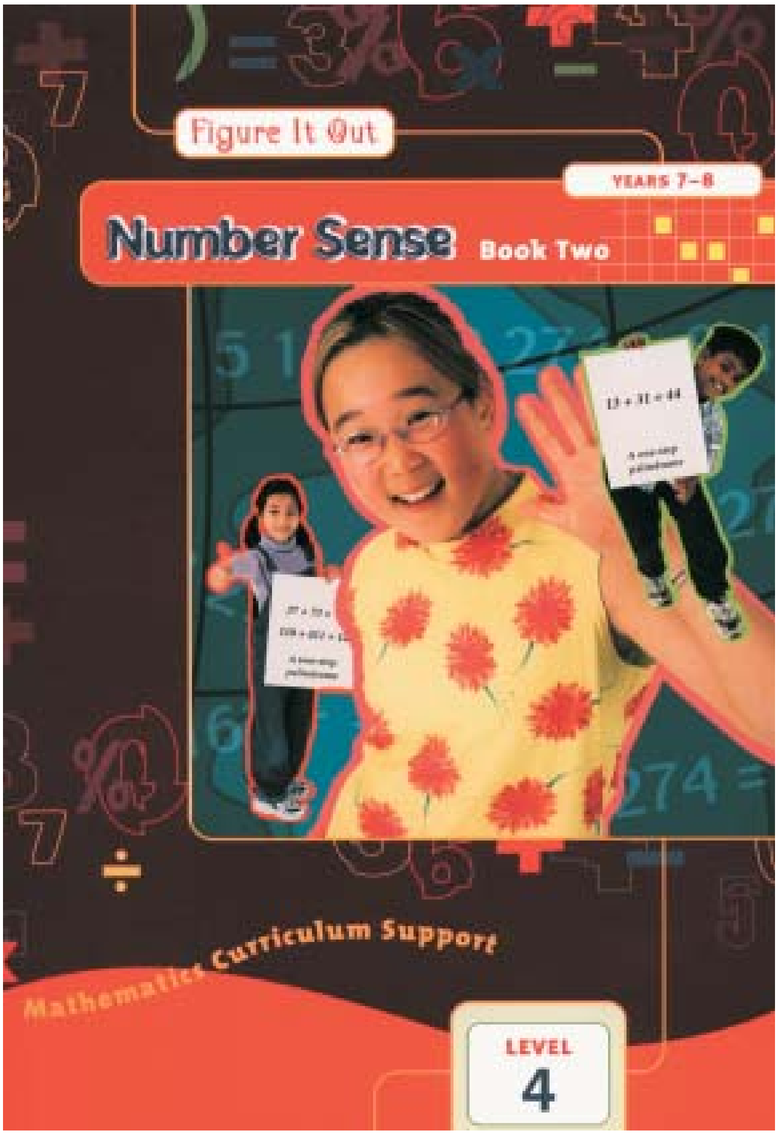
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:

Number (two linking, three level 4, one level 4+)
Algebra (one linking, two level 4, one level 4+)
Measurement (one level 4, one level 4+)
Number Sense (one linking, one level 4)
Geometry (one level 4, one level 4+)
Statistics (one level 4, one level 4+)
Themes (level 4): *Disasters Strike!, Getting Around*

These 20 books will be distributed to schools with year 7–8 students over a period of two years, starting with the six *Number* books that were distributed in October 2002.

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent bookwork, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comment on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/maths/r/curriculum/figure

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Number Sense: Book Two

Page 1 Short Cuts

ACTIVITY

1. a. i. 8 456
   ii. 8 406
   iii. 8 236
   iv. 9 136
   b. Explanations will vary. They will probably include using the place value of 100s or 10s. For example, if the second number was 30 more, the new total would also be 30 more.
2. a. i. 4 272
   ii. 4 283
   iii. 4 245
   iv. 4 075
   v. 4 295
   vi. 3 775
   b. Explanations will vary. They will probably include using the place value of 100s or 10s. For example, if the second number was 30 less, the new difference would be 30 more, or vice versa (the smaller the number subtracted, the greater the difference, and vice versa).

Page 2 Divide and Conquer

ACTIVITY

1. a. It can be divided by 2 and 5. Multiples of 2 are even numbers, and multiples of 5 end in 0 or 5. 100 is even and ends in 0.
   b. It can be divided only by 5. It is odd and ends in 5.
   c. It can be divided only by 2. It is even, and it doesn't end in 5 or 0.
   d. It can't be divided by 2 or 5. It isn't even and doesn't end in 5 or 0.
   e. It can be divided only by 2. It is an even number, and it doesn't end in 5 or 0.
2. a. 17 x 3 = 51. 5 + 1 = 6
   18 x 3 = 54. 5 + 4 = 9
   19 x 3 = 57. 5 + 7 = 12. 1 + 2 = 3
   20 x 3 = 60. 6 + 0 = 6
   21 x 3 = 63. 6 + 3 = 9
   22 x 3 = 66. 6 + 6 = 12. 1 + 2 = 3
   23 x 3 = 69. 6 + 9 = 15. 1 + 5 = 6
   24 x 3 = 72. 7 + 2 = 9

   The pattern (from lowest digital root) is 3, 6, 9, 3, … The digital roots are multiples of 3.
   b. Yes. Their digital roots are multiples of 3. If you work out the digital roots on either side of each of these, the 3, 6, 9 pattern does apply. For example, with 99 x 3:
   97 x 3 = 291. 2 + 9 + 1 = 12. 1 + 2 = 3
   98 x 3 = 294. 2 + 9 + 4 = 15. 1 + 5 = 6
   99 x 3 = 297. 2 + 9 + 7 = 18. 1 + 8 = 9
   100 x 3 = 300. 3 + 0 + 0 = 3

   The results are:
   i. 25 x 3 = 75. 7 + 5 = 12. 1 + 2 = 3
   ii. 99 x 3 = 297. 2 + 9 + 7 = 18. 1 + 8 = 9
   iii. 201 x 3 = 603. 6 + 0 + 3 = 9
   iv. 997 x 3 = 2 991. 2 + 9 + 9 + 1 = 21. 2 + 1 = 3
   c. i. Yes. 6 + 9 = 15. 1 + 5 = 6
   6 is a multiple of 3.
   ii. No. 7 + 3 = 10. 1 + 0 = 1
   10 is not a multiple of 3.
   iii. Yes. 4 + 4 + 4 = 12. 1 + 2 = 3
   3 is a multiple of 3.
   iv. Yes. 2 + 0 + 8 + 5 = 15. 1 + 5 = 6
   6 is a multiple of 3.
   v. Yes. 1 + 1 + 1 + 1 + 1 + 1 = 6
   6 is a multiple of 3.

Page 3 Hundreds

ACTIVITY

1. Expressions will vary. For example, 10 x 10, 10^2, a century, C (Roman), rau (Māori), and so on. You could also use number expressions such as 2 x 50, 60 + 40, 200 ÷ 2, 120 – 20, and so on.
2. **a.** (5 + 5) x (5 + 5) = 100

   b. 36 + 64 = 100 (that is, $6^2 + 8^2 = 100$)

   c. 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 = 100

   d. Solutions will vary. Possible answers include:
   1 + 2 + 34 + 56 + 7 = 100
   21 + 63 + 4 + 5 + 7 = 100
   and $3^4 + 7 + 6 + 5 + 2 - 1 = 100$

   e. $1^3 + 2^3 + 3^3 + 4^3 = 100$

   f. $99 + {}^{99}/_{99} = 100$
3. Answers may vary. A possible strategy could be: Take a 3-digit number and multiply it by 3. If the product contains three new digits (you'll need nine different digits altogether), subtract your original number from this product. If this product contains the remaining unused digits, you have found three 3-digit numbers for your solution. Two possible solutions are: 657, 438, and 219; 981, 654, and 327.

Page 4 Egging On

ACTIVITY ONE

1. **a.** You could work backwards as follows:

Customer	Eggs bought	Eggs left
Egbert	1	0
8	1	1
7	2	2
6	4	4
5	8	8
4	16	16
3	32	32
2	64	64
1	128	128

   b. Henrietta started with 256 eggs.
2. Solutions will vary. You could use the strategy shown above. Keep doubling until you reach the first customer. For **b**, you could add all the purchases together, or better still, double the first purchase.

ACTIVITY TWO

1. **a.** 46

   b. 9

   c. 15
2. Strategies will vary. Possible thinking could be:

   a. 14 + 9 = 23. 2 x 23 = 46

   b. Half of 36 is 18. 18 ÷ 2 = 9

   c. $^1/_4$ of 20 is 5. ☐ ÷ 3 = 5. ☐ = 15

Page 5 Four 4s

ACTIVITY

1. **a.** 2

   b. 11

   c. 1

   d. 24
2. Solutions will vary. For example, solutions for 7 include 7 = (44 ÷ 4) – 4 and 7 = (4! ÷ 4) + (4 ÷ 4). Possible solutions for some of the other numbers include: $1 = {}^4/_4 + 4 - 4$, $5 = \sqrt{4} + \sqrt{4} + {}^4/_4$, $11 = {}^{4!}/_{\sqrt{4}} - {}^4/_4$, and $20 = 4 \times 4 + \sqrt{4} + \sqrt{4}$.
3. Solutions will vary. Two solutions are: $100 = (4! + {}^4/_4) \times 4$ and (4! + 4) x 4 – 4

Page 6 Clothes Spree

ACTIVITY

1. **a.**

T-shirts	$9
Skirts	$17
Sweatshirts	$7
Tops	$1.40

   b.

T-shirts	$9
Skirts	$51
Sweatshirts	$28
Tops	$12.60

2. **a.**

Skirts	$25
Tops	$11.50
T-shirts	$4

   b.

Skirts	$50
Tops	$11.50
T-shirts	$12

3. Answers will vary. Two possibilities are:

Skirts (Threads Galore)	\$50
Three T-shirts (Fitwells)	\$27

\$50 + \$27 = \$77

or

Three T-shirts (Fitwells)	\$27
Two tops (Threads Galore)	\$23
Sweatshirt (Fitwells)	\$28

\$27 + \$23 + \$28 = \$78

Page 7 Time versus Money

ACTIVITY

Solutions will vary. Ben's first idea wouldn't work in this case because more of their items (8 out of 12) round up so they would end up paying more (\$43.30). For Ben's second method, they would have to work out which items to group together so that overall it would cost them less than the extra 2 cents they paid with one bill. One way would be to pay for the first four items (\$11.96, rounded to \$11.95), then the next three items (\$13.45, no rounding required), and then the last five (\$17.82, rounded to \$17.80). With this method, they would actually save 3 cents, instead of paying 2 cents extra.

With Nan's method, they would also save 3 cents rather than pay 2 cents extra. The combined items (all those that end in 3, 4, 8, or 9 cents) come to \$23.23, which would be rounded up to \$23.25. Items rounded down individually would add up to \$12.30 altogether. The chicken, which involves no rounding, could be included in the first group or be paid for individually. The total for all items is \$43.20.

However, given that the items are probably all mixed up in the trolley and have bar codes instead of prices on them, the time taken to save 3 cents may not be worth it … So you would probably decide to just pay the extra 2 cents!

Note that, if you pay by EFTPOS or credit card, you would pay the actual total without any rounding.

Page 8 Reading by Numbers

ACTIVITY

1. a. Numbers will vary. Numbers that finish as one-step palindromic numbers include 10–18, 20–27, 29, 30–36, 38, 40–45, and 47. Numbers that finish as two-step palindromic numbers include 19, 28, 37, 39, 46, 48, and 49. A number that finishes as a four-step palindromic number is 87.

   A number that finishes as a six-step palindromic number is 97.

   b. It works for some numbers, but only if the decimal point is kept constantly in the same place. For example:

$$\begin{array}{r} 0.95 \\ \underline{0.59} \\ 1.54 \\ \underline{4.51} \\ 6.05 \\ \underline{5.06} \\ 11.11 \end{array}$$

   However, in the following example, it doesn't work even though the digits used are the same:

$$\begin{array}{r} 9.5 \\ \underline{5.9} \\ 15.4 \\ \underline{45.1} \\ 60.5 \\ \underline{50.6} \\ 111.1 \end{array}$$

2. Answers will vary. Some 3-digit numbers do not produce a palindromic number. (A palindromic number for 196 has not been found in over 200 000 reversals.)

3. 89 finishes as a 24-step palindromic number (8813200023188).

Page 9 The Greenhouse Effect

ACTIVITY

1.

Day	Control	Plant A	Plant B
Start	3.5	2.75	2.25
1	3.625	3	2.75
2	3.75	3.25	3.25
3	3.875	3.5	3.75
4	4	3.75	4.25
5	4.125	4	4.75
6	4.25	4.25	5.25
7	4.375	4.5	5.75
8	4.5	4.75	6.25

2. Day 32: control plant: 7.5; plant A: 10.75; plant B: 18.25. Explanations will vary. For example, plant B grew 4 cm in 8 days, so in 32 days it would grow 16 cm: 2.25 + 16 is 18.25.

3. **a.** Start at 2.75 cm: 3.25, 3.5, 4, 4.25, 4.75, 5, 5.5, 5.75.

   b. 14.75 cm. The plant grows 0.75 cm every 2 days (0.75 x 16 = 12) or 1.5 cm every 4 days (1.5 x 8 = 12). 2.75 + 12 = 14.75

Page 10 On the Campaign Trail

ACTIVITY

1. 3 students want a shorter school day.
   5 students want girls to be able to wear make-up to school.
   9 students want a longer lunch break.
   10 students want to wear jackets in class.

2. Predictions will vary, but Awatea should win. Explanations should relate to numerical data. For example, 10 students want to be able to wear jackets in class. This would give Awatea more votes than the other two candidates. However, 3 of the 30 students have not expressed an opinion. If they voted for Bram, he would win.

Page 11 Same Answer Every Time!

ACTIVITY

1. Numbers will vary. Possible solutions (found through trial and error) include:
   3 subtraction 4 123
   4 subtraction 2 378
   5 subtraction 9 854
   6 subtraction 2 589
   7 subtraction 3 261
   (You may have noticed that the same numbers that are in the shorter subtractions eventually appear in the longer subtractions.)

2. **a.** Yes, it still works, except when all four digits are the same. (Note that a result of 999 is re-formed as 9990 – 0999.)

   b. 3-digit numbers end up as 495, and 2-digit numbers end up as 9.

Page 12 Plant Patterns

ACTIVITY

1. **a.** Charlotte is right. Peter is partly right. There are three choices of plant for the first position in the garden, but because there are then two choices of plant for the second position (and one for the third), there are actually 3 x 2 x 1 = 6 possible arrangements (as Charlotte suggested, and as listed in **b**).

   b. RKF RFK KRF KFR FRK FKR. There are six possible arrangements, which can be written as 3! (3 factorial).

2. There are 24 different ways when a fourth plant is added: 1 x 2 x 3 x 4 = 24, which is 4!
   The 24 different ways are:

RKFH	KFHR	FHRK	HRKF
RKHF	KFRH	FHKR	HRFK
RFKH	KRFH	FRKH	HKFR
RFHK	KRHF	FRHK	HKRF
RHFK	KHFR	FKRH	HFKR
RHKF	KHRF	FKHR	HFRK

3. Five plants is 5! or 120 different ways.
   Six plants is 6! or 720 different ways.
   When a new plant is added, you multiply the new total number of plants by the previous total number of different ways.
   1 x 2 x 3 x 4 x 5 = 120
   120 x 6 = 720 (1 x 2 x 3 x 4 x 5 x 6 = 720)

Page 13 Fair Exchanges

ACTIVITY

1. $24 Australian equals NZ$30.00.
2. 15 000 yen equals NZ$300.
3. £2.50 equals NZ$7.50.
4. 28 francs equals NZ$40.
5. 45 euros equals NZ$93.75.
6. 900 baht equals NZ$50.

INVESTIGATION

Amounts will vary depending on the exchange rate at the time.

Page 14 Energy Levels

ACTIVITY

1. **b.** 1 hr 45 min.
   c. 1 hr 10 min.
   d. 3 hrs 30 min.
   e. 4 hrs
   f. 3 hrs 30 min.
   g. 1 hr 24 min.
   h. 1 hr 33 min.
   i. 3 hrs 7 min.
   j. 2 hrs 20 min.
   k. 4 hrs 40 min.
   l. 2 hrs
2. Walking: 4 hrs 17 min.
   Rowing: 2 hrs 9 min.
   Aerobics: $2\frac{1}{2}$ hrs
3. Television: $6\frac{1}{2}$ hrs
   Reading: 5 hrs
4. Suggested kilojoules will vary.

Page 15 People Power

ACTIVITY

1. Graph **c** is the correct one.
2. **a.** 61%
   b. 12%
   c. 13%
   d. 14%
3. 0.5%. 1% would be about 60 million, and Oceania has half that population.

Pages 16–17 Astronomical Proportions

ACTIVITY ONE

1. On the basis of the pattern, Pluto might be expected to be two to three times the orbit period of Nepture. 300 is therefore a better estimate. (Pluto's actual orbit period is 248.54 years.)
2. **a.** Venus
   b. Jupiter
   c. Mercury
   d. Neptune
   e. Mars
3. One way to estimate this is:
   29.46 is about 30. 0.24 is about $^1/_4$.
   $30 \div {}^1/_4 = 30 \times 4$
   $= 120$
   (The exact answer is 29.46 ÷ 0.24 = 122.75.)

   Another way to estimate this is: Mercury does approximately four orbits a year, and Saturn does one orbit nearly every 30 years. 4 x 30 = 120

ACTIVITY TWO

1. Estimates will vary. Likely estimates and the actual radii (given in brackets) of all the planets shown are:

Mercury	2 000 km	(2 440 km)
Venus	6 000 km	(6 052 km)
Earth	6 000 km	(6 371 km)
Mars	3 000 km	(3 390 km)
Jupiter	70 000 km	(69 911 km)
Saturn	58 000 km	(58 232 km)
Uranus	25 000 km	(25 362 km)
Neptune	25 000 km	(24 624 km)
Pluto	1 000 km	(1 195 km)

2. a. Yes. Pluto (1 195 km) is the size of a large moon.

   b. i.

Ganymede	2 631
Titan	2 575
Callisto	2 410
Io	1 821
Moon	1 737
Europa	1 561
Triton	1 353
Oberon	761

   ii. Earth's moon is extraordinarily large in relation to the size of Earth compared with those of the other planets. (The diameter is one-quarter that of the host planet.) The other large moons are attached to very large host planets.

3. Answers will vary. Although the larger planets tend to have more moons, the rule is not strictly correct. For example, Mars is smaller than Earth, but it has two moons.

Page 18 No Space to Spare

ACTIVITY

1. a. Practical activity

   b. Estimates will vary.

2. a. New Zealand's population density is 14.4 people per square kilometre.

   b. i. 3 876 times

   ii. 1 156 times

   iii. 214 times (213.6 to 1 d.p.)

   c. Monaco

3. a. Answers will vary. 20 000 classrooms with an area of 50 m^2 would fit into a square kilometre.

   b. Answers will vary. In Macao, 3 people would live in the space of a 50 m^2 classroom, and 4 people would live in the space of a 70 m^2 classroom.

   c. In densely populated countries, many people live in high-rise buildings.

Page 19 Pizza Pieces

ACTIVITY

1. All the ways are:
   $0.5 + \frac{1}{2}$
   $0.5 + 0.4 + 0.1$
   $0.5 + \frac{2}{5} + 0.1$
   $0.5 + \frac{3}{10} + \frac{2}{10}$
   $\frac{1}{2} + 0.4 + 0.1$
   $\frac{1}{2} + \frac{2}{5} + 0.1$
   $\frac{1}{2} + \frac{3}{10} + \frac{2}{10}$
   $0.6 + 0.4$
   $0.6 + \frac{2}{5}$
   $0.6 + \frac{3}{10} + 0.1$
   $\frac{7}{8} + 0.125$
   $0.66\dot{6} + \frac{1}{3}$
   $0.75 + \frac{1}{4}$
   $0.4 + \frac{2}{10} + \frac{3}{10} + 0.1$
   $0.4 + \frac{2}{5} + \frac{2}{10}$
   $\frac{2}{5} + \frac{2}{10} + \frac{3}{10} + 0.1$

2. a.–b. Concentrating on ways to make one-half is a good first strategy:
   0.5 and $\frac{1}{2}$ are equivalent.
   $\frac{2}{5}$ plus 0.1 make one-half.
   $\frac{2}{10}$ plus $\frac{3}{10}$ make one-half.
   Combining these with the ways to make one whole gives all the possible answers. For example:
   0.5 or $\frac{1}{2}$ with:
   $\frac{7}{8}$ and 0.125
   $\frac{1}{3}$ and $0.66\dot{6}$
   0.75 and $\frac{1}{4}$
   $0.4 + \frac{2}{10} + \frac{3}{10} + 0.1$

Page 20 Line Up

ACTIVITY

1. a. i. c

   ii. d

   iii. a

   iv. b

   b. 2.5 is a factor in each product, so the order of the other factors matches the order of the arrows (0.5, 0.9, 1.1, 1.9).

2. i. d
   ii. a
   iii. c
   iv. b

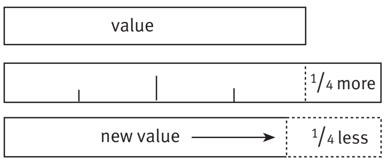
3. 0.4 (2.8 ÷ 7 = 0.4, so 0.4 x 7 = 2.8)
1.4 (9.8 ÷ 7 = 1.4, so 1.4 x 7 = 9.8)
2.5 (17.5 ÷ 7 = 2.5, so 2.5 x 7 = 17.5)
3.1 (21.7 ÷ 7 = 3.1, so 3.1 x 7 = 21.7)
3.6 (25.2 ÷ 7 = 3.6, so 3.6 x 7 = 25.2)

Explanations will vary, but using the family of facts idea is useful. For example, with □ x 7 = 2.8:
2.8 ÷ 7 = □, so □ = 0.4

Page 21 Gains and Losses

ACTIVITY

1. a. Yes.
   b. They have decreased by $^1/_4$ and are now worth only $^3/_4$ of their value of 2 years ago.
2. A possible diagram is:



3. a. $^8/_9$ of the original value
   b. $^{24}/_{25}$ of the original value
   c. $^{35}/_{36}$ of the original value
4. a. The general rule for a fractional increase followed by a decrease of $^1/_n$ is $\frac{n^2 - 1}{n^2}$
   b. $^{9\,999}/_{10\,000}$

Pages 22–23 Balancing Act

ACTIVITY

1. a. 9
   b. 48
   c. 9
   d. 57
   e. 14
   f. 15
   g. $12^1/_2$
   h. $15^1/_3$
2. Solutions will vary, but some possibilities are:
   a. 6 lots of 12; 3 lots of 24; 4 lots of 18; 8 lots of 9
   b. 6 lots of 9; 2 lots of 27; 12 lots of $4^1/_2$
3. Problems and solutions will vary.

Page 24 Double Russian

ACTIVITY

1. Yes, 47 x 67 = 3 149.
2. Numbers will vary. One example is 48 x 68 = 3 264.

48	68	
24	136	
12	272	
6	544	
3	1 088	1 088
1	2 176	2 176
		3 264

1 088 + 2 176 = 3 264

3. a. Yes, this method works with both 3-digit and 4-digit numbers. For example:

286 x 497 = 142 142

286	497	
143	994	994
71	1 988	1 988
35	3 976	3 976
17	7 952	7 952
8	15 904	
4	31 808	
2	63 616	
1	127 232	127 232
		142 142

4 326 x 5 621 = 24 316 446

4 326	5 621	
2 163	11 242	11 242
1 081	22 484	22 484
540	44 968	
270	89 936	
135	179 872	179 872
67	359 744	359 744
33	719 488	719 488
16	1 438 976	
8	2 877 952	
4	5 755 904	
2	11 511 808	
1	23 023 616	23 023 616
		24 316 446

   b. Yes. For example, 48 x 68:

48	68	
24	136	
12	272	
6	544	
3	1 088	1 088
1	2 176	2 176
		3 264

1 088 + 2 176 = 3 264
48 x 68 = 3 264

Teachers' Notes

Overview

Number Sense: Book Two

 Short Cuts

Achievement Objectives

- explain satisfactory algorithms for addition, subtraction, and multiplication (Number, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will also:

- develop accuracy and efficiency in mental calculations
- understand that, when we subtract one number from another, we are measuring the "distance" between them
- adapt or expand a known result to fit a related problem.

ACTIVITY

This activity encourages the students to look for short cuts in calculations by adapting known results to solve new, but related, problems. They will need to distinguish how the new equations differ from those given on the whiteboard.

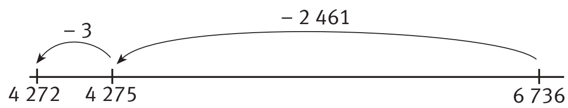
In both question **1** and question **2**, the first number is the same as in the examples on the whiteboard. This means that the students have only to attend to the second number in each case. Encourage them to figure out in their heads how this differs from the second number in the whiteboard example. For the addition statements in question **1**, this is reasonably straightforward. For example, in question **1a i**, 3 294 is 20 more than 3 274 on the whiteboard, so the total or sum will be 20 more than 8 436, namely 8 456. Number lines can be used to help make the relationships clear. For example, in question **1a i**, a possible number line is:

Note that there is no need for the line to be accurate in terms of scale because the focus is on the relationships.

The students may find question **1a iv** a little trickier to do in their heads (although this is still to be encouraged). As the students solve each problem, they are expanding the number of "known" equations that they can use to compare with new problems. This means that, rather than returning to the whiteboard equation each time for comparison, they can adapt one of the solved equations to fit the particular equation that they are working on.

In question **2**, the students are working with the idea that when we subtract one number from another, we are measuring the difference or distance between them. With question **2a i**, for example, they need to realise that 2 464 is 3 more than the 2 461 on the whiteboard. The difference or result will therefore be 3 *less* than the 4 275 shown (because the gap between 2 464 and 6 736 is 3 less than the gap between 2 461 and 6 736). The difference or result will be 4 272. The opposite applies in question **2a ii**. In this case, 2 453 is 8 less than the 2 461 on the whiteboard, so the difference will be 8 greater than the difference for 2 461.

Once again, number lines can be used to help make the relationships clear. For example, in **2a i**, a possible number line is:



The last example (question **2a vi**) presents a slightly greater challenge for mental calculation. 2 961 is 500 more than the 2 461 on the board, so the difference between 6 732 and 2 961 will be 500 less than for the example on the whiteboard.

Finally, having the students explain the strategies they used is important for three reasons:

1. By listening to the reasoning of others, students can learn from their peers.
2. Communicating ideas and strategies is as important in mathematics as it is in any other area.
3. From a teaching point of view, it provides an important window into the students' mathematical thinking.

Page 2 Divide and Conquer

Achievement Objectives

- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)
- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

- All even numbers are divisible by 2 (with no remainder).
- All numbers that end in 0 or 5 are divisible by 5 (with no remainder).
- When the digital root of a number (that is, the single-digit number eventually obtained by adding the digits in a number) is divisible by 3 (that is, it is 3, 6, or 9), the number itself is divisible by 3.

ACTIVITY

For students who are still trying to establish a degree of number sense, a good way to start this activity might be to explore the hundreds board or chart. If they identified all the multiples of 2 (all the even numbers) and then all the multiples of 5, they would see that the numbers at the end of each row (that is, 10, 20, 30, and so on) are multiples of both 2 and 5. With this understanding, they could then try question **1**.

A possible lead-in to question **2** would be to return to the hundreds chart and have the students identify all the multiples of 3. They may notice the diagonal pattern formed on the chart by these multiples. Another pattern that they can also be helped to recognise is that adding the digits in each diagonal pattern always results in the number at the top. For example, taking the first diagonal pattern, the numbers are 3, 12, and

21 (adding 1 and 2 results in 3; adding 2 + 1 also gives 3). Likewise, the second diagonal has the numbers 6, 15, 24, 33, 42, and 51 (all of which add up to 6). From this investigation, the students may be able to conclude that numbers that are multiples of 3 become 3, 6, or 9 when their digits are added. In the case of larger numbers, such as 87, the digits may have to be added more than once to get to a single figure (8 + 7 = 15; 1 + 5 = 6).

Now the students are in a position to work on question **2**. When they try question **2b**, they need to do an example or two either side of the numbers given to check whether the pattern holds.

An interesting extension to question **1** that some students may wish to try has to do with multiplying by 5. Since 5 is half of 10, they may wish to try multiplying larger numbers mentally by halving the number and then multiplying it by 10. Thus, 5 x 140 is 70 x 10. Odd numbers can be multiplied if decimals are used. For example, 5 x 35 is 17.5 x 10, and 5 x 89 is 44.5 x 10.

A further extension activity would be to consider the digital roots of multiples of 9. The students should soon discover that the digital root of such a product is always 9.

Page 3 Hundreds

Achievement Objectives

- demonstrate knowledge of the conventions for order of operations (Number, level 4)
- explain the meaning and evaluate powers of whole numbers (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

- A number can have many names (for example, 2 x 50 is an alternative name for 100).
- Some numbers are square numbers (for example, 4 is 2^2 and 25 is 5^2).
- Some numbers are cubic numbers (for example, 8 is 2^3 and 27 is 3^3).
- Some numbers are prime numbers (for example, 11 and 19).
- "Equation" is also a way of saying that two numbers are equal (rather than just being the record of an operation).
- Students will also develop an understanding of numbers and the ways in which they are represented.

ACTIVITY

In the first question, you can help the students to realise that a number can have many names, in fact, an infinite number of names. For example, 100 (assuming that it is in the usual base 10) can be named using any of the four operation signs: 70 + 30, 120 – 20, 2.5 x 40, 500 ÷ 5, and 50 ÷ 0.5 are all names for 100. The students may accept the challenge of coming up with the most complex name for 100 that they can think of. They could add to the challenge by deciding that a number can be used at most once in an equation (this prevents equations such as 100 + 10 – 20 – 10 + 20 = 100, and so on).

A further extension of this first question would be to investigate how other cultures have represented 100. For example, the early Romans used the letter "C", the Egyptians used a symbol that is similar to our lower-case "e", and the Attic Greeks used a large "X".

The second question requires students either to have or to gain an understanding of square numbers (for example, 6 x 6 = 6^2), cubic numbers (for example, 2 x 2 x 2 = 2^3), and prime numbers (that is, numbers that have only themselves and 1 as factors, such as 11 and 19). The students will also need to realise that when building equations:

- they can use any of the operations;
- they may need to use brackets (in question **2a**, for example);
- they could consider the possibility of constructing fractions (for instance, in question **2f**).

For question **2c**, students who find different solutions could share them with the class. It would be interesting to challenge the class to devise at least three different solutions. Apart from the strategies just mentioned, it will be a matter of the students devising their own strategies to work out solutions to question **2**. Sharing their strategies would also be valuable, even if some were not able to solve a particular problem.

Question **3** is also likely to be a challenge. As with question **2d**, more than one solution is possible. Encourage the students to come up with a strategy (or strategies) to solve the problem rather than simply guessing and checking a string of possible solutions. If, after a time, they are still struggling, you could suggest the strategy outlined in the Answers. This still leaves the students with a lot of investigating to do to produce three different 3-digit numbers, using the digits 1–9 once only, that meet the other conditions of the problem.

In questions **2** and **3**, a calculator is invaluable as a "number cruncher". It enables the students to focus on problem solving rather than on tedious calculation. However, there are some problems in question **2** that the students can probably solve more quickly without the use of a calculator (for example, **2a**, **2b**, and **2f**).

Page 4 Egging On

Achievement Objectives

- pose questions for mathematical exploration (Mathematical Processes, problem solving, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also work backwards to solve a problem.

ACTIVITY ONE

There are various ways that the students might go about solving the problems in question **1**. Perhaps the simplest way is to use a calculator. Firstly, though, the students will need to realise that the scenario involves continuous halving (that is, each customer except for the last one buys half of the available eggs), so getting back to the original number of eggs will require continuous doubling. If there was one egg left for the ninth customer, there must have been two eggs left when the eighth customer arrived at the market stall. A calculator is an excellent tool here. By systematically pushing [x] [2] [=], it is possible to determine the number of eggs that were there when each customer arrived. Then it is just a matter of recording these on a chart, for example:

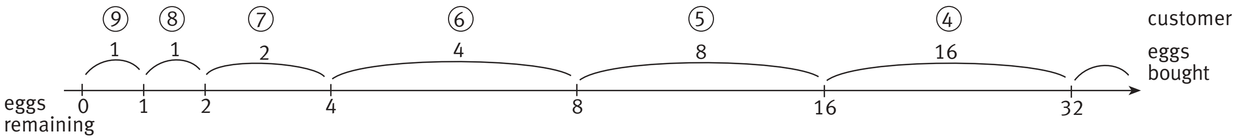
Customer number	9	8	7	6	5	4	3	2	1
Number of eggs on arrival	1	2	4	8	16	32	64	128	256
Number of eggs bought	1	1	2	4	8	16	32	64	128

This shows that Henrietta began with 256 eggs.

The strategy of doubling may suggest to the students that this activity has something to do with powers of 2, and indeed it has. It is possible to work out the number of eggs Henrietta started with, and how many each customer bought, by setting out a table with powers of 2 as below.

Customer number	9	8	7	6	5	4	3	2	1
Number of eggs on arrival	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8
Number of eggs bought	2^0	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7

This shows that the total number of eggs that Henrietta began with was 2^8, which is 256. This can be calculated by going, for example, $2^8 = 2^3 \times 2^3 \times 2^2$ (that is, 8 x 8 x 4) or $2^8 = 2^4 \times 2^4$ (that is, 16 x 16). Notice that the exponents of $2^3 \times 2^3 \times 2^2$ add up to 8 and that in general, $2^a \times 2^b = 2^{(a+b)}$. Notice too that in terms of powers of 2, 2^0 means 1 and 2^1 means 2. The number of eggs each customer bought can be calculated mentally in a similar way. Alternatively, the students may choose to construct a number line to display the required information. Here again, scale is not critical as long as the relationships are clear.



ACTIVITY TWO

The problems in **Activity Two** also require the students to work backwards from the known to the unknown number, although some students may try other strategies that they can explain to the class. Thus, in question **1a**, the known number is 14, and this has resulted from 9 being subtracted from a number. The first step is therefore to add 9 to 14. This gives 23. Now, this 23 represents half of the original number, so if 23 is doubled, the original number of 46 is obtained. The students who have a feeling for algebra may be able to use the following kind of strategy: $14 = (n \div 2) - 9$, which in turn can be written as $14 + 9 = n \div 2$, which can finally be written as $23 \times 2 = n$.

Similar strategies can be used for questions **1b** and **1c**. Working backwards, question **1b** has 36 as the known number, but this is twice the amount of another number, namely, 18. Since the 18 is double the number of the original we are looking for, it is now just a matter of halving 18 to give 9 as the solution. Likewise, in question **1c**, the known number is 20, but the number being sought has something to do with one-quarter of 20, which is 5. The 5 is the result of dividing a number by 3, so the original number can be found by multiplying 5 by 3. Following this logic, it must be 15.

Four 4s

Achievement Objectives

- demonstrate knowledge of the conventions for order of operations (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

- Numbers can be named in a range of ways.
- The square root sign ($\sqrt{}$) signifies that a number is the square of another number (for example, $\sqrt{4}$ is a name for 2 because 2^2 is 4).
- The factorial sign (!) signifies that a number should be multiplied by all whole numbers less than that number (for example, 3! is 3 x 2 x 1 = 6).

ACTIVITY

The tasks in question **1** are designed to give the students some clues about how they can tackle questions **2** and **3**. The students may find it helpful to realise that division can be written in fraction form, and vice versa. For example, 44 ÷ 4 is read as forty-four divided by four. So is $^{44}/_{4}$ (although it can also be read as 44 quarters). Similarly, 4 ÷ 4 is read as four divided by four, and the same is true of $^{4}/_{4}$. As indicated in the Other mathematical ideas and processes above, the students will also need to realise that they can use 4! (that is, 4 factorial) if they wish, and that this means 4 x 3 x 2 x 1, which is 24. The answers indicate two ways in which 4 factorial can be used to construct a four 4s expression for 100.

Note that for many of the numbers up to 20, there is more than one way to construct a four 4s expression. For example, 7 can be named as (44 ÷ 4) – 4, as (4 + 4) – (4 ÷ 4), as (4! ÷ 4) + (4 ÷ 4), or as (4 x 4 ÷ $\sqrt{4}$) – $^{4}/_{4}$. Depending on the time available, all the students could tackle all the numbers up to 20, including finding ways other than those given on the page for naming 9 with the four 4s. Alternatively, different groups of students could be allocated certain numbers to investigate.

Whichever way is used, the students will need time to share and justify their constructions with the rest of the class, and the rest will need to check that the constructions do indeed make the numbers they are claimed to name. The students could also share the strategies they tried (some of which they probably abandoned) in the course of devising the four 4s expressions they came up with.

Strategy-sharing is valuable not only from the point of view of communicating mathematical ideas but also because it helps students to see that they can take control of their own learning in mathematics. It gives them confidence in their own intellectual powers, as opposed to remaining largely dependent on the teacher to tell them what to do.

Probably the whole class, or most of the class, could be challenged with question **3**, that is, to find a four 4s expression for 100. Again, they can share their strategies and constructed expressions.

Page 6

Clothes Spree

Achievement Objectives

- find a given fraction or percentage of a quantity (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- increase and decrease quantities by given percentages, including mark up, discount, and GST (Number, level 5)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

- Finding a fraction of a number is the same as dividing by the reciprocal of the fraction. For example, finding $^1/_3$ of a number is the same as dividing the number by 3.
- Finding a percentage of a number is the same as multiplying the number by either its decimal equivalent (for example, 20% is 0.2, so 20% of \$50 is 0.2 x \$50 = \$10) or its fraction equivalent (for example, 20% is $^2/_{10}$, so 20% of \$50 is $^2/_{10}$ x \$50 = \$10).
- Finding the sale price of an item that has a percentage discounted is the same as finding the complementary percentage of the item. For example, to find the sale price of an item priced at \$50, but with 20% off, simply calculate 80% of \$50.
- Finding the sale price of an item that has a fraction discounted is the same as finding the complementary fraction of the item. For example, if an item priced at \$50 is reduced by $^1/_5$, then the sale price is $^4/_5$ x \$50.
- Students will also develop accuracy, efficiency, and confidence in calculating mentally, on paper, and with a calculator.

ACTIVITY

The questions in this activity can be tackled with greater understanding if the students are able to easily change percentages into fractions and vice versa. It is also likely to be helpful if they know the decimal equivalents. Given such understanding, all the questions can be done mentally, and it is probably wise to encourage the students to do so. At the same time, the questions also provide opportunities to learn some efficient ways to use a calculator to solve problems of this kind. This reinforces the idea that percentages are nothing more than special sorts of fractions, that is, fractions with a denominator of 100. Suggestions for how this might be done are provided below.

First, though, consider the value of being able to change percentages into fractions. If the students know that 50% is half, in the first item of question **1** they just need to take half of \$18 (that is, \$9). For the remainder of the items in this question, they need to know (or be able to work out) that 25% is one-quarter, 20% is one-fifth, and 10% is one-tenth. It may be useful to put aside the problems temporarily while the students construct a chart of the fraction and decimal equivalents of common percentages. A further advantage of doing this is that the students can learn *how* to work out the fraction equivalents. This means that if they forget what some are, they are in a position to figure them out on their own.

Working out such fraction equivalents is basically a matter of simplifying fractions. For example, 25% means 25 out of 100, and can be written as $^{25}/_{100}$. This simplifies to $^1/_4$ through dividing both the top and bottom by 25, or alternatively by dividing the top and bottom by 5 and then 5 again. Mathematically, it is the identity principle at work because the fraction is effectively being divided by 1 in the form of $^{25}/_{25}$. Similarly, 50% can be written as $^{50}/_{100}$ and simplified to $^1/_2$. A completed chart may look like the following:

Percentage	Fraction	Simplified fraction	Decimal
10%	$^{10}/_{100}$	$^{1}/_{10}$	0.1
20%	$^{20}/_{100}$	$^{1}/_{5}$	0.2
25%	$^{25}/_{100}$	$^{1}/_{4}$	0.25
$33^{1}/_{3}\%$	$^{33.\dot{3}}/_{100}$	$^{1}/_{3}$	$0.\dot{3}$
50%	$^{50}/_{100}$	$^{1}/_{2}$	0.5
$66^{2}/_{3}\%$	$^{66.\dot{6}}/_{100}$	$^{2}/_{3}$	$0.\dot{6}$
75%	$^{75}/_{100}$	$^{3}/_{4}$	0.75

You could give the students the chart with only one figure in each row and ask them to complete it.

A decimal column has been added to the chart because the decimal equivalent provides yet another way of calculating prices when a percentage or fraction has been taken off.

There are two main ways of calculating discounts or sale prices using a calculator. The quickest way to calculate the discount on the skirts that have been reduced by 25% is to key in $\boxed{68}\boxed{\text{x}}\boxed{25}\boxed{\%}$, and the calculator immediately gives 17 (that is, $17). It is not necessary to key in $\boxed{=}$. To find the sale price, you subtract the discount ($17) from the full price ($68) to give $51. The quickest way to calculate the sale price directly is to reason that if there is 25% off, the sale price must be 75% of the original price, so key into the calculator the following: $\boxed{68}\boxed{\text{x}}\boxed{75}\boxed{\%}$. This immediately gives 51 (that is, $51).

The second approach is to use the decimal equivalent. Thus, with the same example, simply key in $\boxed{68}\boxed{\text{x}}\boxed{.25}\boxed{=}$, and once again the calculator gives 17. Similarly, to find the actual sale price, key in $\boxed{68}\boxed{\text{x}}\boxed{.75}\boxed{=}$, and the result of 51 is displayed.

When the students come to question **2**, they need to realise that, in the case of the skirts, $75 can be thought of as 3 lots of $25 (which means that one-third of $75 is $25) and, in the case of the tops, the result is not going to be whole dollars (because half of $23 is $11.50).

Finally, question **3** is an interesting investigation involving comparisons of the discounts offered by the two shops and the challenge of working out what would be the best buys. Different pairs of students are likely to devise different solutions, which they can share with classmates. This will inevitably include sharing their mathematical reasoning. It may also include a discussion of whether sale items are the best value for money.

Page 7 Time versus Money

Achievement Objectives

- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)
- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

- Rounding is usually to the nearest whole number (10, 100, and so on) or, in the case of decimals, to the nearest tenth, hundredth, thousandth, and so on.
- With cash payments, money tends to be rounded to the nearest 5 or 10 cents.

ACTIVITY

The activity here is a true investigation in the sense that a number of strategies for paying for the items could be investigated to determine which would yield the cheapest option. The answers for this question provide details about the effect of trying Ben's two proposals and also about the outcome of trying Cherie's suggestion for this particular example. They also point out that, given the reality of shopping in a supermarket, it is unlikely that the time spent trying to save a few cents is actually worth it.

The students may notice that, on the Dollar Saver receipt, one price would not get rounded at all (the No. 9 chicken), eight prices would be rounded up, and just three would be rounded down. It may be useful to identify which items fit into which categories as a basis for investigating Ben's and Nan's propositions. These could then be set out in a chart, for example:

No rounding	Rounding up	Rounding down
Chicken No. 9 $7.65	Cream 500 ml $1.93	Cheese 750 g $5.82
…	Milk 2 L $2.73	Coffee 500 g $3.77
	Bread $1.48	…
	…	

Several extensions of this investigation are possible. The first would be to challenge groups of students to find a grouping of items that minimises the total cost of the groceries. For example, pairing items that would be rounded up individually (rather than putting them all together in one group as in Nan's suggestion) can result in further savings and bring the bill down to under $43.20. Another would be to have groups of students list the prices of 12 or more randomly chosen different items from a local supermarket or grocery store (perhaps from the pamphlets they put in letter boxes) and carry out a similar exercise to determine the effects of rounding. (Note, too, that online grocery shopping is available in New Zealand, so prices of grocery items should be available over the Internet if they are not easily obtained locally.)

A further extension activity would be to have the students investigate what a supermarket stands to gain, say, in a year, if every customer ends up paying 2 cents more than the actual price each time they shop.

Reading by Numbers

Achievement Objectives

- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

- Palindromic numbers are those that read the same backwards or forwards (for example, 121).
- Palindromic numbers can be formed by taking *any* number, reversing the order of the digits and adding them to the original number (for example, 15 + 51 = 66), although this step may need to be repeated several times if the immediate result is not a palindromic number.

ACTIVITY

This activity provides a further opportunity for students to investigate number patterns. For students who are keen to make sense of how numbers work, investigating palindromic numbers can be quite fascinating. After the students have tackled question **2**, you could tell them about the challenge relating to the number 196 mentioned in the Answers.

A useful extension would be to encourage the students to think of other questions about palindromic numbers that they could investigate, for example: Can palindromic numbers be formed from 4-digit numbers? Is there something special about numbers that form one-step palindromic numbers? Does every palindromic number form another palindromic number if the process is repeated?

Another way of thinking about questions is to consider them as conjectures. For example, the students might conjecture that it is probably possible to form palindromic numbers from 4-digit numbers, or at least some of them. They then set out to investigate whether their conjecture holds and, if so, under what circumstances. They may begin with a number such as 4 268 and find that after a number of steps they end up with the palindromic number 2 786 872. They need to see that they cannot generalise from this one instance that palindromic numbers can be made from *any* 4-digit number. More investigation is needed.

The questions lend themselves to using a calculator for the "number-crunching" so that the students can get on with the investigating aspect. Depending on the class, the activities may be done in pairs as this can often generate more interest than working alone. However, some students prefer to work away quietly on their own, so it may be a matter of providing the choice. Whatever they decide, there is likely to be value in the class sharing results because different students will probably investigate different numbers.

The question about decimal numbers is interesting. The answer for this question indicates that provided the position of the decimal point is fixed, some decimal numbers can be turned into palindromic numbers whereas others cannot, even those using the same digits. For instance, 0.95 works (it becomes 11.11), whereas 9.5 does not (it becomes 111.1), and neither does 0.095 (which becomes 2.992) or 0.0095 (which becomes 0.5995). The latter three look like palindromic numbers, but to be true palindromic numbers, they would need to have the decimal point in the middle in each case. The students might like to try the decimal number 0.1289. It doesn't form a palindromic number, but it can be turned into a palindromic number by shifting the decimal point. See if the students can discover this number (it is 1.289). They may also like to try the decimal number 95.95, which can become a palindromic number after a few steps (it becomes 391.193).

The students will probably be able to figure out that if the palindrome-lookalike of a decimal number has an odd number of digits, it can never be a true palindromic number. For example, the decimal number 0.3289 becomes 3.4243 after two steps, but this has five digits and could never be a palindromic number. There is no “middle” where the decimal point could be located.

Page 9 The Greenhouse Effect

Achievement Objectives

- write and solve problems involving decimal multiplication and division (Number, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)
- find, and use with justification, a mathematical model as a problem-solving strategy (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also compare numbers involving decimals (that is, the magnitude of decimal numbers).

ACTIVITY

The questions in this activity involve decimal numbers. To help the students to work with understanding, encourage them to look at the relationships between the rates of growth of the control plant, plant A, and plant B. The students could construct a chart setting out these relationships, maybe along the following lines:

Daily Rate of Growth of the Three Plants			
Plant	Growth in cm	Growth in fractions of cm	
Control	0.125	$^1/_8$	
Plant A	0.25	$^1/_4$	(2 x rate of control plant)
Plant B	0.5	$^1/_2$	(4 x rate of control plant)

The decimals 0.5, 0.25, and 0.125 are common decimals that the students will meet again and again, so an understanding of the fraction equivalents at this stage will stand them in good stead for later work. If time permits, some students may wish to draw up a chart of the decimal equivalents of three-eighths, five-eighths, and seven-eighths (assuming that they already know that two-eighths is one-quarter, four-eighths is one-half, and six-eighths is three-quarters).

When the students come to work out the 8-day growth for each of the plants in question **1**, they are likely to find that the calculator is an invaluable aid. For example, for the control plant, they should key in the starting height and then [+] [.125] [=]. This will obviously give the height at the end of day 1 (that is, 3.625). If they systematically press [=] on the calculator, they can then get the height for each succeeding day, namely 3.75, 3.875, and so on. The students can follow the same procedure for plant A and plant B, except that it will be necessary to add [.25] and [.5] respectively to the starting heights.

Question **2** challenges the students to think about how relationships between numbers can be used to figure out solutions to problems. Different strategies are possible, one of which is indicated in the Answers. Another is to simply use the calculator to multiply the daily growth of each plant by 32 and add on the starting height.

For instance, for the control plant, [32] [x] [.125] [+] [3.5] [=] gives 7.5 centimetres. This equation can be adapted to fit any number of days' growth, for any of the plants, by substituting the appropriate values for number of days, daily growth, and starting height respectively.

With question **3b**, different strategies are also possible. The students may see that under the new plant-food regime, plant A will grow 0.75 centimetres in 2 days or 1.5 centimetres in 4 days. It is then a matter of either multiplying the 0.75 by 16 (16 lots of 2 days is 32 days) on a calculator or multiplying the 1.5 centimetres by 8 (eight lots of 4 days is 32 days) and then adding the starting height of 2.75 centimetres. Either way, the result will be 12 + 2.75, which is 14.75 centimetres.

Page 10 On the Campaign Trail

Achievement Objectives

- find a given fraction or percentage of a quantity (Number, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

- A whole number can be written as different fractions. For example, $^3/_3$ and $^{10}/_{10}$ both equal 1.
- Finding a fraction of a number is the same as dividing by the reciprocal of the fraction. For example, $^1/_8$ of 24 is the same as dividing 24 by 8 (or multiplying by the decimal equivalent, for example, $^1/_8$ of 24 is 0.125 x 24).
- When the fraction is more than a single fraction (for example, $^3/_8$ rather than $^1/_8$), the above procedure can still be followed. For example, $^3/_8$ of 24 is the same as dividing 24 by 8 and then multiplying the result by 3.

ACTIVITY

In question **1**, the students need to calculate the number of pupils out of 30 that are represented by the various fractions. This can be done in several ways, but probably the easiest is to divide by the reciprocals. It is important that the students understand that finding, for example, $^1/_6$ of 30 is the same as calculating $^1/_6$ x 30, which is $^{30}/_6$. This could be written as: $^1/_6 \times 30 = 30 \times {}^1/_6$

$$= 30 \div 6$$

In other words, these are multiplication problems that can be turned into division problems by using the reciprocal of the fractions.

Before the students begin this activity, it may be helpful to develop their confidence in working with reciprocals. The reciprocal of a number is the number that results in 1 when it is multiplied by the original number. For instance, the reciprocal of $^1/_6$ is 6 because 6 x $^1/_6$ is 1. The reciprocal of a fraction can be found by swapping the numerator and the denominator. For example, the reciprocal of $^2/_3$ is $^3/_2$. This also works for whole numbers where the denominator is 1 in fraction form. For example, because 6 = $^6/_1$, the reciprocal is $^1/_6$, and the reciprocal of $^1/_7$ is $^7/_1$ or 7.

Now the next step can be taken, that is, establishing that finding a fraction of any number is the same as dividing the number by the reciprocal of the fraction. You can lead the students to this understanding by considering a simple case, such as finding $^1/_2$ of some number. After asking them to work out half of various numbers, including some larger numbers such as 86 or 124, they can be asked how they did this. They will most likely say that they divided the 86 or 124 by 2, which is equivalent to solving $^{86}/_2$ and $^{124}/_2$ respectively.

It is then a matter of establishing that this same principle works for other numbers. For example, finding $^1/_4$ of 12 is the same as dividing 12 by 4.

With these insights, the students can now return to the activity and divide 30 by 6, 10, 3, and 10 respectively. Finding $^3/_{10}$ of 30 consists of two steps: dividing by 10 and then multiplying the result by 3. This is equivalent to solving $^3/_{10} \times 30 = {}^{90}/_{10}$. Once the proportions are calculated, it is a relatively straightforward matter of arranging them in order from smallest to largest.

In this activity, the use of a calculator is not recommended for two reasons. Firstly, the calculations are simple enough to be done mentally, and secondly, the thirds and sixths could cause some confusion as the decimal equivalents are "messy".

In question **2**, the students need to consider the candidates' policies in the light of Jed's survey results. The Answers provide some indication of how students may tackle this. The key to the election may be the three pupils who didn't express any opinion. In short, there is no one right answer to question **2**. You could conclude the activity by having the students justify their predictions during a whole-class discussion and reflect on the fact that candidates' policies are only one of the factors that influence voter choice.

Page 11 Same Answer Every Time!

Achievement Objectives

- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also:

- develop an understanding of numbers and the ways in which they are represented
- develop accuracy, efficiency, and confidence in calculating.

ACTIVITY

This activity follows on quite nicely from that on palindromic numbers on page **8** of the student book.

The strategy that most students will use to work out solutions to question **1** is trial and error, which is entirely appropriate in this case. The results of the investigations by pairs of students could be shared with the whole class and demonstrated by the pairs to be valid. It might be worth keeping a class tally to see which number of subtraction steps is most common. Examples of three-subtraction through to seven-subtraction Kaprekar numbers are given in the Answers.

Question **2a** is a good example of the kind of mathematical questioning that the mathematics curriculum document proposes students should develop. The example given, namely, 8 089, does in fact become a Kaprekar number after four subtraction steps. This holds for 4-digit numbers generally, with the exception of numbers such as 6 666, in which all the digits are the same. You could ask the students why it wouldn't work with such numbers. A moment's thought will probably have them saying that because the largest number is equal to the smallest number, the result of subtracting such numbers is always zero (6 666 – 6 666 = 0), so it gets you nowhere!

So far, the focus has been on 4-digit numbers. Question **2b** raises the question of whether there is a similar pattern with 3-digit and 2-digit numbers. Again, this is the kind of questioning that the students should be encouraged to engage in and then follow up with an investigation, possibly in pairs.

The students may prefer to investigate 2-digit numbers before moving on to 3-digit numbers because they are simpler to investigate. For example, if one were to begin with 29, the first calculation would be 92 – 29 = 63, the second 63 – 36 = 27, the third 72 – 27 = 45, and the fourth 54 – 45 = 9. This could be taken further (for example, 90 – 9 = 81; 81 – 18 = 63), but the numbers begin repeating themselves.

The students have not been given the Kaprekar numbers in the 2- and 3-digit number cases, so they will not be sure when to stop the process. They will need to investigate several numbers before they arrive at the figures given in the Answers. Once again, it may be useful to keep a class record of Kaprekar numbers suggested by the students until the actual numbers become clear.

As an example of a 3-digit number, take 264. The first calculation would be 642 – 246 = 396, the second 963 – 369 = 594, and the third 954 – 459 = 495. At this point, the last numbers would keep repeating, so it seems that the Kaprekar number for 3-digit numbers is 495. However, here again the students would need to try other examples to check whether this is so.

After they have engaged in this investigation, maybe in pairs, you could invite them to raise any other related mathematical questions. For example, "Could it work with 5-digit numbers?" It appears from examples such as 23 641, 37 218, 69 875, 32 754, and 75 863 that the Kaprekar number is 74 943, whereas for some other 5-digit numbers, for example, 38 275 and 51 936, the Kaprekar number is 63 954.

The calculator comes into its own in this sort of investigation. It makes it possible to accurately investigate a series of numbers in a relatively short time.

Page 12 Plant Patterns

Achievement Objectives

- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

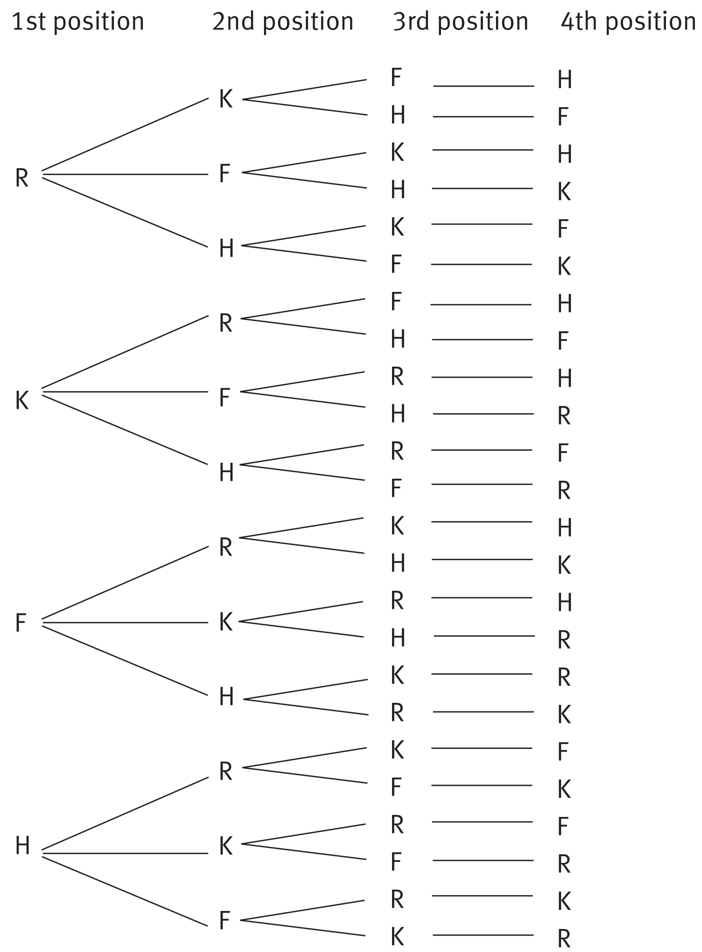
Students will also:

- learn about sample space, that is, the number of arrangements or possible outcomes
- develop an understanding of factorials; for example, 3! (read this as "3 factorial") is 3 x 2 x 1 = 6.

ACTIVITY

The mathematical ideas underlying this activity are those of sample space and factorials. It may be a good idea to begin with just two different plants and invite the students to show how many different ways they can be arranged (that is, two ways). Then move on to the three plants in the activity. Different strategies can be used to work out the number of possible arrangements, including drawings, names, and letters (see the Answers). The students can be encouraged to use a systematic approach to make the task more manageable and to ensure that they do not repeat arrangements or leave some out. They should be able to work out that, with three different plants, six different arrangements are possible. They can share their strategies with the rest of the class.

If they use a similar approach with question **2**, they will eventually find that there are 24 possible arrangements, although at this point, the increased number of plants makes the task of finding possible arrangements rather challenging. Discussion could be useful at this point to clarify what is happening. Essentially, there are four plants the students can choose for the first position in the garden, three for the second place, two for the third place, and whichever is left over for the fourth place, so there are 4 x 3 x 2 x 1 = 24 possible arrangements. A tree diagram may help to make this clear:



To go further, the students need to find the underlying pattern for the number of arrangements. Constructing a table of what they know already may help because listing arrangements or constructing tree diagrams will prove to be very time consuming. For example:

Number of plants	Number of possible arrangements
1	1
2	2
3	6
4	24

At this point, some of the more astute students may be able to see that the number of possible arrangements for a given number of plants is that number of plants multiplied by all the previous numbers of plants. Thus, for four plants, the number of possible arrangements is 4 x 3 x 2 x 1 for the reasons discussed above.

What the students may not yet know is that such numbers are called "factorials" (also mentioned in the notes for page **5** of the student book and see also page **4** of *Number: Book Five*, Figure It Out, Years 7–8, and the notes for this page). If the students are able to recognise this pattern, they have effectively solved the last part of question **3** and can apply this to the case of five or six plants.

The students will now probably be able to work out that the number of possible arrangements for five plants is 120 (that is, 5! or 5 x 4 x 3 x 2 x 1) and that for six plants, it is 720 (that is 6!, which is easily found by multiplying the result for five plants by 6).

Ask students who are having difficulty with five plants to consider the four-plant case and add another plant, say a mānuka, M. Tell them to take any arrangement of the four plants, say FKRH, and ask them how many positions they could place the mānuka in (assuming that the plants must still be in a straight line). The students should recognise that there are five possible positions for the mānuka in this arrangement, namely●F●K●R●H●, or MFKRH, FMKRH, FKMRH, FKRMH, and FKRHM. This will be true for every arrangement of the four plants. There are 24 of these, so there must be 5 x 24 = 120 or 5! = 5 x 4 x 3 x 2 x 1 possible arrangements of five plants.

Page 13 Fair Exchanges

Achievement Objectives

- write and solve problems involving decimal multiplication and division (Number, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will also use different units and their ratios.

ACTIVITY

Converting overseas currency to New Zealand dollars can be tricky, but it is essential if you are overseas and want to work out whether what looks like a bargain is really a good buy.

Several strategies can be used to work out the prices in this activity. One approach is to make use of ratios. For example, question **1** can be thought of as "$^{100}/_{125}$ is the same as $^{24}/_{\square}$" or alternatively as "$^{125}/_{100}$ is the same as $^{\square}/_{24}$". That is, you need to find out what number $\square$ is so that $^{125}/_{100}$ and $^{\square}/_{24}$ are the same. Whichever way it is written, the question is: What does 100 have to be multiplied by to become 125? This can be done by dividing 125 by 100 (it is 1.25, and no calculator is needed for this initial step). To maintain the same ratio, the 24 should now also be multiplied by 1.25. The students should recognise this as $1^1/_4$ and be able calculate mentally that the product is 30 (that is, 1 lot of 24, plus $^1/_4$ of 24). If not, then they can simply use their calculator to work out 24 x 1.25.

Another approach is to calculate the value of 1 unit of foreign currency in New Zealand dollars and then multiply this amount by the foreign currency price. To find the value of 1 unit of foreign currency in New Zealand dollars, divide the New Zealand dollar amount by the foreign currency equivalent. For example, question **2** states that 45 000 yen is equivalent to NZ $900. A calculator quickly shows that 900 ÷ 45 000 = 0.02. So 1 yen = NZ$0.02 (2 cents). For the kimono, multiplying 0.02 by 15 000 (the price in yen) gives a price of NZ$300.

Question **2** can also be done in a similar way to question **1**. It can be set out as follows: $^{45\,000}/_{900}$ is the same as $^{15\,000}/_{\square}$. In this case, it may be easier to use the horizontal approach and ask "What does 45 000 have to be divided by to get 15 000?" The answer is 3. It therefore follows that the 900 also has to be divided by 3. This results in 300. It would have been possible to work vertically and ask "What does 45 000 have to be divided by to get 900?" The answer is 50. Dividing the 15 000 by 50 to maintain the ratio again gives 300.

In question **3**, the students will probably see that the British pound is worth three times the New Zealand dollar, so it is a matter of multiplying the 2 pounds 50 pence by 3. The result is $7.50. This could also be set out as a ratio, that is, $^{1}/_{3}$: $^{\text{2 pounds 50 pence}}/_{\square}$.

Questions **4**, **5**, and **6** can be tackled in similar fashion.

The investigation into how much NZ$100 would be worth in other currencies depends on two things. Firstly, it requires information about exchange rates. The students may be able to suggest that these can be obtained from banks, newspapers, television news programmes, or the Internet. Secondly, it requires the use of ratios once more. For example, if NZ$1 is worth 86 cents in Australian currency (A$0.86), then the ratio will be $^{1}/_{0.86}$: $^{100}/_{\square}$. The 1 has to be multiplied by 100 to get 100 on the top line, so the 0.86 on the bottom also has to be multiplied by 100 to maintain the ratio. That results in A$86. (Note that the exchange rate in question **1** is based on the New Zealand dollar being worth 80 Australian cents. Mathematically, 0.8 : 1.0 is in the same ratio as 1.0 : 1.25 or 100 : 125).

The students may notice that banks quote both a "buy" and a "sell" rate for foreign currency. This reflects bank practice of selling a unit of foreign currency for more than they paid for it.

Page 14

Energy Levels

Achievement Objectives

- find fractions equivalent to one given (Number, level 4)
- express a fraction as a decimal, and vice versa (Number, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

The students will also convert decimal fractions of an hour into minutes.

ACTIVITY

The questions in this activity work with two principal ideas: calculating how long different activities would need to be performed to give the same energy use and converting these times from decimal hours into minutes where necessary.

Question **1** is probably best tackled by considering how many times each of the kilojoule amounts divides into the 2 800 used up during running. The figure obtained for each form of exercise represents the number of hours that the exercise would have to be performed to equate with 1 hour of running. For example, walking uses up 700 kilojoules; 700 x 4 is 2 800, so 4 hours' walking would be the equivalent of 1 hour's running. The speech bubble provides a closely related approach to the same problem.

This is fine for figures that divide evenly into 2 800 (such as the 1 400 for rowing and the 700 for walking). When it comes to the others, however, the decimal fraction obtained will need to be converted into hours and minutes. This is done by multiplying the decimal part of the result by 60 to give the number of minutes. In some cases, this is straightforward and can be done mentally. For example, in the case of horse riding and yoga, which each use up 800 kilojoules per hour, 800 divides into 2 800 exactly 3.5 times. This represents $3^{1}/_{2}$ hours or 3 hours 30 minutes. Swimming is also reasonably straightforward: 1 600 divides into 2 800 kilojoules exactly 1.75 times. This is $1^{3}/_{4}$ hours or 1 hour 45 minutes. (Note that the table of equivalences among decimals and fractions suggested in the notes for page **6** of the student book could be used here. Of course, the calculator can be used, too, for example, 0.75 x 60 = 45, but why use a calculator when a mental calculation is faster?)

The other examples will result in decimal fractions that the students may or may not recognise. If they are not familiar with these decimal fractions, they can use a calculator to work out the number of minutes. For example, jumping rope uses 2 400 kilojoules. 2 400 divides into 2 800 kilojoules $1.1666\dot{6}$ times. $0.1666\dot{6}$ is the decimal fraction equivalent of $^{1}/_{6}$. If the students recognise this, they can then calculate that $^{1}/_{6}$ of 60 minutes is 10 minutes. Otherwise, they can use a calculator to multiply 0.16666 x 60 minutes, which gives 9.99996 minutes. Rounding gives a time of 1 hour 10 minutes required for jumping rope. Times for the other activities can be calculated in a similar fashion.

Question **2** is similar to the first question except that, in this case, the key figure is the 3 000 kilojoules obtained from eating the steak. In each of the cases of walking, rowing, and doing aerobics, the number of times that the kilojoules/hour divides into 3 000 is the number of hours a 50 kilogram person would have to spend on that exercise to burn up the steak kilojoules. Walking, at 700 kilojoules, divides into 3000 kilojoules 4.2857 times, that is, 4 hours and 17 minutes (rounded to the nearest minute). Rowing, at 1 400 kilojoules, would require just half that time, that is, 2 hours 9 minutes (rounded to the nearest minute). Aerobics, at 1 200 kilojoules, divides into 3 000 kilojoules exactly 2.5 times, that is, 2 hours 30 minutes (so aerobics is the easiest of all to calculate).

Question **3** can be calculated using the same kind of strategy as for question **2**. The 200 kilojoules used per hour watching television divides into the 1300 kilojoules gained from eating 6.5 times the amount of fish and chips. This means that one would have to watch television for 6 hours 30 minutes to burn off the fish and chips kilojoules. Less time would be required with reading. The 260 kilojoules per hour used during reading divides into 1 300 kilojoules exactly 5 times, meaning that 5 hours' reading would be required.

For question **4**, a range of student responses is possible and likely. Based on the data provided in questions **1** and **3**, the students will have to estimate roughly how many kilojoules would be used in the other activities undertaken during a typical day. There will clearly be a large amount of conjecture in this. For example, you could ask the students to estimate how much energy the cycling might use, given the figures for other activities. The students should find it interesting to compare their results for different activities and for total daily energy use.

Page 15 People Power

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- express a decimal as a percentage, and vice versa (Number, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also estimate fractions of a circle.

ACTIVITY

For question **1**, the students need to change each population group into fractions of the total human population for comparison with the pie graphs. Encourage the students to attempt this before picking up a calculator. They may notice that Asia makes up half the world's population and that only graph **c** fits this condition. This would be justification enough for giving **c** as the answer to this question. Alternatively, they can create a table, such as the one below, to supply the answers to both questions **1** and **2**. The decimal fractions can be calculated by dividing each population group by 6 047 million.

Region	Population	Fraction of human population
Africa	794 million	0.13
North and South America	833 million	0.14
Asia	3 672 million	0.61
Europe	727 million	0.12
Oceania	31 million	0.005

Note that the rounding effect means that the fractions add up to slightly more than 1. This could be corrected by using more figures, for example, 0.6062 for Asia, and so on.

For question **1**, it is now just a matter of inspecting the pie graphs and looking for one that shows Asia with more than half the population (0.61), and Africa, North and South America, and Europe with approximately the same amount each. As before, the only graph that fits this specification is **c**.

The students could derive the results for question **2** directly from the table above, or an additional column could be added showing the equivalent percentages. It is just a matter of multiplying the decimals by 100 to get the percentages. What this means in practice is that all the digits are simply shifted two columns to the left. For example, 0.13 becomes 13 percent. The percentage column will then be:

Percentage of human population
13%
14%
61%
12%
0.5%

Note that, here again, rounding means that the total adds up to over 100 percent.

This addition to the table also enables a ready solution to question **3**. Oceania has only half of 1% of the human population. This makes sense because, as the answers point out, 60 million would be approximately 1% of the total population (60 is roughly one-hundredth of 6 147), so Oceania's population of 31 million would be approximately half of this 1%.

Pages 16–17

Astronomical Proportions

Achievement Objectives

- write and solve problems involving decimal multiplication and division (Number, level 4)
- make sensible estimates and check the reasonableness of answers (Number, level 4)
- use equipment appropriately when solving mathematical ideas (Mathematical Processes, problem solving, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also use ratio.

ACTIVITY ONE

One approach to question **1** would be to consider the ratios of orbit periods for neighbouring planets to see how the periods are increasing. For example, the ratio of orbit periods for Uranus and Saturn is 84.02 : 29.46, or roughly two times as long. This suggests that Pluto's orbit period might be roughly two or three times as long as Neptune's, which means that 3 000 Earth years would be a highly improbable time for Pluto to take to circle the Sun. The more likely time is 300 years. In fact, as the answer indicates, the time taken by Pluto is considerably less than 300 years. It has been calculated as 247.92 Earth years.

In question **2**, the benchmark figure is the Earth's 365 days to circle the Sun. Most of the periods listed in question **2** can be matched to the planets by informed estimation. For example, the shortest number of Earth days listed is 90, a time that is almost certainly that of Mercury's orbit because it is closest to the Sun. This can be checked, using a calculator, by finding what decimal fraction 90 is of 365. The result (90 ÷ 365) is 0.24. This coincides with the Earth year for Mercury on the table, so Mercury must be the planet that takes about 90 days.

The next highest number of Earth days is 230. This is less than the time it takes Earth to orbit the Sun, so logic suggests that this is the time taken by Venus. To check, once again find the decimal fraction (230 ÷ 365), which is 0.63. This is very close to the figure on the table for Venus. The students then have three orbit periods left to match to the remaining six planets on the table. In each case, the orbit period is greater than that of Earth, so the best strategy will be to estimate how many times 365.25 goes into the number given. For example, the 700 day orbit is almost twice Earth's period. The table reveals that Mars takes almost twice as long (1.88 years) as Earth to orbit the Sun. A calculator check shows that 700 ÷ 365 is 1.9, which is very close to the 1.88 years on the table, so Mars is confirmed as the correct choice. The figures of 4 330 and 60 000 can be approached in the same way.

In question **3**, the students need to consider two figures, the times taken by Mercury and Saturn to orbit the Sun. The data on the table reveal that Mercury orbits the Sun in approximately one-quarter of the year or four times each year. It also shows that Saturn takes approximately 30 years to orbit the Sun. This means that Mercury orbits the Sun approximately 4 x 30 times, that is, 120 times, for every orbit by Saturn. A calculator check (29.46 ÷ 0.24) shows that the figure, based on the data in the table, is 122.75 (a figure that is very close to the estimate).

ACTIVITY TWO

In question **1** of this activity, the students need to compare the planets with either Earth or Uranus. Although the question asks for estimates of the radii of the planets, the students may find it easier to compare the diameters. This will not make any difference for comparative purposes. The students can either estimate by sight or use a ruler to get better estimates. For example, the diameter of Mercury appears to be about one-third that of Earth, so the radius will also be one-third, namely, about 2 000 kilometres. Likewise, Saturn appears to be a bit under two and a half times that of Uranus. Two and a half times 25 000 is about 62 000, so the actual radius could be estimated to be a bit less than 60 000 kilometres. (Note that the Answers also give the actual radius of each planet in kilometres.)

Question **2** involves comparing the radius of Pluto (obtained in question **1** of this activity) with the radii of the larger moons of other planets. The students will probably find that, of the moons listed, only one (Oberon) is smaller than Pluto. This suggests that Pluto could indeed have been a moon. Earth's moon has a radius that is slightly more than one and a half times that of Pluto, and the radius of Ganymede is more than two and a quarter times greater than that of Pluto.

The task of ordering the moons by size may raise in students' minds a question about how the sizes of the various moons relate to the size of the planets they circle. They will probably notice that almost all the larger moons listed circle the large planets. The exception is Earth's moon. What is particularly striking is that Earth's radius is only a little more than three and a half times that of our moon. Compare this with the radius of the largest moon (Ganymede) and its planet (Jupiter). Jupiter's radius is more than 26 times that of Ganymede. Similarly, when the radius of Triton is compared with that of its planet, Neptune (not one of the giant planets), the radius of Neptune is more than 18 times that of Triton. What this shows is that, compared with other moons and the planets they circle, Earth's moon is very large in relation to planet Earth.

Question **3** involves the conjecture that larger planets have more moons. This is a good example of a conjecture or question that does not have a straightforward answer. At first glance, it seems that the notion that larger planets have more moons is correct. Jupiter and Saturn are the giants, and they have 29 and 30 moons respectively. Uranus and Neptune are also big planets and have 21 and eight moons respectively. None of the other planets comes anywhere near having this number of moons. However, Pluto is a midget of a planet and has a moon, whereas Venus, by comparison, is more than five times larger and has no moon. Further, Mars is only approximately half the size of Earth and Venus and it has *two* moons. Nevertheless, these cases are exceptions: 88 of the 92 known moons circle the four largest planets, and 59 of these 88 circle the two giant planets. In percentage terms, 95% of the moons circle the four largest planets, with 64% (that is nearly two-thirds) circling the two giant planets, so one might conclude that the conjecture generally holds.

Page 18 No Space to Spare

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will also:

- use scale (on a map)
- estimate numbers and area.

ACTIVITY

In questions **1a** and **1b**, the students need to locate the scale on a map of their local area so that they can then use their rulers to draw a 1 square kilometre template on that part of the map where they live. Street maps are well suited for this purpose. The scale will frequently be presented as a bar showing the relationship between map distance and the actual distance in metres on the ground. With other maps, the scale might be presented as a ratio, for example, 1 : 20 000, which reports the fact that each unit measured on the map represents 20 000 units on the ground. The students will then need to calculate how many units are required to represent 1 kilometre. For example, in the case of a 1 : 20 000 scale, they could proceed as follows: 1 centimetre on the map represents 20 000 centimetres on the ground. Therefore, 1 kilometre or 1 000 metres will equate to a length of 5 centimetres on the map. To estimate how many people live in that square kilometre, the students may find it useful to work out approximately how many people live in the portion of it that they are most familiar with. For example, a student may add up the people living in her street and then, drawing a grid system over the square kilometre, work out that her street covers about one-twentieth of the square. She would then multiply the number of people in her street by 20 to get an overall estimate. Of course, it may be that the square takes in a park, a large shopping area, or some other feature where people are not living. Conversely, it could be that some streets have an unusually high concentration of people living there, for example, in multi-storey flats. These are factors that the students will need to consider and allow for in their calculations.

Question **2** provides data about population densities in some other places. The students may need to consult a map to see where these other places are, which may in turn provide them with clues as to why they have such dense populations. The smallness of the areas and, in several cases, their being islands are features they may notice.

Many countries, New Zealand being one of them, have large open spaces (mountainous areas, lakes, forests, farmland, and so forth) where few people live, and even our New Zealand cities have relatively low densities by international standards. It is not surprising, then, that when New Zealand's population is divided by the area of the country (3 860 000 ÷ 268 021), the result is just 14.4 people per square kilometre. By dividing the density of the other places listed by 14.4, their population densities can be compared with that of New Zealand. In question **2b**, the students are asked to do this for Macao, Hong Kong, and Malta. Question **2c** can be approached by multiplying New Zealand's population density, roughly 15 people per square kilometre, by 3 000. This gives 45 000, which is very close to the figure of 44 000 given for Monaco.

In question **3**, the students need to measure their classroom's width and length to determine its area (in square metres) and then calculate what proportion of a square kilometre this is. They will need to realise that a square kilometre is an area of 1 000 metres by 1 000 metres, which is 1 000 000 square metres (written 1 000 000 m^2). If they divide their classroom area into 1 000 000, they will find how many times their classroom would fit into a square kilometre. By way of illustration, let's say the classroom was 7 metres by 10 metres. This gives an area of 70 m^2. Divide this into 1 000 000 and you get 14 286 (rounded to whole numbers). So, 14 286 classrooms this size would fit into 1 square kilometre. In Macao, 55 815 people live in each square kilometre, which is the same area as 14 286 classrooms (measuring 70 m^2). This means that approximately four people would live permanently in this classroom space (55 815 ÷ 14 286 = 3.9). Imagining that a classroom this size represents a whole house with a kitchen, bathroom, laundry, living room, and bedrooms may help students to gain a perspective on this. In reality, many more Macao people than this would live in this space because it must be remembered that roads, shops, parks, and so on are included in the land area to which the population density applies. Of course, the main way densely populated places overcome this problem of having so many people in such a small area is to house them in high-rise buildings. To take a simple example, if four people occupy an area the size of the classroom mentioned above, adding an extra floor would mean that two people would occupy this same area on each of the two floors.

Page 19 Pizza Pieces

Achievement Objectives

- express a fraction as a decimal, and vice versa (Number, level 4)
- find fractions similar to one given (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will also explore the equivalence of fractions and decimals.

ACTIVITY

In question **1**, the students need to equate ordinary fractions with decimal fractions so that the pieces can be combined to make two whole pizzas in as many ways as possible. They can probably guess some of the combinations by looking at the sizes and shapes of the pieces. For example, the half and 0.5 pieces look as though they would make a whole pizza; likewise the quarter and 0.75 pieces. However, they may find it helpful to make a chart of the amounts of pizza shown so they can work out all the possible combinations.

Fraction	Decimal fraction	Complementary fraction or fraction needed to make one whole
$^7/_8$*	0.875	$^1/_8$ or 0.125*
$^3/_4$	0.75*	$^1/_4$* or 0.25
$^2/_3$	$0.66\dot{6}$ *	$^1/_3$* or $0.33\dot{3}$
$^6/_{10}$	0.6*	$^4/_{10}$ or $^2/_5$* or 0.4*
$^1/_2$*	0.5*	$^1/_2$* or 0.5*
$^2/_5$*	0.4*	$^3/_5$ or 0.6*
$^1/_3$*	$0.33\dot{3}$	$^2/_3$ or $0.66\dot{6}$ *
$^3/_{10}$*	0.3	$^7/_{10}$ or 0.7
$^1/_4$*	0.25	$^3/_4$ or 0.75*
$^2/_{10}$*	0.2	$^8/_{10}$ or 0.8
$^1/_8$	0.125*	$^7/_8$* or 0.875
$^1/_{10}$	0.1*	$^9/_{10}$ or 0.9

* Pieces of pizza pictured on the student book page

Looking down the chart, there are six different ways a whole pizza can be made from two pieces, or two fractions, of pizza shown in the illustration. The students will need to play around with the fractions to find the ways that a whole pizza can be formed using three or more pieces. For example, the following three pieces could be used: 0.5, 0.4, and 0.1, as could the pieces $^1/_2$, $^2/_{10}$, and $^3/_{10}$.

The answer lists all the possible ways of making a whole pizza, including those using four pieces.

For question **2**, probably the easiest way to make one and a half pizzas using just three pieces is to take one of the pairs that combine to make one pizza (other than the half and 0.5 piece combination itself) and then simply add either the half or 0.5 piece of pizza to this pair. The answer includes a solution that involves using five pieces.

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Line Up

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 4)
- write and solve problems involving decimal multiplication and division (Number, level 4)
- find, and use with justification, a mathematical model as a problem-solving strategy (Mathematical Processes, problem solving, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will also estimate decimal products by rounding.

ACTIVITY

Question **1** will help the students to gain a sense of the magnitude of products of decimal numbers without the need for exact calculations. They will need to be able to round decimal numbers to make approximations through mental multiplication. For example, 2.5 x 1.1 is going to be a bit more than 2.5, whereas 2.5 x 0.9 will be a bit less than 2.5. In both cases, the initial step would be to think of the 1.1 and 0.9 as being close to 1. The expression 2.5 x 1.9 could be thought of as being roughly equal to 2.5 x 2. The result will therefore be a bit less than 5. The students may work out the point on the number line for the expression 2.5 x 0.5 by elimination, that is, by seeing which point is left after the other three expressions have been matched. Alternatively, by equating 0.5 with $^1/_2$, they should be able to work out that 0.5 x 2.5 will be $1^1/_4$ or 1.25.

The tasks in question **3** can be thought of as open number sentences that need to be made true, basically by using the 7 times table. For example, the first one is ☐ x 7 = 2.8, the second ☐ x 7 = 9.8, and so on. The students should be able to do all of these mentally, but they will need to use their number sense (and perhaps their calculators) to check that the decimal point is in the correct place. For instance, because 4 x 7 is 28, the solution to the first number (2.8) must be 0.4. This makes sense because 0.4 of 7 means a bit less than half of 7, which 2.8 satisfies. Some students may see the connection between 2.8 and 9.8 (the next arrow point). If they divide 7 into 9.8, they will find that it goes once, with 2.8 left over. Logically, then, 7 must divide into 9.8 exactly 1.4 times. The next arrowed number, 17.5, can be handled in the same way. That is, 7 goes into 17.5 twice, with 3.5 left over, into which 7 will go 0.5 times. Result: 2.5. The last two numbers can be worked out in a similar way.

Page 21 Gains and Losses

Achievement Objectives

- find a given fraction or percentage of a quantity (Number, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- express quantities as fractions or percentages of a whole (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also:

- use proportion
- explore the equivalence of percentages and decimals.

ACTIVITY

This activity about the changing value of gold krugerrands is similar to another relatively well-known problem involving workers' wages being cut by 10 percent in a time of company hardship but then increased by 10 percent a year later when the company's fortunes improved. Were the workers now back to their previous level of wages? The answer is no! Imagine, to take a nice round figure, that the workers were earning $100 per week. If their wages were reduced by 10 percent (that is, by one-tenth), they would be earning $90 per week. Increasing them again by 10 percent means that 10 percent or one-tenth of 90 would be added. The workers would now be earning $99 per week, not their original $100 per week.

The same is true of the value of the krugerrands. Let's say the krugerrand was worth $8 at the beginning of the previous year and that its value had increased by half or 50 percent. That meant that it would then be worth $12. Now, with the drop in value again, that is, to a level that is just half of last year's value, each krugerrand is now worth just $6. So, over the 2 years, krugerrands have dropped in value from $8 to $6. This drop of $2 on the original value means that they have lost 25 percent or one-quarter of their value. Hence, they are now worth only three-quarters or 75 percent of their value at the beginning of the previous year ($^6/_8$ is the same as $^3/_4$). The diagram on the student page shows visually how and why this drop in overall value occurred.

When the students come to question **2**, they will probably follow similar steps to those that Leyton used in his diagram. This time, however, they will need to divide their diagram into four equal lengths. A convenient length to begin with would be 16 centimetres. When this is increased by one-quarter (that is, 4 centimetres), it becomes 20 centimetres. When this is then decreased by one-quarter (that is, 5 centimetres), it reduces to 15 centimetres.

Now would be a good stage to help the students to see what is happening more generally. With first an increase and then a decrease of half, the result is a loss of one-quarter of the original value. When there is an increase and then a decrease of one-quarter, the result is a loss of one-sixteenth of the value. Thus, a pattern begins to emerge:
$^1/_2 \times ^1/_2 = ^1/_4$
$^1/_4 \times ^1/_4 = ^1/_{16}$
That is, the final decrease is equal to the original increase/decrease amount squared.

These problems can also be approached directly using fractions. If we increase something by one-half, we are effectively multiplying it by $^3/_2$. If we then reduce it by one-half, we are effectively multiplying the result by $^1/_2$, which is equivalent to calculating $^3/_2 \times ^1/_2$, which is $^3/_4$. That is, we now have three-quarters of our original quantity. Similarly, with an increase/decrease of one-quarter, we have $^5/_4 \times ^3/_4 = ^{15}/_{16}$.

The same approach can be used for question **3**. For example, an increase of one-third followed by a decrease of one-third is equivalent to calculating $^4/_3 \times ^2/_3 = ^8/_9$.

Encourage the students to find out (using diagrams and calculation) whether or not the order in which a quantity is increased and decreased is important. For example, an increase of one-third followed by a decrease of one-third will give the same result as the decrease followed by the increase (though the intermediate quantity will clearly be different).

By looking at results such as those above, the students may notice that the numerator of the fraction that results is always one less than the denominator. If they can couple that with the fact that the denominator is found by squaring the increase/decrease amount, then they may be able to arrive at the formula $\frac{n^2 - 1}{n}$ given in the Answers.

In question **4b**, the students need to convert the 1 percent to the fraction $^1/_{100}$. They can then apply the rule that they have discovered in **a** to work out that the "something" has decreased in value by $^1/_{10\,000}$ ($^1/_{100} \times ^1/_{100}$) and is now worth $^{9\,999}/_{10\,000}$ of the original value.

Pages 22–23 Balancing Act

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)
- pose questions for mathematical exploration (Mathematical Processes, problem solving, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- learn that one side of an equation must "balance" the other side (equivalence)
- use ratio and proportion.

ACTIVITY

This is an excellent set of tasks to help the students to recognise that the two sides of an equation name the same number. This is different from the notion that many students develop that an equation is a record of an operation and nothing else. The idea that an equation is also a statement or expression that says two numbers are equal is crucial to work in algebra. It means that if you do something to one side of an equation, you must do the same to the other to maintain the equality.

The students may have some good strategies for approaching question **1**. For example, in the first two examples, they might notice that each weight on the left must be equal to two of those on the right. Another way is to set them out as equations. For example, the first one would be $2 \times 18 = 4 \times \square$ and the next would be $2 \times \square = 4 \times 24$. It is then a matter of figuring out what the product (or total) is on the side with sufficient data and then working out what size weights are needed to get the same total on the other. In the case of $2 \times 18 = 4 \times \square$, the students will soon calculate that the left-hand side is 36. To make the right-hand side 36, they will have to multiply the 4 by 9. Similarly, with $2 \times \square = 4 \times 24$, the right-hand side totals 96, so to have the left-hand side also totalling 96, the 2 must be multiplied by 48.

In question **2**, the students get an opportunity to construct their own set of weights to balance out the two lots of 36 and the three lots of 18. Different students are likely to come up with different possibilities, which can be shared. The answer lists a number of possible sets of weights. Notice that it suggests that the weights need not be whole numbers. For instance, 12 lots of $4^1/_2$ is also equal to 3 lots of 18 (that is, a total of 54). The possible use of weights that are mixed fractions (such as $4^1/_2$) may open up the activities to some nice challenges, especially when the students make up some for a partner to solve. Some of the problems generated in question **3** may be worth posing to the whole class.

Page 24 Double Russian

Achievement Objectives

- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Other mathematical ideas and processes

Students will also use doubling and halving, that is, multiplication and division by 2.

ACTIVITY

This activity shows that, in the absence of a calculator, there is more than one way to calculate efficiently. When the students have completed the investigation, they will see that the method always works. While this does not explain why the method works, it may encourage the students to try other ways of working. For example, based on the idea of doubling, if the students wish to multiply a number by 32, they can simply double the number five times. For example, if they choose 23, then 23 x 32 will give the sequence 46, 92, 184, 368, 736 (with 736 being the final product). This works because 32 is 2^5 or 2 x 2 x 2 x 2 x 2.

Another example of an efficient alternative method may be used in the calculation 50 x 684. Since 50 is half of 100, the calculation can be renamed as $^{1}/_{2}$ x 100 x 684 or, better still (using the commutative principle), as 100 x $^{1}/_{2}$ x 684, which is 100 x 342, giving 34 200. So, multiplying any number by 50 is the same as taking half the number and multiplying the result by 100.

If you wish to give an explanation as to why the method called Russian multiplication works, this will involve an understanding of base 2 or binary numbers. The fact that computers work in base 2 provides another reason for working with numbers in this form. Take the number 47. In base 2, this is expressed as 101111 (because base 2 uses only 0 and 1 as the digits). It means:
$(1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$ or, if you like,
(1 x 32) + (0 x 16) + (1 x 8) + (1 x 4) + (1 x 2) + (1 x 1).

Now, when 47 is halved again and again until 1 is reached, the sequence is 47, 23, 11, 5, 2, 1. Reverse the order and it becomes 1, 2, 5, 11, 23, 47. In Russian multiplication, we are asked to add the numbers that match the *odd* numbers in the sequence. Remember, too, that in base 2, electronically speaking, 1 means "on" and 0 means "off". So, if we line the numbers up, they look like this:

1,	2,	5,	11,	23,	47
1	0	1	1	1	1

The ones represent the odd numbers that are definitely on the list of numbers to be counted, and zero (0) represents the even number that is off the list and doesn't count. You could now return to the original problem and set it out as below, with 47 written vertically (from bottom to top) in base 2:

1	67
1	134
1	268
1	536
0	1 072
1	2 144

This makes it clear why 1 072 is not included in the total: it simply doesn't count.

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