Fraction patterns

Purpose:

The purpose of this multi-level task is to engage students in an investigation that requires them to explore the process of adding fractions and taking reciprocals in order to find a pattern.

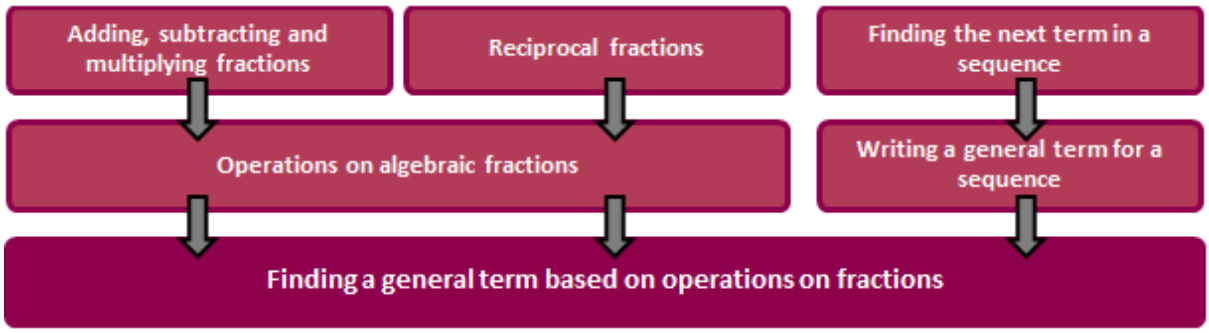
Achievement Objectives:

NA5-3: Understand operations on fractions, decimals, percentages, and integers.

NA5-8: Generalise the properties of operations with fractional numbers and integers.

Description of mathematics:

This background knowledge presumed for this task is outlined in the diagram below:



The task can be presented with graded expectations to provide appropriate challenge for individual learning needs.

Activity:

Task: Investigate the sequence below to find a pattern in the fractions represented by each term.

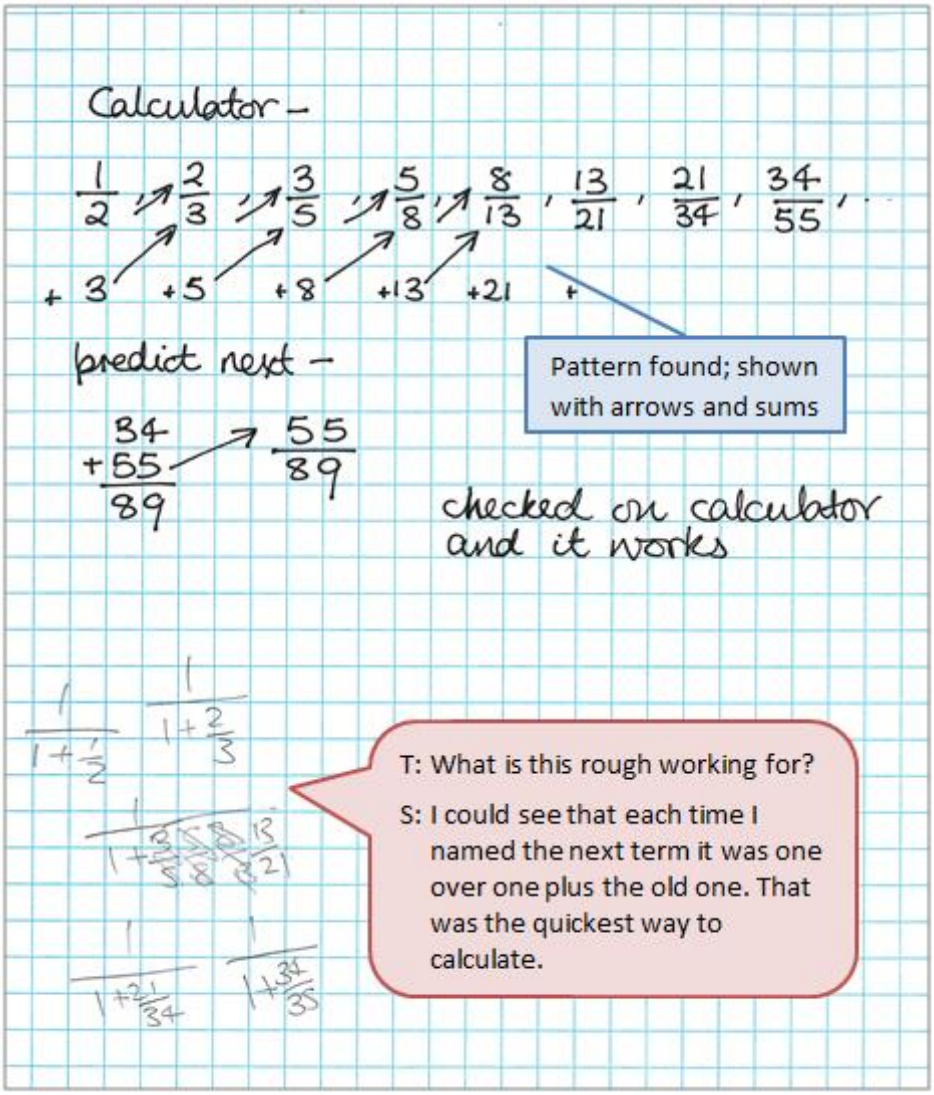
$$\frac{1}{1+1}, \quad \frac{1}{1+\frac{1}{1+1}}, \quad \frac{1}{1+\frac{1}{1+\frac{1}{1+1}}}, \quad \ldots$$

The arithmetic approach

The student is able to calculate the value of terms and give these as a simplified fraction. The student can find a pattern and use this to predict the next term.

Prompts from the teacher could be:

1. Find the value of the first five terms. You may use a calculator to assist you, but you will need to give these terms as simplified fractions.
2. Look for a pattern linking these fractions
3. Predict the next term, using the pattern that you have found.
4. Calculate that term from your initial method and check that this gives you the same fraction as the pattern predicted.

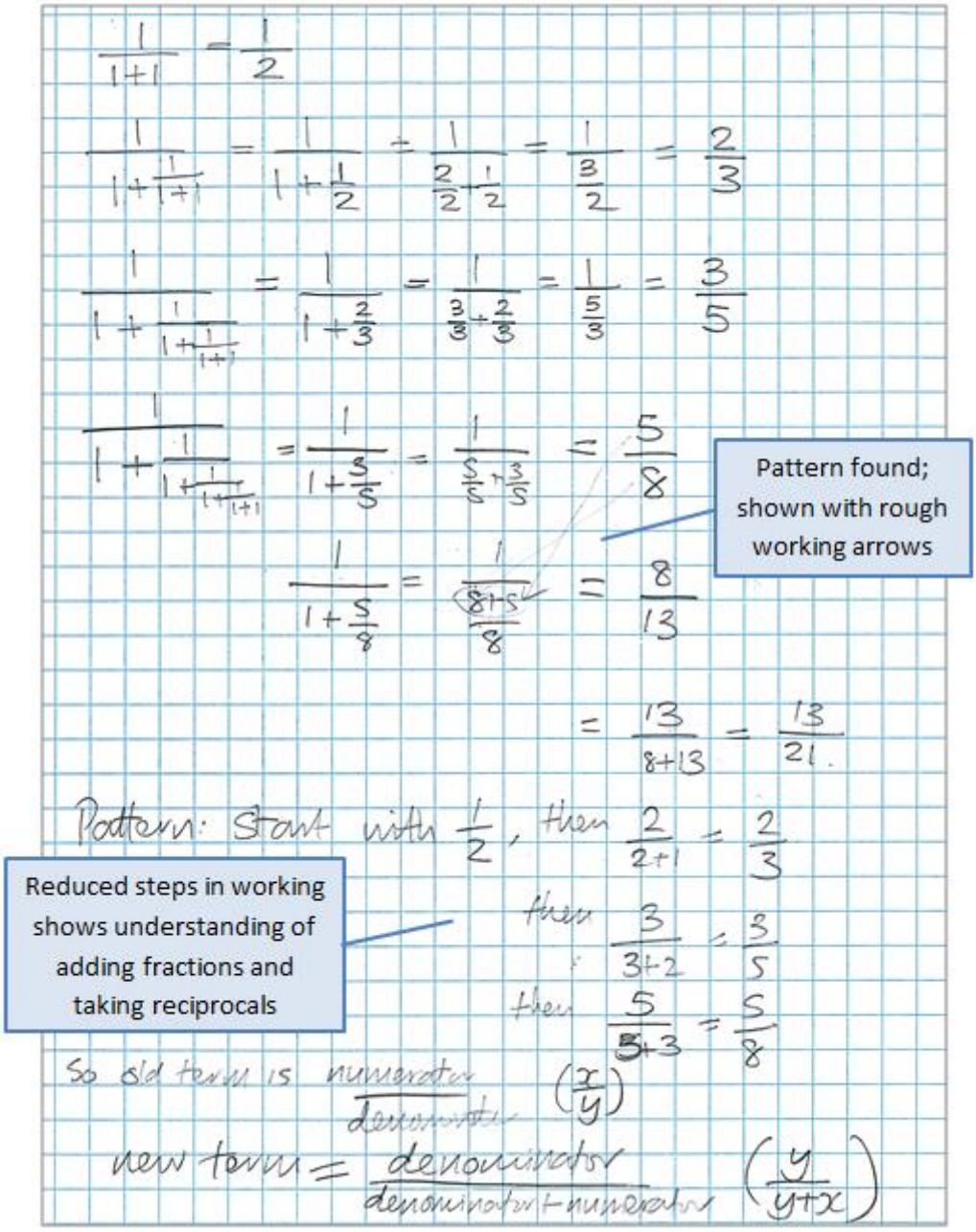


The procedural algebraic approach

The student understands operations on fractions to write each term as a simplified fraction. The student can find a pattern and use this to predict the next term.

Prompts from the teacher could be:

1. Find the value of the first five terms. Rather than using a calculator, set your working out clearly, to look at the process involved in finding each simplified fraction.
2. Look for a pattern linking these fractions
3. Predict the next term, using the pattern that you have found.
4. Calculate that term from your initial method and check that this gives you the same fraction as the pattern predicted.



The conceptual algebraic approach

The student is able to generalise operations on fractions to write a general term for the sequence.

A prompt from the teacher could be:

Can you see that each term is used to generate the next term? Perhaps you could use algebra to look for a pattern or rule to generate the terms.

T: Tell me about why you used the algebraic fraction b/c like this.

S: Well, I could see that each term is one over one plus the term before and I thought if I could write the term before using letters I could tidy it up and see a pattern.

T: And did you see a pattern?

S: No, so I looked at the next term and the next and I could see a pattern.

T: What are the arrows showing?

S: When I realised that because I was adding one to a fraction the number on top ends up being these two added together. And because we are going one over (the reciprocal) the bottom number from one term ends up being on top for the next.