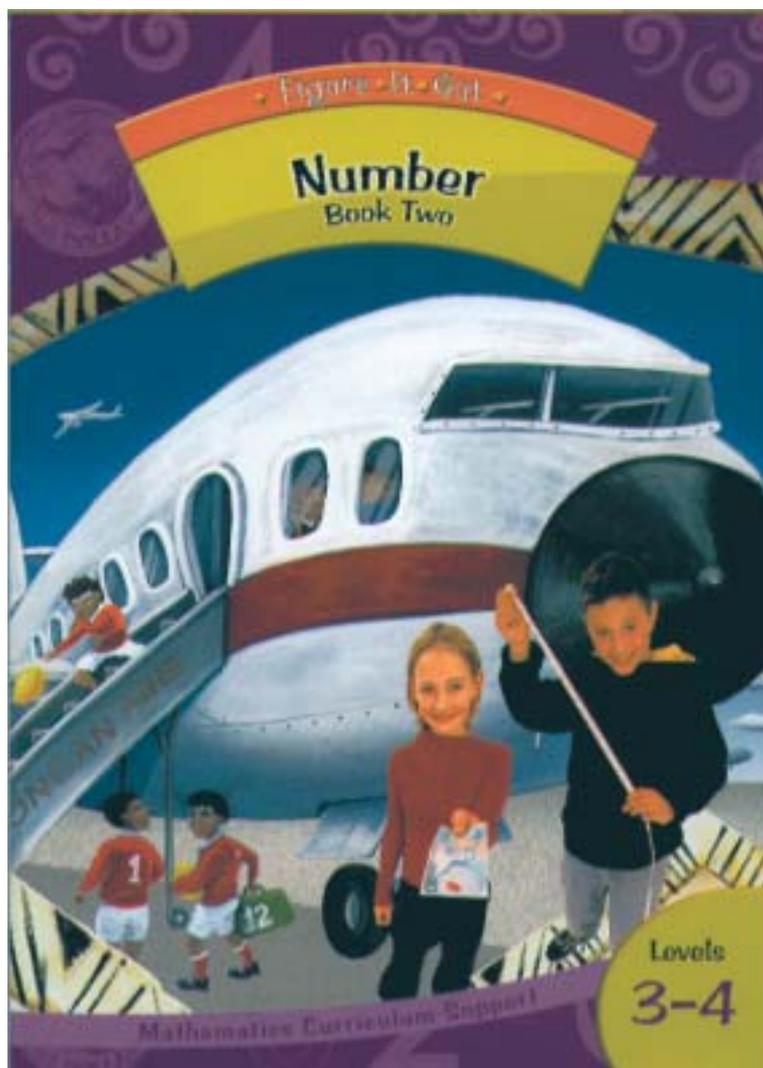


Answers and Teachers' Notes



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Introduction

The books for levels 3–4 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. *Number: Book Two* and *Number: Book Three* have been developed to support teachers involved in the Numeracy Project. These books are most suitable for students in year 6, but you should use your judgment as to whether to use the books with older or younger students who are also working at levels 3–4.

Student books

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students in year 6.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hotspots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure.

Using Figure It Out in the classroom

Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.

Mathematics in the New Zealand Curriculum, page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Number: Book Two

Answers

Page 1: Mystery Hexagons

Activity

- 356 400
- Yes. It works with any 3 pairs of numbers.
- Yes.
- Explanations may vary. Mystery hexagons will work for both addition and multiplication because of the *commutative property*, that is, the order of the addends does not affect the sum, and the order of the factors does not affect the product: $9 \times 6 = 6 \times 9$, $11 + 3 = 3 + 11$. All the factors are used in the final product and all the addends in the final sum, so their order makes no difference. (The mystery hexagons will not work for subtraction and division because these operations are not commutative: for example, $12 \div 4 \neq 4 \div 12$ and $10 - 3 \neq 3 - 10$.)

Pages 2–3: Flying Home

Activity One

- He was using his compatible numbers strategy: $0.29 + 0.01$ add up to 30 hundredths, so $14.29 + 0.01 = 14.3$. And $0.3 + 0.7$ add up to 10 tenths, so $14.3 + 0.7 = 15$.
- Mae: 18.5 kg
Tipu: 4.1 kg
Sali: 8.32 kg
- \$305.35
 - Methods will vary. For example, after Tipu has found the total mass (30.92 kg, which he needs to round to 31 kg), he could round \$9.85 to \$10.00 and then take 0.15 off for each kg and part kg:
 $31 \times 10 - 31 \times 0.15 = 305.35$. (He could break down each step further, for example, $30 \times 10 = 300$ and $30 \times 0.10 = 3.00$.)

- \$100 is about the cost of 10 kg, so they will need to leave about 21 kg behind.

Activity Two

- Answers may vary, but estimates should be about 9 600, using rounding to the nearest 10: 120×80 . (The actual answer to 118×82 is 9 676.)
- Sali: 5.3 kg
Tipu: 27.7 kg
- 123 kg. ($82 + 41$)
 - 10%. (They are 8.2 kg over the usual average body mass. 8.2 is $\frac{1}{10}$ or 10% of 82.)
 - 20%. ($98.4 - 82 = 16.4$, which is twice 8.2, which was 10%.)

Page 4: Oceans Apart

Activity

- Both methods should produce the same answers. For example:

- $1 \times 63 = 63$
 $2 \times 63 = 126$
 $4 \times 63 = 252$
 $8 \times 63 = 504$
 $16 \times 63 = 1\ 008$
 $16 + 4 + 2 = 22$
 $1\ 008 + 252 + 126 = 1\ 386$
- or

	2	2	
1	1	2	6
3	0	6	6
	8	6	

So $22 \times 63 = 1\ 386$.

- 1 632
- 1 944

2. Preferences and reasons will vary. (For example, you may prefer the Chinese method because it uses basic multiplication and addition facts and can be done very quickly once you understand the process.)

Page 5: Food for All

Activity

174. $(14 + [16 \times 9] + [2 \times 8])$. Strategies for 16×9 will vary.)
- 870 pieces of vegetable (174×5), 522 slices of meat (174×3), and 696 pieces of fruit (174×4). (Strategies for multiplying by 5, 4, and 3 will vary.)
- 44 packets. (Each packet feeds $12 \div 3 = 4$ people. $174 \div 4 = 43.5$. Or: $174 \times 3 = 522$. $522 \div 12 = 43.5$)
- 52 200 mL, which is 52.2 L
 27. ($52\ 000 \div 2\ 000 = 26$ or $52\ 000 \div 1\ 000 = 52$. Half of 52 is 26. An extra jug would be needed for the 200 mL.)
- 56 pieces of fruit and 42 biscuits at the top table, 36 pieces of fruit and 27 biscuits at each table seating 9, and 32 pieces of fruit and 24 biscuits at each table seating 8

Pages 6-7: Spring Fever

Activity

- 2.48 m². ($0.66 \times 3 + 0.5 \times 0.5 + 0.5 \times 0.5$)
 - Answers will vary depending on the carpet chosen:
 - \$345
 - \$295.50
 - \$465
 - \$264
- 8.65 m
 - 7 rolls
 - \$64.28. ($\$450 \div 7$, with no rounding. Rounding to \$64.29 would make the total \$450.03.)

- d. Answers will vary, depending on the wallpaper chosen:

- \$215.60
- \$406
- \$325.50
- \$433.65

- 5.8 m. (1.45 for each curtain)
 - Answers will vary, depending on the curtain material chosen:
 - \$229.10
 - \$120.35
 - \$108.75
 - \$150.74
- Answers will vary, depending on the materials chosen.

Page 8: Ageing in Space

Activity

- 16.26 Venus-years old
 - 5.32 Mars-years old
 - 0.84 Jupiter-years old
 - 0.34 Saturn-years old
 - 0.12 Uranus-years old
 - 0.06 Neptune-years old
 - 0.04 Pluto-years old
- 3.39 Saturn-years old (2 d.p.). ($100 \times 365.26 \div 10\ 760.55$)
 - 20.76 Mercury-years old (2 d.p.). ($5 \times 365.26 \div 87.97$)
- 2 486.00 Earth-years old (2 d.p.). ($10 \div 365.26 \times 90\ 803.63$)

Page 9: Meal Deal

Activity

- \$47.10
 - \$12.90
- \$38.60

3. 9
4. She could buy 8 scoops for \$13.60, and she would get \$1.40 change.
5. 15
6. 15
7. 80c
8. \$13.00
9. Answers will vary. Two possible answers are:
 - 1 fillet with chips (\$4.50), 1 sausage (\$1.70), 1 dim sim (70c), 1 crabstick (60c), and 1 hamburger (\$2.50)
 - 2 fish cakes (\$3.00), 1 hot dog (\$1.70), 1 spicy wedges (\$3.00), 1 waffle dog (\$1.80), and 1 tomato sauce (50c)

- b. No. Fractions can only be simplified if the numerator (top number) and the denominator (bottom number) can be divided by the same number, that is, if they have a common factor. For example, fractions such as $\frac{4}{25}$ cannot be simplified because the factors of 4 (2 and 4) do not divide evenly into 25.

2. Answers will vary.

Page 12: Funky Fractions

Game

A game for finding fractions of numbers

Page 10: First to the Draw

Game

A game for working with decimal numbers and place value

Page 11: Sandwich Survey

Activity

1. a. Monday:
 - peanut butter: $\frac{6}{24}$ or $\frac{1}{4}$
 - jam: $\frac{4}{24}$ or $\frac{1}{6}$
 - tomato: $\frac{3}{24}$ or $\frac{1}{8}$
 - luncheon sausage: $\frac{8}{24}$ or $\frac{1}{3}$
 - honey: $\frac{2}{24}$ or $\frac{1}{12}$
 - cheese and chutney: $\frac{1}{24}$
- Tuesday:
 - cheese: $\frac{5}{30}$ or $\frac{1}{6}$
 - jam: $\frac{15}{30}$ or $\frac{1}{2}$
 - tomato: $\frac{1}{30}$
 - egg: $\frac{4}{30}$ or $\frac{2}{15}$
 - peanut butter: $\frac{3}{30}$ or $\frac{1}{10}$
 - cheese and onion: $\frac{2}{30}$ or $\frac{1}{15}$
- Wednesday:
 - peanut butter: $\frac{5}{25}$ or $\frac{1}{5}$
 - lettuce and cheese: $\frac{4}{25}$
 - raisin and nuts: $\frac{6}{25}$
 - ham: $\frac{10}{25}$ or $\frac{2}{5}$

Page 13: Measuring Up

Activity

1. Practical activity. Results will vary.
2. Answers will vary. Possible approximate fractions could be:
 - a. $\frac{1}{2}$
 - b. $\frac{1}{3}$
 - c. $\frac{1}{11}$
 - d. $\frac{1}{1}$
 - e. $\frac{1}{1}$
 - f. $\frac{1}{2}$
3. Estimates will vary.

Page 14: Large or Huge?

Activity

1. a. $\frac{1}{5}$
- b. $\frac{1}{8}$
- c. $\frac{7}{50}$
- d. $\frac{1}{2}$
- e. $\frac{1}{38}$
- f. $\frac{2}{3}$

2. (All part animals have been rounded up to a whole animal.)
- 13 whale sharks
 - 38 African elephants
 - 48 Indian elephants
 - 87 rhinoceros
 - 95 hippopotamuses
 - 159 giraffes
 - 173 crocodiles
 - 238 bison
 - 280 brown bears
 - 760 Siberian tigers
 - 1 267 ostriches
 - 1 697 giant pandas

3. The crocodile's mass

Investigation

Results will vary.

- iii. \$51.53
 - iv. \$78.75
 - v. \$66.15
 - vi. \$39.38
4. a. i. \$45.90
- ii. \$28.44
 - iii. \$41.22
 - iv. \$63.00
 - v. \$52.92
 - vi. \$31.50
- b. i. \$52.98. (\$317.88 sale price – \$264.90 shop cost)
- ii. \$79.50. (\$132.48 – \$52.98)
- c. Discussion will vary.

Page 15: Fraction Distraction

Game

A game for matching fractions, decimals, and percentages

Page 16: Making Money

Activity

1. i. \$12.75
- ii. \$7.90
 - iii. \$11.45
 - iv. \$17.50
 - v. \$14.70
 - vi. \$8.75
2. i. \$38.25
- ii. \$23.70
 - iii. \$34.35
 - iv. \$52.50
 - v. \$44.10
 - vi. \$26.25
3. i. \$57.38
- ii. \$35.55

Page 17: Bargain Busters

Activity

1. a. 25% is $\frac{1}{4}$, so he could round the prices and divide by 4, or he could estimate 10% of the cost, double it, and add half of 10% ($10\% + 10\% + 5\% = 25\%$).
- b. 30% is nearly $\frac{1}{3}$, so he could round the prices and divide by 3, or he could estimate 10% (using place value knowledge) and add 3 lots of 10%.
2. a. i. Estimates may vary, but they should be close to the following:
 Drinks: \$20.70. (The drinks add up to approximately \$23.00. 10% of that is \$2.30, so the approximate cost is \$20.70.)
 Freezer: \$13.60. (20% of \$17 is \$3.40, so the approximate cost is \$13.60.)
 Fruit and vegetables: \$25.50. (25% of \$34 is \$8.50, so the approximate cost is \$25.50.)
 Breakfast food: \$15.40. (30% of \$22 is \$6.60, so the approximate cost is \$15.40.)

- ii. Drinks: \$21.07. (10% of \$23.41 is \$2.34.)
 Freezer: \$13.65. (20% of \$17.06 is \$3.41.)
 Fruit and vegetables: \$24.39. (25% of \$32.52 is \$8.13.)
 Breakfast food: \$14.98. (30% of \$21.40 is \$6.42.)
- b. \$20.30
3. a. No. The price should be \$9.38 or the saving shown as 20%.
 b. With rounding, it is correct. \$12 is the discounted price after taking 20% off \$15.00.

Page 18: How Slow Can You Go?

Activity

1. a. About 140 m
 b. About 12.4 min. (12 min 24 s)
 c. About 120 m/h
2. a. About 3 hrs
 b. About 3.75 m
 c. About 40 min. (The path measures about 20 cm, which is 10 m according to the scale. 10 m is $\frac{2}{3}$ of 15 m, and $\frac{2}{3}$ of 60 min is 40 min.)
3. a. About $1\frac{1}{3}$ km
 b. About 0.75 min. (45 s)

Investigation

Answers will vary.

Page 19: Paddling down the Waikato

Activity

1. 12 hrs 30 min, assuming they avoid all 10 taniwha. (If they dragged the waka for about 500 m round each bend where there is a taniwha, that would take them 150 min. They would paddle for 50 km: $50 \times 12 \text{ min} = 600 \text{ min}$. $150 + 600 = 750 \text{ min}$, which is 12 hrs 30 min.)

2. With 5 friendly taniwha letting them paddle past safely, they would only drag the waka around 5 stretches of 500 m ($5 \times 15 = 75 \text{ min}$) and paddle for 52.5 km ($52.5 \times 12 = 630$). $75 + 630 = 705 \text{ min}$, which is 11 hrs 45 min.

Pages 20–21: The Fish Hooks of Ngāke

Activity

1. a. 270 fish. ($3 \times 10 \times 3 \times 3$)
 b. 360 fish. ($4 \times 15 \times 2 \times 3$)
 c. 960 fish. ($5 \times 16 \times 3 \times 4$)
2. 360 fish. ($5 \times 6 \times 3 \times 4$)
3. a. 2 kūmara
 b. 3 kūmara
 c. 1 kūmara
 d. 3 kūmara

Pages 22–23: Focusing on Water

Activity

1. Strategies will vary. Another possible strategy is:
 $4 \times 100 = 400$, $4 \times 50 = 200$, $4 \times 25 = 100$.
 $400 + 200 + 100 = 700$.
2. Strategies will vary. At least one possible strategy is included for each answer.
- a. 740 L. $(2 \times 9 \times 20) + (2 \times 7 \times 20) + 100$
 $= 360 + 280 + 100$
 $= 740$
- b. 21 L. $3 \times 14 \div 2 = (3 \times 15 - 3) \div 2$
 $= 42 \div 2$
 $= 21$
 or: $\frac{1}{2} \times 14 \times 3 = 7 \times 3$
 $= 21$
- c. 209 L. $19 \times 11 = 10 \times 11 + 10 \times 11 - 11$
 $= 110 + 110 - 11$
 $= 220 - 11$
 $= 209$
 or: $19 \times 11 = (19 \times 10) + (19 \times 1)$
 $= 190 + 19$
 $= 209$

- d. 126 L. 1 tap uses 7 L a min, so
 $4 \times (3 \times 1.5 \times 7) = 126$.
- e. 50 L. $25 + 25 = 50$ or $2 \times 25 = 50$
- f. 105 L. 2 taps for 30 s = 1 tap for 1 min.
 $15 \times 7 = 10 \times 7 + 5 \times 7$
 $= 70 + 35$
 $= 105$
- g. 56 L. $4 \times 14 = 4 \times 15 - 4$
 $= 60 - 4$
 $= 56$
 or: $14 \times 2 \times 2 = 28 \times 2$
 $= 56$
 or: $4 \times 10 + 4 \times 4 = 40 + 16$
 $= 56$
3. 1 307 L (the total of the amounts in question 2)
4. Strategies will vary. One possible strategy is included for each answer.
- a. 350 L. $2 \times 100 + 2 \times 75 = 200 + 150$
 $= 350$
- b. 40 L. $20 + 20 = 40$
- c. 14 L. $2 \times 7 = 14$
- d. 84 L. $4 \times 3 \times 7 = 84$
- e. 520 L.
 $(2 \times 4 \times 20) + (3 \times 6 \times 20) = 160 + 360$
 $= 520$
- f. 115.5 L.
 $3 \times 11 = 33$.
 $15 \times 5.5 = (10 \times 5 + 5 \times 5) +$
 $(10 \times 0.5 + 5 \times 0.5)$
 $= (50 + 25) + (5 + 2.5)$
 $= 75 + 7.5$
 $= 82.5$
 $82.5 + 33 = 115.5$
- g. 35 L. $15 \times 20 \text{ s} = 300 \text{ s}$ or 5 min.
 $5 \times 7 = 35$
5. 1 183 L (the total of the amounts in question 4 + Eliot's 24.5)
6. 124 L. $(1\ 307 - 1\ 183)$

Investigation

Results will vary.

Activity

- Estimates should be between 6 min 25 s and 6 min 30 s. (Their best times add up to 6 min 28 s.)
- Freestyle: William
 Backstroke: William
 Breaststroke: Marie
 Butterfly: Layla
- No. $1 \text{ min } 15 \text{ s} \times 10 = 750 \text{ s}$ or 12 min 30 s. This is more time than 11 min 40 s, so he would not beat the club record.
- Freestyle: William; backstroke: Findlay; breaststroke: Marie; butterfly: Layla.
 Note: William is by far the best freestyle swimmer, so he should be chosen for this stroke rather than for backstroke. Findlay's best time for 100 m backstroke is only 1 s slower than William's.

5.

	Breaststroke	Butterfly
William	17	19
Findlay	18	21
Hine	20	19
Layla	19	18
Marie	17	18

◆ Figure It Out ◆
Number: Book Two
Teachers' Notes

Overview of Number: Book Two

Title	Content	Page in students' book	Page in teachers' book
Mystery Hexagons	Finding a pattern using multiplication	1	12
Flying Home	Solving story problems that involve whole numbers, decimals, and percentages	2–3	13
Oceans Apart	Investigating multiplication strategies	4	15
Food for All	Solving problems involving multiplication and division	5	16
Spring Fever	Solving multiplication and division problems using decimals	6–7	17
Ageing in Space	Solving problems using multiplication and division	8	18
Meal Deal	Multiplying and dividing with decimals	9	19
First to the Draw	Using place value and converting fractions to decimals	10	20
Sandwich Survey	Finding equivalent fractions	11	20
Funky Fractions	Finding fractions of numbers	12	22
Measuring Up	Finding fractions	13	23
Large or Huge?	Using fractions to compare different measurements	14	24
Fraction Distraction	Matching fractions, decimals, and percentages	15	25
Making Money	Finding percentages of amounts of money	16	26
Bargain Busters	Finding percentages of amounts of money	17	27
How Slow Can You Go?	Using speed to calculate distance and time	18	28
Paddling down the Waikato	Using scales and rates to solve problems	19	29
The Fish Hooks of Ngāke	Solving problems involving multiplication	20–21	30
Focusing on Water	Using mental strategies to calculate capacity	22–23	31
Dash and Splash!	Solving problems that involve addition and multiplication	24	32

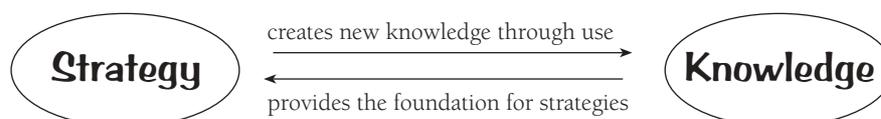
Introduction to Number

There is a remarkable commonality in the way many countries around the world are now teaching arithmetic. Changes in the approaches reflect the evolving demands of everyday life, a greater volume of classroom-based research about how students learn, and a desire to improve general levels of numeracy.

In the past, arithmetic teaching has focused on preparing students to be reliable human calculators. The prevalence in society of machines that calculate everything from supermarket bills to bank balances has meant that students now require a wider range of skills so that they can solve other problems flexibly and creatively.

The Figure It Out series aims to reflect these trends in modern mathematics education. A range of books is provided at different levels to develop both number skills and number sense. The *Number* books are aimed at developing students' understanding of the number system and their ability to apply efficient methods of calculation.

The development of the Figure It Out series has occurred against the backdrop of a strong drive for improved standards of numeracy among primary-aged and intermediate students. A key element of this drive has been the creation of the Number Framework as part of the Numeracy Strategy. The framework highlights this significant connection between students' ability to apply mental strategies to solving number problems and the knowledge they acquire.



The learning activities in the series are aimed at both the development of efficient and effective mental strategies and increasing the students' knowledge base.

Links to the Number Framework

There are strong links to the stages of development in the Number Framework in *Number*, Books Two and Three for level 3 and levels 3–4. The knowledge- and strategy-based activities in the level 3 books are at the early additive part–whole to advanced additive/early multiplicative stages of the Number Framework. Those in the level 3–4 books also link to the advanced multiplicative (early proportional) part–whole stage of the Number Framework.

Information about the Number Framework and the Numeracy Project is available on the NZMaths website (www.nzmaths.co.nz/Numeracy/Index.htm). The introduction includes a summary of the eight stages of the Number Framework. Some of the Numeracy Project material masters are relevant to activities in the Figure It Out student books and can be downloaded from

www.nzmaths.co.nz/Numeracy/materialmasters.htm

Books for the Numeracy Development Project can be downloaded from

www.nzmaths.co.nz/Numeracy/2004numPDFs/pdfs.htm

Terms

Teachers who are not familiar with the Numeracy Project may find the following explanations of terms useful.

The Number Framework: This is a framework showing the way students acquire concepts about number. It comprises eight stages of strategy and knowledge development.

Knowledge: These are the key items of knowledge that students need to learn. Knowledge is divided into five categories: number identification, number sequence and order, grouping and place value, basic facts, and written recording.

Strategies: Strategies are the mental processes that students use to estimate answers and solve operational problems with numbers. The strategies are identified in the eight stages of the Number Framework.

Counting strategies: Students using counting strategies will solve problems by counting. They may count in ones, or they may skip-count in other units such as fives or tens. They may count forwards or backwards.

Part-whole thinking or part-whole strategies: Part-whole thinking is thinking of numbers as abstract units that can be treated as wholes or can be partitioned and recombined. Part-whole strategies are mental strategies that use this thinking.

Partitioning: Partitioning is dividing a number into parts to make calculation easier. For example, 43 can be partitioned into 40 and 3, or 19 can be partitioned into 10 and 9 or thought of as 20 minus 1.

Relevant stages of the Number Framework for students using the level 3 and 3–4 *Number* books:
Stage five: early additive part-whole: At this stage, students have begun to recognise that numbers are abstract units that can be treated simultaneously as wholes or can be partitioned and recombined. This is called part-whole thinking.

A characteristic of this stage is the derivation of results from related known facts, such as finding addition answers by using doubles or “teen” numbers. For example, students at this stage might solve $7 + 8$ by recalling that $7 + 7 = 14$, so $7 + 8 = 15$. They might solve $9 + 6$ by knowing that $10 + 6 = 16$, so $9 + 6 = 15$. They might solve $43 + 35$ as $(40 + 30) + (3 + 5)$, which is $70 + 8 = 78$.

Stage six: advanced additive/early multiplicative part-whole: At this stage, students are learning to choose appropriately from a repertoire of part-whole strategies to estimate answers and solve addition and subtraction problems.

Addition and subtraction strategies used by students at this stage include:

- standard place value with compensation ($63 - 29$ as $63 - 30 + 1$)
- reversibility ($53 - 26 = \square$ as $26 + \square = 53$)
- doubling ($3 \times 4 = 12$ so $6 \times 4 = 12 + 12 = 24$)
- compensation ($5 \times 3 = 15$ so $6 \times 3 = 18$ [3 more])

Students at this stage are also able to derive multiplication answers from known facts and can solve fraction problems using a combination of multiplication and addition-based reasoning. For example, 6×6 as $(5 \times 6) + 6$; or $\frac{3}{4}$ of 24 as $\frac{1}{4}$ of 20 is 5 because $4 \times 5 = 20$, so $\frac{3}{4}$ of 20 is 15, so $\frac{3}{4}$ of 24 is 18 because $\frac{3}{4}$ of the extra 4 is 3.

Stage seven: advanced multiplicative part-whole: Students who are at this stage are learning to choose appropriately from a range of part-whole strategies to estimate answers and solve problems involving multiplication and division. For example, they may use halving and doubling (16×4 can be seen as 8×8) and trebling and dividing by 3 ($3 \times 27 = 9 \times 9$).

Students at this stage also apply mental strategies based on multiplication and division to solve problems involving fractions, decimals, proportions, ratios, and percentages. Many of these strategies involve using equivalent fractions.

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- use words and symbols to describe and generalise patterns (Mathematical processes, developing logic and reasoning, level 4)

Activity

This activity develops students' skills and confidence in communicating mathematical ideas. For this activity, the students need to be able to derive multiplication answers, so it is appropriate for those who are at or beyond the advanced additive stage of the Number Framework. You can support the growth of your students' mathematical vocabulary by modelling, explaining, and encouraging the use of mathematical terms that convey specific meanings. Some of the key number words you could reinforce during this activity are: "factor", "product", "addends", "sum", "decimal", and "commutative property". (See the Answers for an explanation of the commutative property.)

You could introduce this activity to the students by modelling it on the whiteboard. If you don't want the students to use calculators, try limiting the initial factors to whole numbers between 1 and 10. This will make the mental calculations easier, and the students could use strategies such as doubling and halving ($8 \times 5 = 4 \times 10$), tidy numbers ($9 \times 8 = 10 \times 8 - 8$), tens facts ($5 \times 5 = 25$ so 5×50 is 10 times greater; $5 \times 5 \times 10 = 250$), hundreds facts (5×500 is $5 \times 5 \times 100 = 2\,500$), and so on. Draw their attention to 45 as the product of 15 and 3, but point out that this product becomes a factor when it is multiplied by 80 to form a product of 3 600. Working in pairs will generate more mathematical talk.

The students will probably use whole numbers to create their own mystery hexagons in question 2. To ensure that they have understood the process, have each pair of students swap their starting factors with another pair to see if both pairs get the same mystery number for those factors. You could list on the board all the sets of factors successfully used to show that the process works for all factors.

During reporting-back time, reinforce the use of the vocabulary: "Did the mystery hexagon work for all the factors you tried? Whole numbers? Decimals?" Don't accept statements without evidence. Get the students to be specific, for example: "The mystery hexagon worked for three whole number examples and three decimal examples. We think it always works for multiplication." (The answers explain why mystery hexagons will work for both addition and multiplication but not for subtraction or division.)

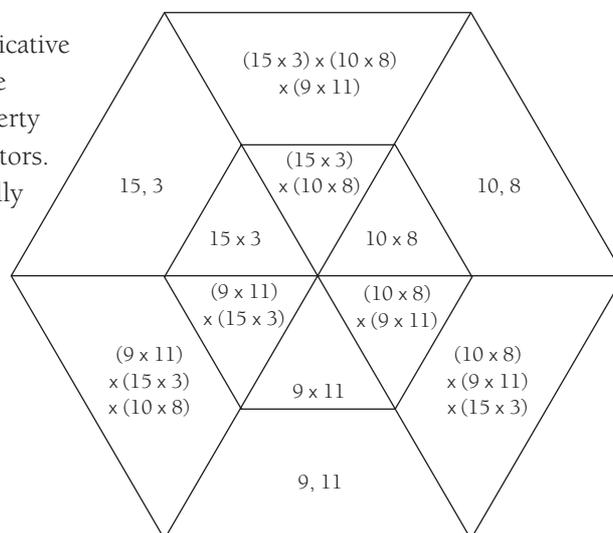
Students who are in transition to advanced multiplicative thinking need to understand what the commutative property is. They can show the commutative property by leaving the products of the inner triangles as factors. Ask them to represent the final products horizontally as mathematical equations.

$$(15 \times 3) \times (10 \times 8) \times (9 \times 11) = 356\,400$$

$$(10 \times 8) \times (9 \times 11) \times (15 \times 3) = 356\,400$$

$$(9 \times 11) \times (15 \times 3) \times (10 \times 8) = 356\,400$$

Ask the students "What's the same?" (the product) and "What's different?" (the order of the factors).



Achievement Objectives

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, levels 3–4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3–4)
- find a given fraction or percentage of a quantity (Number, level 4)

Activity One

In this activity, students work with whole numbers and decimals in a measurement context (mass). Addition and subtraction of decimals is suitable for students who are at or beyond the advanced additive stage of the Number Framework.

Question 1 encourages the students to think of strategies for themselves. Tipu's strategy is based on using known compatible or tidy numbers, but he could also have used strategies such as rounding or a (mental) number line. You could model Tipu's strategy for adding up his luggage and then ask who can explain why Tipu added it the way he did. Ask if anyone could think of another way Tipu could calculate the mass.

Students will often use the first strategy they hear or a strategy offered by a student who is "good at maths". The think, pair, and share approach allows for all students to be heard and have their strategies modified or justified. In this system, the students have individual thinking time when they may not talk to anyone else. They can draw diagrams, jot down ideas, and do calculations. The students then share their solution or strategy with a classmate and justify their methods.

The solutions or strategies can then be shared with the main group by:

- working jointly to present a common strategy
- having one of the pair share their partner's strategy
- sharing their own solution or strategy after justifying it to their classmate.

The best strategy for solving any problem depends on:

1. The numbers involved. Some numbers lend themselves to certain strategies more readily than others. For example, 14.29 and 9.81 can easily be added using compatible numbers knowledge or a rounding strategy. Using a number line would be much more arduous because you would have to show decimal amounts as well as whole number values.
2. The students' preferences. Students tend to use the strategy or strategies they feel comfortable with, as Tipu has with this problem. But don't just accept any strategy that the students use. You should encourage the students to look for and use the most efficient strategy for the problem. For example, a poor knowledge of basic facts may mean a student relies on doubling and halving strategies and skip-counting.

The students' strategies for adding up the mass of each cousin's luggage will vary. The tidy number is good for Tipu's and Mae's luggage ($0.02 + 0.08 = 0.1$). However, the individual items in Sali's luggage do not have convenient masses for using the tidy numbers strategy, so other advanced additive strategies will be better, such as rounding and compensation. Students who are not comfortable with decimal fractions may tend to calculate the whole number portion of the answer first and be unsure how to deal with the remaining decimal portion.

The students will then need to reassess the information provided at the start of the activity (that the airline has a 20 kilogram per person limit on baggage mass) and apply their subtraction knowledge. This is a good opportunity to discuss subtraction strategies, which could include adding on or working backwards using a number line:



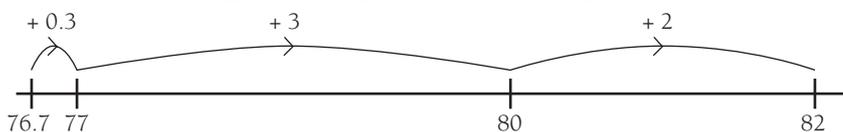
Question 3b again involves trying out different strategies. Remind the students that there is an excess charge of \$9.85 for each “part” kilogram over the limit. One strategy is given in the Answers, but encourage the students to share other methods that they come up with. Again, the group could evaluate the efficiency of different strategies.

Question 4 asks for an estimation only. Encourage the students to use rounding to help them make a quick estimation. Make sure that they understand that estimates are only that and not the full and final answer.

Activity Two

Estimation is also required in question 1. As in the earlier activity, the students could use rounding to help them estimate quickly. The mass is an estimate because the body mass is an average only. Some passengers will be lighter, and some will be heavier.

In question 2, Tipu’s and Sali’s body masses are compared to the airline’s average body mass allowance. Both weigh less than the average. The amount less can be calculated using mental strategies for tidy numbers. For example, an adding-on strategy for Sali: “If I add 0.3 to 76.7 to get 77 and then 3 to get 80 and finally 2 to get 82, then I’ve added 5.3, so Sali weighs 5.3 kilograms less than the average body mass.” Encourage the students to explain their solutions to each other, including the numbers, the units, and a “summary” of what the answer actually shows (as in the conversation above). The tidy number strategy can be represented on an empty number line. For example,



Question 3 involves percentages. The calculation of percentages requires strong multiplication strategies and is generally suitable for students who are at or beyond the advanced multiplicative stage of the Number Framework.

In question 3a, the students need to calculate the mass of a rugby player who is 50 percent above the average passenger’s body mass. Most students will recall that 50 percent equals half.

To find out, in question 3b, what percentage more 90.2 kilograms is than 82 kilograms, the students need to calculate the difference between the two amounts. This is proportional reasoning and requires the students to identify common factors. They then compare the extra mass (8.2 kilograms) to the average mass (82 kilograms). Students with a strong knowledge of place value should see that 8.2 is one-tenth of 82 and that the rugby players therefore weigh 10 percent more on average. The students could also use a calculator to find the amount as a decimal ($8.2 \div 82 = 0.1$) and then multiply this decimal by 100 to calculate the percentage ($0.1 \times 100 = 10\%$).

In question 3c, the students can use the same approach to calculate how much more 98.4 kilograms is than the average mass. Some students may see immediately that 16.4 kilograms is double the extra mass of 8.2 kilograms found in question 3b and will therefore double the percentage ($2 \times 10\% = 20\%$). You can help others to see the relationship by recording the part and whole as a composite. For example,

$$\begin{aligned} \frac{16.4}{82} &= \frac{8.2 + 8.2}{82} = \frac{8.2}{82} + \frac{8.2}{82} \\ &= 10\% + 10\% \\ &= 20\% \end{aligned}$$

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- explain satisfactory algorithms for addition, subtraction, and multiplication (Number, level 4)

Activity

This activity is an example of how you can introduce the historical aspect of mathematics to your students. It presents alternative recording methods to the traditional western multiplication algorithm.

The Egyptian algorithm works using doubles and addition. You could format the example shown in the student book as a table:

Factor (28)	Factor (52)
1	52
2	104
4	208
8	416
16	832
32	1 664

The Chinese algorithm works using multiplication and then addition. Basic multiplication facts are separated using diagonals across a grid. The diagonals are then used to add the products. Note that, in the grid, a single-digit product is preceded by a zero.

Understanding how and why these algorithms work will help develop students' number sense. The Egyptian method is based on the distributive property, which is an advanced multiplicative strategy. For example, 28 distributes into $16 + 8 + 4$.

The Chinese algorithm relies on place value. The students might like to compare this method to the "pencil and paper" algorithm, which is taught at or beyond the advanced multiplicative stage of the Number Framework. In the standard vertical algorithm for multiplication, each digit is multiplied separately and then the digits in the ones column are added together, those in the tens column are added together, and so on. The main difference is that it is formatted diagonally rather than vertically. Note that in either method, the diagonals in some expressions, for example, 84×96 or 212×36 , add up to more than 9. When that happens, the tens figure is added to the ones figure to the left. For example, adjacent diagonal totals of 5 and 13 become 6 and 3.

After the students have worked through this page, you could ask them to discuss in pairs why the two algorithms work. In order to see why the algorithms work, the students need to have sound number sense that includes understanding what multiplication is, that is, what they are doing when they multiply two numbers and how numbers can be partitioned and then recombined during a calculation.

Useful websites that explore the number systems of various cultures include:

www-history.mcs.st-andrews.ac.uk/Indexes/HistoryTopics.html

(Note that this is www-, not www.)

www.ethnomath.org/search/browse.asp?type=cultural

After the students have discussed which method they prefer and why, encourage them to think about what other strategies they could use to find the answer to an expression such as 28×52 . For example, they might use a tens strategy or tidy numbers: $20 \times 50 + 8 \times 50 + 28 \times 2$ or $30 \times 50 + 30 \times 2 - 2 \times 52$.

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, levels 3–4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3–4)

Activity

This activity involves estimation, multiplication, and division. Students at the advanced multiplicative stage of the Number Framework should be able to solve the questions in this activity mentally, while those at the advanced additive stage may need to use a calculator.

Before starting on the questions in this activity, have the students sketch a table plan for te whare kai. This should get them thinking about multiplicative strategies for finding the total number of people in question 1. For example, to find 16×9 , they could use known facts, such as 8×9 , or rounding and compensation, such as $16 \times 10 - 16$.

The students need to apply their answer for question 1 to work out the answer for question 2. They can solve question 2 using a variety of methods, including mental strategies. For example, 174×5 can be seen as half of 174×10 , applying knowledge of the tens strategy. 174×4 can be treated as $174 \times 5 - 174$ or $174 \times 2 \times 2$ (double and double again), and 174×3 can be seen as 58×9 (dividing by 3 and trebling), which in turn can be seen as $58 \times 10 - 60 + 2$. Another strategy for 174×3 is to double 174 and add 174.

In question 3, there are various ways that the students can find to estimate the number of packets needed. For example, they can multiply the 3 biscuits per person by the total number of people to give the total number of biscuits needed. This total can then be divided by 12 to give the number of packets. Alternatively, the students may see that each packet of biscuits will feed 4 people. $174 \div 4 =$ number of packets needed to feed the people at the hui. One strategy is to halve 174, then halve 87.

You may need to remind some students that you cannot buy half a packet, so the answer of 43.5 will need to be rounded up to 44 packets.

In question 4a, the students can calculate their answer in litres or millilitres. Either way, they are looking at 174×300 millilitres. You could encourage the students to estimate this first. Some students may recognise the similarities between 174×300 and 174×3 , which they had to calculate for question 2. Roimata hints at an easy way to answer question 4b. Some students may prefer Roimata's method to dividing by 2 000.

In question 5, the students need to remember that the number of people sitting at different tables varies (14, 9, and 8). They also need to refer back to the proportions of food to people given in questions 2 and 3 and multiply these proportions by the number of people at the various-sized tables.

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

In this activity, the students use number strategies in a measurement context to work out area, length, and rates. Multiplication of simple decimals is appropriate for students at the advanced multiplicative stage of the Number Framework. This stage also includes early proportional thinking.

The students' first task is to calculate the area of wasted carpet. The best method is to visualise how far the carpet would extend past the boundaries of the room and see this as 1 long rectangle and 2 squares.

The students' solutions to questions **1a** and **2a** should be expressed with both quantity and unit, for example, 2.48 m². Reinforce both the name and the symbol for the area unit (for example, metre x metre = square metre) and length unit.

Encourage the students to work out the cost of their chosen carpet without using a calculator. A useful way to calculate rates is to use ratio tables that help record or keep track of mental strategies and enhance accuracy. Ratio tables involve doubling and halving strategies, adding and subtracting, and multiplying. Partial calculations become obvious. A ratio table is similar to a double number line but keeps amounts compartmentalised. For example, for the first carpet:

0.5 m	1 m	2 m	4 m	8 m
\$57.50	\$115	\$230	\$460	\$920

3 metres of carpet at \$115 per metre is $2\text{ m} + 1\text{ m} = 230 + 115$
 $= \$345$.

Similarly, for the first wallpaper in question **2d**, once the students have worked out that they need 7 rolls:

1 roll	2 rolls	4 rolls	8 rolls
\$30.80	\$61.60	\$123.20	\$246.40

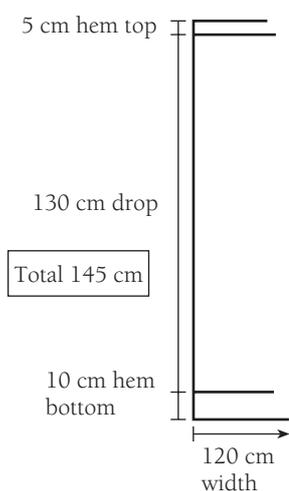
7 rolls of wallpaper at \$30.80 per roll is $8\text{ rolls} - 1\text{ roll} = \$246.40 - \$30.80$
 $= \$215.60$

However, an efficient way of multiplying by 5 is to multiply by 10 and divide by 2, so the students could multiply by 5 and add the cost of 2 rolls.

So, $\$30.80 \times 5 = \$30.80 \times 10 \div 2$
 $= \$308.00 \div 2$
 $= \$154$

$\$154 + \$61.60 = \$215.60$

Note that the chart for rolls of wallpaper needed is based on the perimeter of the room *excluding* doors and windows. (The chart also includes an allowance for wastage when patterns are matched up.)



In question 3, encourage the students to draw a diagram of a curtain for Rebecca's room to obtain the quantities of material she will need.

You may need to remind the students that 100 centimetres equals 1 metre. The material is 120 centimetres wide, so the quantity must be doubled for each window (2 curtains each 120 centimetres wide) and doubled again to account for the two windows. The drop must take into account the window size, a 5 centimetre hem at the top, and a 10 centimetre hem at the bottom: $145 \text{ centimetres} \times 2 \times 2 = 580 \text{ centimetres}$ or 5.8 metres.

You could encourage the students to try out other strategies as they solve these multiplication problems, such as experimenting with rounding, tidy numbers, and known facts. Challenge the students to explain which strategies they feel are best and why.

You could extend question 4 by asking the students to pool their information and work out what is the least or the most that redecorating Rebecca's room will cost.

Page 8: Ageing in Space

Achievement Objectives

- write and solve problems involving decimal multiplication and division (Number, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3–4)

Activity

This activity gives students some interesting facts about how our ages are calculated and relates these facts to other planets and the time they take to orbit the Sun. The activity requires rounding to 2 decimal places, applying a given formula, and manipulating that formula. It differs from other activities in the students' book in that, rather than focusing on mental strategies, it focuses on the ability to understand and work with formulae, to round to 2 decimal places, and to work with large and complex numbers. The numbers involved cannot be easily manipulated using mental strategies, so the students will need to use a calculator. However, they will need to use advanced multiplicative knowledge of number properties, especially for questions 2 and 3, in which they must choose the calculation required to solve each problem. The activity also draws on their number sense because they need to know whether the answer they get on the calculator is a reasonable one.

Question 1 asks the students to round their answers to 2 decimal places. This is a good opportunity to teach or reinforce this skill. If the third digit after the decimal point is 5 or more, the second digit after the point is rounded up. For example, 24.6571 becomes 24.66 (2 d.p.), and 18.995 becomes 19.00 (2 d.p.). (Note that this latter example is not written as 19 because the answer required 2 decimal places.) However, if the third digit after the point is less than 5, the second digit after the point is unaltered. For example, 2605.932 becomes 2605.93 (2 d.p.). To minimise error when doing a calculation, avoid rounding until you reach the final answer.

Before the students start the activity, you may want to discuss with them how the approximately 365.26 days in an Earth year are handled on our calendar. (We have a leap year [in which there are 29 days in February instead of 28] every 4 years to catch up the 4 quarter days.)

Dougal uses a formula to work out his age in planet years:

$$\frac{\text{Dougal's age} \times 365.26}{\text{Earth days for the planet to circle the Sun}} = \text{Dougal's age in the planet's years.}$$

The students use this formula for questions **1** and **2**, inserting the relevant age and planet information. Encourage the students to recognise that multiplying a decimal number by 10 (Dougal’s age) or 100 (his great grandmother’s age) is effectively the same as shifting all the numbers 1 or 2 places respectively to the left. This then takes a step out of Dougal’s formula, which makes the calculation process faster and easier.

For question **3**, in which the known information is Pulcan’s age in Pluto years, the students need to rearrange the formula so that they can calculate the unknown information (Pulcan’s age in Earth years). A student who can “see” the relationship and manipulate the formula on their own is developing the algebraic thinking needed to calculate the “unknown”.

Students who need help with this question may follow the rearrangement of the formula more easily if you show it to them in a 3-step process. The original formula with Pulcan’s age as the known information is:

$$\frac{\text{Pulcan's age in Earth years} \times 365.26}{\text{Earth days for Pluto to circle the Sun}} = \text{Pulcan's age in Pluto years (10)}.$$

Step 1:

Reverse the formula:

$$\text{Pulcan's age in Pluto years (10)} = \frac{\text{Pulcan's age in Earth years} \times 365.26}{\text{Earth days for Pluto to circle the Sun}}.$$

Step 2:

Change the division on the right-hand side to multiplication on the left:

$$\text{Pulcan's age in Pluto years} \times \text{Earth days for Pluto to circle the Sun} = \text{Pulcan's age in Earth years} \times 365.26.$$

Step 3:

Change the 365.26 from multiplication on the right-hand side to division on the left-hand side:

$$\frac{\text{Pulcan's age in Pluto years (10)} \times \text{Earth days for Pluto to circle the Sun}}{365.26} = \text{Pulcan's age in Earth years}.$$

From the table at question **1**, we know that Pluto takes 90 803.63 Earth days to circle the Sun.

So the formula is now:

$$\frac{10 \times 90\,803.63}{365.26} = \text{Pulcan's age in Earth years}.$$

So Pulcan is 2 486.00 (2 d.p.) years old in Earth years.

Page 9: Meal Deal

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3–4)

Activity

This activity is appropriate for students at the advanced multiplicative stage of the Number Framework and will give them a lot of practice using mental strategies for all the four operations in a familiar context. The students will need to understand place value across the decimal point. For example, in question **4**, which requires $\$15.00 \div 1.70$, the students should recognise that the answer must be less than 10. The students also need to recognise that the digits on either side of the decimal point are connected and that the digits to the right of the point are not whole numbers. In question **6**, for example, $5 \times \$1.70$ is not $\$5.35!$

All the questions in this activity can be calculated using part–whole mental strategies, such as tidy numbers and doubling and halving. The students could work independently and check their own work with a calculator or use the think, pair, and share strategy (see the notes for pages 2–3) where they work independently and then check their solutions with a classmate. Each pair of students needs to explain and justify their strategies to each other, then find a solution that is acceptable to both. In case they are both right or both wrong, they should share their joint solution with another pair as a check. Most mistakes are likely to occur from missing a particular item or quantity from the list rather than from incorrect mental strategies.

You could easily extend this activity by having the students think of their own questions about orders at the Fine Seafood Restaurant.

Page 10: First to the Draw

Achievement Objective

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)

Game

This game develops students' knowledge of fractions, decimals, and place value. Students at the advanced multiplicative stage of the Number Framework need to have a sound knowledge of place value before they can progress to later stages.

One way of introducing the game is to divide the class into teams of 5, with each student responsible for two of the digit cards. You call out the First to the Draw card, and the team selects the holders of the digit cards that make up the correct answer. The first team to come forward with the answer (in place value order) wins a point. You record the answer, and the cardholders can reinforce their decimal vocabulary (tenths, hundredths, thousandths) by rereading the answer aloud. For extra points, ask the teams to read the answer another way (for example, 654 hundredths can be read as 6 ones and 54 hundredths or 65 tenths and 4 hundredths) before drawing the next decimal card.

To support those students who have a less developed knowledge of decimal place value and grouping, record the number on a place value chart. For example, for 23 ones and 18 hundredths:

hundreds	tens	ones		tenths	hundredths	thousandths
	2	3	.	1	8	

Students who master the whole-class game quickly can be introduced to the individual version of it, leaving you more time to work with the students who need support with decimal place value. Be aware that addition is required to make some of the decimals in the individual game, for example, 6 tenths and 24 hundredths is 0.84.

The students can extend this game by creating their own First to the Draw cards, to which they may need to add a second set of digit cards (if they use a digit more than once).

Page 11: Sandwich Survey

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- find fractions equivalent to one given (Number, level 4)
- collect appropriate data (Statistics, level 4)

Activity

The focus of this activity is finding equivalent fractions. It is appropriate for advanced multiplicative students and those in transition to the advanced proportional stage of the Number Framework. For earlier work on equivalent fractions, see *Number*, Figure It Out, Level 3, page 9 and *Number: Book Three*, Figure It Out, Level 3, pages 22–23.

Before the students start looking at equivalent fractions, you probably need to check that they understand the meaning of a “whole”, which in this context is Claire and Nandan’s whole class. To reinforce the idea of the whole, you could work through Monday’s list of sandwich fillings with the group or class, writing each filling as a fraction of 24 and then adding the fractions verbally to get $\frac{24}{24}$. To help the students in the addition, use tidy numbers, such as complements to 10: $\frac{6}{24} + \frac{4}{24} = \frac{10}{24}$, $\frac{8}{24} + \frac{2}{24} = \frac{10}{24}$; $\frac{10}{24} + \frac{10}{24} = \frac{20}{24}$; $\frac{20}{24} + \frac{1}{24} + \frac{3}{24} = \frac{24}{24}$. When adding fractions verbally, students rarely make a mistake: “6 twenty-fourths plus 4 twenty-fourths equals 10 twenty-fourths.” It is in recording that they will try to overrun the addition and add the numerators and denominators to mistakenly give answers like $\frac{10}{48}$.

Model the simplification of fractions by initially reminding the students that equivalent fractions are other names for the same amount. For example, “What’s another name for $\frac{50}{100}$? $\frac{4}{8}$?” ($\frac{1}{2}$ is the simplest way to express these amounts.)

Look at the two speech bubbles on the page and model the simplification of $\frac{6}{24}$. Students with a good grasp of multiplication tables find it easy to generate common factors between the numerator and denominator. “Who can think of a number that is a factor for 6 and 24?” “What multiplication factors equal 6? 24?” At this stage, it is a good idea to list all the *common factors*, 2, 3, and 6, and then emphasise the usefulness of selecting the largest common factor that the two numbers have in common (the *highest common factor*).

$$\frac{6}{24} = \frac{6 \times 1}{6 \times 4} = \frac{1}{4}$$

Question **1b** is designed to reinforce the fact that only fractions with a common factor can be simplified.

Question **2** is a good opportunity to introduce students to statistical investigations or to revise this process with them. The following information can be applied to other investigations in this book, such as those suggested for the activities on pages **14**, **18**, and **22–23** of the students’ book.

Investigations should develop students’ thinking and their ability to ask questions, as well as to make and test ideas, collect data, develop conclusions, and communicate the results of their mathematical explorations. An investigation is an ideal opportunity for group work. Often students try to be too wide in their approach to conducting surveys and need assistance to define a problem, given manageable amounts of time and resources. Before they start their investigation, discuss the learning outcomes and expectations. Such a discussion can help you develop assessment criteria with the students and provide them with focus and direction. Some things to consider are content, presentation, and participation:

- Is there a clear purpose and conclusion for the mathematical investigation?
- Did the group obtain and use appropriate, reliable information?
- Is the mathematics correct?
- Is the content communicated in a clear and interesting way? (written, oral, graphs, diagrams, use of information technology)
- Is the presentation medium appropriate for the content?
- Did all the group participate?
- Did the group develop its own investigation strategy?
- Was teacher support needed?
- Was the investigation completed during the time allowed?

Not all students will have sandwiches in their lunchboxes, so a category of no sandwiches may need to be created. Also, students may have more than one kind of sandwich per lunch. These students may choose one of their sandwich fillings for the survey, or the “whole” could be changed to focus on the number of sandwich fillings instead of the number of classmates.

The sandwich-filling investigation is easily managed as a whole-class activity and can be linked to objectives in statistics that relate to collecting and displaying data as well as health objectives in food and nutrition.

Page 12: Funky Fractions

Achievement Objectives

- find a given fraction or percentage of a quantity (Number, level 4)

As extension:

- use a systematic approach to count a set of possible outcomes (Statistics, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3–4)

Game

This game is self-explanatory and relies on students having a strong knowledge of multiplication facts and the ability to apply them to division and find fractions of numbers. Only single-unit fractions are used, so the students only need to divide the whole number by the denominator. For example, $\frac{1}{3}$ of 12 is $12 \div 3 = 4$.

Early additive students will use additive part-whole mental strategies when calculating fractions of whole numbers. For example, $\frac{1}{4}$ is half of a half, and $\frac{1}{8}$ is half of a quarter, so to find $\frac{1}{4}$ of 24, “half of 24 is 12, and half of 12 is 6, so $\frac{1}{4}$ of 24 is 6”; to find $\frac{1}{8}$ of 40, “half of 40 is 20, half of 20 is 10, and half of 10 is 5, so $\frac{1}{8}$ of 40 is 5”.

Students who are advanced additive or beyond will be able to solve the fractions of whole numbers using known multiplication and division facts, for example, $\frac{1}{5} \times 30$ is $30 \div 5 = 6$.

Funky Fractions is a game of tactics because not all the whole numbers on the game board are equally divisible by the denominators shown on the dice. All 9 numbers are divisible by 2, there are 7 numbers divisible by either 3 or 6, 5 numbers divisible by 4, 2 numbers divisible by 5, and 2 by 8.

An interesting challenge for the students would be to calculate the comparative fairness of the two player tables, assuming that all dice throws are equally fair. To do this, they would need to calculate all the possible outcomes of their throws.

		Fraction on dice					
		$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$
Whole number	54	27	18	•	•	9	•
	42	21	14	•	•	7	•
	40	20	•	10	8	•	5
	36	18	12	9	•	6	•
	30	15	10	•	6	5	•
	28	14	•	7	•	•	•
	24	12	8	6	•	4	3
	18	9	6	•	•	3	•
	12	6	4	3	•	2	•

The students could then summarise this information about the factors and the dice throw outcomes and compare it to the player tables for positive outcomes. There are 32 possible positive outcomes. There is a $\frac{3}{32}$ chance of getting a 9, a $\frac{2}{32}$ or $\frac{1}{16}$ chance of getting a 4, and so on.

Positive outcomes: Player A

4: $\frac{2}{32}$	10: $\frac{2}{32}$
20: $\frac{1}{32}$	3: $\frac{3}{32}$
5: $\frac{2}{32}$	18: $\frac{2}{32}$
8: $\frac{2}{32}$	6: $\frac{5}{32}$
7: $\frac{2}{32}$	14: $\frac{2}{32}$

Positive outcomes: Player B

9: $\frac{3}{32}$	3: $\frac{3}{32}$
4: $\frac{2}{32}$	2: $\frac{1}{32}$
12: $\frac{2}{32}$	6: $\frac{5}{32}$
15: $\frac{1}{32}$	18: $\frac{2}{32}$
10: $\frac{2}{32}$	7: $\frac{2}{32}$

In terms of probability, player B is slightly more likely to win than player A, given the combination of whole numbers on the tables. This is unlikely to be an issue when the students are playing the game, but it would be an interesting exercise for the students to record the winning table for each game to determine whether some player tables are more likely to win than others, given the statistical probability of the outcomes. This would require a minimum of 30 tests. The students could then design tables of their own.

You could extend students who quickly master the game by asking them to design their own game board and player tables. This could lead them to investigate which numbers can be divided by 2, 3, 4, 5, 6, and 8 and could be linked with objectives in probability, identifying possible outcomes and assigning numerical probabilities for covering a particular number on the player's table in order to design a "fair" table.

Alternatively, the game could be played again, giving more emphasis to the fractions by using a dice that showed mixed fractions instead of unit fractions, for example, $\frac{2}{3}$. ($\frac{2}{3}$ of 24 is $24 \div 3 \times 2 = 16$.)

Page 13: Measuring Up

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- express quantities as fractions or percentages of a whole (Number, level 4)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

This activity investigates length measured in standard and non-standard units. Rounding is used to develop fractional numbers so accurate measurement is not critical. However, it's a good opportunity to discuss when precise measurements are important and how to select the "right measurement tool for the job". (See level 3, "Use Reasoned Measurement", in *The New Zealand Curriculum Exemplars: Mathematics*.) The activity is suitable for students who are advanced multiplicative or beyond.

The students need to think about sensible rounding, including rounding numerators and denominators to numbers that make the fractions easier to simplify. They will find the tidy numbers strategy useful in connection with the rounding required in question 2.

Students with a strong knowledge of multiplication tables (and common factors) will quickly simplify the fractions. Two examples where the numerator and denominator will be the same, making one whole, are:

- a person's arm span (from finger tip to finger tip) is approximately equal to their height
- the circumference of a person's fist is approximately equal to the length of their foot.

You may need to remind the students that if the numerator and the denominator are the same, the fraction equals one whole. (See also *Number: Book Three, Figure It Out, Level 3*, pages 22–23.)

The ratios in this activity (for example, wrist to neck is 1:2) are expressed as fractions, which is more appropriate for students at this level. It also provides them with practice in simplifying fractions. The students may be surprised to find out that the ratios or fractions are more or less standard for most people.

Page 14: Large or Huge?

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

This activity builds on the previous page in that the students are expressing comparisons as simplified fractions. However, this time they are dealing with larger numbers, so they also need to draw on their knowledge of place value to simplify the fractions. The focus on mass and ratios links to the measurement strand of the curriculum. This activity is suitable for students who are advanced multiplicative or beyond. Students who use additive strategies may be successful, but the process will be time-consuming and inefficient.

In question 1, the masses of pairs of different animals are compared and written as simplified fractions. The students can simplify the fractions using a variety of strategies that combine their knowledge of place value and common factors. For example, for the bison to Indian elephant (800 kg to 4 000 kg) the fraction is $\frac{800}{4\,000}$. Dividing the denominator and numerator by 100 leaves $\frac{8}{40}$. Dividing the denominator and numerator by the highest common factor (8) leaves $\frac{1}{5}$.

In question 2, the masses of the other animals are compared to that of the blue whale. The easiest way to calculate how many of each animal equals the blue whale's mass is to use a calculator, but it is good practice for the students to estimate their answers first. The basic equation is: blue whale's mass \div mass of other animal = number of other animals. For example, an estimate for $190\,000 \div 5\,000$ could be based on $200 \div 5 = 40$. (Another strategy for this is to see $\div 5$ as $\times 2 \div 10$. So the estimate could be based on $190 \times 2 \div 10 = 38$.) The exact calculation is $190\,000 \div 5\,000 = 38$ African elephants. Note that standard rounding procedures are not used for this problem. All part animals need to be counted as 1 whole animal. For example, 3.2 elephants \approx 4 elephants, and 3.8 elephants \approx 4 elephants.

The equation for question 3 is: number of students \times average mass per student = mass of class. For 30 students, this is $30 \times 35 = 1\,050$ kg, so the closest mass is the crocodile at 1 100 kg.

Investigation

See the comments on investigations in the notes for page 11.

Another statistic the students might like to compare is speed (the fastest land animal to other animals). There is also a wealth of human statistics available from the *Guinness Book of Records* and Olympic and Commonwealth Games records.

Achievement Objectives

- find fractions equivalent to one given (Number, level 4)
- express a fraction as a decimal, and vice versa (Number, level 4)
- express a decimal as a percentage, and vice versa (Number, level 4)

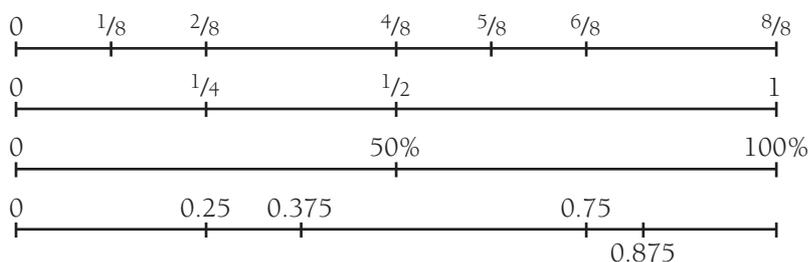
Game

This game uses and develops the students' knowledge of equivalency between fractions, decimals, and percentages. (See *Number: Book Three*, Figure It Out, Levels 3–4, page 24, which also deals with equivalent fractions, decimals, and percentages.)

You could introduce the cards (and model the vocabulary, for example, 2 tenths or 0 point 2) before the students start playing the game by dealing out all the domino cards to the class (or group). The domino cards show fraction, decimal fraction, and percentage equivalents for a half, quarters, thirds, fifths, eighths, and tenths.

Make a “multilevel” number line or table showing a range of the fractions, decimals, and percentages recorded on the dominoes. Group particular amounts together, such as halves, quarters, and eighths; fifths and tenths; then thirds separately to remind the students that thirds are recurring decimals or percentages ($\frac{1}{3} = 0.33333 \dots$, $\frac{2}{3} = 0.6666 \dots$) and are shown by a dot over the reoccurring part of the number, for example, $0.\dot{3}$.

Multilevel number line for halves, quarters, and eighths:



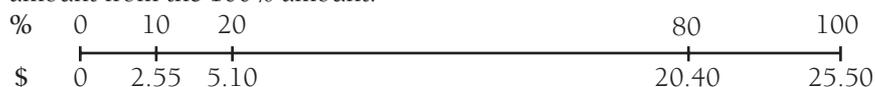
Ask the students if they have a card that matches the missing fractions, decimals, or percentages on each number line. Record the equivalent numbers, making sure that everyone has a turn. Show them how to use the calculator to divide the numerator by the denominator to calculate the decimal and to multiply the decimal value by 100 to calculate the percentage. This is particularly useful for eighths, for example, $3 \div 8 = 0.375$, $0.375 \times 100 = 37.5$ percent. (You could tell the students that percent means 1 part in every hundred and link this to the number of cents in a dollar. You could also discuss with students that they can easily multiply by 100 mentally by moving the numbers 2 places to the left.)

Many students are able to instantly recall equivalent fractions, decimals, and percentages for particular amounts such as halves and quarters. This recall can be supported by using doubling and halving strategies to develop other equivalents. For example, “I know $\frac{1}{4}$ is 25 percent, so $\frac{3}{4}$ is $25 + 25 + 25 = 75$ percent” or “I remember $\frac{1}{8}$ is 0.125, so I just need to double it and double that again and add one more 0.125 to get $\frac{5}{8} = 0.625$.”

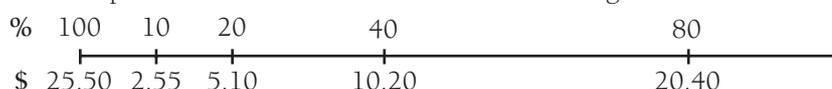
Reinforce the students' knowledge of place value when dealing with the equivalency of 10 percent and its multiples (20, 30, 40 ...). One-tenth = $\frac{1}{10}$ or 0.1, which is $\frac{10}{100} = 10$ percent. Double this for two-tenths: $\frac{2}{10} = 0.2$, which is $\frac{20}{100} = 20$ percent, and so on.

Remind your students of the relationship between tenths and fifths to support conversion between these fractions. For example, “I know $\frac{1}{5}$ is $\frac{2}{10}$, so $\frac{3}{5}$ is $\frac{6}{10}$, which is 60 percent.”

Double number lines based on percentages and money amounts are a good scaffolding tool for advanced multiplicative students and those in transition to advanced proportional. For example, in the double number line below, the students can find the value of 80% by subtracting the 20% amount from the 100% amount:



Another possible double number line involves finding 10% and then doubling:



After class or group discussion, redistribute the cards and play the game with the whole group, reinforcing the discussion points. Once the students have mastered the rules, send groups away to play the game independently. Keep playing the game yourself with those students who need more support.

Page 16: Making Money

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)

Activity

The main focus of this activity is finding percentages and applying these in a practical context. Although some questions involve adding and subtracting money amounts, students will need to be at or beyond the advanced multiplicative stage of the Number Framework and learning to apply their multiplicative strategies to percentage problems.

In question 1, the percentage involved is 50 percent (%). This is easy to calculate because 50% is equivalent to half. Although the students should be able to use mental strategies to answer these questions, you could use this page to teach them how to use the percentage button on a calculator. To do this, they enter the manufacturing cost, press the multiply button, then $\boxed{5}$ and $\boxed{0}$, then the percentage button. There are various ways that the students could use a calculator without a percentage button: they could enter the manufacturing cost and then multiply by 50 and divide by 100; divide the manufacturing cost by 2 to get a half (because 50% is a half); or multiply the manufacturing cost by 0.5 (which is also 50% or a half).

Question 2 can be done by addition: for each item, the cost to the clothing shop (the wholesale price) equals the manufacturing cost plus the manufacturer's 50% profit.

To answer question 3, the students need to refer back to their answers for question 2. They can use the same strategy or calculation as they did for question 1, but they need to add the 50% in each case to the price paid by the clothing shop.

If the students are using calculators for questions 2 and 3, they can calculate the price that the retailer pays for the sweatshirt by doing two calculations:

1. Find 50% of the manufacturing cost
2. Add this 50% amount to the manufacturing cost.

Discuss with the students a quicker way to do this. They are adding half or 50% of the manufacturing cost to get the cost to the retailer, so the cost to the retailer is $1\frac{1}{2}$ times the manufacturing cost.

This cost can be quickly calculated in one calculation: the manufacturing cost $\times 1.5$ (which is $1\frac{1}{2}$) or times 150%. Students who use an efficient calculation like this show that they have the number sense to understand what they are doing when they add a certain percentage.

A mental strategy for question **4a** is to calculate $\frac{1}{10}$ of the customer cost ($\frac{1}{10}$ of \$57.38 is \$5.738), then double that to get $\frac{2}{10}$ or 20% (\$11.476 \approx \$11.48). Remind the students to round to the nearest cent at the end of their calculations to minimise inaccuracies.

The students may notice that the 20% discount is more than half the shop's original profit. Encourage them to understand that a smaller percentage of a larger amount can be more than a larger percentage of a smaller amount.

A systematic way for the students to keep track of their calculations for this activity is to record the information in a table.

Type of sweatshirt	Manufacturing cost	Company profit	Clothing shop (wholesale)	Clothing shop (profit)	Retail price	20% discount	Sale price	Sale profit
Three buttons	\$25.50	\$12.75	\$38.25	\$19.13	\$57.38	\$11.48	\$45.90	\$7.65
Short sleeves	\$15.80	\$7.90	\$23.70	\$11.85	\$35.55	\$7.11	\$28.44	\$4.74
Narrow cuff	\$22.90	\$11.45	\$34.35	\$17.18	\$51.53	\$10.31	\$41.22	\$6.87
Hooded style	\$35	\$17.50	\$52.50	\$26.25	\$78.75	\$15.75	\$63.00	\$10.50
Two transfers	\$29.40	\$14.70	\$44.10	\$22.05	\$66.15	\$13.23	\$52.92	\$8.82
Basic style	\$17.50	\$8.75	\$26.25	\$13.13	\$39.38	\$7.88	\$31.50	\$5.25

Question **4b i** involves subtracting the original cost price of the items from their sale price (after the 20% discount) to find the shop's sale profit on each type of sweatshirt. The students then multiply this amount to take into account the total number of individual items sold (5 three buttons, 2 short sleeves, and 1 basic style) and add the products together to find the shop's total profit for the morning.

Question **4b ii** involves looking back at the original non-sale profit for the three types of sweatshirts and subtracting the total sale profit found at **4b i** from the total original or pre-sale profit.

During the discussion in **4c**, encourage the students to justify their reasoning with examples and to consider what profit margin a business would need to cover wages, electricity, rent, and so on and whether it's acceptable in some circumstances to sell products below cost in order to reduce losses. (Or should a garment be left on the shelf in the hope it will sell eventually?)

Page 17: Bargain Busters

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, levels 3–4)
- find a given fraction or percentage of a quantity (Number, level 4)

Activity

Students who attempt this activity should be advanced multiplicative or beyond. The activity involves finding percentages of money amounts, but estimation also plays a key part. There are several mental strategies that use the students' knowledge of unit fractions, place value, compensation from tidy numbers, and equivalent fractions.

Some possible strategies for question **1** are given in the Answers.

Question **2a** requires the students to add up the items in each group, estimate the discount for each group, and then calculate the total discount for all the food. The students need to remember that some items are listed together, for example, 3 bottles for \$1.99 each, and this must be taken into account when they are adding up all the items.

In question 3, the students can again use a variety of strategies to estimate whether each advertisement is correct. For example, in question 3a, they could find a quarter of \$12.50 (halve then halve again) or divide by 4 or find 10 percent, double it, then add half of 10 percent. In question 3b, they could find 10 percent and double it.

The students need to discuss what is reasonable in terms of rounding to decide whether the advertising in question 3b is correct or not. They will have a variety of answers that need to be justified mathematically.

Page 18: How Slow Can You Go?

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3–4)
- perform measurement tasks, using a range of units and scales (Measurement, level 3)

Activity

Rate problems such as those in this activity are difficult to model with concrete materials, so students who attempt this activity should be strongly multiplicative and at least at the advanced multiplicative stage of the Number Framework.

This activity provides basic scientific information on distance, time, and speed and requires students to use this information in their calculations. The use of ratio tables can enhance the students' understanding of the relationship between time, distance, and speed. The formulae that they need to use are: distance = speed \times time, speed = distance \div time, or time = distance \div speed.

In question 1a: distance = 5 metres per minute \times 28 minutes
= 140 metres

In 1b: time = 62 metres \div 5 metres per minute
= 12.4 minutes (or 12 minutes 24 seconds)

In question 1c, the speed is converted from metres per minute to metres per hour by multiplying by 60 (because there are 60 minutes in 1 hour): 2 metres per minute \times 60 = 120 metres per hour.

Question 2 is ideal for using ratio tables (or double number lines):

15 min	30 min	1 hr	2 hr
3.75 metres	7.5 metres	15 metres	30 metres

So, in 2a: 15 metres + 30 metres = 45 metres, so 1 hour + 2 hours = 3 hours.

In question 3a, the students need to remember that 10 minutes is $\frac{1}{6}$ of an hour because the speed is in kilometres per hour.

Distance = speed \times time
= 8 kilometres per hour \times 10 \div 60 hours (or 8 \div 6)
= $1\frac{1}{3}$ kilometres

In 3b: time = distance \div speed
= 0.1 kilometres \div 8 kilometres per hour
= 0.0125 hours
= 0.0125 \times 60 minutes
= 0.75 minutes \times 60 seconds
= 45 seconds

Investigation

The investigation could be linked with the investigation on page 14 of the students' book. See also the comments on investigations in the notes for page 11.

Page 19: Paddling down the Waikato

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3–4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3–4)

Activity

This activity includes questions on length, time, and rates. It builds on the previous page in that it uses number sense and number calculations to make calculations involving rate. Students who are advanced additive could answer the rate questions using strategies such as repeated addition. In this case, such strategies will work, but they will be inefficient. Students who are advanced multiplicative and beyond should be using multiplication strategies for most of the questions.

Most students will assume for question 1 that the paddlers will try and avoid all the taniwha. Each bend, whether paddling or dragging, is assumed to be 500 metres, and the total journey is 55 kilometres. The total time travelled = number of 500 metres walked \times 15 minutes + kilometres paddled \times 12 minutes.

If Wiremu, Tāmoko, and Ngāhuia drag the waka overland for 10 bends, they will walk 10×500 metres (5 kilometres). It takes 15 minutes to drag the waka over each 500 metres: 10×15 minutes = 2 hours 30 minutes. This leaves 50 kilometres ($55 - 5 = 50$) to paddle.

Total time = 2 hours 30 minutes + 50×12 minutes
= 2 hours 30 minutes + 10 hours
= 12 hours 30 minutes

In question 2, the students take into account the effect of the 5 friendly taniwha, but they can use similar reasoning to that for question 1.

You could use the information on the page to explore various scenarios of walking versus paddling. Encourage the students to justify their answers with mathematical information. For example, "We made them walk 5 bends. That's $2\frac{1}{2}$ kilometres, so that would take 1 hour 15 minutes. That leaves 52.5 kilometres to paddle at the rate of 12 minutes per kilometre, so that would take 10 hours 30 minutes. So the total time would be 11 hours and 45 minutes."

You could give the students a blank version of this table to fill in:

Bends walked	0	1	2	3	4	5	6	7	8	9	10
Distance walked (km)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Drag time (min)	0	15	30	45	60	75	90	105	120	135	150
Distance paddled (km)	55	54.5	54	53.5	53	52.5	52	51.5	51	50.5	50
Paddle time (hrs/min)	11 hrs	10 hrs 54 min	10 hrs 48 min	10 hrs 42 min	10 hrs 36 min	10 hrs 30 min	10 hrs 24 min	10 hrs 18 min	10 hrs 12 min	10 hrs 6 min	10 hrs
Total time (hrs/min)	11 hrs	11 hrs 9 min	11 hrs 18 min	11 hrs 27 min	11 hrs 36 min	11 hrs 45 min	11 hrs 54 min	12 hrs 3 min	12 hrs 12 min	12 hrs 21 min	12 hrs 30 min

The table allows the students to see the relationships between the times and distances. It can also be used to develop an algebraic formula that describes the relationship between the distance walked and the time taken and the distance paddled and the time taken (using kilometres and minutes). For example, when comparing the distance walked and drag time: “What do you multiply 1 by to get 30? 2 by to get 60? 2.5 by to get 75?” (The students should be able to identify the factor of 30.)

When comparing the distance paddled and the paddle time, ask: “What do you multiply 55 by to get 660?” (12) “Does this factor work for all the relationships?” This can be quickly tested with a calculator by multiplying all the distances paddled by 12. (This will establish the relationship if the students are struggling with the mental calculations.)

Total time in minutes = $30 \times \text{distance walked (kilometres)} + 12 \times \text{distance paddled (kilometres)}$.

Pages 20–21: The Fish Hooks of Ngāke

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3–4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3–4)
- solve problems of the type $\square + 15 = 39$ (Algebra, level 3)

Activity

For this activity, which involves number properties, students should be at least advanced additive. It is most suitable for those who are advanced multiplicative.

The relationship between the waka, men, lines, hooks, and fish in questions 1 and 2 is multiplicative: $\text{waka} \times \text{men} \times \text{lines} \times \text{hooks} = \text{fish}$. Encourage the students who cannot establish an abstract relationship to draw a diagram.

The multiplicative relationship for each part of question 1 is:

1a: $3 \times 10 \times 3 \times 3$

1b: $4 \times 15 \times 2 \times 3$

1c: $5 \times 16 \times 3 \times 4$

Question 2 is phrased differently (as the students are not told the number of men), but the multiplicative relationship is the same: $5 \times 6 \times 3 \times 4$.

In question 3, the students need to establish a relationship between the different kinds of fish in comparison to kūmara. They may use a variety of strategies to do this. In the process, they will develop algebraic understanding. (See also *Algebra*, Figure It Out, Level 3, page 21.)

The students could start by making links between the blue fish and the other types of fish and then adjust the blue fish and kūmara equation. For example, 3 blue fish equal 2 green fish, which in turn equal 4 yellow fish, which equal 12 red fish. The diagram shows that 2 blue fish = 4 kūmara, so therefore 3 blue fish = 6 kūmara. So now the basic equation can be:

6 kūmara = 3 blue fish = 2 green fish = 4 yellow fish = 12 red fish. From this, the students can use division to find the number of kūmara for each type of fish:

6 kūmara \div 3 = 3 blue fish \div 3, so 2 kūmara = 1 blue fish;

6 kūmara \div 2 = 2 green fish \div 2, so 3 kūmara = 1 green fish;

6 kūmara \div 4 = 4 yellow fish \div 4, so $6 \div 4$ (or $1\frac{1}{2}$) kūmara = 1 yellow fish;

6 kūmara \div 12 = 12 red fish \div 12, so $6 \div 12$ (or $\frac{1}{2}$) kūmara = 1 red fish.

Another way is for students to solve question **3a** and then use this information to solve **3d**, then **3b**, then **3c**. Students who are having difficulty could model the situation with coloured counters.

Pages 22–23: Focusing on Water

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

Activity One

Students doing this activity should be advanced additive or in transition to advanced multiplicative. The use of capacity and rates reinforces a variety of mental multiplicative part-whole strategies. You could use the think, pair, and share model (see the notes for pages **2–3**) to get the students to share their strategies and also to discuss why some methods are easier to use than others. See the Answers for some suggested strategies. Other appropriate strategies include tidy numbers (for example, for question **2c**, $19 \times 11 = 20 \times 11 - 11$), standard place value and partitioning (for example, for **2f**, $15 \times 30 = 10 \times 30 + 5 \times 30$), and doubling and halving (for example, for **2g**, $4 \times 14 = 8 \times 7$).

Some of the questions require the students to use their answers from earlier questions, so encourage them to check their answers as they go along to avoid getting an incorrect answer because of an earlier miscalculation. Also, in question **4d**, the students need to remember that they are dealing with 4 people (the family consists of 5 people, including Ryan, who has already been dealt with in question **4c**).

Investigation

This investigation has strong links to environmental education and science strands as well as to statistics. See also the comments on investigations in the notes for page **11**.

The students will need to select appropriate units and equipment for measuring. The frequency and size of the drips will vary between groups, but it is assumed that, for a given tap, the drip rate will stay the same. More extensive ideas for focusing on water investigations are available in the following resources and websites:

M^cCabe, Elizabeth (1998). “Testing the North River”. *Connected 2* 1998.

Building Science Concepts Books 1 and 15.

www.rsnz.govt.nz/education/nwp/content/index.htm

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3–4)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

This activity relies on advanced multiplicative strategies and involves estimation, rounding, addition, and multiplication, some with decimals. The swimming context involves length, time, and rates.

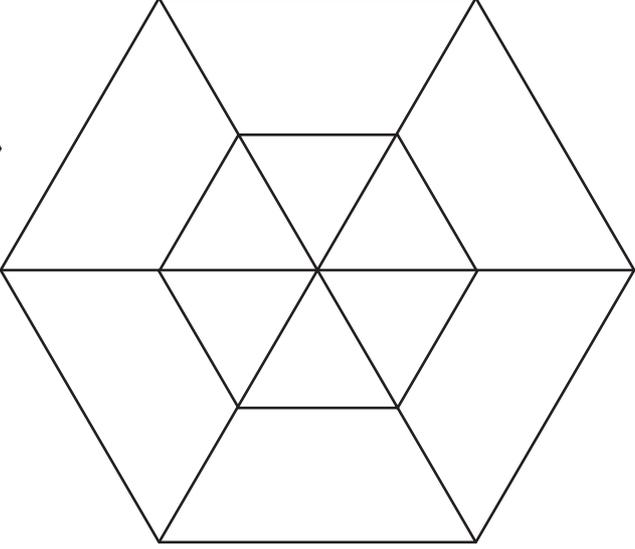
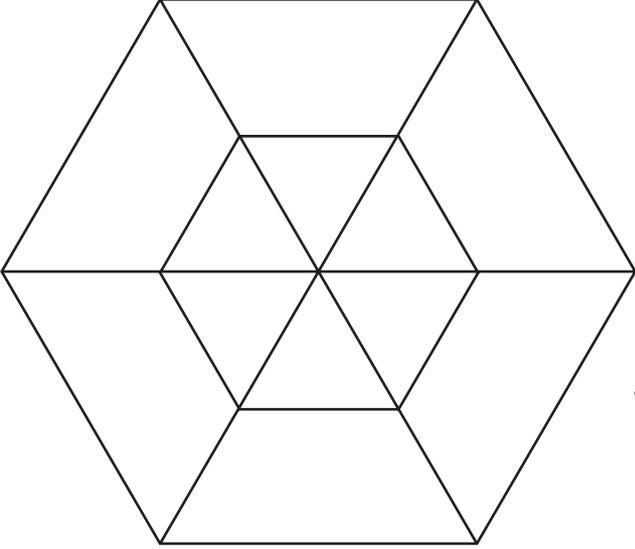
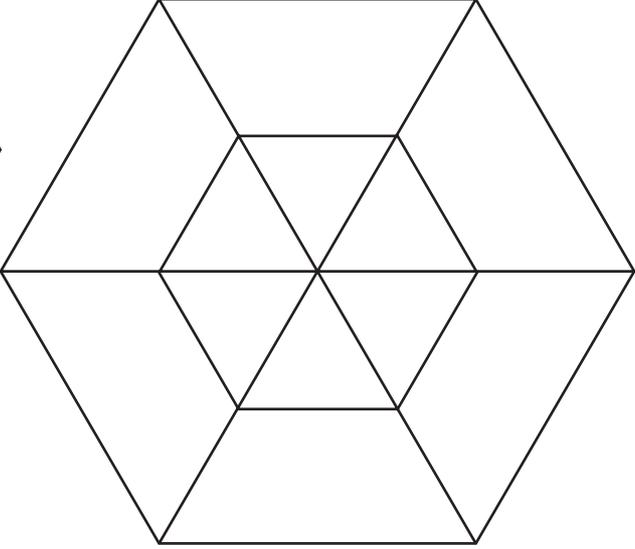
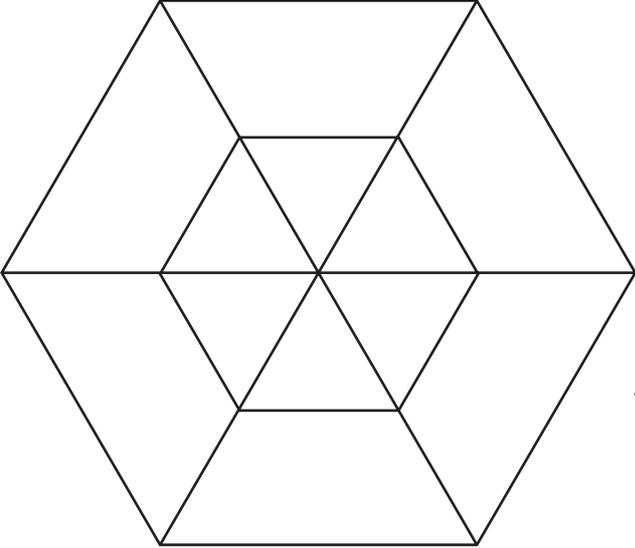
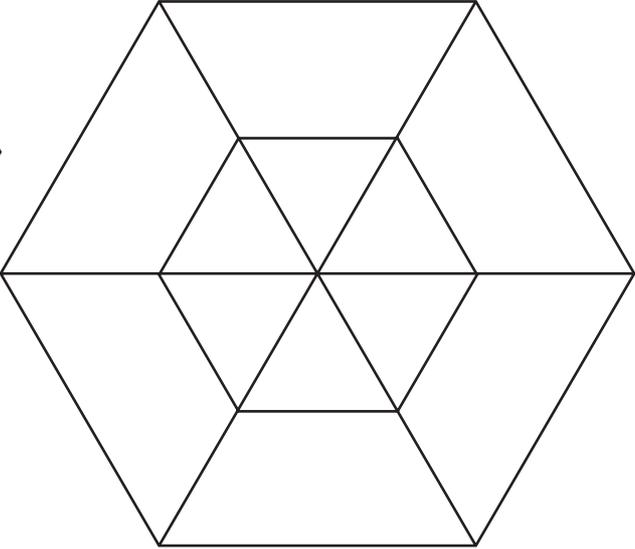
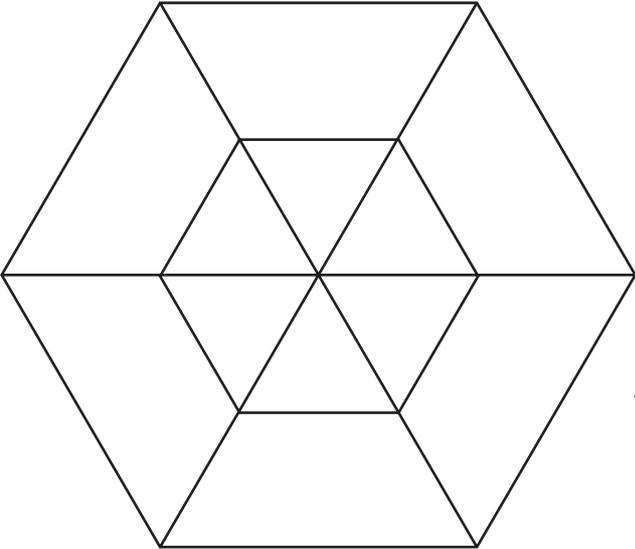
In question 1, the students are asked to estimate. To discourage them from precisely adding the times, have them share their strategies. These could include rounding and compensation, looking for the nearest multiple of 10, and looking for compatible numbers. You may have to remind them that there are 60 seconds in a minute (not 100), so when the seconds total more than 60, the next minute starts. (This also applies to question 3.)

In question 3, Findlay swims 100 metres backstroke in 1 minute and 15 seconds. A kilometre is 10 times greater than 100 metres, so the time should also be multiplied by 10: this can be seen as either 750 seconds or 10 minutes and 150 seconds, both of which amount to $12\frac{1}{2}$ minutes.

In question 4, the students need to select the fastest swimmer for each stroke, remembering that each swimmer can swim only once. (This is not stated, but is generally the case.)

Question 5 involves standard rounding procedures to the nearest second (0.5 or greater “rounded up”).

Copymaster: Mystery Hexagons



Copymaster: First to the Draw

3 ones,
8 hundredths, and
9 thousandths

3.089

4 tenths and
2 hundredths

0.42

318 hundredths

3.18

271 tenths

27.1

97 thousandths

0.097

6 ones and
83 thousandths

6.083

6 tenths and
24 hundredths

0.84

3 tenths and
76 hundredths

1.06

2 ones and
61 thousandths

2.061

9 ones and
345 thousandths

9.345

28 tenths and
3 hundredths

2.83

36 tenths and
9 hundredths

3.69

654 hundredths	3 ones and 5 tenths	25 thousandths	48 tenths
6.54	3.5	0.025	4.8
23 ones and 18 hundredths	89 tenths	34 thousandths	81 thousandths
23.18	8.9	0.034	0.081
7 tens, 4 hundredths, and 8 thousandths	81 ones, 9 hundredths, and 3 thousandths	16 hundredths and 4 thousandths	54 hundredths and 8 thousandths
70.048	81.093	0.164	0.548

Copymaster: Fraction Distraction

Cut along the dotted lines.

$\frac{1}{4}$	25%	0.5	one-half
20%	0.2	$\frac{2}{5}$	40%
0.25	$\frac{1}{2}$	50%	0.75
one-fifth	0.4	two-fifths	$\frac{2}{3}$
two-thirds	0.125	one-eighth	60%
0.7	$\frac{3}{8}$	0.375	70%
$12\frac{1}{2}\%$	$\frac{3}{5}$	three-fifths	90%

Cut along the dotted lines.

75%	$\frac{3}{4}$	$33\frac{1}{3}\%$	one-third
$\frac{1}{10}$	10%	$12\frac{1}{2}\%$	$\frac{1}{8}$
three-quarters	$\frac{1}{3}$	$0.\dot{3}$	$\frac{1}{5}$
$66\frac{2}{3}\%$	0.1	one-tenth	$0.\dot{6}$
0.6	$\frac{3}{10}$	30%	$\frac{7}{10}$
seven-tenths	$37\frac{1}{2}\%$	three-eighths	$\frac{1}{8}$
$\frac{9}{10}$	0.8	80%	one-quarter

Player A	
4	10
20	3
5	18
8	6
7	14

Player B	
9	3
4	2
12	6
13	18
10	7

Player A	
4	10
20	3
5	18
8	6
7	14

Player B	
9	3
4	2
12	6
13	18
10	7

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