Patterns of Performance and Progress on the Numeracy Development Projects: Findings from 2006 for Years 5–9 Students

Jenny Young-Loveridge
University of Waikato
<jenny.yl@waikato.ac.nz>

The purpose of this study was to analyse the data from diagnostic interviews conducted with year 5–9 students by teachers who participated in the professional development programme of the Numeracy Development Projects (NDP) in 2006. By the end of a year on the NDP, just under half of the year 6 students were able to use a range of additive strategies to solve addition and subtraction problems (stage 6). Between one-third and one-half of the year 8 students were able to use a range of multiplicative strategies to solve problems with multiplication, division, and fractions (stage 7). These findings raise issues about some of the achievement objectives in the draft New Zealand Curriculum (2006) and the need to provide further intensive support for teachers if the majority of students are to meet these new objectives at the levels stated in the new curriculum document. The proportion of students at the upper stages of the Framework has increased over time, but this coincides with an increase in the proportion of students from high-decile schools taking part in the NDP and correspondingly a decrease in the proportion of students from low-decile schools. The analysis of effect sizes for comparisons between younger students after the NDP with slightly older students before they began the NDP shows that the impact of the NDP was greatest for Pasifika students, who had the largest effect size, on average (0.40). The average effect size for students from low-decile schools was 0.38, while that for Māori students was 0.35, slightly greater than that for European students (0.33). This analysis suggests that when comparisons are made between students within the same subgroup, those who have traditionally had lower levels of achievement (Māori and Pasifika students and those from low-decile schools) seem to benefit the most from participating in the NDP. Previous comparisons, between Māori/Pasifika and European students at identical stages on the Framework initially, showed that European students made the greatest progress in terms of gains in stages on the Framework. Likewise, simple comparisons between these subgroups on initial and final stages on the Framework showed that European students began at higher stages on the Framework and made greater gains than Māori or Pasifika students. Overall, the data suggests that the achievement gap, while not necessarily narrowing as a result of participation in the NDP, is being prevented from becoming larger, and this effect is greatest for Pasifika students and students from low-decile schools. Analysis of students' performance on basic facts and place value suggests that a focus on building students' knowledge of basic facts and an understanding of place value may lead to improved performance on the operational domains of the Framework.

Introduction

The New Zealand Numeracy Development Projects (NDP) have now been underway for more than seven years. Like other educational reform initiatives worldwide, the NDP were set up to improve mathematics teaching and learning at primary and secondary levels (Ministry of Education, 2001). Analysis of the data on students' mathematics achievement gathered by teachers as part of the professional development (PD) programme has been a valuable source of information for shaping PD in subsequent years (see Young-Loveridge, 2005, 2006). This paper reports on the results for the NDP for 2006, focusing particularly on students in years 5–9.

Method

Participants

In 2006, 37,144 year 5–9 students were assessed at the beginning and end of the year in which their teachers participated in one of the PD programmes for the NDP. Almost two-thirds (64.4%) of the cohort was European, while close to one-fifth (18.6%) was Māori. The remainder of the cohort consisted of Pasifika (7.4%), Asian (4.9%), and students from other ethnicities (4.7%). Compared to the national picture, this cohort included disproportionately more students from high-decile (39.3%) and medium-decile (43.7%) schools, and disproportionately fewer students from low-decile (17.0%) schools. (Note: The decile ranking of the school is used as an indicator of socio-economic status, with low-decile schools constituting the lowest 30% in socio-economic status, medium-decile schools the middle 40%, and high-decile schools the highest 30%.) The cohort was balanced in gender, with 51.3% being boys and 48.7% girls. Appendix A (p. 154) shows the composition of each year group in the 2006 cohort, as well as those for the years 5–9 cohorts that participated in the NDP between 2002 and 2006. (Note: Year 9 data is included for just 2005 and 2006 – the years since the Secondary Numeracy Pilot Project 2005 [see Harvey, Higgins, Maguire, Neill, Tagg, & Thomas, 2006].)

Procedure

Students were interviewed individually by their own teachers, using the NumPA (Numeracy Project Assessment, Ministry of Education, 2006) diagnostic assessment near the beginning of the school year (initial), and again near the end of the year (final). Data from these assessments was forwarded to a secure website for later analysis. Only students with both initial and final data were included in the analysis for this paper.

Results and Discussion

Performance of Students Participating in the NDP

The percentages of students in years 5–9 at each stage on the Number Framework at the beginning and end of the school year are presented in Appendix B (p. 155) for all three operational domains (addition-subtraction [additive domain], multiplication-division [multiplicative domain], and proportion-ratio [proportional domain]) and for the knowledge domains of fractions, place value, and basic facts. It was interesting to note that by the end of the school year, just under half (49.2%) of the year 6 students had reached stage 6 (advanced additive) on the additive domain, an expectation currently at level 3 of the draft New Zealand Curriculum (see Ministry of Education, 2006). By the end of year 8, that proportion had increased to just under two-thirds (65.3%), still somewhat short of the substantial majority one would hope to see for an achievement objective at a particular curriculum level. The 2006 cohort (see Appendix A, p. 154) included disproportionately more students from high-decile (39.3% instead of 30%) and medium-decile schools (43.7% instead of 40%) and disproportionately fewer students from low-decile schools (17.2% instead of 30%). Hence the proportions of students at stage 6 and stage 7 may be greater than is typical of a more representative cohort. By the end of year 8, just over two-fifths (41.1%) of the students had reached stage 7 (advanced multiplicative) on the multiplicative domain, an expectation currently at level 4 of the draft curriculum. This rather disappointing result has major implications for secondary schools and the need for students to be multiplicative if they are to succeed with algebra (Lamon, 2007).

It may be that, over time, as more teachers become familiar with the Framework and assessment tools and more students have learned mathematics with the assistance of an NDP-trained teacher for

the whole (or at least the majority) of their school careers, the numbers of students at upper stages of the Framework will increase. The results of the longitudinal study show greater proportions of students at these levels in schools that have been using NDP tools and resources for several years, with approximately 70% of year 6 students at stage 6 or higher (see Thomas & Tagg, 2006), which is about 20% more students than at the end of one year of NDP professional development. However, the 47% of year 8 students in the longitudinal study findings at stage 7 (see Thomas & Tagg, 2005) is only slightly more than the 41% found in the present study. It seems likely that the longitudinal results are biased favourably by schools that self-select their continued participation in the longitudinal study on the basis of the success they experience in working with NDP tools and resources.

Looking Back over the Last Five Years

Appendix C (p. 158) shows the proportion of students at different stages on the Framework for the three strategy domains plus the fractions domain at the end of the school year in which their teachers were trained to use NDP tools and resources. The proportion of year 6 students reaching stage 6 on the additive domain has increased steadily from 36.3% in 2002 to 49.2% in 2006, an increase of almost 13%. However, the increase in the proportion of year 8 students reaching stage 7 on the multiplicative domain is only 7.3% (from 33.8% to 41.1% over the same years). The cohort has changed from comprising students from mostly low-decile and medium-decile schools in 2002 (33.0% low-decile and 47.6% medium-decile) to comprising students from mostly medium-decile and high-decile schools in 2006 (43.4% medium-decile and 39.3% high-decile). Hence the increase in proportions of students at the upper stages is likely to be related to these changes in cohort composition.

Differences between Initial and Final Stages on the Framework

Table 1 shows the average stages on the Framework at the beginning and end of the year on the three strategy domains, the differences between subgroups, and the associated effect sizes for comparisons between those subgroups. It is evident from Table 1 that on the additive and multiplicative domains, the students gained just over half a stage on the Framework, on average. On the proportional domain, the gain was closer to a whole stage. There were small differences between subgroups (for example, Pasifika students gained slightly more, on average, than European or Māori students). However, these findings were confounded by the fact that the Framework does not consist of an interval scale. On the additive domain, steps at the lower stages of the Framework are smaller than at upper stages. Hence the average stage gain can appear greater for students who start lower on the Framework. The findings for the multiplicative and proportional domains were further confounded by the fact that, in order to maintain appropriate relativities with stages on the additive domain of the Framework, some steps on the Framework consist of a range of stages (for example, on the multiplicative domain, there is no stage 1, and the first step is stages 2–3 [count from one]; the proportional domain has stage 1 as its first step, but the second step is stages 2–4 [equal sharing]; fractions has no stage 1 but has stages 2–3 [unit fractions not recognised]). For the purpose of calculating means, standard deviations, and effect sizes, the first step on the multiplicative domain and the second step on the proportional domain were allocated a value of 3, resulting in much greater variability in students' stages and contributing to larger pooled variance for the calculation of effect sizes. For these reasons, the means, standard deviations, and effect sizes need to be treated with caution.

Comparing Groups

It is clear from Table 1 that European students started higher on the Framework than students with Māori or Pasifika ancestry (for example, 5.19 compared with 4.88 or 4.66 respectively, on the additive

domain in 2006). This difference increased slightly by the end of the school year for Māori, but only on the additive (0.31 to 0.33) and multiplicative (0.39 to 0.43) domains. The effect sizes for these differences were about one-third of a standard deviation for the European to Māori comparison (0.34 and 0.36 on the additive domain initial and final). In 2006, the differences between European and Pasifika students reduced slightly over the school year from the initial to the final assessments. The effect sizes for these differences were just over half a standard deviation for the European-Pasifika comparison (0.57 and 0.53 on the additive domain initial and final), but on all three strategy domains, the effect sizes were smaller for the final assessment than for the initial one. The comparison between students from high- and low-decile schools yielded effect sizes of just over half a standard deviation, and this increased slightly for the final assessments compared to the initial ones.

Table 1

Average Stage on the Framework, Differences between Subgroups and Associated Effect Sizes for Initial and Final Stages on Each of the Three Strategy Domains 2005 & 2006

Domain	European	Māori	Difference European–Māori	Effect Size	Pasifika	Difference European–Pasifika	Effect Size	High Decile	Low Decile	Difference High–Low	Effect Size
2005											
Additive											
Initial	4.99	4.68	0.31	**0.34**	4.50	0.49	**0.54**	5.04	4.54	0.50	**0.54**
Final	5.49	5.19	0.30	**0.34**	5.00	0.48	**0.54**	5.56	5.06	0.50	**0.54**
Gain	0.50	0.51			0.50			0.52	0.53		
Multiplicative											
Initial	5.18	4.77	0.41	**0.36**	4.54	0.64	**0.56**	5.24	4.61	0.63	**0.55**
Final	5.88	5.44	0.44	**0.38**	5.23	0.65	**0.57**	5.96	5.31	0.65	**0.55**
Gain	0.70	0.67			0.69			0.72	0.70		
Proportional											
Initial	4.63	4.00	0.62	**0.37**	3.59	1.04	**0.61**	4.71	3.75	0.96	**0.56**
Final	5.58	4.94	0.65	**0.43**	4.69	0.89	**0.60**	5.68	4.79	0.89	**0.58**
Gain	0.95	0.93			1.10			0.96	1.04		
2006											
Additive											
Initial	5.19	4.88	0.31	**0.34**	4.66	0.53	**0.57**	5.26	4.75	0.52	**0.54**
Final	5.68	5.35	0.33	**0.36**	5.19	0.48	**0.53**	5.78	5.23	0.55	**0.59**
Gain	0.49	0.47			0.53			0.52	0.48		
Multiplicative											
Initial	5.40	5.01	0.39	**0.34**	4.68	0.71	**0.62**	5.45	4.84	0.61	**0.52**
Final	6.05	5.62	0.43	**0.38**	5.40	0.65	**0.58**	6.14	5.48	0.66	**0.57**
Gain	0.65	0.61			0.71			0.69	0.64		
Proportional											
Initial	5.00	4.36	0.64	**0.38**	3.99	1.01	**0.61**	5.05	4.10	0.95	**0.56**
Final	5.86	5.25	0.61	**0.42**	5.01	0.85	**0.59**	5.97	5.03	0.94	**0.63**
Gain	0.86	0.89			1.02			0.92	0.93		

The Impact of the NDP on Students' Performance

The results show that students reached higher levels on the Framework at the end of the year than they had at the beginning of the year. However, it is likely that some of the progress that students made was as a result of "normal" aging rather than because they had participated in the NDP. In a traditional experimental study, the progress (as measured by the difference between pre-test and post-test scores) of the intervention group (those taking part in the project) would be compared with the progress of the control group (those not taking part in the project). There were good reasons why a traditional "control" group was not used in this instance (see Young-Loveridge, 2005). For example, there would have been ethical issues if the opportunity to participate in the NDP had been withheld from some teachers and their students for the purpose of creating a control group. There would also have been logistical problems in training non-participant teachers to assess their students simply for the purpose of comparison with students whose teachers did participate in the NDP. The teachers' assessment of their students was an important dimension of the PD programme for the NDP, so getting outside researchers to interview a control group of students would have introduced a confounding variable that could have been responsible for any differences between intervention and control groups, thus defeating the purpose of taking such a step. However, using students at the next year level up (the adjacent year group) before they participated in the NDP allowed comparisons to be made between students who had experienced a year of the NDP and those who not yet participated (pre-NDP).

For example, year 2 students who were at the end of a year of the NDP were compared with year 3 students who were at the beginning of a year of the NDP. On average, the year 2 students were about one-quarter of a year younger at the final assessment than the year 3 students at their initial assessment, so this provides a conservative measure of "control" for the "intervention" group. Figure 1 shows the comparison for the additive domain between younger students after a year of the NDP and older students before they began on the NDP. It is clear from Figure 1 that there were few differences between younger students after the NDP and older students before the NDP for the first few years at school. It is possible that using a control group that was more closely matched age-wise to the intervention group might have revealed in greater differences. However, Figure 1 also shows that, by year 5, there was an obvious advantage in participating in the NDP.

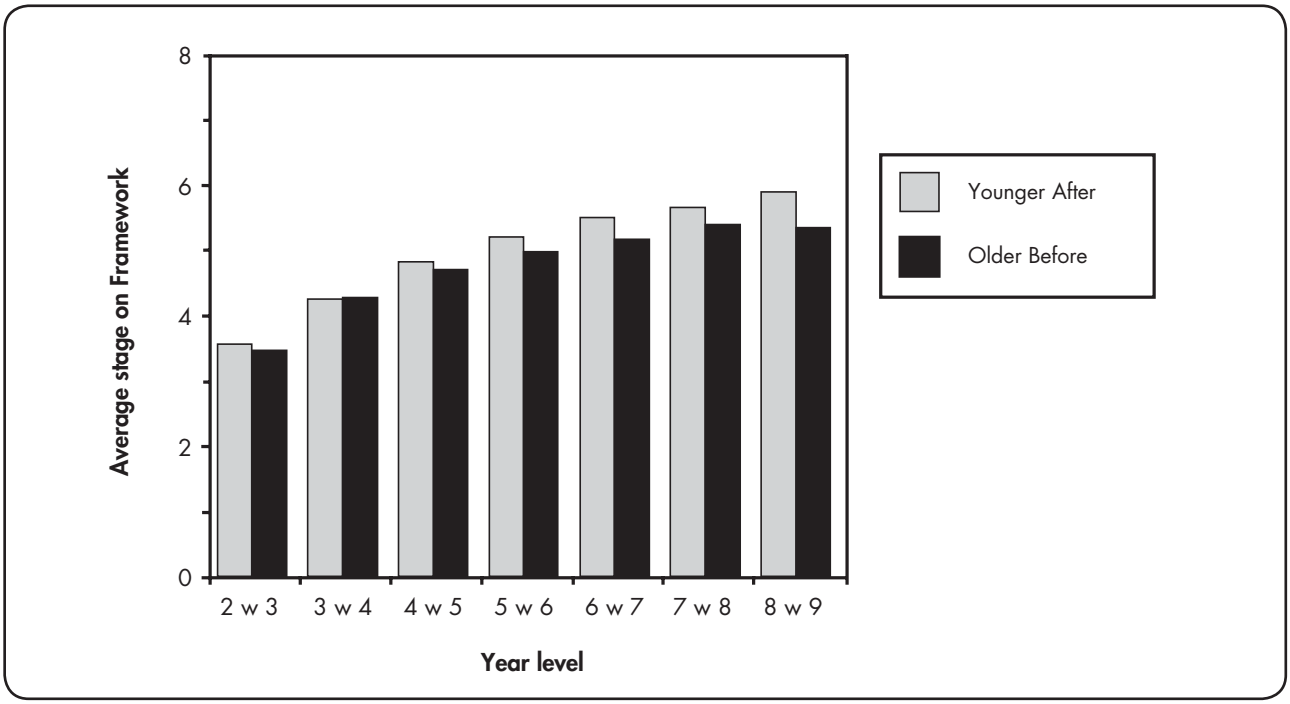

Figure 1: Average stage on the **additive** domain for comparisons between younger year group after a year of the NDP and older year group before they began the NDP. Note: the "w" in the horizonal axis labels for this and following figures means "with".

Figures 2 and 3 show the comparisons for the multiplicative and proportional domains between younger students after participating in the NDP and older students before participating in the NDP.

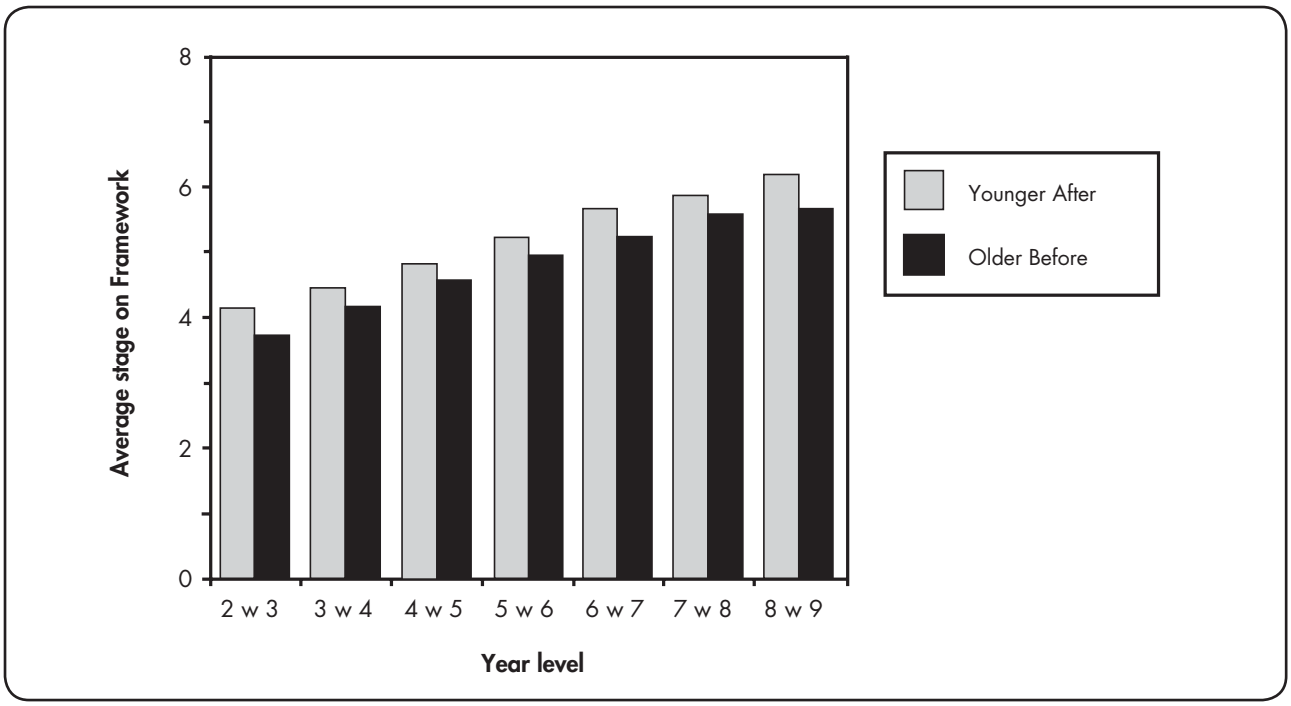

Figure 2: Average stage on the **multiplicative** domain for comparisons between younger year group after a year of the NDP and older year group before they began the NDP

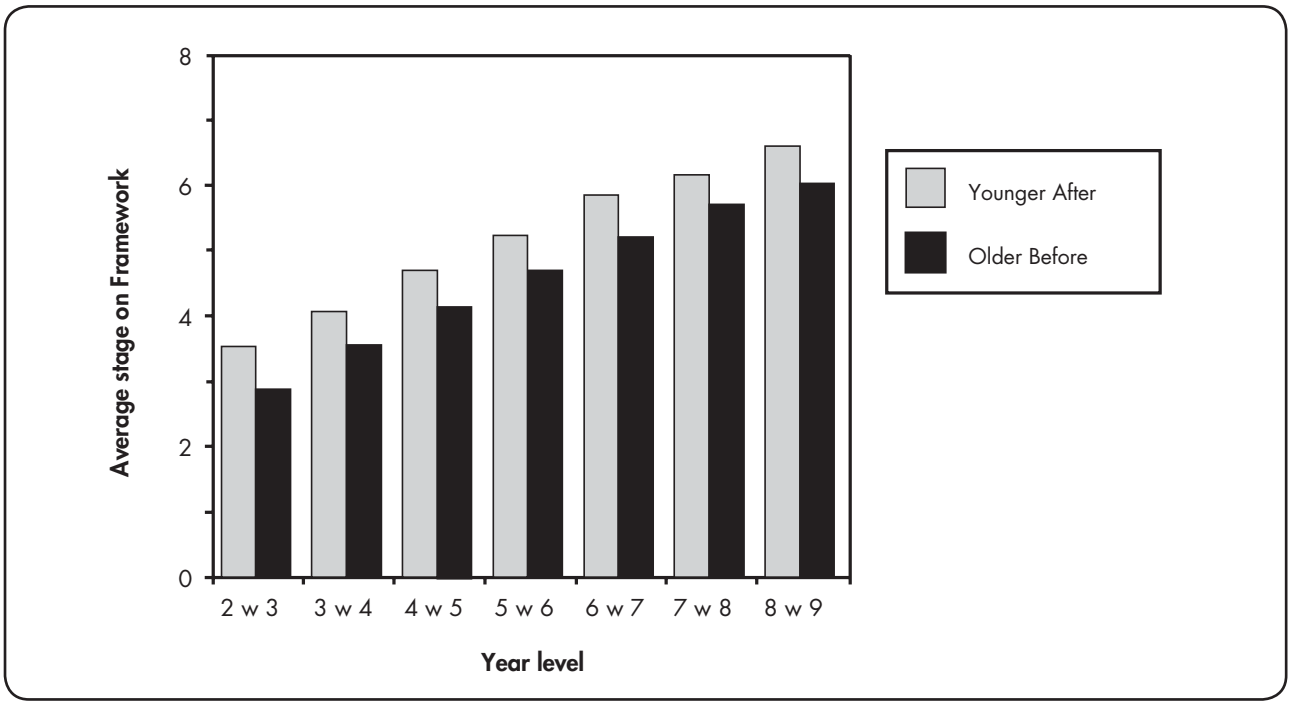

Figure 3: Average stage on the **proportional** domain for comparisons between younger year group after a year of the NDP and older year group before they began the NDP

It is clear from Figures 2 and 3 that there is an advantage in participating in the NDP right from year 2, but this may have been the result of comparing a small number of mathematically proficient students at that year level with a larger more heterogeneous group of slightly older students (see Appendix B (p. 155), which shows that the proportion of students for whom data was not entered or not applicable decreased with year level for all but Numeral Identification, where the opposite pattern was found). The effect was particularly pronounced for the proportional domain.

The Differential Impact of the NDP on Students from Different Subgroups

Appendix D (p. 160) shows the data used to calculate effect sizes for each of the comparisons for the cohort overall and for different subgroups between younger students after participating in the NDP and older students before participating in the NDP. The magnitude of these effect sizes indicates how much students who participate in the NDP benefit relative to slightly older students from the same subgroup who had not yet begun the NDP. It is clear from Appendix D that some of the effect sizes are quite large (that is, half a standard deviation or greater) and that certain subgroups had more of these larger effect sizes than others. For example, 11 out of the 42 effect sizes for Pasifika students were half a standard deviation or greater (see Figure 4, which shows Pasifika students on the multiplicative domain), and this was also the case for students from low-decile schools. In contrast, only five out of 42 effect sizes for European students were half a standard deviation or greater, and the same was true for Māori students and students from high-decile schools. Effect sizes of 0.40 or greater were most frequent for the adjacent year-group comparisons of Pasifika students (25 of 42) and students from low-decile schools (20 out of 42). Effect sizes of 0.40 or greater were least frequent for European (12 of 42), students from high-decile schools (14 of 42), and Māori students (15 of 42). Effect sizes were greater for the multiplicative and proportional domains, with 63% (multiplicative) and just over 77% (proportional) of effect sizes greater than one-third of a standard deviation. Some of the effect sizes for Māori, Pasifika, and low-decile students were more than three-quarters of a standard deviation (the highest effect size for each group was 0.87, 0.82, 0.76, and 0.61 for low-decile, Māori, Pasifika, and European students respectively).

A summary of Appendix D is shown below in Table 2, with the effect sizes averaged across 2005 and 2006 and across the comparisons for adjacent years (from year 2 with year 3 to year 8 with year 9) for the additive, multiplicative, and proportional domains. The overall average across the three domains was 0.34. The average effect sizes were above the overall average for Pasifika students (0.40), Māori (0.35), and students from low-decile schools (0.38). This data suggests that the achievement gap, while not necessarily narrowing as a result of participation in the NDP, was prevented from becoming larger, and this effect was greatest for Pasifika and students from low-decile schools, on average.

Table 2
Summary of Effect Sizes for Younger Students after the NDP and Older Students before the NDP Averaged across Comparisons between Adjacent Years from Year 2 to Year 9 and across 2005 and 2006

Comparison	Additive	Multiplicative	Proportional	Average Overall
Years 2–9				
Overall	0.24	0.39	0.39	**0.34**
European	0.24	0.38	0.37	**0.33**
Māori	0.24	0.40	0.42	**0.35**
Pasifika	0.26	0.45	0.48	**0.40**
Low Decile	0.24	0.44	0.45	**0.38**
High Decile	0.27	0.38	0.38	**0.34**

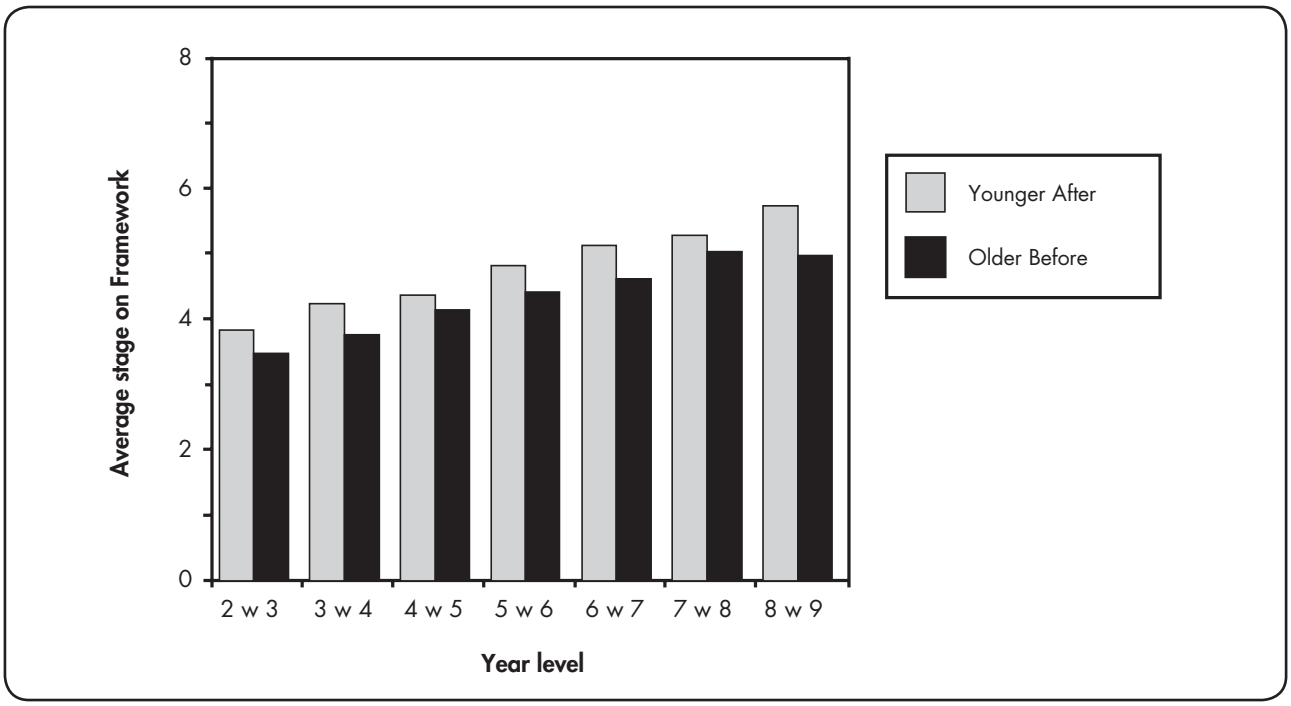

Figure 4: Average stage on the **multiplicative** domain for comparisons between **Pasifika** younger year group after a year of the NDP and older year group before they began the NDP

Patterns of Progress

Appendix E (p. 165) shows the percentages of students who progressed to higher stages relative to their initial stage on the Framework as a function of ethnicity. This analysis compares the progress of students from each subgroup, all of which began at an identical stage on the Framework. In general, European students made greater progress than Māori or Pasifika students. However, for some comparisons, Pasifika or Māori made the greatest progress. For example, for those students who began the NDP at stage 3 or below on the additive domain, Pasifika students made the greatest progress, with 72.0% moving up to a higher stage compared with 71.2% of European and 69.3% of Māori students (see Figure 5). On the multiplicative domain, Pasifika students made greater progress than Māori students when their initial stage on the Framework was 4, 5, or 6 (see Figure 6).

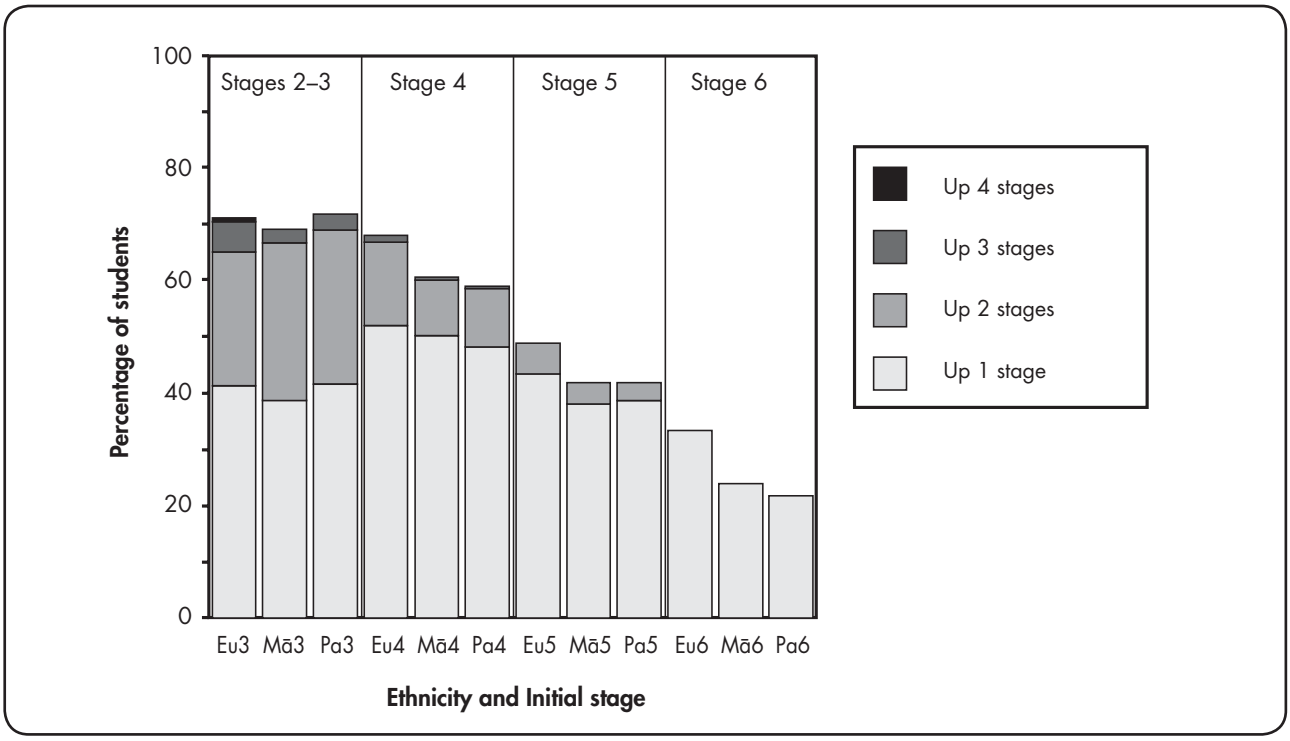

Figure 5: The percentage as a function of **ethnicity** of year 5–9 students who progressed to a higher stage on the **additive** domain relative to their initial stage on the Framework

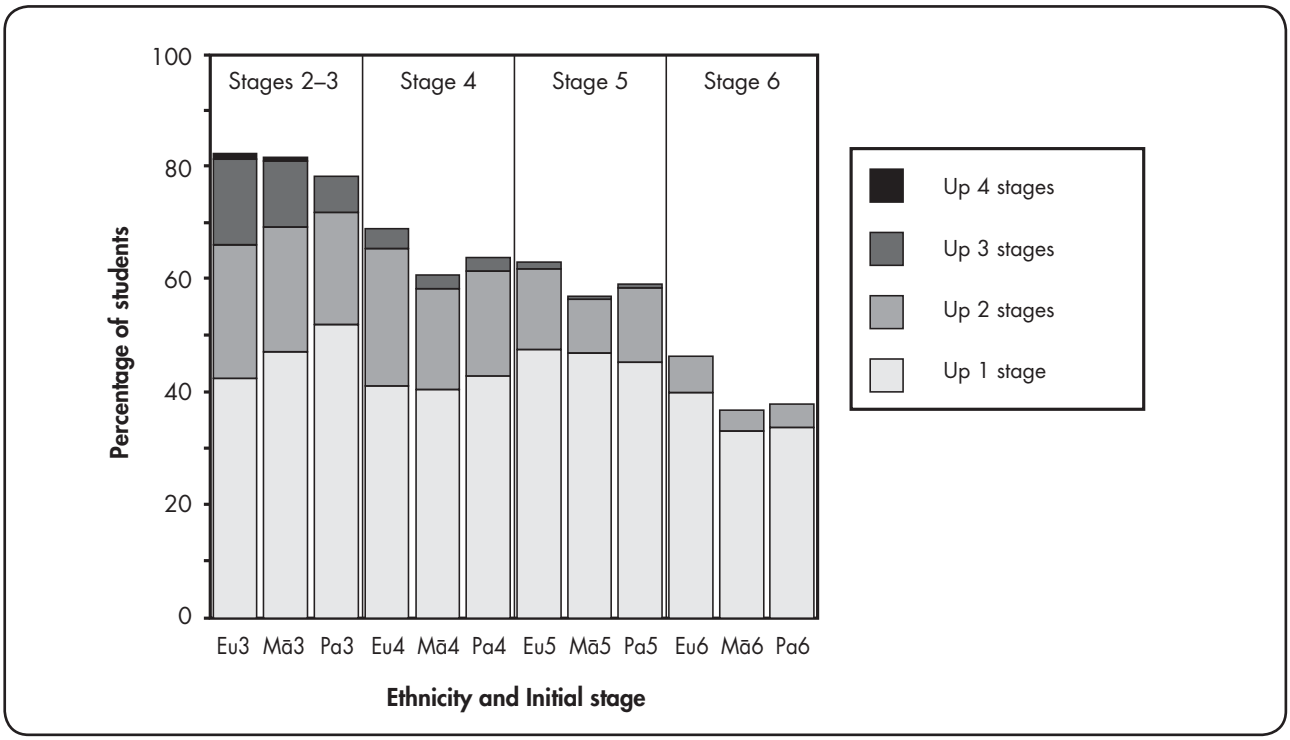

Figure 6: The percentage as a function of **ethnicity** of year 5–9 students who progressed to a higher stage on the **multiplicative** domain relative to their initial stage on the Framework

Appendix F (p. 168) shows the percentages of students who progressed to a higher stage relative to their initial stage on the Framework as a function of school-decile level. The patterns are fairly consistent, with students from high-decile schools making the greatest progress and those from low-decile schools making the least, relative to the same initial stage on the Framework.

Looking at Patterns over Time

Figure 7 shows the percentage of years 5–8 students at stage 7 or higher on the multiplicative domain at the beginning and end of a year on the NDP between 2002 and 2006. It is clear from Figure 7 that only a very small proportion of year 5 students reached stage 7 or higher on this domain. However, the improvements appear to increase with year level, as can be seen in the magnitude of the difference in proportion of students at stage 7 or higher finally compared to initially. Year 6 and year 7 were very similar, but there was a substantial improvement in year 8.

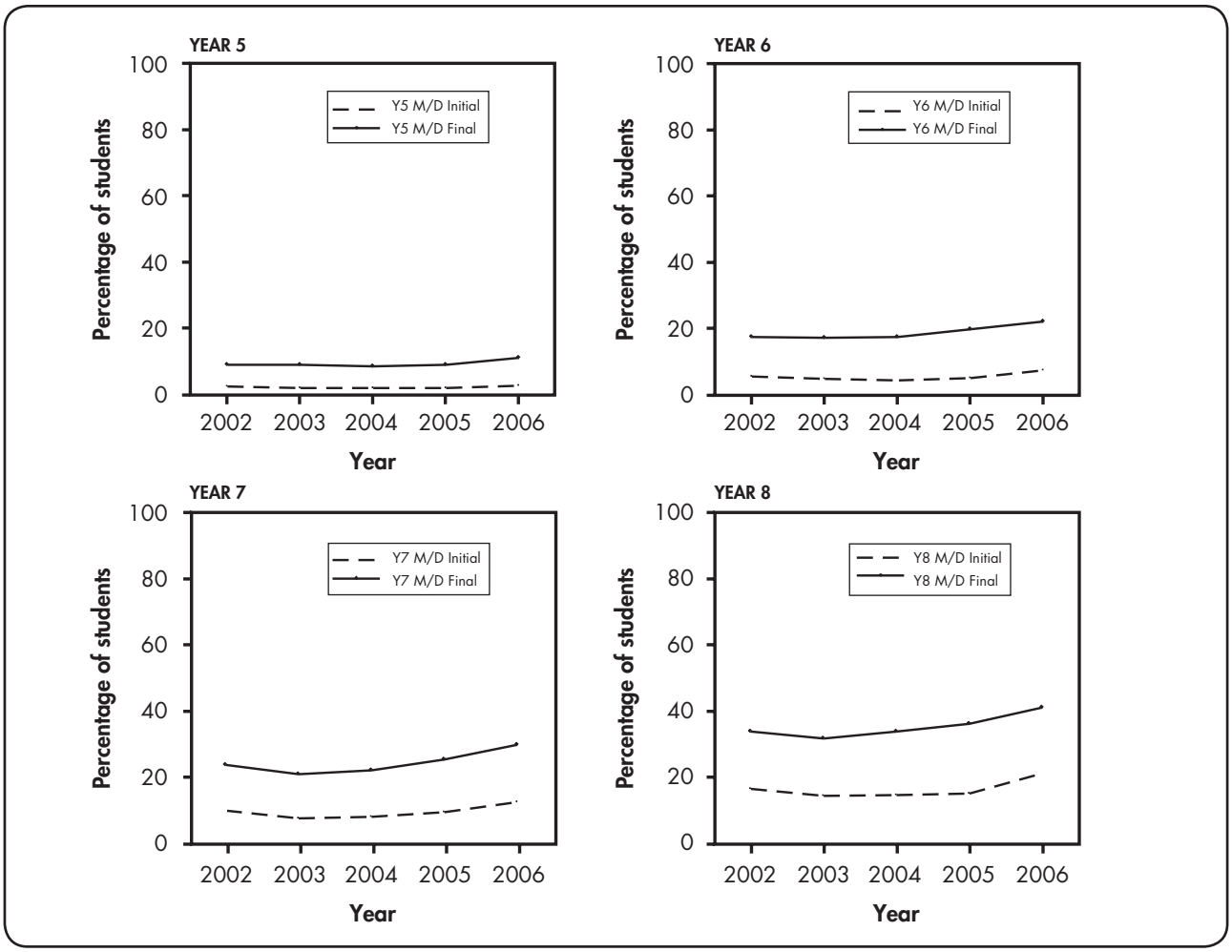

Figure 7: The percentages of year 5–8 students at stage 7 or higher on the **multiplicative** domain at the beginning (initial) and end (final) of a year of the NDP between 2002 and 2006

A similar pattern is evident in Figure 8, which shows the proportion of year 5–8 students at stage 7 or higher for the multiplicative and proportional domains by the end of a year on the NDP. It is clear from Figure 8 that the gaps between years 5 and 6 and between years 7 and 8 were greater than the gap between years 6 and 7. The reasons for this pattern are not obvious. It could be that a change of school for many students at the beginning of year 7 requires a major adjustment that slows down their progress in mathematics learning. By the end of the following year, students have had a chance to settle in to their new environment and adapted to the different expectations at intermediate school (for example, less emphasis on method), and as a result, their progress returns to former levels. This warrants further investigation.

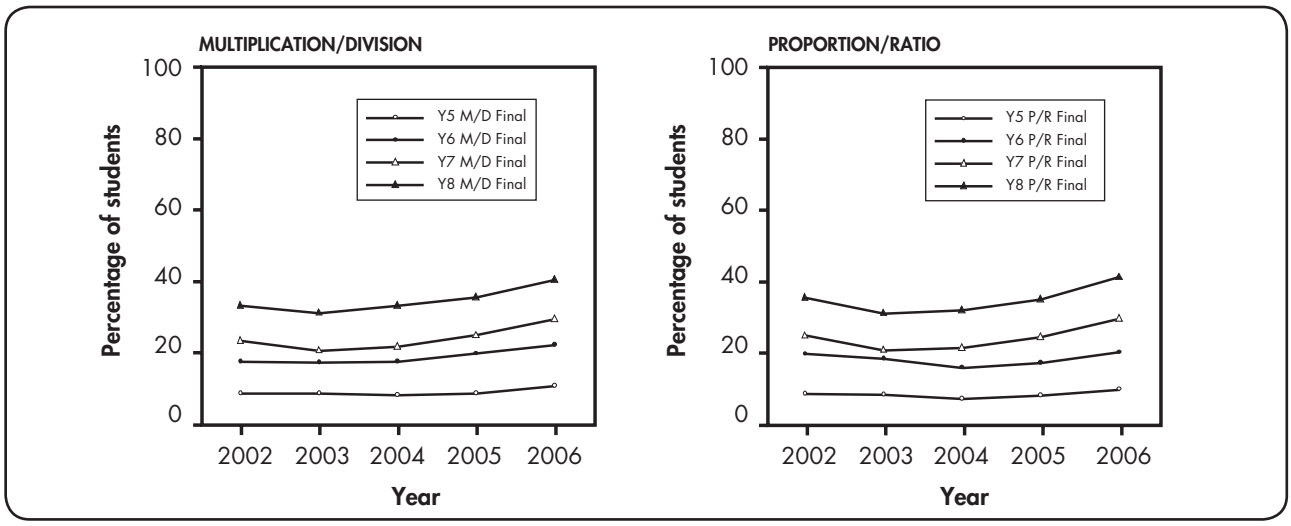

Figure 8: The percentages of year 5–8 students at stage 7 or higher on **multiplicative** and **proportional** domains at the end of a year of NDP between 2002 and 2006

The Challenges of Becoming a Multiplicative Thinker

Appendix G (p. 171) shows comparisons between students who started below stage 7 but reached stage 7 or higher by the end of a year of the NDP and those who were initially below stage 7 but did not progress to stage 7; these are shown for each of the three domains of the Framework. The biggest difference between those who reached stage 7 on the additive domain and those who did not was in their performance at stage 8 on the multiplicative domain, which almost one-third (30.4%) of the students who made progress reached, compared with only 5.4% of the no-progress students. As the stage 8 multiplicative domain tasks involved doing division with decimals and the stage 7 additive domain tasks involved doing subtraction with fractions and decimals, this result is not altogether surprising.

A similar pattern was found for the differences between those who reached stage 7 on the proportional domain and those who did not. On the multiplicative domain, what distinguished those who reached stage 7 from those who did not was their performance on the proportional domain, where more than two-thirds (68.2%) of the students who made progress reached stage 7 or higher, compared with only one-quarter (28.1%) of no-progress students.

Focus on Students Experiencing Difficulties: Persistent Counters

Appendix H (p. 173) presents the percentages of year 5–9 students who continued to use counting strategies for additive domain problems and compares these results with the corresponding percentages of students who reached stage 5, early additive part–whole thinking, by the end of a year on the NDP. The fact that 3846 students persisted in using counting, despite the best efforts of their teachers to help them acquire part–whole strategies, is of considerable concern. Comparison of these students with those in the same year group who had reached stage 5 shows that important components of knowledge are missing or are very weak in persistent counters. For example, knowledge of place value, basic facts, and number sequence forwards and backwards were areas where students at stage 5 showed considerably greater competence than did persistent counters (those at stages 0 to 4). Close to two-thirds of year 7 students (62.0%) were below stage 5 on place value knowledge, whereas only one-quarter (25.6%) of year 7 students were able to use simple part–whole strategies to solve addition and subtraction problems. More than half (55.6%) of the year 5 persistent counters were at stage 4 or lower on basic facts, whereas only one-fifth (21.3%) of year 5 students were able to use simple part–whole strategies to solve addition and subtraction problems.

The Relationships between Basic Facts Knowledge and Proficiency on Other Domains

Appendix I (p. 174) shows the percentages of year 5–9 students at various stages on the basic facts domain of the Framework after a year of the NDP and the stages they had reached on other domains of the Framework in that time. More than 3000 students were still at stage 4 or below on basic facts, despite a year of the NDP. A higher proportion of these students were in year 5 (37.3%), between one-fifth and one-quarter of the students were in years 6 and 7 (20.2% and 23.7% respectively), and only small proportions were in years 8 and 9 (13.5% and 5.2% respectively). These students knew some small number combinations for sums totalling 10 or less (for example, 2 + 3, 5 + 4, 6 + $\square$ = 10), some single-digit doubles (for example, 6 + 6, 9 + 9), and sums of 10 combined with single-digit quantities (for example, 10 + 4, 7 + 10). However, what distinguished these students from those at stage 5 was that they were unable to combine different addends to get totals between 10 and 20 (for example, 8 + 6, 6 + 9) or to recall number facts from the five-times table (for example, 8 x 5, 5 x 7). Nor were they able to subtract single-digit quantities from "teen" numbers (for example, 17 – 9, 15 – 6) or to recall other multiplications (6 x 7, 8 x 4), all of which are characteristics of students at stage 6. Recall of division facts (for example, 56 ÷ 7, 63 ÷ 9), a feature of stage 7 students, was also beyond their capabilities.

An examination of performance on the operational domains of students below stage 5 on basic facts showed that the majority of these students solved addition/subtraction problems by counting on (40.3% were at stage 4) or by simple partitioning and recombining of quantities (45.0% were at stage 5). Only 7.2% reached stage 6 (advanced additive part–whole) or higher. On the multiplicative domain, these stage 2–4 students tended to either skip count (44.6% were at stage 4) or used repeated addition (27.2% were at stage 5). On the proportional domain, half of them were able to share 12 beans into thirds (50.8% were at stages 2–4) or work out that if 4 + 4 + 4 = 12, then one-third of 12 beans is 4 beans (27.8% were at stage 5). About one-third of them could name unit fractions (33.9% were at stages 2–3), and another third could order unit fractions (38.3% were at stage 4). Between half and two-thirds of them could count by tens to work out how many $10 notes would be needed to buy items costing $80 and $230 (60.9% were at stage 4). It was interesting to note that 9.8% had reached a higher stage on basic facts at the initial assessment. However, this might have been the result of their teachers' inexperience with the assessment tool at the beginning of the PD programme. Whereas only 40% of them had reached stage 4 on basic facts initially, after a year of the NDP this had increased to 75.2%.

Students who were assessed as being at stage 5 on the basic facts domain were more likely than those at stages 2–4 to be able to reach stage 6 on other domains, but only about one-third (34.7%) of them reached stage 6 on the multiplicative domain, and about one-quarter reached stage 6 on the additive and proportional domains (25.1% and 24.6% respectively). Only about one-sixth reached stage 6 on the fractions or place value domains (17.7% and 16.9% respectively). Approximately half (between 45.0% and 60.9%) of the students at stage 6 on the basic facts domains reached stage 6 or higher on the other domains. Of those who reached stage 7 on basic facts, between 75% and 90% reached stage 6 or higher on other domains. Students needed to be at stage 8 on basic facts (able to recall division facts as well as addition, subtraction, and multiplication facts) to be virtually guaranteed of reaching stage 6 or higher on other domains on the Framework.

The Relationships between Place Value Knowledge and Proficiency on Other Domains

Appendix J (p. 176) shows the percentages of year 5–9 students at various stages on the place value domain of the Framework after a year of the NDP and the stages they had reached on other domains of the Framework in that time. It is clear from Appendix J that until students are able to count by tens (stage 4), they have little chance of succeeding on any but the simplest addition/subtraction problems. Those at stage 5 on the place value domain (who were able to give the number of tens in

230 and could identify 6.8 on a number line) were more likely to be able to use at least two different mental strategies to solve multi-digit problems from the additive or multiplicative domains or to find a fraction of a number (between 42% and 55% of the students, depending on the domain). Substantially more of the students at stage 6 (who were able to give the number of hundreds in 26 700 and find the number three-tenths more than 4.8) could do this (between 69% and 82% of the students). Close to 90% or more of students at stage 7 on the place value domain (who were able to find the number of tenths in 4.67 and order decimals of varying lengths) were at stage 6 or higher on the other domains. Students at the highest stage on the place value domain (stage 8) were able to find the number of hundredths in 2.097, round 7.649 to the nearest tenth, give three numbers between 7.59 and 7.6, and name 137.5% as a decimal. These students had such an extensive understanding of the number system that they tended to be at stage 7 or higher on all other domains. Students needed to be at stage 8 on the place value domain (able to convert between fractions, decimals, or percentages) to be virtually guaranteed of reaching stage 6 or higher on other domains on the Framework. This is consistent with Ross's (1989) assertion that to understand place value, students must co-ordinate and integrate knowledge about the notational system used to record numbers as well as about part–whole relationships among numerical quantities. Coming to understand these ideas is difficult, and the concepts develop slowly over a number of years (Ross, 1989; Verschaffel, Greer, & De Corte, 2007). As Appendix B (p. 155) shows, even by the end of year 9, only about one-third of students have a well-developed understanding of decimals (that is, the students were at stages 7–8).

General Discussion

The findings presented here include some good and some not-so-good news. It is heartening to see that Pasifika students particularly (and Māori students also) did better as a result of the NDP than they would have otherwise. This is evident in the larger effect sizes found when comparing both younger Pasifika students after a year of the NDP and slightly older Pasifika students before they began the NDP with corresponding European students. The reasons for the larger effect sizes for Pasifika are not entirely clear. It may be that a combination of the schooling improvement initiatives that have been in place now for several years (for example, the Manurewa Enhancement Initiative, see Young-Loveridge, 2005) and recent home–school partnership projects with Pasifika communities have helped schools serving low-decile communities improve the mathematics learning of their students. The schooling improvement initiatives have provided teachers with support above and beyond the NDP itself. As Sowder (2007) has pointed out, "the key to increasing students' mathematical knowledge and to closing the achievement gap is to put knowledgeable teachers in every classroom" (p. 157).

The percentages of students overall reaching the upper stages on the Framework on the multiplicative and proportional domains after a year of the NDP were considerably lower than those reflected in the achievement objectives of the draft New Zealand Curriculum (see Ministry of Education, 2006). For example, according to level 3 of the draft curriculum, most year 6 and 7 students should have a flexible range of additive strategies for dealing with addition and subtraction problems, yet fewer than half of the year 6 students and barely half of the year 7 students in the 2006 cohort reached stage 6, advanced additive part–whole thinking, by the end of the year. At level 4 of the draft curriculum, most year 8 and 9 students are expected to have a flexible range of multiplicative strategies for dealing with multiplication, division, and fraction problems. Yet only between one-third and one-half of year 8 and year 9 students reached stage 7, advanced multiplicative, by the end of the year. The expectation that students at level four should be multiplicative thinkers is based on research evidence showing that students cannot engage with algebra effectively if they are not multiplicative thinkers (for example, Lamon, 2007; Wu, 2002). Hence, it is important not to compromise the expectations for particular curriculum levels.

The mismatch between the achievement objectives in the draft curriculum and the proportion of students at particular Framework stages found in this analysis signals the need for further intensive efforts to improve mathematics teaching and learning at the upper primary and intermediate levels.

The Ministry of Education's fee subsidy scheme, which provides some financial support to offset the costs of teachers doing further university study in mathematics education, may help, but far more publicity is needed as well as support from schools if the scheme is to have an appreciable impact on teachers' understanding of the upper stages of the Framework. A revised version of *Book 1: The Number Framework* was published earlier this year (see Ministry of Education, 2007a), and the combination of array and number-line models more effectively captures the complexity of multiplicative thinking in particular. More recently, a revised version of *Book 6: Teaching Multiplication and Division* was released (see Ministry of Education, 2007b). It remains to be seen whether these revised books make a difference to teachers' understanding of the upper stages on the Framework. As Young-Loveridge, Taylor, Hāwera, & Sharma (this volume) point out, fractional number and multiplicative thinking are extraordinarily complex, and it might be naïve to expect teachers to acquire a deep and connected understanding of these areas after only one or two years of professional development. As Ward, Thomas, and Tagg (this volume) have shown, even teachers who have worked with the NDP over several years do not necessarily have a strong understanding of fractional number. Consistent with this are the findings of Lamon's (2007) research showing that it took considerably more than two years for new ways of teaching fractional number to have a beneficial impact on students' understanding, and that was when the focus was explicitly on fractional number.

It is not just a matter of improving teachers' subject matter knowledge of mathematics or their pedagogical content knowledge in mathematics (Ball, Hill, & Bass, 2005; Ball, Lubienski, & Mewborn, 2001; Moch, 2004; Shulman, 1986). Teacher's beliefs about mathematics learning, and in particular whether they have a *conceptual orientation* or a *calculational orientation*, may have a substantial impact on the way that they teach mathematics in their own classrooms (see Philipp, 2007). A conceptual orientation leads to different kinds of classroom discussions from a calculational orientation. A teacher with a calculational orientation is interested in the calculations students perform to get their answers, whereas a teacher with a conceptual orientation is more interested in students' explanations of their reasoning (Philipp, 2007). According to Hiebert and Grouws (2007), conceptual understanding is promoted by teaching that draws students' attention explicitly to concepts (including the connections between mathematical facts, procedures, and ideas) and ensures that students "struggle" with important mathematical ideas (that is, they expend effort in making sense of mathematics and working out something that is not immediately apparent). It would be interesting to know the extent to which teachers participating in the NDP move to a conceptual orientation.

The NDP's increased emphasis on communicating mathematical ideas and reasoning is not without problems. As previous research has shown, students whose teachers have participated in the NDP don't necessarily value the opportunities to share their strategies with other students or learn from others' approaches (see Young-Loveridge, Taylor, & Hāwera, 2005). Several researchers (for example, Lubienski, 2007; Zevenbergen, 2001) have found that students from lower socio-economic backgrounds are not as comfortable as students from higher socio-economic backgrounds in contributing to discussions about their ways of reasoning about mathematics. However, Hunter (2005, 2006) has shown that teachers can help students in low-decile schools learn how to contribute to a community of inquiry and can assist them to become effective communicators about their mathematics thinking and learning. More work may need to be done in this area to help narrow the gap between European and Māori/Pasifika students and between students at high- and low-decile schools.

The analysis presented here suggests that knowledge of basic facts may be an important requirement for using an advanced part–whole strategy (stage 6) on the operational domains. These findings are consistent with those of other researchers who have investigated the relationship between arithmetical fact retrieval and general measures of arithmetic performance (Cumming & Elkins, 1999; Dowker, 2005; Geary & Brown, 1991; Geary, Brown, & Samaranayake, 1991; Ostad, 1998). Memory processes and the inhibition of incorrect responses seem to play an important part in number fact retrieval (Barrouillet, Fayol, & Lathuliere, 1997). However, researchers have shown that it is possible to improve students' memory for number facts with training (for example, Dowker, 2005; Pauli, Bourne, & Birbaumer, 1998). The importance of basic fact knowledge was highlighted in the latest NEMP (National Education Monitoring Project) study (Flockton, Crooks, Smith, & Smith, 2006), which showed a moderate overall decline (a 5% difference) in average performance on number task components at the year 4 level for the 2005 cohort, compared with that of 2001 (see Crooks & Flockton, 2002). The NEMP researchers have attributed the overall decline to the 9% decline on the large number of task components (71) that involved recall of facts or simple calculations with the four basic operations. This conclusion was supported by their finding of a drop (from 56% to 36%) in the proportion of year 4 students who said they practised mathematics facts or tables in their own time, which the NEMP researchers took as a clear indication of a reduced emphasis on basic facts by teachers.

However, it is important to acknowledge the possibility that students tried to work out the answers using some kind of strategy (either a counting-on or a part–whole strategy), which meant that they often missed the presentation of the next item (4 seconds later) because they weren't quite ready for it. The tasks were presented by the computer and not monitored by an adult, so it is impossible to know just how many of the NEMP problems were solved by the straight recall of basic facts and how many were solved by counting on or by deriving the answer using knowledge of a different basic fact. This warrants further investigation through a probe study. Gray and Tall (1994) use the term "procept" to refer to the amalgamation of both process and concept and the way that multiple procepts can represent the same object (for example, "6" as 1, 2, 3, 4, 5, 6, or ..., 4, 5, 6, or 3 + 3, or 4 + 2, or 10 – 4). They distinguish between meaningful known facts that are generated by flexible thinking and facts are are remembered simply by rote. It is the flexible thinking that is reflected in derived facts that should be the goal of mathematics teaching, rather than the simple recall of facts without meaning. Interestingly, the 2005 year 4 NEMP students outperformed the 2001 cohort (by 3%) on more complex problem-solving tasks, including algebra, logic, finding patterns, estimation, and identifying sequences, despite the lower socio-economic status of the cohort (compared to the 2001 sample, the 2005 NEMP sample included considerably fewer year 4 students from high-decile schools [32% vs 41%], and slightly more year 4 students from low-decile schools [28% vs. 27%]).

Even though most primary schools have now been given an opportunity to participate in one of the PD programmes of the NDP, there is an ongoing challenge to continue teachers' professional learning about mathematics teaching and learning. An enormous amount of effort has been put in to developing the people who can work with teachers to bring about change in their ways of approaching the teaching and learning of mathematics. It is hoped that this effort can continue, so that the gains that have been made are not lost in the future. It is clear from the results that teachers need a great deal more help in coming to understand multiplicative thinking and proportional reasoning.

References

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, Fall, 14–22, 43–46.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of Research on Teaching, 4th ed.* (pp. 433–456). Washington, DC: AERA.

Barrouillet, P., Fayol, M., & Lathuliere, E. (1997). Selecting between competitors in multiplication tasks: An explanation of the errors produced by adolescents with learning difficulties. *International Journal of Behavioural Development*, 21, 253–275.

Crooks, T. & Flockton, L. (2002). *Mathematics Assessment Results 2001 (National Education Monitoring Report 23).* Wellington: Ministry of Education.

Cumming, J. J. & Elkins, J. (1999). Lack of automaticity in the basic addition facts as a characteristic of arithmetic learning problems and instructional needs. *Mathematical Cognition*, 5, 149–180.

Dowker, A. (2005). *Individual Differences in Arithmetic: Implications for psychology, neuroscience and education.* Hove, UK: Pschology Press.

Flockton, L., Crooks, T., Smith, J., & Smith, L. F. (2006). *Mathematics Assessment Results 2005 (National Education Monitoring Report 37).* Wellington: Ministry of Education.

Geary, D. C. & Brown, S. C. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in gifted, normal, and mathematically disabled children. *Developmental Psychology*, 27, 398–406.

Geary, D. C., Brown, S. C., & Samaranayake, V. A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology*, 27, 787–797.

Gray, E. M. & Tall, D. O. (1994). Duality, ambiguity, and flexibility: A "proceptual" view of simple arithmetic. *Journal for Research in Mathematics Education*, 25, 116-140.

Harvey, R., Higgins, J., Maguire, T., Neill, A., Tagg, A., & Thomas, G. (2006). *Evaluations of the 2005 Secondary Numeracy Pilot Project and the CAS Pilot Project.* Wellington: Ministry of Education.

Hiebert, J. & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 371–404). Reston, VA: NCTM.

Hunter, R. (2005). Reforming communication in the classroom: One teacher's journey of change. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds), *Building Connections: Research, theory and practice* (pp. 451–458). (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, July, Melbourne.)

Hunter, R. (2006). Structuring the talk towards mathematical inquiry. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds), *Identities, Cultures and Learning Spaces.* (Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia, Canberra, 1–5 July, pp. 309–317.)

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Towards a theoretical framework for research. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 629–667). Reston, VA: NCTM.

Lubienski, S. T. (2007). Research, reform, and equity in U.S. mathematics education. In N. S. Nasir & P. Cobb (Eds), *Improving Access to Mathematics: Diversity and equity in the classroom* (pp. 10–23). New York: Teachers College Press.

Ministry of Education (2001). *Curriculum Update 45: The numeracy story.* Wellington: Ministry of Education.

Ministry of Education (2006). *The New Zealand Curriculum: Draft for consultation 2006.* Wellington: Ministry of Education.

Ministry of Education (2007a). *Book 1: The Number Framework: Revised edition 2007.* Wellington: Ministry of Education.

Ministry of Education (2007b). *Book 6: Teaching Multiplication and Division: Revised edition 2007: Draft.* Wellington: Ministry of Education.

Moch, P. L. (2004). Demonstrating knowledge of mathematics: Necessary but *not* sufficient. *Curriculum and Teaching Dialogue, 6* (2), 125–130.

Ostad, S. A. (1998). Developmental differences in solving simple arithmetic word problems and simple number-fact problems: A comparison of mathematically normal and mathematically disabled children. *Mathematical Cognition*, 4, 1–19.

Pauli, P., Bourne, L. E., & Birbaumer, N. (1998). Extensive practice in mental arithmetic and practice transfer over a ten-month retention interval. *Mathematical Cognition*, 4, 21–46.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 257–315). Reston, VA: NCTM.

Ross, S. H. (1989). Parts, wholes, and place value: A developmental view. *Arithmetic Teacher, 36* (6), 28–32.

Shulman, (1986). Those who understand: Knowledge growth in teaching, *Educational Researcher, 15* (2), 4–14.

Sowder, J. T. (2007). The mathematics education and development of teachers. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 157–223). Reston, VA: NCTM.

Thomas, G. & Tagg, A. (2005). Evidence for expectations: Findings from the Numeracy Development Projects longitudinal study. In *Findings from the New Zealand Numeracy Development Project 2004* (pp. 21–34). Wellington: Ministry of Education.

Thomas, G. & Tagg, A. (2006). Numeracy Development Projects longitudinal study: Patterns of achievement. In *Findings from the New Zealand Numeracy Development Projects 2005* (pp. 22–33). Wellington: Learning Media.

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 557–628). Reston, VA: NCTM.

Ward, J., Thomas, G., & Tagg, A. (2007, this volume). What do teachers know about fractions? In *Findings from the New Zealand Numeracy Development Projects 2006*. Wellington: Learning Media.

Wu, H. (2002). *Chapter 2: Fractions (Draft).* Retrieved from http://www.math.berkeley.edu/~wu/ on 5 February 2007.

Young-Loveridge, J. (2005). Patterns of performance and progress. In *Findings from the New Zealand Numeracy Development Project 2004* (pp. 5–20, 115–127). Wellington: Ministry of Education.

Young-Loveridge, J. (2006). Patterns of performance and progress on the Numeracy Development Project: Looking back from 2005. In *Findings from the New Zealand Numeracy Development Projects 2005* (pp. 6–21, 137–155). Wellington: Learning Media.

Young-Loveridge, J., Taylor, M., & Hāwera, N. (2005). Going public: Students' views about the importance of communicating their mathematical thinking and solution strategies. In *Findings from the New Zealand Numeracy Development Project 2004* (pp. 97–106). Wellington: Ministry of Education.

Young-Loveridge, J., Taylor, M., Hāwera, N., & Sharma, S. (2007, this volume). Year 7–8 students' solution strategies for a task involving addition of unlike fractions. In *Findings from the New Zealand Numeracy Development Projects 2006*. Wellington: Learning Media.

Zevenbergen, R. (2001). Mathematics, social class, and linguistic capital: An analysis of mathematics classroom interactions. In B. Atweh, H. Forgasz, & B. Nebres (Eds), *Sociocultural Research on Mathematics Education: An international perspective* (pp. 201–215). Mahwah, NJ: Erlbaum.