**Achievement Objectives**

**Number: Levels 4 and 5**

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<tr>
<th>Level</th>
<th>Number strategies and knowledge AO2:</th>
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<tr>
<td></td>
<td>Understand addition and subtraction of fractions, decimals, and integers.</td>
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<thead>
<tr>
<th>Level</th>
<th>Number strategies and knowledge AO3:</th>
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<tbody>
<tr>
<td></td>
<td>Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions, and decimals</td>
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<thead>
<tr>
<th>Level</th>
<th>Number strategies and knowledge AO4</th>
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<tbody>
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<td>Apply simple linear proportions, including ordering fractions.</td>
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<thead>
<tr>
<th>Level</th>
<th>Number strategies and knowledge AO5</th>
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<tbody>
<tr>
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<td>Know the equivalent decimal and percentage forms for everyday fractions.</td>
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<tr>
<th>Level</th>
<th>Number strategies and knowledge AO6</th>
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<tbody>
<tr>
<td></td>
<td>Know the relative size and place value structure of positive and negative integers and decimals to three places.</td>
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**Level 5**

- Number strategies and knowledge AO3:
  - Understand operations on fractions, decimals, percentages, and integers.

**Strategies being developed**

- Find equivalent fractions by splitting, e.g. \( \frac{1}{3} = \frac{20}{60} \), by splitting each quarter into fifths.

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| What would the part be called, if you cut...? One third into 4 pieces \( \left( \frac{1}{3} \times 4 = \frac{12}{3} \right) \) One fifth into 3 pieces One sixth into 2 pieces One half into 6 pieces \( \frac{1}{6} = \frac{2}{12}, \frac{1}{5} = \frac{3}{15}, \frac{3}{5} = \frac{9}{25}, \frac{7}{8} = \frac{28}{60}, \frac{3}{10} = \frac{100}{300} \) | *Teaching Fractions, Decimals and Percentages (Book 7)*
Introduction (35-37) | Order decimals to three places, for examples, 6.25 and 6.3 | *Teaching Number Knowledge (Book 4)*
Number Fans (4)
Place Value Houses (5)
More Reading of Decimal Fractions (9)
Who Wins? (21) |
| Figure It Out
N3.1 *Fun with Fractions* (9)
N3.1 *More Fractions* (10)
N3.1 *Racing to New Heights* (14)
N3.3 *Fraction Frenzy* (22)
N3-4.1 *A Watery Mission* (3)
N3-4.2 (11) *Sandwich Survey*
NS&AT 3.1 *Fraction Tagging* (18)
N7/8 L2 *Boxed Biscuits* (24)
PR 3-4.1 *Paper Partitions* (6) | *Figure It Out*
N 3 *Decimal Day* (15)
N 3.2 *Jumping Along* (20)
N 7/8 L2 *Expanding With Decimals* (17)
N 7/8 4.3 *Awesome Athletes* (13)
N 7/8 4.5 *Give it a Heave!* (3)
N 7/8 4.5 *Gentle Giants* (18) |
### Transition: Advanced Additive to Advanced Multiplicative

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<tr>
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<tr>
<td>Order fractions using equivalence and benchmarks, e.g. $\frac{2}{3} &lt; \frac{7}{16}$ because $\frac{2}{3}$ is $\frac{10}{16}$ less than $\frac{1}{2}$ and $\frac{7}{16}$ is $\frac{1}{16}$ less.</td>
<td>Which fraction is bigger and by how much? $\frac{2}{3}$ or $\frac{3}{5}$ ($\frac{2}{5}$), $\frac{4}{5}$ or $\frac{3}{4}$ ($\frac{1}{4}$), $\frac{5}{8}$ or $\frac{2}{3}$ ($\frac{1}{3}$), $\frac{5}{8}$ or $\frac{7}{12}$ ($\frac{1}{12}$), $\frac{5}{6}$ or $\frac{7}{12}$ ($\frac{1}{12}$), $\frac{5}{6}$ or $\frac{1}{3}$ ($\frac{1}{3}$), $\frac{17}{12}$ or $\frac{5}{4}$ ($\frac{1}{4}$), $\frac{7}{9}$ or $\frac{11}{13}$ ($\frac{1}{13}$).</td>
<td>Teaching Number Sense and Algebraic Thinking (Book 8) Estimating with Fractions (15) Fractions (16) Figure It Out NS&amp;AT 3-4.1 Close Ties (14)</td>
<td>Order fractions, including halves, quarters, thirds, fifths, and tenths</td>
<td>Teaching Number Knowledge (Book 4) Creating Fractions (6) More Geoboard Fractions (7) Non-unit Fractions (7) Packets of Lollies (8) Reading Decimal Fractions (8) Card Ordering (12) Arrow Cards (13) Rocket - Where Will I Fit (15) Who Has More Cake? (18) Who Gets More? (20) Bead Strings (17) Figure It Out N 2-3 Circle Segment (17) N 2-3 Fabulous Folding (18) N 2-3 How Many? (20)</td>
</tr>
<tr>
<td>Find fractions of lengths, areas, volumes and other continuous quantities using reunitising, e.g. three quarters of one half is three eighths</td>
<td>$\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ = $\frac{1}{8}$ $\frac{2}{3}$ of $\frac{1}{2}$ = $\frac{2}{3}$ = $\frac{1}{3}$ $\frac{2}{3}$ of $\frac{2}{3} = \frac{4}{6} = \frac{1}{2}$ $\frac{2}{3}$ of $\frac{3}{4}$ = $\frac{3}{16}$ $\frac{1}{5}$ of $\frac{1}{3}$ = $\frac{1}{15}$</td>
<td>Figure It Out PR 3+ Puzzling Patterns (1) PR 3+ Shaping Up (2) PR 3+ What Do You See? (6) PR 3-4.1 Tri Fractions (4)</td>
<td>Record the results of mental calculations using equations and diagrams, for example, empty number line</td>
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### Transition: Advanced Additive to Advanced Multiplicative

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| Find fractions of whole number amounts using multiplication and division, e.g. \( \frac{2}{3} \) of 36 = \( \square \) (\( \frac{2}{3} \times 36 \)). | \( \frac{3}{5} \) of \( 60 = \square \), \( \frac{5}{6} \) of \( 64 = \square \), \( \frac{5}{10} \) of \( 27 = \square \), \( \frac{5}{8} \) of \( 72 = 27 \), \( \frac{2}{3} \) of \( 48 = 28 \) | *Teaching Number Sense and Algebraic Thinking (Book 8)*  
*Whole Numbers Times Fractions* (22)  
*Fractions Times Whole Numbers* (23) | Recall fraction \( \leftrightarrow \) decimal \( \leftrightarrow \) percentage conversions for halves, thirds, quarters, fifths, and tenths | *Teaching Number Knowledge (Book 4)*  
*Equivalent Fractions, Decimals and Percentages* (21)  
*Bead Strings* (17) |
| Multiply fractions by other fractions, e.g. \( \frac{2}{3} \times \frac{1}{2} = \frac{6}{12} = \frac{1}{2} \) | \( \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} \), \( \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \), \( \frac{3}{5} \times \frac{2}{3} = \frac{6}{15} = \frac{2}{5} \), \( \frac{5}{8} \times \frac{1}{2} = \frac{5}{16} \), \( \frac{2}{3} \times \frac{1}{5} = \frac{2}{15} \), \( \frac{5}{6} \times \frac{2}{3} = \frac{10}{18} = \frac{5}{9} \), | *Teaching Number Sense and Algebraic Thinking (Book 8)*  
*A Fraction Times a Fraction* (24)  
*When Big Gets Smaller* (24) | *Figure It Out*  
N3.2 *Heading for Home* (24)  
N3.3 *Marble Marvels* (21)  
N3-4.2 *Funky Fractions* (12)  
N3-4.2 *Measuring Up* (13)  
N3-4.3 *Sporting Fractions* (16)  
NS & AT 3.2 *On Top of the World* (22)  
NS7/8 4.2 *Mystery Fractions* (21)  
N7/8 L2 *Placing Points* (18)  
N7/8 4.3 *Linking Lollies* (1)  
N7/8 4.3 *Football Fractions* (4)  
PR 3+ *Star Clusters* (5)  
PR 3-4.1 *Fraction Extraction* (8) | *Figure It Out*  
N3.1 (13) *Friendly Fractions*  
N 3-4 (11) *A Long Look at Decimals*  
N 3-4.2 (15) *Fraction Distraction*  
N 3-4.3 (24) *Hidden Help*  
BF 3-4 (21) *Mystery Decimals*  
BF 3-4 (24) *Decimal Spotting*  
N 7/8 L2 (9) *Seeing Double*  
N 7/8 L2 (20) *Getting the Point* |
## Transition: Advanced Additive to Advanced Multiplicative

### Domain: Ratios and Proportions

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</table>
| Rename improper fractions as mixed numbers using division, and position improper fractions on a number line. | $\frac{27}{4} = 6 \frac{3}{4}, \frac{43}{5} = 8 \frac{3}{5}, \frac{23}{3} = 7 \frac{2}{3}, \frac{65}{9} = 7 \frac{2}{9}, \frac{76}{8} = 9 \frac{1}{2}, \frac{100}{3} = 33 \frac{1}{3}$ | *Teaching Number Sense and Algebraic Thinking (Book 8)*
Fractions Greater Than 1 (17)  
*Figure It Out*  
PR 3-4.1 Fraction Line-up (2) | Recall equivalent fractions for halves, thirds, quarters, fifths, and tenths with numbers to 100 and with 1 000 | *Teaching Number Knowledge (Book 4)*  
Super Liquorice (19)  
Little Halves and Big Quarters (19)  
Equivalent Fractions, Decimals and Percentages (21)  
The Same But Different (30)  
*Figure It Out*  
N 3.1 Fun With Fractions (9)  
N 3.1 More Fractions (10)  
N 3.1 To Market, To Market (11)  
N 3.3 Fraction Frenzy (22)  
N3-4.2 Sandwich Survey (11)  
N 7/8 L.1 Chocolate Chip Feast (22)  
N 7/8 L.2 Classy Courtyards (22)  
PR 3-4.2 The Equivalence Game (18) |
## Transition: Advanced Additive to Advanced Multiplicative

**Domain:** Ratios and Proportions

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| Solve division problems that have fraction answers, e.g. $8 \div 3 = 2 \frac{2}{3}$, and connect division with the numerator and denominator of the answer, e.g. $4 \div 5 = \frac{4}{5}$ | $5 \div 4 = \square$, $8 \div 3 = \square$, $12 \div 5 = \square$, $5 \div 6 = \square$, $3 \div 7 = \square$, $13 \div 8 = \square$, $1 \div \square = \frac{1}{7}$, $7 \div \square = 2 \frac{1}{3}$, $2 \div \square = \frac{2}{3}$, $17 \div \square = 1 \frac{7}{10}$, $\square \div 3 = 3 \frac{1}{3}$, $\square \div 5 = 1 \frac{4}{5}$, $\square \div 6 = 1 \frac{5}{6}$, $\square \div 9 = 4 \frac{2}{9}$, | *Figure It Out*
N3.1 [Friendly Fractions](13)  
N 7/8 4.5 [Revisiting Remainders](1) | Round whole numbers and decimals with up to two places to the nearest whole number or tenth | *Teaching Number Knowledge (Book 4)*  
Sensible Rounding (28)  
Swedish Rounding (28)  
*Figure It Out*
N 3-4 .3 [Paddle On](6)  
N 7/8 4.5 [Body Mass](10) |
| Convert fractions to decimals, and percentages and vice versa. | $3 \div 2 = 1 \frac{1}{2} = 1.5 = 150\%$  
$5 \div 4 = 1 \frac{1}{4} = 1.25 = 125\%$  
$3 \div 8 = \frac{3}{8} = 0.375 = 37.5\%$  
$2 \div 3 = \frac{2}{3} = 0.6 = 66.\%$  
$\frac{7}{5} = \square$, $\frac{5}{7} = \square$ | *Teaching Fractions, Decimals and Percentages (Book 7)*  
Deci-mats (41-44)  
*Figure It Out*
BF 3-4 [Mystery Decimals](21)  
BF 3-4 [Decimal Spotting](24)  
N 3-4.1 [Waves Win](8)  
N 3-4.1 [Bottle Up](10)  
N 3-4.1 [A Long Look at Decimals](11)  
NS 7/8 4.2 [Pizza Pieces](19)  
N 7/8 L2 [Seeing Double](9)  
N 7/8 L2 [Getting the Point](20)  
N 7/8 4.3 [Conversion Cousins](2)  
PR 3+ [Discount Deals](8) | |
### Transition: Advanced Additive to Advanced Multiplicative

#### Strategies being developed

- Estimate and find percentages of whole number amounts using benchmark percentages, e.g. 65% of $80 as 50% is $40, 10% is $8, 5% is $4, so $40 + $8 + $4 = $52

#### Problem progression

- 10% of 48 so 30% of 48
- 50% of 64 so 25% of 64
- 10% of 28 so 5% of 28
- 50% of 56, 10% of 56 so 60% of 56
- 10% of 35, 5% of 35 so 15% of 35
- 50% of 140, 10% of 140, 5% of 140 so 65% of 140

#### References

- *Teaching Fractions, Decimals and Percentages* (Book 7)
  - Hot Shots (47-49)
- *Figure It Out*
  - N 3.2 Better Buy Bargains (18)
  - N 3-4.1 Hot Shots (12)
  - N 3-4.2 Making Money (16)
  - NS&AT 3-4.1 Pondering Percentages (12)
  - NS 7/8.L1 Playzone Discount (16)
  - NS 7/8 4.2 People Power (15)
  - N 7/8 4.3 Involving Interest (6)
  - N 7/8 4.3 New Car Capers (14)
  - N 7/8 4.5 Bargain Bonanza (14)
  - N 7/8 4.6 Spending on Sport (10)
  - PR 3+ Getting Tough (14)

### Domain: Ratios and Proportions

#### Strategies being developed

- Add and subtract fractions with related denominators, e.g. \( \frac{1}{3} + \frac{5}{12} = \frac{14}{12} = \frac{1}{2} \)

#### Problem progression

- \( \frac{3}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2} \)
- \( \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2 \)
- \( \frac{4}{5} - \frac{3}{5} = \frac{1}{5} \)
- \( \frac{3}{4} + \frac{5}{8} = \frac{11}{8} = \frac{3}{8} \)
- \( \frac{9}{10} - \frac{5}{10} = \frac{3}{10} \)
- \( \frac{2}{3} + \frac{5}{6} = \frac{9}{6} = \frac{1}{2} \)
- \( \frac{7}{8} - \frac{1}{8} = \frac{3}{8} \)

#### References

- *Teaching Fractions, Decimals and Percentages* (Book 7)
  - Comparing Apples with Apples (38)
- *Teaching Number Sense and Algebraic Thinking* (Book 8)
  - Estimating with Fractions (15)
- *Figure It Out*
  - N 3.3 Stacking Up (20)
  - N 7/8 4.5 Egyptian Fractions (23)
  - PR 3-4.1 Galloping Greyhounds (1)
## Transition: Advanced Additive to Advanced Multiplicative

### Domain: Ratios and Proportions

#### Strategies being developed

- Add and subtract decimals.

#### Problem progression

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<th>Solution</th>
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<tr>
<td>$1.2 + 3.8$</td>
<td>$4$</td>
</tr>
<tr>
<td>$0.75 + 1.25$</td>
<td>$2$</td>
</tr>
<tr>
<td>$5 - 2.25$</td>
<td>$2.75$</td>
</tr>
<tr>
<td>$0.5 + 1.25$</td>
<td>$1.75$</td>
</tr>
<tr>
<td>$2.5 - 1.75$</td>
<td>$0.75$</td>
</tr>
<tr>
<td>$0.375 + 1.625$</td>
<td>$2$</td>
</tr>
<tr>
<td>$5.2 - 1.68$</td>
<td>$3.52$</td>
</tr>
<tr>
<td>$2.673 + 1.327$</td>
<td>$4$</td>
</tr>
<tr>
<td>$5.2 - 1.68$</td>
<td>$3.52$</td>
</tr>
</tbody>
</table>

#### References

- *Teaching Fractions, Decimals and Percentages (Book 7)*
  - Pipe Music with Decimals (38-41)
  - How Can Two Decimals so Ugly..? (45-46)

- *Figure It Out*
  - N 3.2 Target Time (16)
  - N 3.2 Dallying with Decimals (17)
  - N 3-4.3 Riding the Waves (2)
  - N 7/8 4.3 Going for Gold! (12)
  - Pr 3+ Make 1.5 (18)

#### Solve measurement problems with related fractions, by recognising equivalent fractions, e.g.

How many sixths are in one and one half?

- $(1 \frac{1}{2} \div \frac{1}{6} = \frac{9}{6} \div \frac{1}{6} = 9)$

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<tbody>
<tr>
<td>$\frac{1}{4}$'s in $\frac{1}{2}$</td>
<td>$\frac{3}{6} \div \frac{1}{6} = \frac{3}{4}$</td>
</tr>
<tr>
<td>$\frac{1}{10}$'s in $\frac{4}{5}$</td>
<td>$\frac{1}{10} \div \frac{4}{5} = \frac{1}{8}$</td>
</tr>
<tr>
<td>$\frac{1}{6}$'s in $\frac{5}{3}$</td>
<td>$\frac{1}{6} \div \frac{5}{3} = \frac{1}{10}$</td>
</tr>
<tr>
<td>$\frac{1}{7}$'s in $\frac{4}{2}$</td>
<td>$\frac{3}{7} \div \frac{4}{2} = \frac{3}{10}$</td>
</tr>
<tr>
<td>$\frac{3}{8}$'s in $\frac{5}{2}$</td>
<td>$\frac{5}{8} \div \frac{5}{2} = \frac{3}{6}$</td>
</tr>
<tr>
<td>$\frac{5}{12}$'s in $\frac{10}{3}$</td>
<td>$\frac{5}{12} \div \frac{10}{3} = \frac{5}{8}$</td>
</tr>
</tbody>
</table>

#### References

- *Teaching Number sense and Algebraic Thinking (Book 8)*
  - Dividing Fractions (21)
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| Show the order of decimal numbers by developing a number line scale | On a 0-5 scale show: 2.5, 1.25, 4.9, 3.75, 0.67 On a 0-2 scale show: 0.2, 1.8, 0.66, 1.95, 1.125 | **Teaching Number sense and Algebraic Thinking (Book 8)**  
Scales on Number Lines (19)  
Whole Number Rounding (19)  
Confusing Fractions and Decimals (20) |
| Solve simple rate problems using multiplication, e.g. Picking 7 boxes of apples in $\frac{1}{2}$ hour is equivalent to 21 boxes in $1\frac{1}{2}$ hours. | Washing cars:  
4 in 1 hour = 32 in 8 hours  
7 in 2 hours = 35 in 10 hours  
3 in $\frac{1}{2}$ hour = 18 in 3 hours  
9 in $1\frac{1}{2}$ hours = 36 in 4 hours  
5 in $\frac{3}{4}$ hour = 40 in 6 hours | **Figure it Out**  
N 3.3 Numbers on the Line (2)  
N 3-4.1 More Thinking (21)  
N 3-4.2 Paddling Down the Waikato (19)  
N 3-4.3 Challenge Time (4)  
N 3-4.3 Paddle On (6)  
N 3-4.3 Feel the Beat (11)  
NS 7/8.1 Grocery Grapplers (20)  
NS 7/8.1 Shopping Around (22)  
N 7/8.4.3 Kapa Haka Hāngi (17)  
PR 3+ Speed Read (10)  
PR 3+ Demolition Dollars (16)  
PR 3+ Painting by Numbers (17)  
PR 3+ Tiring Teamwork (21) |
| Find equivalent ratios using multiplication and division and express them as equivalent fractions, e.g. 16:8 as 8:4 as 4:2 as 2:1 and $\frac{16}{24} = \frac{8}{12} = \frac{4}{6} = \frac{2}{3}$ | 10:15 as 2:3 and $\frac{10}{15} = \frac{2}{3}$  
20:10 as 10:5 as 2:1 and $\frac{20}{10} = \frac{10}{5} = \frac{2}{1}$  
12:36 as 6:18 as 3:9 as 1:3 and $\frac{12}{36} = \frac{6}{18} = \frac{3}{9} = \frac{1}{3}$  
18:27 as 6:9 as 2:3 and $\frac{18}{27} = \frac{6}{9} = \frac{2}{3}$  
45:15 as 9:3 as 3:1 and $\frac{45}{15} = \frac{9}{3} = \frac{3}{1}$  
16:48 as 8:24 as 4:12 as 2:6 as 1:3 and $\frac{16}{48} = \frac{8}{24} = \frac{4}{12} = \frac{2}{6} = \frac{1}{3}$  
$\frac{18}{27} = \frac{6}{9} = \frac{2}{3} = \frac{1}{3}$  
$\frac{45}{15} = \frac{9}{3} = \frac{3}{1} = \frac{1}{3}$  
$\frac{16}{48} = \frac{8}{24} = \frac{4}{12} = \frac{2}{6} = \frac{1}{3}$  
$\frac{18}{27} = \frac{6}{9} = \frac{2}{3} = \frac{1}{3}$  
$\frac{45}{15} = \frac{9}{3} = \frac{3}{1} = \frac{1}{3}$  
$\frac{16}{48} = \frac{8}{24} = \frac{4}{12} = \frac{2}{6} = \frac{1}{3}$ | **Teaching Fractions, Decimals and Percentages (Book 7)**  
Mixing Colours (50-52) |
| **Figure it Out** | N 3-4.1 Stretch and Grow (4)  
N 3-4.1 Bean Brains (9)  
NS&AT 3.1 Run Like the Wind (12)  
NS&AT 3-4.2 Lunchtime Mardi Gras (18-20)  
NS 7/8.4.2 Balancing Act (22)  
N 7/8.4.5 Bargain Packs (15)  
N 7/8.4.6 Hypertufa Tiles (17)  
PR 3+ Chocolate Choices (4)  
PR 3+ Pop Star Pics (20)  
PR 3-4.1 Smart Sizes (21)  
PR 3-4.1 The Right Gear (20) |