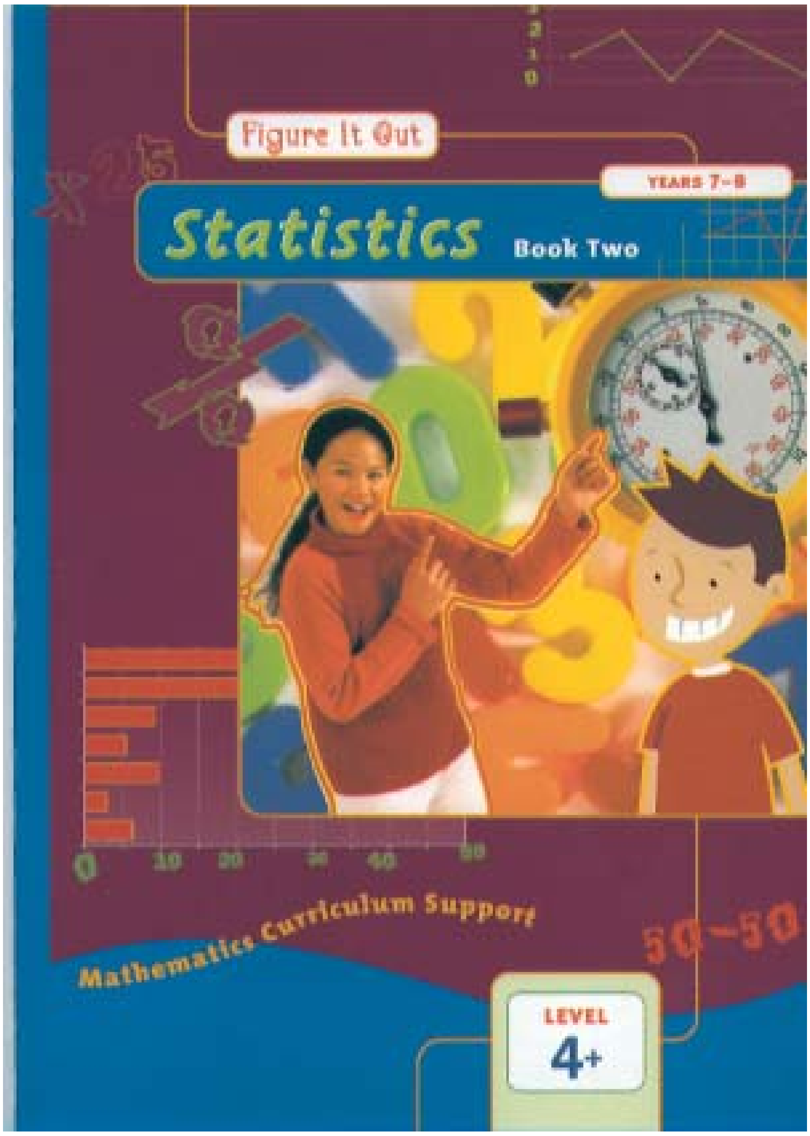
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:
Number (two linking, three level 4, one level 4+, distributed in November 2002)
Number Sense (one linking, one level 4, distributed in April 2003)
Algebra (one linking, two level 4, one level 4+, distributed in August 2003)
Geometry (one level 4, one level 4+, distributed in term 1 2004)
Measurement (one level 4, one level 4+, distributed in term 1 2004)
Statistics (one level 4, one level 4+, distributed in term 1 2004)
Themes: *Disasters Strike!, Getting Around* (level 4, distributed in August 2003)

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent bookwork, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comment on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Statistics: Book Two

Page 1 Social Sounds

ACTIVITY

1. Answers will vary. Possible methods include selecting, for example, every fifth person from the school roll; numbering all the students on the school roll and using random numbers from a calculator to select a sample; or, for each class, drawing the names of two boys and two girls from a hat. Methods must include a degree of randomness and be representative, that is, include people from all the levels and groups involved.
2. **a.** Surveys need to be confidential, use clear, specific, neutral language, have a brief introduction, and have a question that finds the type(s) of music that those surveyed prefer. The students surveyed should be able to ask for more than one type of music. (Just because they like one type doesn't mean they dislike others.)

   b. Practical activity. You could collate the data in a table, using tally marks.
3. **a.** The data should be collated and summarised in a table and could then be graphed. Because the data is discrete, a bar graph would be a suitable kind of graph. It will show the relative popularity of different kinds of bands and singers. You could report your findings to the whole class or the council, using tables and graphs.

   b. Answers need to explain how the mixture of music will reflect the preferences of the students. The programme should have something for everyone while emphasising the kind of music preferred by the majority.

Pages 2–3 Guess the Mass

ACTIVITY ONE

1.–2. Practical activities. Answers will vary depending on the raw data, but the estimates and the actual mass will probably be closer for the later trials.

ACTIVITY TWO

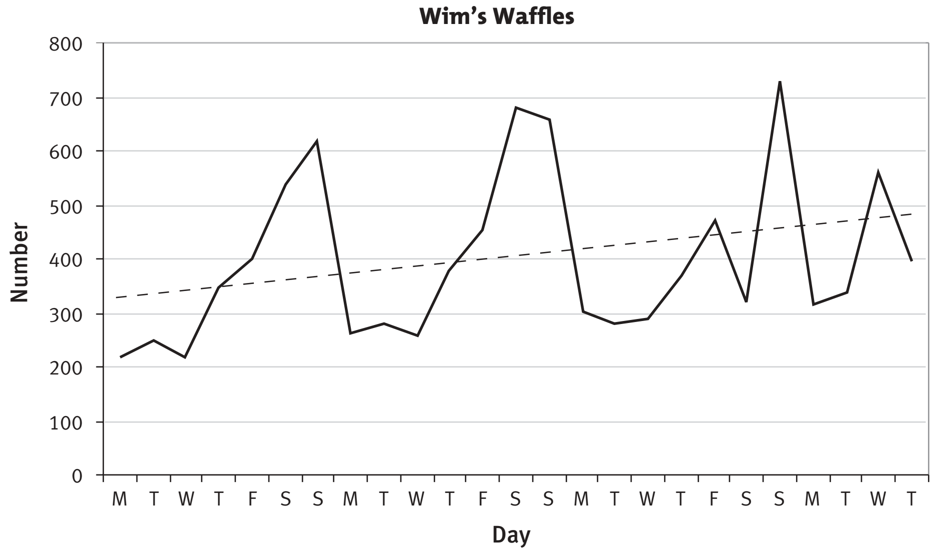
1.–3. Practical activities. As all students are using the same data, everyone should get the same results. If the class is getting more accurate with its estimates, the box-and-whisker plots will change so that:

- the box becomes shorter (showing that the students' estimates are getting closer together);
- the median mark in the box gets closer to the real mass of the object;
- the whiskers get shorter (though it only takes one person to make one long).

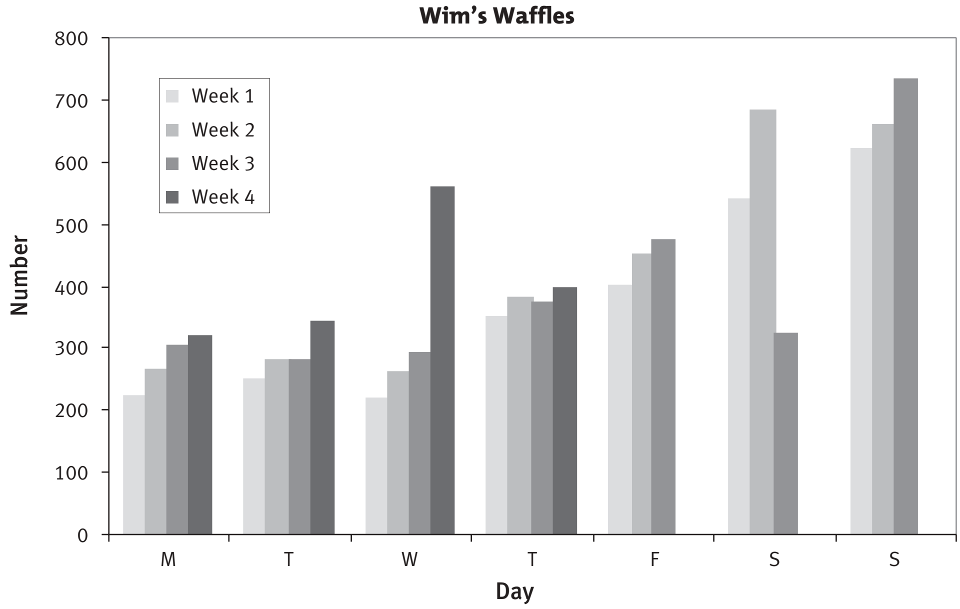
Wim's Waffles

ACTIVITY

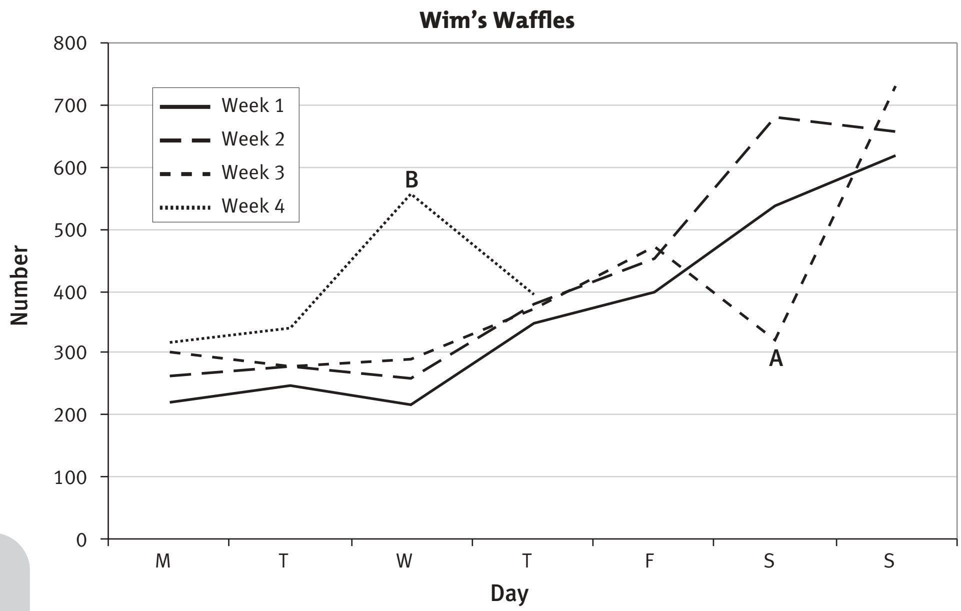
1. **a.** This graph includes a computer-generated trend line (dotted), which is useful for answering question **3**.



b.



c. The labels A and B identify the two days mentioned in **2b** and **2c**.



2. a. Saturday and Sunday

   b. Possible answers include: a cold, rainy day; roadworks blocking the footpath outside; a broken machine; or a major event in another part of town.

   c. The fourth Wednesday (which had increased sales). Possible reasons include a large event nearby, a public holiday, a special promotion, or a really hot day.

3. a. The trend is that the number of sales is generally increasing.

   b. The seasonal pattern is that Wim sells most waffles at the weekend.

4. Predictions will vary, but reasonable predictions are: Friday, 480–520; Saturday, 650–750; and Sunday, 700–800. You may have looked at the trend numerically for each day (not forgetting the poor sales on the third Saturday) or predicted from the graph in question **1**.

5. Answers should be between 2 700 and 3 500. You should make your estimate after adding up the sales over 7 days to get totals for weeks 1–4 and taking note of the increasing trend. Lower and higher totals are acceptable if you have included an exceptional reason, such as a public holiday.

Page 5 **Often Absent**

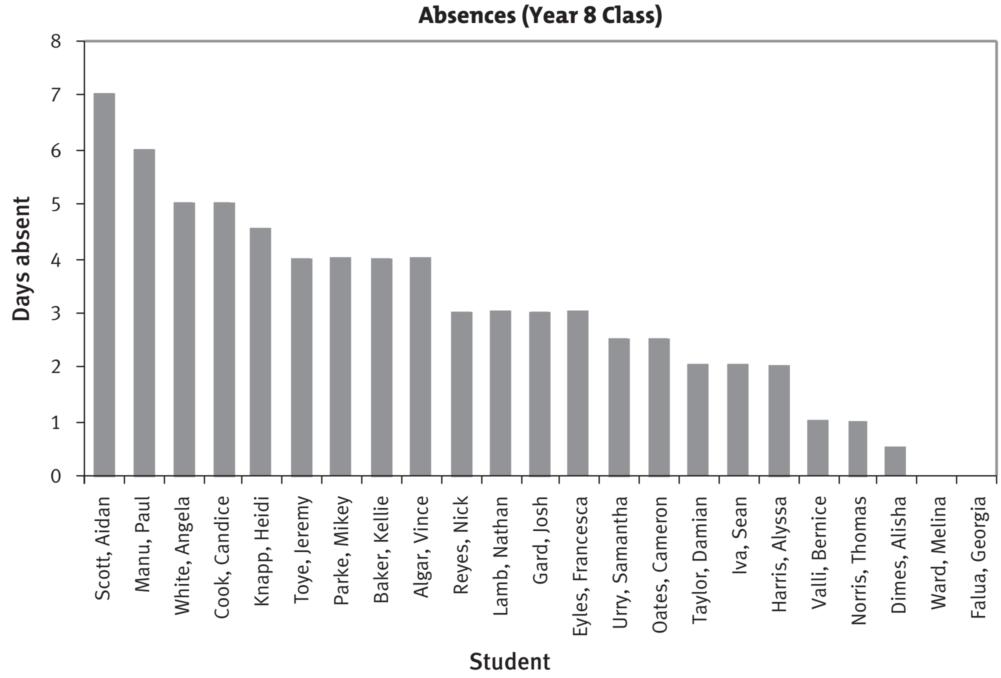
ACTIVITY

1. 4.5

2. a.

Algar, Vince	4
Baker, Kellie	4
Cook, Candice	5
Dimes, Alisha	0.5
Eyles, Francesca	3
Falua, Georgia	0
Gard, Josh	3
Harris, Alyssa	2
Iva, Sean	2
Knapp, Heidi	4.5
Lamb, Nathan	3
Manu, Paul	6
Norris, Thomas	1
Oates, Cameron	2.5
Parke, Mikey	4
Reyes, Nick	3
Scott, Aidan	7
Taylor, Damian	2
Toye, Jeremy	4
Urry, Samantha	2.5
Valli, Bernice	1
Ward, Melina	0
White, Angela	5

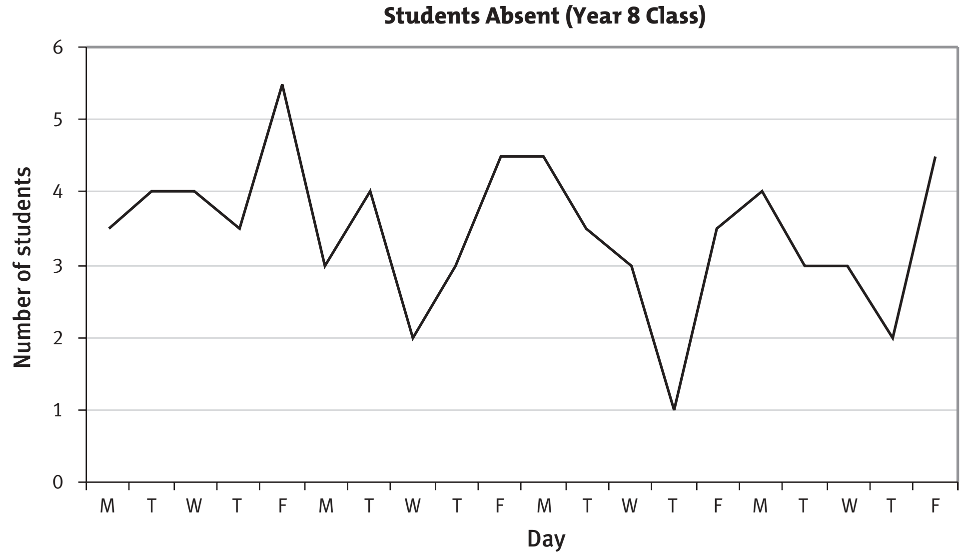
b.



3. a.

	Mon.	Tue.	Wed.	Thu.	Fri.
Week 3	3.5	4	4	3.5	5.5
Week 4	3	4	2	3	4.5
Week 5	4.5	3.5	3	1	3.5
Week 6	4	3	3	2	4.5

b.



4. Yes, she should be concerned if the attendance of this class is representative of attendance across the school. Reasons for concern include that:

- the trend is for students to be away on Fridays
- only 2 students in this class had no absences
- on no day was the whole class present
- the absence rate of this class is 15%.

If she finds that this class is not typical, she should find out why its absence pattern is different from that of other classes.

Pages 6–7 Action and Reaction

ACTIVITY

1. **a.–b.** Practical activity. Results will vary.
2. **a.** Results will vary. Reaction times will probably improve with practice.

   b. Results will vary. The mean or median will be the most useful measure. The mean takes account of every individual trial; the median is unaffected by extremes.

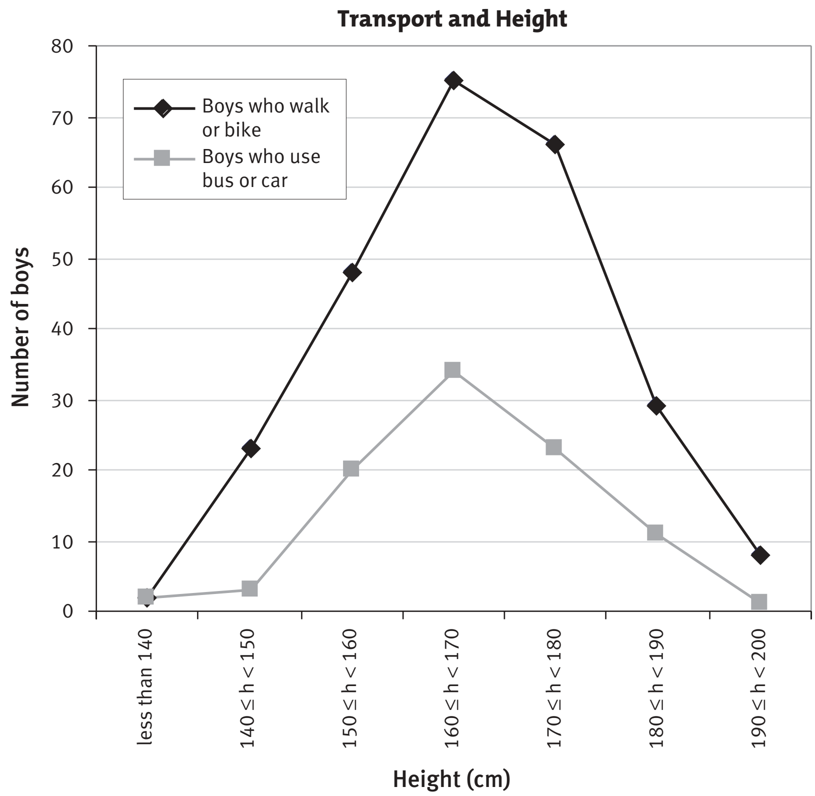
   c. Results will vary. Your conclusion needs to be supported by the mean or median (or both).
3. Trial 7 has the longest measurement recorded but it is still reasonable, so it should be retained. Trial 8 is unusually low compared with all the other measurements recorded, so it is probably an error. It could be truncated.
4. The box-and-whisker plot clearly shows the median, quartiles, and range. The histogram shows how the results were clustered and whether there were any rogue results.
5. Graphs will vary.

Page 8 Walking Tall?

ACTIVITY

1. **a.** Practical activity

   b.

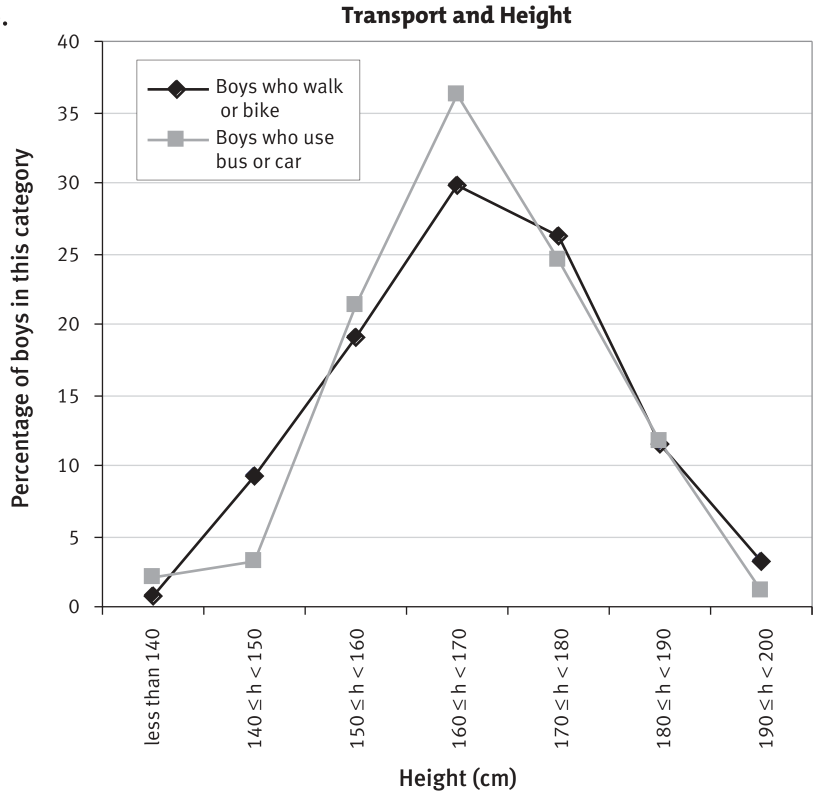


   c. No. Both lines rise and fall at the same points, so the two distributions are very similar. The walk and bike line goes higher than the bus and car line only because there are more students in this group.

2. a.

Percentage of all walkers and bikers	Percentage of all bus and car users
0.8	2.1
9.2	3.2
19.1	21.3
29.9	36.2
26.3	24.5
11.6	11.7
3.2	1.1

b.



c. Answers will vary but may include these points:

- There is a very similar percentage of tall boys in both groups.
- A slightly smaller percentage of the boys who are 170 cm or taller get a ride to school.
- Of the short boys (those less than 160 cm), a slightly higher percentage is in the group that walks or bikes.
- The sample is probably too small to prove much.

3. Investigations and results will vary.

Page 9 Price Hike

ACTIVITY

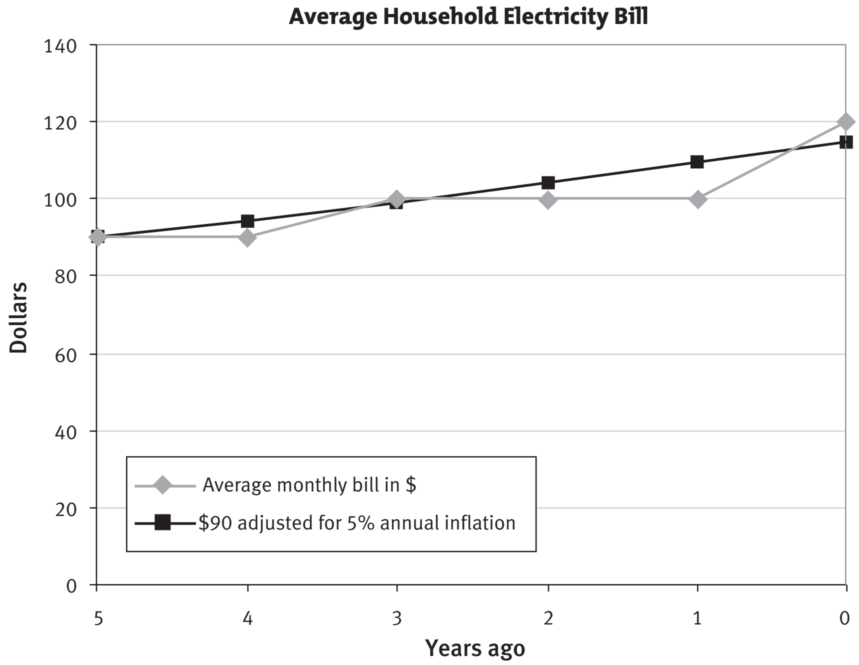
1. **a.** 11.1%. (100 ÷ 90)

   b. $120. (100 x 1.2)

2.

Number of years ago	5	4	3	2	1	0
Average monthly bill in $	90	90	100	100	100	120
$90 adjusted for 5% annual inflation	90	94.50	99.23	104.19	109.40	114.87

3. **a.–b.**



4. Answers will vary, but possible points include the fact that, for 4 years, prices have been at or below the rate of inflation, and running costs have increased. Also, inflation will have caught up with the price increase after a year or so.

5. Answers will vary and may include the fact that the increased power price is above the rate of inflation, that this is the second price rise in 3 years, and that the price of electricity has increased by 33.3% ($^1/_3$) in 5 years.

Pages 10–11 Population Pyramids

ACTIVITY

1. **a.** Answers and explanations will vary. Possible answers and explanations include:

   i. Yes, this is a fair statement. Up to 74 years, the male and female bars are almost exactly the same length, but the four bars that represent women aged 75 and above are longer than the bars that represent men. This is especially true for the bars that represent ages 89 and above.

   ii. Yes, this is a fair statement, assuming that those who are 65 years and older are retired. The lengths of the six bars that represent this part of the population add up to very close to 25% of the total.

   iii. This may be true, but there is not enough data in the graph to be sure. About 28% of the population is aged 20–44 (the age range of most parents when their children are born). About 22% of the population is aged 19 or less. This means that there is fewer than 1 child for each adult in the 20–44 year band.

   iv. Yes, this is a fair statement. Except for the age bands that represent those who are aged 80 years and older, each band represents between 4.6 and 6.4% of the population.

   b. Statements will vary.

2. Sentences will vary. Generally the median age of the population is increasing, so, as time goes by, there will be proportionally more older people and proportionally fewer younger people.

INVESTIGATION

Answers will vary.

Pages 12–13 Testing Times

ACTIVITY

1. a.

10	0
9	5, 7
8	0, 5, 9, 9
7	0, 8, 8, 8
6	1, 5, 5
5	1, 1, 1, 1, 4, 9
4	3
3	5, 9, 9
2	0, 4, 5
1	9
0	6, 6

b. 100 – 6 = 94

c. Mean = 56.8, median = 56.5, mode = 51

d. The two 6s. (These marks are 13 less than the next lowest mark.)

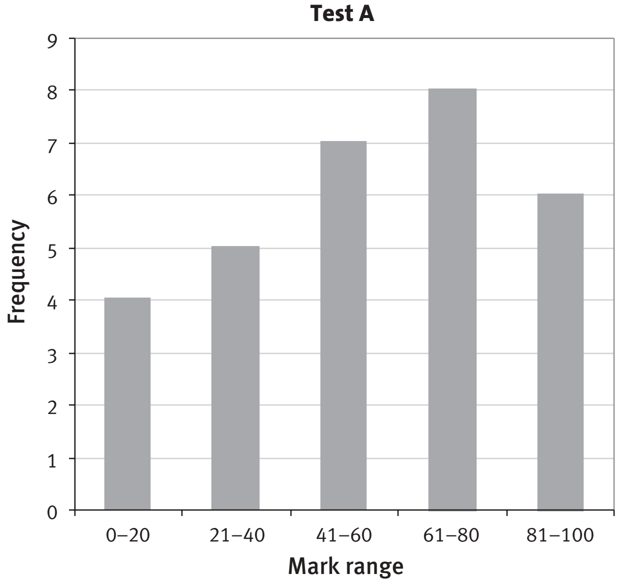
e. Answers will vary but could include the following points:

- the spread (range) is very great
- $^2/_3$ of the class got 51 or better
- there are three fairly distinct groups within the class (78 and over, 51–65, and 20–39).

2. a.

Mark range	Tally	Frequency
0–20	IIII	4
21–40	𝍸	5
41–60	𝍸 II	7
61–80	𝍸 III	8
81–100	𝍸 I	6

b.



c. 61–80

d. This graph is less useful because all the individual data is lost by grouping. The stem-and-leaf plot groups the data without losing any of the original information.

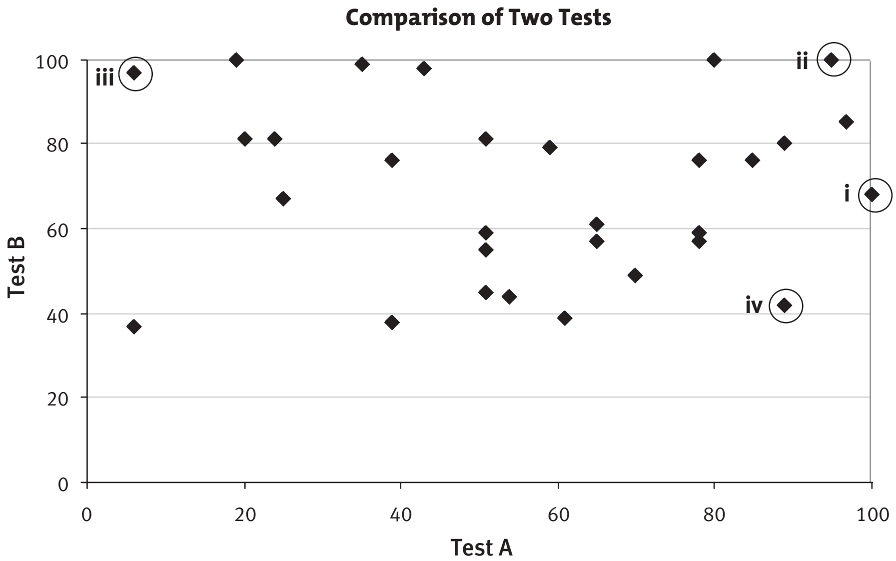
3. a.

Test B		Test A
0, 0, 0	10	0
7, 8, 9,	9	5, 7
5, 1, 1, 1, 0	8	0, 5, 9, 9
9, 6, 6, 6	7	0, 8, 8, 8
8, 7, 1	6	1, 5, 5
5, 7, 7, 9, 9	5	1, 1, 1, 1, 4, 9
2, 4, 5, 9	4	3
7, 8, 9	3	5, 9, 9
	2	0, 4, 5
	1	9
	0	6, 6

b. Range: 63 (100 – 37 = 63); mean: 69.5; median: 72; mode: 76, 81, 100

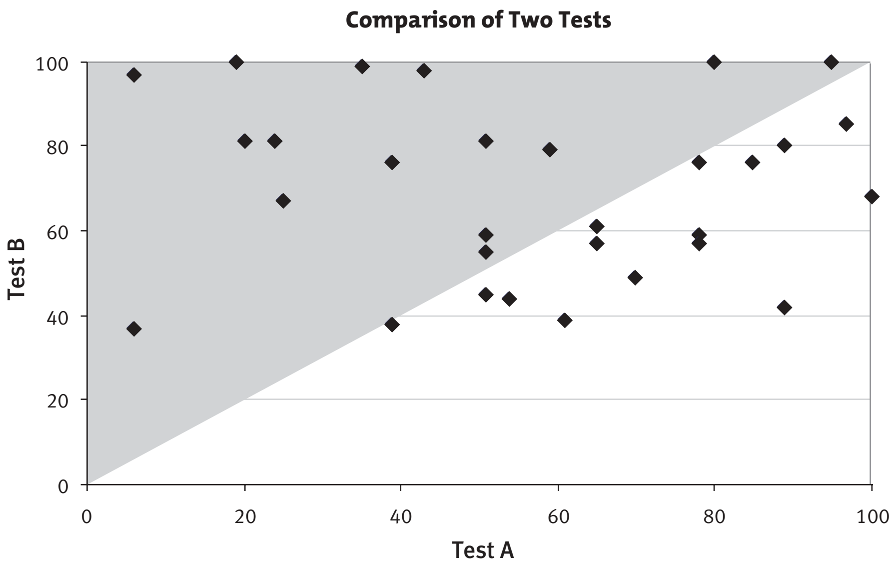
c. Yes. The results indicate improvement: a much smaller range, a higher mean, a higher median, and higher modes.

4. (The labelled points on this graph are the answers to question **5a**.)



5. **a.** See the labelled points on the graph for question **4b**.

   b. They are in the area above the diagonal line that runs from (0, 0) to (100, 100). The shape is a right-angled triangle.



   c. Exactly half ($^{15}/_{30}$) of the students got better results in the second test, so neither more nor fewer did better.

6. The scatter plot in question **4** is the most data rich. Every plotted point tells a story. None of the original data has been lost in creating the graph. It clearly compares the outcomes in the two tests, and it is the only graph that connects the scores of individual students. (The two stem-and-leaf plots keep the data intact but not the connection between an individual's first and second scores.)

Pages 14–15 Just Average

ACTIVITY ONE

1. **a.** 30
   b. 20
   c. 40
   d. 20
2. **a.** 20
   b. 2. (17 x 6 = 102; 102 – 100 = 2)
   c. 50
3. **a.** Rules will vary, but one rule would be: First find the average of the set of numbers you start with. The extra number will be equal to this average plus 5 times the number of numbers that are now in the set.
   b. Practical activity

ACTIVITY TWO

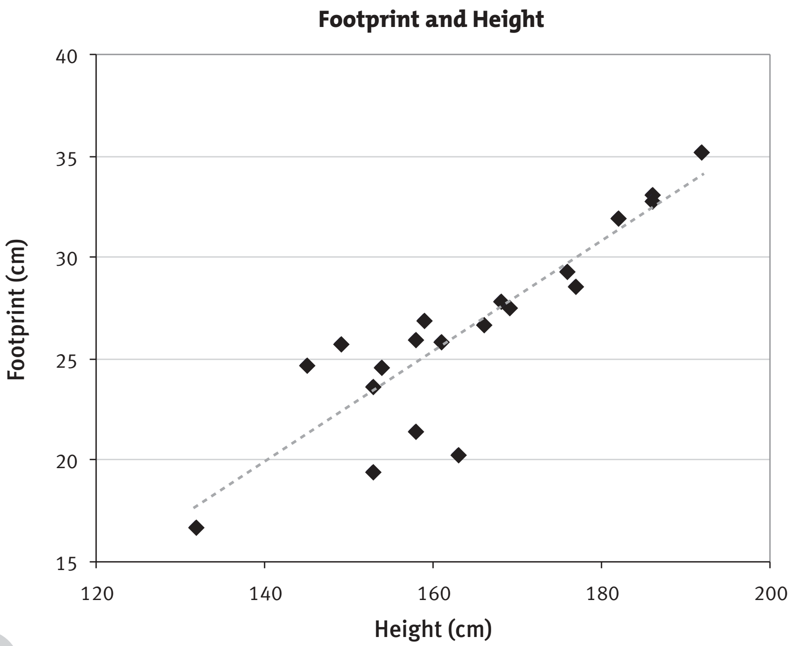
1. **a.** 215 runs. (Over 8 games, his average is 37, so he has made 8 x 37 = 296 runs in total. But his average for the first 3 games was 27, so he scored 3 x 27 = 81 runs in those games. This means that he scored 296 – 81 = 215 runs in games 4–8.)
   b. 1 run. (After playing his 9th game, his average was 33, so his total runs are 9 x 33 = 297. This means that he scored 297 – 296 = 1 run in the 9th game.)
   c. Over the 10 games, he scored 297 + 73 = 370 runs, so his average at that point was 370 ÷ 10 = 37 runs.
   d. 70. (If he is to average 40 over 11 games, he will need to score a total of 11 x 40 = 440 runs. As he has 370 after 10 games, he needs to score 440 – 370 = 70 runs in his final game for the season.)

   There is not enough information to be able to say with confidence that he is or is not likely to reach his goal. If we consider only his average, it appears *unlikely* that he will, but it is possible that there are several 70s "hidden" in the average (there could be three in games 4–8), and he may even be returning to form, in which case it would be *possible* that he will reach his goal.
2. Half of any normally distributed population will be below the *median* on any measure because, by definition, the median *is* the mid-point. For a large population, the median and the *mean* (average) will be extremely close. So of course this means that half the population will not reach the average life expectancy, regardless of what it is. The statement shows that the speaker does not understand the meaning of the word "average".

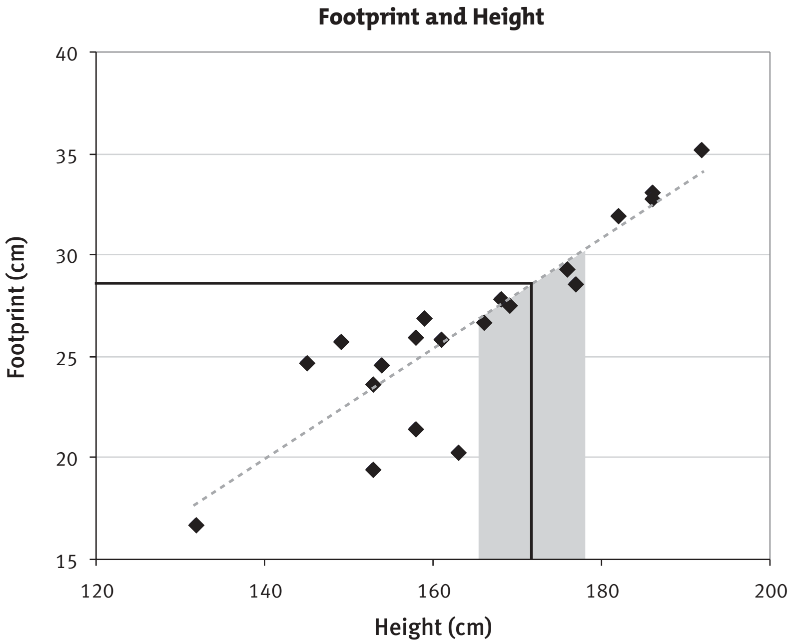
Page 16 Suspect on Foot

ACTIVITY

1. **a.** The additional 10 measurements will vary.
   b. Graphs will vary, but they should be similar to this one:



c. Somewhere between 165 and 178 cm would be reasonable. (See the graph below, where the shaded area shows this range.)



2. Answers will vary. A possible answer is: mark the line that best fits the trend of the data on the graph; find 28.4 cm on the footprint axis; go horizontally to the line you have drawn, then down to the height axis; and read the approximate height off your graph. Allow a reasonable range on either side of this value because there is no precise link between foot size and height.

Page 17 Slater Mazes

ACTIVITY

1. **a.** Exit B has a $^1/_2$ chance because there are 2 paths the slater could follow to get there, whereas there is only 1 path to each of the others. (So exits A and C have a $^1/_4$ chance.)

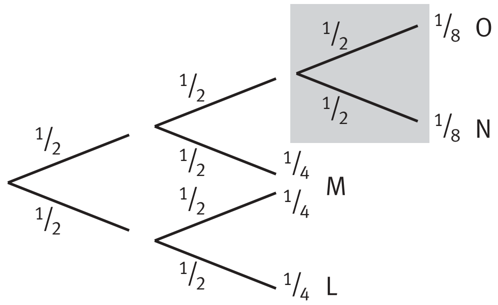
   b. Proposals will vary. The best proposals will have a prize for just one exit, A or C; Jack won't make much money if he rewards every player. The prize will need to cost less than 4 entries to the game, or Jack will be likely to make a loss, not a profit.

2. D and G have a $^1/_8$ chance each. (There are 8 different paths to the exits, but only 1 of them ends up coming out at D and 1 at G.)

   E and F have a $^3/_8$ chance each. (3 of the 8 possible paths end up coming out at E and 3 at F.)

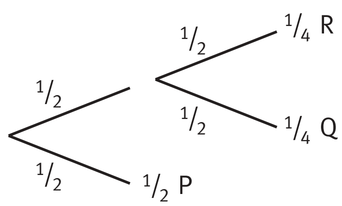
   H and K have a $^1/_6$ chance each. (There are 6 different paths to the exits, but only 1 of them ends up coming out at H and 1 at K.)

   I and J have a $^1/_3$ chance each. (2 of the 6 possible paths come out at each of these exits.)

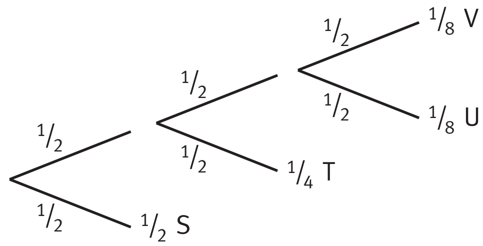


L has a $^1/_4$ chance, M has a $^1/_2$ chance ($^1/_4 + ^1/_4$), and N and O each have a $^1/_8$ chance. This maze is different from the others in that one branch has an extra set of choices. (The tree diagram models the choices, with the extra set of choices shaded.)

3. P has a $^1/_2$ chance, Q and R each have a $^1/_4$ chance. The choices are modelled in this tree diagram:



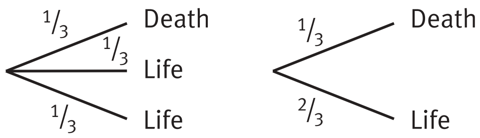
S has $^1/_2$ a chance, T has a $^1/_4$ chance, and U and V each have an $^1/_8$ chance. The choices are modelled in this tree diagram:



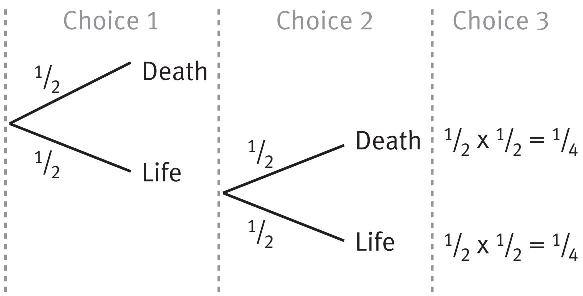
Pages 18–19 Rough Justice

ACTIVITY

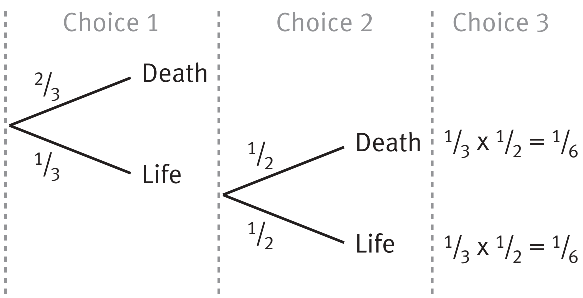
1. a. i. The probability of survival is $^2/_3$. (2 of the 3 doors lead to riches.)



ii. The probability of survival is $^1/_4$. (There is $^1/_2$ a chance of choosing the first door safely, then $^1/_2$ a chance of choosing the next door safely. $^1/_2 \times {}^1/_2 = {}^1/_4$.)



iii. The probability of survival is $^1/_6$. (There is a $^1/_3$ chance of making the first choice safely, then a $^1/_2$ chance of making the second choice safely. $^1/_3 \times {}^1/_2 = {}^1/_6$.)



b. Room **i** for theft; room **ii** for assault, and room **iii** for murder. Murder is the most serious crime, followed by assault and then theft.

2. a–b. Answers will vary.

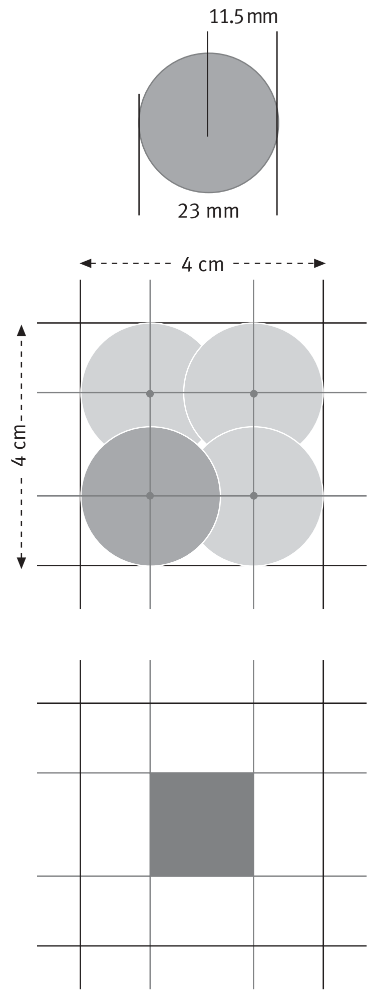
Page 20 Unlucky Lines

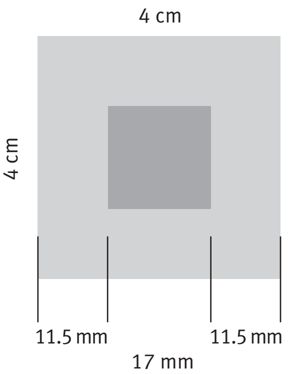
ACTIVITY

1. a. Practical activity

b. Answers will vary, but it is likely that about $^1/_5$ (0.2) of all attempts are successful.

c. The diameter of a \$1 coin is 23 mm, so its centre is 11.5 mm (half of 23 mm) from its edge. This means that the centre of the coin can never be closer than 11.5 mm from the edge of the square, or it will touch. So no matter what size the squares of the grid are, there is always a "lose" area 11.5 mm wide along each side and a square "win" area is left in the middle. The diagram shows how this works for a grid with 4 cm squares:





The total area of a 4 cm by 4 cm square is 40 x 40 = 1 600 mm^2. Of this, only the 17 mm by 17 mm grey square in the middle is the "win" area. 17 x 17 = 289 mm^2.

This means that the chances of the centre of the coin landing on the winning area are: $^{289}/_{1\,600}$ = 0.18, or 18%.

2. 80 mm x 80 mm, or very close to this. (78.5 mm, correct to the nearest 0.1 mm, will give a probability of 0.5.)

Page 21 On a Plate

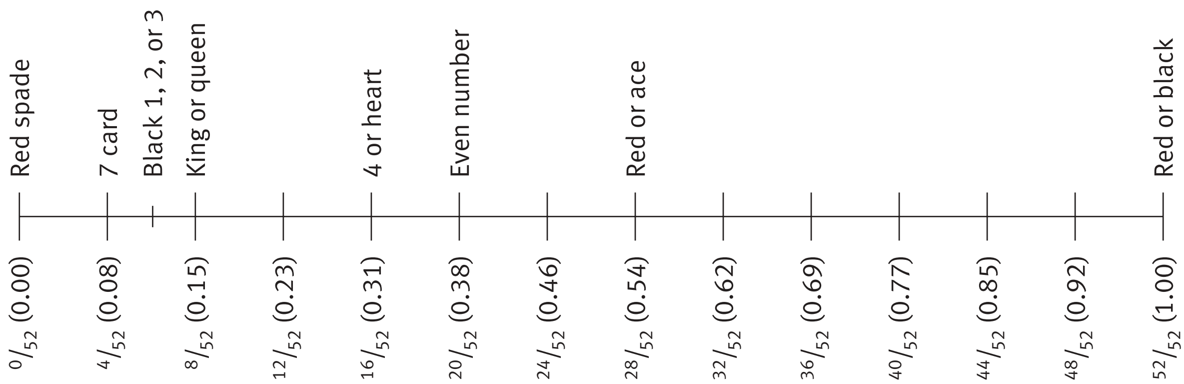
ACTIVITY

1. 6 759 324 plates. (Think about all the plates with an A as the first letter. There are 9 999 in the range AA1–AA9999. This means that there are 26 x 9 999 in the range AA1 to AZ9999. So there were 26 x 9 999 = 259 974 plates starting with A. There will also be 259 974 plates starting with B, 259 974 starting with C, and so on. So altogether, using this system, there are 26 x 259 974 = 6 759 324 plates.)
2. 17 558 424. (Starting with AAA, you can have AAA, AAB, AAC … [26 combinations], ABA, ABB, ABC … [26 combinations], through to AZA, AZB, AZC … [26 combinations], so there are 26 x 26 combinations starting with A. There are 26 possible starting letters, so the number of 3-letter combinations is 26 x 26 x 26 = 17 576. Each letter combination can have 999 possible 3-digit combinations. The total number of combinations is therefore 17 576 x 999 = 17 558 424.)
3. 92 years. (17 558 424 ÷ 190 000 = 92.4)
4. 17 538 444. ([26 x 26 x 26 – 20] x 999)

Page 22 Card Sharp

ACTIVITY

1. a. 7: 0.08 ($^{4}/_{52}$); red spade: 0 ($^{0}/_{52}$); 4 or heart: 0.31 ($^{16}/_{52}$); red or ace: 0.54 ($^{28}/_{52}$); even number: 0.38 ($^{20}/_{52}$); king or queen: 0.15 ($^{8}/_{52}$); black 1, 2, or 3: 0.12 ($^{6}/_{52}$); a red or a black: 1.0 ($^{52}/_{52}$).



   b. Answers will vary.

2. a. 0.08 ($^{4}/_{52}$), 0.08 ($^{4}/_{52}$), 0.15 ($^{8}/_{52}$)
   b. 0.08 ($^{4}/_{52}$), 0.25 ($^{13}/_{52}$), 0.31 ($^{16}/_{52}$)
   c. 0.08 ($^{4}/_{52}$), 0.25 ($^{13}/_{52}$), 0.5 ($^{26}/_{52}$), 0.77 ($^{40}/_{52}$)
3. "Or" means that the number of successful outcomes is increased (because there are more ways of winning), so the use of "or" always pushes the probability closer to 1.
4. The use of "and" always decreases the probability of success, that is, it pushes the probability closer to 0. This is because success is being made dependent on meeting further conditions. Someone who is tall *and* blond *and* blue-eyed *and* left-handed is much harder to find than someone who is just tall or just left-handed.

Page 23 Paper, Scissors, Rock

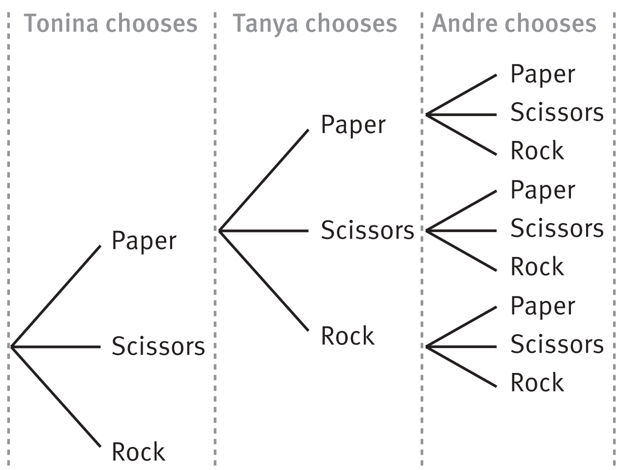
ACTIVITY

1. Results may vary, but Tanya should win. (See the answer to question **3b**.)
2. a.

PPP	SPP	RPP
PPS	SPS	RPS
PPR	SPR	RPR
PSP	SSP	RSP
PSS	SSS	RSS
PSR	SSR	RSR
PRP	SRP	RRP
PRS	SRS	RRS
PRR	SRR	RRR

(P is paper, S is scissors, and R is rock.)

b. A tree diagram would be suitable. The following is $^1/_3$ of a complete tree. It shows all the outcomes from the 1st column of the list under **a**, above.



3. a. No. The players do not have an even chance of winning.

   b. The probability of Tonina winning is $^3/_{27}$, the probability of Tanya winning is $^{18}/_{27}$, and the probability of André winning is $^6/_{27}$.

   c. Ideas may vary, but Tonina should get twice as many points for each win as André, and André should get 3 times as many points for a win as Tanya. For example, Tonina could get 6 points, André 3 points, and Tanya 1 point.

4. Practical activity

Page 24 Birth Months

ACTIVITY

1. Practical activity
2. Answers will vary, but the mathematics shows that the chance of 2 or more people out of 5 having the same birth month is considerably greater than half (0.618). $(1 - {}^{12}/_{12} \times {}^{11}/_{12} \times {}^{10}/_{12} \times {}^{9}/_{12} \times {}^{8}/_{12})$
3. Answers will vary. Most people are surprised to get a match in circumstances like this, but in reality, it is hard to avoid a match.

Teachers' Notes

Overview

Statistics: Book One

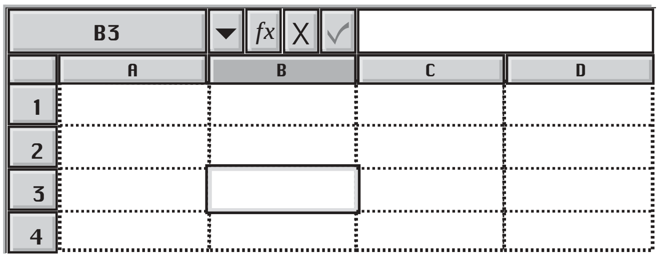
An Introduction to Computer Spreadsheets and Graphs

Computers are valuable tools when it comes to teaching statistics and, if possible, your students should be using them. Working with computers, the students can collate data efficiently, sort it in any order they like, experiment with different types of graph, and perform statistical calculations. They can do all of this without having to re-enter data and without the need for good graphics skills. If they are using computers, you can expect more of your students.

However, students are apt to think that if they have produced a nice-looking graph, their job is done. They need to understand the process by which a graph is created, what makes for a good graph, and how to read the story in the picture. Therefore, even if your students have ready access to computers, you should ensure that they also practise creating and interpreting hand-drawn graphs.

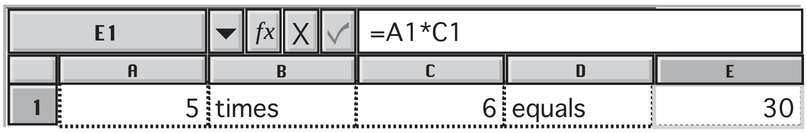
If you are not familiar with computer spreadsheets and graphs, you should read the next two sections. Guidance on specific kinds of graph is given in the notes for individual activities.

Spreadsheets for Beginners
A spreadsheet identifies every *cell* by its *reference*. In this example, cell B3 is selected:

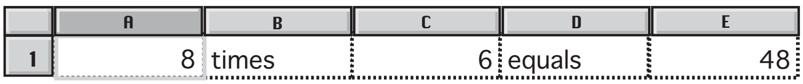


Once you have highlighted (selected) a cell, you can type a word, a number, or a *formula* into it. While words and numbers are static pieces of data, formulae cope with change. It is the ability of spreadsheets to handle formulae that makes them such a powerful tool.

Put the number 5 into cell A1, then type the word "times" into cell B1, the number 6 into cell C1, and the word "equals" into cell D1. Into cell E1, type the formula =A1*C1. (All formulae begin with the "=" sign.) Now press Enter or click on the green tick:



The asterisk in the formula acts as a times sign. Now put the cursor back into cell A1 and type in a new number to take the place of the 5. Press Enter.



If you try putting other numbers in cells A1 and C1, you will see that the formula recalculates the value automatically. This means that formulae are great for checking out patterns or when using trial and improvement to solve a problem. If you delete the words "times" and "equals" and put another pair of numbers into cells A1 and D1, you will see that the formula still works. Although the words helped show what the spreadsheet was doing, they didn't feature in the formula, so removing them did not affect the mathematics.

Note these common symbols:

- / (instead of ÷) for division
- ^ (found above 6 on the keyboard) for powers (for example, 2^3 for 2^3).

There are numerous functions that can be used for calculating the sum, average, and so on. To find them, select Function from the Insert menu (PCs) or Insert Function from the Edit menu (Macintoshes).

Graphs From Spreadsheets

In many of the activities in this book, the students are expected to enter data into a spreadsheet (rather than a hand-drawn table) and then to create a graph. Be aware that spreadsheet programs refer to graphs as *charts*.

First enter your data into a spreadsheet, and then highlight the cells that contain the data you want graphed, including the column headings. (When creating the graph, the program will ignore all parts of the spreadsheet that are not highlighted.)

Click on Insert and select the Chart option. This will bring up a chart *wizard* (assistant), which will guide you through the entire process: selection of graph type, title, labels, formats, and where the finished graph is to be placed.

Experiment with different types of graph and see what they look like; if you don't like what you get, click on Back and choose another type. Remember, none of the graphing options will be available unless you have selected some data cells in your spreadsheet.

The three main types of graph that students will meet in this book are bar graphs (including histograms), line graphs, and scatter plots. Check out the other types available. You will soon discover that it is possible to create some attractive-looking kinds of graphs that do, however, have limited uses.

You can change many features of a graph by double-clicking on the feature concerned (for example, a label, an axis, or a line), in which case you will be given a list of options to choose from. (Macintosh users should hold down Control and click once to get a similar list.) The notes on histograms (below) and the notes that go with the various student activities will help you with many of these formatting issues.

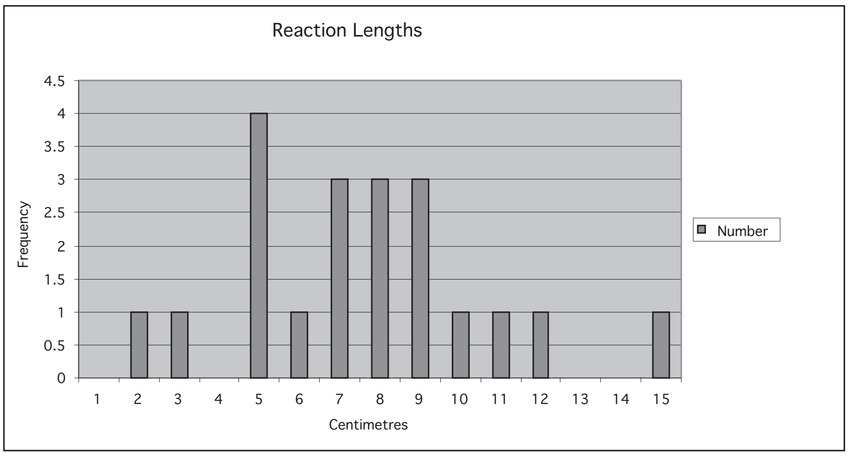
Unfortunately, there are times when, no matter what you do, you can't get the result you want. In such cases, you or your students will need to decide whether to live with the technical shortcomings or create the graph by hand.

Histograms

The histogram is one of the most frequently used kinds of graph. Unfortunately, a well-formatted histogram is not one of the standard chart options. The following example (based on a graph from page 7 in the student book) shows how to produce a reasonable finished product.

First enter the data in the spreadsheet, as below. Select only the cells under Number, choose Column (clustered) from the various chart options, give your graph a title and labels, choose its location, and then press OK. What you get is a bar graph that should look similar to this:

Length	Number
11	0
12	1
13	1
14	0
15	4
16	1
17	3
18	3
19	3
20	1
21	1
22	1
23	0
24	0
25	1



Follow these steps to turn your bar graph into a reasonably satisfactory histogram:

- Remove the legend (the box with "Number" or "Series 1" in it) at the right-hand side by clicking on it and pressing Delete. (Because there is only one series [data set], the legend is unnecessary.)
- Remove the space between the bars by double-clicking on any bar to open the Format Data Series dialog box, choosing the Options tab, and then typing 0 into the space that says Gap Width.

- Remove the in-between values from the vertical axis (as these are meaningless when the values on the scale are whole numbers) by double-clicking on any of the numbers on that axis to open the Format Axis dialog box, choosing the Scale tab, and making the Major Unit and the Minor Unit both 1.
- Correct the labels on the horizontal axis in the following way. Select the entire chart by clicking once anywhere on or within its frame. Choose Chart from the menu bar, then Source Data, then the Series tab. Click the cursor in the panel that says Category (X) Axis Labels, then highlight the cells in the spreadsheet that contain the values that belong on the that axis (11–25). Click on OK. There appears to be no way of getting these labels beneath the tick marks (which is where they should be). You could resolve this problem by deleting the labels entirely, printing the graph, and then adding the labels by hand. To delete the labels, click on the axis, choose Format Axis/Font, and make the font white and the background transparent.
- For the sake of clarity, remove the grey background by double-clicking anywhere on the grey to open the Format Plot Area dialog box. Select None (under Area) and click OK.
- Your graph should now look similar to this. It’s not perfect, but it is definitely an improvement.

Page 1

Social Sounds

Achievement Objectives

- plan a statistical investigation arising from the consideration of an issue or an experiment of interest (Statistics, level 4)
- collect appropriate data (Statistics, level 4)
- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)
- make statements about implications and possible actions consistent with the results of a statistical investigation (Statistics, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

This activity introduces students to the issues that arise with any statistical survey. They are likely to get the greatest benefit if there are other groups doing the same task at the same time. The different approaches, pitfalls, and conclusions could become the subject of a report-back and debriefing at the end. As the teacher, you have plenty of scope for varying this task to suit the needs of your students.

The important concepts are described in the following paragraphs. Introduce these to the students as the need arises. They should become familiar with all these words, and by the time they conclude the activity, they should have developed some understanding of what each of them means in a statistical context.

The *population* is the entire group that is of interest (for example, every student in Brown River Primary School). A *survey* is a questionnaire that is used to gather data from a *sample* of the population. A *census* is the alternative to *sampling*; it involves collecting data from the entire population.

When sampling, *representativeness* is critical if the data collected and the decisions made are to prove reliable. To be representative, a sample needs to match the population on factors such as gender and ethnicity. A sample that is not representative is said to be *biased*.

For a sample to be genuinely without bias, every member of the population should have the same chance of being selected. This means that the person making the selection must use a method that, once in place, picks the names *for* them. For example, students doing this activity could select, say, every fifth name on the school roll. Alternatively, they could use *random numbers* (found from a calculator, computer, or telephone directory).

It is worth emphasising that people (adults as well as children) are not capable of making a genuinely random selection of names or numbers simply by choosing one that comes to mind and then another. There are always biases in a selection made this way, even if they are hidden.

Pages 2–3 Guess the Mass

Achievement Objectives

- collect and display time-series data (Statistics, level 4)
- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)
- collect and display comparative samples in appropriate displays such as back-to-back stem-and-leaf, box-and-whisker, and composite bar graphs (Statistics, level 5)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY ONE

In this whole-class activity, students start with a hypothesis and then collect data that may prove or disprove it. The line graphs that result are time-series graphs because they plot data that has been collected over time.

When they have graphed the data, the students need to examine the two lines and say whether their estimation skills are improving. Ask them what features of the graph will give them the information they need. In this case, it does not matter whether the lines are going up or down. (This is determined by the mass of the potatoes selected for weighing and the order in which they were weighed.) What matters is whether the two lines are *converging* because it is the gap between them that represents the difference between the estimated mass and the actual mass.

You could extend the activity by asking the students to predict what might happen if they were to estimate and weigh 5 more potatoes.

ACTIVITY TWO

This activity provides a suitable context for drawing and interpreting box-and-whisker plots from class data. While some types of graph (for example, bar graphs) plot the entire data set, a box-and-whisker plot summarises a distribution.

Box-and-whisker plots are a relatively recent and very useful statistical tool. The value of this kind of graph is that it shows five key values from a data set (the maximum, minimum, median, and quartiles) in a strikingly simple visual form. By looking at a box-and-whisker plot, a viewer can instantly get a feel for the spread of the distribution and, when two or more plots are placed in the same graph, can easily compare the spread of the different distributions.

Once the whole class's data has been collated by a couple of volunteers, the students need to calculate the median and quartiles. They can find the median by counting from either end of the distribution until they reach the middle. If the number of estimates is even, there will be two values sharing the middle ground, and the median will be the mean of these two.

The lower quartile is the median of the lower half of the data, while the upper quartile is the median of the upper half of the data. See how many different ways your students can state the implications of this division. For example, "50 percent of the data lies between the two quartiles."

There are different approaches to finding the median, depending on whether the data set has an even or odd number of numbers. Similarly, the number of data items affects the finding of the quartiles. Here are two examples:

- The set {12, 16, 17, 23, 27, 43} has 6 numbers, so the median is halfway between the third and fourth numbers, namely, 20. The quartiles are the middle values out of {12, 16, 17} and {23, 27, 43}, namely, 16 and 27.

- In the set {12, 16, 17, 23, 27, 43, 45}, the median is 23, and the quartiles are the middle values out of {12, 16, 17} and {27, 43, 45}, namely, 16 and 43. Some authorities say to include the median when you are finding the quartiles, in which case, they become the middle values out of {12, 16, 17, 23} and {23, 27, 43, 45}, namely, 16.5 and 35. Either way, the important idea is that we divide the data set into four equal sections as best we can.

Note that the lower quartile is sometimes called the first quartile and may be abbreviated as LQ or 1Q, and the upper quartile is sometimes called the third quartile and may be abbreviated as UQ or 3Q. 2Q is, of course, the median.

Having found the median and quartiles, the students use these (along with the minimum and maximum values) to create the box-and-whisker plot. Make sure they understand that the width (thickness) of the box on a box-and-whisker plot is of no consequence. They should choose a width that looks sensible, as they would for a bar graph, and then use the same width for all plots in the same graph. Whether the plots run horizontally or vertically is also of no significance.

When your students have completed their graphs, ask them to make as many true statements as they can about what each plot says and then to focus on a comparison between the plots. By doing this, they will be able to spot significant changes in the distributions of the estimates from potato 1 through to potato 5. By also plotting the actual mass of each potato on the same graph, the students can examine the estimates in relation to the actual values.

Are the students getting better with practice at estimating mass? Things to look for include:

- The boxes are getting shorter. 50 percent of the class's estimates are represented by this part of the plot.
- The whiskers are getting shorter. They represent the bottom and top 25 percent of the estimates. One "way out" estimate (outlier) can give a very long whisker.
- The distance from the median bar to the mark indicating the actual mass is becoming less.

Page 4 Wim's Waffles

Achievement Objectives

- collect and display time-series data (Statistics, level 4)
- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

When data is collected at intervals over a period of time, it is known as *time-series data*. Examples include the temperature (measured each hour during the school day) or the height of a bean plant (measured first thing each morning). This type of data is best shown on a line graph. The special value of time-series graphs is that they show what is happening over time and make it very easy for the viewer to spot trends and patterns. In this activity, the data plotted is the number of waffles Wim sells each day over a period of weeks.

Encourage your students to look for these features:

- The long-term trend. Ignoring all the little ups and downs, is the data remaining steady over time, or is it generally increasing or decreasing?

- Seasonal (cyclical) variations. Can you observe and describe a pattern that repeats itself each "season"? A season might be a week (as in this case, where data is collected each day), a year, a day, or any other period of time.
- Spikes. These are conspicuous variations from the seasonal pattern for which there are probably good explanations. For example, in week 4, Wim's Wednesday sales are exceptionally high; we don't know the reason for this, but Wim probably does.
- Random variation. This term covers fluctuations from the observable seasonal pattern for which there is probably no discernible explanation.

Question **1** asks for three types of graph. To draw them, the students need to set up two different spreadsheets. The first, for question **1a**, needs to have the data for the month in a single column. The second, for questions **1b** and **1c**, needs to have the data in the same format as the table in the students' book.

To create the time-series graph for question **1a**, select both columns of data (day and sales) from the spreadsheet, then click on Insert on the menu bar, select Chart, then the Line Chart option. Work your way through the formatting steps, adding a title and labels for the two axes.

Notice that the labels on the horizontal axis are between the "tick marks" and that the data points are plotted above the gaps rather than above the tick marks. You can change this by double-clicking on the horizontal axis, choosing the Scale tab, and then removing the tick symbol from beside "Value (Y) axis crosses between categories". To align the names of the days of the week vertically, click on the Alignment tab and either move the pointer to the vertical position or type 90 into the Degrees box. Click OK.

You can create the graphs for questions **1b** and **1c** by selecting all the cells that contain the data and then choosing the appropriate graph type.

Questions **2–3** ask the students to examine their graphs for seasonal variations (questions **2a** and **3b**), spikes (questions **2b–c**), and long-term trends (question **3a**). Once these have been identified and described, questions **4** and **5** ask the students to predict future sales and explain and justify their predictions. As they do the questions, they will find that the different graphs they have drawn are all useful for different purposes.

Note that the graph given for question **1a** in the Answers section shows a dotted *trend line*. Although not asked for, this line will help the students when they come to question **3a**. You can get the computer to add this line by going to the Chart menu, choosing Add Trendline, selecting the Linear option, and then clicking OK. But before showing them how to do this, get your students to print their graph out and add a trend line by hand. Get them to place their clear plastic ruler so that about half of the plotted points are above the ruler's edge and half are below. The distances of the points from the edge of the ruler, above and below, should also balance each other out as far as possible.

Once a trend line has been drawn, the students can base their predictions (for questions **4–5**) on it, but it is important that they also take seasonal variations into account. For example, when predicting the sales for Friday in week 4, the students must recognise that, generally, Fridays are just above the trend line. As with any prediction, the students should recognise that their answer is nothing more than a reasoned (intelligent) guess. Sales could plummet if the weather is poor, and in any case, no set of sales figures will keep on increasing forever.

Page 5 Often Absent

Achievement Objectives

- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)
- make statements about implications and possible actions consistent with the results of a statistical investigation (Statistics, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

This activity shows students that, when data that is found in one form (an attendance register) is translated into other forms (spreadsheets or different graphs), the patterns and the issues can become much clearer.

As with the previous activity, the students will have to create two spreadsheets if their spreadsheet for question **3a** is done in table form, where each column represents a day and each row a week. If the students read ahead to question **3b**, they will realise that, for a time-series graph covering the 4 weeks, the days of the week need to be in a column (the Monday–Friday sequence repeated for each week) and the absence data in another (preferably adjacent) column. For guidance on creating time-series graphs, see the notes on Wim's Waffles (page **4** of the students' book).

For the context of this activity to make sense to the students, they will need to view the data from the perspective of the teacher or principal. You could discuss with them how worthwhile it might be for a teacher (or principal) to keep attendance information on a spreadsheet.

Pages 6–7 Action and Reaction

Achievement Objectives

- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)
- collect and display time-series data (Statistics, level 4)
- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)
- find, and authenticate by reference to appropriate displays, data measures such as mean, median, mode, inter-quartile range, and range (Statistics, level 5)
- collect and display comparative samples in appropriate displays such as back-to-back stem-and-leaf, box-and-whisker, and composite bar graphs (Statistics, level 5)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In question **1**, students use a ruler for an interesting purpose: to measure reaction time. In question **2a**, they graph the data they have collected.

Question **2b** asks students to compare the mean, median, and mode. Each of these is a measure of *central tendency*, that is, the way the values in a distribution tend to cluster around a middle point. The mean, median, or mode for a distribution is a single number chosen to represent the entire data set. In any given situation, one of these measures will prove more useful than the others. For a fuller treatment of the mean, see the activity Just Average (pages **14–15** of the students' book) and the accompanying notes.

Question **3** introduces students to *truncation*. Sometimes outliers are valid and should be retained; sometimes they are not and should be discarded. In question **3**, trial number 7 is within the bounds of what is reasonable, so it should be retained, but trial 8 is unusually low so is probably an error. In diving competitions, scored by 10 judges, the highest and lowest scores are discarded. Discuss with your students why this automatic truncation might give a better (fairer) result.

Sometimes people choose to limit the length of each of the whiskers on a box-and-whisker plot to $1^1/_2$ times the length of the box. Data values outside these limits can be represented by a cross and considered outliers. Using this guideline in question **4**, there would be no outliers at the low end of the data, but the 25.7 at the upper end would count as one.

Question **4** asks the students to compare two different graphs. Histograms have an advantage over box-and-whisker plots in that they show where the clusters and the gaps are, and every member of the data set contributes to the shape. Box-and-whisker plots, on the other hand, only identify five individual members of the data set, but because they show the median and quartiles (neither of which is obvious on a histogram), they give a very good idea of the spread of the distribution.

Histograms look similar to bar graphs, but the bars on a histogram always touch. They are used when data *of the same kind* (either discrete or continuous) is grouped into equal intervals. For guidance on formatting computer-generated histograms, see the introductory section of these notes (pages **18–20**).

Page 8 Walking Tall?

Achievement Objectives

- plan a statistical investigation arising from the consideration of an issue or an experiment of interest (Statistics, level 4)
- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this activity, students investigate a hypothesis by comparing data obtained from two different groups. They learn that as a general principle, when comparing data from samples of different sizes, they should use percentages rather than raw data.

When they look at the graph they have created for question **1b**, your students should recognise that it is difficult to compare the two sets of data because the number of students in each group is very different. When they convert the frequencies in each interval to percentages and graph these percentages (in questions **2a–b**), they can make comparisons more easily.

To do question **2**, the students need to extend their spreadsheet and create a graph using columns 1, 4, and 5. Note that the values given in the students' book have been rounded to 1 decimal place, which is fairly standard for percentages. To do this in the spreadsheet, select the cells that contain the numbers to be rounded, click on Format on the menu bar, choose Cells, then the Number tab, and finally, select "Number" from the list of alternatives. Change the number in the small "Decimal places" window to 1.

Encourage your students to use a formula to convert the raw data to percentages. The following spreadsheet shows how this can be done using =B2/B$9*100. Note the "$" that has been inserted into the B9 cell reference. This lets the program know that the value found in cell B9 (in this case, the total number of boys who walk or bike) is a *constant* (that is, it remains the same in all calculations using this formula).

D2 = =B2/B$9*100

Walking Tall?.xls

	A	B	C	D	E
1	Height (in centimetres)	Boys who walk or bike	Boys who use a bus or car	Percentage of all walkers and bikers	Percentage of all bus and car users
2	less than 140	2	2	0.8	
3	140 ≤ h < 150	23	3		
4	150 ≤ h < 160	48	20		
5	160 ≤ h < 170	75	34		
6	170 ≤ h < 180	66	23		
7	180 ≤ h < 190	29	11		
8	190 ≤ h < 200	8	1		
9		251	94		

Use the cursor to grab the bottom right-hand corner of cell D2 and drag it down the next 7 lines. Release the mouse button, and the maths is done! Use a similar process to complete the last column.

If you want to select cells from non-adjacent columns (which can be important when graphing just part of the data in a spreadsheet), select the first group of cells you need, hold down the CTRL key (or the Apple key, if using a Macintosh) and select whatever other cells are required.

Question 3 may be the first time that your students have encountered the use of a hypothesis as the basis for a statistical investigation, so discuss the concept with them. When developing a hypothesis or framing a question, they should think about the kind of data they would need to collect in order to prove or disprove it and how they would use that data to reach a conclusion. The following is a useful framework for an investigation.

1. Clarify the hypothesis or question. Ask: What is the investigation really about? Can I break it into steps? What information do I already have, and what further information do I need?
2. Devise a strategy for the investigation. Ask: Where can I find the data I need? How will I gather it? What resources or equipment do I need?
3. Carry out the plan. Collect data, make calculations, and create suitable graphs.
4. Draw conclusions. Analyse the calculations and graphs, outline the findings, and give an answer or conclusion to the investigation.
5. Report back. This can be done in verbal or written form and may include tables, charts, and displays.
6. Evaluate by asking: Have I investigated the problem as thoroughly as I could? What problems did I encounter, and how did I overcome them? What would I do differently next time?

Price Hike

Achievement Objectives

- evaluate others' interpretations of data displays (Statistics, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)
- increase and decrease quantities by given percentages, including mark up, discount, and GST (Number, level 5)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this activity, students calculate percentages in different ways and meet compounding percentages, perhaps for the first time. Percentage is an extremely useful concept, but students often have trouble applying it correctly.

In question **1a**, the students use percentages to compare two amounts. They should divide the "new" (current) value by the "old" (original) value.

Question **1b** is about increasing an amount by a given percentage. There are different ways of doing this:

- Find 20 percent of the amount and then add it to the original amount.
- Multiply by 120 and divide by 100.
- Multiply by 1.2, which is 120 ÷ 100 in its simplest form.

It is important that students come to see that multiplying by a percentage greater than 100 is the same as calculating a percentage increase and then adding it to the original amount. You may choose to discourage the use of the percentage key on calculators, because students tend to use it without understanding what it does.

In question **2**, a spreadsheet formula can be used to adjust the $90 for the annual rate of inflation. In the spreadsheet below, the formula =B3*1.05 has been entered in cell C3, then the cursor used to drag the bottom right-hand corner of this cell across the row to fill the cells D3 to G3 with the inflation-adjusted values. Other formulae would have worked equally well: =B3 + B3*0.05, or =B3*105/100.

C3 =B3*1.05

Household Electricity Bill.xls

	A	B	C	D	E	F	G
1	Number of years ago	5	4	3	2	1	0
2	Average monthly bill in $	90	90	100	100	100	120
3	$90 adjusted for 5% annual inflation	90.00	94.50	99.23	104.19	109.40	114.87

In question **3**, the students create a time-series graph using the spreadsheet data from question **2**. The graph should have two lines on it: one showing the average monthly electricity bill and the other showing $90 adjusted for 5 percent annual inflation. One unusual feature of this graph is that the horizontal axis is reversed and ends with 0. The introductory section on histograms (pages **19–20**) explains how to get the axis labelled correctly.

Questions **4** and **5** involve interpreting data in different ways, depending on a person's purpose or perspective. See the Answers section for possible interpretations.

As an extension, the students could collect graphs from newspapers or magazines and see whether they could interpret them differently from the accompanying article. They could, for example, put themselves in the position of the boss instead of the employee, the house seller compared with the house buyer, the politician compared with the voter, or the manufacturer compared with the consumer.

Pages 10–11 Population Pyramids

Achievement Objectives

- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)
- evaluate others' interpretations of data displays (Statistics, level 4)
- make statements about implications and possible actions consistent with the results of a statistical investigation (Statistics, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

The population pyramids in this activity show changes and projected changes to the make-up of our population over a 200-year period.

The pyramids are back-to-back histograms placed on their sides. Back-to-back graphs such as these are useful for comparing two related sets of data in the same format, in this case, male and female population data for several different years. Examine with your students how the axes are drawn and labelled and how the bars are drawn.

You should explain to your students that, although the pyramids are similar to histograms, each bar represents the proportion of the population, not the number of people, in that particular age band. This means that the sum of the lengths of the bars in any population pyramid will be 100 whether the population is 1 000 000 or 100 000 000. You could further explain this point by asking the students to compare the bottom bars of the 1901 and 2001 pyramids. They may think that, because the bars are much longer for 1901, there were a lot more children around then. What they show, however, is that about 11.5 percent of the population was aged 0–4 in 1901, but only 6.6 percent was in 2001. An interested student could investigate whether 11.5 percent of the 1901 population was more or less than 6.6 percent of the 2001 population.

The pyramids for 1901 and 2001 contain hard data; those for 2051 and 2101 are projections based on assumptions. These assumptions concern the three main factors that determine whether a population increases or decreases over time. You could ask your students what they think these would be. They are: fertility (the birth rate), life expectancy (mortality), and migration. You could go on to discuss with your students what other factors affect these factors and which of the three is the most volatile (likely to change in unpredictable ways).

The footnote to the 2051 pyramid explains that it is based on what Statistics New Zealand calls its Series 4 projections. This set of projections is one of eight. The series assumes that:

- women will have 1.85 children each on average (below the 2.1 children required for the population to replace itself without migration)
- life expectancy at birth will increase by about six years between 2001 and 2051
- there will be a net migration gain of 5 000 people each year (the average annual level for the last 100 years) from 2007 onwards.

Statistics New Zealand has put its data from 1951 to the present day and its Series 4 projections from the present day to 2051 onto its website as a downloadable database. Using this database, your students can find the data for each age band, by gender, for each year within the range. They can call up a population pyramid for any year or can select a group of cells from the database and create their own graphs without having to re-enter data. They can also find out more about the processes involved in making demographic projections. The main website is at www.stats.govt.nz. The Series 4 data can be found by using the following link and then clicking on the icons for Series 4 tables.xls and Age-sex pyramids.xls: www.stats.govt.nz/domino/external/web/prod_serv.nsf/htmldocs/National+Population+Projections+Tables

Statistics New Zealand have education officers who produce resources such as the *Statzing!* leaflets (available for both primary and secondary levels) and send them to schools. The major CensusAtSchool project carried out by Statistics in 2003 provides an enormous database of real information that is relevant to young people aged 8–15 and that you can access. The New Zealand database is part of a much wider international database that you can also access. Check out the Schools' Corner at www.stats.govt.nz/schoolscorner

CROSS-CURRICULAR LINKS

Social Studies

The students' book suggests an investigation that could form part of a unit on either social change or the future.

Achievement Objectives

Students will demonstrate knowledge and understandings of:

- how people organise themselves in response to challenge and crisis (Social Organisation, level 4)
- causes and effects of events that have shaped the lives of a group of people (Time, Continuity, and Change, level 4)

Pages 12–13 Testing Times

Achievement Objectives

- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)
- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)
- find, and authenticate by reference to appropriate displays, data measures such as mean, median, mode, inter-quartile range, and range (Statistics, level 5)
- collect and display comparative samples in appropriate displays such as back-to-back stem-and-leaf, box-and-whisker, and composite bar graphs (Statistics, level 5)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

ACTIVITY

Testing Times shows how a different interpretation can often be put on data, depending on the measures used to interpret or display it.

Stem-and-leaf graphs are introduced in question **1**. They are particularly useful when working with 2-digit numerical data because they reduce the amount of transcribing while revealing information about the spread or shape of the distribution. When a stem-and-leaf graph is rotated 90 degrees, it is very much like a histogram.

It is easy to find the median or quartiles from a stem-and-leaf graph. First, count the total number of data items in the distribution. To find the median, count from the top or bottom (or both) until you locate the middle number. To find the quartiles, count from the top or bottom until you locate the number that is one-quarter of the way from the beginning or the end of the distribution. The mode is the number that appears most often. As always, the mean can only be found by calculation.

Question **2** asks the students to draw a bar graph, using the same data, so that they can compare the relative usefulness of the two types of graphs. See the Answers for a discussion of their comparative merits. The stem-and-leaf graph in question **3** is back-to-back in a way that is similar to the population pyramids in the previous activity.

The value of these tasks is that they demonstrate the fact that no single graph or measure can "do it all"; each reveals something different. If Ms Lewis were using only the back-to-back stem-and-leaf graph or the median or mean, she would almost certainly be pleased with her students' progress. But a scatter plot gives less reason for such confidence.

While other graphs typically summarise data, a *scatter plot* displays each individual item separately. The location and density of clusters give the "macro" picture, but the story of each individual piece of data is also present in the individual points. Scatter plots are particularly suited to bivariate data, that is, situations where we have two pieces of information about the same person, creature, or object.

In this case, one group of points represents those who did better in the first test than the second, and another represents those in the reverse situation. It turns out that exactly half of Ms Lewis's class improved, while the other half regressed. This is explained in the Answers.

Pages 14–15 Just Average

Achievement Objectives

- find, and authenticate by reference to appropriate displays, data measures such as mean, median, mode, inter-quartile range, and range (Statistics, level 5)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)
- generalise mathematical ideas and conjectures (Mathematical Processes, developing logic and reasoning, level 5)

ACTIVITY ONE

By now, your students will have had some experience of calculating the means of different data sets. This task challenges them to investigate the way in which each item in a distribution influences the mean (the one value selected to represent the whole distribution). To do this, they must work backwards from the mean to the data set.

Note that the term *average* is used here instead of *mean* because it is more commonly used in such contexts.

The conversation on page **14** sets up the activity, and the students should read it to find the following "mean messages":

- Although the mean is a kind of "middle", the amount of numbers above and below the mean can be very different. (Unlike the median, the mean does not usually split a distribution into two equal halves.)
- The mean of a distribution in which all the numbers are similar can be exactly the same as the mean of a distribution in which the numbers are very different.
- Zero is a number, and the inclusion of 0 in a distribution affects the mean.

The "mean messages" in question **1** are:

- If you add or subtract the same amount to or from every number, you do the same to the mean.
- Changing the order of the numbers in a distribution has no effect on the mean.
- If you multiply or divide each number in a distribution by an amount, the mean is multiplied or divided by the same amount.

- If you alter the numbers in a distribution in compensatory way (for example, if you add 8 to one number and take 8 off another), the mean will not change.

You could encourage your students to answer questions **1–2** by reasoning and then to check their reasoning by calculation.

Questions **2** and **3** challenge the students to think in reverse and to generalise their findings.

ACTIVITY TWO

Like the questions in the first activity, question **1** helps students to understand the effect on the mean of adding further numbers to a data set. The Answers work through the mathematics of the cricket game step by step. Question **2** challenges students to take care when using the concept of "below average".

Page 16 Suspect on Foot

Achievement Objectives

- use data displays and measures to compare data associated with different categories (Statistics, level 5)
- find, and use with justification, a mathematical model as a problem-solving strategy (Mathematical Processes, problem solving, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

This activity introduces students to the idea that scatter plots can be used to find relationships between two sets of data (in this case, between height and footprint length). Data of this kind is said to be *bivariate*.

Because scatter plots plot a point for each individual piece of data, it is easy to see whether there are any patterns in the data. If there were none, the data would be spread evenly across the entire area of the graph. Wherever the points "bunch" (that is, where they are more dense) or form a line of any sort, the pattern suggests a relationship.

This leads to the concept of *correlation*, that is, the relationship or connection between two sets of data. For instance, does height increase as foot size increases? If so, there is said to be a positive correlation between these two variables. If one thing decreases as another increases, there is said to be a negative correlation. Often there is no correlation at all.

Where there is a correlation, the students should be wary of assuming that a change in one variable is *causing* a change in the other. Correlation does not imply causation. A third factor may be influencing both variables. For example, a person may find there is a correlation between their indigestion and their consumption of sun-dried tomatoes. But the cause of the problem may be the olives that they always eat when they eat the tomatoes.

When the students are entering their data in a spreadsheet (question **1b**), they should work down the spreadsheet until all 30 rows of data are in place. (The data in the students' book has been shown side by side to save space.)

Question **1c** asks the students to use their graph to make predictions about height based on footprint length. Ask them to manually put a trend line on their scatter plot before they get the computer to do this so that they can compare the two lines. For information on trend lines, see the notes for Wim's Waffles (page **4** of the students' book).

When estimating the height of the burglar, the students should give a range rather than a precise value. This recognises the fact that, while there is some connection between height and footprint, it can't be expressed as a precise rule.

Once they have established that there is some kind of relationship, the students could extend their trend line to predict the height of someone with much bigger feet than those shown. This is known as *extrapolation* (the making of predictions that go beyond the range of available data). Making predictions within that range is known as *interpolation*.

Page 17 Slater Mazes

Achievement Objectives

- determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (Statistics, level 5)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY ONE

This activity involves calculating the *theoretical probability* for the possible outcomes of an experiment. Although the slaters would not necessarily go down the various possible paths in the way suggested, the activity provides an interesting context for a probability investigation, especially as the mazes are, in effect, tree diagrams.

The activity reinforces the fact that some situations demand the ability to multiply and add fractions. However, if the students have a major problem with the fraction operations, it is possible to sidestep this by asking them to think in terms of a whole number of slaters (for example, 24) starting at the top and then tracking them through the various paths. Each time the slaters reach a junction where the path divides into two equally likely paths, half go down one and half go down the other. So if 24 approach a fork, 12 will go one way and 12 the other. If one of these groups of 12 approaches a second fork, 6 will go one way and 6 the other.

If they find this activity difficult, you could set it up as a game, using a dice and counter or a coin and counter to simulate the slater decision-making process. The students can play the game many times, keep details of the results, and collate them with the results of other students.

To do this, you will need copies of each maze on paper or card. The students place a counter at the starting point to represent a slater and then toss a coin. If the coin is a "head", the slater chooses the left path; if a "tail", the right path. The students toss the coin again each time the slater is faced with a choice. (Remind them that going back up the maze is not allowed.) When the slater reaches an exit, the students put a tally mark by that exit or on a table that has a column for each exit.

Suggest to the students that they play each game 24 times and record how many times the slater comes out of each exit. Ask them to examine their results and suggest, for the particular maze that they used, why they got the results they did. If there are others playing the game, suggest that they collate all their results so that a larger data set is obtained. The greater the number of results, the more likely it is that they will mirror the expected (theoretical) probabilities.

Pages 18–19 Rough Justice

Achievement Objectives

- determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (Statistics, level 5)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- make conjectures in a mathematical context (Mathematical Proceses, developing logic and reasoning, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this fantasy scenario, students work out the theoretical probability of each outcome. To do this, they should be learning to use fractions, including fractions of fractions and the addition of fractions. If you wish, however, you could adapt the activity so that the students use whole numbers, as suggested for the previous activity (page **17** of the students' book). In the adapted version, the students would visualise a lot of hapless victims making their choices over time and would keep a tally of their decisions.

Given that the prisoners have no external clues on which to base their decisions, the alternatives are always considered equally likely. If 12 prisoners were to face the choices provided in layout **ii**, 6 could be expected to go straight to the tiger and the other 6 into the room with 2 doors. Of these 6, 3 could be expected to choose the next room with the tiger in it while 3 would go free with a bag of gold. So out of the 12 who started, we could expect 9 to get eaten while 3 would go free.

If you used this simplified approach, you could go on to encourage your students to think in terms of fractions: $^1/_2$ would be condemned at the first door, and of the $^1/_2$ who end up in the next room, $^1/_2$ would be condemned while "$^1/_2$ of the $^1/_2$" would go free. That is, $^1/_4$ would go free while $^1/_2 + ^1/_4 = ^3/_4$ would be condemned.

Tree diagrams are an alternative way of modelling the probabilities in this activity. See the Answers for these. You will need to explain the rationale for multiplying the probabilities along each branch of the tree. If the probability of choosing a branch is $^1/_2$ and the probability of choosing a branch off the first one is $^1/_3$, the probability of choosing both branches is "$^1/_3$ of a $^1/_2$" or $^1/_6$.

The *experimental* probabilities in this activity could be explored as a game, as suggested for Slater Mazes, this time using a dice to make the decisions. If there are 2 alternatives, the numbers 1–3 could indicate choice 1, and 4–6 could indicate choice 2. If there are 3 alternatives, assign 2 numbers to each of the 3 different choices. By collating their results with those of other students, they can quickly obtain a sizeable body of results. The students can then be encouraged to explain the reasons behind their results. In this way, they can make the link between experimental results and theoretical probability.

In question **2**, students may design complicated room arrangements and then find that they are unable to determine the theoretical probabilities for them. If so, they may need to use experimental probability and go through the process as outlined for question **1**.

Page 20 Unlucky Lines

Achievement Objectives

- determine probabilities of events based on observations of long-run relative frequency (Statistics, level 5)
- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- find, and use with justification, a mathematical model as a problem-solving strategy (Mathematical Processes, problem solving, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this activity, students find the experimental probability of a coin landing between the lines on a grid and then try and discover the theoretical basis for their results. As always with practical experiments involving probability, they should realise the importance of obtaining a large number of results (the larger the better). Collating results from a number of students is a good way of getting a large body of results in a short space of time.

Students' explanations for the results of this experiment (question **1c**) will vary according to their level of mathematical understanding. They will need good reasoning skills to justify their explanations mathematically. A full explanation is given in the Answers.

The students will probably want to use a trial-and-improvement approach to question **2**. Starting in this way, they will soon realise that the mathematical solution is related to the area that can be occupied by a winning coin compared with the total area of each square in the grid. Once they realise that the winning area is defined by the locus of the centre of the coin, the question becomes a measurement problem as much as a probability one. (See the diagram in the Answers.)

For your reference, this table shows how the probability changes as the size of the grid increases:

Side of square	Win area	Win area as a fraction	Win area as a decimal
40 mm	289 mm^2	$^{289}/_{1\,600}$	0.18
50 mm	729 mm^2	$^{729}/_{2\,500}$	0.29
60 mm	1369 mm^2	$^{1\,369}/_{3\,600}$	0.38
70 mm	2209 mm^2	$^{2\,209}/_{4\,900}$	0.45
80 mm	3249 mm^2	$^{3\,249}/_{6\,400}$	0.51

Note that, although the concept of locus is not mentioned in the achievement objectives, it is included as a suggested learning experience for Geometry, level 4.

On a Plate

Achievement objectives

- find all possible outcomes for a sequence of events, using tree diagrams (Statistics, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

This activity introduces students to a systematic approach for finding the number of possible outcomes in a situation, which can be used when tree diagrams are inappropriate because the number of options is too great. The activity also shows that a large number of possibilities can be generated from a small number of choices.

If your students are new to problems involving arrangements, you may wish to introduce the activity using a reduced number of letters and digits, for example, 2 letters followed by up to 2 digits. This makes it easy for the students to list all the arrangements and to begin looking for patterns.

Sometimes the order in which a number of items is arranged is very important; in other situations, it is irrelevant. If a cook is shopping for a meal, it makes no difference what order they write the items on a shopping list. But when that cook is writing out the menu, the order does make a difference. When order doesn't matter, we use the word *combination* to refer to the different possibilities. When order does matter, we refer to the different possibilities as *permutations*.

In non-mathematical contexts, "arrangement" is an alternative for "permutation", but "combination" is often used to cover all situations, whether order matters or not. Don't feel that you have to discuss these technicalities with your students, but you should try and ensure that when you use "combination" in a mathematical context, you use it in the mathematically correct sense!

When finding all possible outcomes, the students first need to decide whether order is important – the approach used for combinations is different from the approach for permutations. On a number plate, 12 is different from 21, so this activity is about permutations.

It may not take the students long to realise that questions **1** and **2** can be solved by multiplying together the number of choices at each step. This is sometimes known as the *multiplication principle*. Plenty of simple examples can be devised, using menus, to illustrate the way in which this principle works. Family Feast and Catch of the Match, pages 17–18, *Statistics: Book One*, Figure It Out, Years 7–8, would be an excellent introduction to this activity. The teachers' notes for these two activities discuss permutations in greater detail.

Page 22 Card Sharp

Achievement Objectives

- determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (Statistics, level 5)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- generalise mathematical ideas and conjectures (Mathematical Processes, developing logic and reasoning, level 5)

ACTIVITY

In this activity, the students calculate probabilities and mark them on a number line. This helps them to visually compare the probabilities of various outcomes. The main concept (and one that is likely to be new to many students) is that the word "or" always increases the probability of success (because the number of acceptable outcomes is increased), whereas "and" always reduces the probability of success (because there are now more conditions to satisfy). An explanation of the principle is given in the Answers for question **4**.

Although the students are told to work with a classmate, they could equally well work by themselves and then discuss their answers. You should have a pack of cards available for students to refer to.

When the students are counting the possible outcomes for an event, they will need to take care not to double-count any. For example, when they are considering the probability of getting a red card or an ace, they will find there are 26 red cards and 4 aces (a total of 30). However, because there are 2 red aces (cards that are red *and* aces), these cards must only be counted once.

When doing question **1**, the students need to draw a number line that goes from 0 to 1. They can divide this line into either a multiple of 10 parts or a multiple of 13 parts. If they use a line with 10 or 20 divisions, they will need to convert the fractions to decimals before they can locate an event on the line. If they use 13, 26 or 52 divisions, they will find the placement easier, but they may have trouble understanding that all these awkward-looking fractions lie between 0 and 1.

In the activity, the card is returned and the pack is re-shuffled before another card is taken. Interested students may like to explore the situation in which the card is not returned to the pack before another is taken. They could investigate this by using a small number of cards (for example, 2 aces and 3 kings) and looking at what happens when they draw 2 cards without replacing them. They could then consider the probability of getting 2 aces in a row, starting with a full pack. If the first card is returned before the second is taken, the probability is $^4/_{52} \times {}^4/_{52} = {}^{16}/_{2704} = 0.0059$. If the first card is not returned, the probability is $^4/_{52} \times {}^3/_{51} = {}^{12}/_{2652} = 0.0045$. The probability of getting 2 aces in a row is therefore greater if the first card is replaced before the second is taken.

Page 23 Paper, Scissors, Rock

Achievement Objectives

- find all possible outcomes for a sequence of events, using tree diagrams (Statistics, level 4)
- determine probabilities of events based on observations of long-run relative frequency (Statistics, level 5)
- predict the outcome of a simple probability experiment, test it, and explain the results (Statistics, level 5)
- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

Question **1** asks students to play a variation of the old paper, scissors, rock game while keeping a record of their results so that they can estimate the probabilities involved and determine the fairness of the scoring system.

In question **2a**, the students should use a method that ensures they don't miss any of the possible outcomes. The simplest method is to number the players 1–3, assume that player 1 chooses paper, then list each of player 2's choices, matched each time with all of player 3's choices. This gives {PPP, PPS, PPR, PSP, PSS, PSR, PRP, PRS, PRR}. By replacing player 1's choice with scissors, then rock, the list of 9 outcomes becomes a complete list of 9 x 3 = 27 outcomes.

A tree diagram is the best alternative to a list of outcomes (question **2b**). The first set of branches will represent the 3 choices open to player 1, the second set to player 2, and the third set to player 3. The students should label the end of each branch with the outcome it represents. The top branch will be PPP, and the bottom branch RRR.

Tree diagrams can get very crowded and difficult to draw. The students should therefore use pencil and do a little planning before they start. Sometimes an incomplete tree (as in the Answers) is sufficient to establish the pattern, and there is little added value in completing the entire tree if the other sections have an identical structure. Most students will find that they can draw a tree diagram more quickly and more tidily if they use a computer drawing program. For other activities involving tree diagrams, see Family Feast and Catch of the Match, pages 17–18, *Statistics: Book One*, Figure It Out, Years 7–8.

In question **3**, the students should realise that there is a different number of winning outcomes for each player. Therefore, the game is not fair. In question **3c**, the students can change the number of points assigned to each of the winning outcomes so that each player does get a fair deal.

As in some of the earlier activities in this book, the students can explore the probabilities at stake by accumulating experimental results. This can be an effective way for them to gain confidence with probability.

Birth Months

Achievement Objectives

- determine probabilities of events based on observations of long-run relative frequency (Statistics, level 5)
- determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (Statistics, level 5)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)
- generalise mathematical ideas and conjectures (Mathematical processes, developing logic and reasoning, level 5)

ACTIVITY

This activity is a simplified version of the well-known "birthday paradox". This asserts that if you put 23 people together in a room, there is a 50 percent probability that at least 2 of them will share the same birthday. If there are 30 in the room, the probability is 70 percent. And if there are 50 in the room, the probability is 97 percent. This is not what people intuitively expect. In fact, given that there are 365 days in the year, they tend to be extremely surprised when they find themselves in the company of someone who shares their birthday. This is why it is referred to as a paradox: something that seems impossible but is nevertheless true.

The students in this activity work with months (12 possibilities), which is much easier than actual birthdays (365 possibilities). They carry out what is, in effect, a simulation designed to discover experimentally the probability that 2 (or more) out of a group of 5 will share the same birth month.

When 30 results have been recorded, the students use them to calculate the *long-run relative frequency* of the desired outcome (that is, at least 2 cards have matching months). The calculation is done by putting the number of "successes" over the number of trials.

Long-run relative frequency is especially useful in situations where the theoretical probability of an outcome can't be calculated (though in this case, it can). After 30 trials, the long-run relative frequency should be fairly close to what would be obtained by calculation. As always, the key to a more accurate value is a larger pool of results. The more times a simulation (experiment) is performed, the closer the relative frequency will be to the theoretical probability.

You may have students in your class who are interested in knowing how to find the theoretical probability of finding people with shared birth months in a group of 5. If so, start by explaining what is meant by *complementary* events. Two events are complementary if, when one does not happen, the other does. So the sum of the probabilities of two complementary events is always 1.

Sometimes it can take many calculations to find the probability of an event, and it may be a lot simpler to instead calculate the probability of the event *not* happening. You then take the probability of the event *not* happening away from 1 to find the probability of the event happening. Simple! In this activity, it is easiest to consider the probability that none of the 5 people share the same birth month.

Person 1 could have a birthday in any month in the year ($^{12}/_{12}$). Person 2's birthday could then fall in any of the remaining 11 months ($^{11}/_{12}$). Person 3's birthday could fall in any of the remaining 10 months ($^{10}/_{12}$), person 4's in the remaining 9, and person 5's in the remaining 8 ($^{9}/_{12}$ and $^{8}/_{12}$). So the probability that there are no matches is $^{12}/_{12} \times {}^{11}/_{12} \times {}^{10}/_{12} \times {}^{9}/_{12} \times {}^{8}/_{12} = 0.382$. The probability that at least 2 out of the 5 do share a birth month is found by taking this probability away from 1, that is, 1 – 0.382 = 0.618, or 62 percent (to the nearest whole percent).

ACKNOWLEDGMENTS

Learning Media and the Ministry of Education would like to thank Tim Burgess and Brenda Bicknell (Senior Lecturers, Department of Technology, Science and Mathematics Education, Massey University) for developing these teachers' notes and reviewing the answers.

The photograph on the front cover is by Mark Coote and Adrian Heke. The illustration of the boy on the front cover and the side strip on page 2 are by Ali Teo. The side strip on page 17 is by Steven Chen.

Series Editor: Susan Roche
Editor: Ian Reid
Series Designer: Bunkhouse graphic design
Designer: Rose Miller

Published 2004 for the Ministry of Education by
Learning Media Limited, Box 3293, Wellington, New Zealand.

Dewey number 519.5
ISBN 0 7903 0164 4
Item number 30164
Students' book: item number 30158