

Key rates

Purpose:

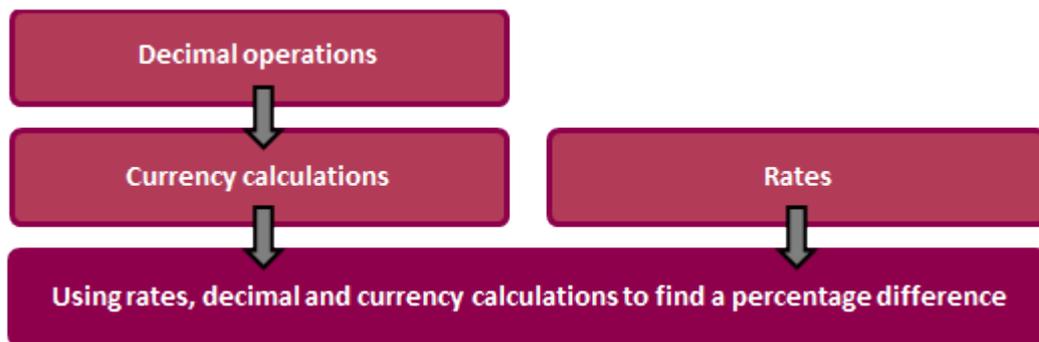
The purpose of this multi-level task is to engage students in an investigation that requires them to use rates, decimal and currency calculation to find a percentage difference.

Achievement Objectives:

NA5-4: Use rates and ratios.

Description of mathematics:

The background knowledge presumed for this task is outlined in the diagram below:



The task can be presented with graded expectations to provide appropriate challenge for individual learning needs.

Activity:

Task: In March of 2014, NZ and China brokered a deal to allow direct exchange between their two currencies (NZ\$ and CNY, the Chinese Yuan). This removed the need to use in intermediate step of exchanging into US\$.

Use the exchange rates below to investigate possible savings if a sum of NZ\$ is changed into CNY and then back into NZ\$ with and without this direct exchange deal. Find the percentage of the sum involved that is 'lost' within the transactions.

NZ\$ → US\$ 0.846

US\$ → NZ\$ 1.168

US\$ → CNY 6.213

NZ\$ → CNY 5.314

CNY → US\$ 0.160

CNY → NZ\$ 0.188



The arithmetic approach

The student is able to use currency exchange multipliers to convert between currencies and to find the percentage cost of the exchange, with guidance.

Prompts from the teacher could be:

1. Note that if I have NZ\$100 and I change it into US\$, I would expect to get US\$84.60. Use this information to work out how the exchange multipliers work
2. Try modelling a transfer of NZ\$100 000 into CNY and back to NZ\$ (once with US\$ in between and once without).
3. Find how much of the NZ\$100 000 was lost in the transactions and express this lost value a percentage of the original amount.

NZ\$ 100 000 with us\$ in between

$\times 0.846 \rightarrow$ US\$ 84 600
 $\times 6.213 \rightarrow$ CNY 525 619.80
 $\times 0.160 \rightarrow$ US\$ 84 099.17
 $\times 1.168 \rightarrow$ NZ\$ 98 227.83

lost NZ\$ 100 000 - 98 227.83 = 1772.17
as a % $\frac{1772.17}{100\ 000} \times \frac{100}{1} = 1.77\%$

NZ\$ 100 000 directly

$\times 5.314 \rightarrow$ CNY 531 400
 $\times 0.188 \rightarrow$ NZ\$ 99 903.20

lost NZ\$ 100 000 - 99 903.20 = 96.80
as a % $\frac{96.80}{100\ 000} \times \frac{100}{1} = 0.097\%$

Concrete values used to illustrate process to follow

Found loss as a percentage of original value

The procedural algebraic approach

The student is able to use currency exchange multipliers to convert between currencies and to find the percentage cost of the exchange.

Prompts from the teacher could be to note that if I have NZ\$100 and I change it into US\$, I would expect to get US\$84.60.

The image shows a handwritten algebraic approach on grid paper, divided into two columns. Each column starts with 'NZ \$X' and shows a sequence of currency conversions with arrows indicating the direction. The left column converts NZ\$ to US\$, then US\$ to CNY, then CNY to US\$, and finally US\$ to NZ\$. The right column converts NZ\$ to CNY, then CNY to NZ\$. Below each column, the student calculates the loss as a decimal and then converts it to a percentage. Two callout boxes provide additional context: one points to the final NZ\$ value in the right column, stating 'Found loss as a proportion of original value', and another points to the percentage calculation in the right column, stating 'Conversion of decimal to percentage'.

Step	Left Column (NZ\$ to NZ\$)	Right Column (NZ\$ to NZ\$)
Start	NZ \$X	NZ \$X
1	US 0.846X	
2	CNY 5.256X	
3	US 0.841X	
4	NZ 0.982X	
5		CNY 5.314X
6		NZ 0.999X

Calculation	Left Column	Right Column
Loss (Decimal)	$X - 0.982X = 0.018X$	$X - 0.999X = 0.001X$
Loss (%)	$\% \text{ Loss is } 0.018 \times 100 = 1.8\%$	$\% \text{ Loss is } 0.001 \times 100 = 0.1\%$

Found loss as a proportion of original value

Conversion of decimal to percentage

The conceptual algebraic approach

The student is able to use currency exchange multipliers to convert between currencies and to find the percentage cost of the exchange.

Prompts from the teacher should encourage the student to look at the exchange multiplier as a rate. The 0.846 for NZ\$ into US\$ means there are 0.846 US\$ per NZ\$.

T: Tell me what you mean by these diagonal slashes.

S: The one over "per" is because "per" means "out of", so I was making a dividing line. The other ones are cancelling lines, because US dollars over US dollars cancels out.

Exchange Rate: eg 0.846 US\$ ~~per~~ NZ\$

NZ to CNY and back, with US\$

$$\frac{\cancel{\text{US}}}{\cancel{\text{NZ}}} \cdot \frac{\cancel{\text{CNY}}}{\cancel{\text{US}}} \cdot \frac{\cancel{\text{US}}}{\cancel{\text{CNY}}} \cdot \frac{\cancel{\text{NZ}}}{\cancel{\text{US}}}$$

Treating rate as a fraction

Makes a new overall rate out of substages

This is 4 transactions, similar to one of:
 $0.846 \times 6.213 \times 0.16 \times 1.168 = 0.9823$
 So a loss of $100 - 98.23 = 1.77\%$ 98.23% of original

NZ to CNY and back without US\$

$$\frac{\cancel{\text{CNY}}}{\cancel{\text{NZ}}} \cdot \frac{\cancel{\text{NZ}}}{\cancel{\text{CNY}}}$$

This time, 2 transactions, similar to one of:
 $5.314 \times 0.188 = 0.999$ 99.9% of original

So a loss of $100 - 99.9 = 0.1\%$

Clear understanding of conversions between decimals and percentages