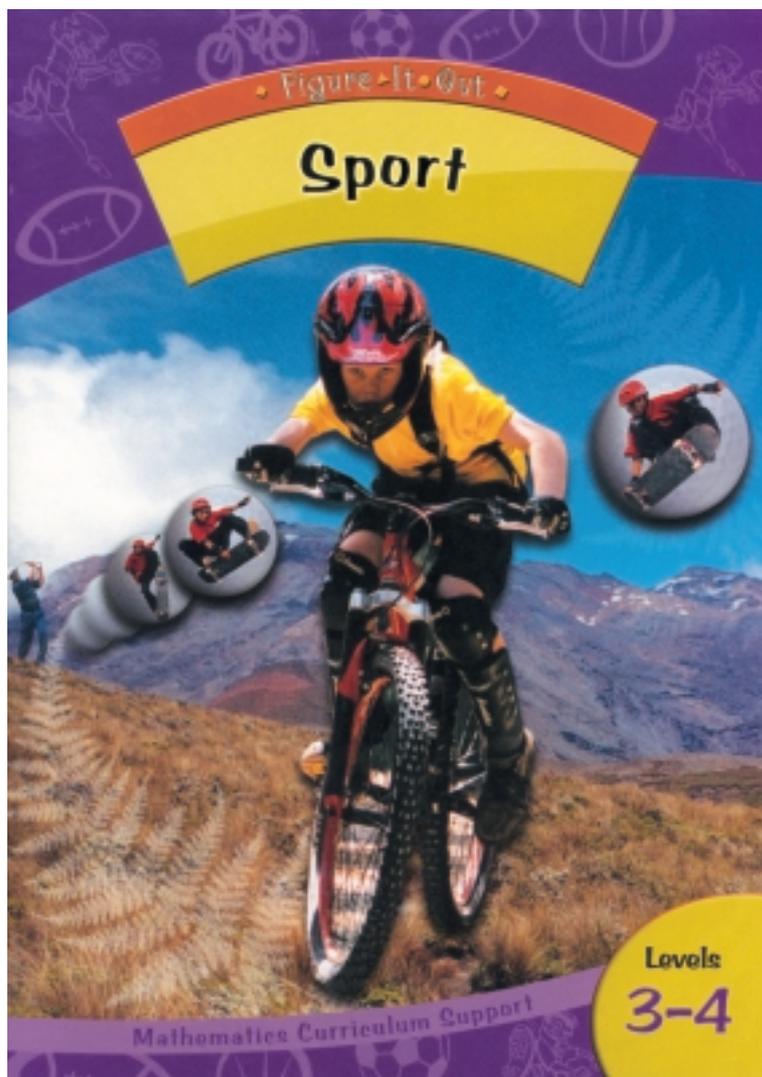


Answers and Teachers' Notes



Contents

Introduction	2
Answers	3
Teachers' Notes	11
Copymasters	29

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum*. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students. The teachers' notes are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

There are eight booklets for levels 3–4: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (levels 3–4) are suitable for most students in year 6. However, teachers can decide whether to use the booklets with older or younger students who are also working at levels 3–4.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.

Mathematics in the New Zealand Curriculum, page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

Sport

Answers

Page 1: Can't Catch Me!

Activity

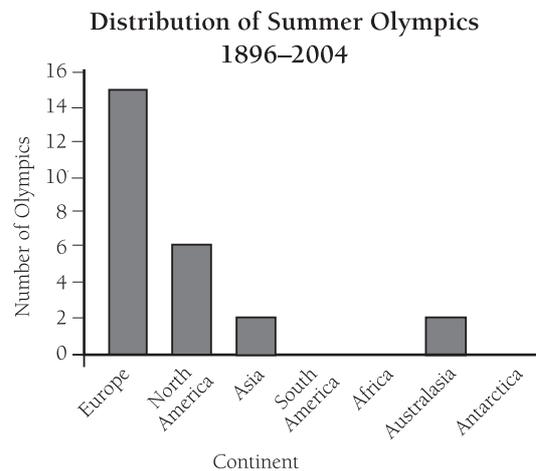
1.
 - a.–c. Practical activities. Teacher to check
 - d. If actual times do not match estimates, it may be because you did not take into account that the longer the distance, the slower most people run. Also, if you are not used to running, you may be tired after the first run, and this may affect your speed on the next run.
2.
 - a.–b. Teacher to check. Graphs and running times will vary, but the slope of the graphs should get slightly steeper for the longer distances.
3. Research activity. Patterns may include: the longer the distance, the slower the speed; men tend to run faster than women.

Pages 2–3: Olympic Locations

Activity

1.
 - a. Both Summer and Winter Olympics used to be held in the same year, at 4-yearly intervals. The pattern changes after 1992, with the summer and winter games held alternatively every second year. Both games are still held every 4 years, as before. Reasons for this could include spreading out the cost of travel for people going to watch both games and spreading out the costs for countries supporting their sportspeople financially.
 - b. Because it was 100 years since the first Olympics, which Greece hosted

2.
 - a. A possible graph is:

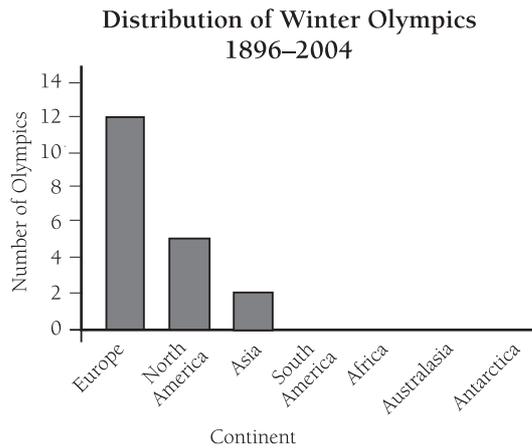


A possible table could look like this:

Continents	Number of Summer Olympics
Africa	0
Antarctica	0
Asia	2
Australasia	2
North America	6
South America	0
Europe	15

- b. Europe has hosted the most Summer Olympics (14).
 - c. Africa, South America, and Antarctica have hosted the least (0).
 - d. Answers will vary. Europe has hosted the summer games more than any other continent, partly because the Olympic revival movement started in Europe but also because Europe contains many countries with sufficient wealth to host the games. This would include being able to pay for the security required for athletes and important visitors. Antarctica is not suitable for any games, and most places in Africa and South America probably lack the funding required.

3. a. A possible graph is:



A possible table could look like this:

Continents	Number of Winter Olympics
Africa	0
Antarctica	0
Asia	2
Australasia	0
North America	5
South America	0
Europe	12

- b. Europe has hosted the most Winter Olympics (12).
- c. Africa, South America, Australasia, and Antarctica have hosted the least (0).
- d. Answers will vary. The events held in the Winter Olympics are restricted to countries that have cold temperatures or mountainous areas. These events also require expensive equipment and are therefore most popular in wealthy countries.

You may have worked out the points like this: The Greens had one draw and 1 point. The points system is unlikely to involve half points, so they must have had no wins and two losses. A draw is therefore worth 1 point. A win must be worth more than a draw. The Reds had one win and a total of 2 points, so it seems logical to assume that a win is worth 2 points. (It also means that the Reds must have had no draws and two losses.)

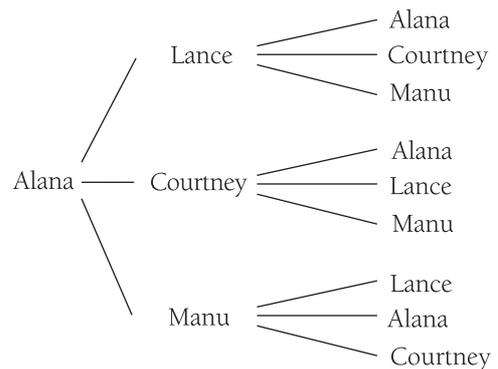
For every loss by a team, there must be a corresponding win. The Reds and Greens had a total of four losses between them, but only two wins by the Reds and Blues are recorded. The Greens had no wins, as shown by their one-point total. This means that the Yellows must have won two games. The total number of draws must be even because two teams met each time. The wins and losses account for four games, so the draws must account for the other two games.

The Blues had two draws and the Greens one, a total of three. Therefore, the Yellows' other game must have been a draw. With two wins and one draw, the Yellows have a total of 5 points.

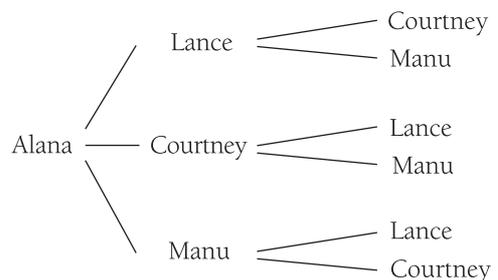
Page 5: Pass It On!

Activity

1. a. 3 possible passes (to Lance, Courtney, or Manu)
- b. 9, as shown by this tree diagram:



- c. If you can't pass the ball back, there are only six possible second passes:



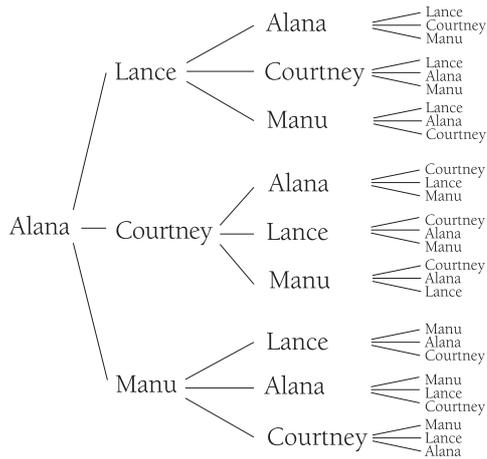
Page 4: Luck of the Draw

Activity

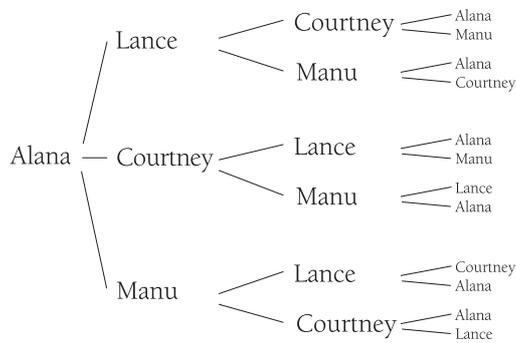
1. Reds vs Blues, Reds vs Greens; Yellows vs Blues, Yellows vs Greens
2. a. The Yellows, with 5 points
- b. Answers will vary, but the completed table should look like this (the numbers in bold were already on the table):

	Wins	Draws	Losses	Points
Reds	1	0	2	2
Yellows	2	1	0	5
Blues	1	2	0	4
Greens	0	1	2	1

2. a. 27 ways:



b. There would only be 12 ways:



3. a. 5

b. 20

4. a. 6

b. 42

c. 56

Pages 6-7: Great Golf

Activity

1. a. A positive score, such as +2, shows how many shots more than the par for the hole the player had. A negative score, such as -1, shows how many shots less than par the player had.
- b. The usual notation is 0 (neither over nor under par).

2. a.

Hole	Par	Adam		Susan	
		Shots	Par score	Shots	Par score
1	3	2	-1	5	+2
2	4	6	+2	4	0
3	5	4	-1	7	+2
4	5	6	+1	5	0
5	4	7	+3	4	0
6	3				
7	5				
8	4				
9	3				
Subtotal	36				
10					
11					
12					
13					
14					
15					
16					
17					
18					
Subtotal					
Total					

Both players are equal after five holes (25 shots each).

b. Both players are 4 shots over par (+4).

3.

Hole	Par	Adam		Susan	
		Shots	Par score	Shots	Par score
1	3	2	-1	5	+2
2	4	6	+2	4	0
3	5	4	-1	7	+2
4	5	6	+1	5	0
5	4	7	+3	4	0
6	3	3	0	2	-1
7	5	5	0	8	+3
8	4	4	0	4	0
9	3	5	+2	2	-1
Subtotal	36	42		41	
10					
11					
12					
13					
14					
15					
16					
17					
18					
Subtotal					
Total					

Susan is winning by one shot.

4. a. What hole each par is on will vary. The total par for these nine holes is 36.
- b. Scorecards will vary, but the second nine should add up to 45 for Adam and 43 for Susan. A possible scorecard is:

Hole	Par	Adam		Susan	
		Shots	Par score	Shots	Par score
1	3	2	-1	5	+2
2	4	6	+2	4	0
3	5	4	-1	7	+2
4	5	6	+1	5	0
5	4	7	+3	4	0
6	3	3	0	1	-2
7	5	5	0	8	+3
8	4	4	0	5	+1
9	3	5	+2	2	-1
Subtotal	36	42		41	
10	4	5	+1	4	0
11	5	8	+3	7	+2
12	4	3	-1	4	0
13	3	4	+1	5	+2
14	4	5	+1	6	+2
15	4	5	+1	3	-1
16	5	6	+1	5	0
17	3	4	+1	5	+2
18	4	5	+1	4	0
Subtotal	36	45		43	
Total	72	87	+15	84	+12

- c. Adam is +15 and Susan is +12. Susan has won the game.
5. a. Discussion will vary, but their scores would be likely to be under par because they would probably not have the high scores on some holes that Adam and Susan had.
- b. Practical activity

Page 8: Keeping in Shape

Activity One

1. a. There are 20 hexagonal faces and 12 pentagonal faces.
- b. There are 60 vertices and 90 edges.
2. Practical activity. Teacher to check

Activity Two

1. It becomes a hexagon.
2. 20

Activity Three

1. Octagons and triangles
2. 36 edges and 24 vertices

Page 9: Offside!

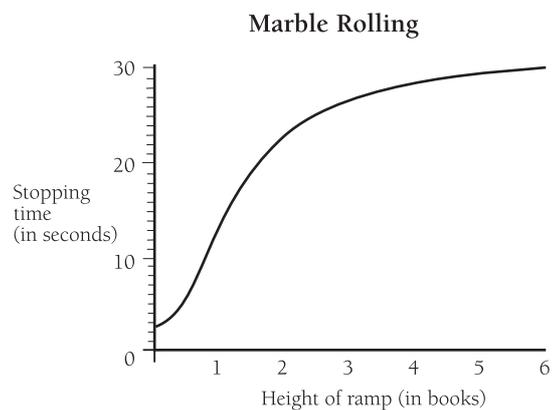
Activity

1. a. approximately 10 m²
- b. 150 m²
- c. approximately 140 m²
- d. 150 m²
2. a. 33% (rounded)
- b. 67% (rounded)
- c. 64% (rounded)
- d. 96% (rounded)

Pages 10–11: On the Slippery Slope

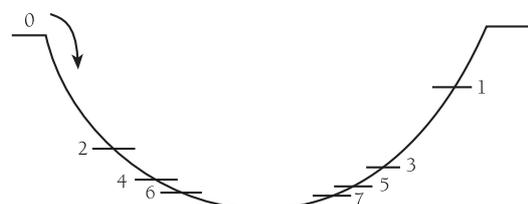
Investigation

The points on the graph for Quinten's first investigation should follow an "S" curve, like this:



Quinten's other investigations:

- a. The mark a marble rolls up to on each side follows a pattern like this:



With each roll of the marble, the distance between the points gets smaller and smaller.

- b. The steeper the ramp becomes, the faster the marble rolls, and therefore the further it rolls after leaving the ramp. However, as the ramp gets steeper, friction affects the rolling distance, and the gain in distance becomes less and less each time the ramp is made steeper.

Page 12: Roundabout Rugby

Activity

1. 15 games
2. a. A suggested timetable could be:

9.00–9.30	A vs B	C vs D
9.40–10.10	A vs E	B vs F
10.20–10.50	E vs C	D vs F
11.00–11.30	A vs D	B vs C
11.40–12.10	E vs F	
12.10–12.40	Lunch	
12.40–1.10	F vs C	B vs E
1.20–1.50	A vs C	B vs D
2.00–2.30	A vs F	D vs E

Many other draws are possible.

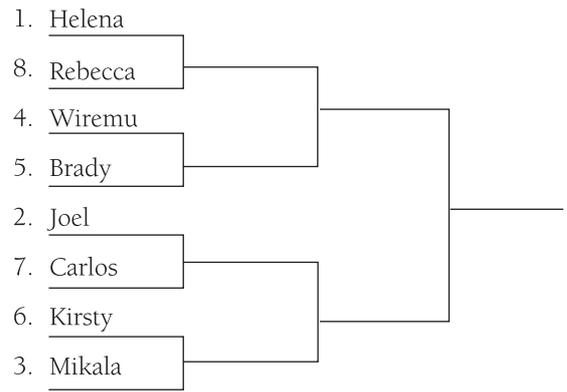
- b. This would take a total of 5 hours 30 minutes ($5\frac{1}{2}$ hours), the minimum possible time.

Page 13: It's a Knockout

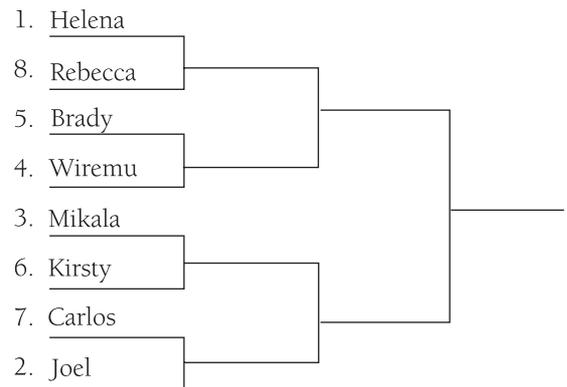
Activity

1. a. Alistair: 4th
Joanna: 2nd or 3rd
Steven: 2nd or 3rd
- b. The top-ranked player usually plays the lowest ranked player in the first round. The position of Miriama on the draw suggests that Alistair is the 4th seed, and therefore Joanna and Steven must be the 2nd and 3rd seeds.
- c. 3 games

2. Two possible draws are:



or



Pages 14–15: Which Way Now?

1. a.

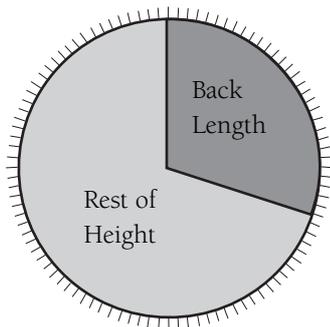
Compass direction	Landmark
60 m SE	edge of lake
35 m NE	top of Giraffe Hill
70 m S	stream
25 m SW	bench
40 m SE	top of Hippo Hill
60 m SW	swamp
75 m NW	forest
40 m N	stream
20 m NW	oak tree

- b. Answers will vary.

2. Practical activity

Activity

1.
 - a. Practical activity. Answers will vary. Teacher to check
 - b. Graphs will vary. In this case, a pie graph or a strip graph are good for showing approximate proportions as an overall trend. For example, a pie graph of the average measurements of a group of classmates on a 100 circle could look like this:



This could be shown more easily as a strip graph:



2. The *mode* is the most common height. In this case, there are a small number of measurements with no most common measurement, so the use of a mode is inappropriate. The *median* is the middle number when they are placed in numerical order:
 1.75 1.80 1.91 1.92 1.93 2.00 2.10 2.11 2.15
 The median in this data is 1.93 m.
 The *mean* is found by totalling the measurements and dividing this total by the number of measurements:
 $1.75 + 1.80 + 1.91 + 1.92 + 1.93 + 2.00 + 2.10 + 2.11 + 2.15 = 17.67$
 $17.67 \div 9 = 1.96 \text{ m}$
 The median and mean are reasonably close, so they both give an indication of where the middle is. (You may decide to make the shirts and tops a little longer so that they fit all the team members.)

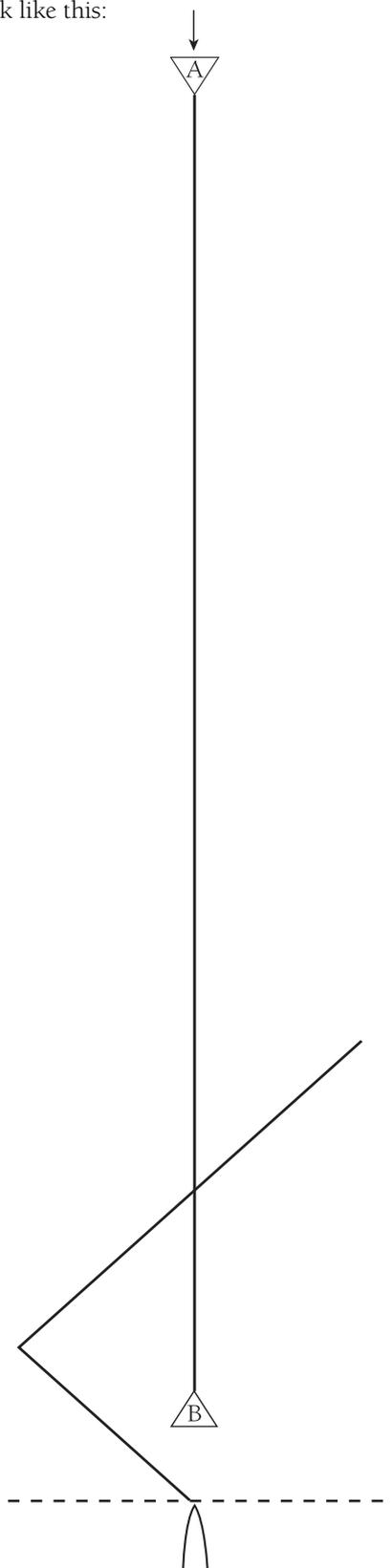
Activity

1.
 - a. 100–113 strokes
 - b. 10–11 times
 - c. Answers will vary. A higher stroke rate usually means faster, but this depends on how much power the paddlers create with each stroke.
2. Answers will vary. The crew members would need to pace themselves over the longer distance so that they do not run out of energy. This means that they would go more slowly overall.
3.
 - a. 4.14 minutes per kilometre (rounded to 2 d.p.)
 Speed would vary depending on wind direction, points at which there are crew changeovers, and so on. A well-trained crew would try to pace itself so that it keeps up a consistent speed.
 - b. The women’s average was 5.31 minutes per kilometre (to 2 d.p.), so the men’s waka ama was faster.

Pages 18–19: Sailing with Maths

Activity

- 34 262 metres (18.5 nautical miles)
- 1 080.3 metres
- 4. Teacher to check. The first part of your drawing should look like this:



- The yacht would travel faster, and there would be less need for tacking with the wind behind the yacht. The yacht might make only one tack on a downwind leg compared with several on an upwind leg.

Page 20: Scrum Power

- For the All Blacks, the mean is 110.38 kg, the median is 111 kg, the mode is 103 kg, and the range is 18 kg.

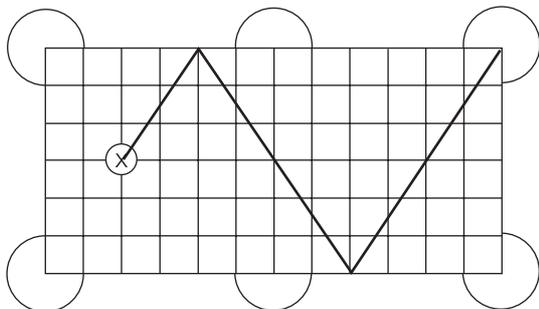
For the Wallabies, the mean is 112 kg, the median is 114.5 kg, the mode is 116 kg, and the range is 17 kg.
 - Sports commentators usually use the total pack mass and the mean as an indicator of scrum strength.
 - Heavier players may be able to push harder, but fitness and the way they push (technique) also have a lot to do with pushing strength.
- The total mass of each scrum will tell you which of the two packs as a whole has the greater pushing power. Mount Mission has a total mass of 323 kg and Runny River 320 kg. The statistics for the two teams are:

Measure	Mount Mission	Runny River
Mean	40.375 kg	40 kg
Median	37.5 kg	40 kg
Mode	38 kg	39kg and 41 kg (there are two modes)
Range	31 kg	2 kg

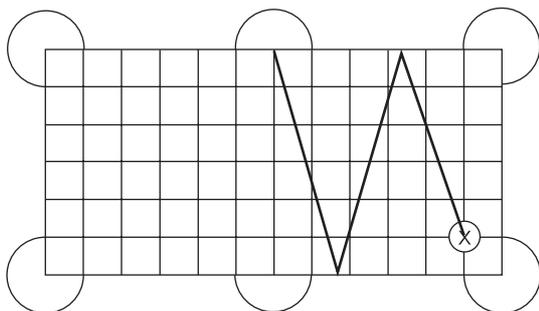
Mount Mission has one player (65 kg) that skews the mean. It tells us that the average mass of the Mount Mission forwards is bigger than that of the Runny River forwards. The 65 kg also means that the range is not useful as a statistic. However, the mode tells us that most of the Runny River forwards are bigger than most of the Mount Mission forwards. The same is true of the median. Since the Mount Mission scrum has one very big person and the rest are comparatively small, the median is probably the most accurate statistic to use.

Page 21: Pool Power

1.
 - a. Bottom right
 - b. Bottom left
 - c. Bottom middle
 - d. Bottom left
2.
 - a.



b.



Pages 22-23: Gearing Up

Activity

1.
 - a.
 - i. 2 times
 - ii. 3 times
 - iii. 4 times
 - iv. 2.5 times
 - b. You need to divide the number of sprockets in the chain-ring cog by the number of sprockets in the free-wheel cog.
2.
 - a. Answers will vary. Teacher to check. (A ten-speed bike has two.)
 - b. Answers will vary. Teacher to check. (A ten-speed bike has five.)
3.
 - a. The chain moves from cog to cog, at the front or back or both.
 - b. The number of gears will vary, but you can work this out by multiplying the number of

cogs on the crank by the number of cogs on the free-wheel cluster. (With a ten-speed bike, this is $2 \times 5 = 10$ gears.)

4.
 - a. Answers will vary. Teacher to check
 - b. Low gears are normally best for climbing hills. In general, the fewer turns the free wheel makes with each rotation of the chain-ring wheel, the easier the bike is to pedal.
 - c. High gears, or those when the chain is around the larger cogs, are best for cycling fast. High gears give a lot of turns of the free wheel for each rotation of the chain-ring wheel. Although a lot of effort is needed to rotate the chain-ring wheel, it results in more turns of the free wheel and therefore greater speed.

Page 24: Hitting Fours

1. You can work this out from the totals line in the table:

Score off shot	Number of shots	Runs
1	58	58
2	28	56
3	4	12
4	22	88

(He scored a total of 214 runs.)

2. 88 out of 214 runs came from boundaries (fours). This is just over $\frac{2}{5}$ or 40% of his runs.
3. Mathew ran the whole length of the pitch for every single, two, or three that he scored. That was a distance of $(58 + 56 + 12) \times 18 = 2\,268$ m. For each boundary, Mathew may have run only one or two lengths of the pitch, a distance of between $22 \times 18 = 396$ m and $44 \times 18 = 792$ m. If the other batters during Mathew's innings had scored the same runs as he did, the total distance he would have run is between $(2\,268 + 396) \times 2 = 5\,328$ m and $(2\,268 + 792) \times 2 = 6\,120$ m. However, as the other batters probably scored fewer runs than Mathew, it would be more realistic to base your estimate on the lower figure of 5 328 m.

♦ Figure It Out ♦

Sport

Teachers' Notes

Overview: Sport

Title	Content	Page in students' book	Page in teachers' notes
Can't Catch Me!	Measuring and estimating speed	1	12
Olympic Locations	Interpreting data	2–3	13
Luck of the Draw	Finding relationships	4	13
Pass It On!	Exploring possible outcomes	5	14
Great Golf	Investigating integers	6–7	16
Keeping in Shape	Exploring two- and three-dimensional shapes	8	17
Offside!	Calculating areas and percentages	9	18
On the Slippery Slope	Measuring time and distance and describing relationships	10–11	18
Roundabout Rugby	Making organised lists	12	19
It's a Knockout	Using systematic thinking	13	20
Which Way Now?	Measuring with direction and distance	14–15	21
Reach for the Sky	Collecting, displaying, and interpreting measurement data	16	22
Waka Ama	Solving problems of time and distance	17	23
Sailing with Maths	Using scale and compass directions	18–19	24
Scrum Power	Solving problems of mass	20	25
Pool Power	Investigating angles	21	26
Gearing Up	Investigating ratios	22–23	27
Hitting Fours	Presenting data in displays	24	28

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- read and interpret everyday statements involving time (Measurement, level 3)
- sketch and interpret graphs on whole number grids which represent everyday situations (Algebra, level 4)

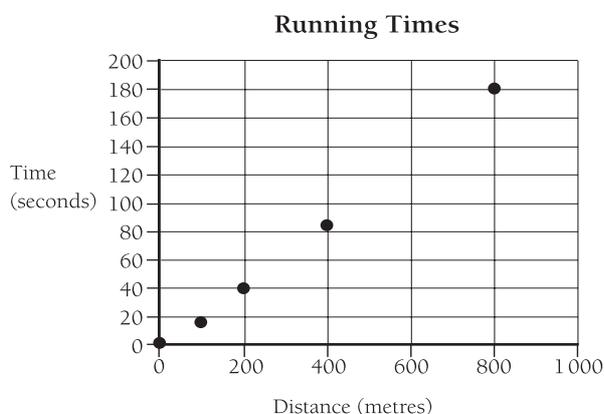
Activity

In this activity, students extrapolate (predict values outside the range of the data they have) to estimate how long they will take to run certain distances. For example, in question 2, the students use their recorded times for running 100, 200, and 400 metres. This is the range of their data. From this data they are asked to predict their time over 800 metres, which is outside this range.

Students also work with extrapolation and interpolation (predicting values within a data range) on page 19 of *Algebra*, Figure It Out, Levels 3–4 and on pages 18–19 of *Measurement*, Figure It Out, Levels 3–4.

The time that most 10- to 11-year-old students take to run 200 metres is more than double their 100 metre time. Similarly, their 400 metre time is over double their 200 metre time. This is because they are more tired and have to pace themselves over the whole distance.

A graph of distance against their running time in question 2 would follow this trend:



If the students join the points on the graph, they will find that it isn't a straight-line graph, that is, the relationship between time and distance is not linear. The relationship is not linear because the further the students run, the slower their speed.

As an extension, the students could use interpolation to predict the time they would take to run 300, 500, 600, or 700 metres.

In question 3, students research world records. In the world records as at February 2001 (shown on the following page), the men's 200 metre time is less than double the 100 metre time. This is because sprinters do not reach top speed until they are at least 50 metres into the race and they lose time on their start. There is no start time involved in the second 100 metres.

The table below shows the world records at February 2001 for 100 metres, 200 metres, 400 metres, 800 metres, and 1 500 metres:

Distance in metres	Women's record	Men's record
100	10.49	9.79
200	21.34	19.32
400	47.60	43.18
800	1:53.28	1:41.11
1 500	3:50.46	3:26.00

Students may like to get some idea of the speed of these runners by watching a video of an athletics event or trying to ride a bike at 10 seconds per 100 metres (36 km/h), which is approximately the world record speed over 100 metres.

Pages 2-3: Olympic Locations

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)

Activity

For question 1, if the students have difficulty establishing a pattern in the years on which the Olympics were held, suggest that they take five consecutive dates from the display. This will help them focus on the numbers rather than the extraneous information. Ask: "What do you do to 1968 to get 1972 that you also do to 1972 to get 1976?" (Add 4 years.)

This pattern of fours was interrupted after the 1992 Olympic Games. Since then, the Olympics have alternated between the winter and summer games every 2 years. Suggested reasons for this change are given in the Answers.

The best graph for questions 2a and 3a, which focus on the continental locations of the Summer and Winter Olympics, is a bar graph, as shown in the Answers. Explanations for venue choices are also given in the Answers. The students may not have thought about Antarctica as a continent, so this could be a useful discussion point. They should be able to come up with plenty of reasons why it has never been considered as an Olympic venue.

Page 4: Luck of the Draw

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3-4)
- record, in an organised way, and talk about the results of mathematical exploration (Mathematical Processes, communicating mathematical ideas, level 3-4)

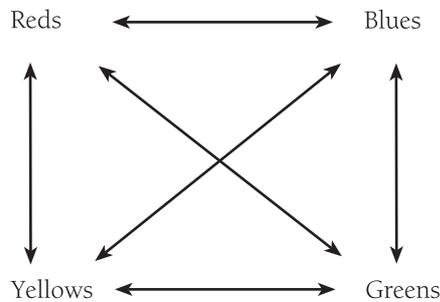
Activity

In this activity, students need to work systematically and use logic and reasoning to work out the tournament draw and who won, lost, and drew their games. To establish the games played in the tournament, the students might use a variety of strategies, including:

i. Making an organised list:

Reds versus Yellows
 Blues
 Greens
Yellows versus Blues
 Greens
Blues versus Greens

ii. Drawing a diagram:



From question 1, the students will have realised that six games were played in all. To answer question 2, they will also need to work out how many points each team got for a win and for a draw. It is safe to assume that the teams got more for a win than a draw and more for a draw than a loss. In question 2b, the students discuss how they worked out the points table. A rationale for this is given in the Answers, although some students may have worked it out in other ways.

As a further challenge, the students may like to work out how many games are played in a five-team, round-robin competition. They may like to pose a similar problem to the one in this activity for their classmates to solve.

Page 5: Pass It On!

Achievement Objective

- use a systematic approach to count a set of possible outcomes (Statistics, level 3)

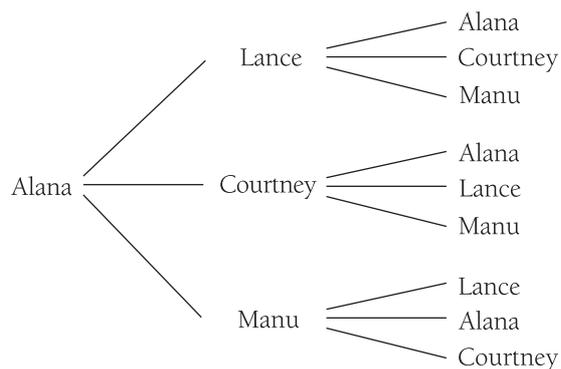
Activity

In this activity, students find all the possible outcomes of an event. They will need to be able to do this for other statistics work, when they work out the probability of a chosen event occurring. For example, to find out the chances of getting two heads when two coins are tossed (H = heads and T = tails):

- Find out all the possible outcomes: HH, HT, TH, TT.
- Work out the chances of getting HH: 1 in 4 or 25%.

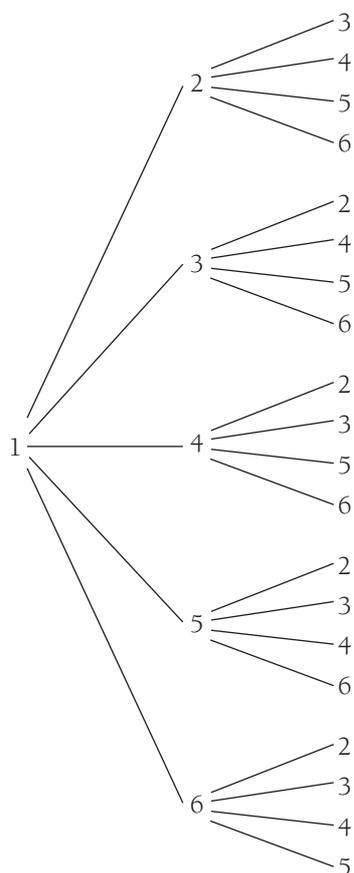
The students could use a tree diagram to find all the possible two-pass options available.

For example, for question **1b**:



The tree diagram also makes it easy to see how the number of two-pass combinations decreases with a no-pass-back rule. For question **1c**, one option (Alana) is removed from each arm of the diagram, leaving only six possibilities.

After the students have drawn several tree diagrams, they may notice a pattern in the numbers and use this to find a more efficient way to calculate the number of combinations of passes. For example, for question **3b**, there are six players. The tree diagram is:



$5 \times 4 = 20$ possible second passes

The number of possible first passes is one less than the number of players ($6 - 1 = 5$). The number of possible two-pass combinations is the number of first passes (5) times one less than the number of first passes ($5 - 1 = 4$): $5 \times 4 = 20$.

Applying this method to question **4c**:

There are nine players. The number of possible first passes will be eight ($9 - 1 = 8$). The number of two-pass combinations is $8 \times 7 = 56$.

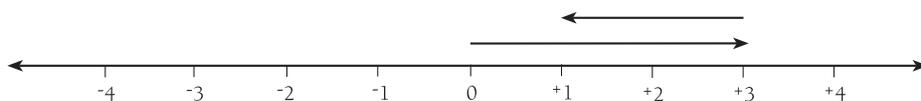
Achievement Objective

- explain the meaning of negative numbers (Number, level 4)

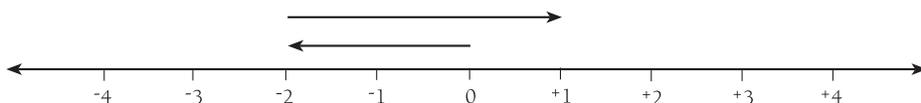
Activity

Sports scores in a game such as golf provide an excellent context for working with positive and negative numbers (integers). This context also helps students to recognise the different representations of zero. For example, a golfer might score +2, -1, and -1 over three holes. The overall result for those three holes is par (or 0), which is often referred to as “all square”.

In this activity, students are performing simple additions of integers by calculating scores in relation to par. Consider a golfer who scores +3 for her first hole and -2 for her second hole. Her current score could be shown using a number line:



As an equation, this can be expressed as $+3 + -2 = +1$, so her overall score for the first two holes is +1 (or one over par). Reversing the scores to get -2 for the first hole and +3 for the second hole can be expressed as:



As an equation, this is $-2 + +3 = +1$, so her overall score is still +1. These examples show that the order of the numbers being added does not affect their sum, which is also the case with counting numbers.

The students may wish to keep track of each golfer’s score from questions 2 and 3 using a counter sliding across an integer number line. For an under-par hole, the students would slide the counter to the left, and for an over-par score hole, they would move it to the right.

Golf scores can also be used for students to investigate adding and subtracting negative numbers. Removing a negative score has the same result as adding a positive score, and removing a positive score has the same result as adding a negative score. The students can check their ideas about adding and subtracting integers using a calculator. For example, $+3 + -2 =$ can be calculated by pushing $3 + 2 +/- =$. The key $+/-$ changes the sign of a number from positive to negative or vice versa. For example, to calculate the example above, $-1 - -2 = +1$, on a calculator, the students would press $1 +/- - 2 +/- =$.

Achievement Objective

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)

Activity One

The students will need to use a systematic approach to count the number of faces on the ball. Some may notice that the netball or soccer ball can be divided into two symmetrical halves, which makes the counting easier.



Alternatively, the students could mark each face with chalk as they count. They will find that the ball has 20 hexagonal faces and 12 pentagonal faces. This polyhedron can be described by the notation (6, 6, 5), which means that around each corner are two hexagons (6 sides) and one pentagon (5 sides). This pattern of surrounding each corner with two hexagons and one pentagon gives the “recipe” for building the polyhedron from shapes.

For question **1b**, counting the edges and vertices can be a time-consuming exercise. As a more efficient method, the students could think about how many sides of either a hexagon or a pentagon meet at each edge. Two sides are used to make each edge. The polyhedron is made of 20 hexagons and 12 pentagons. This is $(20 \times 6) + (12 \times 5) = 180$ sides. Dividing this by two gives the number of edges, 90. A polygon has the same number of sides as corners, so the hexagons and pentagons have a total of 180 corners available to form the vertices of the ball. Three corners are used to form each vertex, so the total number of vertices is $180 \div 3 = 60$.

Activity Two

Truncated solids are very common in nature. In the case of the soccer ball, the original polyhedron is an icosahedron that has 20 triangular faces. Each corner is chopped through to form a new face. This creates 12 new faces, the pentagons, while the 20 triangles are transformed into hexagons.

Activity Three

Truncating a cube has a similar result to truncating an icosahedron. The eight vertices are cut through to form eight triangular faces. The six square faces with their vertices removed become octagons. The students can use similar methods to those above to find the number of edges and vertices of the truncated cube. Six octagons and eight triangles have a total of $(6 \times 8) + (8 \times 3) = 72$ sides and corners. Two sides make each edge, so the number of edges is $72 \div 2 = 36$. Two octagons and one triangle meet at each vertex of the truncated cube, using up three corners of the shapes making up the truncated cube; $72 \div 3 = 24$ is the number of vertices.

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- calculate perimeters of circles, rectangles, and triangles, areas of rectangles, and volumes of cuboids from measurements of length (Measurement, level 4)

Activity

The students will first need to calculate the total area of the court in square metres. The court is 15 metres wide by 30 metres long, so the total area is $15 \times 30 = 450$ square metres. Questions **1b** and **1d** are fairly straightforward. Each third is $10 \times 15 = 150$ square metres (or a third of the total area: $\frac{1}{3}$ of 450 is 150 square metres). To work out the area of the shooting circles for questions **1a** and **1c**, the students will have to count squares and parts of squares. They will find that each shooting circle is about 10 square metres. This is close to the area of the shooting circle calculated using the formula πr^2 (9.8 square metres).

For question **2**, the students need to express as a percentage the areas of the court that each player can go into. For example, for question **2c**, the players can go into two-thirds of the court minus the shooting circle. This is $(2 \times 150) - 10 = 300 - 10 = 290$. The students could calculate the percentage as follows: $290 \div 450 \times 100 = 64.4\%$.

Achievement Objectives

- perform measuring tasks using a range of units and scales (Measurement, level 3)
- read and interpret everyday statements involving time (Measurement, level 3)
- use graphs to represent number, or informal, relations (Algebra, level 3)

Investigation

The students will need to use cardboard that is flexible enough to curve evenly along its length. Strong cartridge paper or poster card is about the right thickness. As well as being 80 centimetres long, the cardboard needs to be at least 30 centimetres wide so that the marble is less likely to roll off the side. The students also need to make sure that when the cardboard is set up between the books, it doesn't slope to one side because this could make it roll off the side.

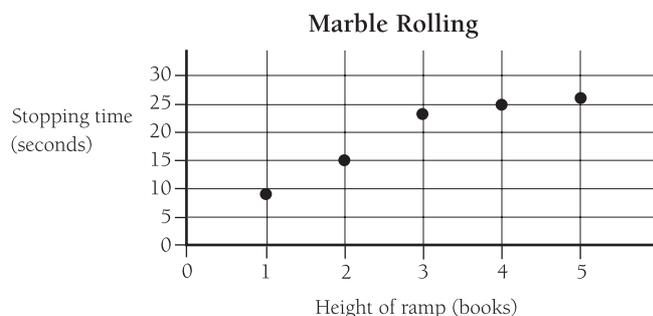
Discuss the following potential problems with the students before they start the investigation:

- How will they decide when the marble is finally at rest? (This should be when it stops moving up and down the ramp.)
- How will they cope with variable measurements? (One approach is to take three or five measurements and use the middle measurement.)
- How will they record their data as they gather it? (A table is ideal, and they can enter it into a spreadsheet in order to graph it.)

Here are the results of one experiment:

Height of ramp (in books)	Stopping time (in seconds)
1	9
2	15
3	23
4	25
5	26

When graphed, the data looked like this:



The line on the graph shows that increasing the height of the ramp causes an increase in the rolling time of the marble, but this increase tapers off as the height gets beyond a certain point.

Larger marbles will probably roll for a slightly longer time than smaller marbles, given the same height ramp. The distance travelled up the ramp is high on the first rolls but becomes increasingly smaller. The final up and down movements are very small.

Page 12: Roundabout Rugby

Achievement Objectives

- use a systematic approach to count a set of possible outcomes (Statistics, level 3)
- find all the possible outcomes for a sequence of events, using tree diagrams (Statistics, level 4)

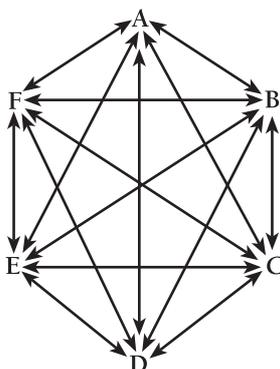
Activity

The students first need to establish which games need to be played, given the condition that each team plays each other team only once.

Below are some ways of doing this.

	Teams					
	A	B	C	D	E	F
A						
B	●					
C	●	●				
D	●	●	●			
E	●	●	●	●		
F	●	●	●	●	●	

An arrow diagram:



An organised list:

AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF.

All of these methods reveal that 15 games must be played. The students can also use multiplication to work out the total number of games. There are six teams. Each team plays each other team once, so each team will play five games. Six teams playing five games is $6 \times 5 = 30$ games. But this counts each game twice (it counts the Cats playing the Eagles and the Eagles playing the Cats, but this is actually only one game), so half of 30 is the number of games played: $30 \div 2 = 15$ games.

The students then need to make up a timetable in which no team has more than two consecutive games. A table is a useful way to do this.

West Field	East Field	Games to be played	
A vs B	C vs D	A vs B	B vs F
...	...	A vs C	C vs D
		A vs D	C vs E
		A vs E	C vs F
		A vs F	D vs E
		B vs C	D vs F
		B vs D	E vs F
		B vs E	

The students will need to work systematically to ensure that their draw meets all the requirements. Many different draws are possible. The students will not always be able to have a game on both fields at the same time. The minimum time is eight games of 30 minutes, a 30 minute lunch break, and six 10 minute breaks between games. This is a total of 330 minutes or $5\frac{1}{2}$ hours.

Page 13: It's a Knockout

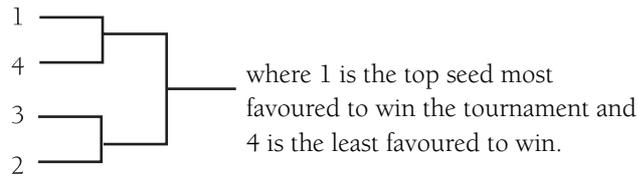
Achievement Objectives

- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, levels 3–4)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, level 3)

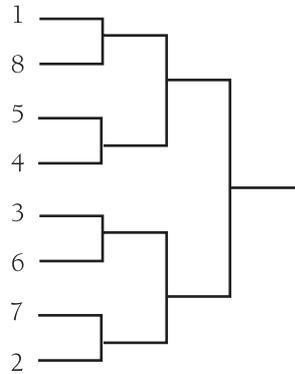
Activity

Seeded knockout tournaments are organised so that two of the top players don't meet in the early rounds of the competition. This avoids the possibility of a top seed, who may be a crowd favourite, being eliminated by another top seed in the first round.

In a four-player tournament, the draw might be organised like this:



In an eight-player, seeded tournament, the draw might be:



An interesting pattern is that the seedings for each pair of players in the first round add to nine. Note also that the only place seeds 1 and 2 can play each other is in the final, should they reach it. Seeds 2 and 3 can only ever meet in the semi-final, and the same applies to seeds 1 and 4.

Two possible eight-player draws are given in the Answers.

Pages 14-15: Which Way Now?

Achievement Objectives

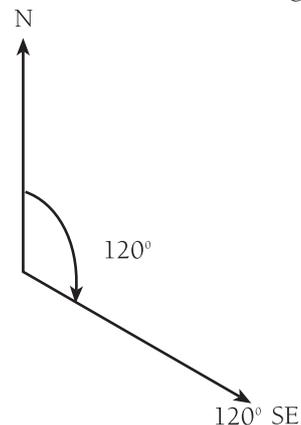
- draw and interpret simple scale maps (Geometry, level 3)
- specify location, using bearings or grid references (Geometry, level 4)

Activity

The map on the student page is a drawing, not a scale map, so the students will need to work from the copymaster provided at the back of these notes.

The needle on a compass is magnetised and points to the magnetic north pole. Although true north is about 23° off magnetic north, compasses do suffice for finding directions that do not need to be pinpoint accurate. This makes a compass ideal for outdoor pursuits such as tramping. (Some orienteering competitions require competitors to adjust for the difference between magnetic and true north.)

Directions given in degrees are relative to north in a clockwise direction. So 120° south-east contains an exact bearing (120° clockwise from north) and a reference (south-east).



Scale is a difficult concept for students at this level to understand. It is perhaps best illustrated by drawing a 10 centimetre line on an overhead transparency. The projector can be moved closer or further away from the screen to produce different scale factors. For example, if the image of the line is 1 metre long, then it is 10 times longer than the original, so the scale factor is 1 centimetre : 10 centimetres.

The students will use the scale and their compass or protractor to find their way around the map. If they are using a compass, they will need to align north on the map with north on their compass. If they are using a protractor, they can measure the number of degrees from north that the direction is in and apply this to their map. For example, south-east is 135° clockwise from north.

The students can also use the grid lines on their maps. The vertical grid lines run from north at the top of the page to south at the bottom, and the horizontal grid lines run from west on the left of the page to east on the right.

When they are designing their own orienteering course in question 2, the students should use two methods to document the directions: recording a series of compass directions and distances and drawing the path on the scale map of the school. The map can be kept as a check for other groups of students who attempt to follow the orienteering instructions.

Page 16: Reach for the Sky

Achievement Objectives

- make sensible statements about an assertion on the basis of the evidence of a statistical investigation (Statistics, level 3)
- make statements about implications and possible actions consistent with the results of a statistical investigation (Statistics, level 4)

Activity

Ask the students how they can use their own measurements to work out the likely back and waist-to-knee measurements of the Nikau Knights. If they work out what fraction (or proportion) of their own height their back measurement is and what fraction of their own height their waist-to-knee measurement is, they can use these fractions to work out the measurements of the Nikau Knights.

The students will need to gather data from at least 10 other students in order to estimate what proportion of their total height their back and waist-to-knee measurements are.

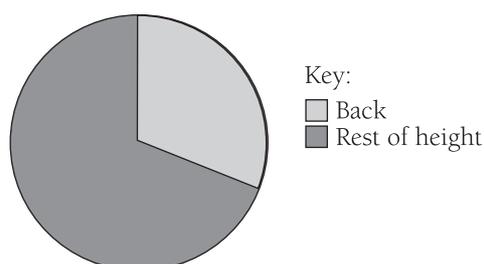
For example, for the back measurements:

Student	Back length	Total height	Back length as a fraction of total height	Back length as a percentage of total height
a	35	105	$\frac{35}{105}$	33%
b	43	119	$\frac{43}{119}$	36%
c	30	100	$\frac{30}{100}$	30%
d	37	115	$\frac{37}{115}$	32%
			Average %	32.75%

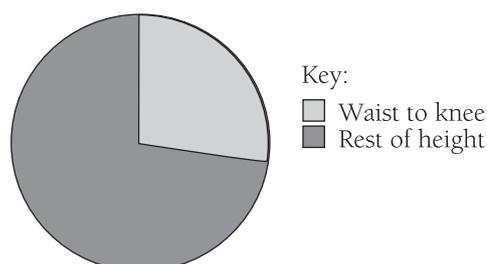
Because this problem involves proportions, the students will need to use a graph that emphasises proportions of a whole. A pie or strip graph will do this.

Below are two pie charts showing how the data could be displayed:

Back Length Compared with Height



Waist-to-knee Length Compared with Height



For question 2, the heights of the basketball players are deliberately skewed to prompt students to consider which statistic, the mode, median, or mean (average), gives the best indicator of a team member's expected height. Comments on the appropriateness of each statistic are provided in the Answers.

As an extension to question 2, you could ask the students to work out what the back length and waist-to-knee measurement of the uniforms should be.

For example, using the mean, $\frac{1}{3}$ of 1.96 is about 65 centimetres (back) and $\frac{1}{4}$ of 1.96 is 49 centimetres. The uniforms need to fit all the team members, so the students may decide to make the shirts and tops a little longer than the average measurement.

Page 17: Waka Ama

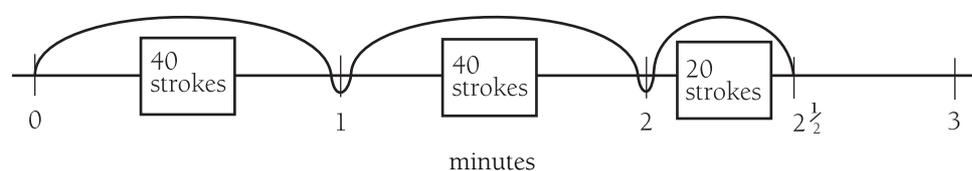
Achievement Objectives

- read and interpret everyday statements involving time (Measurement, level 3)
- perform calculations with time, including 24-hour clock time (Measurement, level 4)

Activity

To start the activity, the students could simulate a stroke rate of 40 strokes per minute to get some feel for how fast that is. They could change sides every 10 strokes. One student could call the stroke, using words to set the approximate pace: "1 001 stroke, 1 001 stroke, 1 001 stroke, 1 001 stroke, ... 1 001 stroke, change, 1 001 stroke, 1 001 stroke, ..." This will help the students to model the questions.

To answer question 1a, the students might draw a diagram like this:



So the crew took approximately 100 strokes.

The students can use this information to answer question 1b. If the crew took 100 strokes and changed sides every 10 strokes, they changed sides $100 \div 10 = 10$ times. Alternatively, they may reason that 40 strokes per minute means four side changes, so 100 strokes means 10 side changes.

A higher stroke rate results in a faster boat speed only if the crew is able to maintain the same power output per stroke. This is difficult to do, and often a faster stroke rate means that each stroke is less powerful. Also, a faster stroke rate may mean that the crew are tired and are not well co-ordinated, so the boat may actually go slower. Question 2 continues this idea by looking at how tiredness over a longer distance results in lower average speed.

Questions **3a** and **3b** are difficult because they involve the concept of average speed. The students may need the following scaffolding questions to help them. (They could also use double number lines.)

For question **3a**:

“How many minutes altogether is 2 hours and 4 minutes?” (You may need to explain to the students that 2:04:06 means 2 hours, 4 minutes, and 6 seconds.)

“How many minutes would the crew have taken to cover one-third of the distance?”

“How many kilometres was one-third of the distance?”

“How long would it have taken them to cover 1 kilometre?”

For question **3b**:

“How many minutes did the women take to cover each 11 kilometres? Compare that to the time taken for the men to cover 10 kilometres.”

“How long would the men take to cover 11 kilometres?”

“Which crew was fastest?”

Pages 18-19: Sailing with Maths

Achievement Objectives

- draw and interpret simple scale maps (Geometry, level 3)
- carry out measuring tasks involving reading scales to the nearest gradation (Measurement, level 4)

Activity

Students should find this sailing activity interesting, especially as it involves the scenario of an actual America’s Cup course.

In calculating the length of the course in question **1**, encourage the students to use multiplication rather than repeated addition. The total length of the course can be easily found by multiplying the total distance in nautical miles ($3 \text{ nm} \times 6$ plus 0.5 nm for the extra distance to the committee boat at the start and finish of the race) by $1\,852$ metres. The students may find other ways of working this out correctly.

Calculations with speed are quite difficult, and the students will need some help in finding solutions to question **2**. Scaffolding questions such as these will help:

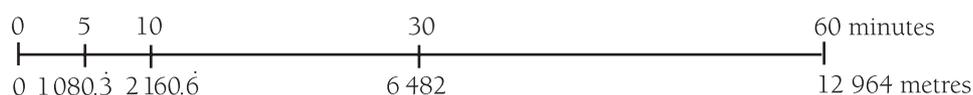
“How far does a boat travelling at 1 knot travel in 1 hour?” ($1\,852$ metres)

“How far is that in a minute?” ($1\,852 \div 60 = 30.8\dot{6}$ metres)

“How far is that in 5 minutes?” ($30.8\dot{6} \times 5 = 154.\dot{3}$ metres)

“If that is the distance for a boat travelling at 1 knot, how far would a yacht travelling at 7 knots go in 5 minutes?” ($154.\dot{3} \times 7 = 1\,080.\dot{3}$ m)

This could be shown as a double number line:

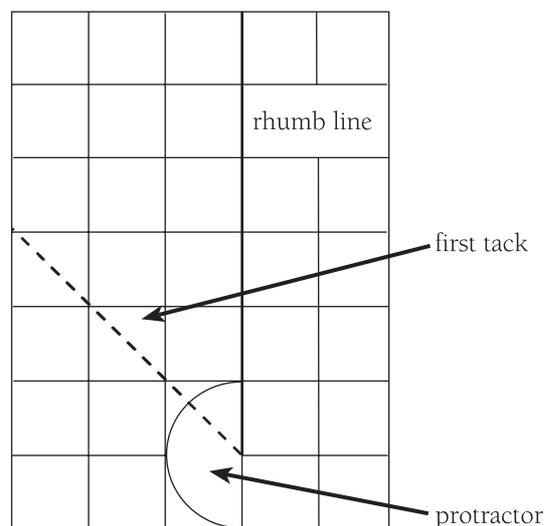


For question 3, the students will need to use the scale and the measurements given on the course map to draw the course on square grid paper.

For question 4, ensure that the students know how to measure tacking at 45° using their protractor and grid paper.

They also need to realise for question 4c that the tacking course that they plot is as efficient as possible, that is, it is as short as possible but does not have the yacht turning too often, as this will slow it down. They can discuss these points when they compare their course with a classmate's.

Points about speed and tacking when sailing downwind are given in the Answers.



Page 20: Scrum Power

Achievement Objectives

- make sensible statements about an assertion on the basis of the evidence of a statistical investigation (Statistics, level 3)
- make statements about implications and possible actions consistent with the results of a statistical investigation (Statistics, level 4)

Activity

This activity is based on real data from test matches between the New Zealand All Blacks and the Australian Wallabies. The students could get some feel for how big these forwards are by working out how many of their own classmates would make up a player of that mass. Given that most 10-year-olds weigh about 40 kilograms, most of the players weigh almost three times as much as one student.

The number of pieces of data is small (eight in each set), so finding the mean, mode, and median will only give rough indications of the expected mass of each forward pack because with small samples, each statistic can be distorted quite badly.

The mean or average is found by adding all of the data and dividing this total by the number of data readings or records. So the mean for each pack of forwards is:

All Blacks: $110 + 112 + 121 + 114 + 103 + 103 + 113 + 107 = 883$, and $883 \div 8 = 110.4$ kilograms

Wallabies: $118 + 102 + 116 + 119 + 107 + 105 + 116 + 113 = 896$, and $896 \div 8 = 112$ kilograms

The students will find the median by putting all the masses in order and finding the middle one. There are eight numbers, so the middle will be halfway between the fourth and fifth numbers.

So for each team, the median is:

All Blacks: 103, 103, 107, 110, 112, 113, 114, 121. Halfway between 110 and 112 is 111.

Wallabies: 102, 105, 107, 113, 116, 116, 118, 119. Halfway between 113 and 116 is 114.5.

The mode in this case is not useful. The Australian pack has a single most common mass of 116 kg. This could be taken as an indication of strength, but it does not take the lower masses into account. In the same way, the mode for the All Blacks is 103 kilograms, which is the lowest mass listed and takes no account of the much heavier masses.

Although heavier packs of forwards are usually able to push harder, many other factors come into the strength of a scrum, for example, fitness and technique. However, the mean is probably the best indicator when comparing pack masses. This is the statistic that broadcasters use.

Question 2 explores how the mean can be distorted by an outlier or extreme score. The presence of a 65 kilogram player in the Mount Mission team skews the mean mass. This gives the Mount Mission pack a mean mass of 40.4 kilograms, while the Runny River pack has a mean mass of 40 kilograms. This is a false picture of pack strength because six of the Mount Mission pack are lighter than their Runny River counterparts. In this case, the median gives a better idea of the pack strength: 38 kilograms for Mount Mission and 40 kilograms for Runny River.

Page 21: Pool Power

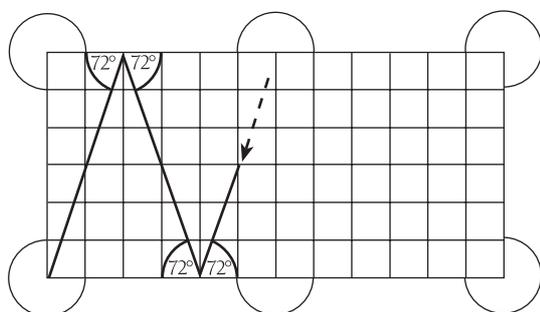
Achievement Objectives

- design and make a pattern which involves translation, reflection, or rotation (Geometry, level 3)
- describe the reflection or rotational symmetry of a figure or object (Geometry, level 4)

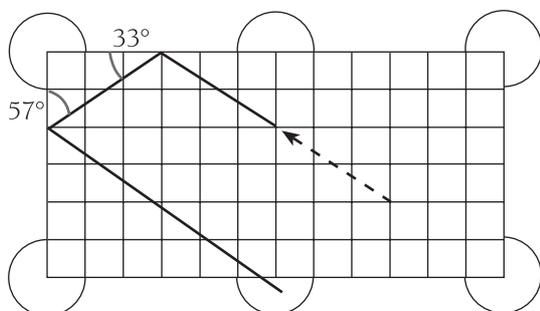
Activity

You will find a copymaster for this activity at the end of these notes. The students will use similar skills to those used for question 4 on page 19 to work out the path of each ball.

For example, for question 1b, they will draw the path of the ball like this:

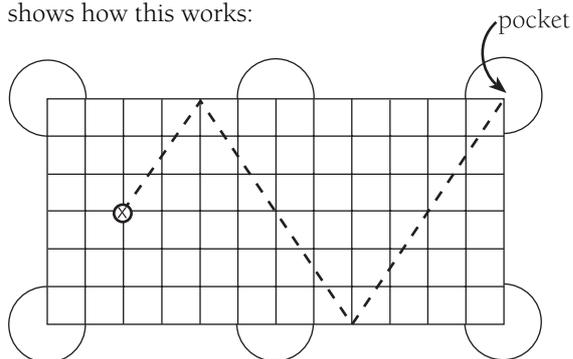


The students will notice that if the ball is bouncing off cushions on opposite sides of the pool table, the size of the angles will be the same. If the ball bounces off two cushions that are at right angles to each other, the two angles will add up to 90° . For example, for question 1c:



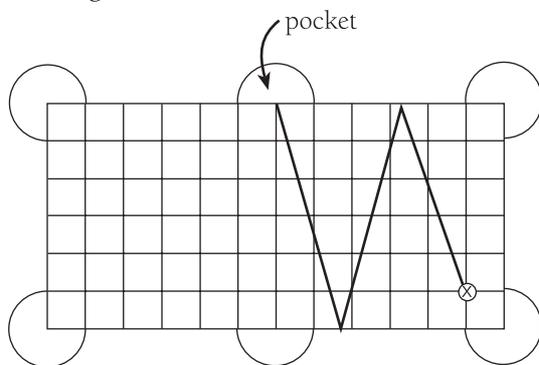
In question 2, the students work backwards, which is a significant problem-solving strategy. Rather than randomly drawing an arrow from the pool ball in the direction of a cushion, it is more effective for the students to use the grid pattern to work out where the ball hits each cushion. In example 2a, the ball must travel 10 squares to the right and strike two cushions. Obviously, the top and bottom cushions are the easiest to bounce off. The ball starts its journey in the centre of

the table width-wise, so it will cross the width of the table two and a half times. $10 \div 2\frac{1}{2} = 4$, so the ball should travel four squares to the right on each journey across the table. In the first journey, it travels halfway across the table, so it travels only two squares to the right. The diagram below shows how this works:



On the pool table in **2b**, the ball must travel five squares to the left while rebounding off at least two cushions. To do this, it will cross the width of the table three times. $5 \div 3 = 1\frac{2}{3}$, so the ball should travel $1\frac{2}{3}$ squares to the left for each crossing to end up in the top centre pocket.

The diagram below shows how this works:



Pages 22-23: Gearing Up

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- write and solve problems involving decimal multiplication and division (Number, level 4)

Activity

Gearing on bikes is an excellent context for students to explore ratio. The general rule for working out the number of rotations is to divide the number of sprockets on the chain-ring cog by the number of sprockets on the free-wheel cog (back wheel). This is because each sprocket on the chain-ring cog is matched by a corresponding sprocket on the free-wheel cog through the links on the chain. So in the case of **1a i**, $32 \div 16 = 2$ gives the number of times the free wheel will rotate for every turn of the chain wheel.

On a 10-speed bike, the 10 gears are created by two different chain-wheel cogs and five different free-wheel cogs. The selectors on the bike allow the chain to slip between these various cogs. This gives 2×5 different combinations, that is, 10 different gearing ratios.

The students will find it easier to count the number of sprockets on each wheel of their 10-speed bike if they turn the bike upside down. Using Twink or a similar correction fluid, they can mark one sprocket on each wheel to show where they start counting from. They should use a calculator

to work out the ratio divisions because the results often have difficult remainders. Encourage the students to round the decimals in each gear ratio to make them easier to interpret.

In general, the fewer turns the free wheel makes with each rotation of the chain wheel, the easier the bike is to pedal. This requires less effort, which makes low gears ideal for hill climbing. However, biking fast is difficult in low gears. High gears give a lot of turns of the free wheel for each rotation of the chain wheel. Although a lot of effort is needed to rotate the chain wheel, it results in more turns of the free wheel and therefore greater speed.

Page 24: Hitting Fours

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

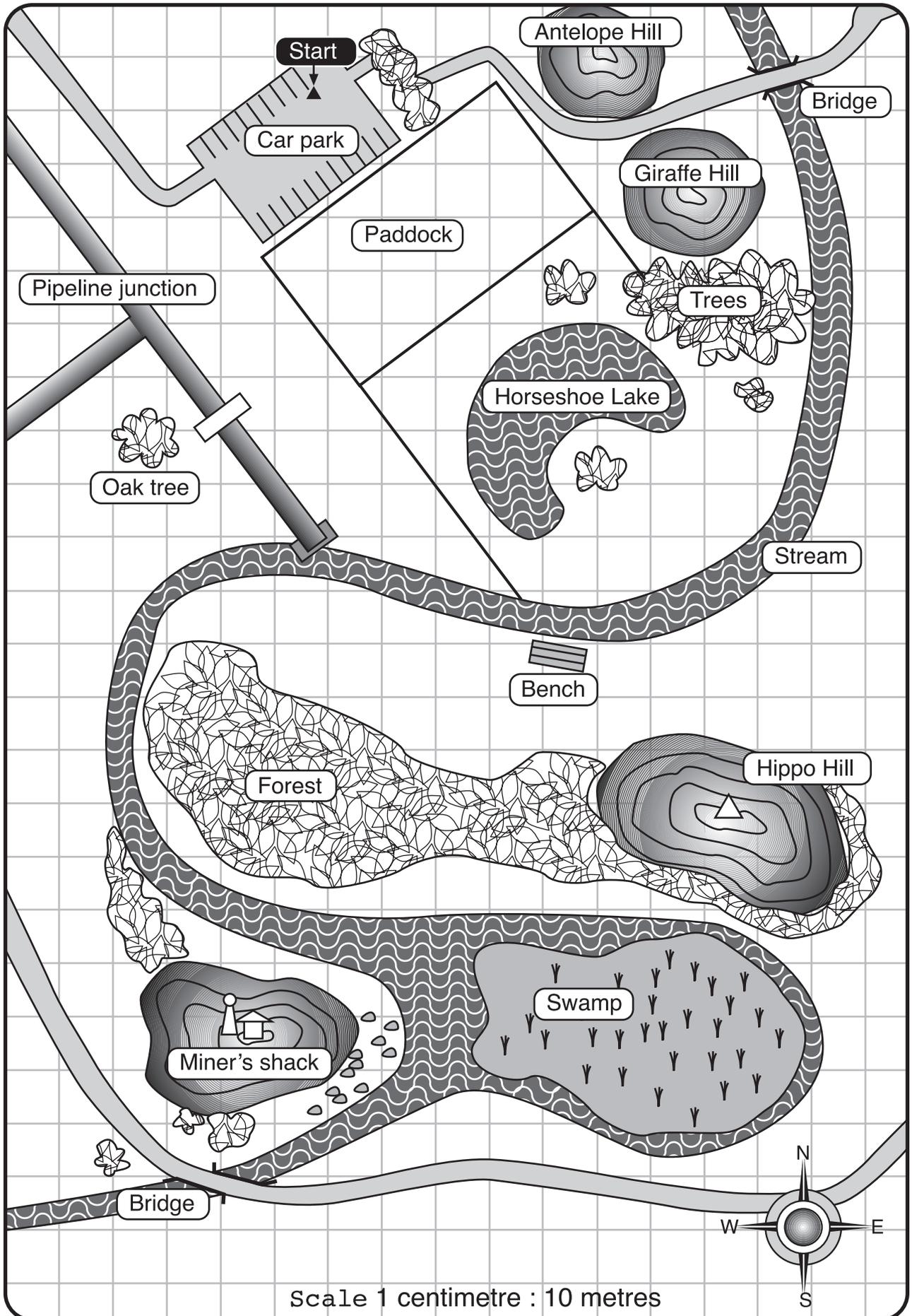
The scoring wheel shown on this page is a little different from those shown on TV. In TV broadcasts, the “wagon wheel” is compiled as if the batter hit shots from only one end. This is done so that patterns can be found in the areas the batter scores in.

The wheel on this page shows the runs scored from each bowler. For example, 4K means a hit for four from R. King’s bowling and 1W means a single taken from C. Walsh’s bowling.

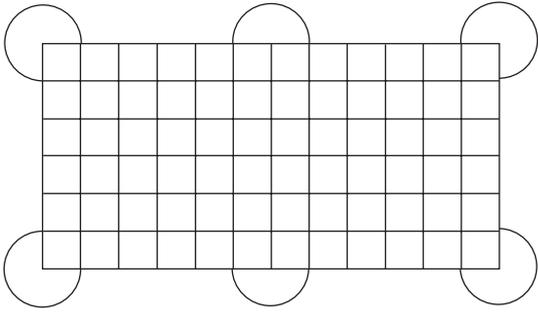
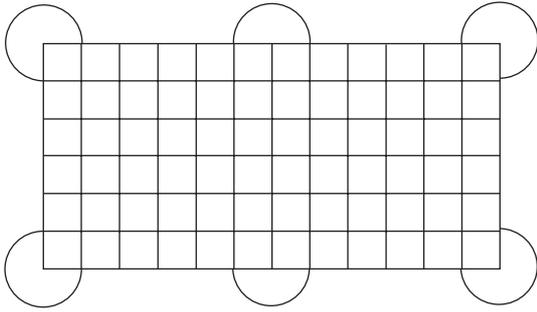
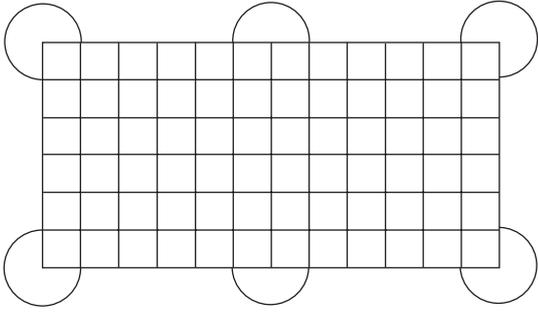
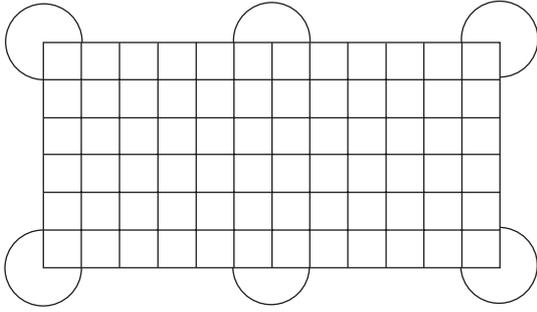
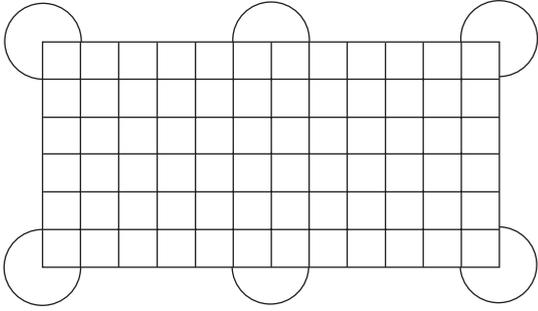
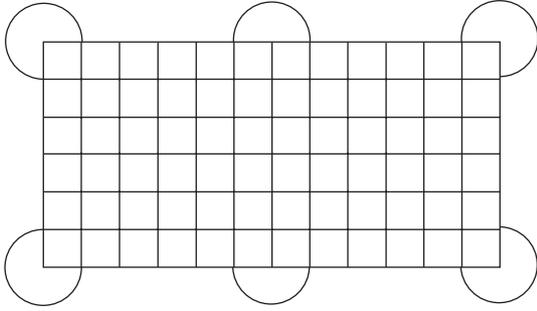
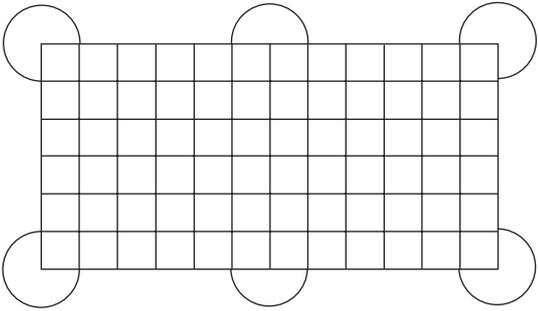
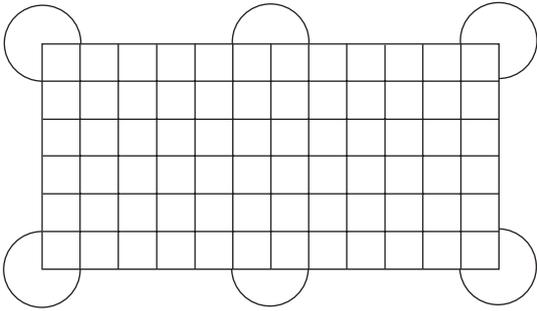
The students can work out the score from Mathew Sinclair’s innings fairly easily because he hit a large number of boundaries (fours). They should use multiplication to work out the total of the fours. Mathew hit 22 fours for a total of 88 runs. Similarly, they can find the number of runs scored from twos and threes by multiplying.

To estimate the distance that Mathew ran during his innings, the students will need to recognise that he ran 18 metres for every non-boundary run he scored (ones, twos, threes), that is, 126 lengths of 18 metres, which is 2 268 metres. For each four he scored, Mathew may have averaged about one and a half lengths running, which gives $22 \times 1\frac{1}{2} \times 18$, which is 594 metres. (You may need to remind the students that when Mathew hits a four, he would run until he was sure that the ball had hit the boundary.) The students also need to take into account that Mathew was not always facing the bowler and that he had to run when the other batter was scoring runs. The batters at the other end during Mathew’s innings may have scored fewer runs in total than he did, so the students might consider that doubling $2\,268 + 594$ to get $2\,832 \times 2 = 5\,664$ metres is an exaggerated estimate. It is interesting to reflect that this distance was covered at sprinting pace while carrying a bat and wearing protective gear!

Copymaster: Which Way Now?



Copymaster: Pool Power



Acknowledgments

Learning Media would like to thank Vince Wright, School Support Services, School of Education, The University of Waikato, for developing the teachers' notes. Thanks also to Ken Benn and Kathy Campbell for reviewing the answers and notes.

In the main illustration on the cover and contents page, the illustration of the fern is by Dawn Barry copyright © Dawn Barry; the panorama of Tongariro is by Meredith Biberstein copyright © Meredith Biberstein; the photograph of the golfer Michael Campbell is reproduced courtesy of *The New Zealand Herald* copyright © *The New Zealand Herald*; and the photograph of the golf ball is from the *Sporting Goods* image disc copyright © PhotoDisc, Inc. 1997.

All other photographs in the main cover illustration are by Adrian Heke.
The line art of the soccer player and the rugby player in the background on the cover, the contents page, and pages 2, 3, and 11 is by Dylan Coburn.
The map on page 29 is by Esther Chua.
These photographs and all illustrations are copyright © Crown 2001.

Series Editor: Susan Roche
Designer: Dawn Barry

Published 2001 for the Ministry of Education by
Learning Media Limited, Box 3293, Wellington, New Zealand.
www.learningmedia.co.nz

Copyright © Crown 2001
All rights reserved. Enquiries should be made to the publisher.

Dewey number 510.76
ISBN 0 478 12984 X
Item number 12984
Students' book: item number 12983