Inside Irregular Polygons

Purpose:
The purpose of this multi-level task is to engage students in using a given rule, to deduce another rule. This is an example of the deductive reasoning required for forming successful geometric proofs.

Achievement Objectives:
GMS-5: Deduce the angle properties of intersecting and parallel lines and the angle properties of polygons and apply these properties.
AO elaboration and other teaching resources

Description of mathematics:
The background knowledge and skills that need to be established before and/or during this task are outlined in the diagram below:

This task may be carried out with practical exploration, and/or by generalising with the rule that has been established from the internal angles of a triangle. The approach should be chosen in sympathy with students' skills and depth of understanding.

Activity:
Task: To show that the sum of the internal angles of any triangle is 180°, a triangle can be torn into three parts so that the three internal angles can then be lined up. Use this practical idea, to find a relationship, or rule, between the number of sides of any polygon and the sum of its internal angles.
The arithmetic approach
The student is able to find a pattern that leads to a rule, using practical exploration.

Prompts from the teacher could be:

2. Label each of the internal angles differently and photocopy this page.
3. Each of the photocopied polygons can be cut out. For each shape, cut out the internal angles of the polygon.
4. For each polygon, arrange and paste the internal angles to show the sum of these angles. (note: If they make a straight line, the sum is 180° and if they make a full rotation, the sum is 360°.)
5. Find a rule that links the number of sides of an irregular polygon to the sum of its internal angles.
The procedural algebraic approach
The student is able to use a given rule, to find a new rule, using practical exploration and then generalising, with guidance.

1. Prompts from the teacher could be:
2. By breaking a triangle up into three parts, each having a complete vertex, arrange the parts to show that the sum of the internal angles of a triangle is 180°.
4. Place lines inside each of these polygons to show how they may be made up of just triangles.
5. Use the idea that the sum of the internal angles of a triangle is 180°, to find the sum of the internal angles of each of the irregular polygons.
6. Find a rule that links the number of sides of an irregular polygon to the sum of its internal angles.

![Image of triangle angles]

<table>
<thead>
<tr>
<th>number of sides</th>
<th>number of triangles</th>
<th>total of the internal angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>1 \times 180°</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2 \times 180°</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3 \times 180°</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>4 \times 180°</td>
</tr>
</tbody>
</table>

So if the angles inside each triangle = 180°
The angles inside each polygon add up to the number of triangles that make up the polygon \( \times \text{180°} \).
The conceptual algebraic approach

The student is able to use a given rule to find a new rule, using practical exploration and then generalising, independently.

Prompts from the teacher could be:

1. By breaking a triangle up into three parts, each having a complete vertex, arrange the parts to show that the sum of the internal angles of a triangle is 180°.
3. Use the idea that the sum of the internal angles of a triangle is 180°, to find the sum of the internal angles of each of the irregular polygons.
4. Find a rule that links the number of sides of an irregular polygon to the sum of its internal angles.

<table>
<thead>
<tr>
<th>Sides</th>
<th>Sum of internal angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>180° (See above)</td>
</tr>
<tr>
<td>4</td>
<td>180° + 180° = 360°</td>
</tr>
<tr>
<td>5</td>
<td>360° + 180° = 540°</td>
</tr>
<tr>
<td>6</td>
<td>540° + 180° = 720°</td>
</tr>
</tbody>
</table>

"This pattern would keep going. Start with a triangle has sum of it 180°. Every extra side adds on another 180°"