thousandths	2%	0%	0%	2%	0%	1%
N =	124	29	25	108	98	206

Table 39 – *continued*

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Final						
0–1: Counts by ones	1%	0%	0%	1%	0%	0%
2: Counts in ones	1%	0%	4%	1%	1%	1%
3: Counts in fives and ones	1%	7%	4%	3%	1%	2%
4: 10s to 100, orders to 1000	25%	38%	44%	23%	35%	29%
5: 10s to 1000, orders to 10 000	52%	48%	48%	49%	50%	50%
6: 10s, 100s, 1000s, orders whole numbers	11%	7%	0%	11%	10%	11%
7: Tenths in and orders decimals	8%	0%	0%	9%	3%	6%
8: Tenths, hundredths, and thousandths	1%	0%	0%	3%	0%	1%
N =	124	29	25	108	98	206

Table 40
Performance of Year 5 Students on the Basic Facts Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Non-grouping	1%	3%	0%	0%	2%	1%
2: Facts to 5	3%	10%	0%	2%	5%	3%
3: Facts to 10	7%	3%	4%	5%	8%	6%
4: Within 10, doubles and teens	26%	45%	28%	28%	26%	27%
5: Addition, multiplication for 2, 5, 10	39%	31%	32%	37%	34%	35%
6: Subtraction and multiplication	15%	7%	32%	19%	18%	19%
7: Division	7%	0%	4%	6%	7%	6%
8: Common factors, multiples	2%	0%	0%	4%	0%	2%
N =	124	29	25	108	98	206
Final						
0–1: Non-grouping	0%	7%	0%	0%	2%	1%
2: Facts to 5	2%	0%	0%	2%	0%	1%
3: Facts to 10	1%	3%	4%	3%	1%	2%
4: Within 10, doubles and teens	12%	17%	8%	4%	22%	13%
5: Addition, multiplication for 2, 5, 10	45%	48%	32%	46%	37%	42%
6: Subtraction and multiplication	24%	14%	52%	24%	29%	26%
7: Division	15%	10%	4%	17%	9%	13%
8: Common factors, multiples	2%	0%	0%	5%	0%	2%
N =	124	29	25	108	98	206

Table 41
Performance of Year 6 Students on the Additive Domain

	Ethnicity			Gender		
	NZE	Māori	Pasifika	Male	Female	Total
Initial						
0: Emergent	0%	0%	0%	0%	0%	0%
1: One-to-one counting	0%	0%	0%	0%	0%	0%
2: Counting from one on materials	0%	0%	0%	0%	2%	1%
3: Counting from one by imaging	0%	0%	0%	0%	0%	0%
4: Advanced counting	13%	25%	24%	10%	18%	14%
5: Early additive part–whole	63%	38%	29%	43%	64%	53%
6: Advanced additive part–whole	24%	38%	48%	43%	16%	30%
7: Advanced multiplicative part–whole	0%	0%	0%	3%	0%	2%
N =	54	16	21	58	55	113
Final						
0: Emergent	0%	0%	0%	0%	0%	0%
1: One-to-one counting	0%	0%	0%	0%	0%	0%
2: Counting from one on materials	0%	0%	0%	0%	0%	0%
3: Counting from one by imaging	0%	0%	0%	0%	0%	0%
4: Advanced counting	2%	13%	10%	5%	5%	5%
5: Early additive part–whole	35%	38%	43%	26%	44%	35%
6: Advanced additive part–whole	57%	38%	48%	57%	45%	51%
7: Advanced multiplicative part–whole	6%	13%	0%	12%	5%	9%
N =	54	16	21	58	55	113

Table 42
Performance of Year 6 Students on the Multiplicative Domain

	Ethnicity			Gender		
	NZE	Māori	Pasifika	Male	Female	Total
Initial						
Not rated	0%	0%	0%	0%	2%	1%
2–3: Counting from one	4%	6%	5%	3%	5%	4%
4: Advanced counting	15%	25%	14%	10%	24%	17%
5: Early additive part–whole	37%	44%	29%	36%	33%	35%
6: Advanced additive part–whole	35%	25%	29%	34%	33%	34%
7: Advanced multiplicative part–whole	9%	0%	24%	16%	4%	10%
8: Advanced proportional part–whole	0%	0%	0%	0%	0%	0%
N =	54	16	21	58	55	113

Table 42 – *continued*

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Final						
Not rated	0%	0%	0%	0%	0%	0%
2–3: Counting from one	0%	0%	0%	0%	0%	0%
4: Advanced counting	6%	6%	10%	3%	9%	6%
5: Early additive part–whole	20%	38%	29%	24%	22%	23%
6: Advanced additive part–whole	48%	25%	38%	34%	51%	42%
7: Advanced multiplicative part–whole	24%	31%	24%	36%	18%	27%
8: Advanced proportional part–whole	2%	0%	0%	2%	0%	1%
N =	54	16	21	58	55	113

Table 43
Performance of Year 6 Students on the Proportional Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
Not rated	0%	0%	5%	2%	2%	2%
1: Unequal sharing	2%	0%	0%	0%	4%	2%
2–4: Equal sharing	35%	56%	43%	34%	44%	39%
5: Early additive part–whole	30%	25%	24%	29%	22%	26%
6: Advanced additive part–whole	20%	13%	19%	21%	24%	22%
7: Advanced multiplicative part–whole	13%	6%	10%	14%	5%	10%
8: Advanced proportional part–whole	0%	0%	0%	0%	0%	0%
N =	54	16	21	58	55	113
Final						
Not rated	0%	0%	5%	2%	0%	1%
1: Unequal sharing	0%	0%	0%	0%	0%	0%
2–4: Equal sharing	13%	19%	33%	16%	18%	17%
5: Early additive part–whole	30%	31%	19%	22%	27%	25%
6: Advanced additive part–whole	31%	31%	33%	24%	44%	34%
7: Advanced multiplicative part–whole	26%	19%	10%	34%	11%	23%
8: Advanced proportional part–whole	0%	0%	0%	2%	0%	1%
N =	54	16	21	58	55	113

Table 44
Performance of Year 6 Students on the Fractions Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
Not assessed	0%	0%	0%	0%	2%	1%
2–3: Non-fractions	6%	25%	29%	14%	13%	13%
4: Assigns unit fractions	20%	19%	48%	21%	35%	27%
5: Orders unit fractions	54%	50%	24%	43%	42%	42%
6: Co-ordinates numerators/denominators	15%	6%	0%	16%	9%	12%
7: Equivalent fractions	6%	0%	0%	7%	0%	4%
8: Orders fractions	0%	0%	0%	0%	0%	0%
N =	54	16	21	58	55	113
Final						
Not assessed	0%	0%	0%	0%	0%	0%
2–3: Non-fractions	0%	0%	5%	2%	0%	1%
4: Assigns unit fractions	6%	19%	43%	16%	16%	16%
5: Orders unit fractions	46%	63%	52%	41%	56%	49%
6: Co-ordinates numerators/denominators	31%	0%	0%	21%	20%	20%
7: Equivalent fractions	15%	19%	0%	19%	5%	12%
8: Orders fractions	2%	0%	0%	2%	2%	2%
N =	54	16	21	58	55	113

Table 45
Performance of Year 6 Students on the Place Value Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Counts by ones	0%	0%	0%	0%	0%	0%
2: Counts in ones	0%	0%	0%	0%	0%	0%
3: Counts in fives and ones	0%	0%	10%	2%	2%	2%
4: 10s to 100, orders to 1000	43%	50%	62%	34%	53%	43%
5: 10s to 1000, orders to 10 000	35%	31%	24%	36%	31%	34%
6: 10s, 100s, 1000s, orders whole numbers	15%	19%	5%	21%	15%	18%
7: Tenths in and orders decimals	7%	0%	0%	7%	0%	4%
8: Tenths, hundredths, and thousandths	0%	0%	0%	0%	0%	0%
N =	54	16	21	58	55	113

Table 45 – *continued*

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Final						
0–1: Counts by ones	0%	0%	0%	0%	0%	0%
2: Counts in ones	0%	0%	0%	0%	0%	0%
3: Counts in fives and ones	0%	0%	0%	0%	0%	0%
4: 10s to 100, orders to 1000	7%	25%	48%	21%	16%	19%
5: 10s to 1000, orders to 10 000	50%	44%	43%	33%	51%	42%
6: 10s, 100s, 1000s, orders whole numbers	30%	19%	10%	28%	31%	29%
7: Tenths in and orders decimals	9%	13%	0%	14%	2%	8%
8: Tenths, hundredths, and thousandths	4%	0%	0%	5%	0%	3%
N =	54	16	21	58	55	113

Table 46

Performance of Year 6 Students on the Basic Facts Domain

	Ethnicity			Gender		
	NZE	**Māori**	**Pasifika**	**Male**	**Female**	**Total**
Initial						
0–1: Non-grouping	0%	0%	0%	0%	0%	0%
2: Facts to 5	0%	0%	0%	0%	2%	1%
3: Facts to 10	0%	0%	0%	0%	0%	0%
4: Within 10, doubles and teens	11%	13%	5%	7%	11%	9%
5: Addition, multiplication for 2, 5, 10	50%	44%	33%	40%	49%	44%
6: Subtraction and multiplication	28%	25%	24%	29%	29%	29%
7: Division	11%	19%	33%	24%	7%	16%
8: Common factors, multiples	0%	0%	5%	0%	2%	1%
N =	54	16	21	58	55	113
Final						
0–1: Non-grouping	0%	0%	0%	0%	0%	0%
2: Facts to 5	0%	0%	0%	0%	0%	0%
3: Facts to 10	0%	0%	0%	0%	2%	1%
4: Within 10, doubles and teens	4%	6%	5%	3%	4%	4%
5: Addition, multiplication for 2, 5, 10	35%	38%	24%	33%	27%	30%
6: Subtraction and multiplication	37%	31%	38%	22%	45%	34%
7: Division	20%	19%	33%	40%	18%	29%
8: Common factors, multiples	4%	6%	0%	2%	4%	3%
N =	54	16	21	58	55	113

Table 47
Effect Sizes for Impact of NDP on Demographic Subgroups of Year 0–1 Students

	Add.	Mult.	Prop.	FNWS	Fract.	PV	BF	N =
NZ European	1.14	0.38	0.40	1.16	0.42	0.98	0.83	239
Māori	0.97	0.30	0.32	1.12	0.30	1.02	0.89	74
Pasifika	0.99	0.36	0.36	1.07	0.36	0.95	0.71	91
Male	1.08	0.47	0.45	0.95	0.48	1.00	0.75	234
Female	1.09	0.29	0.31	1.15	0.32	0.88	0.86	196
Total	1.08	0.39	0.39	1.04	0.41	0.95	0.80	430

Table 48
Effect Sizes for Impact of NDP on Demographic Subgroups of Year 2 Students

	Add.	Mult.	Prop.	FNWS	Fract.	PV	BF	N =
NZ European	0.93	0.65	0.66	0.79	0.73	0.79	1.05	195
Māori	0.83	0.60	0.44	0.96	0.53	0.68	0.68	55
Pasifika	1.01	0.78	0.94	0.65	1.07	0.90	0.72	29
Male	0.94	0.65	0.62	0.76	0.75	0.80	0.99	241
Female	0.95	0.72	0.72	0.88	0.74	0.89	0.94	257
Total	0.95	0.69	0.67	0.81	0.74	0.84	0.96	498

Table 49
Effect Sizes for Impact of NDP on Demographic Subgroups of Year 3 Students

	Add.	Mult.	Prop.	FNWS	Fract.	PV	BF	N =
NZ European	0.85	0.50	0.50	0.65	0.68	0.59	0.76	195
Māori	0.80	0.71	0.46	0.74	0.63	0.97	1.13	55
Pasifika	1.11	1.68	1.29	0.97	1.78	1.04	1.25	29
Male	0.86	0.58	0.57	0.63	0.72	0.63	0.87	180
Female	0.82	0.68	0.60	0.84	0.77	0.74	0.80	145
Total	0.84	0.63	0.59	0.71	0.74	0.68	0.84	325

Table 50
Effect Sizes for Impact of NDP on Demographic Subgroups of Year 4 Students

	Add.	Mult.	Prop.	FNWS	Fract.	PV	BF	N =
NZ European	0.66	0.66	0.81	0.67	0.84	0.68	0.64	124
Māori	0.72	0.65	0.69	0.71	0.82	0.77	0.74	29
Pasifika	0.51	0.69	0.70	0.40	1.01	0.54	0.87	25
Male	0.56	0.61	0.62	0.45	0.74	0.63	0.59	121
Female	0.61	0.67	0.80	0.76	0.88	0.66	0.72	125
Total	0.57	0.63	0.70	0.59	0.81	0.63	0.65	246

Table 51
Effect Sizes for Impact of NDP on Demographic Subgroups of Year 5 Students

	Add.	Mult.	Prop.	FNWS	Fract.	PV	BF	N =
NZ European	0.60	0.63	0.68	0.43	0.79	0.50	0.50	54
Māori	0.70	0.82	0.94	0.60	1.21	0.71	0.51	16
Pasifika	0.66	0.75	0.67	0.23	0.96	0.45	0.43	21
Male	0.63	0.70	0.59	0.40	0.75	0.45	0.49	108
Female	0.57	0.62	0.84	0.47	0.78	0.62	0.37	98
Total	0.60	0.65	0.70	0.43	0.76	0.51	0.43	206

Table 52
Effect Sizes for Impact of NDP on Demographic Subgroups of Year 6 Students

	Add.	Mult.	Prop.	FNWS	Fract.	PV	BF	N =
NZ European	0.91	0.71	0.63	0.32	0.75	0.71	0.53	54
Māori	0.44	1.00	0.86	0.29	0.88	0.56	0.31	16
Pasifika	0.19	0.24	0.19	0.23	0.79	0.56	0.00	21
Male	0.50	0.64	0.55	0.09	0.60	0.51	0.35	58
Female	0.83	0.80	0.72	0.69	0.87	0.81	0.55	55
Total	0.64	0.70	0.63	0.37	0.71	0.61	0.44	113