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# Powers of Investigations

We are investigating the powers of 2 and other numbers.  
We are solving problems involving the powers of 2 and other numbers.

## Exercise 1 – The 64<sup>th</sup> Chess Square

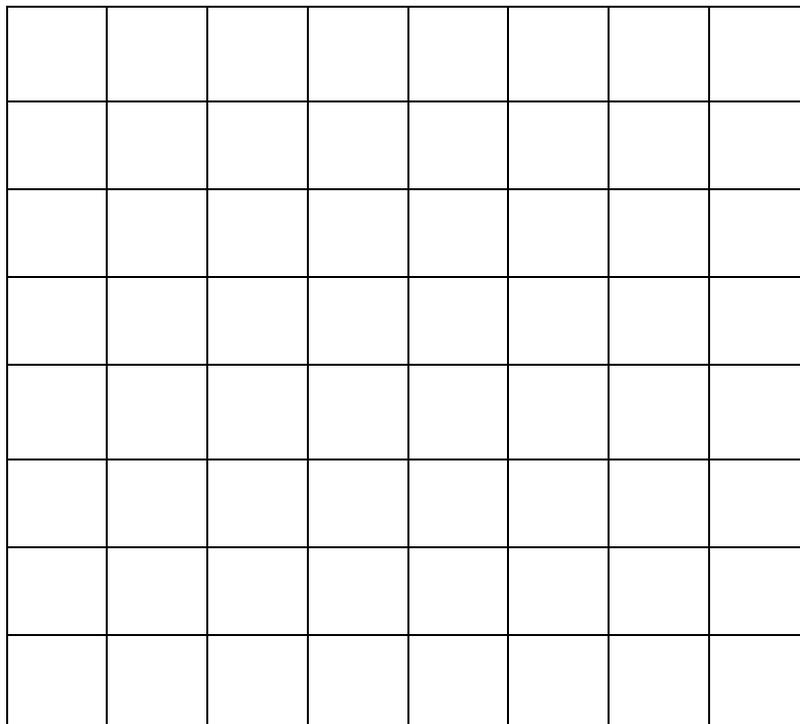
You will need a calculator.

Once upon a time a very rich Arabian Prince was in serious trouble. A man on a donkey helped him and saved his life. The Prince was so grateful he said to the man, "You can have anything in the world as a reward for saving my life. What do you choose?"

The man replied,

"I want you to place a grain of wheat on the first square of a chess board. In the next square you are to place double this or 2 grains of wheat. On the third square you are to place double this ( 4 grains) and so on until you have reached the last square. I would like all the wheat you place on the last square in return for saving your life if it so pleases your Highness."

Use the board below, or any other method, to estimate the amount of wheat the rich Prince should give to the man.



Is this possible? Why or why not?

## Exercise 2 – Powerful Leap Frog!

You will need ten red counters and ten green counters and a piece of blank paper to draw the square lilly pads as in the diagram.

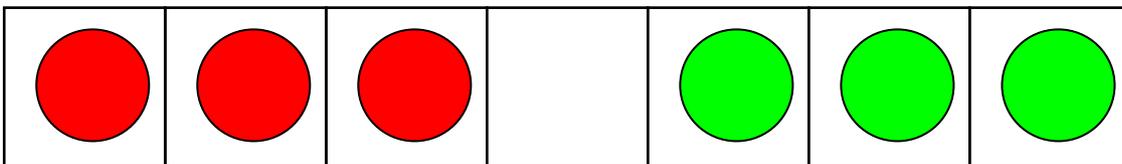
### Task

Frogs can walk from one pad to the next and they can jump over another frog two squares. The red frogs walk and jump to the right only and the green frogs walk and jump to the left only. All the red frogs move in such a way as to get to the lilly pads where the green frogs are and the green frogs move to get to the other lilly pads occupied by the red frogs.

Get someone who knows the problem to show you how to move if you are unsure about how to move and record..

Complete the chart and try to work out the patterns that emerge. Make predictions about what the numbers are and move the frogs carefully recording the "walks" and the jumps" to see if you are correct.

Frogs on one side	Total frogs	Number of Walks	Number of Jumps	Total Walks and Jumps
1	2	2	1	3
2	4			8
3				



Write the rule that relates the numbers of walks and the number of frogs.

Write the rule that relates the numbers of jumps and the number of frogs.

Write the rule that relates the total of walks and jumps and the number of frogs.

### Exercise 3 – Regions in a Circle • All is not what it seems!

You will need a sheet of paper and a pencil.

Sketch a circle and mark two points as in the diagram below. Construct the connecting line and count the regions that are so formed. This easy part of the table has been completed for you.

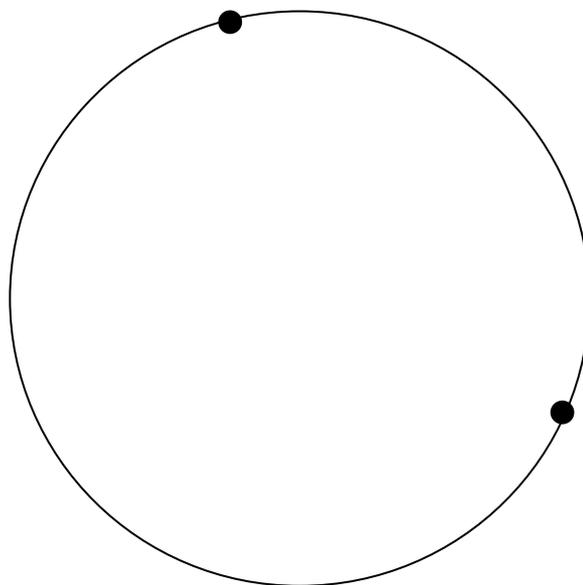
Now add another point, somewhere, and connect it with straight lines to all other points. Complete the table and start to look for patterns.

Repeat for 4, 5 points. Complete the table as you proceed looking for patterns and making predictions. Use a graph for a visual view of the data if you wish.

Test your rule with 6, 7 and even 8 points.

The aim is to generate a rule that predicts the number of regions.

Number of Points	Number of Regions
2	2
3	
4	
5	
6	
7	



Thinking Space.

My Rule

## Exercise 4 – Paper Folding

You will need a sheet of paper and a pencil.

Mark an X on one side of the piece of paper you wish to fold. This is the top side.

Fold the paper in half and open it up. The fold forms a valley when you open it up.

This has been recorded for you as 1 fold, 1 valley.

Fold the paper back in half and then fold again. This is called fold number 2. Open it up to find 2 valleys and 1 hill or ridge. This has been recorded for you as well.

Refold and then do fold number 3 and open up and count all the valleys and hills.

Record your information and start looking for patterns in the data.

Repeat folding for 4, 5, 6 and more if you can.

Try and write rules for the number of valleys, hills and totals of everything.

Folds	Valleys	Hills	Total Valleys and Hills
2	1	0	1
3	2	1	3
4			
5			
6			

**Write your rules her**

My rule for the number of valleys is

My rule for the number of hills is

My rule for the total number of hills and valleys is

Explain a pattern you see in the data you have collected.

Did you expect this result?



## Exercise 6 – Martian Maths • $1_2 + 1_2 = 10_2$

A very very old mathematician's joke reads "There are 3 types of mathematicians; those that can count and those that can't." After you have done this activity you will understand a similar joke that says "There are  $11_2$  types of mathematicians; those that can count and those that can't."

Counting can be done in a very large number of ways. The simplest is based on the number two. Martians count this way. All of the Martians I have met had one finger on each hand and one finger on each foot. They are very good at pointing.

A Martian only uses 0's and 1's when counting. The following table shows the start of counting using Martian numbering. See if you can finish the pattern. Look carefully.

Human	Martian		Human	Martian
Zero	00000		14	
1	00001		15	
2	00010		16	
3	00011		17	10001
4	00100		18	
5	00101		19	
6	00110		20	
7			21	
8	01000		22	
9			23	
10			24	
11			25	
12	01100			
13				11011

Explain the Martian counting system in your words.

My words!

How many can you count to on one hand Earthling using the Martian System.

How many can you count to on two hands Earthling?

What is the largest number you can count to using your body bits?

## Exercise 7 – The Magic Powers of 3.

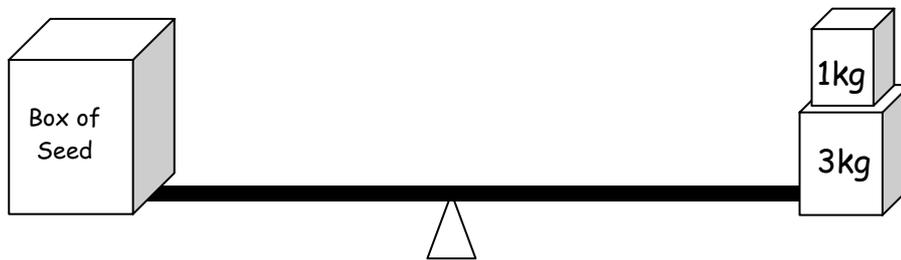
You will need a calculator with the  $x^y$  or an  $y^x$  button.

$3^0$	$3^1$	$3^2$	$3^3$			$3^6$	
1	3	9		81			

Complete the table above to find the first seven powers of 3.

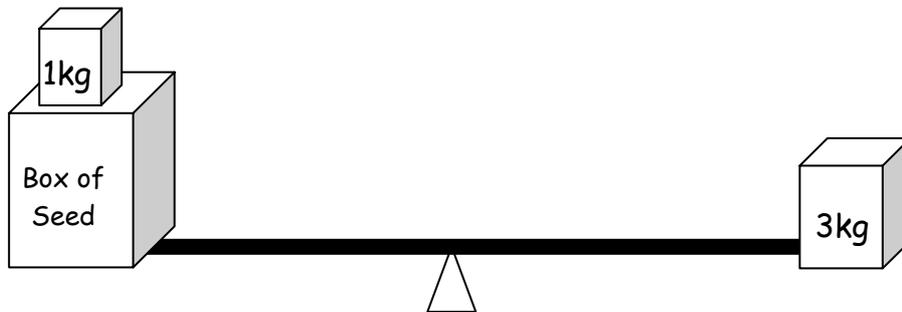
Holly the Wholesaler uses a 1kg, 3kg, 9kg, 27kg and a huge 81kg mass to measure out boxes of seed she sells. She uses a balance scale and puts a 1kg and a 3kg mass on one side and adds seed to the box on the other side until it balances.

The diagram shows this. This is like **adding** 1 and 3.



Here Holly has measured a 4kg box of seed using a 1kg and a 3kg mass.

Holly uses the same masses to measure a 2kg box of seed by putting the 1kg mass on the same side as the box of seed. This is like **subtracting** 1 from 3.



This diagram shows how this is done.

Show how to weigh these measures using the 1kg, 3kg, 9kg, 27kg and/or 81kg masses.

Weight	Description
12kg	
13kg	
14kg	
25kg	
47kg	
74kg	
How could you measure 190kg of seed using these masses? How could you measure a half a kilogram of seed?	

## Exercise 8 – It All Adds Up!

You need your thinking hat here, a buddy and maybe some multilink blocks.

A **sequence** is a list of numbers with a **secret** formula linking each term.

Mathematicians become very good at figuring out these secrets. Here is a chance for you to do some figuring and learn how to discover some secrets of your own.

**Secret Pattern #1...Difficulty Rating is like playing Aussie at Netball.**

Add the counting numbers to make the following series called in the table **SUM1**.

Counting Numbers	1	2	3	4	5	6						n
<b>SUM1</b>	1	3	6	10								?

Look carefully and talk about the pattern. Predict the number in each gap and fill in the gaps when you agree that this is correct.

Now for the tricky n (or any) number. What is the sum of the first 10 counting numbers? Of 20 numbers? Of 50 numbers? Of 999 numbers? Of any number n?

**Secret formula for adding the n Counting Numbers is**

What the secret for adding the first n **even** numbers? First n **odd** numbers?  
First n multiples of 3? Multiples of 7? Multiples of y?

**Secret Pattern #2...Difficulty Rating is like playing Aussie at League.**

As we add the powers of 2 we can make the following series called here **SUM2**.

Powers of 2	$2^0$	$2^1$	$2^2$	$2^3$	$2^4$	$2^5$	$2^6$	$2^7$				$2^n$
	1	2	4	8	16	32						
<b>SUM2</b>	1	3	7	15								?

Look carefully and talk about the pattern. Predict the number in each gap and fill in the gaps when you agree that this is correct.

Now for the tricky n (or any) number. What is the sum of the first 10 powers of 2?  
Of 20 powers? Of 100 powers? Of 999 powers? n powers of 2?

**Secret formula for adding the n Powers of 2 is**

What is the secret for the powers of 3, the powers of 4, the powers of 5 or the powers of just any number y?

## Exercise 9 – Adding Fractions Step by Step

You just need a ruler, a wall and a buddy or two.

We are going to add this rather long sum of smaller and smaller numbers.

$$\text{SUM} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \dots \text{ all the way to infinity}$$

### Firstly

You must guess what you think is the answer. Have a careful think about this. It might help to draw the problem. You can use a calculator but it will take a long time. You could graph the problem. You could make up a table. You can guess or invent another way that might give you the exact answer.

My/Our answer is

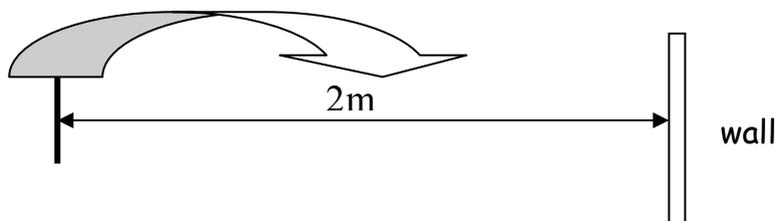
SUM =

Guess or REASON is

Here is a practical way that might help you get the answer.

### Instructzione

a. Find a wall and measure a line exactly 2 metres long out from the wall.



b. Stand at the point 2m away from the wall and take a big 1m step. What you have just done is halve the distance 2m and you are now 1m from the wall.

c. Now do a smaller step and halve the distance between you and wall. This is  $\frac{1}{2}$ m. So far you have done  $1 + \frac{1}{2}$

d. Now do another step halving the distance once again. What is the sum you have done so far?

e. Repeat stepping towards the wall by halving the distance every time you step a step.

**Where are you going to end up? Will you ever get there?**

**Now can you add that long SUM?**

**Create your own maths problem that could be solved in a similar way.**

## Exercise 10 – Alias Smith and Number

You will need a calculator with the  $x^y$  or an  $y^x$ . button.

This exercise could also be named:-

"A number by any other name would be just as sweet".

Use your calculator and the "power" or  $x^y$  button to calculate these problems.

1)  $2^4 =$

2)  $4^2 =$

3)  $8^{1.3333334} =$

4)  $16^1 =$

5)  $64^{0.66666666} =$

6)  $256^{0.5} =$

7)  $65536^{0.25} =$

8)  $1.23^{13.3933} =$

9)  $0.25^{-2} =$

What did you find out?

There is a very very large number of ways that 16 can be written.

### Believe it or not!

Every one of the infinite large number of numbers you might think of can be written in an infinitely large number of ways.

Here is your chance to discover some of those different ways for the number 2. Use your guessing intelligence and the  $x^y$  button to find these tricky NUMBERS.

1)  $\text{number}^1 = 2$

2)  $4^{\text{number}} = 2$

3)  $8^{\text{number}} = 2$

4)  $16^{\text{number}} = 2$

5)  $0.5^{\text{number}} = 2$

6)  $\text{number}^{0.5} = 2$

7)  $65536^{\text{number}} = 2$

8)  $\text{number}^{13} = 2$

9)  $\text{number}^{-2} = 2$

**NOW....choose any number you like and write it out in 9 different ways.**

My number is

The 9 ways are....

# Powers of Investigations

## Answers

### Exercise 1: The 64<sup>th</sup> Chess square

$2^{64} = 1.845 \times 10^{19}$  grains of wheat would be needed for the last chess square. Weigh 10 grams of wheat and count the grains to help estimate the total tonnage and use the internet to find out the wheat harvest of the world. All good research practice.

### Exercise 2: Powerful Leap Frog

The number of walks is the even numbers.  $n$  frogs,  $2n$  walks or in “kid speak” “just double the number of frogs”. This is good algebraic thinking. The number of jumps is the square numbers giving for  $n$  frogs we need  $n^2$  jumps or similar. The Total then is just  $(n+1)^2 - 1$  seen from looking at the patterns in the table. This is not exactly easy but is a great group problem.

### Exercise 3: Regions in a Circle • All is not what it seems

Rule for  $n < 6$  is Number of Regions =  $2^{(n-1)}$

The rule for  $n > 6$  is unknown.

### Exercise 4: Paper Folding

The rule for the number of valleys is  $2^F$

The rule for the number of hills is  $2^F - 1$

The rule for the total number of valleys and hills is  $2^{F+1} - 1$ .

### Exercise 5: The Powers Charts

Chart produced.

### Exercise 6: Martian Maths • $1 + 1 = 10$

Sensible explanation that can generate the correct sequence.

What is the largest number you can count to using base 2 with one hand. Answer is a 16, an 8, a 4, a 2 and a 1. One more is  $2^6$  so answer is  $2^6 - 1 = 63$ . With 2 hands the biggest number is  $2^{11} - 1 = 2047$ .

With all 10 fingers and toes the largest number is 2,097,151. This gets even bigger if you include using the head, an arm or two and the legs, a pair of ears, a nose and a tongue!

### Exercise 7: The Magic Powers of 3

$12L = R9 + 3$ ,  $13L = R9 + 3 + 1$ ,  $14 + 1 + 3 + 9L = R27$ ,  $25 + 3L = R 27 + 1$ ,

$47 + 1 + 9 + 27L = R81 + 3$ ,  $74 + 1 + 9L = R81 + 3$

190 can be weighed in parts. Eg A sensible solution might be  $6 \times 30\text{kg}$  bags and a 10kg bag depending on what you can lift. Justify your solution!

One half is a tricky idea. One way is to measure with a 1kg and divide the pile of wheat being weighed into two equal parts. Another is to share the 1kg of wheat just weighed onto each side of the scale until it balances. The learning is the thinking and the communication of ideas.

### Exercise 8: It All Adds Up

Triangular numbers are fun to explore and will reveal the secret formula for adding  $n$  counting numbers. This is  $n(n + 1)/2$  or the middle number  $\times$  the number of numbers or  $T_n + T_{n+1}$  divided by 2 where  $T_n$  is the  $n$ th triangular number.

The even numbers is double this formula, the odd numbers is the even numbers subtract n lots of 1 which reduces to  $n^2$  of course. The multiples of 3 is 3 times  $n(n + 1)/2$  and likewise 7 and y.

The secret formula for adding n powers of 2 is the  $(n + 1)$ th power of 2 minus 1 or  $2^{(n+1)} - 1$  which is easily seen from the table. The powers of 3, 4 and beyond are much more difficult but is for powers of y  $1/(y-1)(y^{(n+1)}-1)$  and it is a useful exercise to show this is true.

For the powers of 3 the formula is  $1/2(3^{n+1} - 1)$ , the denominator always being one less.

## **Exercise 9: Adding Fractions step by Step**

Answers will vary.

## **Exercise 10: Alias Smith and Number**

Answers to first group of questions 1 to 9 is 2

Answers to second group of questions 1 to 9 are as follows

- 1) 2                      (2) 0.5                      (3) 0.3333334                      (4) 0.25  
5) -1                      (6) 4                      (7) 0.0625

8) Somewhere between 1.054 and 1.055, 1.05476607648 to be more precise

9) .707106781187 or somewhere near.

The students will be solving these by trial and error!