

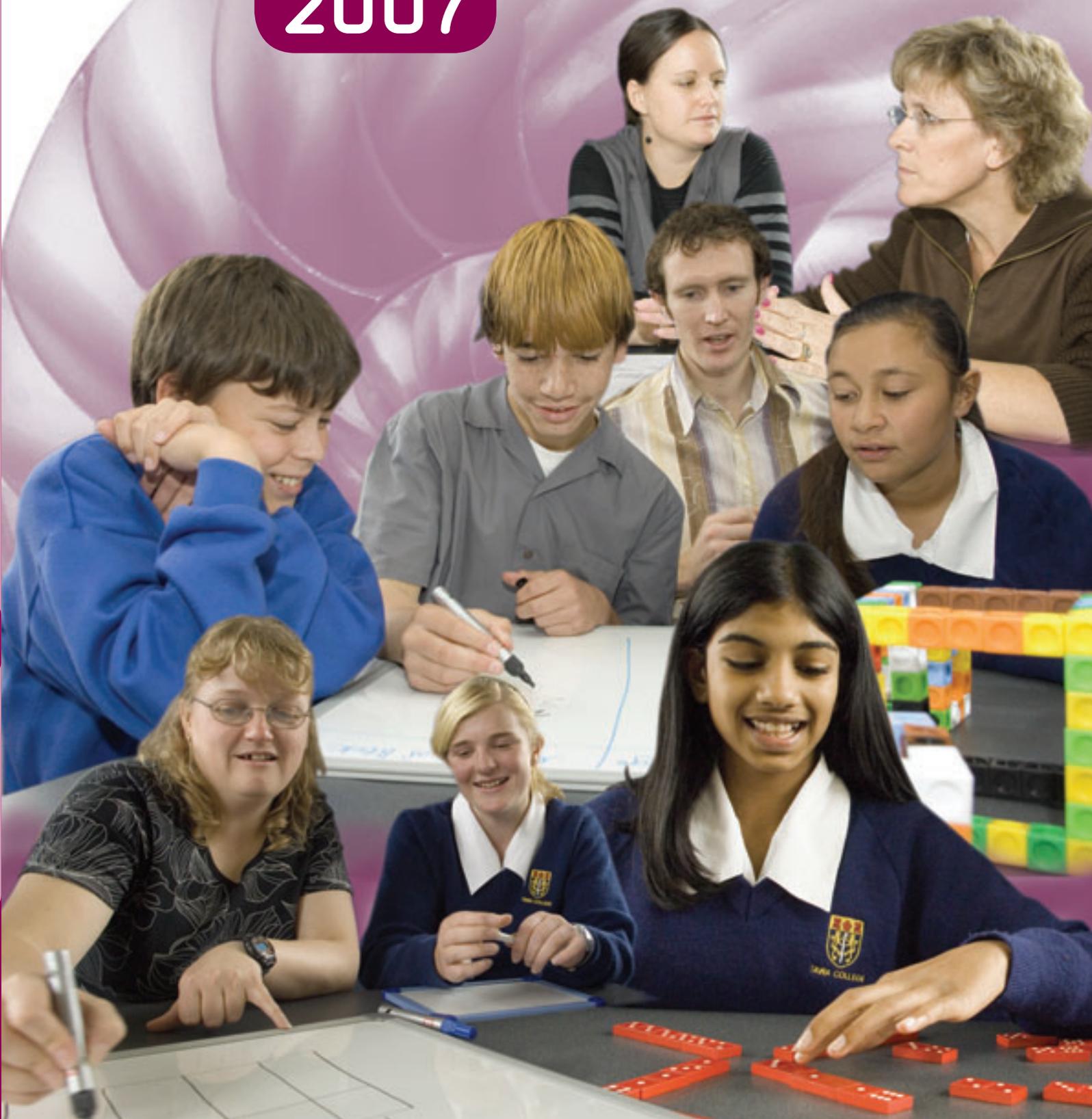


MINISTRY OF EDUCATION

Te Tāhuhu o te Mātauranga

Findings from the New Zealand Secondary Numeracy Project

2007



Findings from the Secondary Numeracy Project 2007

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Foreword

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Findings from the New Zealand Secondary Numeracy Project 2007

Foreword

The Secondary Numeracy Project (SNP) was introduced in selected secondary schools in 2005 with the aim of providing teachers with two years of professional development that would improve the effectiveness of their teaching and enable students to develop a deeper understanding of mathematics. This compendium is a collection of the research undertaken alongside the SNP in the third year of its implementation.

The papers in this compendium reflect the continuing development of the SNP as more schools engage. Some papers follow on from earlier reports that analysed the impact on years 9 and 10, while other papers explore previously uncharted waters. Facilitation and professional development in wharekura (Māori-medium secondary schools) within the context of Te Poutama Tau features for the first time, while there is also a progress report on attempts to construct a written diagnostic assessment to complement the Numeracy Development Project (NDP) diagnostic interview. Research into the connection between number and algebra gets more exposure, and cautionary attempts are made to detect differences in NCEA student performance in year 11 as a result of the SNP and to identify what impact the SNP may have on teaching and learning in year 11 mathematics.

The SNP continues to grow. The development of a mathematical pedagogy has opened opportunities for more students to understand mathematics. As students show improved understanding of mathematics, so teachers continue to explore and develop their pedagogy. And the cycle continues. There will always be more work to do. What has been achieved so far can be applauded.

Student Performance and Progress

In “Performance of SNP students on the Number Framework” (p. 5), Andrew Tagg and Gill Thomas present their third year of analysis of the progress of students in SNP, measured against the Number Framework. With SNP in its third year of implementation, comparisons could be made on the effects of the project for students from 2005 to 2007.

Tagg and Thomas’s analysis indicates that the SNP continues to have a consistently positive impact on student achievement in year 9. For schools new to the project in 2007, significant shifts in performance between the beginning and the end of the year were achieved in the proportion of the student population that could perform in the top three stages of the additive domain and the top two stages of the multiplicative domain, and in the proportional domain. These figures are remarkably similar to the gains achieved in the first two years of the SNP for these domains. Consistent with previous findings, New Zealand European students performed better than Māori or Pasifika students, and students from high-decile schools performed better as a group than students from medium- or low-decile schools. Male students performed slightly better than female students in the multiplicative domain, while female students performed better on the basic facts domain.

Schools that entered the project in 2005 and 2006 were also expected to assess their students at the end of their year 10 courses. The results for 2007 followed the pattern of 2006 and indicate that, while these year 10 students performed better than the year 9 students in all aspects except the proportional domain, the differences in performance were small. Further investigation may reveal reasons for these differences. It may be related to schools consolidating their efforts in year 9 during the second year so it may be too soon to detect change in year 10 performance. Tagg and Thomas also allude to the possibility that the sample of year 10 students may not have the same characteristics as the initial year 9 cohort.

Tagg and Thomas draw attention to the proportion of students who still perform at stage 5 or lower on the Number Framework for both the strategies that they bring to bear and the knowledge that they can recall quickly to help solve a problem. As an example, 50% of year 9 students start the year at or below stage 5 in the fractions and place value knowledge domains. While these reduce to 31% and 28% by the end of year 9, the future learning for these students in mathematics is in jeopardy.

Impact of the SNP on Teaching and Learning of Year 11 Mathematics

In “An investigation into the impact of the Secondary Numeracy Project on student performance in two NCEA Level 1 mathematics achievement standards” (p. 17), Roger Harvey focused on two NCEA Level 1 achievement standards, AS90147: Use straightforward algebraic methods and solve equations, and AS90151: Solve straightforward number problems in context. These achievement standards were chosen because their content has the most direct connection to the content of the SNP.

In view of the comments from teachers in Harvey and Smith’s other paper (see below) that the greatest impact of the SNP on their teaching occurred in courses that do not generally use achievement standards as their assessment tool, it would seem fair to assume no discernable effect would be detected in the achievement standards results. In fact, Harvey has detected some differences in performance, with a modest improvement for SNP students in the algebra standard and no significant difference in the number standard. He cautions against jumping to any conclusions, noting many factors that could be affecting the data. Further study is suggested to collect a comprehensive picture of student attainment in mathematics.

In “Teachers’ views on the impact of the Secondary Numeracy Project on the teaching of year 11 classes” (p. 27), Roger Harvey and Derek Smith investigate whether the professional development within the SNP has had an effect on teaching practice in year 11 classrooms. A group of 17 teachers from five schools were selected to participate in the study. They completed a written survey that allowed them to express their views on the impact of the SNP on their teaching practice. The teachers surveyed taught courses focused on achievement standards and courses focused on unit standards. Most of these courses were assessed across the range of achievement standards and unit standards available for year 11 qualifications.

An extended use of discussion in developing mathematical ideas with students was noted by a number of the teachers. Many teachers reported a more judicious use of calculators. The majority of teachers noted a positive influence on student engagement in learning mathematics. Comments indicate a greater willingness on the part of students to ask questions and develop an understanding of mathematics. The effect is not, however, universal.

The study also found that teachers of year 11 mathematics courses focused on unit standards tended to report greater changes in their practice, perhaps because they see benefits in adopting different pedagogical practices to ensure they meet the needs of individual students who have historically found mathematics demanding.

Written and Oral Assessments of Secondary Students’ Number Strategies

A key component of the assessment tools used in the SNP is the diagnostic interview with students that teachers use to establish a student’s strategy stage on the Number Framework. In “Written and oral assessment of secondary students’ number strategies: developing a written assessment tool” (p. 32), Gregor Lomas and Peter Hughes describe the first stage of an attempt to develop a written strategy Number Framework stage assessment tool (WSSAT) that is designed to give teachers the same information about students’ stages from a pencil-and-paper assessment as they would get from

the oral interview. The WSSAT has been designed to closely align with the SNP strategy section of the Number Framework, but it also draws on several other research sources for diagnostic questions on place value, decimals, and number sense. It was envisaged that it would primarily be used in the secondary school environment, so the WSSAT attempted to accurately assess for only stages 5–8 of the Framework. If students did not meet stage 5 criteria, they were assigned a category covering stages 1–4. An oral assessment tool closely aligned to the SNP and NDP Global Strategy Stage (GloSS) assessment tool was also used for comparison purposes in this research.

The written assessment was given to 278 year 9 SNP students, and on the following day, the oral assessment was given to 27 of these students. A comparison of the results of the oral assessment with nationally available data indicated that the oral assessment used could be regarded as a reliable indicator of the students' strategy stages. However, no such claim could be made of the written assessment. The stages determined by the written assessment were generally lower than those of the oral assessment, and in six of the 27 cases, the variation was by more than 2 stages.

Despite this somewhat disappointing finding, there is still profit in further development of this written tool. The items in the assessment are of broader mathematical interest than those currently being used in the Numeracy Project Assessment (NumPA) diagnostic tool and GloSS, incorporating questions requiring estimation and rounding appropriately in a context. Even in its current form, the WSSAT has commendable internal consistency – students were displaying command of early items before weakening at the later stages. The results from the WSSAT also related well to the Assessment Tools for Teaching and Learning (AsTTle) measure of achievement in mathematics and English that was used by the school to place students in banded classes. But further work on the items in the assessment and the criteria for assigning stages is needed before WSSAT can stand alongside the current assessment tools used in the SNP.

Solving Equations: Students' Algebraic Thinking

In "Solving equations: students' algebraic thinking" (p. 39), Chris Linsell reports on research in progress that investigates linkages between students' numeracy stages and the strategies they use for solving linear equations.

A team of researchers and teachers is developing a diagnostic tool that identifies the strategies that students employ to solve linear equations. The tool has both a knowledge and strategy section, somewhat reminiscent of the NumPA. The algebraic diagnostic tool was used in 2007 with 450 year 7–10 students, and with a further 400 students at the beginning of 2008. Ten strategies for solving linear equations have been identified: using known basic facts; counting techniques; using inverse operations; guess and check; cover up; working backwards; working backwards, then using known facts; working backwards, then using guess and check; formal operations; and using a diagram. While some of the strategies are clearly more sophisticated than others, no definitive hierarchy amongst the strategies has yet been suggested.

Preliminary findings indicate that there is a high correspondence between a student's numeracy stage and the most sophisticated strategy that student uses for solving equations, with multiplicative part-whole thinking required for access to the strategies of working backwards and formal operations. Indeed, numeracy stage is a better predictor of a student's ability to use a strategy than the amount of algebra teaching that has occurred. However, although the numeracy stage is a good predictor, it is not foolproof: some students who have mastered computational algorithms but score low on aspects of the numeracy assessment have been able to use sophisticated equation-solving strategies.

A final point of interest in these findings is that students tend to be able to employ their most sophisticated strategy in both a formal symbolic question and an equivalent question that is described in a context using words or diagrams. The consequence is that it may be unnecessary to present students with symbolic equations to detect their most sophisticated solution strategy.

Support for Pāngarau Teachers Working in Wharekura

In their “Evaluation of support for pāngarau teachers working in wharekura” (p. 45), Pania Te Maro, Robin Averill, and Joanna Higgins have compiled a case study that examines the impact of a pilot project of professional development and support in the wharekura Te Poutama Tau (the Māori-medium version of the SNP) and pāngarau (mathematics) on nine teachers working in wharekura in the Hawkes Bay, Taranaki, Waikato, Wellington, and Whanganui regions.

This pilot project was developed as a consequence of recommendations by Tony Trinick and Makoare Parangi, in their 2006 research report, about the needs of wharekura teachers of pāngarau and a wish to create a professional development initiative that would support and ease the workload of these teachers and reduce their professional isolation. The project used three modes of delivery: hui, facilitator visits to schools, and video conferencing. Although video conferencing was perceived initially by the participants as the least useful aspect of the modes of delivery, participants came to the view that all the delivery modes were important and tended to complement each other. Each aspect contributed to teacher growth and assisted in the development of a supportive social and professional network.

Facilitators play a crucial role in the professional development structure of the NDP (which include the SNP). This Te Poutama Tau case study includes feedback from the wharekura teachers on the essential characteristics for effective facilitation. Amongst others, participants agreed that an effective Te Poutama Tau facilitator is able to give positive and affirming feedback; is inclusive and sharing; displays humour and humility; is accommodating, kind, empathetic, available, and approachable; has perseverance and expects it of others; shares a passion for mathematics and Te Poutama Tau; and is passionate about the importance of the project for Māori students. Such characteristics are not peculiar to the facilitation of Te Poutama Tau in wharekura, but they were highly valued by the participants in this study. The report provides an analysis of the achievement data of 125 students, who showed impressive achievement gains on the Number Framework (Te Mahere Tau).

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Performance of SNP Students on the Number Framework

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This paper presents the third year of analysis of the performance and progress of students in the Secondary Numeracy Project (SNP), measured against the Number Framework. With the SNP in its third year of implementation in 2007, comparisons can be made with the effects of the SNP for students in 2005 and 2006. The findings indicate that the SNP continues to have a positive impact on student achievement in year 9. Consistent with previous findings, significant shifts were achieved in raising the proportion of the student population rated at the top stages of all domains of the Number Framework. These gains are very similar to those achieved in previous years. Demographic factors were again shown to impact on the performance of students. In particular, New Zealand European students performed better than Māori students, who in turn performed better than Pasifika students, and students from high-decile schools performed better than those from medium-decile schools, while the performance of students from low-decile schools was weaker than either of the other deciles. The end-of-year results of year 10 students were very similar to those of year 9 students, despite these students having been exposed to an additional year of SNP practices. The proportion of students in both year 9 and year 10 who remain at stage 5 or lower on both the strategy and knowledge domains is again concerning.

Background

Since 2000, the Numeracy Development Projects (NDP) have aimed to raise the mathematics achievement of students by improving the quality of the teaching and learning of mathematics. The Secondary Numeracy Project (SNP) provides similar support to that provided to primary and intermediate teachers through the NDP, although the structure of the SNP support is organised differently. In NDP schools, the support is provided by external facilitators, who run workshops for the teachers and provide in-class modelling and feedback. In the SNP, a member of the school's own mathematics department is provided with training and operates as an in-school facilitator (ISF). The role of the ISF includes running workshops for other teachers, observing and modelling lessons, and providing support to peers in teaching numeracy. The ISFs are supported by a regional facilitator and work as part of a cluster of participating SNP schools. In small secondary schools, an ISF from one school may also act as the facilitator for one or two other nearby schools.

In 2005, the SNP pilot project provided professional development in 42 schools, aimed at enhancing the teaching of year 9 mathematics. In 2006, teachers in the same 42 schools were supported in consolidating their teaching of year 9 mathematics and in developing their teaching of year 10 mathematics. An additional 37 schools participated in the SNP for 2006. These schools received support of a similar nature to that given in the pilot project the previous year. 2007 was the third year of implementing the SNP, with 47 new schools participating.

This paper analyses the results of year 9 students in schools participating in the SNP for the first time in 2007 and of year 10 students in the schools that first participated in 2005 or 2006. The aim of this research is to quantify any improvement made in students' number knowledge and strategies. The research questions specifically addressed in this paper are as follows:

- What progress have year 9 students in first-year SNP schools made on the Number Framework in 2007?
- How does this progress compare to that made by year 9 students in 2005 and 2006?

- What is the numeracy profile of year 10 students in schools that are in their second or third year of the SNP?
- What demographic factors impact on the progress and performance of SNP students?

Method

Participants

The results reported in this paper were obtained from the online numeracy database on 17 December 2007 by downloading all the data entered by SNP schools. The schools were required to enter both initial and final data on the three strategy domains and the four knowledge domains of the Number Framework for year 9 students, while for year 10 students, only final data on the seven domains was required. Students were only included in this analysis if complete data had been entered for them.

Unless otherwise stated, this paper describes the results of the 5093 year 9 students in schools participating in the SNP for the first time in 2007 and the 2355 year 10 students in schools that have implemented the SNP for at least two years. Table 1 summarises the number of complete sets of data available for each year level in schools in each year of participation in the SNP. The 5093 year 9 students in first-year schools represent 83% of the year 9 students in those schools for whom complete initial data was entered. (Some schools had not managed to complete the entry of this data in time to be included in this analysis.)

Table 1
Summary of SNP Results by Year Level and School's Year in Project

	Year 9	Year 10
First year	5093	178
Second year	825	2261
Third year	243	94
Total	6161	2533

Table 2 comprises a breakdown by year, gender, and ethnicity of the SNP students included for analysis. National data from 2007 is provided for comparison (Ministry of Education, 2008). Sixty-three percent of the SNP students were of New Zealand European origin, 17% identified as Māori, and approximately 7% identified as Pasifika. There were more female year 9 students than male (54% compared with 46%).

Table 2
Profile of SNP Students and Year 9 and 10 Students Nationally by Ethnicity and Gender

Ethnicity	Year 9				Year 10			
	Male		Female		Male		Female	
	SNP	National	SNP	National	SNP	National	SNP	National
NZ European	58%	57%	66%	58%	63%	58%	69%	58%
Māori	20%	23%	15%	23%	18%	21%	14%	22%
Pasifika	8%	9%	6%	9%	9%	9%	8%	9%
Asian	7%	8%	7%	8%	4%	8%	3%	8%
Other	7%	2%	6%	2%	5%	2%	6%	2%
Total	2232	31 817	2861	29 636	1207	31 843	1148	30 175

The decile groups of the participating students were not evenly distributed. Only 8% of the students in each year level came from low-decile (1–3) schools, 38% of the year 9 students and 24% of the year 10 students came from high-decile (8–10) schools, and the remainder came from medium-decile (4–7) schools.

Analysis

An independent samples T-test was used to compare the means of variables with only two categories (gender and year level), while an ANOVA (analysis of variance) test was used to compare the means of variables with three or more categories (decile band and ethnicity). Where statistically significant differences are described between groups, a difference has been verified to at least the 1% significance level, either by the T-test or by a post-hoc analysis using Tukey's honestly significant difference test. It needs to be noted that, in some instances, significantly different mean gains and effect sizes may be smaller than other gains and effect sizes shown that are not statistically significant due to differences in sample size.

All the tables (including the appendices) show rounded percentages. Percentages less than 0.5% are therefore shown as 0% and, where there are no students represented, the cell is left blank. Due to rounding, percentages in some tables may not total to 100.

Effect sizes, where used, have been calculated by dividing the average difference between two groups by the pooled standard deviation of the two groups. Effect sizes of 0.2 are considered "small", effect sizes of 0.5 are "medium", and effect sizes of 0.8 or higher are "large" (Cohen, cited in Coe, 2002). For the purposes of this paper, effect sizes of 0.2 or less are described as small, effect sizes between 0.2 and 0.8 are described as medium, and effect sizes of 0.8 or higher are described as large.

Findings

The results discussed in this paper are divided into three sections. The first section describes the performance and progress of year 9 students in schools participating in the SNP for the first time in 2007 and compares this with the performance of year SNP 9 students in previous years. The second section compares the 2007 end-of-year performance of year 10 students in second- or third-year SNP schools with the 2007 end-of-year performance of year 9 students in first-year SNP schools. The final section analyses the impacts of demographic factors (gender, decile, and ethnicity) on the performance and progress of all students.

Performance of Year 9 Students

This section describes the performance and progress of year 9 students in schools participating in the SNP for the first time in 2007 and compares this with the performance of year 9 SNP students in previous years.

Table 3 shows the initial and final percentages of year 9 students in first-year SNP schools at each stage of the three strategy domains. The final results of year 9 SNP students from 2005 and 2006 are provided for comparison (Tagg & Thomas, 2007).

Table 3
Performance of Year 9 Students on the Strategy Domains

Stage	Additive				Multiplicative				Proportional			
	2007 initial	2007 final	2006 final	2005 final	2007 initial	2007 final	2006 final	2005 final	2007 initial	2007 final	2006 final	2005 final
0-3	1%	0%	0%	1%	3%	1%	0%	0%	1%	0%	0%	1%
4	11%	4%	5%	5%	10%	4%	5%	6%	14%	6%	6%	6%
5	42%	27%	29%	26%	26%	14%	16%	16%	30%	22%	24%	23%
6	36%	42%	44%	46%	36%	35%	35%	32%	18%	18%	19%	17%
7	10%	21%	22%	23%	20%	31%	29%	30%	32%	41%	38%	41%
8	0%	5%	n/a	n/a	5%	14%	14%	16%	5%	13%	12%	12%
N =	5093	5093	5807	3975	5093	5093	5807	3975	5093	5093	5807	3975

The percentages of the 2007 year 9 students rated at stage 7 or 8 of the additive, multiplicative, and proportional strategy domains increased from 10% to 26%, 25% to 45%, and 37% to 54% respectively. Correspondingly, the percentages of students still rated as using counting strategies (stage 4 or below) decreased by 8% on the additive and multiplicative domains and 9% on the proportional domain. A comparison with the final scores from 2005 and 2006 shows that the performance of students was similar on all three domains.

Table 4 gives the initial and final percentages of the 2007 year 9 students in first-year SNP schools at each stage of the four knowledge domains and shows a similar pattern to Table 3, with progress being made on all domains.

Table 4
Performance of the 2007 Year 9 Students on the Knowledge Domains

Stage	FNWS		Fractions		Place Value		Basic Facts	
	Initial	Final	Initial	Final	Initial	Final	Initial	Final
0-3	1%	1%	3%	1%	1%	0%	2%	1%
4	3%	1%	10%	6%	8%	3%	4%	2%
5	33%	22%	38%	25%	41%	25%	18%	12%
6	63%	77%	24%	23%	28%	28%	49%	39%
7	N/A	N/A	18%	31%	13%	20%	25%	41%
8	N/A	N/A	7%	14%	9%	23%	2%	5%
N =	5093	5093	5093	5093	5093	5093	5093	5093

While the proportions of students at the higher stages of all seven domains increased and the proportions of students at the lower stages decreased, it is concerning that between 15% and 32% of students on each domain finished year 9 still rated at stage 5 or below. This finding is consistent with SNP results from previous years (Tagg & Thomas, 2006, 2007). The mathematics and statistics learning area of *The New Zealand Curriculum* (Ministry of Education, 2007) includes objectives in the Number and Algebra strand that link closely to the Number Framework. Students in year 9 are expected to

be working at curriculum levels 4 or 5, which are equivalent to stages 7 and 8 of the Framework. Expectations provided on the nzmaths website (Maths Technology Ltd, n.d.) identify year 9 students still rated at stage 5 or below as “at risk” and describe them as being “sufficiently below expectations that their future learning in mathematics is in jeopardy” (Maths Technology Ltd, n.d.).

Figures 1–3 show the percentages of year 9 students gaining stages on each of the three strategy domains in 2005, 2006, and 2007, linked to their initial stage. The results from 2007 are very similar to those from 2005 and 2006. Generally, a higher percentage of students at lower initial stages made gains, although the proportion of students initially rated at stage 6 on the proportional domain making gains (57%) was slightly higher than that of students initially rated at stage 5 (49%).

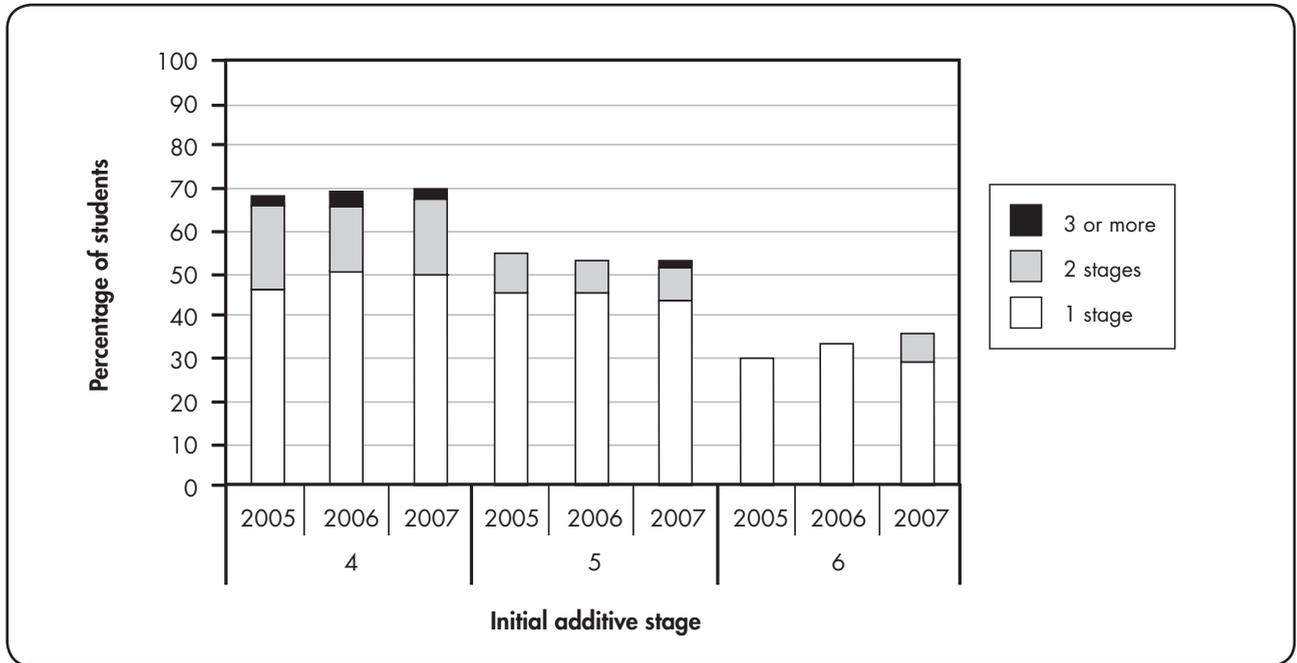


Figure 1. Number of stages gained from initial additive stage for year 9 students in 2005, 2006, and 2007

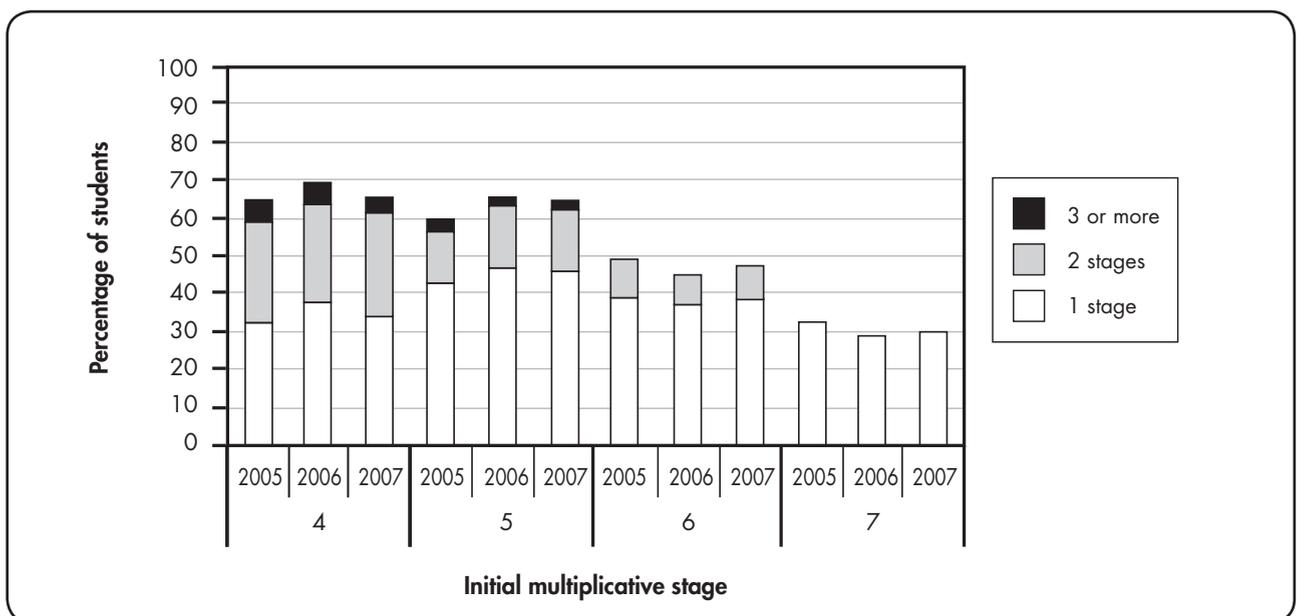


Figure 2. Number of stages gained from initial multiplicative stage for year 9 students in 2005, 2006, and 2007

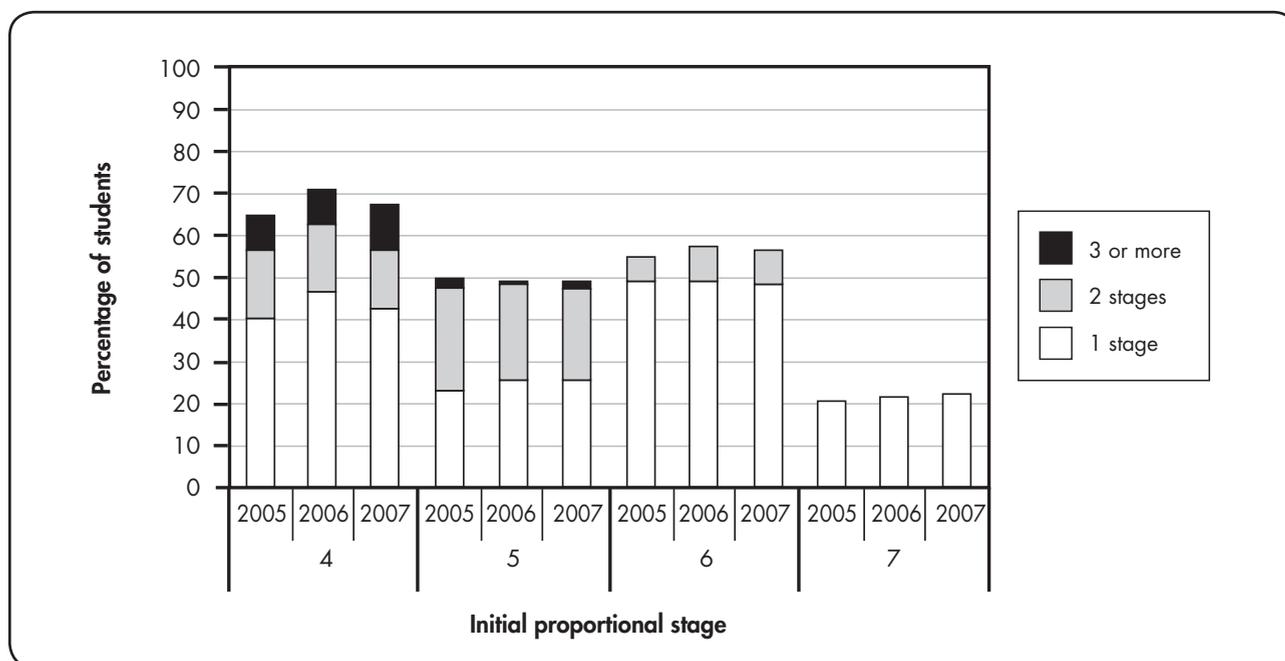


Figure 3. Number of stages gained from initial proportional stage for year 9 students in 2005, 2006, and 2007

Effect sizes were calculated to examine the magnitude of the impact of the SNP on year 9 students in 2007. Table 5 presents the mean stages and effect sizes for the seven domains.

Table 5
Effect sizes for Comparisons of Initial and Final Scores of Year 9 SNP Students in 2007

Domain	Mean Score		Mean Gain	Effect Size
	Initial	Final		
Additive	5.43	5.95	0.52	0.56
Multiplicative	5.76	6.34	0.58	0.52
Proportional	5.65	6.25	0.60	0.44
FNWS	5.58	5.74	0.16	0.28
Fractions	5.64	6.20	0.56	0.47
Place value	5.69	6.33	0.64	0.55
Basic facts	5.93	6.33	0.40	0.42

Mean gains of at least half a stage were made on all domains apart from FNWS (0.16) and basic facts (0.40). The apparent lack of progress on the FNWS domain can be explained by a ceiling effect, with 63% of all year 9 students rated at the top stage of this domain (stage 6) at the initial assessment. The relatively small gains on the basic facts domain may be partly explained by the fact that this was the domain with the highest mean initial stage (5.93), meaning that a ceiling effect again impacts on progress for many students.

An independent samples T-test was carried out, and effect sizes were calculated to compare the performance of year 9 students in first-year SNP schools with the 1068 year 9 students from second- or third-year SNP schools. The results are shown in Table 6. The shaded cells represent comparisons where the difference was not statistically significant at the 1% significance level.

Table 6
Comparisons of Performance of Year 9 students in First-year and Second/Third-year SNP Schools

Domain	Assessment	Mean Score		Difference	Effect Size
		2nd/3rd yr	1st yr		
Additive	Initial	5.46	5.43	0.03	0.03
	Final	5.74	5.76	-0.02	-0.04
Multiplicative	Initial	5.50	5.65	-0.15	-0.01
	Final	5.39	5.58	-0.19	-0.17
Proportional	Initial	5.48	5.64	-0.16	-0.10
	Final	5.56	5.69	-0.13	-0.20

The differences between the initial and final additive scores and the initial multiplicative scores of the two groups were not statistically significant. While the differences between the mean final stages of the two groups on the multiplicative and proportional domains were statistically significant, the effect sizes for the differences between the groups were small (0.2 or less). This indicates, in practical terms, that there is little difference in mean strategy stage and that the performance of year 9 students in SNP schools in the years following the initial implementation of the project remains at a similar level to that of year 9 students in schools in their first year of the SNP.

Comparison of Results of Year 9 (First-year SNP Schools) and Year 10 (Second/Third year SNP Schools)

This section compares the 2007 end-of-year performance of year 10 students in second- or third-year SNP schools with the 2007 end-of-year performance of year 9 students in first-year SNP schools.

Table 7 shows the percentages of these year 9 and year 10 students at each stage of the three strategy domains at the final assessment.

Table 7
Performance on the Strategy Domains of Year 9 Students in First-year and Year 10 Students in Second/Third-year SNP Schools

Stage	Additive		Multiplicative		Proportional	
	Yr 9	Yr 10	Yr 9	Yr 10	Yr 9	Yr 10
0-3	0%	0%	1%	0%	0%	1%
4	4%	6%	4%	5%	6%	7%
5	27%	22%	14%	13%	22%	24%
6	42%	45%	35%	31%	18%	16%
7	21%	22%	31%	33%	41%	37%
8	5%	5%	14%	16%	13%	15%
N =	5093	2355	5093	2355	5093	2355

Similar percentages of year 9 and year 10 students reached the top two stages of each of the three strategy domains with, for example, 26% of the year 9 students and 27% of the year 10 students reaching at least stage 7 on the additive domain. At the lower stages, the difference between the percentages

of students remaining at stage 4 or below was 2% or less on each domain. A T-test and analysis of effect sizes indicates that the differences between the two groups on the additive and proportional domains are not statistically significant ($p < 0.01$) and that, on all three domains, the effect size is small (less than 0.2). More specifically, the effect sizes for the difference between year 9 and year 10 students are 0.02 for the additive domain, 0.06 for the multiplicative domain, and -0.05 for the proportional domain. It is concerning that year 10 students who have had two years of teaching in schools that are implementing the SNP achieve at the same level as year 9 students who have had one year of exposure to the SNP.

Table 8 below shows the percentages of year 9 and year 10 students at each stage of the four knowledge domains at the final assessment.

Table 8
Performance on the Knowledge Domains of Year 9 Students in First-year and Year 10 Students in Second/Third-year SNP Schools

Stage	FNWS		Fractions		Place Value		Basic Facts	
	Yr 9	Yr 10	Yr 9	Yr 10	Yr 9	Yr 10	Yr 9	Yr 10
0-3	1%	1%	1%	1%	0%	0%	1%	1%
4	1%	2%	6%	8%	3%	4%	2%	3%
5	22%	22%	25%	28%	25%	26%	12%	15%
6	77%	76%	23%	23%	28%	26%	39%	39%
7	N/A	N/A	31%	29%	20%	17%	41%	39%
8	N/A	N/A	14%	12%	23%	27%	5%	3%
N =	5093	2355	5093	2355	5093	2355	5093	2355

The pattern of performance on the knowledge domains mirrors that of performance on the strategy domains, with similar proportions of year 9 and year 10 students reaching the higher stages of each domain. The proportions of students still rated at stage 4 or below were also similar for the two year groups on all four domains. The effect sizes for the differences between the two years levels were again small (less than 0.2), with the largest being on the fractions (-0.12) and basic facts (-0.13) domains, where year 9 students outperformed year 10 students.

Impacts of Demographic Factors

This section analyses the impacts of demographic factors (gender, decile, and ethnicity) on the performance and progress of all students.

The results from the SNP in 2005 and 2006 showed that the comparative performances of demographic subgroups of students in the SNP were similar to those found in previous NDP research (Tagg & Thomas, 2006, 2007; Young-Loveridge, 2006). The 2007 results show a similar pattern to previous findings (see appendices A and B (pp. 59–65) for a detailed breakdown of the performance of students from each demographic subgroup). This section describes the results on the multiplicative and basic facts domains by demographic subgroups. These domains were chosen by the researchers because they consider these domains to be central to numeracy at years 9 and 10. Appendix C (p. 66) provides details of effect sizes between the demographic subgroups of SNP students for all seven domains. Table 9 shows the mean initial and final stages of the demographic subgroups of year 9 students on the multiplicative domain as well as the mean final stages of the year 10 students.

Table 9
Mean Stages on the Multiplicative Domain of Demographic Subgroups

	Year 9 Initial	Year 9 Final	Year 10 Final
Male	5.86	6.41	6.55
Female	5.68	6.29	6.26
Low decile	5.50	5.92	5.94
Medium decile	5.68	6.31	6.37
High decile	5.92	6.47	6.65
NZ European	5.86	6.44	6.49
Māori	5.50	6.04	6.22
Pasifika	5.35	5.89	5.81
Total	5.76	6.34	6.41

The pattern of comparative performance of demographic subgroups on the multiplicative domain continues to reflect that found in previous years. Male students had a higher mean stage than females, the mean stage of New Zealand European students was higher than that of either Māori or Pasifika students, and the mean stage of students from high-decile schools was higher than that of students from medium-decile schools, with both being higher than that of students from low-decile schools. It should be noted that the multiplicative domain was the only domain on which the mean final stage of year 9 males was significantly higher than that of year 9 females. Year 9 females had a higher mean final stage on the basic facts domain than males, while on all other domains, there was no significant difference between genders. The subgroup of year 9 students with the lowest mean final stage was Pasifika students (5.89), while the highest was students from high-decile schools (6.47). This finding is consistent with the results for year 10 students, where Pasifika students had the lowest mean stage (5.81) and students from high-decile schools had the highest mean stage (6.65). The mean final multiplicative stages of year 9 and year 10 students were similar, with the difference less than 0.2 of a stage in all instances.

To investigate the significance of the impact of demographic factors on students' performance on the Number Framework further, effect sizes were calculated for comparisons between males and females; between students from low-, medium-, and high-decile schools; and between New Zealand European, Māori, and Pasifika students. The results of this analysis for all seven domains are included in full in Appendix C (p. 66). Table 10 shows the effect sizes for demographic factors on the multiplicative domain. The shaded cells represent comparisons where the difference was not statistically significant at the 1% significance level.

Table 10
Effect Sizes for Comparisons of Demographic Subgroups on the Multiplicative Domain

	Year 9 Initial	Year 9 Final	Year 10 Final
Male/Female	0.15	0.11	0.27
High/Medium decile	0.21	0.16	0.26
High/Low decile	0.38	0.53	0.66
Medium/Low decile	0.16	0.35	0.40
NZ European/Māori	0.31	0.38	0.26
NZ European/Pasifika	0.45	0.52	0.64
Māori/Pasifika	0.14	0.14	0.36

Table 10 shows that the effect sizes for comparisons of final results for individual year levels varied from 0.11, for the difference between year 9 male and female students, to 0.66, for the difference between year 10 students from high- and low-decile schools.

Table 11 shows the mean gains and effect sizes for the impact of the SNP on year 9 students on the multiplicative domain, analysed by demographic subgroup. The results of this analysis for all seven domains are included in full in Appendix C (p. 66).

Table 11

Effect Sizes for Gains Made on the Multiplicative Domain by Demographic Subgroups of Year 9 Students

	Mean Initial Stage	Mean Final Stage	Mean Gain	Effect Size
Male	5.86	6.41	0.55	0.48
Female	5.68	6.29	0.61	0.56
Low decile	5.50	5.92	0.42	0.38
Medium decile	5.68	6.31	0.63	0.55
High decile	5.92	6.47	0.55	0.52
NZ European	5.86	6.44	0.58	0.53
Māori	5.50	6.04	0.54	0.49
Pasifika	5.35	5.89	0.54	0.52
Total	5.76	6.34	0.58	0.52

The overall effect size for the impact of the SNP on year 9 students' performance on the multiplicative domain was 0.52. The largest mean gain (0.63) was for students from medium-decile schools, while the highest effect size (0.56) was for female students. In an interesting contrast to previous results, both the smallest mean gain (0.42) and the lowest effect size (0.38) were for students from low-decile schools. Previous findings had indicated that subgroups with lower mean initial stages tended to make greater gains (Tagg & Thomas, 2007; Thomas, Tagg, & Ward, 2003). However, despite the fact that only Pasifika students had a lower mean initial stage, the gains made by students from low-decile schools in 2007 were 0.12 of a stage smaller than those of any other subgroup.

Table 12 shows the effect sizes for demographic factors on the basic facts domain. The effect sizes on this domain tended to be smaller than those on the other six domains. The shaded cells represent comparisons where the difference was not statistically significant at the 1% significance level.

Table 12

Effect Sizes for Comparisons of Demographic Subgroups on the Basic Facts Domain

	Year 9 Initial	Year 9 Final	Year 10 Final
Male/Female	-0.24	-0.17	0.15
High/Medium decile	0.44	0.33	0.14
High/Low decile	0.61	0.59	0.47
Medium/Low decile	0.11	0.21	0.32
NZ European/Māori	0.34	0.36	0.11
NZ European/Pasifika	0.44	0.34	0.44
Māori/Pasifika	0.09	-0.03	0.33

The three largest effect sizes calculated on the basic facts domain were for the differences between high- and low-decile students. The largest effect size calculated for final assessment results was 0.59, for the difference between year 9 students in high- and low-decile schools. The fact that the effect sizes for the differences in final results tended to be smaller than those for initial results would seem to indicate that the differences between subgroups are reduced over the course of the SNP. Overall, year 9 students performed better on the basic facts domain than year 10 students, though the effect size was small (-0.13).

Concluding Comment and Key Findings

Year 9 students in schools participating in the SNP for the first time made progress, by the final assessment, on all three strategy domains. Specific results are as follows:

- The percentages of year 9 students rated at stage 7 or 8 on the additive, multiplicative, and proportional domains increased from 10% to 26%, 25% to 45%, and 37% to 54% respectively.
- The percentages of students still rated stage 4 or lower on the additive, multiplicative, and proportional domains decreased from 12% to 4%, 13% to 5%, and 15% to 6% respectively.

These year 9 students also made progress on the four knowledge domains. By the final assessment, the percentage of students at the top two stages of the domain had increased from 25% to 45% for fractions, from 22% to 43% for place value, and from 27% to 46% for basic facts. The percentage of students at the top stage of the FNWS domain increased from 63% to 77%. The percentages of students rated at stage 5 or below of the domain decreased from 37% to 24% for FNWS, from 51% to 32% for fractions, from 50% to 28% for place value, and from 24% to 15% for basic facts.

While year 10 students performed better than year 9 students on the multiplicative domain, year 9 students performed better on the basic facts domain. The effect sizes of the differences between year levels were small in all instances. The lack of progress in year 10 requires further investigation because previous studies (for example, Higgins, 2004; Thomas & Tagg, 2004; Young Loveridge, 2006) have shown that, in all other year levels, students build on previous achievement.

Demographic factors continue to impact on the performance of students in 2007. Specific factors are as follows:

- New Zealand European students performed better than Māori or Pasifika students; the performance of year 9 Māori students was slightly better than that of year 9 Pasifika students.
- Year 10 male students performed better than year 10 female students. Year 9 male and female students performed similarly, with males performing better on the multiplicative domain and females performing better on the basic facts domain.
- Students from high-decile schools performed better than students from medium-decile schools, who in turn performed better than students from low-decile schools.

References

- Coe, R. (2002). *It's the effect size, stupid: What effect size is and why it is important*. Paper presented at the annual conference of the British Educational Research Association, University of Exeter, England, 12–14 September. Retrieved December 2006 from: www.leeds.ac.uk/educol/
- Higgins, J. (2004). *An evaluation of the Advanced Numeracy Project 2003: Exploring issues in mathematics education*. Wellington: Ministry of Education.
- Maths Technology Ltd (n.d.). *Norms and benchmarks*. Retrieved December 2006 from: www.nzmaths.co.nz/Numeracy/Lead_Teacher/PLC/Norms_and_Benchmarks/index.aspx
- Ministry of Education (2007). *The New Zealand Curriculum*. Wellington: Ministry of Education.
- Ministry of Education (2008). *Students by year level 2007*. Retrieved April 2008 from: www.educationcounts.govt.nz/statistics/schooling/july_school_roll_returns/students_by_year_level/rolls_by_year_level_as_at_1_july_2007
- Tagg, A., & Thomas, G. (2006). Performance on the Number Framework. In *Evaluations of the 2005 Secondary Numeracy Pilot Project and the CAS Pilot Project* (pp. 12–35). Wellington: Learning Media.
- Tagg, A., & Thomas, G. (2007). Performance of SNP students on the Number Framework. In *Evaluations of the 2006 Secondary Numeracy Project* (pp. 29–39). Wellington: Learning Media.
- Thomas, G., & Tagg, A. (2004). *An Evaluation of the Early Numeracy Project 2003: Exploring issues in mathematics education*. Wellington: Ministry of Education.
- Thomas, G., Tagg, A., & Ward, J. (2003). *An Evaluation of the Early Numeracy Project 2002: Exploring issues in mathematics education*. Wellington: Ministry of Education.
- Young-Loveridge, J. (2006). Patterns of performance and progress on the Numeracy Development Projects: Looking back from 2005. In *Findings from the New Zealand Numeracy Development Projects 2005* (pp. 6–21, 137–155). Wellington: Learning Media.

An Investigation into the Impact of the Secondary Numeracy Project on Student Performance in Two NCEA Level 1 Mathematics Achievement Standards

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In 2007, the Secondary Numeracy Project (SNP) operated in some secondary schools for the third year. This study investigated whether the professional development has had an effect on student achievement in two NCEA Level 1 mathematics achievement standards. Analysis of results for these standards found little apparent difference between the attainments for students in SNP schools compared with the rest of the cohort.

Background

The Secondary Numeracy Project (SNP) was formally introduced into secondary schools in 2005. Teachers of year 9 mathematics classes in 42 schools undertook professional development in the teaching of mathematics to year 9 classes. Particular focuses of the professional development were: introducing the Number Framework, which specifies a progression of strategies and knowledge in number; using the Numeracy Development Projects (NDP) diagnostic assessment (NumPA) to ascertain students' number strategies, and administering a whole-class test to establish their knowledge; and expanding the range of pedagogical practices used in mathematics classrooms. The results of the SNP diagnostic assessment were used to establish detailed knowledge of the students' number knowledge and skills, and the project focused on extending students' understanding by building on what they already knew. Teachers in the participating schools received continued support in 2006 to embed good practice into their teaching of year 9 students and to extend this practice to their teaching of year 10 students. In 2007, these teachers received further support to consolidate their development of mathematics pedagogy.

In 2007, the students who were taught as year 9 students in 2005 had progressed to year 11 and, by the end of that year, would potentially have experienced "numeracy-project-aware" teaching for their three years of secondary schooling.

NCEA Level 1 is the first national qualification undertaken by students in New Zealand secondary schools. Most students in year 11 undertake study and assessment towards NCEA Level 1 (although in many schools, this opportunity may be offered to selected year 10 students). At NCEA Level 1, mathematics is not formally assessed through one overarching assessment – instead, it is assessed through a collection of smaller blocks of related content called "standards". Two separate but interconnected systems are used for assessment, namely unit standards and achievement standards. Each of these standards has a weighting of typically between 2 and 4 credits – a full-time course in mathematics consists of about 24 credits. All unit standards are internally assessed, and students who meet the required standard are awarded credits.

There are nine achievement standards at NCEA Level 1; six of these are externally assessed in an end-of-year examination, and three are internally assessed. For the achievement standards, students are awarded grades of not achieved, achieved, merit, or excellence.

Research Question

This study reports on the impact of the SNP on student achievement in year 11 mathematics. Specifically, the research question was:

- What impact on achievement at year 11 in two external assessments can be found for students in SNP schools?

Rationale

Where the standards are internally assessed, that is, the unit standards and three of the achievement standards, data are sent to the New Zealand Qualifications Authority (NZQA) only if the student has achieved the standard at some level. Analysis of trends in performance for internally assessed standard was outside the scope of this study, so the internally assessed standards have not been analysed in this study. However, this could be the focus of a future study.

The two externally assessed achievement standards, Use straightforward algebraic methods and solve equations (Achievement Standard 90147) and Solve straightforward number problems in context (Achievement Standard 90151), were used for this research because these are the achievement standards where the skills developed through the SNP are most likely to have the greatest impact. Trends in the assessment for these two standards were compared for the two groups (SNP and non-SNP students) over the two years of 2006 and 2007.

The Sample

Table 1

Distribution of Candidates by Decile for 2007 Achievement Standard 90147: Use straightforward algebraic methods and solve equations

	Decile										Total
	1	2	3	4	5	6	7	8	9	10	
SNP schools: number of schools in study by decile	2	4	3	5	4	6	5	4	5	3	41
SNP schools: number of candidates from each decile	74	205	260	329	489	657	507	604	1210	207	4542
SNP schools: percentage of candidates from each decile	1.6	4.5	5.7	7.2	10.8	14.5	11.2	13.3	26.6	4.6	100.0
All secondary schools: number of candidates from each decile	823	1363	1925	3246	3990	4108	4912	4064	4604	6413	35 448
All secondary schools: percentage of candidates from each decile	2.3	3.8	5.4	9.2	11.3	11.6	13.9	11.5	13.0	18.1	100.0*
Non-SNP schools: number of candidates from each decile	749	1158	1665	2917	3501	3451	4405	3460	3394	6206	30 906
Non-SNP schools: percentage of candidates from each decile	2.4	3.7	5.4	9.4	11.3	11.2	14.3	11.2	11.0	20.1	100.0

* Percentages may not add up to 100 due to rounding.

Table 1 shows the distribution of candidates by decile for the 2007 Achievement Standard 90147: Use straightforward algebraic methods and solve equations. The table shows that the SNP schools come from the full range of deciles. However, deciles 1, 3, and 10 appear to be comparatively under-represented. A comparison of the proportion of candidates from the SNP schools with the national distribution of students shows that the SNP sample significantly over-represents decile 9 candidates and under-represents decile 10 candidates.

The number of candidates at non-SNP schools for each decile was found by subtracting the number of candidates at SNP schools from the number of secondary students.

Research Method

Data were obtained from the NZQA for two NCEA Level 1 achievement standards and the decile ratings of the secondary schools. The data supplied were: performance of students from the SNP schools, performance by decile of all secondary school candidates, and decile ratings for the SNP schools for 2006 and 2007 (NZQA, personal communications, 2008). The specific achievement standards investigated were: Use straightforward algebraic methods and solve equations (Achievement Standard 90147) and Solve straightforward number problems in context (Achievement Standard 90151). The performance of students at the 41 SNP schools¹ was compared with the national patterns of achievement for all other students in the 2006 and 2007 student cohorts.

Analysis of achievement of students in these two externally assessed standards showed that there was a very strong relationship between the decile rating of the school and the performance of the candidates. This disparity of achievement by social economic status is consistent with data from previously reported studies (for example, Timperley & Alton-Lee, 2008). This relationship can be seen in Figure 1, which shows performance of candidates by decile for the 2007 Achievement Standard 90147: Use straightforward algebraic methods and solve equations.

In order to create a weighted sample of results that matched the decile distribution of candidates in the SNP schools, the performance of candidates in the non-SNP schools, by decile, was weighted by the proportion of candidates from SNP schools of that decile.

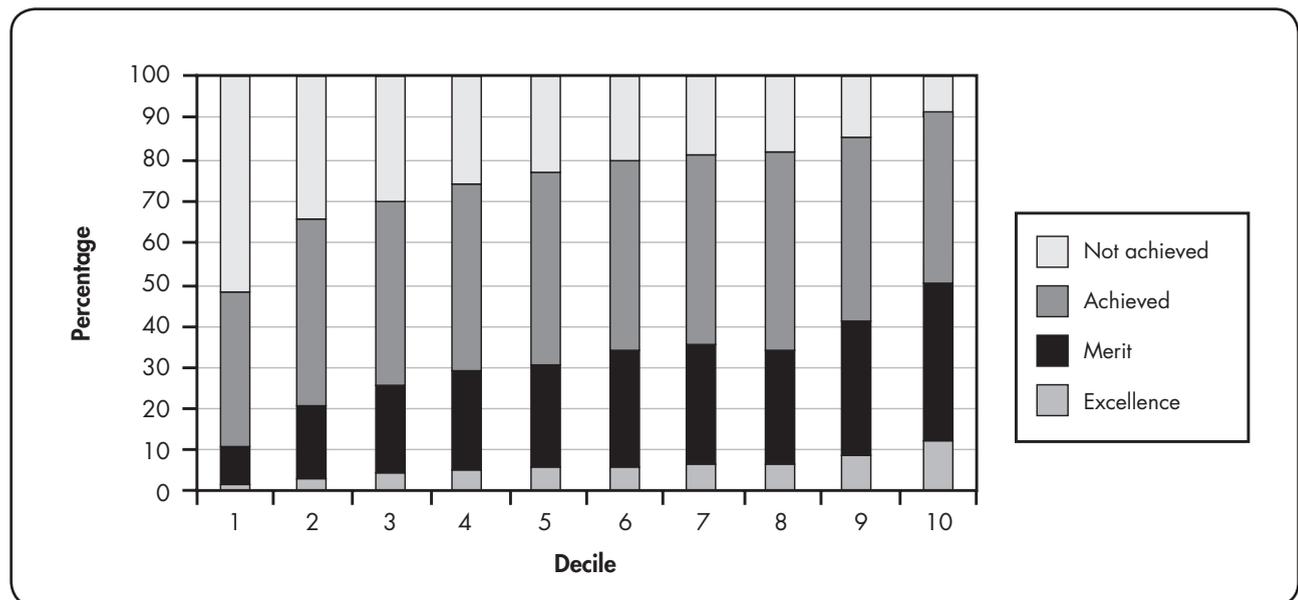


Figure 1. Performance of candidates by decile for 2007, Achievement Standard 90147: Use straightforward algebraic methods and solve equations

¹ One school that completed the SNP in 2005 withdrew from the project in 2006, so the data from this school have not been included with the SNP schools.

Results

Achievement Standard 90147: Use straightforward algebraic methods and solve equations

In the material that follows, "SNP" refers to the students in the 41 schools who first participated in the SNP in 2005. "Non-SNP" refers to those students who were in schools that did not participate in the SNP in 2005.

Table 2

Results for Achievement Standard 90147: Use straightforward algebraic methods and solve equations

	Number	Not Achieved	Achieved	Merit	Excellence
2006 all candidates	35 415	12 495	12 720	6704	3496
2006 SNP	4915	1871	1726	871	447
2006 non-SNP	30 500	10 624	10 994	5833	3049
2007 all candidates	35 448	13 013	15 700	4173	2562
2007 SNP	4542	1866	1935	474	267
2007 non-SNP	30 906	11 147	13 765	3699	2295

Table 3

Comparison of Performance of Students in SNP Schools with Weighted National Distribution of Students in Non-SNP Schools for Achievement Standard 90147: Use straightforward algebraic methods and solve equations

	% Not Achieved	% Achieved	% Merit	% Excellence
2006 SNP	38.1	35.1	17.7	9.1
2006 weighted non-SNP	34.0	35.8	19.5	10.8
2007 SNP	41.1	42.6	10.4	5.9
2007 weighted non-SNP	37.1	44.8	11.4	6.8

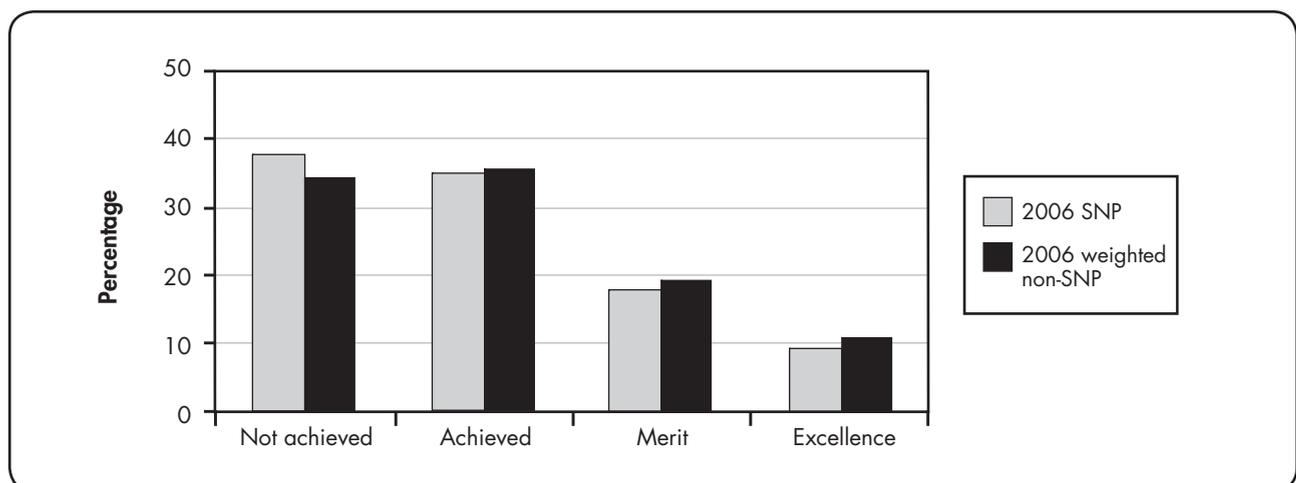


Figure 2. 2006 proportion of students gaining grades for Achievement Standard 90147: Use straightforward algebraic methods and solve equations

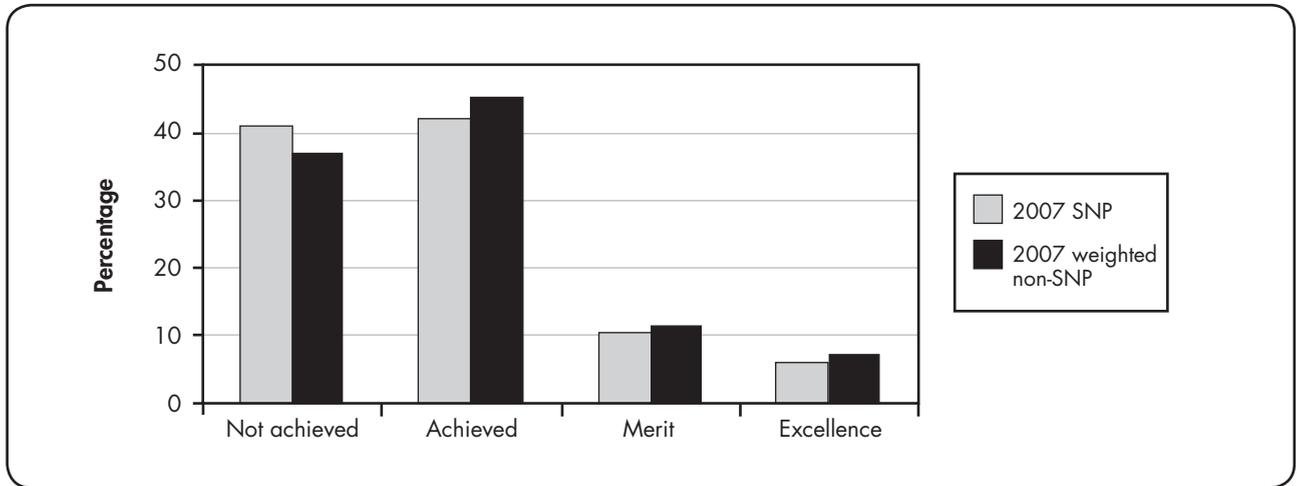


Figure 3. 2007 proportion of students gaining grades for Achievement Standard 90147: Use straightforward algebraic methods and solve equations

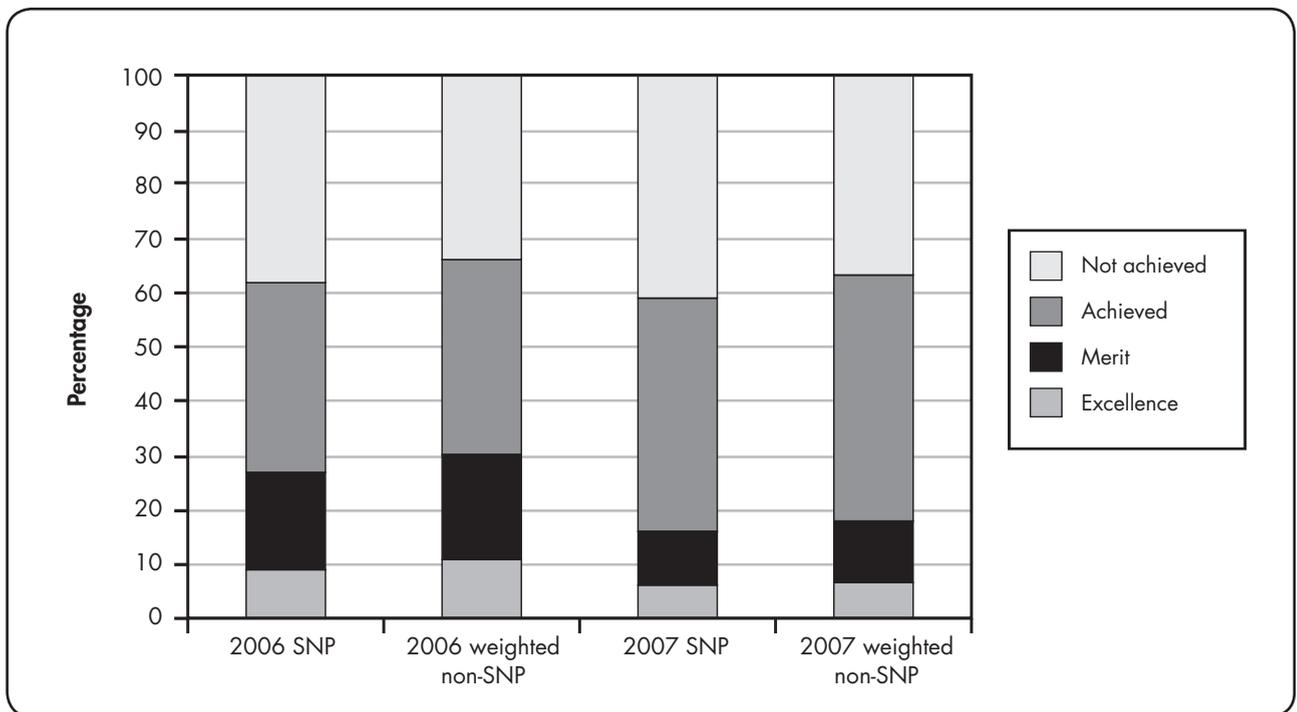


Figure 4. 2006 and 2007 proportions of students gaining grades for Achievement Standard 90147: Use straightforward algebraic methods and solve equations

Table 4
Analysis of Results for Achievement Standard 90147: Use straightforward algebraic methods and solve equations

	Percentage of Students		
	Gaining achieved, merit, or excellence	Gaining merit or excellence	Gaining excellence
2006 SNP	61.9	26.8	9.1
2007 SNP	58.9	16.3	5.9
Progress for SNP	-3.0	-10.5	-3.2
2006 weighted non-SNP	66.0	30.2	10.8
2007 weighted non-SNP	62.9	18.1	6.8
Progress for weighted non-SNP	-3.1	-12.1	-4.0
Growth = progress SNP – progress non-SNP	0.1	1.6	0.8

Results for Achievement Standard 90147: Use straightforward algebraic methods and solve equations are shown in tables 2–4 and figures 1–4. It is clear that students at both SNP and non-SNP schools were less successful in this standard in 2007 than in 2006. In particular, there was a marked decline in 2007 in the proportion of students gaining merit or better compared with 2006. The data also show that, in both 2006 and 2007, students in non-SNP schools were more successful than those in the SNP schools.

Table 4 has been used to compare the performance between the two groups in greater depth. The term “progress” has been used to quantify the improvement in attainment from 2006 to 2007. In every case, for both the SNP and non-SNP students, this figure is negative, indicating that, as a group, the 2007 students found this assessment more demanding than did the 2006 cohort.

“Growth” is the measure of how much students in the SNP schools improved their performance compared with the students in the non-SNP schools. For Achievement Standard 90147: Use straightforward algebraic methods and solve equations, the growth is small and positive for all three measures, although the 0.1 for the proportion gaining an achieved grade or better may be regarded as negligible. This indicates that over the two years, 2006–2007, the students in the SNP schools showed a modest improvement in the proportion who gained merit or excellence when compared with the students in the non-SNP schools.

For this standard, the variation in results between 2006 and 2007 for the SNP schools is largely explained by the change in performance in the whole cohort. However, there may have been a small improvement in performance for SNP students when compared with the rest of the cohort.

Achievement Standard 90151: Solve straightforward number problems in context

Table 5

Results for Achievement Standard 90151: Solve straightforward number problems in context

	Number	Not Achieved	Achieved	Merit	Excellence
2006 all candidates	37 878	5894	19 217	9124	3643
2006 SNP	5563	973	2889	1197	504
2006 non-SNP	32 315	4921	16 328	7927	3139
2007 all candidates	37 295	7330	16 751	10 643	2571
2007 SNP	5144	1148	2332	1314	350
2007 non-SNP	32 151	6182	14 419	9329	2221

Table 6

Comparison of Performance of Students in SNP Schools with Weighted National Distribution of Students in Non-SNP Schools for Achievement Standard 90151: Solve straightforward number problems in context

	% Not Achieved	% Achieved	% Merit	% Excellence
2006 SNP	17.5	51.9	21.5	9.1
2006 weighted non-SNP	16.1	51.4	23.4	9.1
2007 SNP	22.3	45.3	25.5	6.8
2007 weighted non-SNP	20.3	45.3	28.0	6.4

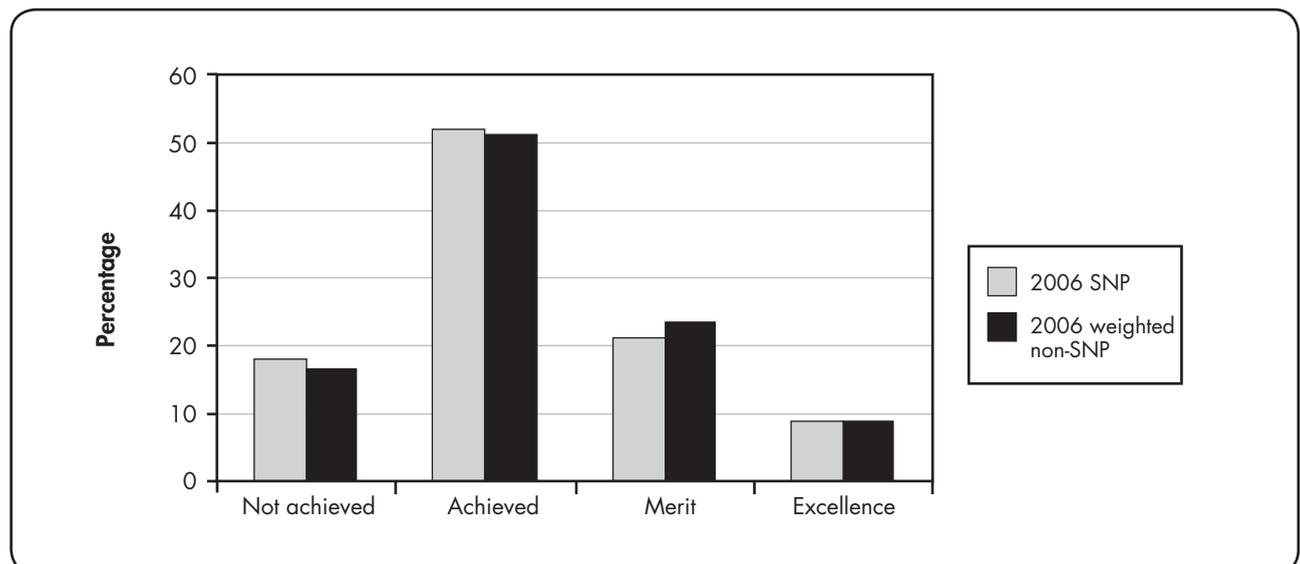


Figure 5. 2006 proportion of students gaining grades for Achievement Standard 90151: Solve straightforward number problems in context

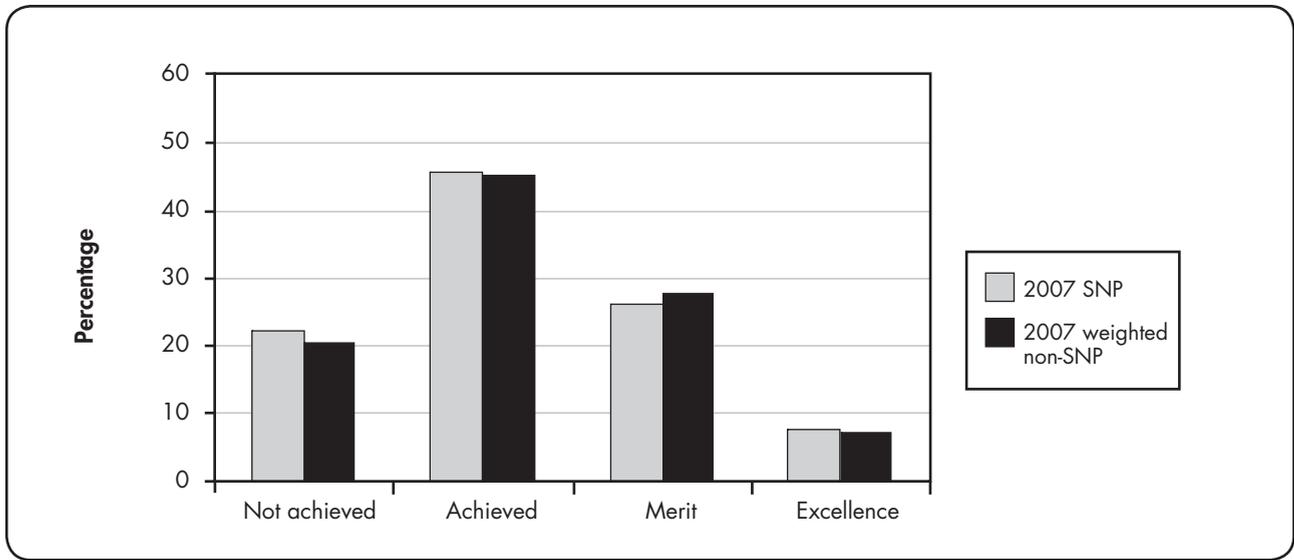


Figure 6. 2007 proportion of students gaining grades for Achievement Standard 90151: Solve straightforward number problems in context

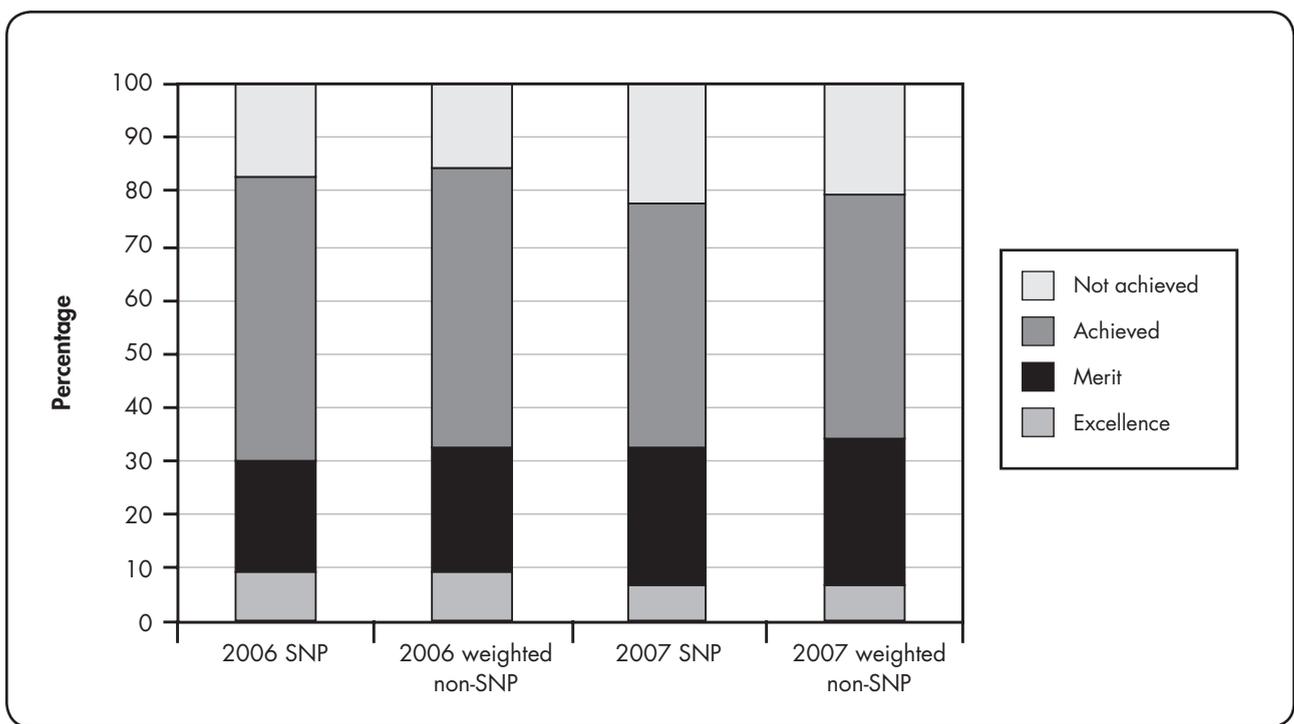


Figure 7. 2006 and 2007 proportions of students gaining grades for Achievement Standard 90151: Solve straightforward number problems in context

Table 7

Analysis of results for Achievement Standard 90151: Solve straightforward number problems in context

	Percentage of Students		
	Gaining achieved, merit, or excellence	Gaining merit or excellence	Gaining excellence
2006 SNP	82.5	30.6	9.1
2007 SNP	77.7	32.3	6.8
Progress for SNP	-4.8	1.8	-2.3
2006 weighted non-SNP	83.9	32.5	9.1
2007 weighted non-SNP	79.7	34.4	6.4
Progress for weighted non-SNP	-4.2	1.9	-2.7
Growth = progress SNP – progress non-SNP	-0.6	0.1	0.5

Results for Achievement Standard 90151: Solve straightforward number problems in context are displayed in tables 5–7 and figures 5–7. These results for the two cohorts (SNP and non-SNP) indicate very similar patterns in both 2006 and 2007. However, the results also show that, compared with 2006, a lower proportion of 2007 students attained excellence and a lower proportion gained achieved or better. However, a greater proportion achieved merit or excellence.

Analysis of the growth in Table 7 shows a slight relative increase in attainment in the proportion of students gaining excellence for the SNP schools and a slight relative decrease in the proportion attaining achieved or better.

Overall for this achievement standard, the variation of results between 2006 and 2007 students in SNP schools matches very closely to the variation of results for all other candidates.

Discussion

The analysis of results shows that, for students involved in the SNP, there may have been a very modest improvement of achievement in Achievement Standard 90147: Use straightforward algebraic methods and solve equations, but there was no apparent difference in the results for Achievement Standard 90151: Solve straightforward number problems in context.

Interpretation of these results needs to be treated with caution. The SNP aims to enhance students' conceptual understanding and achievement in number and algebra strategies, whereas the standards were developed at a time when procedural approaches to answering questions was the focus. The SNP approach includes more judicious use of calculators. However, assessments for the achievement standards are conducted with students being able to use calculators freely. It is possible that ways of showing thinking that are encouraged through the SNP may not yet have been recognised as valid in the marking of the achievement standards assessments.

The assumption that is required to compare the results for the two years is that the cohorts of students in the two groupings will be roughly comparable year to year. This may or may not be true. School-based decisions, such as the setting of the criteria for which students do which courses, may influence the quality of the student population. It is noticeable that between 2006 and 2007, there was a decline of approximately 7.5% in the number of students from SNP schools entered for each of the

achievement standards, whereas the comparable numbers in non-SNP schools has remained relatively stable. This change in student numbers may have had an impact on the profile of performance for the SNP students.

The groups of students taking these assessments does not include the students doing the “alternative” courses. It must be remembered that the teachers of alternative courses in general reported greater impact on the teaching and learning of mathematics than the teachers of the achievement standards classes.

One of the SNP schools changed their decile rating between 2006 and 2007; acknowledging this factor in the make-up of the weighted sample had a noticeable effect on the results for growth for each standard. While the use of decile-weighted results enabled more robust scrutiny of the information, there could be other important factors that need to be considered in order to make a more reliable analysis of the sample compared with the rest of the cohort.

This study has focused on the achievement of students on two externally assessed achievement standards. It is important that these results are not over-generalised. Harvey and Smith (this volume) found that the SNP had a greater effect on the teaching of year 11 students who were doing courses predominantly assessed by unit standards. It is likely that other studies focusing on different standards could produce different conclusions.

A more in-depth study, using students’ assessment scripts for these standards, is likely to give more detailed information about the ways in which students in SNP schools carry out mathematical tasks. Tracking these data may provide information that is of great use in ensuring continued improvements in the ways in which mathematics is taught.

Recommendation

It is recommended that research be carried out on student performance using examination scripts for year 11 external assessments to better assess the way in which the SNP has impacted on student performance.

References

- Harvey, R., & Smith, D. (this volume). What views do teachers hold on the impact of the Secondary Numeracy Project on the teaching of year 11 classes? In *Findings from the New Zealand Secondary Numeracy Project 2007*. Wellington: Learning Media.
- Timperley, H., & Alton-Lee, A. (2008). Reframing teacher professional learning: An alternative policy approach to strengthening valued outcomes for diverse learners. *Review of Research in Education*, 32, 328–369.

Teachers' Views on the Impact of the Secondary Numeracy Project on the Teaching of Year 11 Classes

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In 2007, the Secondary Numeracy Project operated in some secondary schools for the third year. This study investigates whether the professional development has had an effect on teacher practice in year 11 mathematics classes and on student achievement. The study finds that the impact of the project on teacher practice varies, with those teachers who are teaching year 11 mathematics classes that focus on NCEA unit standards tending to report greater changes in practice than their colleagues who are teaching year 11 classes focusing on NCEA achievement standards.

Background

The Secondary Numeracy Project (SNP), which was formally introduced into 42 secondary schools in 2005, offered professional development to teachers of year 9 mathematics classes. (For further background, see Harvey, this volume.) These teachers received continued support in 2006 to consolidate their teaching of year 9 students and to extend this practice to their teaching of year 10 students. In 2007, these teachers received further support to consolidate their development of mathematics pedagogy.

An extended time frame is usually a requirement for effective teacher professional development (Timperley, Wilson, Barrar, & Fung, 2007). The SNP requires extensive involvement from teachers in the first year and then ongoing professional development over subsequent years. This study investigates the impact of the professional development over time in one area of teacher practice.

Belonging to a supportive community enables teachers to develop their knowledge and practice (Anthony & Walshaw, 2007). Through the SNP, members of schools' mathematics departments experience professional development as a team, with one member of each department being given additional support in order to lead the project within their school. It is through the operation of such departments that the professional issues of teaching can be addressed, a process that may result in enhanced pedagogical practices in mathematics teaching at all levels.

In 2007, the students who were taught as year 9 students in 2005 had progressed to year 11 and, by the end of that year, would potentially have experienced "numeracy project aware" teaching for their three years of secondary schooling.

This study reports on teachers' views of the impact of the SNP on the teaching of year 11 mathematics. Specifically, the research question is:

- What views do teachers hold on the impact of the SNP on the teaching of year 11 mathematics?

Research Method

Five schools that had been part of the initial 2005 professional development were involved in this study. The schools were selected by convenience, in that they were schools where one of the researchers had a professional link. From each school, the school's Head of Mathematics selected three or four year 11 mathematics classes to participate in the study, giving a total of 17 teachers in the study. The classes selected in each school included at least one class that was doing a course that was assessed by NCEA achievement standards and at least one class that was being assessed by NCEA unit standards. The rationale for this distinction in the mathematics classes was that, at year 11, most schools offer students two different types of mathematics courses to pursue. Students who have been successful in year 9 and 10 mathematics usually study the mathematics course predominantly assessed using NCEA achievement standards (mostly assessed externally). Those who found year 9 and 10 mathematics demanding usually study a mathematics course mainly assessed using NCEA unit standards (assessed internally).

Teachers of the classes involved in the study were asked to complete a written survey to ascertain the impact of the SNP on their practices in teaching their year 11 classes and the impact of their approaches on students' achievement. The teacher practice aspect of the survey was organised into 15 sections, including increased use of group work, differentiated teaching, and greater emphasis on understanding key ideas. The participating teachers were asked to indicate if the SNP had impacted on their teaching of years 9, 10, and 11 students and to comment about the impact on their teaching of year 11 students. In addition, the participants were invited to describe any other changes to their teaching of year 11 students that they felt resulted from their involvement in the SNP. Similarly, the participants were asked to rank and describe the impact of the SNP on their year 11 students in four areas, including increased questioning by students and less reliance on calculators. The final section of the survey gave the participants the opportunity to describe any barriers they felt there were to implementing the SNP with year 11 classes and asked them to include any other comments they might have about the impact of the project.

Results

Impact on Teachers' Practice

Teachers were asked open-ended questions about their practice and about their perception of the impact of their practice on the students' involvement, understanding, and achievement. The researchers have made an overall judgment of the teachers' responses by aggregating responses to individual questions, as shown in Table 1.

Table 1
Teachers' Responses about the Impact of SNP on Their Teaching

Main Assessment Used in Course:	Achievement Standards	Unit Standards	Combined Results
Impact:			
Very little impact	3	1	4
Modest positive impact	2	1	3
Positive impact	3	6*	9

* Includes one class completing a two-year NCEA Level 1 course

Note: One survey was not rated because the teacher was new to the school in 2007.

Analysing the overall results by course reveals the strong association between the classes taking the unit standards course and the impact of the SNP. This link will be discussed later in this paper.

For discussion in this paper, the teachers' responses have been grouped by themes.

1. *Students Sharing Their Strategies*

A number of teachers reported extending their use of discussion in developing ideas with their students:

Teacher-student discussion as a class forum. (Teacher 3)

I ask students to explain a lot more of their work to me when helping them one-on-one rather than just [jumping] in and [showing] them what to do. I ask students to explain their working to each other. (Teacher 5)

I usually ask the question "Why?" or "How did you get that?" Probably because of the numeracy project. (Teacher 11)

Students [are] encouraged to give their ideas and solutions and to discuss them with others in the class. (Teacher 2)

Despite many of the teachers embracing greater use of discussion, a small minority of the teachers cautioned against placing too much emphasis on class discussion:

If a student comes up with a difficult or complex strategy, I tend to cut it off and encourage easier, more effective strategies. Too many strategies and the year 11s complain about being confused, so I stick with what strategies are effective and limit them deliberately. (Teacher 7)

2. *Calculator Use*

A focus of the SNP is on developing mental strategies for calculation and on using calculators only when it is appropriate. Many teachers reported more judicious use of calculators in their classes. However, as calculators may be used in almost all assessment situations, some students still depend heavily on their calculators for support:

Students definitely use the calculator less and use it to save time and energy/brain power rather than not knowing what to do. Year 11 is still quite heavily reliant on a calculator. (Teacher 11)

Limiting the use of calculators and placing an emphasis on "the art of approximation" prior to solving problems. (Teacher 14)

Less dependence on calculators. More confidence in estimation. (Teacher 12)

3. *Changes in Student Participation, Work, and Understanding*

The teachers were asked to "Describe any changes to student participation, work, and understanding that you believe are due to the Numeracy Project." The majority of the teachers cited positive influences on student participation in year 11 classes:

My students gradually obtain a better understanding of number. Certain algebra techniques like equation solving and expanding using grids have assisted skills mastery. Some other topics have been neglected. (Teacher 2)

The students want to understand and aren't satisfied if they don't. (Teacher 11)

Students are definitely more prepared to give strategies and ideas. (Teacher 13)

Focused learning, willingness to ask questions and not be afraid of maths. (Teacher 15)

[Students] are more involved and interested [now] because they are able to understand rather than just learn to get the right answer. (Teacher 17)

These statements indicate that, in many classrooms, students are more actively involved in their mathematics learning. However, not all teachers noted this change:

Have not been cognisant of change in this year 11 class. (Teacher 16)

4. *Barriers to Implementation*

Teachers were asked to list factors that prevented them from making greater use of the practices that had been advocated in the SNP. Workload was the most commonly cited factor. Other frequently cited factors were the exam expectations of the school, the students, and the community. Factors that are primarily beyond the scope of the SNP include student behaviour, student motivation issues, and staff turnover.

Additional Comments

There is a big leap from strategy stage 7/8 to year 11 achievement standards. Many of our students [are] still working at strategy stage 5/6 and yet [are] expecting to do the achievement standard course. (Teacher 11)

... this course [unit standards] exists [because] there is a lack of numeracy among certain year 11s. If the numeracy project had worked, this course would be redundant. (Teacher 16)

I feel my students (at year 11) have missed picking up on the basic principles at a younger age, so the further on they get in their schooling years, the further behind they get. Their confidence in maths is poor, so they are hesitant to think they can do maths without a calculator. (Teacher 13)

It has opened my eyes to some new ideas and techniques. It has made me aware of how much we need physical resources and how few we have. It has made me aware of how much assessment drives our teaching. (Teacher 4)

I like the questioning I am able to do now. (Teacher 9)

Our year 9 programme has greatly assisted my year 11 teaching. (Teacher 15)

This year group was our first (we are still learning), and the students were new to numeracy ideas as well. When the students come through having had the numeracy project from year 1, I believe the impact will be higher. (Teacher 11)

Discussion

This study was designed to look for impact on practices of teaching and learning in year 11 mathematics classes, despite year 11 not being the focus of the SNP. Many of the teachers have indicated that they have made adjustments to their practice at year 11 that have built on the ideas that they have trialed in their year 9 and 10 mathematics classes.

It is notable that, in general, the teachers of the unit standards mathematics classes felt that the SNP had had a greater impact on their teaching of these year 11 students than it had on the teaching of their peers in achievement standards classes. A possible reason for this is that there is a greater focus on number in the unit standards courses compared with the achievement standards courses and, hence, the emphasis on mental strategies is more easily applied in the unit standards courses. Another reason could be that the teachers of unit standards classes recognise that their students found year 9 and 10 mathematics demanding, and so these teachers are able to see benefits in adopting different pedagogical strategies.

The survey indicated that there is considerable variation in the degree to which teachers feel that their teaching has been influenced by their participation in the SNP. In order to sustain and extend the changes instigated by the SNP (changes that many teachers may consider to be modest), it may

be beneficial for mathematics departments to have explicit discussions about the ways in which individual teachers have enhanced their teaching of year 11 mathematics classes.

A goal of the SNP is to change the way in which mathematics is taught in secondary schools. Such changes to the methods of teaching are unlikely to be achieved quickly or easily. The changes that teachers have made to date should be acknowledged and used to support the continued growth of teacher practice.

Recommendation

Mathematics departments in their third and subsequent years of involvement with the SNP should be encouraged to explicitly discuss the ways in which the pedagogies of the SNP can enhance the teaching of mathematics in the senior secondary school.

Ongoing support should be provided to enable mathematics teachers to continue exploring and developing their practices.

References

- Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in mathematics/pāngarau: Best evidence synthesis iteration [BES]*. Wellington: Ministry of Education.
- Harvey, R. (this volume). An investigation into the impact of the Secondary Numeracy Project on student performance in two Level 1 mathematics achievement standards. In *Findings of the New Zealand Secondary Numeracy Project 2007*. Wellington: Learning Media.
- Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2007). *Teacher professional learning and development: Best evidence synthesis iteration [BES]*. Wellington: Ministry of Education.

Written and Oral Assessment of Secondary Students' Number Strategies: Developing a Written Assessment Tool

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This paper examines the first trial of a Written Strategy Stage Assessment Tool (WSSAT) designed for use within the Secondary Numeracy Project (SNP) in secondary schools. WSSAT was designed to identify students' strategy stages and to provide formative data for teachers to use in their planning and teaching. A year 9 cohort of SNP students took the written assessment, which focuses mainly on number strategy concepts. A small sample of these students was also interviewed by a numeracy expert to identify each student's strategy stage, using GloSS (Global Strategy Stage) oral-type protocols. Results from the written assessment gave relatively consistent measures of stages in terms of the criteria set to establish the stages. Comparison of the written and oral assessment results gave data on the relationship between the two measures. The stages established by the written assessment did not match the stages indicated by the oral assessment. They were generally lower, often by several stages. These results raise issues around the criteria set for establishing stages using the written assessment tool.

Background

Assessment practices for establishing strategy stages within the Numeracy Development Projects (NDP) have, from their inception, focused on diagnostic interviews for both summative and formative purposes. This has avoided issues of students' reading levels, particularly with very young students, and has encouraged the teasing out of students' understandings via oral interactions that move beyond the assessment script of the NDP diagnostic interview. The NDP oral assessment tools (NumPA [Numeracy Project Assessment] and GloSS [Global Strategy Stage]) establish the stages and indicate "where to next" in terms of teachers' specific planning for strategy (and knowledge) teaching and learning. Both tools have worked satisfactorily in primary school environments, in which the teachers are mainly generalist teachers with a single class. With the establishment of the SNP in secondary schools, involving specialist mathematics teachers who work with multiple classes of students each day, the need for a written assessment tool has been recommended by facilitators and teachers.

It is anticipated that such a tool would:

- reduce the amount of teacher time taken per student
- provide a record of students' work that fulfils the same purposes (formative and summative) as the NumPA diagnostic interview
- provide a standardised "marking" schedule.

Such a schedule would enable the assessment to be marked consistently by someone new to the SNP or even by a person with little knowledge of numeracy, such as a parent or teacher aide. It would also reduce possible individual variability that might occur when conducting diagnostic interviews.

Another benefit would be an enhancement in the accuracy of the assessment, particularly in the multiplicative and proportional domains. These domains take on a significant focus in secondary schools, but Thomas, Tagg, and Ward's (2006) data set the accuracy of various types of numeracy assessments in these domains at 76%.

A written tool was not seen as a replacement for GloSS or other GloSS-type oral assessments or for their use for “in-action” checking of students' progress to inform the teacher's next teaching action. Rather, a written tool was seen as replacing the NumPA as an initial or final assessment of a whole class of secondary-school students.

On this basis, a written assessment tool for the part–whole strategy stages was developed for trialling in secondary schools. Although, in these schools, reading skills were expected to be at a level that would not impact negatively on the student's ability to respond, care was taken in the trial assessment tool to limit the amount and difficulty level of written material. The development took place within the SNP community and was informed by interaction and feedback in a similar way to many other previous developments within the NDP.

Method

The participants in the study were drawn from the year 9 cohort of a large, Auckland, decile-3 secondary school of mixed ethnic composition. The school had participated in the SNP in 2006 and 2007. The written assessment was given to the majority (278) of the year 9 cohort, with only one small class of special needs students being excluded at the school's request. The oral assessment developed for use in this research was given to a subset (27) of the year 9 cohort, who were drawn from a range of the bands in which the school organised their classes. This banding was based on Assessment Tools for Teaching and Learning (asTTle) measures of the students' general achievement in mathematics and English (see Table 1). The subset sample comprised students whose participation in the study would least inconvenience the school, with the interviewer working around the school's programme to order to access students.

Table 1

Classes in Bands (High to Low), Showing Student Roll Numbers and the Number of Students Taking the Written and Oral Assessments

Class Name	9A1	9A2	9B1	9B2	9B3	9B4	9C1	9C2	9C3	9C4	9D1	9D2	Total
Roll No.	32	32	31	32	31	31	29	29	27	22	26	14	336
Written Ass. No.	27	28	28	29	23	28	25	14	25	16	24	11	278
Oral Ass. No.	10	–	–	–	10	–	–	–	–	–	4	3	27

Note: Students in the bolded classes used the stage 5–7 version of the written assessment; the students in other classes used the 5–8 version (see below).

The Nature and Structure of the Written Strategy Stage Assessment Tool

The Written Strategy Stage Assessment Tool (WSSAT) is closely aligned with, and arises from, the NDP strategy section of the Number Framework (and to a lesser extent, given the tool's strategy focus, the knowledge section of the Framework). The items (short answers and multi-choice) draw upon the material of the NumPA and GloSS assessment tools as well as on the work of Hart (1981) and Brown, Hart and Dietmar (1984) on place value and decimals and on that of McIntosh, Reys, Reys, et al. (1997) on number sense. The focus of the WSSAT is primarily strategy, with two overlapping versions: one covering stages 5–7 for less advanced students and the other stages 5–8 for more advanced students (this version was the stage 5–7 version with an additional stage 8 component).

Each stage has a set of items, with the students required to correctly answering a specified number of items to signify their achievement of that stage (see Table 2). As with the SNP diagnostic interviews,

the highest strategy stage achieved in the WSSAT trial was taken as the stage for the student. For example, if a student met the criteria for stages 5, 6, and 7, they were classified as being at stage 7, or if they met the criteria for stage 5 and not that for stage 6, but did meet the criteria for stage 7, they were classified as stage 7. Where the students did not meet the criteria for stage 5 or higher, they were assigned to a category covering stages 1–4.

Table 2
The Number of Items per Stage and Criteria for Achieving Each Stage

Stages	1–4	5	6	7	8
Number of items	12	12	11	9	10
Number correct to assign to the given stage	< 9	> 9	> 8	> 7	> 9

Although the WSSAT is a written assessment in the sense that the students read the problems on their own and write their answers down, they do not write down the process by which they arrive at their answers. This is different from a diagnostic interview, in which the questions are communicated orally (read) to the students, printed material is sometimes given as a prompt/reminder, and the students give their answer orally and often talk through the process they used to arrive at their answers. Further prompts can then be given by the interviewer to elicit explanation and clarify points about a student's understandings. Thus, assigning stages by oral assessment relates largely to process, whereas assigning stages in the WSSAT assessment is based on outcomes.

A key issue in the WSSAT is to ensure that the items used force the student participants to use a particular process and restrict their use of any other approach, that is, items that could be answered procedurally are not included. For example, some of the written assessment items use combinations of numbers written as words and in digits, for example, "2 fifths is equivalent to 0.4". The aim is to expose the student's in-depth understanding by stating material in a way that limits the use of procedural methods and requires more understanding of number structure. It is also seen as a way of keeping aspects of "oral" language use within a written format, with "two-fifths" giving a potentially greater access to the underlying meaning than "zero point four" (or possibly "four tenths") in symbolic form, while still requiring the students to be able to connect the underlying meanings. Whether this has worked will be determined in part by the extent to which the assignment of stages via the written assessment matches the oral assessment used in the trial.

For the use of the WSSAT in this study, an answer sheet was provided that was laid out with clear directions for the students to follow. The format ensured ease of marking, giving both a quick impression of a student's stage and more detailed formative data for planning and teaching purposes. The answer sheet had room only for the answers because what was required was the result of mental processes rather than the process itself or procedures such as written algorithms.

A range of items for each stage needed to isolate and encapsulate some of the conceptual aspects and elements of strategy that might be present in an oral exchange. These sets of items have the potential to provide a more detailed and standardised diagnostic map of a student's learning needs than an oral assessment because they allowed the student to attempt all the items, thus demonstrating any partial understandings that the student might have (with some items answered correctly) beyond the point where an oral assessment would stop.

The Oral Assessment

The oral assessment tool used in this trial was developed as a research tool rather than as a tool for general use with students. The tool used some of the GloSS items, but these were supplemented by a few items that offered the potential for allowing a finer measure of a student's position within a stage, that is, beginning, middle, or later (nearly ready to move to the next stage).

Data Collection

The WSSAT was conducted during term 4 in each class's usual classroom setting, mostly under the supervision of the regular mathematics teacher, on the Monday of the week before the students' end-of-year exams. Both the teachers and the students received a set of instructions on how to conduct the assessment, a key point of which was that the students could stop if they felt the items had become too difficult for them. This instruction was an attempt to mirror the practice used in the NDP diagnostic interviews of the assessor only asking questions until the student started to give incorrect responses or showed other evidence of failing to understand what the questions required. At that point, the assessor would bring the interview to a close and assign a stage to the student on the basis of the student's responses up to that point.

Although the research team did not brief the teachers before giving them the written assessment, the liaison teacher who worked with the teachers had been briefed. After the assessment, the teachers collected all the answer sheets and gave them to the researchers. To ensure consistency, only one person marked the assessments. Copies of the marked answer sheets were returned to the school for potential diagnostic use by the school.

The GloSS-type oral assessment was given the next day, Tuesday, to a sample of 27 students drawn roughly, and at the school's convenience, from across the "ability" bands into which the year 9 cohort was organised. The oral interviews were conducted by a person external to the school who had expert knowledge of the NDP and was well experienced in conducting and interpreting oral assessments.

Analysis

The results of the written assessment were first analysed for the internal consistency of the tool in identifying a student's stage, that is, whether a student assigned as being at stage 6 had also been assigned as being at stage 5, and so on. Then they were analysed against three other measures of achievement, one school-based and two based on nationally collected data from the NDP. The stages that the students achieved were compared with:

- the banding of the class they were in to see whether this reflected the school's placement of students on the basis of asTTle results
- the 2007 national, end-of-year, year 9, low-decile stage distribution data from the SNP (Tagg & Thomas, this volume)
- the 2006 national, end-of-year, year 9, stage distribution data from the SNP (Tagg & Thomas, 2007).

The results of the written and oral assessments were compared to establish a relationship between the two forms of assessment. For the purpose of comparison in this study, the oral assessment was assumed to be accurate and thus provided a baseline for the comparison. This assumption was used in order to establish the connection to the existing and "proven" NDP assessment tools and the database of student results arising from their use.

Results

Internal Consistency

Of the 109 students who could be assigned as being at stage 6 on the written assessment, only 14 had not also achieved the criteria for being assigned as being at stage 5. Of these 14, eight had missed the criteria by only one correct response and three by two correct responses. For the 19 students assigned as being at stage 7, only four had not achieved the criteria for one or both of stages 5 and 6. Of these four, two had missed the criteria for stage 6 by only one correct response. These data suggest that the written assessment was largely internally consistent in assigning stages.

A further analysis, conducted for the situation where the criteria for assigning stages were adjusted by reducing one of each component, resolved the ten cases mentioned above and created only seven more. Of the 136 students who could be assigned as being at stage 6 on the adjusted criteria, only nine had not also achieved the criteria for being assigned as being at stage 5, and of the 49 assigned as being at stage 7, only six had not achieved the criteria for one or both of stages 5 and 6. This analysis also supports the internal consistency of the written assessment in assigning stages 5–7. No stage 8 was assigned under either set of criteria levels, so consistency across stages 7 and 8 could not be determined.

Conformity of Assigned Stages with Students Banding into Classes

The stages assigned by both the written and oral assessment generally conformed to the banding of the classes: students in higher band classes achieved more of the higher stages, and students in lower band classes achieved fewer of the higher stages. Additionally, in line with the internal consistency of the written assessment tool, as discussed above, the meeting of the criteria for particular stages also aligned with the banding of the classes, with fewer students meeting the criteria for each stage. For example, of the students in class 9A2, 10 students met the stage 7 criteria, 23 the stage 6 criteria, and 22 the stage 5 criteria, compared with class 9B2, in which no students met the stage 7 criteria, 17 met the stage 6 criteria, and 23 met the stage 5 criteria, while in class 9C2, only one student met the stage 6 criteria and two met the stage 5 criteria. No one in 9D2 met any of the criteria for stages 5–7. (See Table 3 for the written assessment data.)

Table 3
Classes in Band Order, Showing the Number of Students Taking the Written Assessment and the Number of Students Meeting the Criteria for Achieving a Particular Stage

Class	9A1	9A2	9B1	9B2	9B3	9B4	9C1	9C2	9C3	9C4	9D1	9D2	Total
Number of students	27	28	28	29	23	28	25	14	25	16	24	11	278
Number meeting stage 5	24	22	19	23	8	17	4	2	8	5	3	0	135
Number meeting stage 6	23	23	10	17	5	18	5	1	4	0	3	0	109
Number meeting stage 7	4	10	0	2	0	3	0	0	1	0	0	0	20

Note: Students in the bolded classes used the stage 5–7 version of the written assessment; the students in other classes used the 5–8 version.

This data suggests that the asTTle measure of general achievement in mathematics and English that was used to place the students in bands, and classes within those bands, is reflected in the WSSAT measurement of their numeracy achievement and assigning of stages.

However, the results for class 9B3 do not match the results for other classes in that band, being lower for all stages. This may reflect some variation in: the composition of this class compared with others in the same band; the way in which the assessment was actually conducted for this class despite the instructions that were distributed to each teacher; or a teacher/teaching practice difference.

The oral assessment also appeared consistent with the placement of the students in their bands. Of the ten students in 9A1, eight were assigned as being at stage 8 and two at stage 7; of the ten students in 9B3, five were assigned as being at stage 7, four at stage 6, and one at stage 5; and the three students in 9C2 were assigned as being at stage 5. The students in 9C1 did not fit as well, with three being assigned as being at stage 7 and one at stage 5.

Comparison between Oral and Written

The stages determined by the written assessment generally did not match the stages determined by the oral assessment (see Table 4). Only one student at stage 6 was the same. The stages assigned by the written assessment were generally lower, with six students lower by more than two stages (see the bolded figures in Table 4).

Table 4

Comparison of Stages Assigned to Students by the Written Assessment Compared with the Oral

Oral assessment stage	5	6	6	6	7	7	7	8	8
Written assessment stage	1–4	1–4	5	6	1–4	5	6	6	7
Number of students	5	2	1	1	4	2	4	6	2

The assignment of generally lower stages using the written assessment can also be seen in Table 5, where 46% of the students were assigned as being at stages 1–4 using the written assessment, while none of the students assessed orally were assigned as being at less than at stage 5. At the upper stages, only 7% were assigned as being at stage 7 on the written assessment, much fewer than the 41% assigned as being at stage 7 by the oral assessment. Also, no students were assigned as being at stage 8 using the written assessment, while 26% of the students assessed orally were assigned as being at stage 8.

Comparison with 2007 Low-decile Data and 2006 Year 9 Data

The assigning of stages from the written assessment gave rise to a considerably different distribution (see Table 5) from both the low-decile-schools data (averaged across the strategy domains; see Tagg & Thomas, this volume) and the 2006 year 9 data (Tagg & Thomas, 2007). The most significant differences are evident in the percentage of students in the stage 1–4 category, with the written assessment at 46%, compared with 11% and 2% respectively for the oral assessment, and in the comparison of the top two stages, in which the percentages of stages 7 and 8 assigned by the written assessment are very much less than those for the oral assessments. The school in which the written assessment was conducted is decile 3, which might indicate an achievement level below the 2006 year 9 data, but this does not account for the variation from the low-decile-schools data.

Table 5
The Percentage of Stages Assigned to Students for Each Assessment Tool and for the End-of-year, Year 9, Low-decile (2007) and the Year 9 as a Whole (2006)

Stages	0–4	5	6	7	8
Written assessment					
% of students (n = 278)	46	13	34	7	0
Oral assessment					
% of students (n = 27)	0	18	15	41	26
Low-decile data					
% of students (n = 412) (Tagg & Thomas, this volume)	11	29	33	22	5
2006 year 9 data					
% of students (n = 5807) (Ministry of Education, n.d.)	2	14	27	39	18

However, the oral assessment's assigning of stages to students is reasonably close to the 2006 year 9 data percentages, particularly when taking into account the small sample size. This would support the assumption that the oral assessment is accurate and provides a useful baseline for calibrating the written tool.

Discussion

The initial trial of the written assessment tool has highlighted a number of issues. Although the WSSAT is internally consistent for stages 5–7 and can be used to assign students a stage, there is a significant divergence of these stages compared with the data for low-decile-schools at the end of their first year in the SNP, the 2006 year 9 data, and the oral assessment assignment of NDP stages. Thus, the written assessment tool in its current form does not determine a student's numeracy strategy stages with any apparent degree of accuracy. Although these results clearly indicate that further development work is definitely needed, the internal consistency suggests that there is a solid basis from which to conduct such a development.

Further steps are needed to examine the criteria for assigning stages and to see if adjusting these criteria could assist in creating a closer match between the stages assigned by the WSSAT and the national data sets for the current data. This could be done by repeating the above analysis. Following this, and any changes to items or format suggested by teachers from the study school or prompted by expert feedback, such a refined tool would need to be further trialled.

References

- Brown, M., Hart, K., & Dietmar, K. (1984). *Chelsea mathematics tests: Place-value and decimals*. Berkshire, UK: NFER-Nelson.
- Hart, K. M. (Gen. Ed.) (1981). *Children's understanding of mathematics: 11–16*. London: John Murray.
- McIntosh, A., Reys, B., Reys, R., Bana, J., & Farrell, B. (1997). *Number sense in school mathematics: Student performance in four countries*. Perth, WA: MASTE.
- Ministry of Education (n.d.) Curriculum expectations. Retrieved December 2007 from: www.nzmaths.co.nz/Numeracy/Lead_Teacher/PLC/Expectations/index.aspx
- Thomas, G., Tagg, A., & Ward, J. (2006). Numeracy assessment: How reliable are teachers' judgments? In *Findings from the New Zealand Numeracy Projects 2005* (pp. 91–102). Wellington: Learning Media.
- Tagg, A., & Thomas, G. (this volume). Performance of SNP students on the Number Framework. In *Findings of the Secondary Numeracy Project 2007*. Wellington: Learning Media.

Solving Equations: Students' Algebraic Thinking

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This study is part of current initiatives to extend the New Zealand Numeracy Development Projects from number into algebra. The paper describes an approach to linking numeracy with students' strategies for solving linear equations. Preliminary data from diagnostic interviews with approximately 450 year 7–10 students suggests that there is a hierarchy of sophistication of strategies. The most sophisticated strategy that a student is able to use is associated with the student's numeracy stage on the Number Framework.

Introduction

A new curriculum for all subject areas in New Zealand schools was launched on 6 November 2007 (Ministry of Education, 2007). The mathematics and statistics learning area is now divided into three strands rather than the previous six, with number and algebra being one strand. The achievement objectives of this strand are grouped to reflect the structure of the Number Framework (Ministry of Education, 2003), which details the number strategies that students use and the number knowledge required for these strategies. At the lower levels of the new mathematics and statistics learning area, the number and algebra achievement objectives are divided into: number strategies, number knowledge, equations and expressions, and patterns and relationships. The integration of number and algebra into one strand follows debate within the mathematics education community in New Zealand and within international research (see, for example, Carraher & Schiemann, 2007; Kieran, 1992; and Lee, 2001, for an explanation of algebraic thinking).

The Numeracy Development Projects (NDP) have been very successful at raising the achievement of New Zealand students in the number strand (Thomas & Tagg, 2007; Young-Loveridge, 2007), and various initiatives are currently underway to extend the projects into early algebra. Irwin and Britt (2007) examine the impact of the NDP on students' ability to generalise and suggest that students who can access a range of strategies to solve numerical problems are using thinking that is essentially algebraic in nature. These students are the ones who are likely to be able to work with algebraic symbols to express generality. University of Otago College of Education researchers involved in the study reported in this paper are examining students' strategies for solving linear equations and the relationship of these strategies to numeracy.

Background

Many students struggle with introductory algebra, and teachers have little to guide them in assisting their students to learn this important component of mathematics. Little is known about the effect of students' numeracy on the learning of early algebra or about the strategies that students use to solve equations. There is widespread agreement that few students easily understand algebra. The Cockcroft (1982) report highlights the fact that algebra is a source of considerable confusion and negative attitudes among students, while the title of Brekke's (2001) paper, "School algebra: Primarily manipulations of empty symbols on a piece of paper?" sums up the situation for many students.

Arithmetic in schools is often presented as a computation ready to complete, for example, $3 + 5 =$. Students know that pressing the equals button on a calculator performs a calculation on whatever has been entered, so many students understand "equals" as meaning "compute now" rather than

“is equivalent to” (Booker, 1987; Booth, 1988). Linchevski (1995) suggests that, in the transition from arithmetic to algebra, students need to move from a unidirectional view of the equals sign to a multidirectional one.

Closely related to this is the use of an equation as a process rather than as an object that can be operated on (Sfard, 1991). Students initially see equations as the description of an arithmetic process, for example, $2 \times 3 + 4 = x$, and when presented with an equation to solve, for example, $2x + 4 = 10$, they see this, too, as the description of an arithmetic process, with “guess and check” as the natural way of finding x . Even the more sophisticated strategy of solving the equation by working backwards may result from a view of equations as processes, yet this viewpoint is often not revealed until the students encounter equations of the kind $2x + 4 = 3x - 6$.

It is not possible to view this type of equation (unknowns on both sides) as the description of a process giving a result. What is essential is to view the equation as an object to be acted upon in order to be solved (Sfard & Linchevski, 1994). Herscovics and Linchevski (1994), however, present data that shows that many students revert to the strategy of “guess and check” to solve equations of this type. The issue of operating on unknowns is another important factor in understanding why equations with unknowns on both sides cause so many difficulties. Booker (1987) suggests that it is the shift from manipulation of numbers in order to solve an unknown to the manipulation of unknowns themselves that marks the entry into algebra proper.

A student’s numeracy stage (Ministry of Education, 2003) is likely to be important for their understanding of expressions and equations (Irwin, 2003). Equations of the form $x + 3 = 7$ can be solved by advanced counters through guess and check, but they can be solved much more easily by part-whole thinkers, who are able to visualise 7 as $3 + 4$. It can be argued that solving equations such as $3x = 15$ requires multiplicative thinking if a student is to do more than simply follow prescribed algorithms. Furthermore, equations of the kind $2x + 3 = 11$ might require an understanding of numbers beyond simple additive part-whole or multiplicative part-whole thinking.

The Current Study

As part of the NDP, diagnostic tools (including the Numeracy Project Assessment [NumPA] and Global Strategy Stage [GloSS]) have been developed to help assess students’ numeracy stages. Also, during 2006, a group of researchers and teachers working on a Teaching and Learning Research Initiative (TLRI) project developed a diagnostic tool for assessing students’ ability to solve linear equations (Linsell, Savell, Johnston, et al., 2006). The researchers in this study are now using all these tools as they investigate the links between numeracy and students’ strategies for solving linear equations. During 2007, they interviewed approximately 450 year 7–10 students, with another 400 students interviewed early in 2008 before formal data analysis commenced. This paper explains the approach used in the study and some preliminary data.

In the 2006 TLRI study, the initial classification of strategies for solving equations was based on the work of Kieran (1992), whose review of the learning and teaching of algebra describes the strategies that students use. However, in this study, further strategies were added that students were observed using. For example, the use of an inverse operation (1c) (see Figure 1) on a one-step equation was considered to be different to Kieran’s strategy of working backwards (3b) on multi-step equations. It was difficult to distinguish the difference between known basic facts (1a) and inverse operations (1c) because the students often justified their answers by describing the inverse operation, when, in fact, what they had done was use a known fact. One-step equations with larger numbers were therefore used to elicit the use of inverse operations.

The strategy of working backwards was found to be not as homogeneous as had been assumed. Many students partly worked backwards and then used either known facts (3c) or guess and check (3d). When large or decimal numbers precluded the use of these strategies, the students could no longer use the working backwards strategy (3b).

The strategy of using a diagram (5) was also included. This resulted from the study's explorations of questions in context, where a number of students solved equations through direct use of diagrams.

The final classification of strategies is listed in Figure 1. It should be noted that this list of strategies is not intended to be hierarchical, as there is insufficient evidence to make such a claim. In fact, 3c and 3d are clearly less sophisticated strategies than 3b, and at present, the relative sophistication of 5 is not known.

0.	Unable to answer question
1a.	Known basic facts
1b.	Counting techniques
1c.	<i>Inverse operation</i>
2.	Guess and check
3a.	Cover up
3b.	Working backwards
3c.	<i>Working backwards, then known facts</i>
3d.	<i>Working backwards, then guess and check</i>
4.	Formal operations/equation as object
5.	<i>Using a diagram</i>

(Based on Kieren, 1992; the strategies added from this study are in italics.)

Figure 1. Classification of strategies for solving equations

The algebra diagnostic assessment for this study is run in two parts, a knowledge section and a strategy section.

The knowledge section is administered as a written test because supplementary questions are not required. The areas investigated in this section, together with example questions, are:

- understanding of conventions and notation
(If $n = 4$, then what is the value of $3n$?)
- understanding of the equals sign
(Given that $7x + 4 = 15$, find the value of the \square in the equation $7x + 7 = \square$)
- understanding of arithmetic structure
(What is the value of $18 - 12 \div 6$?)
- understanding of inverse operations
(Replace the box with $+$, $-$, \times , or \div : If $z - 27 = 25$, then $z = 25 \square 27$)
- manipulating symbols/unknowns (acceptance of lack of closure)
(Add 3 to $d - 1$).

The strategy section consists of a series of increasingly complex equations, which the students are asked to solve with an explanation of their thinking. There are 12 pairs of parallel questions – ones that are in context (that is, word problems), and ones that are purely symbolic. An example of a pair of parallel questions is:

- Our kapa haka group is made up of some Māori students and 11 Pākehā students. The whole group is divided into four equal-sized groups for practice sessions. Each practice group has 19 students in it. How many Māori students are there in our kapa haka group?

$$\frac{n + 12}{4} = 18.$$

The increasing complexity of the equations can be illustrated by a selection of questions from the symbolic section:

- $n - 3 = 12$ (one-step equation with single digits)
- $n + 46 = 113$ (one-step equation)
- $3n - 8 = 19$ (two-step equation)
- $5n - 2 = 3n + 6$ (unknowns on both sides)
- $2n - 3 = \frac{2n + 17}{5}$ (complex structure)
- We can rearrange the equation $p = r - s$ to make r the subject, $r = p + s$. Similarly if $v = u + at$, then $a = \dots$ (purely symbolic equation).

The questions are presented on cards so that the more difficult questions can be omitted as required without suggesting to the student that they are not coping. Each question is read to the student to minimise the impact of reading difficulties, including difficulties with reading symbolic equations. Calculators and pencil and paper are available for the students to use, but it is stressed to the students that they may use whatever method they choose.

Preliminary Results

From the interviews conducted to date, a number of points have become apparent:

- In all year levels, there are large differences between the students as to the sophistication of the strategies they are able to use. Some students are unable to use more than guess and check or counting techniques. At year 7, the majority of the students are able to use inverse operations for one-step equations, with some being able to work backwards for multi-step equations. At year 10, the majority of the students are able to work backwards for multi-step equations, with only a very small proportion being able to operate formally on equations, treating them as objects.
- The strategies the students were observed using are consistent with Kieran's (1992) classification, but they used other strategies as well. The strategy of working backwards is less homogeneous than previously reported. Many students are only just grasping this strategy and can use it only when the first step reveals a known basic fact to them for the next step. These students use the strategy of working backwards, then known facts. Other students are prevented from fully using working backwards because of a lack of knowledge of multiplication and division facts. These students use the strategy of working backwards then guess and check.
- There is a high correspondence between numeracy stage and the most sophisticated strategy a student is able to use for solving equations. Only students who are at the level of multiplicative

part-whole thinking or above are able to solve equations by working backwards or using formal operations. Numeracy stage appears to be a better predictor of the most sophisticated strategy a student is able to use than the amount of algebra teaching the student has received. There were, however, students who did not score highly on GloSS but who were able to use sophisticated strategies for solving equations. These year 9 and 10 students were invariably very efficient at using algorithms for computations and came from primary schools that did not promote numeracy.

- The most sophisticated strategy a student is able to use is extremely similar for questions that are in context or questions that are fully symbolic. In fact, many students are able to use slightly more sophisticated strategies for questions that are in context.

Discussion

Instead of looking at how hard equations are to solve and whether the students get them right, it appears to be more useful to look at what strategies the students are using to solve equations. The approach used in this study is very similar to that used in other work in the NDP, with strategy being separated out from the knowledge required for strategy use. This approach will allow the classification of the students according to their most sophisticated strategy rather than the most difficult equation they are able to solve. Within numeracy teaching, students are grouped for instruction according to their most sophisticated strategy. It is suggested that a similar approach to grouping students is likely to be beneficial for teaching students to solve equations.

As has been clearly identified (Herscovics & Linchevski, 1994), students have great difficulty with formal operations, which treat equations as objects. Few students, even from year 10, were observed using formal operations; those students who were able to use formal operations could also use the strategy of working backwards. Consistent with the perspective of Filloy and Sutherland (1996), it is suggested that these strategies are not simply alternative approaches to solving equations, but represent different stages of conceptual development. Similarly, and not surprisingly, those students who were able to work backwards could also use the strategy of inverse operations. Formal analysis of the data from this study will allow the construction of a hierarchy of sophistication of strategies.

In many current school programmes, little attention is paid to the students' numeracy stage when attempting to teach them to solve equations. The researchers found that even year 10 students, who have received at least two years' instruction in how to solve equations, are unable to use the more sophisticated strategies if they are below the numeracy stage of multiplicative part-whole thinking. Similarly, students are unable to use inverse operations for one-step equations if they are still at the numeracy stages that involve counting strategies. This strongly suggests that prerequisite numeracy should be considered when designing teaching programmes for algebra.

It would appear to be unnecessary to present students with purely symbolic equations in order to determine their most sophisticated strategy. Questions presented in context gave very similar information and were generally perceived by the students, particularly the younger ones, as being less threatening.

A number of teachers involved in the study have commented that the algebra diagnostic tool is more useful than GloSS for revealing the thinking of students at the upper end of the Number Framework. This is because GloSS focuses on mental strategies (and does not value the use of algorithms), while the algebra tool has a focus on students' understanding of arithmetic structure and the nature of equations.

The preliminary data presented in this paper suggest that the approach adopted is revealing useful information about students' understanding of aspects of algebra. It is anticipated that formal data analysis will establish a hierarchy of strategies, make explicit the connections between numeracy and algebraic strategies, and clarify the role of contexts.

References

- Booker, G. (1987). Conceptual obstacles to the development of algebraic thinking. In J. C. Bergeron, N. Herscovics, & C. Kieran (Eds), *Proceedings of the 11th Conference of the International Group for the Psychology of Mathematics Education* (pp. 275–281). Montreal: International Group for the Psychology of Mathematics Education.
- Booth, L. (1988). Children's difficulties in beginning algebra. In A. F. Coxford & A. P. Schulte (Eds), *The ideas of algebra, K–12: 1988 yearbook* (pp. 20–32.) Reston, VA: National Council of Teachers of Mathematics.
- Brekke, G. (2001). School algebra: Primarily manipulations of empty symbols on a piece of paper? In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds), *The Future of the Teaching and Learning of Algebra* (Proceedings of the 12th ICMI Study Conference, pp. 96–102). Melbourne: University of Melbourne.
- Carraher, D. W., & Schiemann, A. D. (2007). Early Algebra and Algebraic Reasoning. In F. K. Lester (Ed.), *Second handbook of research in mathematics teaching and learning* (Vol. 2, pp. 669–705). Charlotte, NC: Information Age Publishing.
- Cockcroft, W. H. (1982). *Mathematics counts: Report of the Committee of Enquiry*. London: HMSO.
- Fillooy, E., & Sutherland, R. (1996). Designing curricula for teaching and learning algebra. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds), *International handbook of mathematics education* (pp. 139–160). Dordrecht: Kluwer.
- Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics*, 27 (1), 59–78.
- Irwin, K., C. (2003). *An evaluation of the Numeracy Project for years 7–10 2002: Exploring issues in mathematics education*. Wellington: Ministry of Education.
- Irwin, K. C., & Britt, M. (2007). The development of algebraic thinking: Results of a three-year study. In *Findings from the New Zealand Numeracy Development Projects 2006* (pp. 33–43). Wellington: Learning Media.
- Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390–419). New York: Macmillan.
- Lee, L. (2001). Early algebra – But which algebra? In H. Chick, K. Stacey, J. Vincent, and J. Vincent (Eds), *The Future of the Teaching and Learning of Algebra* (Proceedings of the 12th ICMI Study Conference, pp. 392–399). Melbourne: University of Melbourne.
- Linchevski, L. (1995). Algebra with numbers and arithmetic with letters: A definition of pre-algebra. *Journal of Mathematical Behavior*, 14, 113–120.
- Linsell, C., Savell, J., Johnston, N., Bell, M., McAuslan, E., & Bell, J. (2006). Early algebraic thinking: Links to numeracy. Retrieved 30 November 2007 from www.tlri.org.nz/pdfs/9242_summaryreport.pdf
- Ministry of Education (2003). *The Number Framework*. Wellington: Ministry of Education.
- Ministry of Education (2007). *The New Zealand Curriculum*. Wellington: Ministry of Education.
- Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as two sides of the same coin. *Educational Studies in Mathematics*, 22, 1–36.
- Sfard, A., & Linchevski, L. (1994). The gains and pitfalls of reification: the case of algebra. *Educational Studies in Mathematics*, 26, 191–228.
- Thomas, G., & Tagg, A. (2007). Do they continue to improve? Tracking the progress of a cohort of longitudinal students. In *Findings from the New Zealand Numeracy Development Projects 2006* (pp. 8–15). Wellington: Learning Media.
- Young-Loveridge, J. (2007). Patterns of performance and progress on the Numeracy Development Projects: Findings from 2006 for years 5–9 students. In *Findings from the New Zealand Numeracy Development Projects 2006* (pp. 16–32). Wellington: Learning Media.

Evaluation of Support for Pāngarau Teachers Working in Wharekura

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This case study examined the impact that a pilot project of professional development and support had on nine teachers of pāngarau¹ working in wharekura² in the Hawkes Bay, Taranaki, Waikato, Wellington, and Whanganui regions. The relative usefulness of the modes of delivery (hui,³ in-school visits by the facilitator, and video conferencing) and teacher and facilitator perceptions of the impact of the support project on teacher content knowledge and teacher practice were also examined. This evaluation found that the support project had a positive impact on teachers' content and pedagogical knowledge, the students of those participating made impressive achievement gains, and certain facilitator characteristics are considered by participants to be important for generating such progress.

Hutia te rito o te harakeke
Kei whea te kōmako e kō
Kī mai ki ahau
He aha te mea nui o te ao?
Māku e kī atu
He tangata, he tangata, he tangata.

*Pluck the shoots of the flax and it will die.
Then where will the kōmako be?
Say to me
What is the greatest thing in the world?
I will respond
It is people, it is people, it is people.*

Background

This paper discusses wharekura Te Poutama Tau and pāngarau. Wharekura Te Poutama Tau is a numeracy professional development project for teachers of pāngarau at secondary school level in Māori-medium schools. It is based on the Number Framework developed for New Zealand schools as part of the Numeracy Development Projects.

The aims of Te Poutama Tau are to lift the achievement of students in numeracy and to be responsive to Māori goals of language revitalisation and empowerment through education. Building the capability of teachers in the teaching and learning of numeracy is a pathway to attaining these aims (Christensen, 2004; Trinick, 2005, 2006).

The importance of teacher development in raising student achievement is clear (Sowder, 2007). Key elements contributing to students' positive achievement through teacher development in Te Poutama Tau include: the use of diagnostic tools to gather data about students' number knowledge and strategies; using data to set clear goals; focusing on student learning of knowledge and strategies from the Number Framework; the clarity of the Framework; building students' positive attitudes to numeracy; the continuous evaluation and monitoring of goals; and support from school leadership (Trinick, 2005).

Evaluations of Te Poutama Tau have called for: a strong emphasis on teacher professional development at the higher stages of the Framework; linking number knowledge and strategies to other strands of the mathematics and statistics learning area; exploring further ways to improve language use in

¹ Pāngarau: mathematics

² Wharekura: Māori-medium secondary school(s)

³ Hui: congregation, meeting, a coming together

teaching and learning; and improving the outcomes of those students making little or no stage gains (Christensen, 2004; Trinick & Stevenson, 2005, 2006, 2007). These findings are consistent with *Teacher Professional Learning and Development: Best Evidence Synthesis Iteration [BES]* (Timperley, Wilson, Barrar, & Fung, 2007) in terms of using data and expertise to challenge teachers' existing beliefs and providing programmes that focus on student learning rather than on teaching programmes.

A recent study (Trinick & Parangi, 2007) into the conditions of wharekura pāngarau teachers found that a range of issues impacted on their delivery of pāngarau. These issues included: the isolated nature of teaching pāngarau at wharekura level (usually one teacher across all levels); pāngarau teachers carrying teaching loads in other curriculum areas; support and professional development not being available in te reo⁴; no provision of a professional development programme equivalent to the Secondary Numeracy Project (SNP) in English-medium schools; insufficient resources, including people; resources in English needing translation into te reo; only an underdeveloped pāngarau language available to teachers, who are generally second language learners of te reo; and outside commitments to marae, whānau, and hapū⁵. Recommendations arising out of this study were:

- to create and provide assistance for wharekura pāngarau teachers that would be of benefit to them and therefore to wharekura students;
- to develop a range of professional development initiatives and resources;
- to support and ease teacher workload in wharekura.

A pilot project for professional development was designed in an attempt to address issues for wharekura pāngarau teachers raised in the Trinick and Parangi (2007) study. The project took into account: the isolation of wharekura pāngarau teachers; their need to work with others in a similar fashion to SNP teachers; and their need to stay in contact with each other. The project included three modes of delivery:

- Hui. Participants met as a cluster for two days four times in the year to: receive expert, specific, and focused facilitation of wharekura Te Poutama Tau; develop mathematical language, concepts, content, and pedagogical knowledge; discuss needs; have questions responded to; and network. At least one hui was scheduled to occur during a holiday period to minimise disruption to teaching and to alleviate the difficulties of finding classroom relievers.
- In-school visits by the facilitator (one facilitator worked with the nine schools in the pilot project for the year). Facilitator modelling, observations, and sharing of expertise within each wharekura allowed teachers to observe their students in action. These visits occurred at arranged intervals after the hui. The facilitator visited each kaiako⁶ at least once between each hui. Some kaiako received between two and five visits between each hui.
- Video conferencing. This provided a mode of delivery that allowed isolated kaiako to share ideas and network with others in a focused, pre-planned distance-workshop situation. All video conferences were preceded by an email from the facilitator outlining the purpose and what kaiako needed to prepare for showing or discussion. The video conferences occurred approximately every three weeks; altogether, there were six video conferences.

⁴ Te reo: Māori language

⁵ Hapū: a sub tribe of a tribe, made up of a larger extended whānau group.

⁶ Kaiako: teacher(s)

Research Aims

This study explored the impact in nine lower North Island wharekura of the pilot project's support for teachers of pāngarau at year 9. In particular, it explored the impact of this provision of additional support on:

- teachers' content and pedagogical knowledge
- classroom practice.

Exploration of facilitator characteristics was not initially part of this study. However, researcher observations, subsequently backed by participants' comments, indicated the importance of investigating and reporting on the facilitator characteristics that appeared to be most important for the additional support to be optimally effective. This aspect of the professional development project relates to findings of the *Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration [BES]* (Anthony & Walshaw, 2007), which states that interaction between people is tied closely to pedagogy and that productive interaction, as well as enhancing skill and knowledge, also has an impact on identity and disposition. This BES links pedagogical approaches to achievement outcomes and also to social and cultural outcomes that became an aspect of this project.

Participants and Method

The wharekura Te Poutama Tau pilot project involved wharekura in Hawkes Bay, Taranaki, Waikato, Wellington, and Whanganui. All the schools involved in the project in those areas were invited to participate in the study, and all agreed to take part.

Participants in the study included the facilitator and the year 9 pāngarau kaiako from each of the nine wharekura. All the wharekura were small, with fewer than 50 students and up to three staff, and were situated in urban areas. All but one of the wharekura were linked to kura tēina⁷, and all but one included years 9–13. Class sizes ranged from five to twenty-two students. Teaching of pāngarau was in te reo Māori (eight wharekura) and in English (one wharekura).

The facilitator, a Pākehā male, was a speaker of te reo Māori and had taught for over 20 years, including three in wharekura. The kaiako teaching experience in wharekura ranged between two and six years, and each was the sole teacher of pāngarau in their wharekura. None of the kaiako held tertiary mathematics qualifications, and none had teaching qualifications specific to secondary school. All but one of the kaiako also taught in subjects other than pāngarau. One of the kaiako was the teaching principal. Three were male and six female. All kaiako agreed to be involved in the evaluation research and to provide data. Important factors in participants' willingness to be involved appeared to be relationships that were already in place and the use of elements consistent with the principles of a Māori-centred approach (Cunningham, 1998) alongside kaupapa Māori approaches (Bishop & Glynn, 1999). Prior established relationships existed between: researcher-facilitator; facilitator-wharekura; facilitator-kaiako; researcher-wharekura; and researcher-kaiako. The facilitator had strong prior relationships with both the initial group of kaiako and the main researcher. The main researcher was instrumental in further wharekura and kaiako joining the pilot support project.

In keeping with the context of the study and in order to maximise participation and data, quality elements consistent with kaupapa Māori research methodology were included. For example:

- Participation in the study drew on established relationships;

⁷ Kura tēina: primary school

- Steps were taken to establish and develop relationships where they did not already exist (for example, *kanohi ki te kanohi*⁸ meetings between researchers and facilitator and/or kaiako before and during data gathering);
- All aspects of the data gathering were negotiated with the facilitator;
- Data collection methods and timing were negotiated with kaiako and the facilitator;
- The facilitator was consulted regarding the themes that emerged from the data.

Data collection included:

- questionnaires completed by the facilitator and kaiako at the initial hui (May, six completed kaiako questionnaires) and the final hui (December, five completed kaiako questionnaires);
- audio recording of the first day of the final hui, including a video conference between the facilitator, kaiako, Malcolm Hyland (Ministry of Education), and Jim Hogan, the secondary mathematics adviser for the Waikato region;
- two interviews with the facilitator (one early in the project and one towards the end);
- one kaiako interview at the final hui (initially planned as individual interviews, this was held as a group interview in response to the participants' request. The facilitator was not present when his role was discussed);
- informal observation of facilitator-kaiako interactions at the initial and final hui.

Analysis

Data analysis was generative and open. It included finding and coding relationships between concepts and ideas and placing them in manageable chunks relating to the themes that were emerging in line with the research question, thus reducing complexity. The themes emerging from analysis of statements and ideas commonly expressed by kaiako and the facilitator were identified collaboratively by the researchers. The key themes were then discussed with the facilitator to enhance trustworthiness. They were then synthesised to create a story of the outcomes of the evaluation of the wharekura Te Poutama Tau professional development project for teachers.

Findings

Teacher Growth

Many wharekura teachers do not initially see themselves as maths teachers and are still gaining confidence in the wharekura setting. (Facilitator)

All participants reported that the additional support provided through the project generated personal growth in content and pedagogical knowledge. The facilitator commented on the personal teacher growth he had seen in kaiako over the project, particularly regarding their confidence in their practice and in having others observe and discuss their teaching, and in them seeing the big picture of Te Poutama Tau and how it can work for students. He felt all delivery modes of the project contributed to teacher personal growth.

Some schools, some teachers have done heaps and others have done just little bits, but I can see changes in all of them. Just occasionally having opportunities where the kids talk and that they're not telling them all the time, and starting the class with a starter, a warm up, which is something

⁸ *Kanohi ki te kanohi*: face-to-face

they didn't do before. There's some teachers who've made huge changes in that they've completely re-jigged their whole classroom. In the space of less than a year, they've gone from a sort of whole-class, teacher-dominated kind of thing and now they've got three or four groups all operating independently and they're servicing them. (Facilitator)

I think all the teachers will say, particularly with the strategies, it's really made them, that's what's actually improved their understanding, the penny's dropped about how different strategies worked. They're beginning to see that actually the strategy is maths generally. I think that must be one of the biggest – in terms of content knowledge ... insights, because once they themselves see that, they realise, well actually, what they're trying to do with the students is not get them to get answers but to be able to see those patterns and structures and talk about them and explain what's going on. (Facilitator)

Further evidence of teacher growth is indicated by the student achievement data collected before and after the project. Data was collected from 125 students. Data from students at years 8, 9, and 10 showed average gains across all strategy and knowledge domains:

- Year 8 data (22 students, one class) showed greater shifts in their teacher's focus areas (addition/subtraction, multiplication/division, and place value) and smaller shift in other areas. The smallest shift occurred in the fractions domain;
- Year 9 data (77 students) showed greater shifts in proportion/ratio and fractions, with consistency in the size of stage gains across all domains between students at different initial stages;
- Year 10 data (26 students) showed greater gains in multiplication/division, proportion/ratio, and fractions.

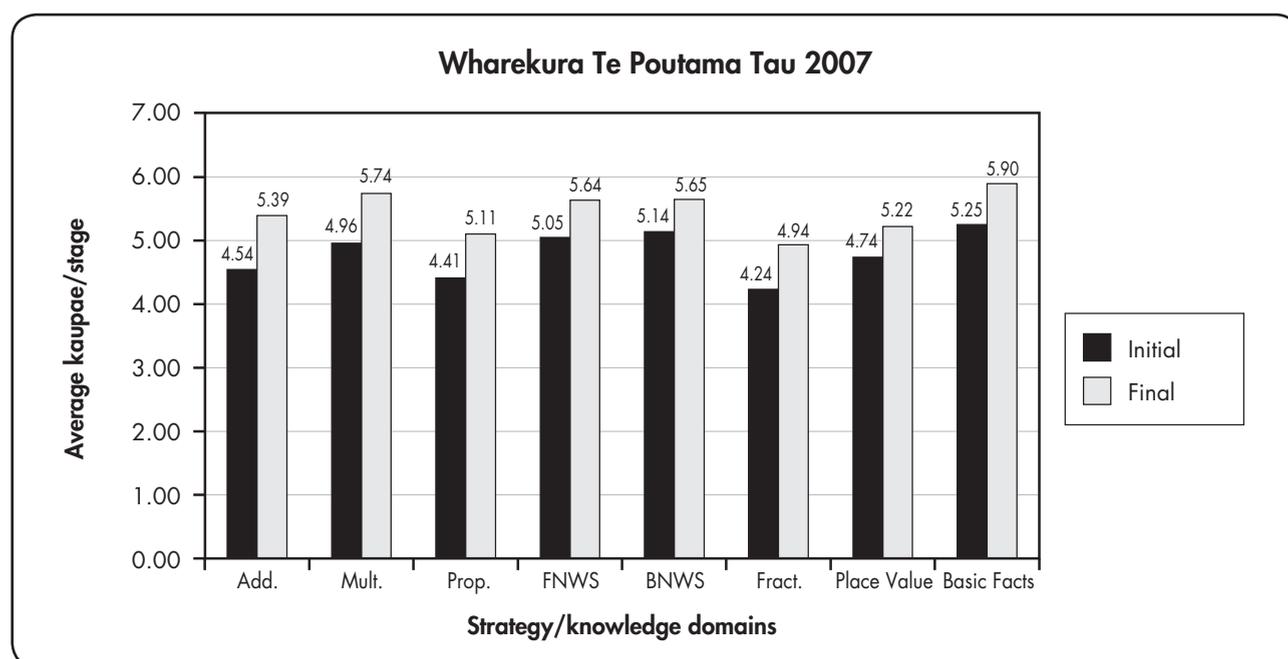


Figure 1. Average stage gain for all students in each of the domains

In general, the students with the lowest initial strategy stages made highest average stage gains (see Figure 1 and Table 1). For example, across the addition/subtraction, multiplication/division, and proportion/ratio domains:

- students initially at stage 4 had average gain of between 1 and 1.5 strategy stages;
- students initially at stage 5 and 6 had an average stage gain of 0.7–1.1 strategy stages.

Table 1
Average Stage Gain for All Students in Each of the Strategy Domains

Initial Stage	Addition/ Subtraction	Multiplication/ Division	Proportion/Ratio	Fractions
3		2.0	1.5	0.7
4	1.0	1.3	1.4	0.6
5	0.8	1.1	0.7	0.7
6	0.8	0.8	0.9	-0.3
7	0	0.3	0.8	

Again, positive average stage gains across all students existed across all knowledge domains (see Figure 1 and Table 2) and the greatest average stage gains were fairly consistently seen for students with the lowest initial strategy stages (for example, 0.7–2.0 stages at level 3 versus 0.5–0.7 stages at level 5). Smaller effects were noted across the knowledge domains of fractions and place value, with students at levels 6 and 7 showing no stage gains in these domains.

Table 2
Average Stage Gain for Each of the Knowledge Domains

Initial Stage	Place Value	Basic Facts
2	2.0	
3	1.8	2.0
4	0.8	1.4
5	0.5	0.7
6	-0.1	0.2
7		0

Delivery Modes

Questionnaire responses indicated an even split amongst participants' views of whether the most useful mode of delivery was the hui or the in-school visits. The following section discusses specific features of each delivery mode.

a) Hui

In his final interview, the facilitator expressed a preference for two four-day hui in the year as opposed to the three-day hui that he had initially planned. He had changed the three-day hui to two days in response to kaiako feedback that three days was too long. Having longer hui was important to the facilitator so that kaiako could have time out away from their kura to enable them to focus on their professional development: "... just push everything aside and focus on getting those main ideas clear" (facilitator).

Another aspect of the hui important to the facilitator was creating a collegial team and maintaining that collegiality:

Another aim is for these teachers who basically work alone in their kura to have colleagues.
(Facilitator)

Kaiako expressed concern at being away for two days in terms of who was in charge of their students and about being behind in their programmes because relievers and/or the students might not follow the programmes.

Always the week before ... are your kids going to be safe ... get a good reliever ... [will they] follow the work I've left them ... worrying about relievers. (Kaiako)

A further factor impacting on participants' views of the usefulness of the hui was the low turnout at the third hui due to its timing (school holidays). However, as a result of this low turnout, two kaiako initiated increased participation and ownership of the project by the kaiako group. The stated motivation for increased ownership and input was the importance of raising student achievement and kaiako seeing the project as being of national importance rather than just about their own development and that of their own students:

I can see this programme can work and the types of mathematical thinking that the kids can have; however, I think that sometimes we have so many other things on that it just doesn't quite work, it needs to be constantly focused on. I think that we as a team of people need to make a commitment to the integration of this programme and the feedback as well! We simply just have to make time! If we fail to do so, we fail our students big time, if you're not in it then neither are our kids. (Kaiako)

b) In-school visits

The facilitator viewed the in-school visits as being an important means of providing one-to-one assistance, modelling, and informing all aspects of the project, ensuring it was responsive to needs:

I think it made a big difference [for kaiako], seeing how their kids reacted when some different work came. I think that's the most powerful part that I've seen of modelling, that the teacher sees how their own kids respond so differently to someone else doing a different kind of work. (Facilitator)

Kaiako also reported finding the in-school visits very important:

[The facilitator] came and showed me how to do a long-term plan. [This was a] watershed moment for me. [He] assessed me and I understood what level I was at, understood my strategies and what strategies I need to learn so that I can help my students. (Kaiako)

I really enjoyed when you took a class and I observed, for me especially, I mean I've been out of maths for a long time, I hadn't done maths for 10 years, so to see it in action again, and the new programme, and looking at the domains that you were covering ... I found that invaluable, beneficial, just watching, observing, using materials, having it visualised, consolidated. (Kaiako to facilitator at final hui)

c) Video conferencing

All participants found the video conferences the least useful aspect of the development. The reasons given for this included that not all kaiako had access to video-conferencing equipment, this mode of delivery was new to many, and some kaiako experienced technical or practical issues⁹ in accessing the video conferences. However, the video conferences were viewed as useful for both networking and sharing ideas:

One of the main reasons for the use of video conferencing was for networking, so that we could all have a chance to get together. At least have that regular opportunity to break down the sort of isolation of each teacher working in the individual classrooms and not really knowing what's going on in other schools. (Facilitator)

[The usefulness of video conferences] was just confirming that I was on the right track, or, oh, I'm way behind ... (Kaiako)

[Seeing] what activities others were using ... (Kaiako)

Yeah, seeing the actual delivery and content that we were using; I thought that was really good. (Kaiako)

Yeah, and looking at what other schools are doing. Oh, I tried this; have a look at this ... (Kaiako)

⁹ Some kaiako needed to travel up to an hour and a half to take part in the video conferences.

d) Additional delivery mode

A fourth important aspect of the project was frequent emailing between kaiako and the facilitator, used for:

- sharing practical information for setting up and ensuring that everyone was prepared for hui and video conferencing (dates, venue, and so on);
- asking and answering questions;
- encouragement and sharing ideas and useful websites (from kaiako and facilitator);
- maintaining rapport.

After the poor turnout at the third hui, email was used by one of the kaiako to encourage full participation in the project.

I can see this programme can work and the types of mathematical thinking that the kids can have; however, I think that sometimes we have so many other things on that it just doesn't quite work, it needs to be constantly focused on. (Kaiako)

The use of email as a tool to prepare sessions with each other was recognised and commented on.

I like how you posed in emails something to think of before the actual video conference so [we] could come to the conference with something that we'd thought about. So that was good to sort of collect ideas when you sent your email. (Kaiako)

Affordances and constraints across delivery modes

The delivery modes in the project were explored through elements that supported the effectiveness of each mode (affordances) and those issues that limited the effectiveness of each mode (constraints) (Table 1). The affordances supporting its effectiveness were: time to share and discuss ideas; prior organisation; and flexibility. The constraints were: issues such as finding suitable relievers to allow attendance at hui and time for discussions during in-school visits; and logistical transportation or technical issues. In spite of the constraints, all kaiako endorsed all modes of delivery as important for their growth and the success of the project (Table 3).

Table 3
Affordances and Constraints of Delivery Modes

Delivery Mode	Affordances	Constraints
Hui	<p>Focused time all together to model, discuss, and participate in numeracy strategy learning and problem solving</p> <p>Prior structure and organisation, including agenda</p> <p>Kaiako bringing activities to share and discuss</p>	<p>Time consuming, including time out of class</p> <p>Transportation logistics</p> <p>Accessing sufficient suitable relievers (particularly problematic for wharekura sharing the same relieving pool)</p>
In-school visits by facilitator	<p>On site, able to experience and discuss the specific context of each kaiako, answer specific questions</p> <p>Students were able to hear facilitator/ kaiako discussions</p> <p>Facilitator flexibility regarding visit timing, use of modelling, and length of visit</p>	<p>Lack of teacher release</p> <p>"Kotahi hāora noa iho taku wā wātea engari i noho ia mō te rā, kāti, i muri hoki."¹⁰ (Kaiako)</p>
Video conferences	<p>Cost- and time-effective</p> <p>Environmentally friendly</p> <p>Prior structure (email and tasks prior to conference)</p> <p>Kaiako bringing activities to share and discuss</p>	<p>Technical and access difficulties</p> <p>Beginners in this mode in 2007; however, experience improved practice.</p>

¹⁰ I only had one hour of release, but he stayed for the day as well as after [school].

The combination of modes

Participants considered the combination of all three modes as important for the wharekura Te Poutama Tau initiative. This was evidenced by a shift in questionnaire responses, from the initial questionnaire, in which most participants indicated they felt only one or two modes would be useful, to the final questionnaire, in which everyone ranked all modes as being important. The modes of delivery were seen to be complementary, and the combination allowed the momentum of the project to be maintained. Regardless of whether kaiako had or used access to the video conferences, all received the emails and associated tasks sent in preparation for them.

Facilitator and kaiako views of the three main delivery modes were examined to explore how each mode promoted teacher growth (pedagogical and content knowledge development) and contributed to the development of a learning community (Table 4). Some aspects were common to all modes (for example, sharing project experiences), and others were specific to particular delivery modes.

He āwhina ā ngā mea katoa, he raruraru ā ngā mea katoa.¹¹ (Kaiako, questionnaire response)

Table 4

Most Important Aspects of Delivery Modes for Teacher Growth

Delivery Mode	Most important aspects for teacher growth (pedagogical)	Most important aspects for teacher growth (content knowledge)	Most important social factors
Hui	<p>Maintaining project momentum</p> <p>Sharing teaching strategies: "Have different approaches to teaching". (Kaiako)</p> <p><i>Having questions answered, individual help</i></p> <p>Modelling (e.g., diagnostic interview)</p>	<p>Maintaining project momentum</p> <p>Sharing teaching strategies: "Have different approaches to teaching". (Kaiako)</p> <p><i>Having questions answered</i></p> <p>"[The usefulness of video conferences] was just confirming that I was on the right track, or, oh, I'm way behind ..." (Kaiako)</p> <p><i>Doing and sharing mathematical activities</i></p>	<p>Maintaining project momentum</p> <p>Sharing teaching strategies: "Have different approaches to teaching". (Kaiako)</p> <p>Individual discussions with facilitator</p> <p><i>Networking</i></p> <p>"He rawe te mahi ā-rōpū."¹² (Kaiako)</p>
In-school visits by facilitator	<p>Focused reflection time</p> <p>Kaiako seeing what their children can do (with the facilitator)</p> <p><i>Individual help (e.g., needs-based, using the diagnostic interview)</i></p> <p>Modelling "Pai ake ki te kite i ngā mahi e kōrerohia nei e te kaiwhakahaere."¹³ (Kaiako)</p>	<p>Focused reflection time</p> <p>Content knowledge development</p> <p><i>Having questions answered</i></p>	<p>Focused reflection time</p> <p>Energising</p> <p>Enhancing sense of common purpose</p>
Video Conferences	<p>Sharing project experiences (teaching ideas): "Yeah, seeing the actual delivery and content that we were using; I thought that was really good." (Kaiako)</p>	<p><i>Doing and sharing mathematical activities. "[Seeing] what activities others were using ..."</i> (Kaiako)</p>	<p>Sharing project experiences: "Kia whakawhiti whakaaro, kia wānanga, kia werohia."¹⁴ (Kaiako)</p> <p><i>Networking</i></p>

Bold indicates re-occurring themes across the categories of teacher growth in content, teacher growth in pedagogy, and social factors. *Italics* indicates themes that occur in more than one mode of delivery.

¹¹ All things had helpful aspects, and all things had problems.

¹² Working as a group is fantastic.

¹³ It's heaps better to see what has been talked about by the facilitator.

¹⁴ So that we can swap thoughts, so that we can discuss them and learn, so that we can challenge and be challenged.

Focus on Te Reo Pāngarau

Evaluations of Te Poutama Tau for 2003, 2004, and 2005 included te reo as an influencing factor in student achievement in pāngarau. Trinick (2005) included te reo proficiency of teachers as well as students as being influential. Christensen (2004) noted the significant correlation between language proficiency and performance in the diagnostic interview. Te reo pāngarau is continuing to develop, particularly at the higher stages, and wharekura teachers and students operating at higher stages learn te reo along with te reo pāngarau at critical thinking, problem-solving levels.

“Te reo pāngarau” does not translate directly as “mathematical words in te reo Māori”. It is a developing concept that includes all the kōrero¹⁵ you use when you are doing mathematics: kupu pāngarau¹⁶, ways of asking and answering questions, within the context of a Māori world view. (Facilitator)

The Māori kupu¹⁷ [are] good. Pāngarau ... the word sums it up. It’s about how one thing connects with another. It’s about relationships. The pānga i waenganui i tēnei.¹⁸ And once you start thinking relationally like that, then mathematics becomes a really powerful way of figuring things out, a very efficient, powerful way of figuring things out. (Facilitator)

Te reo pāngarau was a focus area in all delivery modes, and the facilitator and kaiako commented on their growth in te reo pāngarau.

I was very happy to see that the last hui was almost entirely in te reo Māori. The work that we’ve done in our hui about te reo pāngarau, well basically, it was translation we were doing and at the start what we saw were quite clunky ways of translating stuff, but by the end of it we’d actually refined it down. We got the pukapuka¹⁹ out and found we could actually really refine it down to a good reo. I think that will carry over quite easily. We were dealing with concepts: when an object is proportional, its shape is proportional to another. How do you say that in te reo Māori? Even things like, something is ten times bigger than another and comparisons; we looked at this problem about this boy who was jumping over stepping stones, he jumped on every other stone. Well, how do you translate that? We came up with something; it was a nice succinct way of expressing that concept, and so why wouldn’t they use it. (Facilitator)

Kaiako expressed the need for a more comprehensive dictionary of mathematics terms than currently exists, particularly for terms used at senior secondary school levels. The facilitator identified wanting to consolidate and further develop te reo pāngarau through the project in 2008.

Because when you have the language, then you can make advances in the conceptual understanding – you can’t have one without the other; the mathematical thinking and the mathematical language help develop each other. (Facilitator)

Focus on Facilitator Characteristics

Data showed the importance of investigating and reporting on facilitator characteristics. Kaiako stated that it was essential to have “the right person” as facilitator, so their views of facilitator characteristics, essential for the success of the pāngarau support project, were collected. The important characteristics identified by the participants for this facilitator were: his knowledge and interaction with te āo Māori²⁰; having empathy through having taught in wharekura; having certain personal traits; and knowledge of his discipline area. In response to comments by participants, we asked directly what the characteristics of the facilitator were that contributed to their success in the project. These ideas will be further expanded upon under two broad themes of cultural and personal characteristics and discipline-related characteristics.

¹⁵ Kōrero: words, ideas, written or spoken

¹⁶ Kupu pāngarau: mathematical words

¹⁷ Kupu: words

¹⁸ Pānga i waenganui i tēnei: the relationships between this

¹⁹ Pukapuka: books

²⁰ Te āo Māori: the Māori world

a) Culturally responsive and personal characteristics

Their responses indicate that, in addition to strong numeracy facilitation expertise, facilitators working with kaiako in wharekura must be culturally responsive and empowering:

His ngāwari²¹ nature, aye, just ... ahakoa he Pākehā²², aye, there's something about his wairua²³ that, um, that I find really comfortable, you know; he said, "do you want me to do the uiui²⁴ on you?", and I said, "yup, yup, you can do that, I feel fine." Somebody else, you know, another mathematician wanting to do that to me, I go, "no ... what are you looking for?" And then the kids also felt that way when he came in; we'd finished the maths and I was saying to him, "ok, you need to go now 'cause I need to move on", he goes, "oh, I might just stay", and I'm going, "ok ...", and he stayed, and he joined in the conversation (which was not about maths) with the kids and the kids really responded and appreciated him, so his ngāwari nature makes tons of difference. Aye. He's so easy to get on with. (Kaiako)

And he knows when to be quiet too, aye. That's one of the things I really noticed, too, is that I talk a lot and it's just noise in the kids' heads. And watching him, he gives them time to think. (Kaiako)

I think he, he also, well for me, he empowers me. To actually do what I'm doing, you know, and he doesn't make me feel like oh, you know, kōtiro me mahi koe i tēnei.²⁵ But actually, you know, ka whakanuia i ngā wā katoa ka kitea a ia²⁶... and he likes maths. (Kaiako)

When he came to me one time, and he said, "Now, where are you?" And I was just like ... and he said, "Well, I think that it's probably been a waste of time for you and do you want to give up?" And I was like mmmmm. And he said, "Well, I don't want to give up, shall we start again, we'll throw it all away and we'll start again, eh." And I was like, "oh, is that alright? You know ... it's July now." And he was like, "there's nothing wrong with starting it now, you're just becoming comfortable with the strategies now; let's become comfortable with the planning" ... and I didn't feel stink. (Kaiako)

Facilitator characteristics consistent with te ao Māori begin with the cultural knowledge that one needs as a visitor to wharekura, including how to behave appropriately in terms of protocols; to have flexibility; to know when to let others take the lead; to have knowledge of and use te reo Māori; and to be respectful as well as engender respect and trust:

[At the initial hui] two or three of the teachers got defensive because they had no other strategies. So we said, "Oh well, you're going to have to be ... kaupae tuarima²⁷" and they were very unhappy about this because they felt that they'd been judged as being bad at maths. That brought out kōrero about Poutama Tau not being about judgment at all, but just saying, well, this is where you are now. It actually makes no comment about your ability. (Facilitator)

Having a background within wharekura and understanding issues for kaiako in wharekura was identified as important. This background and understanding included the facilitator establishing relationships (where possible) with kaiako and personal commitment to Te Poutama Tau, kaiako, and Māori students:

So that [not being about judgment] freed people up to be less insecure, more secure about where their content knowledge was, and then to make progress with it. So, before teachers can make progress with their own content, there's actually these other things, these other personal issues of whakamā²⁸ and feeling you're being judged. We've got to get those out of the way before we can make progress with their content. (Facilitator)

²¹ Ngāwari: accommodating, kind

²² Ahakoa he Pākehā: although he is Pākehā

²³ Wairua: spirituality, way of being

²⁴ Uiui: diagnostic interview

²⁵ Kōtiro me mahi koe i tēnei: Girl, you should do this.

²⁶ Ka whakanuia i ngā wā katoa ka kitea a ia: He uplifts you every time you see him.

²⁷ Kaupae tuarima: stage 5

²⁸ Whakamā: shyness, embarrassment

Personal traits that enabled kaiako to feel comfortable with the facilitator include: being able to give positive and affirming feedback; the wairua of the facilitator; having a sense of humour; humility; being inclusive and sharing; persevering and expecting others to persevere; being empathetic, available and approachable; and being ngāwari. The need for the facilitator to be of Māori heritage was not mentioned, and therefore it can be inferred that this was not seen as essential. Kaiako, when asked whether being Pākehā was an issue, stated it was not.

The facilitator also shared his views on the characteristics needed for his role:

You need to be infinitely patient and to put people first if you want them to grow. (Facilitator)

b) Discipline-related characteristics

The characteristics identified by kaiako as important for facilitators included: holding and sharing a passion and knowledge about pāngarau alongside knowledge of how Te Poutama Tau fits into a broader view of mathematics; and passion about the importance of the project for Māori students.

The message I keep on about all the time really, is that Te Poutama Tau is not about numeracy but about pāngarau. I want to use it as a vehicle to get teachers into thinking about getting into the real meat of what pāngarau is really about. I think that's important if they're [going to be] able to support students in getting to those higher levels of pāngarau. (Facilitator)

One of the things that he does, and in his quiet way, so you know all of this electronic stuff ... the resources he's given us on our [data sticks], it's a lot of that stuff he's developed and he just quietly goes oh, here you go. It's huge stuff that he's done; that's quite special to what he does, he just freely gives it. Like, "Take it, take it." And one time he gave it to me, and then he said, "Do you like any of them?" I was, "I like this one and this one." And so, "Well, print it out", and we sat there and make the, all these resources straightaway, we made the games up straightaway. Ten minutes later, we had three games, three new games. (Kaiako)

Recommendations

This study set out to explore the usefulness of various modes of delivery of a pilot project for delivering professional development of Te Poutama Tau to wharekura teachers of pāngarau and the effect that it had on raising teachers' content and pedagogical knowledge and exploring teachers' practice.

It found that while there were constraints that impacted on the delivery of the project, the overall results were positive. The main recommendation is that the momentum of the pilot project should be sustained through the following:

- Ongoing support should be provided to kaiako who took part in the 2007 wharekura support project so that they can consolidate and extend the advances made in pāngarau teaching.
- Further wharekura and kaiako should have the opportunity to benefit from a similar support project.
- Opportunities for links between the 2007 and 2008 cohorts of kaiako participating in the project should be explored.
- The combination of modes of delivery (hui, in-school visits, video conferencing, and email communication) should be maintained.
- The selection process for facilitators for curriculum support projects in wharekura should include consideration of the essential facilitator characteristics found through this study.

Further Questions

Further areas for exploration that emerged from the analysis of this study included:

- How can social networking (using web-based person-person linking) be used to build a network of kaiako between cohorts and within cohorts, and how effective is this mode of delivery when used alongside hui, in-school visits, and video-conferencing?
- Can sharing of video recordings of kaiako practice be used to develop kaiako pedagogical and content knowledge?
- What are the effects on kaiako pedagogical and content knowledge of interactions within and between development groups and other national networking?
- What is the impact on classroom kaiako and taurira²⁹ practice and achievement of the pāngarau support development?

Further analysis of specific aspects of culturally responsive facilitator practice and of the impact on teacher content and pedagogical knowledge of the interaction of the combined delivery modes would also be useful.

Concluding Comments

This evaluation found that the support project was an effective means of assisting with the government's focus on reducing inequalities in the education sector (Ministry of Education, 2006). Both kaiako and facilitator described kaiako content and pedagogical growth as a result of being part of the project:

This far down in the programme, I'm a lot more confident because I know the strategies and have more chance to practise them, yeah, plus I can find them straightaway in the book now and I know, well, now I know which book I'm using. (Kaiako)

I can see now how it all fits together so that I can really get the students learning, and they know it as well. They know if they want to figure something out, they can get out some resources, materials, to help them get a handle on it. (Kaiako)

The project has allowed kaiako, who previously had nobody else in their kura to talk with about pāngarau, to form a collegial network that builds a "professional community that supports new ideas and practice at the same time as challenging existing ones" (Timperley et al., 2007). The provision of expert facilitation consistent with te ao Māori was important in encouraging and enabling kaiako to recognise their own agency in effecting transformation with their students' results, in keeping with the whakatauki chosen specifically to reflect these strengths and concepts.

Acknowledgments

Hutia te rito o te harakeke, kei whea te komako e ko, kī mai ki ahau he aha te mea nui o te ao, māku e kī atu, he tangata, he tangata, he tangata.

Nō reira e āku nui, e āku rahi, koutou e pukumahi ana i mua i te mura o te ahi, e mihi kau ana, e mihi kau ana. Tēnā rā koutou i tere whakaae ki te tangi o ngā pononga nei kia whai wāhi i roto i te arotakenga o tēnei mahi whakahirahira e tū nei hei toka tū moana mō a tātou kaiako, nō reira ka puta ake hei oranga mō a tātou tamariki, mokopuna, kāore he mutunga o ngā mihi ki a koutou. Otirā ki te tohunga o te pāngarau kia eke ai a tātou tamariki ki ngā tihi o ngā maunga, arā ko koe tēnā kua whāngaia. Ki a koutou i whakahuatia te moemoeā kia whakatinana ai, e hika mā, mei kore koutou, kua aha kē tātou?

²⁹ Taurira: student

To you who stand before the flames of the fire, we stand humbly before you and greet you. You who so readily agreed to take part in this evaluation research of this project, which provides a solid rock for our teachers to stand on should the seas be rough, and do so that our young might reach their potential in life, there is no end to our thanks to you. Also to the facilitator who has fed our teachers in order for them to help our children in their search for excellence, our thanks, and to those of you who spoke the dream so that it might be given substance, e hika mā, if it were not for you, what would we do?

Heoi anō rā, tēnā koutou, tēnā koutou, huri noa, tēnā rā tātou katoa.

References

- Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in mathematics/pāngarau: Best evidence synthesis iteration [BES]*. Wellington: Ministry of Education.
- Bishop, R., & Glynn, T. (1999). *Culture counts: Changing power relations in education*. Palmerston North: Dunmore Press.
- Christensen, I. (2004). *An evaluation of Te Poutama Tau: Exploring issues in mathematics education*. Wellington: Ministry of Education.
- Cunningham, C. (1998). *A framework for addressing Māori knowledge in research, science and technology*. Keynote address to Te Oru Rangahau Māori Research and Development Conference, 7–9 July, 1998, Massey University.
- Ministry of Education (2006). *Ngā haeata mātauranga: 2005 annual report on Māori education*. Wellington: Ministry of Education.
- Sowder, J. T. (2007). The mathematics education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157–223). Reston, VA: NCTM.
- Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2007). *Teacher professional learning and development: Best evidence synthesis iteration [BES]*. Wellington: Ministry of Education.
- Trinick, T. (2005). Te Poutama Tau: A case study of two schools. In *Findings from the New Zealand Numeracy Development Project 2004* (pp. 80–88). Wellington: Ministry of Education.
- Trinick, T. (2006). Te Poutama Tau: A case study of two schools. In *Findings from the New Zealand Numeracy Development Projects 2005* (pp. 103–113). Wellington: Learning Media.
- Trinick, T., & Parangi, P. (2007). *Te Poutama Tau evaluation report*. Wellington: Ministry of Education.
- Trinick, T., & Stevenson, B. (2005). An evaluation of Te Poutama Tau 2004. In *Findings from the New Zealand Numeracy Development Project 2004* (pp. 56–65). Wellington: Ministry of Education.
- Trinick, T., & Stevenson, B. (2006). An evaluation of Te Poutama Tau 2005. In *Findings from the New Zealand Numeracy Development Projects 2005* (pp. 34–45). Wellington: Learning Media.
- Trinick T., & Stevenson, B. (2007). Te Poutama Tau 2006: Trends and patterns. In *Findings from the New Zealand Numeracy Development Projects 2006* (pp. 44–53). Wellington: Learning Media.

Appendices (Performance of SNP Students on the Number Framework)

Appendix A: Performance on the Strategy Domains

Table 13
Performance of SNP Students on the Additive Domain

	Ethnicity			Decile Group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Year 9 Initial									
0–3: Counting from One	1%	1%	2%	1%	1%	1%	1%	1%	1%
4: Advanced Counting	10%	14%	20%	16%	12%	8%	9%	12%	11%
5: Early Additive	39%	45%	51%	46%	43%	39%	38%	45%	42%
6: Advanced Additive	39%	34%	25%	29%	35%	39%	40%	33%	36%
7: Advanced Multiplicative	11%	6%	2%	8%	8%	13%	11%	9%	10%
8: Advanced Proportional	0%	1%			1%	0%	1%	0%	0%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 9 Final									
0–3: Counting from One	0%	1%	1%	1%	1%	0%	1%	0%	0%
4: Advanced Counting	4%	5%	8%	8%	4%	3%	4%	4%	4%
5: Early Additive	24%	33%	39%	32%	28%	24%	24%	29%	27%
6: Advanced Additive	43%	44%	36%	42%	42%	43%	44%	41%	42%
7: Advanced Multiplicative	23%	14%	12%	16%	20%	23%	22%	19%	21%
8: Advanced Proportional	5%	3%	3%	1%	5%	7%	4%	6%	5%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 10 Final									
0–3: Counting from One	0%	0%	1%	1%	0%	0%	0%	0%	0%
4: Advanced Counting	6%	6%	12%	11%	6%	7%	5%	8%	6%
5: Early Additive	19%	30%	35%	37%	22%	18%	20%	24%	22%
6: Advanced Additive	46%	44%	41%	37%	49%	35%	41%	48%	45%
7: Advanced Multiplicative	23%	16%	10%	15%	20%	29%	27%	17%	22%
8: Advanced Proportional	6%	3%	1%		3%	12%	7%	3%	5%
Number of students	1550	385	198	179	1614	562	1207	1148	2355

Notes:

- For appendices A–C:
- totals may be affected by rounding
 - NZE = New Zealand European
 - percentages less than 0.5% are shown as 0%
 - where there is no data entered for students, the cell is left blank.

Table 14
Performance of SNP Students on the Multiplicative Domain

	Ethnicity			Decile Group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Year 9 Initial									
0–3: Counting from One	2%	3%	4%	3%	3%	2%	2%	3%	3%
4: Advanced Counting	8%	14%	13%	11%	11%	7%	9%	10%	10%
5: Early Additive	24%	31%	39%	35%	27%	23%	24%	28%	26%
6: Advanced Additive	37%	37%	31%	35%	35%	39%	35%	38%	36%
7: Advanced Multiplicative	22%	13%	13%	14%	19%	23%	23%	18%	20%
8: Advanced Proportional	6%	3%	1%	2%	5%	6%	7%	4%	5%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 9 Final									
0–3: Counting from One	1%	1%	2%	2%	1%	0%	1%	1%	1%
4: Advanced Counting	4%	6%	5%	8%	4%	4%	5%	4%	4%
5: Early Additive	12%	20%	27%	22%	16%	10%	13%	16%	14%
6: Advanced Additive	34%	40%	40%	40%	34%	35%	33%	37%	35%
7: Advanced Multiplicative	34%	23%	21%	22%	30%	35%	32%	31%	31%
8: Advanced Proportional	16%	9%	5%	7%	14%	16%	17%	12%	14%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 10 Final									
0–3: Counting from One	0%	0%	0%	1%	0%	0%	0%	0%	0%
4: Advanced Counting	4%	7%	16%	15%	5%	3%	5%	5%	5%
5: Early Additive	11%	17%	22%	18%	14%	11%	11%	16%	13%
6: Advanced Additive	32%	34%	33%	30%	33%	26%	27%	36%	31%
7: Advanced Multiplicative	35%	29%	22%	27%	33%	37%	35%	32%	33%
8: Advanced Proportional	17%	12%	7%	8%	15%	23%	22%	11%	16%
Number of students	1550	385	198	179	1614	562	1207	1148	2355

Table 15
Performance of SNP Students on the Proportional Domain

	Ethnicity			Decile Group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Year 9 Initial									
0–1: Unequal Sharing	0%	1%	4%	2%	1%	0%	1%	0%	1%
2–4: Equal Sharing	12%	17%	27%	17%	17%	10%	15%	14%	14%
5: Early Additive	27%	40%	37%	39%	32%	25%	27%	32%	30%
6: Advanced Additive	19%	17%	12%	20%	16%	22%	19%	18%	18%
7: Advanced Multiplicative	36%	22%	18%	20%	31%	37%	32%	32%	32%
8: Advanced Proportional	5%	3%	2%	3%	4%	6%	6%	4%	5%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 9 Final									
0–1: Unequal Sharing	0%	0%	1%	0%	0%	0%	0%	0%	0%
2–4: Equal Sharing	5%	9%	14%	13%	7%	5%	7%	6%	6%
5: Early Additive	18%	33%	33%	34%	23%	17%	21%	23%	22%
6: Advanced Additive	18%	19%	23%	17%	19%	18%	16%	20%	18%
7: Advanced Multiplicative	45%	31%	24%	28%	39%	46%	42%	39%	41%
8: Advanced Proportional	14%	8%	6%	8%	12%	15%	14%	12%	13%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 10 Final									
0–1: Unequal Sharing	1%	1%	0%	0%	1%	0%	0%	2%	1%
2–4: Equal Sharing	7%	7%	10%	9%	6%	9%	5%	9%	7%
5: Early Additive	20%	31%	46%	39%	26%	14%	20%	28%	24%
6: Advanced Additive	14%	17%	17%	18%	14%	18%	15%	17%	16%
7: Advanced Multiplicative	41%	32%	24%	28%	40%	34%	40%	35%	37%
8: Advanced Proportional	17%	12%	4%	5%	13%	25%	20%	10%	15%
Number of students	1550	385	198	179	1614	562	1207	1148	2355

Appendix B: Performance on the Knowledge Domains

Table 16

Performance of SNP Students on the FNWS Domain

	Ethnicity			Decile Group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Year 9 Initial									
0-3: To 20	1%	2%	5%	3%	1%	1%	1%	1%	1%
4: To 100	2%	4%	5%	7%	3%	2%	3%	3%	3%
5: To 1000	30%	43%	47%	41%	38%	23%	30%	34%	33%
6: To 1 000 000	68%	51%	43%	49%	58%	74%	65%	62%	63%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 9 Final									
0-3: To 20	0%	1%	2%	1%	0%	1%	1%	1%	1%
4: To 100	1%	3%	4%	2%	2%	1%	2%	1%	1%
5: To 1000	19%	31%	35%	29%	25%	15%	20%	23%	22%
6: To 1 000 000	80%	66%	59%	67%	72%	84%	78%	75%	77%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 10 Final									
0-3: To 20	0%	1%	3%	3%	0%	0%	1%	0%	1%
4: To 100	1%	3%	4%	3%	2%	1%	1%	2%	2%
5: To 1000	19%	24%	36%	31%	23%	15%	15%	28%	22%
6: To 1 000 000	80%	72%	57%	62%	75%	84%	82%	70%	76%
Number of students	1550	385	198	179	1614	562	1207	1148	2355

Table 17
Performance of SNP Students on the Fractions Domain

	Ethnicity			Decile Group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Year 9 Initial									
0–3: Non-fractions	1%	4%	7%	8%	3%	1%	4%	2%	3%
4: Assigned unit fractions	8%	18%	17%	19%	11%	7%	11%	10%	10%
5: Ordered unit fractions	38%	42%	43%	42%	42%	33%	39%	38%	38%
6: Co-ordinated num./denom.	24%	21%	23%	18%	22%	27%	21%	26%	24%
7: Equivalent fractions	21%	12%	9%	10%	17%	22%	17%	19%	18%
8: Orders fractions	7%	3%	1%	3%	5%	11%	8%	6%	7%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 9 Final									
0–3: Non-fractions	0%	2%	2%	3%	1%	0%	1%	1%	1%
4: Assigned unit fractions	4%	11%	9%	11%	6%	3%	7%	5%	6%
5: Ordered unit fractions	23%	34%	31%	35%	29%	18%	27%	24%	25%
6: Co-ordinated num./denom.	24%	21%	25%	21%	23%	24%	20%	25%	23%
7: Equivalent fractions	33%	26%	26%	22%	31%	34%	29%	34%	31%
8: Orders fractions	15%	6%	7%	7%	10%	21%	16%	12%	14%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 10 Final									
0–3: Non-fractions	1%	1%	1%	2%	1%	1%	1%	1%	1%
4: Assigned unit fractions	5%	10%	18%	16%	7%	8%	7%	8%	8%
5: Ordered unit fractions	28%	36%	35%	36%	31%	18%	28%	29%	28%
6: Co-ordinated num./denom.	23%	21%	21%	17%	24%	20%	20%	26%	23%
7: Equivalent fractions	31%	24%	23%	26%	26%	37%	29%	29%	29%
8: Orders fractions	13%	7%	2%	2%	11%	16%	16%	7%	12%
Number of students	1550	385	198	179	1614	562	1207	1148	2355

Table 18
Performance of SNP Students on the Place Value Domain

	Ethnicity			Decile Group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Year 9 Initial									
0–3: Counts in fives and ones	1%	1%	6%	4%	2%	1%	2%	1%	1%
4: 10s to 100, orders to 1000	7%	13%	11%	15%	10%	5%	9%	8%	8%
5: 10s to 1000, orders to 10 000	38%	53%	54%	49%	46%	32%	41%	41%	41%
6: 10s, 100s, 1000s, orders whole numbers	29%	23%	22%	19%	26%	32%	26%	29%	28%
7: Tenths in and orders decimals	15%	7%	5%	8%	10%	18%	13%	13%	13%
8: Tenths, hundredths, and thousandths	10%	3%	2%	5%	6%	13%	10%	8%	9%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 9 Final									
0–3: Counts in fives and ones	0%	0%	2%	1%	1%	0%	1%	0%	0%
4: 10s to 100, orders to 1000	2%	5%	6%	6%	4%	1%	4%	2%	3%
5: 10s to 1000, orders to 10 000	22%	37%	39%	32%	30%	17%	25%	26%	25%
6: 10s, 100s, 1000s, orders whole numbers	27%	31%	31%	34%	30%	24%	27%	28%	28%
7: Tenths in and orders decimals	22%	15%	13%	11%	18%	25%	19%	22%	20%
8: Tenths, hundredths, and thousandths	26%	11%	8%	17%	18%	32%	25%	21%	23%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 10 Final									
0–3: Counts in fives and ones	0%	0%	1%	2%	0%	0%	0%	1%	0%
4: 10s to 100, orders to 1000	3%	3%	7%	4%	4%	3%	3%	4%	4%
5: 10s to 1000, orders to 10 000	24%	29%	40%	32%	27%	22%	20%	33%	26%
6: 10s, 100s, 1000s, orders whole numbers	25%	34%	31%	35%	27%	21%	28%	24%	26%
7: Tenths in and orders decimals	18%	16%	12%	19%	16%	19%	17%	16%	17%
8: Tenths, hundredths, and thousandths	30%	18%	9%	8%	26%	35%	31%	22%	27%
Number of students	1550	385	198	179	1614	562	1207	1148	2355

Table 19
Performance of SNP Students on the Basic Facts Domain

	Ethnicity			Decile Group			Gender		Total
	NZE	Māori	Pasifika	Low	Medium	High	Male	Female	
Year 9 Initial									
0–3: Facts to 10	1%	2%	4%	1%	3%	1%	3%	2%	2%
4: Within 10, doubles, and teens	4%	6%	6%	8%	5%	3%	6%	3%	4%
5: Addition, multiplication for 2, 5, 10	17%	25%	26%	28%	22%	12%	20%	17%	18%
6: Subtraction and multiplication	49%	51%	49%	48%	51%	46%	49%	49%	49%
7: Division	27%	14%	14%	14%	18%	36%	20%	28%	25%
8: Factors and multiples	2%	1%	1%	1%	1%	2%	1%	2%	2%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 9 Final									
0–3: Facts to 10	1%	1%	2%	1%	1%	0%	1%	0%	1%
4: Within 10, doubles, and teens	2%	3%	3%	3%	3%	1%	3%	2%	2%
5: Addition, multiplication for 2, 5, 10	11%	20%	13%	21%	15%	7%	14%	11%	12%
6: Subtraction and multiplication	37%	45%	51%	45%	39%	36%	39%	38%	39%
7: Division	43%	30%	31%	26%	37%	49%	38%	43%	41%
8: Factors and multiples	6%	2%	1%	4%	5%	7%	5%	6%	5%
Number of students	3176	892	326	412	2748	1933	2232	2861	5093
Year 10 Final									
0–3: Facts to 10	1%	1%	1%	1%	1%	0%	1%	1%	1%
4: Within 10, doubles, and teens	2%	3%	8%	7%	2%	2%	3%	3%	3%
5: Addition, multiplication for 2, 5, 10	15%	18%	18%	17%	16%	13%	14%	17%	15%
6: Subtraction and multiplication	38%	38%	51%	47%	40%	35%	35%	43%	39%
7: Division	41%	37%	22%	27%	38%	45%	42%	35%	39%
8: Factors and multiples	3%	2%	3%	3%	5%	1%	3%		
Number of students	1550	385	198	179	1614	562	1207	1148	2355

Appendix C: Effect Sizes for Performance of SNP Students

Table 20

Effect Sizes for Differences between Demographic Subgroups of SNP Students

		Add.	Mult.	Prop.	FNWS	Fract.	PV	BF
Male/Female	Year 9 initial	0.18	0.15	0.01	0.03	-0.08	0.02	-0.24
	Year 9 final	0.04	0.11	0.03	0.04	-0.07	0.00	-0.17
	Year 10 final	0.30	0.27	0.37	0.23	0.19	0.29	0.15
High/Medium	Year 9 initial	0.20	0.21	0.27	0.29	0.35	0.46	0.44
	Year 9 final	0.14	0.16	0.20	0.26	0.39	0.46	0.33
	Year 10 final	0.31	0.26	0.17	0.21	0.28	0.22	0.14
High/Low	Year 9 initial	0.31	0.38	0.47	0.56	0.71	0.67	0.61
	Year 9 final	0.39	0.53	0.59	0.43	0.74	0.62	0.59
	Year 10 final	0.66	0.66	0.48	0.60	0.67	0.58	0.47
Medium/Low	Year 9 initial	0.12	0.16	0.17	0.24	0.36	0.23	0.11
	Year 9 final	0.24	0.35	0.36	0.15	0.35	0.15	0.21
	Year 10 final	0.43	0.40	0.32	0.37	0.42	0.35	0.32
NZE/Māori	Year 9 initial	0.21	0.31	0.34	0.37	0.42	0.49	0.34
	Year 9 final	0.29	0.38	0.44	0.34	0.46	0.50	0.36
	Year 10 final	0.27	0.26	0.21	0.21	0.33	0.25	0.11
NZE/Pasifika	Year 9 initial	0.51	0.45	0.67	0.66	0.59	0.63	0.44
	Year 9 final	0.45	0.52	0.68	0.59	0.40	0.65	0.34
	Year 10 final	0.57	0.64	0.53	0.62	0.56	0.59	0.44
Māori/Pasifika	Year 9 initial	0.32	0.14	0.32	0.23	0.16	0.18	0.09
	Year 9 final	0.17	0.14	0.22	0.19	-0.06	0.15	-0.03
	Year 10 final	0.31	0.36	0.32	0.31	0.22	0.38	0.33
Year 10/Year 9	Final	0.02	0.06	-0.05	-0.01	-0.12	0.01	-0.13
Initial/Final	Year 9	0.56	0.52	0.44	0.28	0.47	0.55	0.42

(Shaded cells represent differences that are not statistically significant [$p > 0.01$].)

Table 21

Effect Sizes for Impact of Project on Demographic Subgroups of SNP Students

	Add	Mult	Prop	FNWS	Frac	PV	BF	N =
NZE	0.57	0.53	0.45	0.28	0.48	0.56	0.41	3176
Māori	0.50	0.49	0.38	0.30	0.43	0.58	0.39	892
Pasifika	0.62	0.52	0.47	0.31	0.65	0.55	0.46	326
Low	0.47	0.38	0.31	0.37	0.45	0.59	0.40	412
Medium	0.58	0.55	0.48	0.29	0.47	0.55	0.46	2748
High	0.55	0.52	0.41	0.25	0.51	0.58	0.39	1933
Male	0.49	0.48	0.43	0.28	0.45	0.52	0.44	2232
Female	0.61	0.56	0.44	0.28	0.49	0.58	0.41	2861
Total	0.56	0.52	0.44	0.28	0.47	0.55	0.42	5093