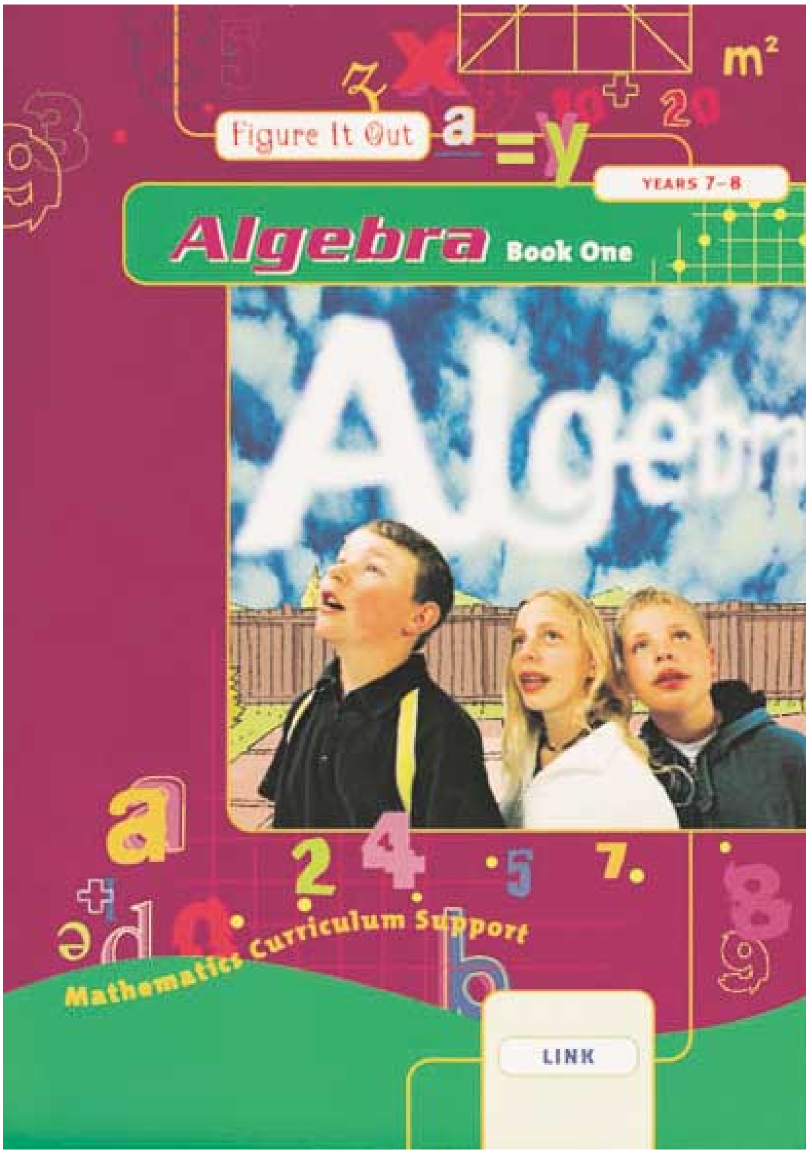
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books
The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:
Number (two linking, three level 4, one level 4+, distributed in November 2002)
Number Sense (one linking, one level 4, distributed in April 2003)
Algebra (one linking, two level 4, one level 4+, distributed in August 2003)
Geometry (one level 4, one level 4+, distributed in term 1 2004)
Measurement (one level 4, one level 4+, distributed in term 1 2004)
Statistics (one level 4, one level 4+, distributed in term 1 2004)
Themes: *Disasters Strike!, Getting Around* (level 4–4+, distributed in August 2003)

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent bookwork, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes
The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comment on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in your classroom
Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Algebra: Book One

Page 1 Something Adds Up

ACTIVITY

1. a. Sets will vary, but in each pair, the ones digits will add up to either 0 or 10, that is, the possible pairs of ones digits are 0 and 0, 1 and 9, 2 and 8, 3 and 7, 4 and 6, or 5 and 5. Note that, for every pair of numbers except those where the ones digits are both 0, the tens digits must add up to 9 (because you are adding another 10 from the ones digits).

   b. Possible answers include:
   The ones digits must add up to a multiple of 10, either 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, or 100.
   If the ones digits add up to 0, the tens digits add up to 10; if the ones digits add up to 10, the tens digits add up to 9; if the ones digits add up to 20, the tens digits add up to 8; and so on.

2. The ones places must add up to 0 or 10.
   If the ones places add up to 0, the hundreds and tens places must add up to 100, for example, 630 + 370.
   If the ones places add up to 10, the hundreds and tens places must add up to 99, for example, 286 + 714.

Pages 2–3 Pyramid Parts

ACTIVITY

1. a. Yes.

   b. The middle number is added twice, and each outside number is added once (2 x 5 + 6 + 4 = 20 or 4 + 2 x 5 + 6 = 20).

2. a. i. 19

   ii. 27

   iii. 24

   iv. 67

   b. A possible explanation is: for 3 tiers, add the outside numbers to 2 times the inside number. For 4 tiers, add the outside numbers to 3 times the sum of the inside two numbers.

3. a. 5

   b. 2

   c. 5

4. a. She noticed that the numbers in opposite corners of a calendar rectangle add up to the same total.

   b. In a calendar, the numbers across a row are in numerical order (1, 2, 3, 4, …) and the difference between each number in a column is 7 (2, 9; 3, 10; and so on). This means that for a 2 by 2 rectangle, *a* is 1 less than *b* but *d* is 1 more than *c*:

a	*b*
c	*d*

   This ensures that $a + d = b + c$.
   For a 2 by 3 rectangle, *a* is 2 less than *c* but *f* is 2 more than *d*:

a	*b*	*c*
d	*e*	*f*

   For a 3 by 4 rectangle, *a* is 3 less than *d* but *l* is 3 more than *i*:

a	*b*	*c*	*d*
e	*f*	*g*	*h*
i	*j*	*k*	*l*

   So it does work for all calendar rectangles with 2 sets of 2 numbers or 2 sets of 3 numbers.

Pages 4–5 Cube Signs

ACTIVITY

1. a. 4 x 5 + 1

   b. Each plus sign has 4 arms, and there is 1 cube in the centre. In the third plus sign, there are

3 cubes in each arm. So, altogether, this sign has 4 x 3 + 1 cubes. The fifth plus sign has 5 cubes in each arm. So, altogether, the sign has 4 x 5 + 1 cubes.

c. 4 x 7 + 1 = 29

d. The seventh plus sign has 4 arms, each with 7 cubes, and there is 1 cube in the centre of the plus sign. Altogether, there are 29 cubes in this sign.

2. a. Kali sees the plus sign as a horizontal strip of 7 cubes and a vertical strip of 7 cubes, which is 2 sets of 7 cubes. But the centre cube has then been counted twice, and so 1 cube must be removed or subtracted. So the short cut is 2 x 7 – 1.

b. 2 x 19 – 1 = 37

3.

Plus sign	Number of cubes	
	Alan's short cut	Kali's short cut
3rd	4 x 3 + 1 = 13	2 x 7 – 1 = 13
5th	4 x 5 + 1 = 21	2 x 11 – 1 = 21
7th	4 x 7 + 1 = 29	2 x 15 – 1 = 29
9th	4 x 9 + 1 = 37	2 x 19 – 1 = 37
20th	4 x 20 + 1 = 81	2 x 41 – 1 = 81
100th	4 x 100 + 1 = 401	2 x 201 – 1 = 401
56th	4 x 56 + 1 = 225	2 x 113 – 1 = 225

4. a. One short cut is 4 x 4 + 2 = 18, and another is 2 x 9 = 18. Other short cuts are possible. In the first short cut, 4 x 4 + 2 = 18, there are 4 arms with 4 cubes, plus 1 extra cube at the bottom and another at the centre. In the second short cut, 2 x 9 = 18, there are 9 cubes in the horizontal arm and 10 – 1 cubes in the vertical arm (because the centre cube has been counted in the horizontal arm).

b. 4 x 75 + 2 = 302 cubes using the first short cut or 2 x 151 = 302 cubes using the second short cut

5. a.–b. The fifth times sign has 4 arms, each with 6 cubes. There is 1 cube in the centre of the times sign, so the short cut 4 x 6 + 1 = 25 does work.

c. Answers may vary. One possible short cut is 4 x 5 + 5 = 25. A model that matches this short cut might have 4 arms, each with 5 identical cubes, and then a further 5 cubes, 1 for the centre and 1 on the end of each arm.
Another possible short cut is 4 x 7 – 3. This is 4 arms of 7, minus 3 for the 3 arms in which the centre cube is subtracted.

d.

Times sign	Number of cubes	
	Kali's short cut	A possible short cut
1st	4 x 2 + 1 = 9	4 x 1 + 5 = 9
3rd	4 x 4 + 1 = 17	4 x 3 + 5 = 17
5th	4 x 6 + 1 = 25	4 x 5 + 5 = 25
10th	4 x 11 + 1 = 45	4 x 10 + 5 = 45
30th	4 x 31 + 1 = 125	4 x 30 + 5 = 125
500th	4 x 501 + 1 = 2 005	4 x 500 + 5 = 2 005

Pages 6–7 Fencing

ACTIVITY

1. a. Tyson is right. A 1-section fence has 2 posts and 3 rails; a 2-section fence has 3 posts and 6 rails; and a 3-section fence has 4 posts and 9 rails. So the number of posts is always 1 more than the number of sections.
A 10-section fence has 10 + 1 = 11 posts.
Each section has 3 rails, so a 10-section fence has 10 x 3 = 30 rails.

b. 37 + 1 = 38 posts, and 37 x 3 = 111 rails.

2. a. 6 sets of 3 or 6 x 3 rails

b. For a fence with 7 posts, you can think of sets of 3 rails being attached to just 6 of the posts (1 fewer than the total number of posts).

c.

Number of posts	Number of rails
4	9 (3 x 3)
5	12 (4 x 3)
27	78 (26 x 3)
95	282 (94 x 3)
168	501 (167 x 3)

3. a. Tyson is right. The 27 rails are attached to 27 ÷ 3 = 9 posts, but another post is needed to complete the fence. So there are 10 posts altogether.

b.

Number of posts	Number of rails
(27 ÷ 3) + 1 = 10	27
(45 ÷ 3) + 1 = 16	45
(93 ÷ 3) + 1 = 32	93
(102 ÷ 3) + 1 = 35	102
(495 ÷ 3) + 1 = 166	495

4. a. A short cut is 5 x 4 rails because 5 of the 6 posts have 4 rails attached.

b.

Number of posts	Number of rails
3	2 x 4 = 8
4	3 x 4 = 12
6	5 x 4 = 20
10	9 x 4 = 36
80	79 x 4 = 316
100	99 x 4 = 396

Number of posts	Number of rails
(16 ÷ 4) + 1 = 5	16
(40 ÷ 4) + 1 = 11	40
(76 ÷ 4) + 1 = 20	76
(120 ÷ 4) + 1 = 31	120
(356 ÷ 4) + 1 = 90	356
(4 000 ÷ 4) + 1 = 1 001	4 000

5. a. 5 x 8 + 1 = 41. A fence with 4 posts has 3 sets of 8 palings and 1 additional paling, which is 3 x 8 + 1 palings. So a 6-post fence will have 5 x 8 + 1 palings.

b.

Number of posts	Number of palings
6	41 (5 x 8 + 1)
10	73 (9 x 8 + 1)
27	209 (26 x 8 + 1)
5 ((33 – 1) ÷ 8 + 1)	33
9 ((65 – 1) ÷ 8 + 1)	65

Page 8 **Thinking Ahead**

ACTIVITY

1. a. i. 10 + 12 = 22, 12 + 14 = 26, 14 + 16 = 30

ii. A possible rule is: an even number plus the number 2 more than itself equals twice the number plus 2. The tenth equation is 20 + 22 = 42.

b. i. 10 – 5 = 5, 12 – 6 = 6, 14 – 7 = 7

ii. Possible rule: twice a number take away itself equals itself.
Tenth equation: 20 –10 = 10

c. i. 5 + 8 = 6 + 7, 6 + 9 = 7 + 8, 7 + 10 = 8 + 9

ii. Possible rule: a number added to the number 3 more than itself equals 1 more than the number plus 2 more than the number.
Tenth equation: 10 + 13 = 11 + 12

d. i. 25 – 15 = 10, 30 – 18 = 12, 35 – 21 = 14

ii. Possible rule: 5 times a number take away 3 times the number equals 2 times the number.
Tenth equation: 50 – 30 = 20

e. i. 5 = 10 ÷ 2, 6 = 12 ÷ 2, 7 = 14 ÷ 2

ii. Possible rule: a number equals 2 times itself divided by 2.
Tenth equation: 10 = 20 ÷ 2

f. i. 5 + 6 + 7 + 8 = 2 x 13, 6 + 7 + 8 + 9 = 2 x 15, 7 + 8 + 9 + 10 = 2 x 17

ii. Possible rule: any 4 consecutive numbers add up to 2 times the sum of the first and last numbers.
Tenth equation: 10 + 11 + 12 + 13 = 2 x 23

g. i. 5 x 7 = (6 x 6) – 1, 6 x 8 = (7 x 7) – 1, 7 x 9 = (8 x 8) – 1

ii. Possible rule: a number times the number 2 more than itself equals the square of 1 more than the number, minus 1.
Tenth equation: 10 x 12 = (11 x 11) – 1

2. a. 1 + 3 = 2 x **2**

2 + **4** = 2 x 3

3 + 5 = **2** x 4

4 + 6 = 2 x 5

9 + **11** = 2 x **10**

b. 100 – 1 = **99**

100 – 11 = **89**

100 – **21** = 79

100 – **31** = 69

100 – 81 = **19**

c. 1 + 1 = 1 x **2**

2 + 4 = 2 x **3**

3 + 9 = **3** x 4

4 + **16** = 4 x 5

9 + **81** = 9 x **10**

Page 9 Guess My Number

ACTIVITY

1. a. 28

   b. 80

2. a. 8

   b. 8

   c. 8

3. Answers will vary.

4. a. 4

   b. i. The answer is always 4.

   ii. When twice a number plus 8 is divided by 2, you always get the original number plus 4. When the original number is subtracted from this, the answer is always 4.

5. Answers will vary.

Page 10 Vedic Digits

ACTIVITY

1.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Multiplication Grid

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

Vedic Grid

2. a. i.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Multiplication Grid

ii.

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

Vedic Grid

The circled numbers are in rows and columns that are multiples of 3, 6, and 9.

iii. A number is a multiple of 9 when its Vedic digit is 9.

iv. A number is a multiple of 3 when its Vedic digit is 3, 6, or 9.

b. A number is a multiple of 6 when it is an even number *and* its Vedic digit is 3, 6, or 9.

3. a. i. The Vedic digit is 9. So 5 472 is a multiple of 3, 6 (5 472 is an even number), and 9.

ii. The Vedic digit is 6. So 7 458 is a multiple of 3 and 6 (7 458 is an even number) but not 9 (the Vedic digit is not 9).

iii. The Vedic digit is 9. So 897 543 is a multiple of 3 and 9 but not 6 (897 543 is not an even number).

iv. The Vedic digit is 6. So 12 876 is a multiple of 3 and 6 but not 9.

b. i. 86$\boxed{4}$

ii. 96$\boxed{9}$21 or 96$\boxed{0}$21

c. 1 617, 1 647, or 1 677

Page 11 Bagged Cubes

ACTIVITY

1. a. 16. (Substitute r = 3 and b = 5 into r + r + b + b: 3 + 3 + 5 + 5 = 16.)

b. 22 **c.** 17 **d.** 24

e. 22 **f.** 17 **g.** 24

h. 16

2. Yes. Each collection has the same number of cubes in it as another collection (**1a** and **1h**, **1b** and **1e**, **1c** and **1f**, **1d** and **1g**). This happens because each pair of matching collections has the same number of each coloured bag in it, but they are arranged in a different order. For example, **1a** is 3 + 3 + 5 + 5 = 16, which is **1h** (5 + 3 + 3 + 5 = 16) in a different order.

3. Each green bag has 2 cubes, each orange bag has 6 cubes, and each black bag has 8 cubes. (You may have used trial-and-improvement, but if you used deductive reasoning, clues iii and iv are a good place to start.)

Pages 12–13 Building Borders

ACTIVITY

1. a. Nikki is correct. An 8-plot garden has 8 x 3 + 1 = 25 boards.

b. Each plot has 4 boards on its edge. One of the boards is also on the edge of the adjacent plot (the one next to it). An additional board is needed at the end of a row of plots to complete the border for the last plot. So, a garden with 2 plots has 2 x 3 + 1 boards, a garden with 5 plots has 5 x 3 + 1 boards, and so on.

c. 1 + 5 x 3. So a 99-plot garden will have 1 + 99 x 3 = 298 boards.

d. The second short cut in the table below is based on the diagram in **1c**.

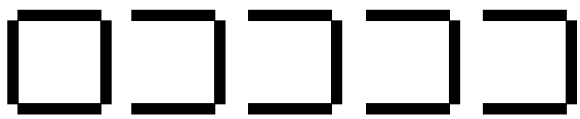
Number of plots	Number of boards: Nikki's short cut	Number of boards: another short cut
5	5 x 3 + 1 = 16	1 + 5 x 3 = 16
8	8 x 3 + 1 = 25	1 + 8 x 3 = 25
99	99 x 3 + 1 = 298	1 + 99 x 3 = 298
30	30 x 3 + 1 = 91	1 + 30 x 3 = 91
156	156 x 3 + 1 = 469	1 + 156 x 3 = 469

2. a. The short cut for the 7-plot garden is 6 x 3 + 4.

b. For a 7-plot garden, 6 plots have 3 boards each (a total of 6 x 3 boards) and the last plot needs 4 boards. So 6 x 3 + 4 = 22 boards are needed altogether.

c. 220 boards (72 x 3 + 4)

3. a. The most likely arrangement is:



(There are other possibilities, but they do not fit in with the horizontal pattern that you have been working with earlier.)

b.

Number of plots	Number of boards
9	4 + 8 x 3 = 28
27	4 + 26 x 3 = 82
42	4 + 41 x 3 = 127
77	4 + 76 x 3 = 232
106	4 + 105 x 3 = 319
293	4 + 292 x 3 = 880

4. a. For a 5-plot garden, 6 boards in the diagram are vertical. There are 2 boards in each of the 5 spaces between the 6 vertical boards. So there are 5 sets of 2 boards or 2 + 2 + 2 + 2 + 2 boards in the 5 spaces. Altogether, there are 2 + 2 + 2 + 2 + 2 + 6 boards.

   b. 2 + 2 + 2 + 2 + 2 boards is the same as 5 x 2 boards. So a simpler short cut is 5 x 2 + 6.

   c.

Number of plots	Number of boards
10	10 x 2 + 11 = 31
13	13 x 2 + 14 = 40
28	28 x 2 + 29 = 85
75	75 x 2 + 76 = 226
126	126 x 2 + 127 = 379
576	576 x 2 + 577 = 1 729

Pages 14–15 Patterns and Spreadsheets

ACTIVITY ONE

1. a. If Mel had no money in her account before week 1, she deposited $12 in week 1 and $7 in each of the following 2 weeks. (There is no indication as to whether the account had money in it before week 1.)

   b. If Mel continues to deposit $7 each week, she will have saved $82 after 11 weeks. (Weekly totals from week 4 are $33, $40, $47, $54, $61, $68, $75, and $82.)

2. a. The value in B3 is 7 more than the value in B2. The value in B2 is 12, so the value in B3 is 12 + 7 = 19.

   b. B8: =B7+7
   B11: =B10+7
   B20: =B19+7
   B43: =B42+7

   c. Columns A and B must be filled down to cell A53 and B53 respectively. The value in cell A53 is 52 and in B53 is $369. This is Mel's savings after 52 weeks.

ACTIVITY TWO

1.

Jeff's Savings (SS)

B3 =B16+45

	A	B	C
1	Month	Savings ($)	
2	1	81	
3	2	126	
4	3	171	
5	4	216	
6	5	261	
7	6	306	
8	7	351	
9	8	396	
10	9	441	
11	10	486	
12	11	531	
13	12	576	
14	13	621	
15	14	666	
16	15	711	
17	16	756	
18	17	801	

2. The spreadsheet shows that after 15 months Jeff has saved $711 and after 16 months he has saved $756. So it takes 16 months for Jeff to save $750.

ACTIVITY THREE

a. $9

b. The value in cell B5 is 9 less then the value in cell B4. The value in B4 is 69, so the value in B5 is 69 – 9 = 60.

c. B6: =B5–9
B8: =B7–9

d. Nothing, if she withdraws the $6 left in week 10. If the formula is filled down to cell B12, the value will show as –3 (negative 3).

ACTIVITY FOUR

a. Practical activity

b. B7: =B6+5
B15: =B14+5
B63: =B62+5
B75: =B74+5

c. 88 paving stones (cell B18)

d. 36 lawn squares (cell A37)

ACTIVITY FIVE

1. The 18th number is 119, and the 36th number is 227. The initial formula, for cell B2, is: =B1+6.

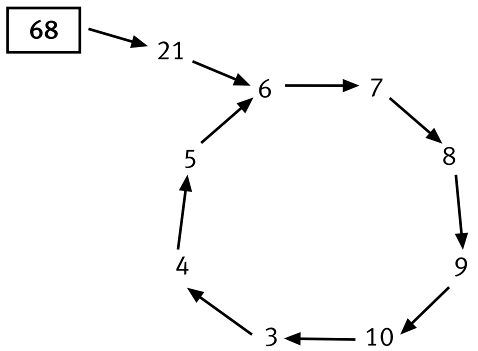
2. The 27th number is 573, and the 43rd number is 893. The initial formula, for cell B2, is: =B1+20.

3. The 24th number is 86.5, and the 87th number is 307. The initial formula, for cell B2, is: =B1+3.5.

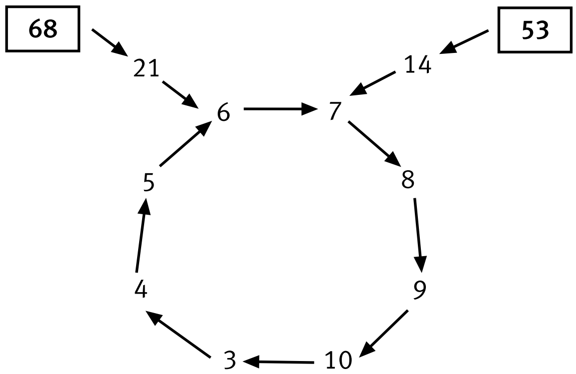
Digit Chains

ACTIVITY

1. a.



b. i.–ii.



c. i. The digit chain for 37 is: 37, 14, 7, 8, 9, 10, 3, 4, 5, 6, 7, 8, 9, 10, …

The digit chain for 48 is: 48, 17, 10, 3, 4, 5, 6, 7, 8, 9, 10, 3, 4, 5, …

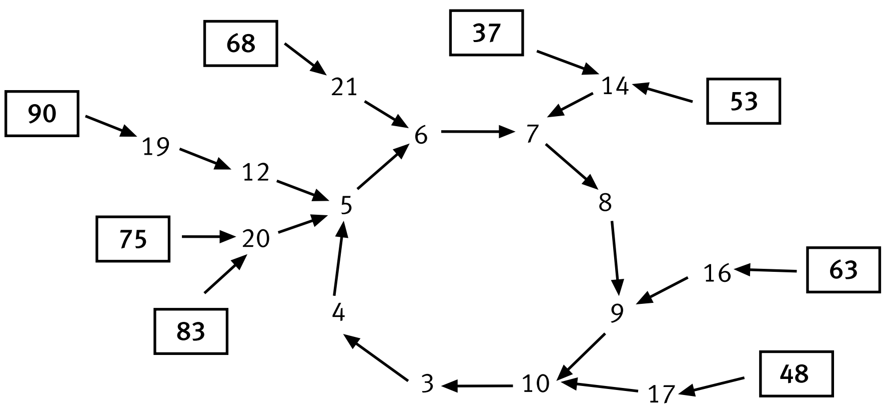
The digit chain for 63 is: 63, 16, 9, 10, 3, 4, 5, 6, 7, 8, 9, 10, 3, 4, …

The digit chain for 75 is: 75, 20, 5, 6, 7, 8, 9, 10, 3, 4, 5, 6, 7, 8, …

The digit chain for 83 is: 83, 20, 5, 6, 7, 8, 9, 10, 3, 4, 5, 6, 7, 8, …

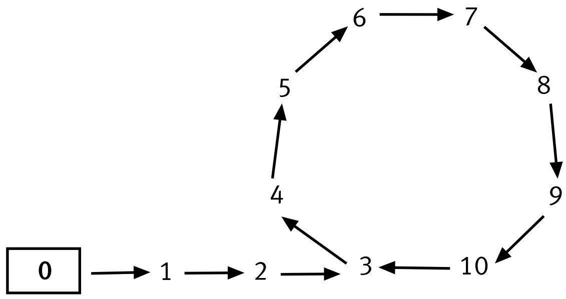
The digit chain for 90 is: 90, 19, 12, 5, 6, 7, 8, 9, 10, 3, 4, 5, 6, 7, 8, …

ii.



d. Each 2-digit starter reduces to a number in the never-ending loop, 3, 4, 5, 6, 7, 8, 9, 10, 3, 4, …

e. Using the digit rule, 0 stays as 0 and begins the chain 0, 1, 2, 3, 4, … So, starter number 1 begins the chain 1, 2, 3, 4, 5, … , and starter number 2 begins the chain 2, 3, 4, 5, 6, … The diagram below shows how this works.



2. a. 134 = 13 tens and 4 ones. So 134 becomes (2 x 13) + (4 + 1) = 26 + 5, which is 31.

b. The number 31 becomes (2 x 3) + (1 + 1). 6 + 2 = 8, so 134 enters the loop at 8.

c. The starter number 100 is 10 tens + 0 ones. This becomes (2 x 10) + (0 + 1) = 21. Then 21 becomes 6, which is where it enters the loop.

The starter number 256 is 25 tens + 6 ones. This becomes (2 x 25) + (6 + 1) = 57. Then 57 becomes 18, which becomes 11, which becomes 4, which is where it enters the loop.

The starter number 751 is 75 tens + 1 one. This becomes (2 x 75) + (1 + 1) = 152, which becomes (2 x 15) + (2 + 1) = 33, which becomes 10, which is where it enters the loop.

d. 2 138 = 213 tens and 8 ones. So 2 138 becomes (2 x 213) + (8 + 1) = 435, which becomes (2 x 43) + (5 + 1) = 92, which becomes (2 x 9) + (2 + 1) = 21, which becomes (2 x 2) + (1 + 1) = 4 + 2 = 6, which is where it enters the loop.

e. Answers will vary.

Page 17 Pizza Order

ACTIVITY

1. **a.** $87.00 (24 + 9 x 7 = 87)

   b. $93.00 (3 lots of 4 pizzas, where each lot is 3 for $24 and 1 at $7. Note that this is $3 cheaper than buying 4 lots of 3 pizzas at $24.)

2.

Number of pizzas	Special offer cost ($)	Working
3	24.00	
4	31.00	(24 + 7)
5	39.50	24 + 7 + 8.50
6	48.00	2 x 24
7	55.00	2 x 24 + 7
8	62.00	2(24 + 7)
9	70.50 *	2(24 + 7) + 8.50
10	79.00	3 x 24 + 7
11	86.00	3 x 24 + 2 x 7
12	93.00 **	3(24 + 7)

* Note that this is cheaper than 3 x 24 = $72.
** Note that this is cheaper than 4 x 24 = $96.

3. **a.** $102.00

   b. $9.00

4. Yes. The cost of 4 pizzas under the previous offer was $31 (from $24 + $7), whereas 4 pizzas under the new offer cost $30.50, a saving of 50 cents.

Pages 18–19 Tiling Teasers

ACTIVITY

1. **a.**

Jewels		
Number of hexagons	Number of squares	Number of rhombuses
1	2	4
2	4	8
3	6	12
4	8	16
5	10	20
6	12	24
7	14	28
8	16	32
9	18	36

   b. A rule is: 2 times the number of hexagons gives the number of squares, and 2 times the number of squares or 4 times the number of hexagons gives the number of rhombuses.

   c. 20 squares and 40 rhombuses

2. **a.**

Yachts	
Number of trapezia	Number of triangles
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32
9	36

   A rule is: the number of triangles is 4 times the number of trapezia.

   b. 40 triangles

3. a.

Townhouses		
Number of trapezia	Number of squares	Number of rhombuses
1	2	0
2	4	1
3	6	2
4	8	3
5	10	4
6	12	5
7	14	6
8	16	7
9	18	8

Rules may vary. A possible rule is: the number of squares is found by multiplying the number of trapezia by 2, and the number of rhombuses is found by taking 1 off the number of trapezia.

b. 20 squares and 9 rhombuses

4. a.

Number of fish	Number of rhombuses
1	7
2	12
3	17
4	22
5	27
6	32
7	37
8	42
9	47

A rule is: the number of rhombuses is 5 times the number of fish plus 2, or each new fish (after the first) adds 5 rhombuses.

b. 52 rhombuses

5. a.

Cogs		
Number of hexagons	Number of rhombuses	Number of triangles
1	6	0
2	11	2
3	16	4
4	21	6
5	26	8
6	31	10
7	36	12
8	41	14
9	46	16

A rule is: 5 times the number of hexagons plus 1 gives the number of rhombuses, and 2 times the number of hexagons minus 2 gives the number of triangles.

b. 51 rhombuses and 18 triangles

Page 20 Which Wheels Where?

ACTIVITY

1. a. There are three possible answers:

16 wheels (2 people with 2 cycle wheels each, 1 person with 4 skateboard wheels, and 1 person with 8 in-line skate wheels);

18 wheels (1 person with 2 cycle wheels, 2 people with 4 skateboard wheels each, and 1 person with 8 in-line skate wheels);

22 wheels (1 person with 2 cycle wheels, 1 person with 4 skateboard wheels, and 2 people with 8 in-line skate wheels each).

b. There are two possible answers:

20 wheels (4 people with 2 cycle wheels each, 1 person with 4 skateboard wheels, and 1 person with 8 in-line skate wheels);

22 wheels (3 people with 2 cycle wheels each, 2 people with 4 skateboard wheels each, and 1 person with 8 in-line skate wheels).

c. 22 wheels. (An extra cyclist and an extra skateboarder added to the first option in **1a** gives the second option in **1b** above.)

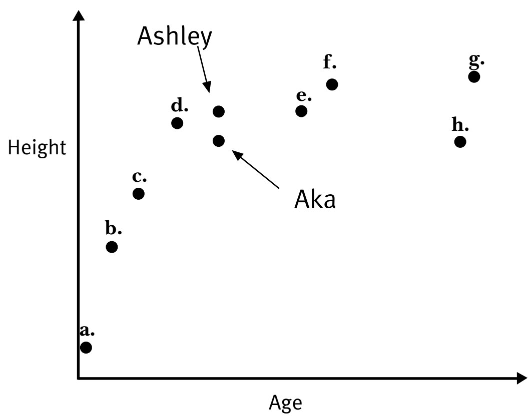
2. 1 cyclist, 1 skateboarder, and 4 in-line skaters

Page 21 Whānau Photo

ACTIVITY

1. a. Rata
   b. Paora
   c. Mere
   d. Manu
   e. Aroha
   f. Hōne
   g. Timoti
   h. Huia
2. Your graph should be similar to the graph below.

Whānau Heights by Age



Pages 22–23 Graphic Details

ACTIVITY ONE

1. 21 tiles. The completed table would look like this:

Number of crosses	Number of square tiles
1	5
2	9
3	13
4	17
5	21

2. a. Yes.
   b. Yes.
   c. No.
   d. No.
   e. Yes.

   Explanation: The number of tiles is the same as 4 times the number of crosses plus 1. If the second co-ordinate does not fit this rule (second number = first number x 4 + 1), then the point will not be on the same straight line as the other points.

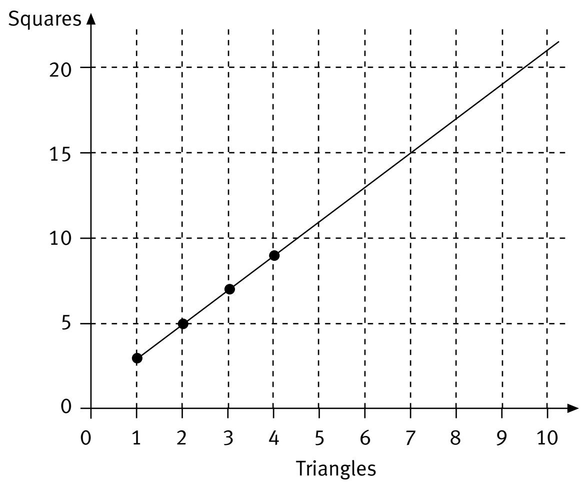
ACTIVITY TWO

1. If you make a table like this before you do your graph, it might look like this:

Triangles	1	2	3	4	...	10
Squares	3	5	7	9	...	21

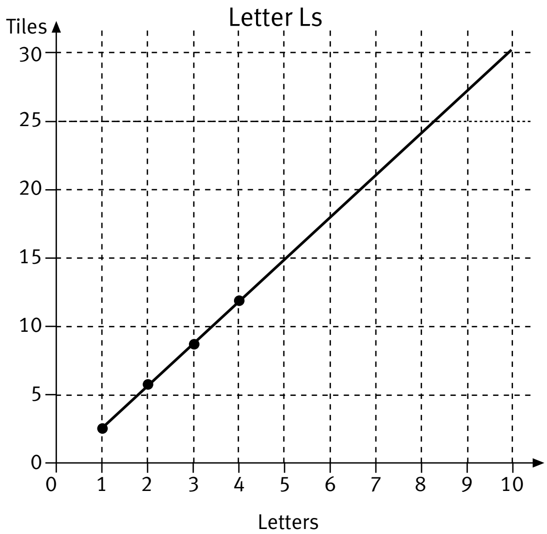
A graph for this is:

Windmills

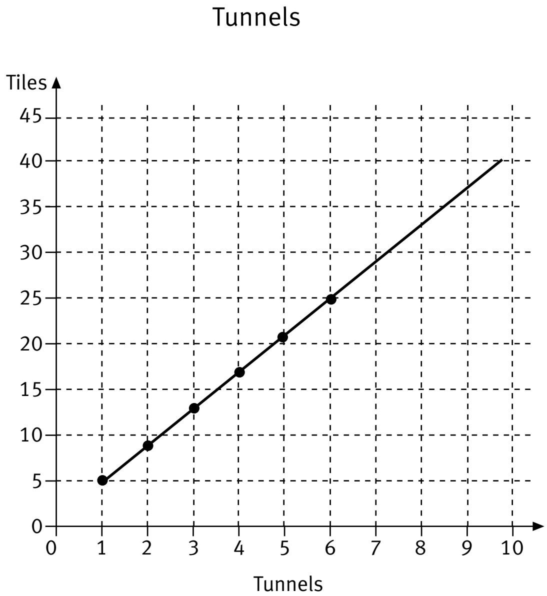


So 21 squares will be needed to surround 10 triangles.
(A rule for this could be expressed as: the number of squares equals 2 times the number of triangles plus 1. This can be written as $s = 2t + 1$.)

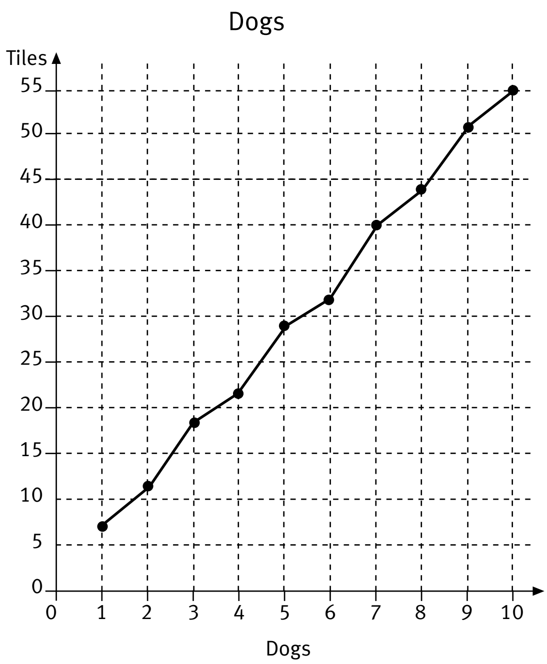
2. a. i.



ii.



iii.



b. i. 30

ii. 41

iii. 55

Page 24 Scooting

ACTIVITY

1. a. Josh

b. Rewi

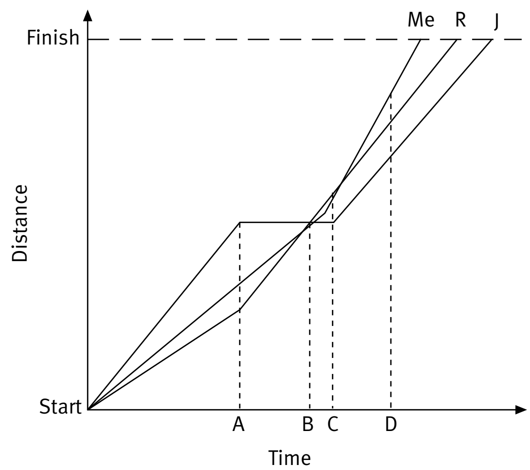
2. Rewi

3. Rewi overtakes Josh.

4. Answers will vary. Josh obviously stopped because time passes, but his distance from the start remains unchanged. He may have fallen off his scooter or stopped to fix a loose wheel.

5. His speed has increased.

6. The graph might look something like this.



A description might then be:
At the beginning of the race, I was behind Josh but ahead of Rewi. Then at time B, Rewi caught up with me and I passed Josh (who had stopped). Rewi got ahead of me until a bit later (at time C), when I sped up and overtook him and won the race.

Teachers' Notes

Overview **Algebra: Book One (link)**

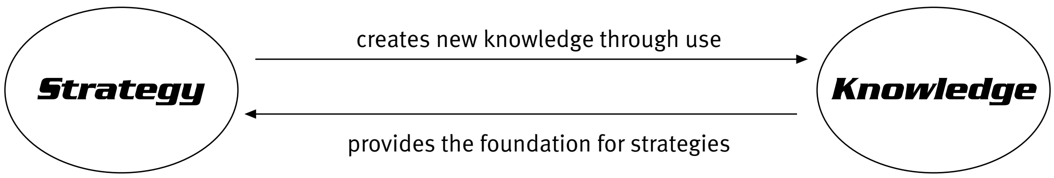
Introduction to Algebra

Teaching and learning algebra has always posed difficulties for teachers and their students. Even today, there is no consensus about when it should be introduced and exactly what should be included in an introduction to algebra. Historically, algebra has formed an important part of the secondary curriculum. However, its inclusion as a strand in the national curriculum statement, *Mathematics in the New Zealand Curriculum*, has meant that teachers at all levels have been grappling with what should be taught at those levels. Internationally, there is a growing consensus that the ideas of algebra have a place at every level of the mathematics curriculum.

One view is that algebra is an extension of arithmetic. Another view is that it is a completion of arithmetic. Some argue that algebra begins when a set of symbols is chosen to stand for an object or situation. Others argue that all basic operations are algebraic in nature: for example, the underlying structure of a part-whole mental strategy for adding 47 to 36 is algebraic because it may involve seeing 47 + 36 as 47 + 33 + 3, giving 50 + 30 + 3 and then 83, and such mental action constitutes algebraic thinking, in spite of the absence of algebraic-looking symbols.

The Figure It Out series aims to reflect the trends in modern mathematics education. So this series promotes the notion of algebraic thinking in which students attend to the underlying structure and relationships in a range of mathematical activities. While the student material includes only limited use of algebraic symbols, the teachers' notes show how mathematical ideas formulated in words by learners can be transformed into symbolic form. Teachers are encouraged to introduce the use of symbols in cases where they themselves feel comfortable and where they think that their students are likely to benefit.

The basis for the material in the students' books is consistent with the basis of the Number Framework, which highlights the connections between strategies students use to explore new situations and the knowledge they acquire.



To help students develop sensible strategies or short cuts for working with new mathematical situations, the activities encourage them to create their own visual and pictorial images to represent mathematical ideas and relationships. As the students use and develop short cuts for particular problems, they come to see that these can be developed into rules for any similar problem. They also learn that they can apply their strategies to a range of similar, as well as new, mathematical situations.

There are four *Algebra* books in this series for year 7–8 students:
Link (Book One)
Level 4 (Book Two)
Level 4 (Book Three)
Level 4+ (Book Four)

Page 1 **Something Adds Up**

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- use a rule to make predictions (Algebra, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 2–3)

ACTIVITY

This activity will help students to increase their understanding of number structure, in particular of decimal (or base 10) representation.

In question **1**, the students should focus separately on the digits in the ones place and the digits in the tens place to help them identify the links between any 2 numbers that add up to 100.

The students may find it helpful to complete tables like the ones below. To do this, they must add the digits in the ones place and then the digits in the tens place. Encourage them to write a rule for the ones and the tens digits in each table.

First number	Second number	Total
80	20	100
70	30	100
60	40	100
50	50	100

Rule: The ones digits add up to 0, and the tens digits add up to 10.

First number	Second number	Total
81	19	100
68	32	100
63	37	100
46	54	100

Rule: The ones digits add up to 10, and the tens digits add up to 9.

In question **1b**, as in the 2-number sums making 100, the students need to focus on the sums of the digits in the ones place and then on the sums of the digits in the tens place. Tables like the one below can help the students to see patterns that lead to the rules shown alongside the table.

First number	Second number	Third number	Total	
30	60	10	100	← The ones digits add up to 0, and the tens digits add up to 10.
32	25	43	100	← The ones digits add to 10, and the tens digits add up to 9.
17	38	45	100	← The ones digits add up to 20, and the tens digits add up to 8.

The students should test their rules by making up different sets of numbers. They also need to be sure that they have covered every possible rule. For example, an argument to show that in the 3-number case there are exactly three possible rules might go like this: “We know that the ones digits have to add up to a multiple of 10. With 3 numbers, the biggest number we could get by adding up the ones digits would be 27 (when each ones digit was 9), so the only multiples of 10 we can get by adding up the ones digits will be 0, 10, and 20 (we can’t reach 30 or any of those above).”

Encourage them to predict rules when 4 numbers add up to 100, then when 5 numbers do, and so on. Each prediction needs to be tested to ensure the rule is sensible. For example, they could ask: “Is it always true that the number of possible rules is equal to the number of numbers being added?”

In question **2**, the students should explore possibilities. If necessary, suggest that they use a table format as above to help with their explorations. They will find it helpful to work systematically with 2 numbers in cases where the digits in the ones place add up to 0, then with 2 numbers where the digits in the ones place add up to 10. Those students who are quick to see the rules might then explore what happens when 3 or even 4 numbers add up to 1 000.

Pages 2–3 Pyramid Parts

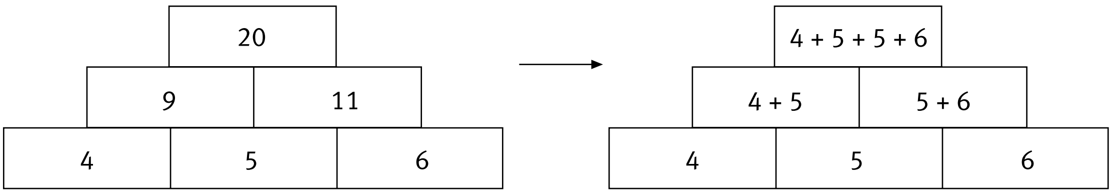
Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- use a rule to make predictions (Algebra, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- solve simple linear equations such as 2❒ + 4 = 16 (Algebra, level 4)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, levels 2–3)

ACTIVITY

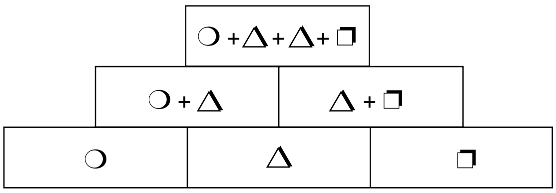
This activity encourages students to think systematically about particular arithmetic structures or equations.

In question **1**, suggest to students who have difficulty seeing how the top pyramid number can be found that they first use only the numbers in the base of the pyramid.



Some students may need to explore several 3-level pyramids in this way before they can suggest a rule for finding the top pyramid number, given the three base numbers. Note that the rule will be easier to discover if they ensure that the three base numbers are all different. Encourage the students to check if their rule works for large whole numbers, decimal numbers, and numbers in which one or more digits are zero.

This could be a good opportunity to introduce the use of symbols (or variables) that stand for any number and to show how they can help us understand what is going on in the general case. For example, if the base numbers are ❍, ❒, and ∆, we obtain the 3-level pyramid:



Here, the symbols ❍, ❒, and △ stand for any numbers and are used in exactly the same way as the algebraic symbols *a, b, c, ... x, y, z* are frequently used.

The first two examples in question **2** give the students an opportunity to consolidate their thinking on how the rule works for a 3-level pyramid and then to use this thinking as the basis for a sensible prediction for how to get the top number for a 4-level pyramid (questions **iii** and **iv**). The students could check their predictions by using only the base numbers to form 4-level pyramids. For example:

1 + 2 + 2 + 2 + 3 + 3 + 3 + 4			
1 + 2 + 2 + 3	2 + 3 + 3 + 4		
1 + 2	2 + 3	3 + 4	
1	2	3	4

In this pyramid, the top pyramid number is 1 + (3 x 2) + (3 x 3) + 4, which is 1 + 6 + 9 + 4 = 20. Once again, symbols could be used to help confirm the rule the students have discovered. Some students may want to see if they can predict what will happen for a 5-level pyramid, a 6-level pyramid, and so on. All predictions need to be checked carefully.

In question **3**, the top pyramid number is the starting point for finding the missing number in the base of the pyramid. The students need to reverse their thinking (although some may find the solution through guessing and checking for the missing base number). In question **3a**, for example, the rule for the 3-level pyramid tells us that 2 x the missing number + 2 + 7 = 19. This means that the missing base number is half of 10 (the answer to 19 – 2 – 7). In question **3b**, the missing base number is 13 – 3 – (2 x 4) = 2. In question **3c**, the missing base number is one-third of the answer to 40 – 6 – 7 – (3 x 4). So the missing base number is one-third of 15, which is 5. Students who are able to reverse their thinking in this way show that they understand the rule linking the top pyramid number with the numbers in the base of the pyramid. Encourage the students to explain their thinking.

In question **4**, the students see if they can spot the relationships between corner numbers in rectangular arrays of different sizes, located on calendars for different months. Encourage the students to talk with others about what they see before they try to write anything. If you explain the links, you will limit the exciting moments when students “see” the links themselves. Searching for mathematical similarities as well as differences is the process that enables students to make mathematical generalisations.

Pages 4–5 Cube Signs

Achievement Objectives

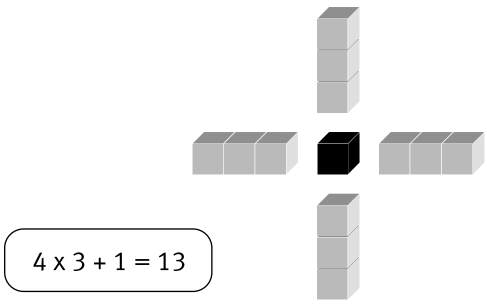
- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, levels 2–3)

ACTIVITY

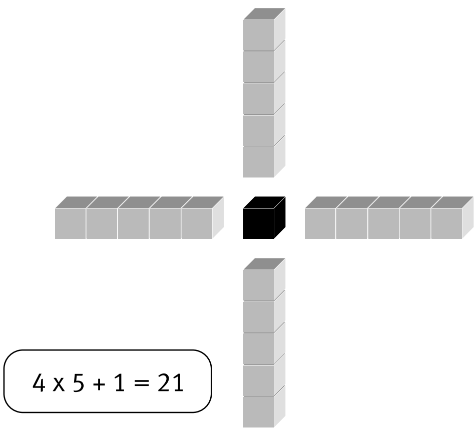
This activity further develops students' abilities to predict patterns through the use of rules or equations. It also introduces them to the idea that the same pattern can be represented by different (but equivalent) equations.

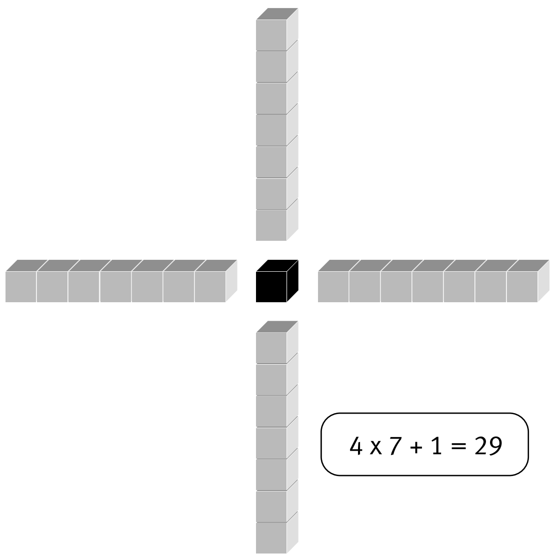
In questions **1–4**, the students develop and explain different short cuts or strategies for working out the number of multilink cubes in particular plus signs. The short cuts arise from different ways of visualising the structure of the plus signs. The students use the short cuts to predict the number of cubes in plus signs of *any* size.

The students need to actually build the plus signs and then see if they can figure out how the physical models can be represented by the short cuts. For example, Alan's third plus sign has 4 arms, each with 3 cubes, with 1 cube in the centre. So the short cut is 4 x 3 + 1.



The fifth plus sign has 4 arms, each with 5 cubes, and 1 cube in the centre, so the short cut is 4 x 5 + 1. The seventh plus sign has 4 arms, each with 7 cubes, and 1 cube in the centre, so the short cut is 4 x 7 + 1.





In question **2**, the students should build the third plus sign with multilink cubes to match the short cut 2 x 7 – 1 = 13. As noted in the Answers, they need to take into account that the centre cube is initially counted twice.

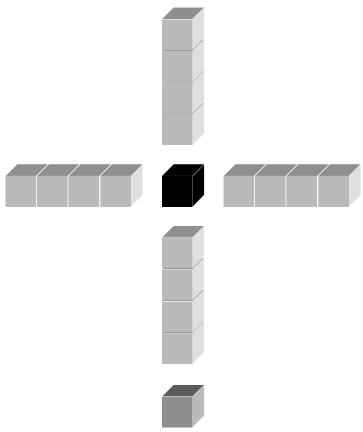
Students who have difficulty seeing that $2 \times 19 - 1$ is Kali's short cut for the ninth plus sign, where the 19 is found from $2 \times 9 + 1$, may need to build models with multilink cubes. They should explain how, for example, the short cut for the fourth plus sign is $2 \times 9 - 1$; how the short cut for the fifth plus sign is $2 \times 11 - 1$; and so on. The table in question **3** provides an opportunity for the students to use the short cuts they have modelled with multilink cubes. They should also see that the different short cuts generate distinct rules that have exactly the same outcomes for particular plus signs of any size. Each rule is dependent on the way we see the cubes that make up the plus signs. As we change our point of view, we find new opportunities to express generality.

As noted in the introduction to these notes, students at this stage are not expected to use algebraic language, but there may well be students in your group or class who are ready to do so. The notes on algebraic language given here and for later pages are to help you to assist these students.

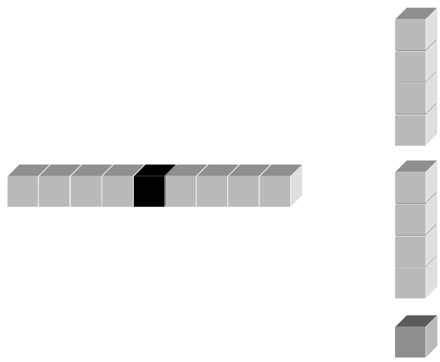
In algebraic language, the rules are $4 \times x + 1$ (Alan's) and $2 \times (2 \times x + 1) - 1$ (Kali's), where the symbol x stands for the number of cubes in each arm of any plus sign. These rules are usually written more simply as $4x + 1$ and $2(2x + 1) - 1$. The multiplication sign is left out so that, for example, $4 \times x$ becomes $4x$ (four times x is the same as four x). $2(2x + 1) - 1$ and $4x + 1$ give the same result for any value of x, so it is sensible to assume that $2(2x + 1) - 1$ can be reduced to $4x + 1$. The algebraic manipulation involved in this is

$$\begin{aligned} 2(2x + 1) - 1 &= (2 \times 2x) + (2 \times 1) - 1 \\ &= 4x + 2 - 1 \\ &= 4x + 1. \end{aligned}$$

In question **4**, encourage the students to look for more than one short cut. Two short cuts for the fourth plus sign are shown below.



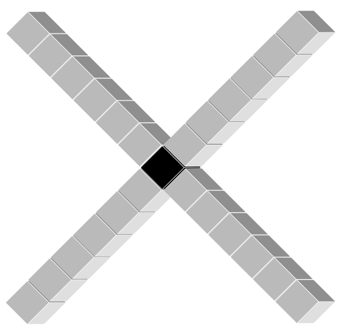
$$\begin{aligned} 4 \times 4 + 1 + 1 &= 4 \times 4 + 2 \\ &= 18 \end{aligned}$$



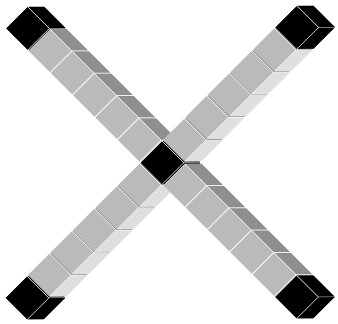
$2 \times 9 = 18$

Again, the best way for the students to identify short cuts is for them to make plus signs with multilink cubes. They need time to experiment and test the short cuts and to see if they can use them to develop a rule that in turn can be used to work out the number of cubes needed for the 75th or 123rd plus sign, and so on. Using more than one rule checks the accuracy of both the rule and the arithmetic in the calculations.

In question **5,** the plus sign has been tilted and the arms extended. This means that the numbering of the plus and times signs are different. For example, Kali's first times sign has the same number of cubes as Alan's second plus sign. Likewise, in the fifth times sign, each arm has 6 cubes, with an additional cube for the centre of the times sign. So the fifth times sign has $4 \times 6 + 1 = 25$ cubes.



Another way to build the fifth times sign is:



Here, each arm has 5 yellow cubes. There are also 5 black cubes, one for the centre and one on the end of each arm. So the short cut is 4 x 5 + 5 = 25.

The table in question **5** provides an opportunity to consolidate the students' use of the different rules that arise out of the different ways of seeing short cuts. The number of cubes for the 500th times sign can be very quickly calculated as 2 005 using either rule, and this demonstrates the power of generalisations and algebraic thinking. The algebraic rule for the xth times sign arising from the short cut 4 x 6 + 1 for the fifth times sign is $4 \times (x + 1) + 1$. This is the same as $4(x + 1) + 1$, which simplifies to $4x + 4 + 1 = 4x + 5$. The algebraic rule arising from 4 x 5 + 5 for the fifth times sign is $4 \times x + 5$, which is the same as $4x + 5$. So the two rules are in fact equivalent.

Pages 6–7 Fencing

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- solve simple linear equations such as 2❐ + 4 = 16 (Algebra, level 4)
- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 3)

ACTIVITY

By this stage, students should be starting to realise that algebra is about the general rule that works rather than about specific numbers. In this activity, students continue to use equations or rules to capture the behaviour of patterns and extend or adapt known rules to cover new, but related, cases.

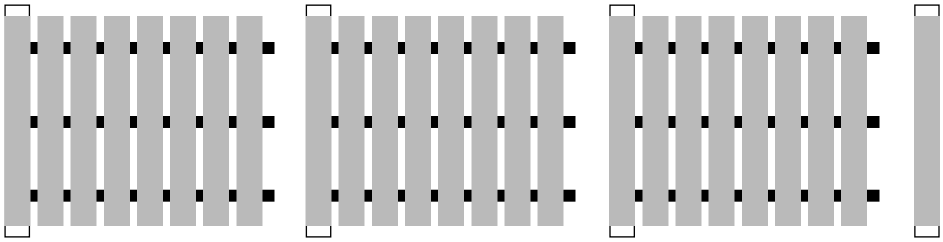
The diagram in question **1** shows that a fence with 3 sections has 4 posts and 9 rails. The students will need to see that each section has 3 rails and that for any fence, there is always 1 more post than there are sections. Some students may need to draw a fence with 1 section, a fence with 2 sections, and so on to help them visualise these relationships. So for a 37-section fence, there are 37 + 1 = 38 posts and 37 x 3 = 111 rails. And for a fence with n sections, there are $n + 1$ posts and $n \times 3$ or $3n$ rails.

In question **2**, attention shifts from sections to the link between posts and rails. Tyson's drawing is designed to help the students to visualise a short cut for counting rails. There are 3 rails for each of the first 3 posts in a 4-post fence. So for a 7-post fence, 6 posts each have 3 rails, giving 18 rails altogether. Some students may need to draw the posts and rails for different fences before they "see" and can use the rule for calculating the number of rails for a fence with *any* number of posts. So a fence with x posts will have $(x - 1) \times 3$ rails. We usually ignore the times sign and write this as $3(x - 1)$. Note that if x is the number of posts, then $(x - 1)$ is the number of sections in the fence.

In question **3**, you might want to ask the students to first check their fence drawings to see if they can explain how to find the number of posts for a fence with 27 rails. This entails visualising sets of 3 rails attached to all but the last post. So for 27 rails, they visualise 9 posts (from 27 ÷ 3), each attached to sets of 3 rails. There is 1 additional post, so altogether there are 9 + 1 = 10 posts in the fence. So a fence with y rails will have (y ÷ 3) + 1 posts. We usually write this in fraction form as $^{y}/_{3} + 1$.

The diagram in question **4** shows a fence with 4 rails in each section between adjacent posts. So a fence with 6 posts can be visualised as having 5 posts each attached to 4 rails. The tables provide opportunities for the students to see how the strategy for a fence with 6 posts can develop into a rule for fences with any number of posts. The algebraic rules for fences with x posts and y rails when there are 4 rails in each section are $y = 4(x - 1)$ when you know the number of posts and $x = {}^{y}/_{4} + 1$ when you know the number of rails. Both of these equations rely on the fact that every post except the last has 4 rails attached.

This extended diagram for question **5** shows the palings on a fence with 4 posts.



All but the last post has 8 palings attached to the rails, which in turn are attached to the post. The last post has just 1 paling attached to it. So a fence with 4 posts has 3 x 8 + 1 palings. As earlier, some students may need to draw fences with different numbers of posts to help visualise such a short cut. For 6 posts, there are 5 x 8 + 1 palings. So for x posts, there will be $(x - 1) \times 8 + 1$ palings. This is usually written as $8(x - 1) + 1$. The students will be using this rule to find the number of palings in question **5b**, even though they are not using the algebraic notation to express it. To find the number of posts in this question, given the number of palings, subtract 1 from the number of palings, divide the answer by 4 to get the number of sections, and add 1 for the extra post (the number of posts is always 1 more than the number of sections).

Page 8

Thinking Ahead

Achievement Objectives

- describe in words, rules for continuing number and spacial sequential patterns (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- solve simple linear equations such as 2❐ + 4 = 16 (Algebra, level 4)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, levels 2–3)

ACTIVITY

These activities provide opportunities for pattern spotting and for writing numerical expressions in alternative ways. Some of the patterns suggest short cuts for making calculations. For example, in question **1f**, the equation 4 + 5 + 6 + 7 = 2 x 11 is used to calculate the sum of 4 consecutive numbers. The rule is: the sum of 4 consecutive numbers is 2 times the sum of the first and last numbers. So, for example, 249 + 250 + 251 + 252 = 2 x (249 + 252), which is 2 x 501 or 1 002. Another rule, the sum of 4 consecutive numbers is 2 times the sum of the middle 2 numbers, would do equally well.

While students are often quick to see patterns, they often have considerable difficulty writing rules for patterns in words. This is especially the case for those with English as a second language. The students are likely to benefit from talking with others about the patterns they notice as they draft the rules. It is this process of talking and explaining that is the key to this work.

As the students become more accomplished at writing the rules in words, it may be possible to introduce the symbolic notations that are customarily used in algebra. However, algebraic notation has its own difficulties. For example, students are often confused by expressions such as $n + 2$ or $n - 3$ and so on, where a literal symbol and a number are combined by an addition or subtraction operation. They prefer to get an answer similar to $2 + 3 = 5$ and sometimes write, incorrectly, that $n + 2 = n2$. For many, it seems a little odd that $n + 2$ is just $n + 2$ when $2 \times n$ is written as $2n$. ($2n$ is the same as $n + n$, as in arithmetic. For example, $2 \times 13 = 13 + 13$.) It is also customary in algebra to put the number before the literal symbol, so we write $4n$ rather than $n4$.

In question **1a**, the pattern is more evident when the equations are written in the following way:

$2 + 4 = 2 + (2 + 2) = 2 + 2 + 2 = 2 \times 2 + 2 = 6$
$4 + 6 = 4 + (4 + 2) = 4 + 4 + 2 = 2 \times 4 + 2 = 10$
$6 + 8 = 6 + (6 + 2) = 6 + 6 + 2 = 2 \times 6 + 2 = 14$
$8 + 10 = 8 + (8 + 2) = 8 + 8 + 2 = 2 \times 8 + 2 = 18$

This may help the students to see that the rule for the pattern can be expressed as "An even number plus the number 2 more than itself equals twice the number plus 2." This is $n + (n + 2) = 2n + 2$, where n is *any* even number.

In question **1b**, the rule for the pattern can be expressed as "Twice a number take away itself equals itself." This is $2n - n = n$. A pattern table may help students see how to figure out the tenth equation in the pattern.

Equation position	Starting number	Pattern for equation
1st	1	$2 \times 1 - 1 = 1$
2nd	2	$2 \times 2 - 2 = 2$
3rd	3	$2 \times 3 - 3 = 3$
4th	4	$2 \times 4 - 4 = 4$
10th	❒	2 x ❒ – ❒ = 10

Patterns in the table show that the number that goes in the ❒ in the tenth equation is 10. So, for example, the 100th equation would be $2 \times 100 - 100 = 100$.

In question **1c**, the rule for the pattern can be expressed as "A number added to the number 3 more than itself equals 1 more than the number plus 2 more than the number." Using algebraic language, the rule is $n + (n + 3) = (n + 1) + (n + 2)$. Note that the left-hand side of the equation is $n + (n + 3) = 2n + 3$, and the right-hand side is $(n + 1) + (n + 2) = 2n + 1 + 2 = 2n + 3$. For this question, and for questions **1d**, **1e**, and **1f**, the first equation is the equation with $n = 1$. So the tenth equation is the equation with $n = 10$. Similarly, the 100th equation is the equation with $n = 100$. For question **1c**, the 100th equation is $100 + 103 = 101 + 102$.

In question **1d**, the rule for the pattern can be expressed as "5 times a number take away 3 times the number equals 2 times the number." This can be written as $5n - 3n = 2n$. The tenth equation is $5 \times 10 - 3 \times 10 = 2 \times 10$, which can be written more simply as $50 - 30 = 20$.

In question **1e**, the rule for the pattern can be expressed as "A number equals 2 times itself divided by 2." It could also be expressed as "A number is half its double." Algebraically, this is $n = 2n \div 2$.

In question **1f**, the rule for the pattern can be expressed as "Any 4 consecutive numbers add up to 2 times the sum of the first and last numbers." Using algebra, this is $n + (n + 1) + (n + 2) + (n + 3) = 2[n + (n + 3)]$. The first equation in the pattern is $1 + 2 + 3 + 4 = 2 \times 5$, with $n = 1$. So the tenth equation is

$$\begin{aligned} 10 + 11 + 12 + 13 &= 2 \times (10 + 13) \\ &= 2 \times 23. \end{aligned}$$

In question **1g**, the rule for the pattern can be expressed as "A number times the number 2 more than itself equals the square of 1 more than the number, minus 1." Algebraically, this is $n(n + 2) = (n + 1)^2 - 1$. The tenth equation is $10 \times 12 = 11 \times 11 - 1$. The 100th equation is $100 \times 102 = 101 \times 101 - 1$.

In question **2**, the students need to find either the value of the missing number or the numbers to make the total value on the left-hand side of the equation equal to the total value on the right-hand side of the equation. Where there are two numbers missing in an equation, the numbers can be found from the pattern in the equations.

In question **2c**, draw the students' attention to the pattern in the second numbers on the left-hand side of these equations. Each second number is the square of the first number. And on the right-hand side, each second number is 1 more than the first number. The final equation is then $9 + 81 = 9 \times 10$.

Page 9

Guess My Number

Achievement Objectives

- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, levels 2–3)
- solve simple linear equations such as 2❒ + 4 = 16 (Algebra, level 4)
- make up and use a rule to create a sequential pattern (Algebra, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

In this activity, the students follow sets of instructions as they work with numbers and arithmetic operations. They use their number knowledge to reason the outcome of sets of instructions on particular numbers and then to generalise the effect of the instructions on *any* number.

In question **1a**, the students need to examine the numbers between 25 and 32 and eliminate those that do not satisfy the given conditions. They first eliminate 30 and 31 because their tens digits are greater than their ones digits. Of the remaining numbers, only 28 gives an even number when it is halved. The other numbers, 26, 27, and 29, give either an odd number or a fraction when they are halved. So the number "I am thinking of" is 28. In question **1b**, the students examine multiples of 5 from 55 to 95. Multiples of 5 end in either 5 or 0. Because we are told that the number is even, we can ignore all the multiples that end in 5. This leaves 60, 70, 80, and 90. Only 80 has digits that add up to 8.

In question **2**, the students will find that when they carry out the three different sets of instructions beginning with 10, the result (8) is the same in each case. The students might like to try writing the equations that capture these operations. They are:

i. $10 \div 2 + 10 - 7 = 8$
ii. $(10 - 3) \times 2 - 6 = 8$
iii. $(10 \times 2 + 4) \div 3 = 8$

(The correct positioning of brackets will be critical in the last two equations.)

Encourage the students to explore starting numbers other than 10. They may be surprised that the results are all different for such numbers.

In question **3**, encourage the students to use at least one multiplication or division in their operations to avoid instructions of the form "Add 2 then subtract 4", and so on.

In question **4**, the students will find they always get 4 as an answer no matter what starting number they use. This surprises many and consequently encourages further exploration as to why this happens. In this case, the effect of the set of instructions on any starting number is the same as doing nothing. This is explained in the Answers.

In question **5**, the students will find it challenging to write sets of instructions that produce the same effect as in question **4**. They will need to experiment and should talk with each other about how they have devised the instructions to ensure that the result is always the same as the starting number.

Page 10 Vedic Digits

Achievement Objectives

- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, levels 2–3)
- use a rule to make predictions (Algebra, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

Vedic mathematics is an ancient form of Hindu mathematics. Vedic techniques are designed to simplify multiplication, divisibility, finding squares, square roots, cubes, cube roots, and other aspects of mathematics, including working with recurring decimals and fractions.

This activity focuses on aspects of divisibility and multiples. For example, a number is divisible by 9 if the sum of its digits is a multiple of 9. The mathematics underlying this is as follows.

Numbers such as 135 can be expressed in expanded form as $1 \times 100 + 3 \times 10 + 5$. This tells us that 135 is not a multiple of 10 because the last digit, 5, is not a multiple of 10. We can also write the expanded form for 135 in a different way: $(1 \times 99 + 1) + (3 \times 9 + 3) + 5$. The links between the two ways of writing 135 are shown below.

$$1 \times 100 + 3 \times 10 + 5$$

$$= (1 \times 99 + 1) + (3 \times 9 + 3) + 5$$

$$= (1 \times 99) + (3 \times 9) + (1 + 3 + 5)$$

Digit sum for 135

Note that the last bracketed term is just the digit sum for 135. So 135 = 1 x 99 + 3 x 9 + 9. We know that 1 x 99 is a multiple of 9 because 9 x 11 = 99. 3 x 9 is also a multiple of 9, and so is the digit sum 1 + 3 + 5 = 9. So we can say that 135 is a multiple of 9. In fact, to decide if any number is a multiple of 9 (or is divisible by 9), all we need to do is to look at the digit sum and see if this digit sum is itself a multiple of 9. Using short cuts like this is what is meant by Vedic mathematics.

Sometimes the digit sum has more than one digit. When this occurs, the students need to repeat this process until the digit sum is a single digit. We have called this digit the Vedic digit for the number. For example, 2 389 has the digit sum 2 + 3 + 8 + 9 = 22. The digit sum for 22 is 2 + 2 = 4. So the Vedic digit for 2 389 is 4, which indicates that 2 389 is not a multiple of 9.

The mathematics above is not included in the activity for the students, but it does provide a background to the activities in which the students use patterns in their completed Vedic grids to devise rules for identifying multiples of 9, 6, and 3 (see questions **1** and **2**).

It may be useful to put the following questions to the students to develop their thinking about multiples and divisibility:

Question: Are all multiples of 6 and 9 also multiples of 3? Why or why not?
Answer: Yes. 6 and 9 are themselves multiples of 3, so the product formed when any (whole) number is multiplied by 6 or 9 will also be divisible by 3 and be a multiple of 3. For example, 12 (a multiple of 6) is also a multiple of 3, and 18 (a multiple of both 6 and 9) is also a multiple of 3.

Question: Are all multiples of 9 also multiples of 6? Why or why not?
Answer: No. A number such as 27 is a multiple of 9 but not of 6. Multiples of 9 are sometimes odd (as with 27) but multiples of 6 are always even.

In question **3a**, the students start by using their rules to check which numbers are multiples of 9, 3, or 6. (They will find that all the numbers are multiples of 3 but that not all are multiples of 6 or 9.) In question **3b**, they use their rules to find the missing digits in a 3-digit number and a 5-digit number that are multiples of 9. There are two possible answers for 96 ❐21 in this question and three possible answers for 1 6❐7 in question **3c**. The three possible missing digits for 1 6❐7 give the Vedic digits 6, 9, and 3 respectively.

Page 11

Bagged Cubes

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)
- solve simple linear equations such as 2❐ + 4 = 16 (Algebra, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 2–3)

ACTIVITY

This activity requires students to evaluate equations that are represented symbolically and, in question **3**, to solve a series of linear equations.

In question **1**, the students substitute 3 for *r*, 5 for *b*, and 4 for *y*. Here, *r* stands for the number of cubes in the red bag, *b* stands for the number of cubes in the blue bag, and *y* stands for the number of cubes in the yellow bag. So, in **1a**, the total number of cubes represented by the bag diagram shown below is 3 + 3 + 5 + 5 = 16.

r	*r*	*b*	*b*

In question **1h**, the bag diagram is:

b	*r*	*r*	*b*

The total number of cubes in these bags is 5 + 3 + 3 + 5 = 16, which is the same as the total number of cubes in the bags shown in question **1a**. Only the order has changed. Whichever diagram is used, there are 2 red bags and 2 blue bags and a total of 16 cubes.

Question **3** is considerably more challenging, and the students have to find a way to deduce (logically reason) the number of cubes in each bag. They will probably do best if they begin with clues iii and iv because these two equations both involve only two (and the same two) unknown quantities: in this case, black and orange but not green.

[Black] [Black] [Black] is equivalent to [Orange] [Orange] [Orange] [Orange]

and

[Orange] + ❐❐ is equivalent to [Black] .

So each black bag shown for clue iii can be replaced by 1 orange bag and 2 extra cubes. So,

[Orange + ❐❐] [Orange + ❐❐] [Orange + ❐❐] is equivalent to [Orange] [Orange] [Orange] [Orange]

The students can deduce from this that 1 orange bag has 6 cubes. Each black bag has 2 more cubes than an orange bag (clue iv), so there are 6 + 2 = 8 cubes in each black bag. In clue iii, 1 orange bag is equivalent to 3 green bags. So each green bag has 6 ÷ 3 = 2 cubes. Note that although clue i is not needed to solve this puzzle, it is consistent with the other clues, that is, the solution satisfies the equation 6 + 2 + 6 + 2 = 8 + 8. Note also that the clues are effectively symbolic equations. For example, we could represent clue i as $g + o + g + o = b + b$ or $2g + 2o = 2b$.

If necessary, you could assist students who are struggling with question **3** by telling them the number of cubes in one of the coloured bags and seeing if they can then deduce the number of cubes in each of the other two coloured bags.

Pages 12–13 Building Borders

Achievement Objectives

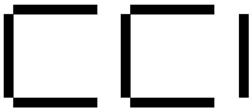
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, levels 2–3)

ACTIVITY

This activity is very similar to that on page **6** of the students' book and again requires students to break down a pattern in order to get to grips with its behaviour mathematically.

In question **1a**, the students build an 8-plot garden by extending the arrangement they are given for a 5-plot garden. The arrangement they make helps them visualise the garden as 8 sets of 3 boards and 1 extra board.

In question **1b**, the students are asked to explain how the geometrical structure of the arrangement of the boards matches the numerical expression 8 x 3 + 1. This expression is a short cut because it provides a quick way to count the number of boards in an 8-plot garden. The following diagrams show how similar short cuts work for a 2-plot garden and then a 5-plot garden. The students may need to use sticks to build "gardens" with different numbers of plots so that they can see clearly how the short cuts work.

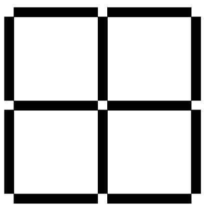


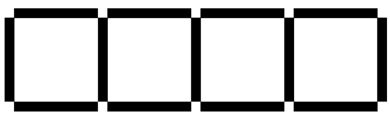
Short cut is 2 x 3 + 1

Short cut is 5 x 3 + 1

In question **1c**, the diagram for a 5-plot garden has been reversed. So, instead of visualising 5 x 3 + 1 boards, we now see 1 + 5 x 3 boards. Although they are based on different ways of visualising the boards, the two short cuts have the same numerical value, in this case, 16. In fact, rotating the diagram in question **1c** through 180 degrees, by turning the page upside down, will give the diagram Nikki used, so the two short cuts are clearly equivalent. Once the students visualise a 5-plot garden as having 1 + 5 x 3 boards, they may reason that a 99-plot garden has 1 + 99 x 3 boards. Such reasoning involves the rule:
the number of boards = 1 + number of plots x 3. The rule based on the short cut 5 x 3 + 1 for a 5-plot garden is:
the number of boards = number of plots x 3 + 1.
Note that the rules are expressed here in words rather than in symbols because the students are not expected to use the symbolic language of algebra at this stage. If they were to use it and there were x plots, then the number of boards, y, can be expressed by the rules $y = 3 \times x + 1$ or $y = 1 + 3 \times x$. However, $3 \times x$ is usually written more simply as $3x$, so the rules are $y = 3x + 1$ or $y = 1 + 3x$.

Note that the short cuts used in this activity will only work for plots in a row. Other arrangements may use fewer boards. For example, the students may discover that the arrangement of 4 plots in a 2 by 2 grid uses only 12 boards, whereas 13 boards are needed for 4 plots in a row.

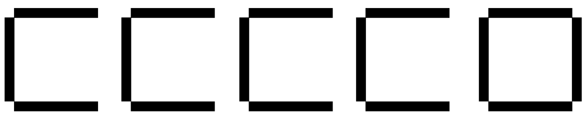




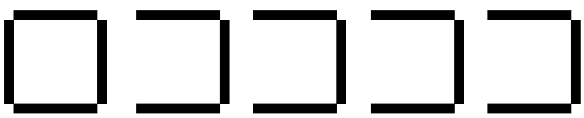
In the 2 by 2 grid, the fourth plot requires only 2 boards to complete it. In general, rectangular arrangements of plots will require fewer boards than plots arranged in single rows.

In question **2**, the arrangement helps us visualise the 7-plot garden as having the first 6 of the 7 plots with 3 boards each and the last plot with 4 boards. So, altogether, we see 6 x 3 + 4 boards. Therefore, a garden with 5 plots has 4 x 3 + 4 = 16 boards, and a garden with 73 plots has 72 x 3 + 4 = 220 boards.

An arrangement that helps us visualise a 5-plot garden as having 4 x 3 + 4 boards is:



So, in question **3**, an arrangement that helps us visualise a 5-plot garden as having 4 + 4 x 3 boards is:



We can express Tau's rule in words as "The number of boards = (number of plots – 1) x 3 + 4."

Using x for the number of plots and y for the number of boards, the rule is $y = (x - 1) \times 3 + 4$, or $y = 3(x - 1) + 4$. The algebraic rule for the reversed arrangement is $y = 4 + 3(x - 1)$.

Finally, in question **4**, another arrangement is given to help the students visualise the boards needed for a 5-plot garden. There are 5 sets of 2 boards (shown horizontally) and 6 vertical boards. So, for a 10-plot garden, we can reason that there are 10 sets of 2 horizontal boards and 11 vertical boards. So the rule is: the number of boards = (number of plots x 2) + (number of plots + 1). Using x for the number of plots and y for the number of boards, the algebraic rule is $y = x \times 2 + x + 1$, or $y = 2x + x + 1$.

If the students are finding question **4c** difficult, it may help to introduce a further two columns to the table to help make the pattern and calculations clearer:

Number of plots	Number of vertical boards	Number of horizontal boards	Total number of boards

Note that we have expressed rules for the number of boards in a garden with any number of plots in several different ways:
$y = 3x + 1$
$y = 1 + 3x$
$y = 3(x - 1) + 4$, which simplifies to $y = 3x - 3 + 4$ or $y = 3x + 1$
$y = 4 + 3(x - 1)$, which simplifies to $y = 4 + 3x - 3$ or $y = 1 + 3x$
$y = 2x + x + 1$, which simplifies to $y = 3x + 1$.

The rules all simplify to either $y = 3x + 1$ or $y = 1 + 3x$, which shows that all the short cuts we have considered share the same underlying mathematical structure.

Pages 14–15 Patterns and Spreadsheets

Achievement Objectives

- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- state the general rule for a set of similar practical problems (Algebra, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY ONE

In this activity, the students learn how to work with spreadsheet formulae to make calculations that might take considerable time if they were carried out manually. Note that the way your spreadsheets work may be slightly different from the spreadsheets that are shown. You will need to check to see what differences there are before you begin this activity with your students.

In question **1**, the students must first notice that Mel saves regular $7 amounts each week starting from week 2. They calculate the total savings in 4 weeks by adding $7 to the savings in week 3. For 5 weeks, the total amount saved is the savings in week 4 plus $7, and so on. Pressing the following sequence of calculator buttons on most calculators will make these calculations:

[12] [+] [7] [=] [=] [=] ...

(On some calculators, the sequence is 12 + + 7 = = = ... or 7 + + 12 = = = ...) A new total amount saved is displayed after each press of the equals button.

In question **2**, the students need to look for the pattern in the cell formulae. They see that the formula in cell B3 is =B2+7 and then must predict that the formula in cell B8 is =B7+7, the formula in B11 is =B10+7, and so on. These predictions are based on the rule: the value in any cell is the value in the previous cell plus 7.

The power of spreadsheet calculations is demonstrated when the students use their spreadsheet to find Mel's savings in 52 weeks. In an instant, the computer makes 51 calculations from cell B3 for week 2 to cell B53 for week 52, to give the total savings in 52 weeks as $369.

ACTIVITY TWO

The students here use their knowledge and understanding of spreadsheets to devise a spreadsheet, such as the one shown in the answers, that will indicate how long it takes Jeff to save $750.

ACTIVITY THREE

In this activity, Mārama makes regular withdrawals of $9 from her account. Note that in week 11, cell B12 will show the savings as –3, that is, an overdraft of $3.

The students might modify this spreadsheet to explore different starting amounts and regular withdrawal amounts.

ACTIVITY FOUR

The students may need to draw pathways on square grid or graph paper, one with 4 lawn squares that can be extended to 5 lawn squares, and so on as necessary, before they see the rule "Add 5 to the previous number of paving stones." They use this rule to create the spreadsheet formula for cell B3 and so on. The answers to questions **3** and **4** are found by using Fill Down for the Number of paving stones column.

ACTIVITY FIVE

Here, the students design their own spreadsheets and use them to answer the questions. The spreadsheets should look something like the following. In each case, the formula shown at the top is for cell B12.

Number pattern 1 (SS)

B12 =B11+6

	A	B	C
1	Position of number	Number	
2	1	17	
3	2	23	
4	3	29	
5	4	35	
6	5	41	
7	6	47	
8	7	53	
9	8	59	
10	9	65	
11	10	71	
12	11	77	
13	12	83	

Number pattern 2 (SS)

B12 =B11+20

	A	B	C
1	Position of number	Number	
2	1	53	
3	2	73	
4	3	93	
5	4	113	
6	5	133	
7	6	153	
8	7	173	
9	8	193	
10	9	213	
11	10	233	
12	11	253	
13	12	273	

Number pattern 3 (SS)

B12 =B11+3.5

	A	B	C
1	Position of number	Number	
2	1	6	
3	2	9.5	
4	3	13	
5	4	16.5	
6	5	20	
7	6	23.5	
8	7	27	
9	8	30.5	
10	9	34	
11	10	37.5	
12	11	41	
13	12	44.5	

Note that each spreadsheet above has been extended only to the 13th row or the 12th number in the sequence. The students will need to extend these further using the Fill Down command.

Page 16 Digit Chains

Achievement Objectives

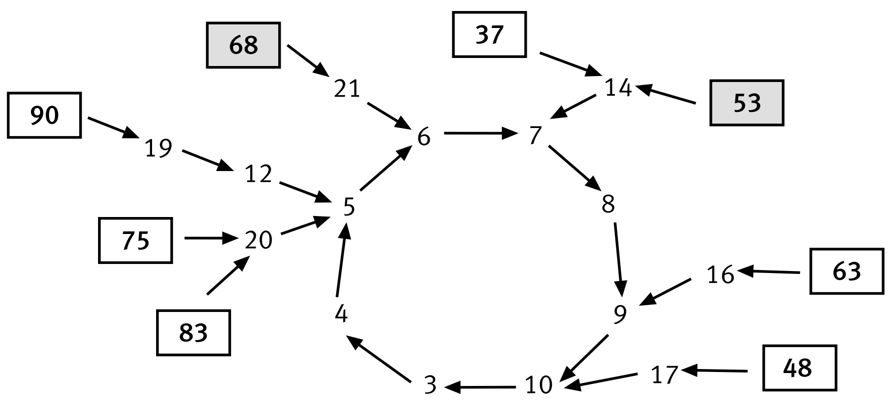
- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, levels 2–3)
- state the general rule for a set of similar practical problems (Algebra, level 3)
- make up and use a rule to create a sequential pattern (Algebra, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 2–3)

ACTIVITY

Most of the early difficulty that students have with digit chains arises from not understanding the given rule to make the numbers in the digit chain. It may be that their difficulties are related to confusion with the notion of *digit* as distinct from *number*. It is likely that some discussion will be needed so that all can see how, for example, 68 can be transformed into 21 and then to 6 and so on.

The interesting and surprising aspect of working with digit chains, which are easily made by devising a rule for each digit, is that they often reduce to a number that is part of a loop. The digit rules for question **1** produce a loop comprising the sequence 3, 4, 5, 6, 7, 8, 9, 10, 3, 4, 5, … The loops given in the Answers show how the digit rule works for the starting numbers 68 and 53.

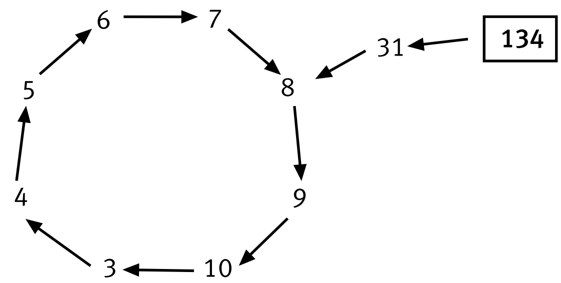
In question **1c**, the students create a separate digit chain for each starting number. When this is complete, they try to combine their separate digit chains so that they can clearly see the effect of the digit rule.



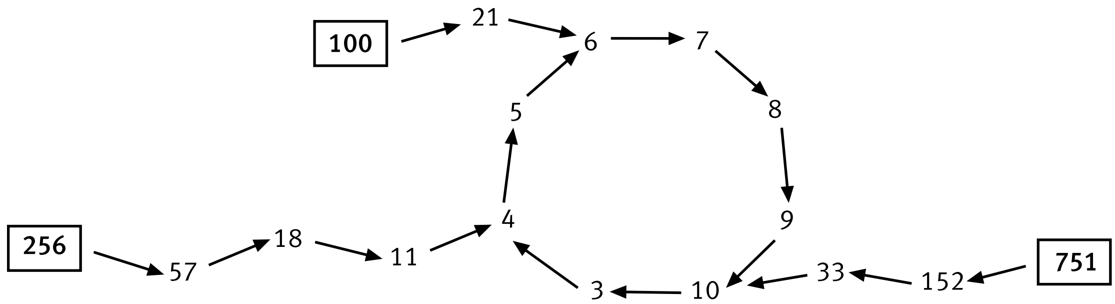
Note that three of the given starter numbers, 75, 83, and 90, all enter the loop at 5. The students could explore other 2-digit starters to see how many they can find entering the loop at 5. They might also do this for other numbers in the loop.

Before the students begin question **1e**, they might try to predict what happens to 0, 1, and 2, which are not in the loop. As shown in the Answers, they will find that they form a separate string or sequence of numbers, which enter the loop at 3.

In question **2**, the students extend their exploration of digit chains with the given rule to 3- and then 4-digit starter numbers. A diagram for 134 is:



The 3-digit numbers enter the loop in the following way:



In question **2e**, the students try to show how a 4-digit number they choose fits into the chain. The process may involve a number of steps, each of which provides an opportunity for the students to show their place-value knowledge and their skills in working with numbers. For example, 26 384 is 2 638 tens and 4 ones. So it becomes (2 x 2 638) + (4 + 1) = 5 276 + 5, which is 5 281. This becomes (2 x 528) + (1 + 1) = 1 056 + 2, which is 1 058. And 1 058 becomes (2 x 105) + (8 + 1) = 219, which becomes (2 x 21) + (9 + 1) = 52. The number 52 becomes (2 x 5) + (2 + 1) = 13, which finally becomes (2 x 1) + (1 + 3) = 6 where it enters the loop.

Page 17 Pizza Order

Achievement Objectives

- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 2–3)
- record, in an organised way, and talk about the results of mathematical exploration (Mathematical Processes, communicating mathematical ideas, levels 2–3)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, levels 2–3)

ACTIVITY

This activity asks students to combine regular prices and special deals in order to calculate the total cost for an order of pizzas.

In question **1,** Sally thinks (reasonably) that the first 3 pizzas will cost $24 and that the cost of each subsequent pizza in her order is $7. The total cost is then $24 + 9 x $7 = $24 + $63 = $87. However, it turns out that Sally has misunderstood the special offer. The total offer is for sets of 4 pizzas where each set of 4 pizzas costs a total of $24 + $7 = $31. This means that 12 pizzas will actually cost 3 x $31 = $93, not $87 as Sally had first thought.

In question **2**, the students complete the table so they can see how to calculate the cost of any number of pizzas.

There are two ways that the offer could be used to calculate the cost of 9 pizzas and also of 10 and 12 pizzas. Firstly, 9 pizzas could be charged as 2 sets of 4 pizzas and 1 additional pizza. The cost for this is (2 x $31) + $8.50 = $70.50. An alternative is to charge for 3 sets of 3 pizzas costing 3 x $24 = $72. So the first charging method is the cheaper for 9 pizzas. The situation is similar for 12 pizzas. Here, it is cheaper to pay for 3 sets of 4 pizzas (3 x $31 = $93) than for 4 sets of 3 pizzas (4 x $24 = $96). For 10 pizzas, the offer could be used to buy 2 sets of 4 pizzas at $31 per set, plus 2 pizzas at $8.50 each, for a total of $79. Alternatively, the offer could be used to buy 3 sets of 3 pizzas at $24 per set, plus 1 pizza at $7, for a total of $79. In this example, both approaches give the same total price. More able students could examine charging possibilities for other numbers of pizzas to see if there are acceptable alternatives.

In question **4**, the students examine a new offer. Under the initial offer, 4 pizzas cost $24 + 7 = $31. Now the cost is $30.50, which is a saving of 50 cents. The students could extend the table they made for question **2** earlier to further compare prices under the two offers:

Number of pizzas	First offer	Second offer	
	Cost	Cost	Working
3	24.00	25.50	3 x 8.50
4	31.00	30.50	30.50
5	39.50	39.00	30.50 + 8.50
6	48.00	47.50	30.50 + 2 x 8.50
7	55.00	56.00	30.50 + 3 x 8.50
8	62.00	61.00	2 x 30.50
9	70.50	69.50	61 + 8.50
10	79.00	78.00	61 + 2 x 8.50
11	86.00	86.50	61 + 3 x 8.50
12	93.00	91.50	3 x 30.50

Note that the second offer is the cheaper for all except 3, 7, or 11 pizzas. By extending the table to 15 pizzas, the students might explore to see if this pattern continues to jump in fours.

Pages 18–19 Tiling Teasers

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, levels 2–3)

ACTIVITY

In this activity, students consider patterns built from basic geometric units. While the first questions ask the students to extend patterns in a straightforward way, later questions ask them to consider a pattern in which new elements are introduced or where only part of the initial shape is repeated.

In question **1**, the students count the number of squares and rhombuses (shapes with 4 equal sides that in this case are not squares) for jewels with 1, 2, and 3 hexagons and enter their results in a table. They look for patterns in the table and write a rule for each pattern. Having completed the table, the students then use the rules to predict the number of squares and rhombuses for 10 hexagons. The patterns here produce simple rules: the number of squares is 2 times the number of hexagons, and the number of rhombuses is 4 times the number of hexagons.

As earlier, some students may be ready to write these rules using algebraic symbols. So for n hexagons: $s = 2 \times n$ and $r = 4 \times n$, where s is the number of squares and r is the number of rhombuses. These rules are usually written more simply as $s = 2n$ and $r = 4n$. The symbols s and r stand for the number of squares and the number of rhombuses respectively, not for the actual square and rhombus shapes. Note that $r = 2s$ is a third rule, this time linking the *number* of rhombuses with the *number* of squares. This rule can be expressed in words as "The number of rhombuses is 2 times the number of squares."

In question **2**, each yacht has 1 trapezium (a 4-sided shape or quadrilateral with 1 pair of parallel sides). Each yacht also has 4 triangles. So a rule is: the number of triangles is 4 times the number of trapezia (or yachts). In algebraic notation, we can write $n = 4 \times t$, or better, $n = 4t$, where n is the number of triangles and t is the number of trapezia (or yachts).

In question **3**, the number of squares is 2 times the number of trapezia (or townhouses), and the number of rhombuses is the number of trapezia minus 1. These can be expressed algebraically as $s = 2t$ and $r = t - 1$. Here, s stands for the number of squares, r stands for the number of rhombuses, and t stands for the number of trapezia (or townhouses).

In question **4**, the number of rhombuses is 5 times the number of fish plus 2. This can be expressed algebraically as $r = 5 \times f + 2$ or $r = 5f + 2$, where r stands for the number of rhombuses and f stands for the number of fish, although this is likely to prove challenging for many students. They may instead come up with a rule something like the following: 1 fish needs 7 rhombuses, and each additional fish adds 5 rhombuses. Using this rule, they could then predict that 10 fish would need 7 + 9 x 5 = 52 rhombuses.

In question **5**, the number of rhombuses is 5 times the number of hexagons plus 1, and the number of triangles is 2 times the number of hexagons minus 2. These rules can be expressed respectively as $r = 5n + 1$ and $t = 2n - 2$, where r stands for the number of rhombuses, n stands for the number of hexagons, and t stands for the number of triangles.

Once again, such rules will elude some students. Equally acceptable is a rule such as: the first cog needs 1 hexagon, 6 rhombuses, and 0 triangles; each additional cog adds 1 hexagon, 5 rhombuses, and 2 triangles. Using this rule to calculate the number of rhombuses and triangles for 10 cogs or hexagons, we find:
rhombuses: 6 + 9 x 5 = 51
triangles: 0 + 9 x 2 = 18.

Page 20 Which Wheels Where?

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)
- record, in an organised way, and talk about the results of mathematical exploration (Mathematical Processes, communicating mathematical ideas, levels 2–3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 2–3)
- solve simple linear equations such as 2❒ + 4 = 16 (Algebra, level 4)

ACTIVITY

In this activity, students work with simple linear equations in order to draw conclusions about the number and types of skatepark users. Encourage the students to approach the questions in a systematic way rather than guessing and checking solutions.

In question **1**, recording in a systematic way in a table may help the students to reason logically.
In question **1a**, the students should reason that because there are just 4 people involved and at least 1 must be a cyclist, 1 a skateboarder, and 1 an in-line skater, there are three possibilities. There could be 2 cyclists, or 2 skateboarders, or 2 in-line skaters. The table below shows the three possible solutions.

Number of people	Number of 2-wheel cycles	Number of 4-wheel skateboards	Number of 8-wheel in-line skates	Total number of wheels
4	2	1	1	2 x 2 + 4 + 8 = 16
4	1	2	1	2 + 2 x 4 + 8 = 18
4	1	1	2	2 + 4 + 2 x 8 = 22

In question **1b**, the total number of people involved is 6. Note that the data in the first row in the table above can be modified in two ways, represented by the first two rows in the table below. The second row in the table above can be modified to produce the third row in the table below. This produces the same set of riders as the second row in the table below, so effectively there are only two possibilities. Changes made to the numbers in the third row in the first table would not meet the new conditions.

Number of people	Number of 2-wheel cycles	Number of 4-wheel skateboards	Number of 8-wheel in-line skates	Total number of wheels
6	2 + 2	1	1	4 x 2 + 4 + 8 = 20
6	2 + 1	1 + 1	1	3 x 2 + 2 x 4 + 8 = 22
6	1 + 2	2	1	3 x 2 + 2 x 4 + 8 = 22

The number of wheels of each type can then be calculated by considering the number of users of each type of transport as listed in the table.

In question **1c**, David and Francie have different types of wheels. This situation is represented by the data in the second row in the table above, where either David or Francie rides a cycle and the other rides a skateboard.

In question **2**, using a table in a systematic way will ensure that the students will consider all the possibilities. In such tables, a good strategy is to initially consider the machine with the greatest number of wheels, in this case, the in-line skates. The students should make the number of in-line skates as large as possible and then systematically consider other possibilities.

Number of people	Number of 2-wheel cycles	Number of 4-wheel skateboards	Number of 8-wheel in-line skates	Total number of wheels
6	1	1	4	2 + 4 + 4 x 8 = 38
6	1	2	3	2 + 2 x 4 + 3 x 8 = 34
6	2	1	3	2 x 2 + 4 + 3 x 8 = 32

The students do not need to go beyond the data in the first row of the table because different combinations of riders with fewer sets of in-line skates will always include fewer than 38 wheels.

Page 21 Whānau Photo

Achievement Objectives

- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 2–3)

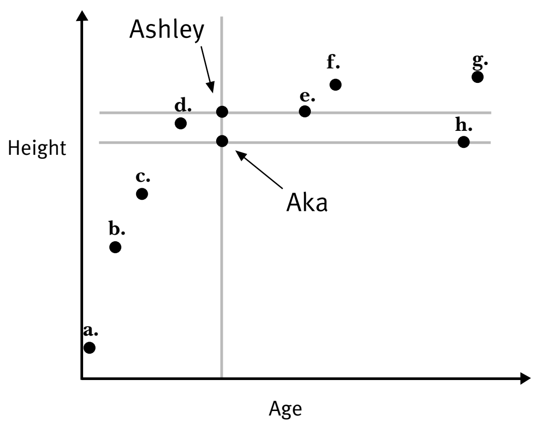
ACTIVITY

This activity shows the students how the distinctive features of an everyday situation, provided by a photograph of a whānau, can be represented graphically. These features, or variables, are age and height. The lack of any scale for age and height means that the students must focus on the relationships between the variables. Note that for the children represented by the points a, b, c, and d, height is mostly dependent on age, but for the adults, e, f, g, and h, height is related primarily to gender rather than to age.

In question **1**, the students should be encouraged to explain their reasoning for choosing the person represented by each point.

In question **2**, the students should also explain their reasoning for positioning the points for Aka and Ashley. In the graph below, any point on the horizontal line through h (Whetū), represents a person who is the same height as Whetū. Aka is the same height as Whetū, so the point for Aka is on the horizontal line. Aka is also older than Manu (d) so the point for Aka is on the horizontal line but to the right of the point for d.

Whānau Heights by Age



Ashley is the same age as Aka, so the point for Ashley lies on the vertical line that passes through the point for Aka. Ashley's height is also halfway between Hotu's height (f) and Tungia's height (e). So the point for Ashley lies on the vertical line in a position midway between the points e and f.

Pages 22–23 Graphic Details

Achievement Objectives

- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- state the general rule for a set of similar practical problems (Algebra, level 3)

ACTIVITY ONE

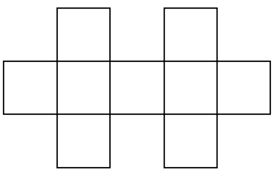
In this activity, the students record in a table the data taken from the tile designs and then draw a graph to illustrate any patterns in the data. If the students have not encountered co-ordinates already, you will need to take the time to introduce the concept of co-ordinate pairs (ordered pairs), how to write them, and how to graph and interpret them. The two key principles to emphasise are:

i. the order of the numbers within the brackets matters
ii. when plotting the point on a graph, the first number defines horizontal movement from the origin (0,0) and the second number defines vertical movement.

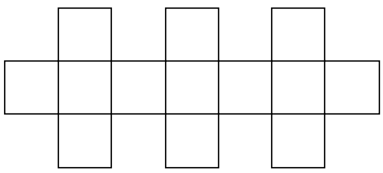
Each point on the graph has a pair of co-ordinates representing the data in the table. The points lie on a straight *line*, so the algebraic relationship for the pattern is called a *linear* relationship.

Graphing ordered pairs is particularly useful when it comes to seeing patterns. If there is a rule connecting the first and second number in each pair, this will show up as an obvious line, curve, series of lines, or area. In order to predict if the points (6,25), (10,41), (9,36), (7,30), and (8,33) lie on the line, some students may choose to extend the line on the graph. This is an appropriate approach, but such students will need to be very accurate in their sketching. For example, although the points (9,36) and (7,30) are not on the line, the points (9,37) and (7,29) are, and the difference may be difficult to read. A generous-sized graph, for example, with a scale limited to 12 on the horizontal scale and 45 on the vertical scale, would assist students who use this method. Students who approach this question algebraically will need to test each pair of co-ordinates using the following relationship for the pattern: the number of tiles is equivalent to the number of crosses times 4 plus 1.

So, for example, the design below with 2 crosses has 2 x 4 + 1 = 9 tiles, and the design with 3 crosses has 3 x 4 + 1 = 13 tiles.



2 crosses



3 crosses

A similar design with 9 crosses has 9 x 4 + 1 = 37 tiles. So the point (9,37) lies on the line, but, as mentioned above, (9,36) does not lie on the line. Similarly, the point (7,29) lies on the line, but (7,30) does not.
The students may express this linear relationship algebraically. For x crosses, there are y tiles where $y = x \times 4 + 1$, or $4x + 1$. When $x = 1\,000$, then $y = 4 \times 1\,000 + 1$, or 4 001. This means that 1 000 crosses will require 4 001 tiles.

ACTIVITY TWO

In this activity, students collect data from the tile designs. They record the data in a table to help identify the co-ordinates for points on the graphs illustrating the rules for the patterns of tiles in the designs.

In question **1**, the students collect and record the data for the designs. They use their graph to find the number of square tiles (21) for a design with 10 triangle tiles. The table below shows the pattern leading to the rule for the linear relationship.

Number of triangle tiles	Number of square tiles	Co-ordinates of points	Pattern
1	3	(1, 3)	2 x 1 + 1 = 3
2	5	(2, 5)	2 x 2 + 1 = 5
3	7	(3, 7)	2 x 3 + 1 = 7
4	9	(4, 9)	2 x 4 + 1 = 9
10	21	(10, 21)	2 x 10 + 1 = 21

The rule for the pattern is: the number of square tiles is 2 times the number of triangle tiles plus 1. Algebraically, this is $y = 2x + 1$, where x stands for the number of triangle tiles and y stands for the number of square tiles.

The tables of data and rules for the designs in question **2** are as follows:

i.

Number of Ls	Number of tiles	Co-ordinates of points	Pattern
1	3	(1, 3)	3 x 1 = 3
2	6	(2, 6)	3 x 2 = 6
3	9	(3, 9)	3 x 3 = 9
4	12	(4, 12)	3 x 4 = 12
10	30	(10, 30)	3 x 10 = 30

The rule for the pattern is: the number of tiles is 3 times the number of Ls. Algebraically, this is $y = 3x$, where x stands for the number of Ls and y stands for the number of tiles.

ii.

Number of tunnels	Number of tiles	Co-ordinates of points	Pattern
1	5	(1, 5)	4 x 1 + 1 = 5
2	9	(2, 9)	4 x 2 + 1 = 9
3	13	(3, 13)	4 x 3 + 1 = 13
4	17	(4, 17)	4 x 4 + 1 = 17
10	41	(10, 41)	4 x 10 + 1 = 41

The rule for the pattern is: the number of tiles is 4 times the number of tunnels plus 1. Algebraically, this is $y = 4x + 1$, where x stands for the number of tunnels and y stands for the number of tiles.

iii.

Number of dogs	Number of tiles	Co-ordinates of points
1	7	(1,7)
2	11	(2,11)
3	18	(3,18)
4	22	(4,22)
5	29	(5,29)

It is not possible to draw a straight line through the points in the table above, so there is no linear rule for the pattern. The following table shows how the number of tiles for successive designs in the pattern increases by 4, then 7, then 4, then 7, and so on.

Number of dogs	Number of tiles	Increase for number of tiles
1	7	+ 4
2	11	+ 7
3	18	+ 4
4	22	+ 7
5	29	

Using this pattern gives the sequence for the number of tiles as 7, 11, 18, 22, 29, 33, 40, 44, 51, 55, …
The tenth number is 55. The alternating increments of 4 then 7 occur because the dog designs are alternately tail to tail and head to head. When they are tail to tail, the tail and hind legs of one dog are also the tail and hind legs of the next dog. Three tiles are shared in this way. When they are head to head, no tiles are shared with the next dog.

Page 24 **Scooting**

Achievement Objectives

- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 2–3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 2–3)

ACTIVITY

This activity asks the students to interpret line graphs illustrating a scooter race between Josh and Rewi. Where the line graph for one racer is steeper than the line graph for the second racer, we can conclude that the first racer is going faster than the second. We know this because the graph indicates that the first racer is covering more distance than the second racer during the same time period.

Some students incorrectly interpret such line graphs as “going up a hill”. They interpret the graph in terms of its physical features rather than in terms of the underlying mathematics relating to the concept of *rate*.

In figure 1, a vertical line through both graphs can help the students to see who is leading at a particular time. Similar vertical lines can be drawn at different times to help the students decide who is in front at that particular moment. Here, the vertical line shows that Josh has travelled further than Rewi for the same elapsed time.

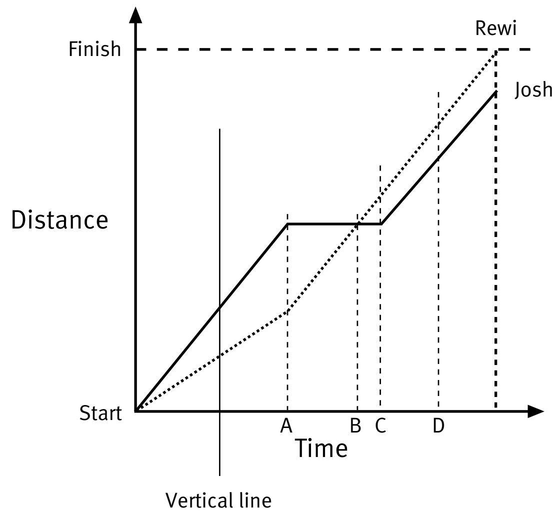


Figure 1

Figure 2 shows how horizontal lines can also be used to work out who has taken the longer time to cover a particular distance. Here, the horizontal line shows that Rewi has taken longer than Josh to cover the same distance.

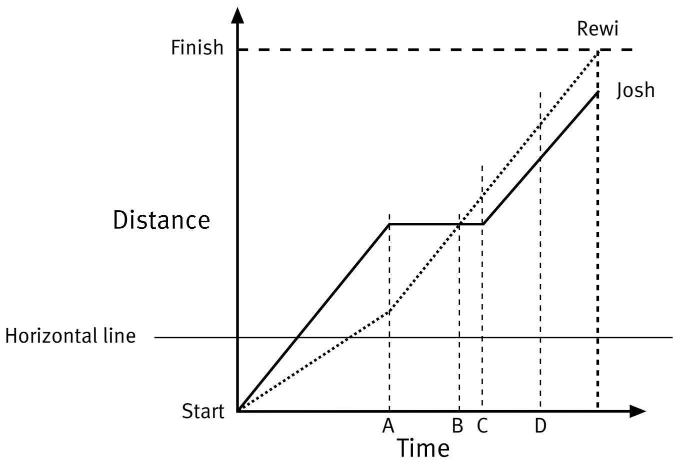
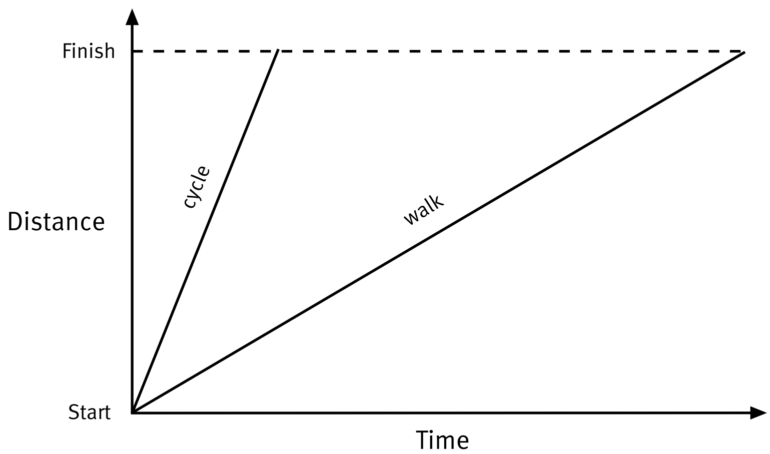


Figure 2

When they have completed question **6**, the students might like to draw lines showing the progress of someone who walked the race route as well as someone who, for example, cycled the route. Possible lines of progress for the individuals are shown below.



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