

Number in Statistics 1

AC

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AA

AM

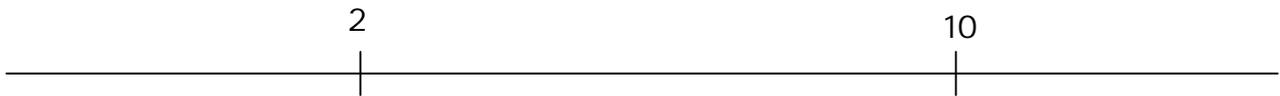
AP

We are solving problems in statistical contexts
We are finding reasons and proofs in mathematical contexts
We are exploring the properties of numbers

Exercise 1: Midway Island

This exercise is all about finding the number that is exactly midway between two other numbers.

Philbert wants to find the number midway between 2 and 10. He uses a number line to mark the two numbers.



He says "I could fold the paper to find the point in the middle."

Explain why this could work.

He says "I could use a ruler and measure the same distance to both points."

Explain how this might work.

"I could use a compass and make arcs to find the midway point" he said.

See if you can do this and explain why it works as well.

"And $10 + 2$ is 12 and half of this is 6 so this is the midway point" he said. "Another way to do this is $10 - 2$ is 8 and half of 8 is 4. The midpoint is 4 down from 10 or 4 more than 2. The answer is 6.

Explain how these methods work.

Where is Midway Island?

Exercise 2: Half-time

This exercise is all about finding the number that is exactly midway between two other numbers. You may use a number line to help you if you wish.

Find the number that is exactly halfway between these two.

Easy whole numbers

1) 2, 12 (2) 5, 15 (3) 11, 13

4) 12, 20 (5) 14, 18 (6) 17, 27

and some answers with halves

7) 18, 19 (8) 20, 25 (9) 35, 40

10) 39, 100 (11) 41, 50 (12) 50, 101

and some big numbers

13) 555, 1000 (14) 70, 1000 (15) 90, 900

16) 100, 1000 (17) 121, 232 (18) 256, 512

some questions with halves

19) $3\frac{1}{2}$, $5\frac{1}{2}$ (20) $1\frac{1}{2}$, $7\frac{1}{2}$ (21) $\frac{1}{2}$, $3\frac{1}{2}$

22) $10\frac{1}{2}$, $20\frac{1}{2}$ (23) 0, $11\frac{1}{2}$ (24) $6\frac{1}{2}$, $19\frac{1}{2}$

and some tricky numbers that might need some help. Skip them if you wish.

25) -1, 3 (26) 4, -6 (27) -201, +201

28) 3.2, 4.3 (29) 0.1 and 0.01 (30) -1, +1

Your turn. Write 5 problems for a friend like the ones above.

1.	2.	3.	4.	5.
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Write your answers here my friend.

1.	2.	3.	4.	5.
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Describe two ways you use to find midway points

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Exercise 3: This is Really Mean

This exercise is about finding the number that is in the middle of two numbers using two different number methods.

First Method

Jasmine likes adding so she adds both numbers and then just halves the answer. She says she halves the answer (or divides the answer by 2) because there are 2 numbers.

She says "For example, take the number 12 and 34. Adding these gives 46. Halve that number to get 23. This also called the mean".

Show what Jasmine has done on the empty number line.

Second Method

Freddie likes subtracting and he works out the difference between the numbers by subtracting them. He explains "If I take half of this difference and subtract it from the big number I get the middle or mean. I can do it using the little number also."

Show Freddie's working for the numbers 12 and 24 on the empty number line.

Show what Freddie means by "I can do it using the smaller number also."

Do these problems using one of the methods above.

1) 6, 14

(2) 12, 28

(3) 18, 42

4) 3, 17

(5) 6, 34

(6) 9, 51

7) 24, 48

(8) 36, 72

(9) 28, 56

10) 29, 87

(11) 37, 93

(12) 123, 429

Exercise 4: It's All about Balance

You may need a balance beam.

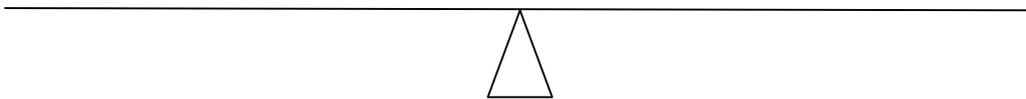
This exercise is about finding the number that is in the middle of two numbers using a balance beam.

Once upon a time, long before statistics was born people who made things needed to find the middle of beams, blocks, castle walls, sticks, stones, bones, the day, the night, the year and a few other odd things. They often needed to find the middle of a stick of wood because it is a good place from which to lift it.

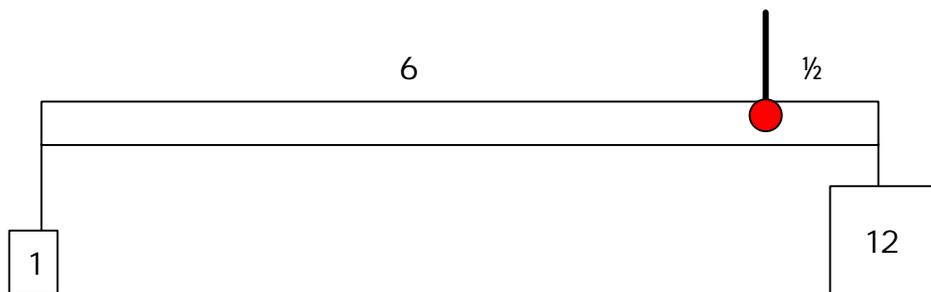
Explain why the middle of something is usually a good place to lift it.

When is it not a good place to choose?

By doing this they invented a balance beam and discovered a whole lot of things that a balance beam could be used for. Here is a simple balance beam.



Here is another one. The one above is exactly the same as a seesaw. The one below is a hanging balance beam and can be used for weighing things.



Here the long part (6) has a little block (1) and the short part of the beam ($\frac{1}{2}$) has a big block (12) and it all balances perfectly. What is happening here? Who can you be on a seesaw with and make it all balance?

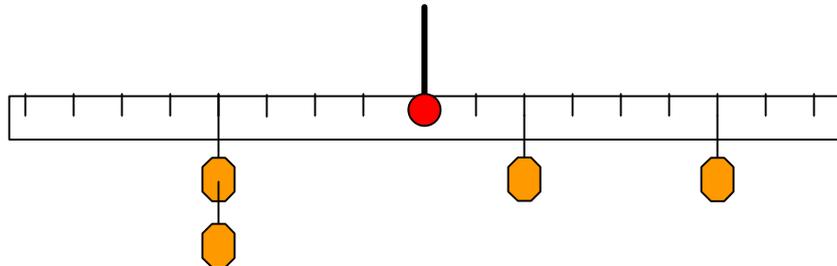
Explain why the second diagram all balances. Explain why it works.

Have a play with the balance beam and work out how it all works. You will need it for the next exercise.

Exercise 5: Well, Balance Me!

You will need a balance beam.

This exercise is about finding the place that balances two weights on a balance beam. Set up your balance like the diagram.



Here two weights on the right side of the balance beam are placed at 2 and at 6. On the left side both weights are placed at the same place 4. Make this on your balance beam to see if it all balances nicely.

We can write the one above like this

two at 4 on the left BALANCES one at 2 and one at 6

Explain why it should balance nicely.

Now for the fun part!

Move the two weights on the right side into new positions carefully so that the beam still balances. Leave the left side alone!

Write four more statements that describe what you find out.

two at 4 on the left BALANCES one at ___ and one at ___

two at 4 on the left BALANCES one at ___ and one at ___

two at 4 on the left BALANCES one at ___ and one at ___

two at 4 on the left BALANCES one at ___ and one at ___

Now you can move the left weights as well to find more balance equations.

Write four more statements that describe what you find out.

two at ___ on the left BALANCES one at ___ and one at ___

two at ___ on the left BALANCES one at ___ and one at ___

two at ___ on the left BALANCES one at ___ and one at ___

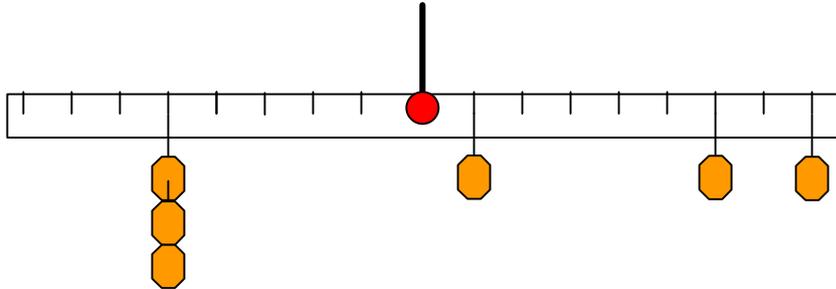
two at ___ on the left BALANCES one at ___ and one at ___

The next exercise explores more weights on the beam.

Exercise 6: Balance Madness

You will need a balance beam.

This exercise is about finding the place that balances more than two weights on a balance beam. Here is one system that will balance.



We can write this one above like this

three at 5 on the left **BALANCES** one at 1 and one at 6 and one at 8

Your turn!

Find more balance positions where the weights on the right are spread out and the weights on the left are all at the same place.

Write statements that describe what you find out.

See if you can balance the beam with two weights on the left balancing three on the right and they are all at different positions.

Write your answer here.

How many different ways can you make this arrangement work?

Be creative. Make up a really complicated balance and describe what you have done.

Bon Chance!

Exercise 7: Getting a Bit Mean!

You may need a calculator.

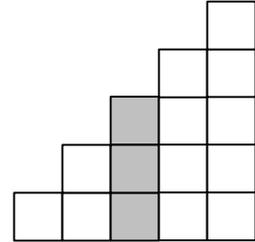
This exercise is about calculating the mean of a group of numbers that have a pattern which should help you to find the answer. You may use the calculator to check you are correct if you have doubts.

What is the mean of the numbers 1, 2, 3, 4 and 5?

We can model these numbers like this.

Now perhaps we can see the middle number.

The shape shows that 3 is in the middle of this nicely arranged pile.



Explain in your own way why 3 should be the middle or mean number for this group.

Here are some other groups of numbers all based on what you have just done. Find the mean of each set of numbers. You can draw a picture if it helps you.

1) 11, 12, 13, 14, 15 (2) 51, 52, 53, 54, 55 (3) 111, 112, 113, 114, 115

4) 33, 34, 35, 36, 37 (5) 9, 8, 7, 6, 5 (6) 20, 21, 22, 23, 24

If I started with 1, 2, 3, 4, 5 what did I do to get each set of numbers in questions 1 to 6?

What happened to the mean in each case?

Find the mean of the following:

7) 2, 4, 6, 8, 10 (8) 3, 6, 9, 12, 15 (9) -2, -4, -6, -8, -10

10) 10, 20, 30, 40, 50 (11) 4, 8, 12, 16, 20 (12) 5, 10, 15, 20, 25

If I started with 1, 2, 3, 4, 5 what did I do to get each set of numbers in questions 7 to 12?

What happened to the mean in each case?

Complete these sentences.

If I add the same number to all data the mean is

If I multiply all data by the same number the mean is

Mean eh!

Exercise 8: Noah Means Well

You do not need a calculator.

You do need to know the mean of the data group 1, 2, 3, 4, 5, 6, 7 is 4. (Explain why).

These problems are in pairs and they are related to the number 4. Think carefully and creatively and you should be able to solve them easily. You can check using the calculator tomorrow.

What are the means of:

1) 6, 12, 18, 24, 30, 36, 42 (2) 106, 112, 118, 124, 130, 136, 142

The means are _____ for problem (1) and _____ for problem (2).

Explain how each of these means are related to the number?

Relation is _____ for problem (1) and _____ for problem (2).

3) 7, 14, 21, 28, 35, 42, 49 (4) 257, 264, 271, 278, 285, 292, 299

The means are _____ for problem (3) and _____ for problem (4).

Explain how each of these means are related to the number?

Relation is _____ for problem (3) and _____ for problem (4).

5) 6, 7, 8, 9, 10, 11, 12 (6) 18, 21, 24, 27, 30, 33, 36

The means are _____ for problem (5) and _____ for problem (6).

Explain how each of these means are related to the number?

Relation is _____ for problem (5) and _____ for problem (6).

7) 13, 14, 15, 16, 17, 18, 19 (8) 26, 28, 30, 32, 34, 36, 38

The means are _____ for problem (7) and _____ for problem (8).

Explain how each of these means are related to the number?

Relation is _____ for problem (7) and _____ for problem (8).

So what is the mean of the n multiples of 7?

JOKE :) How many of each kind of animal did Moses take onto the Ark?

Exercise 9: Generally Mean Problems

This exercise is about calculating the mean of a group of numbers that are related. You do not need a calculator.

- 1). What is the mean of:
 - a) the first 21 counting numbers?
 - b) the next 21 counting numbers after those?
 - c) the first 21 odd numbers?
 - d) the first 21 even numbers?
 - e) the first 21 multiples of 5?

- 2) Is the mean of two numbers even or odd if the two numbers are:
 - a) both odd?
 - b) both even?Is the mean of a set of numbers even or odd if there are:
 - c) an odd number of odd numbers?
 - d) an even number of odd numbers?Explain why your answers are correct for every problem in questions (1) and (2).

- 3) Tarsha gets 60%, 70%, and 80% in her first three tests. There is one more test and the mean of all 4 tests is her score for the year. What is:
 - a) the most she can possibly score for the year?
 - b) the least she can possibly score for the year?
 - c) her score if she gets 75% in test number 4?

- 4) Belisha has scored a mean of 75% in the first three tests of four.
 - a) What will be his score for the year if he misses the last test?
 - b) What does he need to score in the last test to keep his mean at 75%?
 - c) What does he need to score in the last test to average over 80%?

- 5) The class has 27 students and so far the class average for the all three tests that everyone sat is 60%. In the last test only 20 students sit but overall the class kept the 60% average. Students who do not sit score zero and are included in the average.
 - a) What was the class average for the last test?
 - b) What is the maximum class average they could possibly score?
 - c) Is the teacher mean?

Joke:) There are two types of mathematics teachers; those who can count; those who can not count and the mean ones.

Number in Statistics 1

Answers

Exercise 1: Midway Island

Explanation could include folding or reflecting preserves lengths. Ruler is a measured way of solving the same problem as folding. Compass arcs of the same length make two points that are on the line joining all such points, one of which is the midpoint. Midpoint, mean, halfway are good language points and have links to coordinate geometry studies in senior years. The number methods may need modelling.

Exercise 2: Half-time

Any method that works is a good method. Students should be asked to use different methods to encourage diversity of thought. The last group are starters for much discussion.

- | | | |
|----------------------|----------------------|----------------------|
| 1) 7 | (2) 10 | (3) 12 |
| 4) 16 | (5) 16 | (6) 22 |
| 7) $18\frac{1}{2}$ | (8) $22\frac{1}{2}$ | (9) $37\frac{1}{2}$ |
| 10) $69\frac{1}{2}$ | (11) $45\frac{1}{2}$ | (12) $75\frac{1}{2}$ |
| 13) $777\frac{1}{2}$ | (14) 535 | (15) 495 |
| 16) 550 | (17) 176.5 | (18) 384 |
| 19) $4\frac{1}{2}$ | (20) $3\frac{1}{2}$ | (21) $1\frac{1}{2}$ |
| 22) $15\frac{1}{2}$ | (23) $5\frac{3}{4}$ | (24) 13 |
| 25) 1 | (26) -1 | (27) 0 |
| 28) 3.75 | (29) 0.055 | (30) 0 |

Exercise 3: This is Really Mean

Various answers. Draw these on number lines to illustrate the thinking.

- | | | |
|--------|---------|----------|
| 1) 10 | (2) 20 | (3) 30 |
| 4) 10 | (5) 20 | (6) 30 |
| 7) 36 | (8) 54 | (9) 42 |
| 10) 58 | (11) 65 | (12) 276 |

Exercise 4: It's All About Balance

The middle is a good place to lift because it is the centre of gravity and only one lifting force is needed to balance the weight. It is not a good place to choose when it is a tiger, or a ring, or a bucket of water for example.

The second beam balances because the turning effect of the 1×6 is the same as the turning effect of the $12 \times \frac{1}{2}$. The clockwise and counter clockwise turning effects balance out.

Exercise 5: Well, Balance Me!

This beam balances because $2 \times 4 = 1 \times 2 + 1 \times 6$.

Other problems: various.

Exercise 6: Balance Madness

Various answers all verified by using the balance.

Exercise 7: Getting a Bit Mean

Various explanations for why.

- | | | |
|---------------------|---------------------|---------------------|
| 1) 13, added 5 | (2) 53, added 50 | (3) 113, added 100 |
| 4) 35, added 30 | (5) 7, added 4 | (6) 22, added 19 |
| 7) 6, $\times 2$ | (8) 9, $\times 3$ | (9) -6, $\times -2$ |
| 10) 30, $\times 10$ | (11) 12, $\times 4$ | (12) 15, $\times 5$ |

If I add the same number to all data the mean is increased by the same amount.

If I multiply all data by the same number the mean is multiplied by the same number.

Exercise 8: Noah Means Well

The mean of the data is 4.

- | | | |
|------------------------------|----------------------------------|-----------------------------|
| 1) 24, $\times 6$ | (2) 124, $\times 6$ and + 100 | (3) 28, $\times 7$ |
| 4) 278, $\times 7$ and + 250 | (5) 9, add 4 | (6) 27, + 4 then $\times 3$ |
| 7) 16, + 12 | (8) 32, + 12 and then $\times 3$ | |

mean of the n multiples of 7 is $7n/2$ or half way!

(Joke, NONE!, It was Noah not Moses!)

Exercise 9: Generally Mean Problems

- | | | |
|------------------------------------|-------------------------------|--|
| 1(a) 11 th number is 11 | 1(b) 32 or 11 plus 21 | 1(c) 11 th odd number is 21 |
| 1(d) 11th even number is 22 | 1(e) 11th multiple of 5 is 55 | |

- 2(a) even, both means 2! 2(b) even, both means 2! (2c) even plus a half
2(d) even, both the last cases include the number property of even or multiple of 2 which of course are divided out when averaged.

- | | | |
|-------------|------------|--------------------|
| 3(a) 77.5% | 3(b) 52.5% | 3(c) 71.25% |
| 4(a) 56.25% | 4(b) 75% | 4(c) more than 95% |
| 5(a) 81% | 5(b) 70% | 5(c) Yes! |