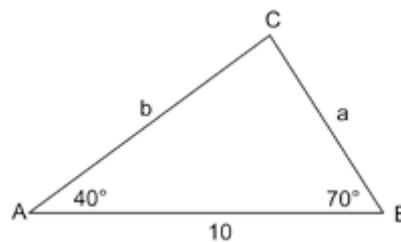
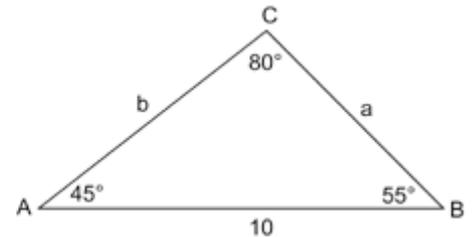
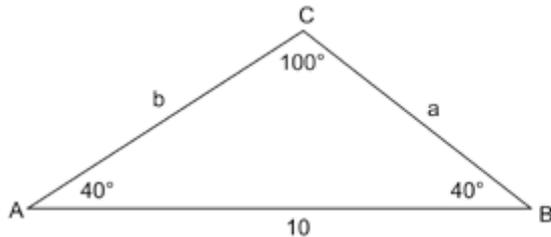


I'm So Sorry I Ate Chocolate Copymaster 1

The Sine Rule I

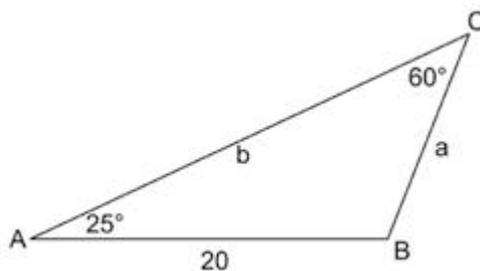
1. Find the lengths of the unknown sides in the following triangles.



2. How big could the side lengths of a triangle be that has angles of 30° and 40° , and one side of length 15m?

Check your answers by constructing the triangles using ruler and protractor.

3. A triangle has two sides of length 4cm and 5cm. The angle between these two sides is 60° . Use the Sine Rule to find the magnitude of the other angles and side of this triangle.



[Hint: $\sin(X - Y) = \sin X \cos Y - \cos X \sin Y$]

What answers would you get if the 60° angle was not between the sides of length 4cm and 5 cm?

4. David was working on the triangle below and found that $a = 40.98$.
Why is this clearly wrong?
What is the correct answer?
What mistake do you think David made?

5. Chen Jian wants to find the distance between two Kauri trees but there is a lot of undergrowth between them and she's not able to measure the distance directly. So she finds a shrub near the first Kauri where the bush is less dense. She measures the distance between that Kauri and the shrub and sees that they are 25.2m apart. Then she measures the angle subtended at the shrub by the two trees and the angle subtended at the first tree by the shrub and the second tree. These angles are 43° and 56° , respectively.

How far apart are the two trees?

(You may assume that the ground is pretty flat in this area.)

6. Hari wants to find the distance across a parallel sided ravine. He decides to find this distance by using the Macrocarpa on one side of the ravine as one vertex of a triangle and two sticks on the other side as the other two vertices. He measures the distance between the two sticks; and the angles subtended at one stick by the tree and the other stick.

The distance between the two sticks is 36.2m and the two angles are 47° and 65° .

Can Hari find the distance across the ravine with this data? If so, find that distance. If not, what further information will he need in order to be able to find the required distance?

Answers to Copymaster 1

- (i) Since $A = B$, $a = b$. Using the Sine Rule we see that

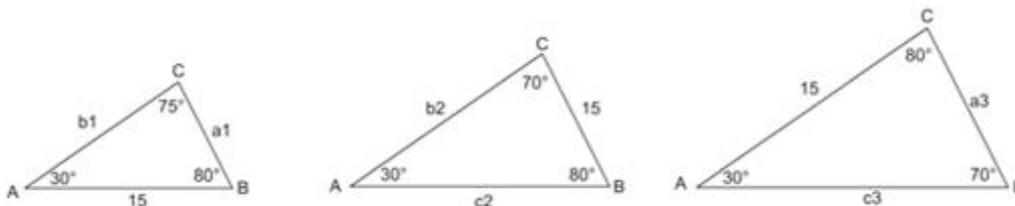
$$\frac{a}{\sin 40^\circ} = \frac{10}{\sin 100^\circ}$$

Therefore $a = 10 \sin 40^\circ / \sin 100^\circ = 6.53 = b$.

(ii) $a = 10 \sin 45^\circ / \sin 80^\circ = 7.18$; $b = 10 \sin 55^\circ / \sin 80^\circ = 8.32$.

(iii) The missing angle is 70° , so $b = 10$. $a = 10 \sin 40^\circ / \sin 70^\circ = 6.84$.

2. There is only one triangle with angles 30° , 70° and 80° . However, there are three triangles with those angles that have one side of length 15.



Case 1: $a_1 = 15 \sin 30^\circ / \sin 80^\circ = 7.62$; $b_1 = 15 \sin 70^\circ / \sin 80^\circ = 14.31$

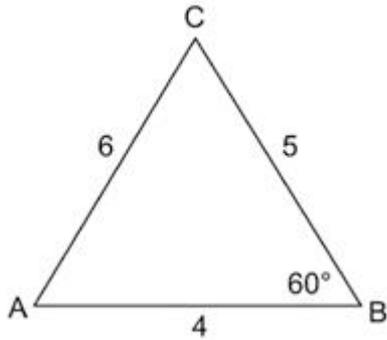
Case 2: $b_2 = 15 \sin 70^\circ / \sin 30^\circ = 28.19$; $c_2 = 15 \sin 80^\circ / \sin 30^\circ = 29.54$

Case 3: $a_3 = 15 \sin 30^\circ / \sin 70^\circ = 7.98$; $c_3 = 15 \sin 80^\circ / \sin 70^\circ = 15.72$

For Case 1, draw a line AB of length 15 and use your protractor to mark off angles of 30° and 70° at A and B. The lines that form these angles should meet at 80° at C. (It is worth checking this.) Now measure $a_1 = BC$ and $b_1 = AC$.

The other cases can be checked in a similar way.

3.



From the diagram, $b/\sin 60^\circ = 5/\sin A = 4/\sin C$ and $A + C = 120^\circ$.

So $5/\sin A = 4/\sin (120^\circ - A)$.

Expanding using the Hint we get

$$5 \sin 120^\circ \cos A - 5 \cos 120^\circ \sin A = 4 \sin A.$$

Since $\sin A$ is not zero, we can cancel out the $\sin A$ term to give

$$5 \sin 120^\circ \cos A = \sin A (4 + 5 \cos 120^\circ)$$

Simplifying gives

$$\tan A = 5 \sin 120^\circ / (4 + 5 \cos 120^\circ) = 2.88675.$$

Hence $A = 70.89^\circ$. From here $C = 180 - 60^\circ - 70.89^\circ = 49.11^\circ$.

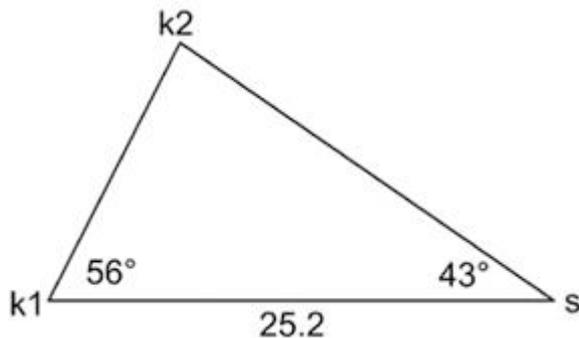
From the first equation we get

$$B = 5 \sin 60^\circ / \sin 70.89^\circ = 4.58.$$

4. A is less than C, therefore a must be less than c. So there is clearly an error here.

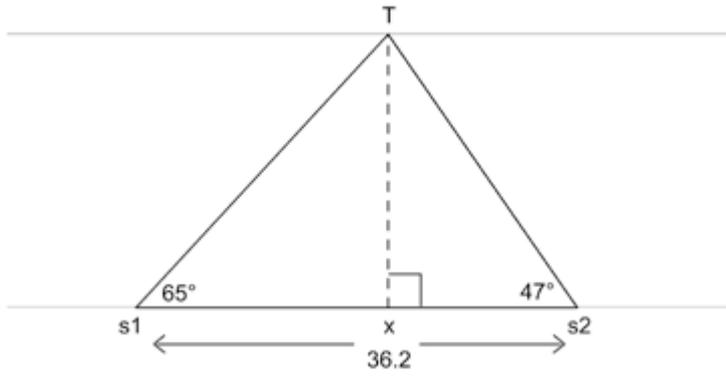
The answer should be $a = 20 \sin 25^\circ / \sin 60^\circ = 9.76$. Perhaps David made an error in his use of the Sine Rule. Now $20 \sin 60^\circ / \sin 25^\circ = 40.98$, so that is where he went wrong.

5.



The angle opposite the side of length 20m is $180^\circ - 43^\circ - 56^\circ = 81^\circ$, so $K1K2 = 20 \sin 56^\circ / \sin 81^\circ = 16.79\text{m}$.

6. The width of the ravine can be found using these measurements. In the diagram, S1 and S2 are the two sticks and T is the tree. We first find S1T (or S2T if you prefer). Then we can find TX using the properties of right angled triangles.



$$S1T = 36.2 \sin 47^\circ / \sin 68^\circ = 28.55\text{m.}$$

In triangle S1TX, $TX/S1T = \sin 65^\circ$, so $TX = S1T \sin 65^\circ = 28.55 \sin 65^\circ = 25.88\text{m.}$