

## Inside Irregular Polygons

### Purpose:

The purpose of this multi-level task is to engage students in using a given rule, to deduce another rule. This is an example of the deductive reasoning required for forming successful geometric proofs.

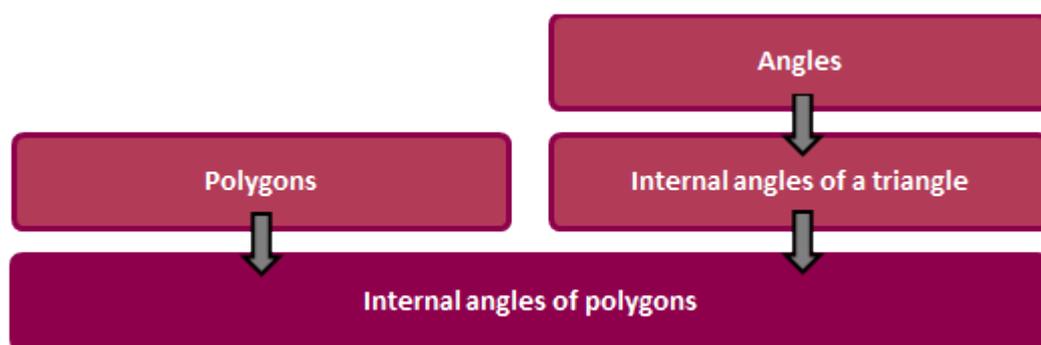
### Achievement Objectives:

GM5-5: Deduce the angle properties of intersecting and parallel lines and the angle properties of polygons and apply these properties.

AO elaboration and other teaching resources

### Description of mathematics:

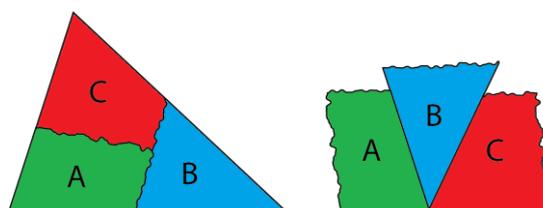
The background knowledge and skills that need to be established before and/or during this task are outlined in the diagram below:



This task may be carried out with practical exploration, and/or by generalising with the rule that has been established from the internal angles of a triangle. The approach should be chosen in sympathy with students' skills and depth of understanding.

### Activity:

Task: To show that the sum of the internal angles of any triangle is  $180^\circ$ , a triangle can be torn into three parts so that the three internal angles can then be lined up. Use this practical idea, to find a relationship, or rule, between the number of sides of any polygon and the sum of its internal angles.



## The arithmetic approach

The student is able to find a pattern that leads to a rule, using practical exploration.

Prompts from the teacher could be:

1. Start by drawing a series of irregular polygons.
2. Label each of the internal angles differently and photocopy this page.
3. Each of the photocopied polygons can be cut out. For each shape, cut out the internal angles of the polygon.
4. For each polygon, arrange and paste the internal angles to show the sum of these angles. (note: If they make a straight line, the sum is  $180^\circ$  and if they make a full rotation, the sum is  $360^\circ$ .)
5. Find a rule that links the number of sides of an irregular polygon to the sum of its internal angles.

Irregular Polygons (colour coded angles)      Pasted together angles (from photocopy)

3 sides,  $180^\circ$

4 sides,  $360^\circ$

5 sides,  $540^\circ$

6 sides,  $720^\circ$

7 sides  $2 \times 360 + 180 = 900^\circ$

8 sides,  $3 \times 360 = 1080^\circ$

(hard to see 3 rotations)

Rule: Start with 3 sides have  $180^\circ$ . Every extra side is another  $180^\circ$

T: Tell me about why you have laid out the angles in this way.

S: I put the brown and dark green ones on top and could see that they went round another half turn after a whole one but they covered up the orange and yellow. So I thought if I slide it down and go round another point I can add those turns together to show  $540^\circ$ .

T: And why did you paste the hexagon angles this way?

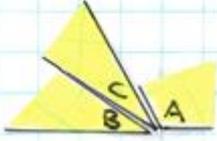
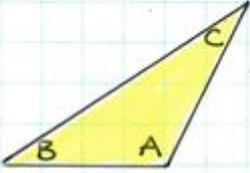
S: I would have done another slide like before, but these neatly made two lots of  $360^\circ$ .

## The procedural algebraic approach

The student is able to use a given rule, to find a new rule, using practical exploration and then generalising, with guidance.

1. Prompts from the teacher could be:
2. By breaking a triangle up into three parts, each having a complete vertex, arrange the parts to show that the sum of the internal angles of a triangle is  $180^\circ$ .
3. Draw a series of irregular polygons.
4. Place lines inside each of these polygons to show how they may be made up of just triangles.
5. Use the idea that the sum of the internal angles of a triangle is  $180^\circ$ , to find the sum of the internal angles of each of the irregular polygons.
6. Find a rule that links the number of sides of an irregular polygon to the sum of its internal angles.

Triangle: Internal Angles add up to  $180^\circ$



number of sides		number of triangles	total of the internal angles
3		1	$1 \times 180^\circ$
4		2	$2 \times 180^\circ$
5		3	$3 \times 180^\circ$
6		4	$4 \times 180^\circ$

So if the angles inside each triangle =  $180^\circ$   
The angles inside each polygon add up to the number of triangles that make up the polygon  $\times 180^\circ$

## The conceptual algebraic approach

The student is able to use a given rule to find a new rule, using practical exploration and then generalising, independently.

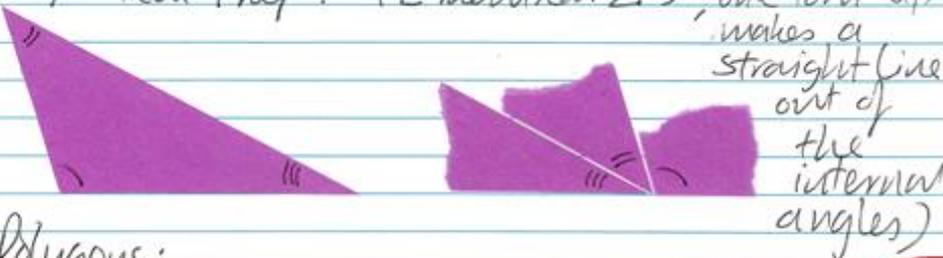
Prompts from the teacher could be:

1. By breaking a triangle up into three parts, each having a complete vertex, arrange the parts to show that the sum of the internal angles of a triangle is  $180^\circ$ .
2. Draw a series of irregular polygons.
3. Use the idea that the sum of the internal angles of a triangle is  $180^\circ$ , to find the sum of the internal angles of each of the irregular polygons.
4. Find a rule that links the number of sides of an irregular polygon to the sum of its internal angles.

*Internal*  
Sum of  $^n$  Angles in Polygons:

Triangle: Internal Angles sum to  $180^\circ$

Practical Proof: (2 identical  $\triangle$ 's, one turn up, makes a straight line out of the internal angles)



Polygons:

Sides	Diagram	Sum of internal angles
3		$180^\circ$ (see above)
4		$\triangle + \triangle$ $180^\circ + 180^\circ = 360^\circ$
5		$360^\circ + 180^\circ = 540^\circ$
6		$540^\circ + 180^\circ = 720^\circ$

This pattern would keep going;  
Start with triangle has sum of int  $\angle$ 's  $180^\circ$   
every extra side adds on another  $180^\circ$ .

T: Tell me about what you were thinking here.

S: Well, each extra side, like to go from a 5 to a 6 sided shape is just one more triangle. You can draw a 5 sided shape and stick a triangle on.

T: So this observation helped you find the rule?

S: Yeah, coz each extra side means the angle sum is 180 more.