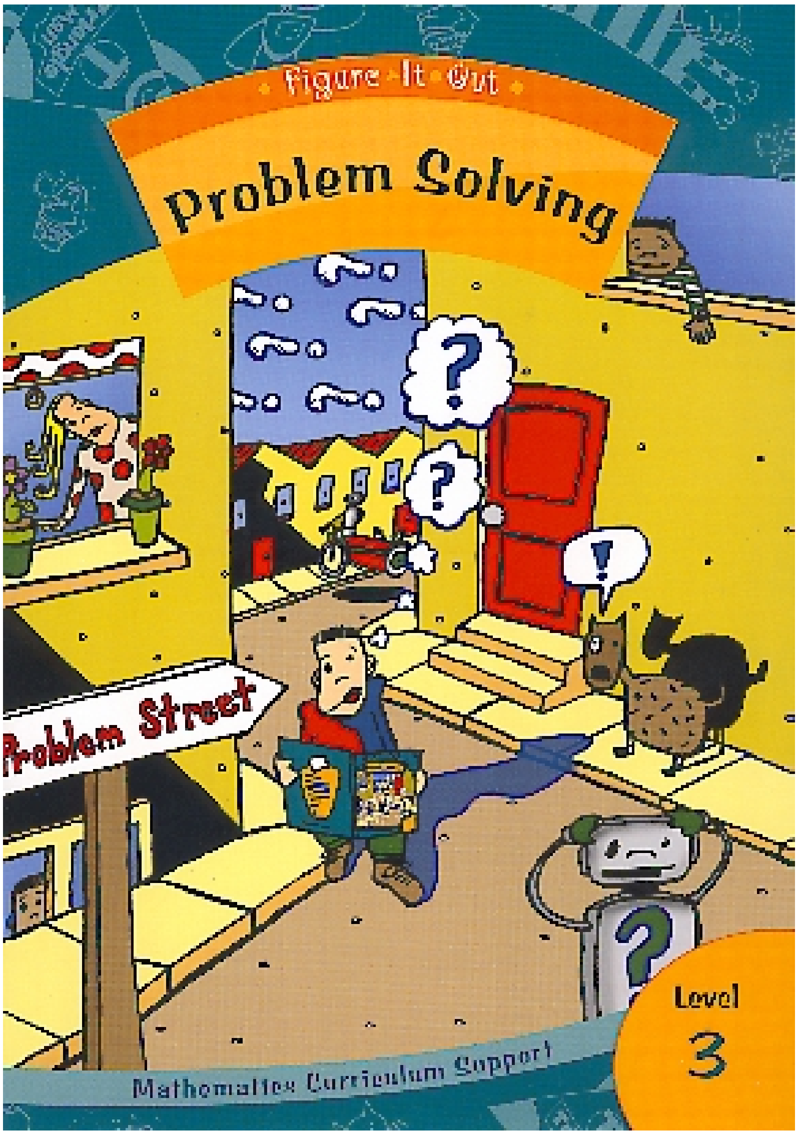
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum*. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students.

There are eight booklets for level 3: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (level 3) are suitable for most students in year 5. However, teachers can decide whether to use the booklets with older or younger students who are also working at level 3.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> (*Mathematics in the New Zealand Curriculum*, page 7)

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out
Problem Solving
Answers

Page 1: All Sorts

1. $600
2. Yellow
3. Many answers are possible. The centre number must be 6. The 2 numbers at either end of each line must add to 12.
4. $100

Page 2: Size-wise

1. **a.** 87 420

   b. 02 478 (using 0 as a place holder with no value) or 20 478

   c. **i.** 28 704 and 28 047 or 82 704 and 82 047
   ii. Yes – whichever set of numbers you didn't use for Simon in **i**

   iii. Explanations will vary. One way to do this is to eliminate digits using an organised list. (See Teachers' Notes for an example.)

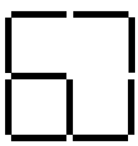
2. **a.** Only tower heights of (2, 3, 6) and (2, 4, 5) are possible.

   b. (1, 1, 9), (1, 2, 8), (1, 3, 7), (1, 4, 6), (1, 5, 5), (2, 2, 7), (3, 3, 5), and (3, 4, 4)

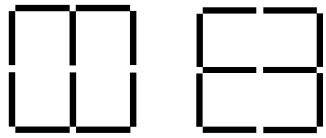
3. $^1/_2$ hour
4. $500

Page 3: Pattern Teasers

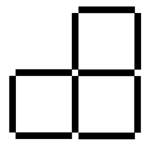
1. **a.** One solution would be:



   b. Two rectangles



   or any combination of 3 squares, for example:



2. There are a number of possible addition equations, including 235 + 746 = 981, 168 + 327 = 495, 139 + 428 = 567, 278 + 316 = 594, 596 + 142 = 738, and 219 + 438 = 657. Manipulating the digits in each of these equations leads to others, such as 462 + 357 = 819, 782 + 163 = 945, and 281 + 394 = 675.
3. 30
4. 6

Page 4: Birthday Surprise

1. 64 lollies
2. **a** and **g**, **b** and **h**, **c** and **e**, **d** and **f**
3.

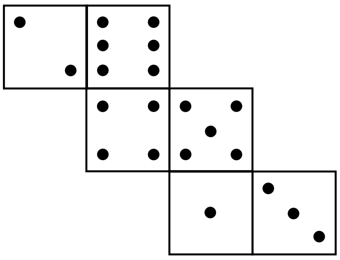
Digit	Number of tabs to be ordered
0	11
1	21
2	20
3	20
4	20
5	20
6	20
7	20
8	20
9	20

4. **a.** 12 metres

   b. The minimum number of bends is 6 (if 6 lengths of plastic piping are cut) and the minimum number of joins is 8.

Page 5: Exercise Time

1. 6

2. 

3. Although the answer could be any multiple of 5 and 6, for example, 30, 60, or 90, the most likely answer is 30.

4. Yes. The red rabbit will reach 600 in 600 ÷ 30 = 20 jumps. The purple rabbit will reach 600 in 600 ÷ 20 = 30 jumps. The pink rabbit will reach 600 in 600 ÷ 25 = 24 jumps. 30, 20, and 25 are all factors of 600.

Page 6: Layer upon Layer

1. **a.** 30

   b. 55

2. A

3. **a.** 20

   b. The perimeter will be twice as long. The area will be 4 times as big.

4. 14

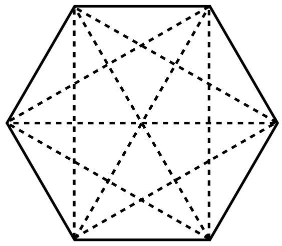
Page 7: Digit Detail

1.

79	169	259	349	439	529	619	709	808	907
88	178	268	358	448	538	628	718	817	916
97	187	277	367	457	547	637	727	826	925
	196	286	376	466	556	646	736	835	934
		295	385	475	565	655	745	844	943
			394	484	574	664	754	853	952
				493	583	673	763	862	961
					592	682	772	871	970
						691	781	880	
							790		

2. $2 (4 x 50c) or 20c (4 x 5c) if the coins are identical; 50c (20c + 20c + 5c + 5c or 20c + 10c + 10c + 10c) or $1 (50c + 20c + 20c + 10c) if the coins are not identical

3. No. A triangle has no diagonals and a square has only 2 diagonals. Any polygon with more than 5 sides has many more diagonals than sides or vertices (corners). For example:



hexagon

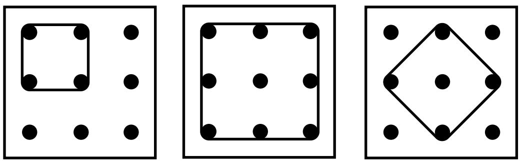
4. **a.**

Digit	Lines of symmetry
0	2
1	1
2	0
3	1
4	0
5	0
6	0
7	0
8	2
9	0

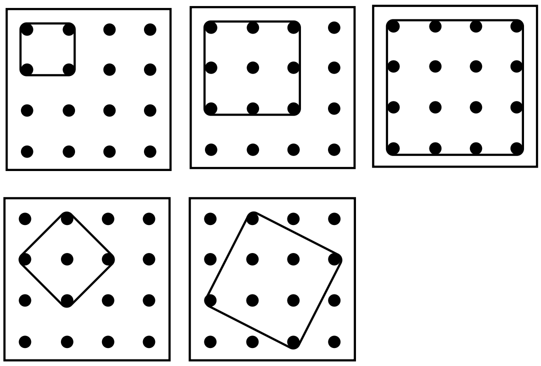
b. 2 and 5

c. 0, 2, 5, and 8

5. a. 3



b. 5



Page 8: Upside Down

1. a. $^{1}/_{16}$ (0.0625)

   b. $^{1}/_{64}$

2. 4 x 20c; 50c, 10c, 10c, and 10c; or 50c, 20c, 5c, and 5c

3. a. 3 481

   b. You get the same answer.

   c. When the book is turned upside down, you are looking at exactly the same set of numbers being added together as before because 8 and 1 have half-turn symmetry, 9 is a half-turn image of 6, and 2 is a half-turn image of 5.

4. Yes. The first bar is showing 4 out of 12 units, the second is showing 6 out of 12, and the third 2 out of 12. Expressed as fractions, this is $^{4}/_{12} = {}^{2}/_{6}$, $^{6}/_{12} = {}^{3}/_{6}$, and $^{2}/_{12} = {}^{1}/_{6}$. These are the same fractions shown by the pie graph.

   Students' stories will vary.

Page 9: Ps for Ever

1. 4 x 4 and 3 x 6. Other solutions are possible if side lengths with fractions are included, for example, 2.5 x 10.

2. Four 5 kg bags at a cost of $13 (The extra kilogram could be saved for another occasion.)

3. a. 16

   b. 351

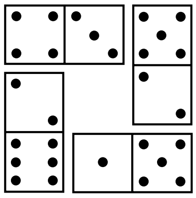
4. Possible solutions: 9 postcards, 6 postcards and 2 letters, 3 postcards and 4 letters, or 6 letters

Page 10: Locomotive Magic

1. a. 6 full turns

   b. anticlockwise

2. One solution:



3. One solution: The farmer takes the chicken to the other side of the river and leaves it there. He goes back and gets the fox. He leaves the fox on the other side of the river and takes the chicken back. He leaves the chicken on the bank, takes the corn over to the other side, and leaves it with the fox. He then goes back and fetches the chicken.

   Another solution follows the above for moving the chicken, but the trips for the corn and the fox are reversed.

4. One solution is to follow these steps:

   - Fill the 7 L container.
   - From the 7 L container, fill the 3 L container and then empty the 3 L container. Do this twice. There is now 1 L remaining in the 7 L container.
   - Pour the remaining litre into the 3 L container.
   - Refill the 7 L container.
   - From the 7 L container, top up the 3 L container until it is full. There are now exactly 5 L of water in the 7 L container.

Another solution:

- Fill the 3 L container. Pour this 3 L into the 7 L container. Do this twice. (There are now 6 L in the 7 L container.)
- Fill the 3 L container. Top up the 7 L container. This leaves 2 L in the 3 L container.
- Empty the 7 L container. Pour the remaining 2 L from the 3 L container into the 7 L container. Fill up the 3 L container again.
- Add the 3 L from the 3 L container to the 2 L in the 7 L container. There are now exactly 5 L of water in the 7 L container.

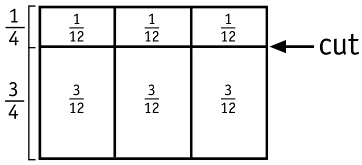
You may find other solutions.

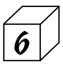
Page 11: Challenging Times

1. 24 minutes

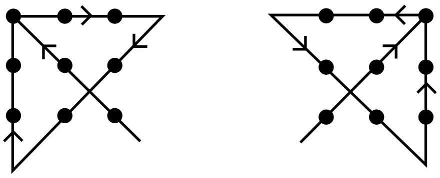
2. **a.** No. The 2 larger pieces at each end are bigger than the 4 other pieces joined together.

b.



3. She needs to move the  from the third stack to the first. (Each stack will then total 18.)

4. Two possible solutions:



Page 12: Thirsty Work

1. $1

2. 25 pins (in a 4 x 4 arrangement)

3. 46 cm

4. Beth

Page 13: Simple Solutions

1. **a.** (1 x 8) + 1 = 9

(12 x 8) + 2 = 98

(123 x 8) + 3 = 987

(1 234 x 8) + 4 = 9 876

b. (12 345 x 8) + 5 = 98 765

(123 456 789 x 8) + 9 = 987 654 321 (If you are checking this on your calculator, note that calculators that only fit 8 digits on the screen show the answer as 98765433.)

2. Note that, including Mrs Brown, there were 7 people present.

Number eaten by each person	Total buns eaten	Buns left over
0	0	60
1	7	53
2	14	46
3	21	39
4	28	32
5	35	25
6	42	18
7	49	11
8	56	4

3. To get a sure result, only 2 weighings are needed, based on weighing groups of 3 marbles.

You could take the risk of weighing 2 lots of 4, and if they balance, the ninth ball is the light one, and no more weighings are needed. However, if this is not the case, more than 2 weighings are necessary.

4. Top left-hand corner square

Page 14: Arrow Antics

1. 3

2. **a.** In each case, the digits total 9.

b. The pattern is decreasing by 1. (When the single-digit number gets to 1, the next number is 9 and the pattern continues.)

c. The pattern is groups of 4 odd or even digital sums decreasing by 2:

Multiples of 7	7	14	21	28	35	42	49	56	63	70	...
Digital sum	7	5	3	10→1	8	6	13→4	11→2	9	7	...

3. **a.** 27

b. 24

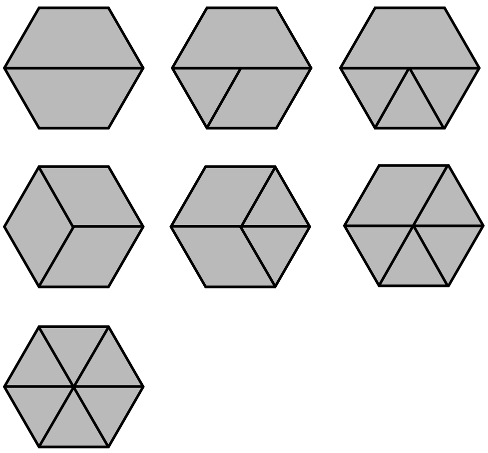
4. 3

Page 15: Missing Links

1. Yes, but only if the jeweller opens and closes 3 links of one chain and uses these to connect the other chains. Cost = 3 x 50c + 3 x 70c
= $3.60

2. There are an infinite number of answers. Solutions include: 2.5 ÷ 2; 5 ÷ 4; 7.5 ÷ 6; 10 ÷ 8; 12.5 ÷ 10; 15 ÷ 12 … (Notice the pattern: the dividend increases by 2.5, and the divisor increases by 2.)

3.



4. **a.** 176; 1 776; 17 776

b. 177 776

444 444

There is always one more 4 in the second factor than there are sevens in the answer.

Page 16: Darting for Cover

1. Possible scores:

12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 36

2.

$$\begin{array}{r} 101 \\ -\ \ 91 \\ \hline 10 \end{array}$$

3. **a.** 24

b. 120

4. Sam: netball

Harley: soccer

Rupert: volleyball

Vijay: rugby

Narissa: hockey

Page 17: Over a Barrel

1. 10 kg

2. The circle represents 4, the triangle represents 5, and the square represents 9.

3. **a.** 7

b. 7

4. There are 10 different pizzas that could be made:

ham – salami – olives

ham – salami – peppers

ham – salami – pineapple

ham – olives – peppers

ham – olives – pineapple

ham – peppers – pineapple

salami – olives – peppers

salami – olives – pineapple

salami – peppers – pineapple

olives – peppers – pineapple

Page 18: Colourful Calculations

1. Shape **a**

2. 50

3.

$$\begin{array}{r} \boxed{1}\boxed{5} \\ \boxed{4}\,\overline{)\boxed{6}\boxed{0}} \end{array}$$

4. 11 red, 7 blue, 17 yellow, and 2 green

Page 19: Stretching Exercises

1. a. 186 + 187 = 373

   b. 17 x 18 = 306

   c. 11 ÷ 10 = 1.1

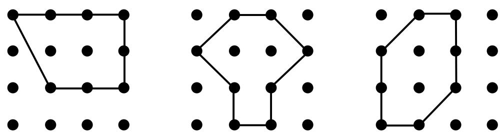
2. 16 October

3. a. 5 square units

   b. 8

   c. 2

   d. Three possible shapes:



   They all have an area of 5 square units.

4. a. $\boxed{\text{on/c}}\ \boxed{5}\ \boxed{+}\ \boxed{5}\ \boxed{-}\ \boxed{2}\ \boxed{-}\ \boxed{2}\ \boxed{=}$ or

   $\boxed{\text{on/c}}\ \boxed{2}\ \boxed{+}\ \boxed{+}\ \boxed{=}\ \boxed{=}$

   b. $\boxed{\text{on/c}}\ \boxed{5}\ \boxed{+}\ \boxed{5}\ \boxed{-}\ \boxed{2}\ \boxed{=}$ or

   $\boxed{\text{on/c}}\ \boxed{2}\ \boxed{+}\ \boxed{+}\ \boxed{=}\ \boxed{=}\ \boxed{=}$

Page 20: Mixed Fruit

1. a. 0, 1, 2, 3, 4, or 5

   b. The most likely answer is 1, and the next most likely is 2.

2. 6 (8 if all the blocks are connected)

3. $0.35

4. a. Answers will vary, but most solutions will include 15 as the central number, for example, 7, 9, 15, 21, 23 or 7, 11, 15, 19, 23. Solutions that do not include 15 must give an average of 15, for example, 9, 11, 13, 19, 23 or 7, 9, 17, 19, 23.

   b. Some other solutions include:
   9 + 11 + 15 + 17 + 23 = 75
   9 + 13 + 15 + 17 + 21 = 75
   11 + 13 + 15 + 17 + 19 = 75

Page 21: Talk About: One

1. Answers will vary. Comments could include:

   a. The triangle has sides, but the circle does not appear to have any.

   b. Both are closed figures with reflectional and rotational symmetry, and you can find the centre of both shapes.

2. a. The digits shift 1 place to the right in relation to the decimal point.

   b. The digits shift 2 places to the right when dividing by 100.

   c. The digits shift 3 places to the right when dividing by 1 000.

3. Three people sharing 5 custard squares get more ($1\frac{2}{3}$ compared to a $1\frac{1}{2}$).

4. No. You cannot form a square because there are only 14 small cubes. There are not enough small squares to form a 4 x 4 square, and there are too many for a 3 x 3 square.

Page 22: Talk About: Two

1. Smaller. 0.79 is $\frac{79}{100}$ and 0.8 is $\frac{80}{100}$.
   (A number line would show that 0.79 comes before 0.8.)

2. 14 faces, 36 edges, 24 vertices

3. The pie graph (graph **b**). The ratios of the strip and pie graphs are $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{3}{8}$. The bar graph has 4 different lengths.

4. a. 500 mL bottle: 50 g; 1.5 L bottle: 100 g (0.1 kg); 3 L bottle: 1 kg

   b. 2.5 kg

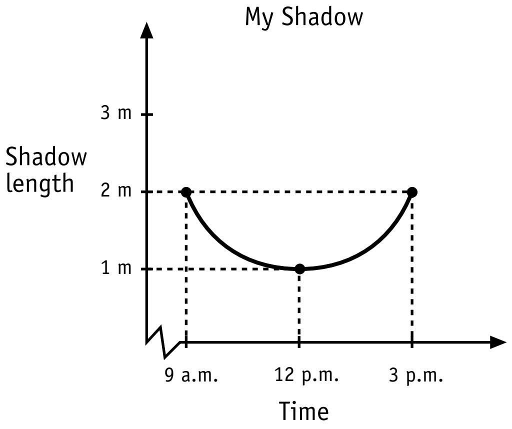
Page 23: Talk About: Three

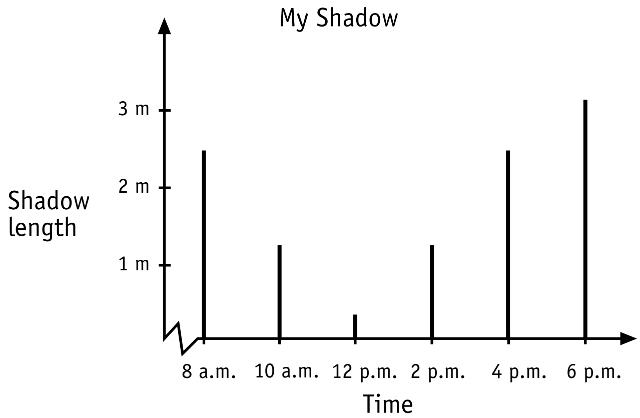
1. a. When you add an odd number to an even number, the result is always odd.

   b. Two odd numbers added together always give an even result.

2. False. Triangles also tessellate.

3. Answers will vary, but 2 possible graphs are:





4. **a.** Answers will vary. A class of students (average mass of 30–40 kg per student) is likely to have a combined mass much greater than most bulls (450–650 kg), although a prize show bull can have a mass as great as 1 tonne.

   b. Answers will vary. You might find out the weight of bulls through the Internet, a library, or a rural veterinary service.

Page 24: Talk About: Four

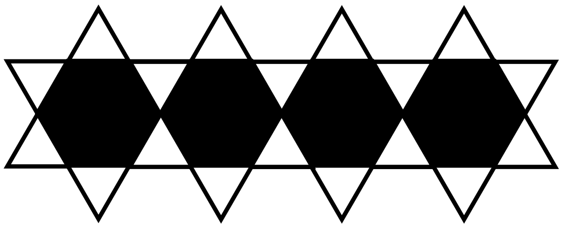
1. **a.** Dividing by 10 is the same as multiplying by 0.1.

   b. Multiplying by 0.25 is the same as dividing by 4.

2. The volume of box **a** is $2.5 \times 2.5 \times 1 = 6.25$ cm^3 and the volume of box **b** is $1.5 \times 2 \times 3 = 9$ cm^3. To find the volume, multiply the height by the width by the depth.

3. 56 cubes

4. **a.**



   b. 82

Figure It Out

Problem Solving

Teachers' Notes

Overview: Problem Solving

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About Problem Solving

The nature of this booklet is different to the other seven in the Figure It Out series in that it focuses on students' ability to solve non-routine problems rather than focusing specifically on mathematical content. Some ways in which the problems could be used are:

- "Problem of the day" examples for the introductory part of a lesson
- Homework examples for students and parents to work on together
- Examples that students can use to write their own problems.

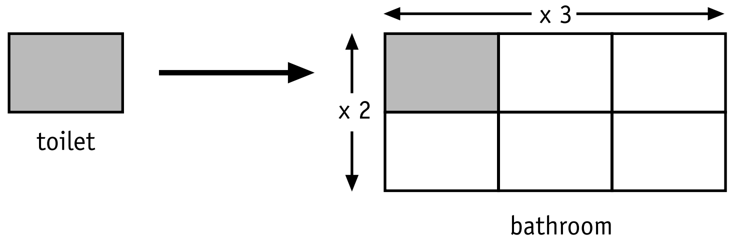
These teacher's notes contain suggestions about how each problem can develop effective problem-solving strategies, powerful reasoning, and communication. The notes do not list the achievement objectives for each problem because effective problem solving involves the combined application of all of the mathematical processes.

In the notes, important strategies are shown as a way of describing how a particular problem might be solved. This is not necessarily the only productive method, and you will need to be receptive to the ideas of your students.

Page 1: All Sorts

Problem One

A physical model of the problem will help students to visualise the comparative area of the toilet and bathroom. Let one square tile represent the area of the toilet:



Therefore, the cost of tiling the bathroom is six times that of the toilet.

The problem can easily be extended by providing different dimensions for enlargement. For example, "The kitchen is twice the size of the bathroom. How much longer and wider could it be than the toilet?"

This would mean that the kitchen has 12 times the area of the toilet. A table is a good way to organise the possibilities:

Width	1	2	3	4	6	12
Length	12	6	4	3	2	1

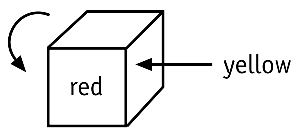
More able students may realise that the dimensions need not be restricted to whole numbers. For example, the kitchen might be five times wider and 2.4 times longer than the toilet.

Problem Two

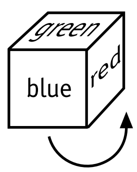
Three-dimensional problems like this are extremely difficult to solve unless some form of model is used. This may be a cube or the net for a cube. Following the pictorial clues and using coloured pens to shade the appropriate faces on the model make solving the problem easier.

Another approach is to use a process of elimination. The left-hand view shows that neither red nor green can be opposite blue because they are adjacent to it. Similarly, the middle view shows that pink is also adjacent to blue. This leaves purple and yellow as the only possibilities left to be opposite blue.

If you roll the right-hand cube forward so that red is facing:



then make a quarter turn to the right:



you can see that the bottom face is purple and the face at the back is yellow. This means that the front face must be blue and the top face must be green. So yellow is opposite blue.

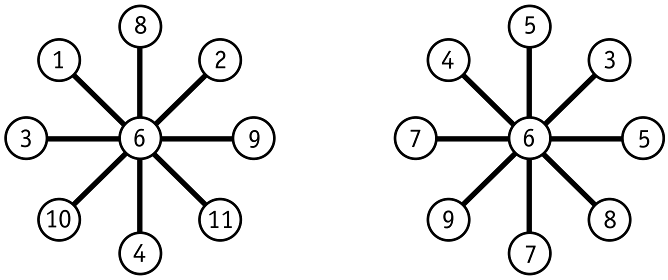
Students will enjoy creating their own cube puzzles for others to solve.

Problem Three

Finding out which number is in the centre circle is the most difficult part of the problem. Students may work this out by trial and improvement, or they may divide 54 (the total of all the numbers in the circle) by nine (the number of circles). This gives six, which is a good choice for the centre circle.

A more rigorous approach is to realise that each row of three circles adds to 18, so the total is 4 x 18 = 72. The circle numbers total 54, and the difference of 18 (72 – 54) is the result of the centre number being counted three extra times. So the centre number must be six.

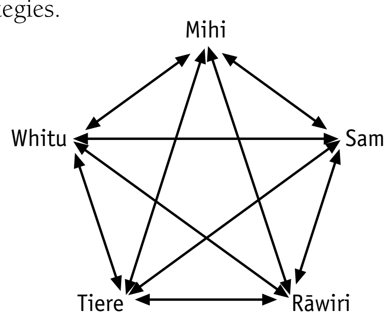
Once this is established, students can find a large number of possible answers. The numbers on opposite ends of a line will always add to 12. For example:



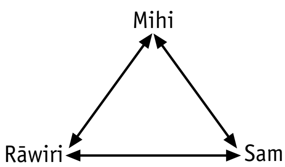
Similar problems are easy to make by changing the conditions. For example, tell students the circle numbers add to 63 and each line of circle numbers adds to 21.

Problem Four

An important deduction is that each child in the Wīpere family will have four other children to buy for, so the total number of presents will be 5 x 4 = 20. Students may find this using a number of strategies.



An arrow diagram is particularly useful, especially if students first use an arrow diagram to solve a simpler problem, for example, fewer children in the family:



A matrix or table is useful when students assume the roles of the Wīpere children to act out the problem.

		Giver				
		Mihi	Sam	Rāwiri	Tiere	Whitu
Receiver	Mihi		✔	✔	✔	✔
	Sam	✔		✔	✔	✔
	Rāwiri	✔	✔		✔	✔
	Tiere	✔	✔	✔		✔
	Whitu	✔	✔	✔	✔	

Encourage students to generalise by asking questions such as: “What if there were six children in the family?” “What about seven children?”

Another variation, which will lead to a set of triangular numbers, is “What if the children give a present only to brothers and sisters who are younger than them?” Students may want to know the age of the Wīpere children, though this is irrelevant. The oldest child will give four presents, the next oldest three presents, and so on. Only four children, not five, will give presents in this scenario because the youngest has no one to give a present to. The number of presents will be 4 + 3 + 2 + 1 = 10. If there were six children in the family, the number of presents would be 5 + 4 + 3 + 2 + 1 = 15.

Page 2: Size-wise

Problem One

Parts **a** and **b** require students to apply their knowledge of place value. Arranging the digits in order so that the largest digit is in the place with the highest positional value (10 000), the next largest is in the thousands column, and so on, produces the largest number. This process is reversed to obtain the smallest number. An interesting concept in **b** is the use of zero as a place holder. Putting zero in the ten-thousands column, as in 02 478, effectively means that it has no place value and therefore does not increase the positional value of the two, as would be the case in 20 478.

Part **c** is difficult and requires students to think analytically. Trial and improvement is unlikely to be a productive strategy on its own. Since the subtraction answer has only hundreds, tens, and ones digits, only three or four of the five digits are significant.

```
  □□□□□□
- □□□□□□
  ------
  ↑  6 5 7
```

both digits must be the same

Encourage students to record strategies in the different scenarios that they try. Eliminating digits using an organised list is a useful way to do this. In this method, digits are crossed out as they are used.

top number digits	2 7 0 4 8
bottom number digits	2 7 0 4 8

Beginning with the ones digits of the answer, 7, gives only two possible arrangements of digits in the ones place.

First possibility:

```
  □□□□□7
- □□□□□0
  ------
     6 5 7
```

top digits	2 ~~7~~ 0 4 8
bottom digits	2 7 ~~0~~ 4 8

This leads to a dead end because it is not possible to use the remaining digits to get a difference of five in the tens column.

Second possibility:

```
  □□□□□4        □□□0 4
- □□□□□7      - □□□4 7
  ------        -------
    6 5 7         6 5 7
```

top digits 2 7 0 ~~4~~ 8 | top digits 2 7 ~~0~~ ~~4~~ 8

bottom digits 2 ~~7~~ 0 4 8 | bottom digits 2 ~~7~~ 0 ~~4~~ 8

Since only three or four digits are involved, the obvious option now is to use seven and zero to end up with six in the hundreds column:

```
  □□7 0 4
- □□0 4 7
  -------
    6 5 7
```

top digits 2 ~~7~~ ~~0~~ ~~4~~ 8

bottom digits 2 ~~7~~ ~~0~~ ~~4~~ 8

This means that the eight and the two can be reversed to form two possible answers, 28 704 and 28 047 or 82 704 and 82 047.

Problem Two

Students may want to begin by experimenting with multilink cubes to solve this problem. However, using triplets of numbers to represent the tower heights is more efficient. For example, (3, 2, 6) can represent three towers with heights of three cubes, two cubes, and six cubes respectively. Students can then work systematically to find the solution. First establish that (3, 2, 6) is the same triplet as (6, 3, 2) and (2, 6, 3) because order is not important in the problem.

The lowest possible tower is two cubes high, so students can work out different arrangements on this basis:

(2, 3, 6) or (2, 4, 5) or ~~(2, 5, 4)~~ or ~~(2, 6, 3)~~ or ~~(2, 7, 2)~~

same as (2, 4, 5) | same as (2, 3, 6) | two towers are the same height

The middle number cannot be eight or higher because one of the other towers would then have to be one cube. So there are only two possibilities that have two as the lowest tower.

Trying three as the lowest tower gives:

(3, 4, ❑) or (3, 5, ❑)

must be greater than three

This leads to (3, 4, 4) and (3, 5, 3) – neither of which is allowed because each tower must be a different height.

If we try to have four as the lowest tower, only seven cubes remain to make the other two towers. At least one of these towers would end up being three or less, so no solutions can be found with four as the lowest tower.

Therefore (2, 3, 6) and (2, 4, 5) are the only possible answers.

Part **b** removes the constraints of having to have different heights and towers that use more than one cube, so the possibilities are all those shown in the answers.

Encourage students to investigate solutions to similar problems, for example, “Make four towers out of a total of 20 cubes”.

Problem Three

In the first instance, students will need to work out how long a one-way walk takes. The walk to and from school takes $1^1/_2$ hours, so a one-way walk takes half that amount of time, which is $^3/_4$ hour or 45 minutes. A bus trip one way with a walk the other way takes 1 hour, so a one-way bus trip must take 1 hour minus $^3/_4$ hour (45 minutes), which is $^1/_4$ of an hour (15 minutes). If Frank caught the bus both ways, it would take 2 x $^1/_4$ = $^1/_2$ hour (30 minutes).

Students may enjoy variations on this problem, such as: "Frank travels to school by either walking, cycling, or taking the bus. If he walks to and from school, it takes him 1 hour in total. The bus takes half the time it takes for him to cycle. If he walks one way and buses back, it takes him 40 minutes in total. How long does it take him to cycle to and from school?"

Problem Four

Students need to identify what information is important and what is relevant before attempting to solve the problem. For example, the T-shirt context is irrelevant because the question only relates to the money taken.

$200 worth of T-shirts at $4 each means that 50 were sold in the morning.
Twice as many (100) were sold in the afternoon, at $3 each. $100 x $3 = $300
This means that $200 + $300 = $500 worth of T-shirts were sold over the whole day.

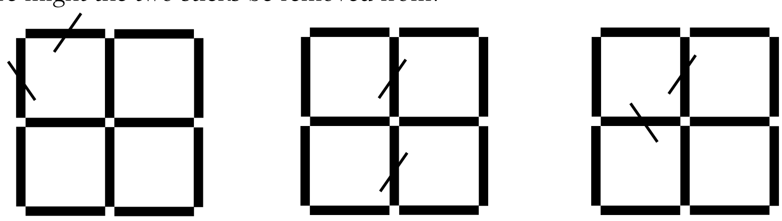
Once students have experienced this type of problem, encourage them to write their own. An example might be: "Jenny sold 40 bags of lollies at $2.50 each in the morning and sold half as many at twice the price in the afternoon. How much money did she take in the day?"

Page 3: Pattern Teasers

Problem One

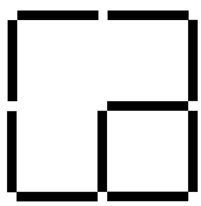
Students may want to solve this problem using nursery sticks as a physical model. The problem can be approached systematically in a number of ways.

Where might the two sticks be removed from?



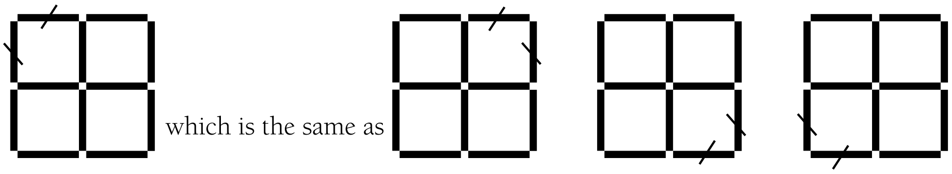
Are the remaining shapes squares?

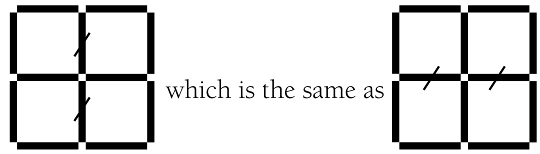
Removing two sticks leaves 10 sticks behind, which limits the possible sizes of the squares.
A 2 x 2 square takes eight sticks. A 1 x 1 square takes four sticks. A combination of one each of these sizes seems the most likely, with two sides of the 1 x 1 square shared:

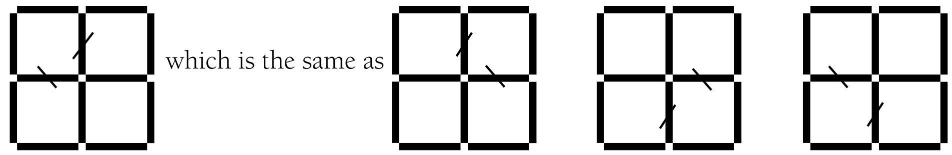


Students need to know the meaning of polygon to answer part **b**. The origin of polygon is "many angles". Its meaning is a closed shape with an unspecified number of angles. The polygons in part **b** therefore must be bounded by sticks. This limits the places where two sticks can be removed from because every stick must be connected to another stick at both ends.

The sticks that can be removed are:

i. 

ii. 

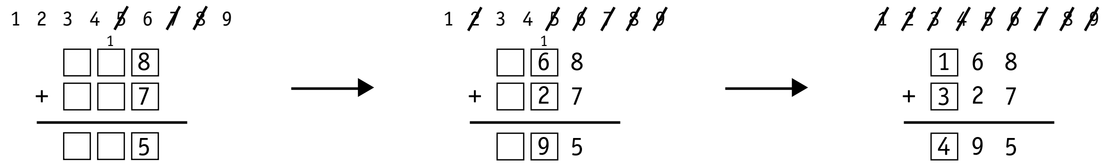
iii. 

Option i leaves three squares.
Option ii leaves two rectangles.
Option iii leaves two squares (as in part **a**).

Problem Two

Encourage the students to record the strategies they use in developing their solutions. They could start by filling in the ones column. Remind them that putting six and seven in the ones column will result in a 10 being carried over to the tens column.

One strategy might be:



Note that in the second step of the example given above, nine is the crucial digit to place. It can’t go in the tens place of either addend because the carried 10 would mean that a digit is used twice.

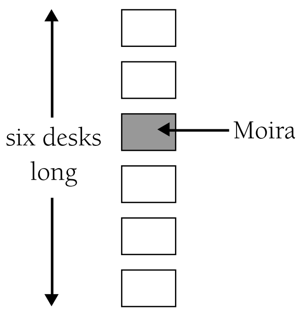
Once students have found one solution, they can easily produce another by rearranging the columns.

$$\begin{array}{r}168\\+\ 327\\\hline 495\end{array} \quad \text{also gives} \quad \begin{array}{r}681\\+\ 273\\\hline 954\end{array}$$

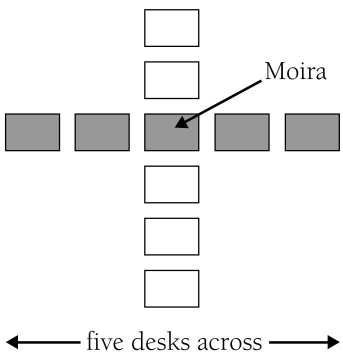
Problem Three

Students may like to use square tiles to represent the desks. They can build up the classroom as they consider the clues:

two desks in front and three behind



two desks to the right and two desks to the left



So, the full classroom is 6 desks x 5 desks = 30 desks. This is an example of a multiplication array. Students' understanding of arrays can be enhanced by trying similar problems. For example, "In Angelo's classroom, there are twice as many desks in each row across as there are in each line going down. There are 32 desks in total. How long is each row?"

Problem Four

Making a table is a useful strategy for solving this problem.

Years	0 (now)	1	2	3	4	5	6
Mr Smith	30	31	32	33	34	35	**36**
Sidney							

← this is
← three times this

So

Years	0 (now)	1	2	3	4	5	**6**
Mr Smith	30	31	32	33	34	35	**36**
Sidney	6	7	8	9	10	11	**12**

So Sidney is currently 6 years old.

Similarly, students may use a trial and improvement strategy:

Sidney is 3 so In 6 years' time, he'll be 9. so Mr Smith will be 3 x 9 = 27. He is 30 now!

Sidney is 10 so In 6 years' time, he'll be 16. so Mr Smith will be 48. That makes him 42 now!

Students can improve their attempts until they find Sidney's current age.

Page 4: Birthday Surprise

Problem One

This problem is ideal for using a number of different strategies in combination. For example, consider a trial and improvement, working backwards, table strategy. Students may realise, perhaps through trial and error, that only multiples of eight in the first column give a whole number in the last column.

Lollies in jar	$^1/_2$ eaten	$^3/_4$ of $^1/_2$ eaten	8 left?
20	10	7.5	$2^1/_2$ ✘
80	40	30	10 Too high
56	28	21	7 Too low
64	32	24	8 ✔

This could be recorded in an equation like this:

(❑ ÷ 2) ÷ 4 = 8

Students may use an undoing (opposite) operation to solve it:

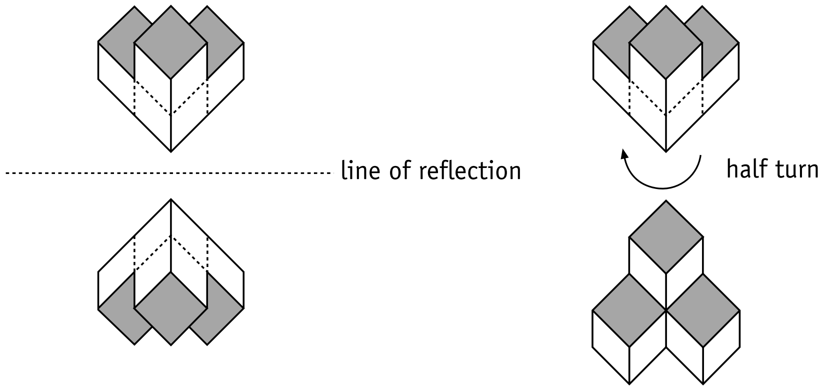
(❑ ÷ 2) ÷ 4 = 8 so ❑ ÷ 2 = 8 x 4
so ❑ = 32 x 2
so ❑ = 64

Writing equations is not a natural method for most students. If students do use the strategy, it is worth encouraging them because it demonstrates that they have a useful understanding of algebraic ideas.

Problem Two

Although students may prefer to build each model using multilink cubes, ask them as a first step to try to recognise the matching models. They can identify the characteristics of a shape and look for a matching model.

For example, **a** is an L shape with one arm of three cubes and the other of two cubes. Shape **g** has similar characteristics. The top view of shape **b** can be represented like this, $\begin{smallmatrix} & 2 \\ 1 & 1 \end{smallmatrix}$, where the number shows the number of cubes in each column. Shape **h** has the same representation as **b** if it is turned a quarter turn anticlockwise. Similarly, **c** can be represented as $\begin{smallmatrix} 2 & \\ 1 & 1 \end{smallmatrix}$, which is a quarter turn anticlockwise of shape **e**. This leaves shape **d** as the same as shape **f**. Notice that **d** and **f** are reflections or half turns of each other:



Problem Three

Students may find it helpful to use a hundreds board. Encourage them to work systematically to make their counting more efficient. They could use a table.

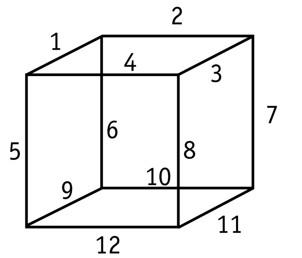
House numbers	Digits									
	0	1	2	3	4	5	6	7	8	9
1–10	1	2	1	1	1	1	1	1	1	1
11–20	1	10	2	1	1	1	1	1	1	1
21–30	1	1	10	2	1	1	1	1	1	1
31–40	1	1	1	10	2	1	1	1	1	1
41–50	1	1	1	1	10	2	1	1	1	1
51–60	1	1	1	1	1	10	2	1	1	1
61–70	1	1	1	1	1	1	10	2	1	1
71–80	1	1	1	1	1	1	1	10	2	1
81–90	1	1	1	1	1	1	1	1	10	2
91–100	2	2	1	1	1	1	1	1	1	10
Total	11	21	20	20	20	20	20	20	20	20

Patterns within the table allow it to be completed easily. Students can check they have completed the table correctly by finding out how many digits are needed in total. There are nine single-digit numbers, 90 two-digit numbers, and one three-digit number.

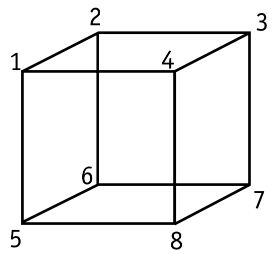
This gives a total of (9 x 1) + (90 x 2) + 3 = 192.

Problem Four

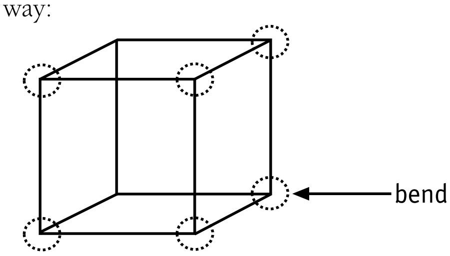
a. A cube has 12 edges. Each edge is 1 metre long, so Priscilla needs 12 metres of pipe.



b. The cube will need a join at each corner (vertex). Since a cube has eight vertices, it will have eight joins.

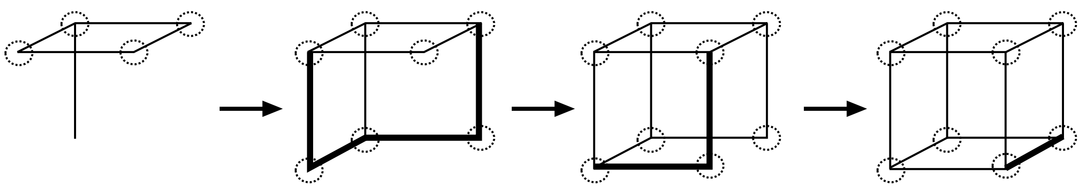


The number of bends depends on how many lengths of plastic tubing are used. If all 12 edges are cut separately, there are no bends. If six lengths are cut, the cube can be made in the following way:



Two of the lengths form the top and bottom squares. The top and bottom squares require three bends each, giving a total of six bends.

As an extension, you could ask students to work out the minimum number of lengths needed to make a cube. The answer is four. The cube can be made in this way:



In this case, eight bends are needed.

Page 5: Exercise Time

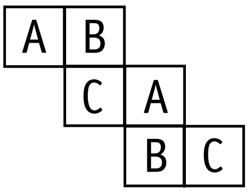
Problem One

Students will need to know that 1 tonne = 1 000 kilograms. So a 3 tonne log has a mass equivalent to 3 000 kilograms. Since one elephant can pull 550 kilograms, six elephants can pull 6 x 550 = 3 300 kilograms.

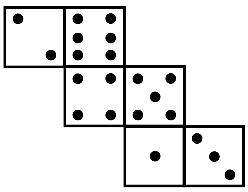
Students could also divide the total mass of the log by the mass each elephant can pull.
3 000 ÷ 550 = 5.45, so six elephants are needed.

Problem Two

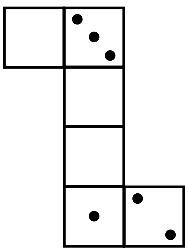
The net shown is not the T shape most commonly used to form a cube. Students may need to build a cube from the net so that they realise how it fits together. Once they know this, they can visualise what the opposite faces will be.



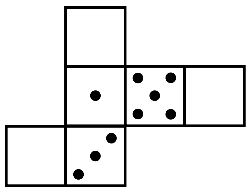
The pairs of opposite faces are shown as A, B, and C. Each pair must have a total of seven dots, so the complete net must be:



Students may enjoy creating their own dice net puzzles, such as:



or



Some students may need to make a blank net and fold it to establish which faces are opposite.

Problem Three

This problem involves the concept of common multiples. Room 8 can form an exact number of teams of five for netball, so the possible numbers of students are 5, 10, 15, 20, 25, 30, 35, … They can also form an exact number of teams for mini hockey, so the possible numbers of students are 6, 12, 18, 24, 30, 36, …

The common multiples of 5 and 6 give the number of possible students in Room 8. That is, 30, 60, 90, … Classes usually comprise about 30 students, so this is the most likely answer.

Providing similar types of common multiple problems will help students to generalise the mathematics involved. For example, "Terry has a stamp collection. He can arrange three stamps on a page of his album with none left over. He can also arrange four on a page with none left over or five on a page with none left over. He has fewer than 100 stamps. How many has he got?" (60).

Problem Four

As with **Problem Three**, this involves the concept of common multiples. The landing marks of each rabbit are:

Red rabbit	30, 60, 90, 120, 150, 180, …
Purple rabbit	20, 40, 60, 80, 100, 120, …
Pink rabbit	25, 50, 75, 100, 125, 150, …

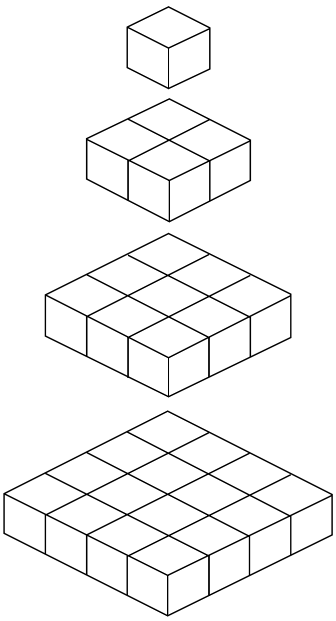
Although students could continue the multiple patterns to see whether the rabbit lands on the 600 metre mark, dividing by the jump distance is more efficient: 600 ÷ 30 = 20 is the number of jumps for the red rabbit, 600 ÷ 20 = 30 is the number of jumps for the purple rabbit, and 600 ÷ 25 = 24 is the number of jumps for the pink rabbit. In each case, there is no remainder, so the rabbits all land on the 600 mark.

As an extension, give the students other rabbits to consider. For example:
"The green rabbit jumps 40 centimetres each time." (600 ÷ 40 = 15 jumps)
"The brown rabbit jumps 75 centimetres each time." (600 ÷ 75 = 8 jumps)
"The blue rabbit jumps 45 centimetres each time." (600 ÷ 45 = 13.33 jumps) The blue rabbit will not land exactly on the 600 centimetre mark.

Page 6: Layer upon Layer

Problem One

Many students will need to build the model with cubes. This is good visualisation in itself because it involves interpreting a diagram of a solid object. Some students may realise that they can calculate the number of cubes in each layer of the building.



1 x 1 = 1

2 x 2 = 4

3 x 3 = 9

4 x 4 = 16

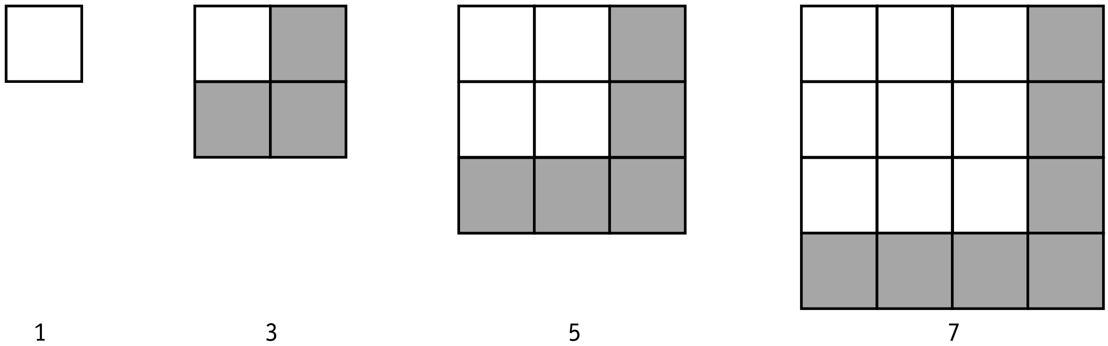
Total = 30

The next layer will be a 5 x 5 square, so it will be made of 25 cubes, giving a total of 55 cubes. Students may be interested in the pattern created by the square numbers, which are central to this problem:

1   4   9   16   25

+ 3   + 5   + 7   + 9

Each difference is the number of extra blocks needed to make each successive square:



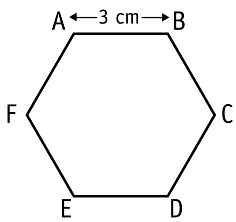
The first block, which becomes a corner square, ensures that each difference is an odd number.

Problem Two

Each side of the pentagon is 2.5 centimetres long, so the total perimeter is 5 x 2.5 = 12.5 centimetres. Twice around the pentagon will be 25 centimetres.
Multiplication and division can be used to solve the problem efficiently instead of repeatedly adding 25. Two possible solution strategies are:

1. 4 x 25 = 100 cm (eight times around)
   (6 x 4) x 25 = 600 cm (48 times around)
   600 + 25 = 625 (50 times around)
   Since 50 round trips end exactly on 625 centimetres, Wiremu Wēta's journey begins and ends at A.
2. 625 ÷ 25 = 25 double trips, which is 50 trips in total.
   There is no remainder, so Wiremu Wēta begins and ends on A.

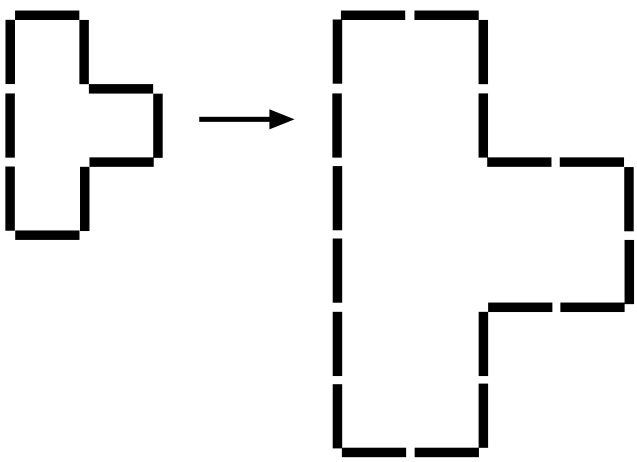
Students may enjoy variations on this type of problem. For example:



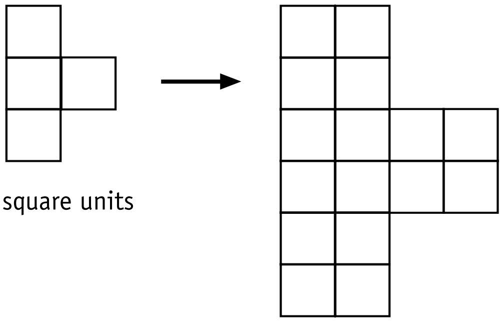
Wiremu Wēta begins at A and walks clockwise for 726 centimetres round and round this regular hexagon.
Which letter does he finish at? (Answer: C)

Problem Three

Nursery sticks can be used to model the figure and its enlargement.



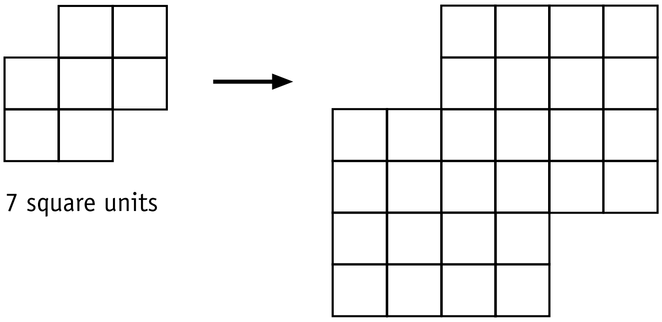
Ten sticks were needed to make the original figure, so twice as many sticks (20) are needed to make the enlarged figure. The diagram below shows that the area of the shape increases four times.



4 square units

16 square units

Students may wish to investigate whether this also occurs with any closed figure when the side lengths are doubled. For example:



7 square units

28 square units

A generalisation is that when the side lengths of a figure are doubled, its area increases by a factor of four.

Problem Four

Some students may look for a pattern from left to right if they do not read the question carefully. The different pattern this shows is a useful discussion point.

Following the pattern from right to left shows:

14 15 12 13 10 11

− 1 + 3 − 1 + 3 − 1

Following the pattern from left to right shows:

14 15 12 13 10 11 8

+ 1 − 3 + 1 − 3 + 1 − 3

Students may need more experience using the difference technique to analyse number sequences. For example:

1 3 6 10 15 ? (triangular numbers)

+ 2 + 3 + 4 + 5

1 2 4 8 16 ? (powers of two)

+ 1 + 2 + 4 + 8

Page 7: Digit Detail

Problem One

Encourage students to look for a systematic strategy to find all the possible solutions. They could use an organised list, a tree diagram, or a table. Here is how a table might be used to find all the possible solutions.

Hundreds digit	Tens digit	Ones digit	Number
0	7 8 9	9 8 7	79 88 97
1	6 7 8 9	9 8 7 6	169 178 187 196
2	5 6 7 8 9	9 8 7 6 5	259 268 277 286 295
3	4 5 6 7 8 9	9 8 7 6 5 4	349 358 367 376 385 394
4	3 4 . . .	9 8 . . .	439 448 . . .

If students find this problem difficult, start with a digit problem that has fewer solutions, such as:
"The digits of 57 add to 12. What other two-digit numbers have a digit sum of 12?"
"What three-digit house numbers have a digit sum of 21?"
Using digit cards may be helpful for some students as they try various possibilities.

Problem Two

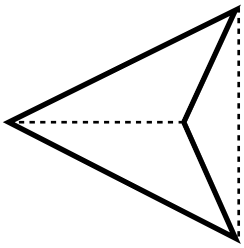
Students might use a strategy of trying all the denominations of coins available and eliminating those that don't work.

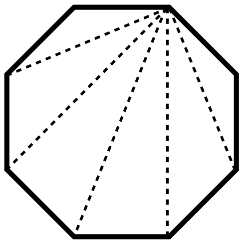
Coin	Four coins with same total?
5c	Smallest coin, therefore impossible
10c	Two 5c coins is the only exchange possible
20c	5c + 5c + 5c + 5c
50c	20c + 20c + 5c + 5c 20c + 10c + 10c + 10c
$1	50c + 20c + 20c + 10c
$2	50c + 50c + 50c + 50c

Only 20 cents and $2 can be made with four identical coins. However, 50 cents can be made with four coins in two ways and $1 can be made in one way.

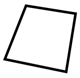
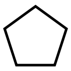
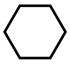
Problem Three

Students will need to understand that a diagonal connects a corner (vertex) of a polygon with any corner that is not adjacent to it. For example, these are diagonals:



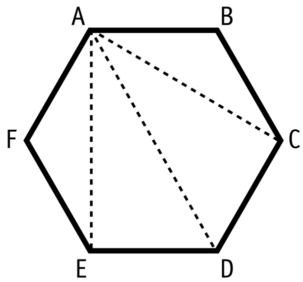


Students need to approach the problem systematically and organise the results in a table or list. This will help them to identify patterns.

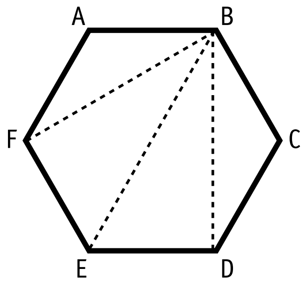
Shape	Sides/corners	Diagonals
	3	0
	4	2
	5	5
	6	9
.	.	.
.	.	.
.	.	.

The table will show that the number of diagonals increases markedly with each side added, so the sides/corners and diagonals will not be equal for any other polygon.

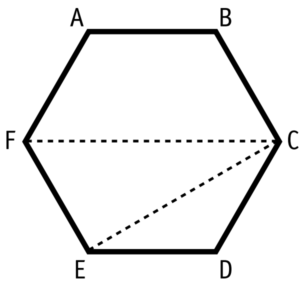
Students may be interested in looking at ways to work out the number of diagonals for a given polygon. Consider a hexagon:



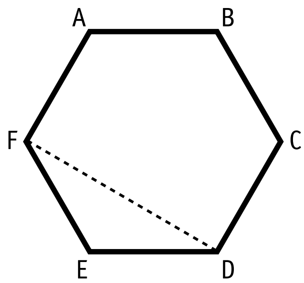
Three diagonals connect with vertex A.



Three additional diagonals connect with vertex B.



Two further additional diagonals connect with vertex C.



One more diagonal connects with vertex D.

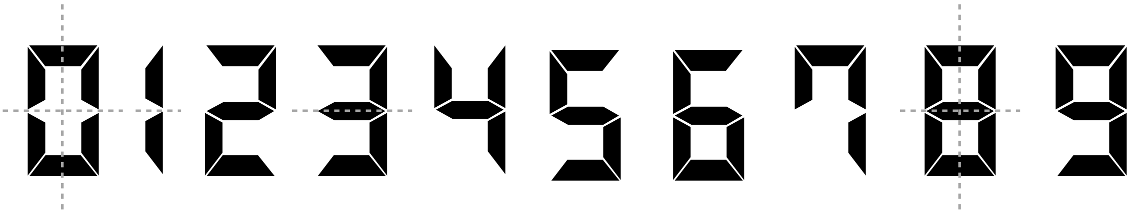
So a hexagon has 3 + 3 + 2 + 1 = 9 diagonals.

This thinking can be used to show that a heptagon (seven-sided polygon) has 4 + 4 + 4 + 3 + 2 + 1 = 14 diagonals.

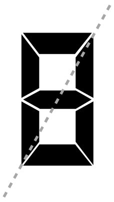
Problem Four

The digits shown in this problem are slightly different from those shown on some calculators. Some displays show the figures on a slight slant.

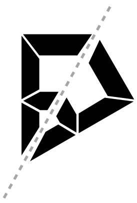
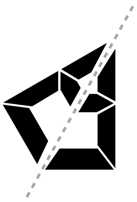
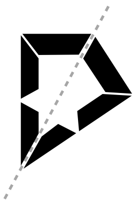
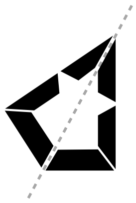
Some students may need a mirror to find the lines of symmetry by looking for mirror positions where part of the digit is masked yet the digit appears whole, as in the diagram below.
Vertical and horizontal lines are the easiest to find.



Some students may be confused by diagonal lines of symmetry. For example:

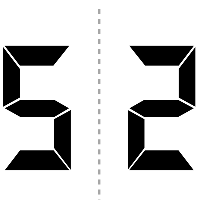
 and  are not lines of symmetry because

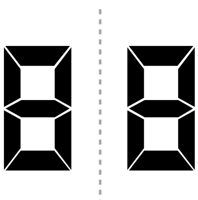
when viewed in a mirror (depending on which way the mirror faces), the digits appear as:

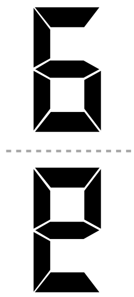
 or  and  or 

for 8 for 0

When students put the mirror beside the digits, they will see that some digits are mirror images of other digits while others are their own image.

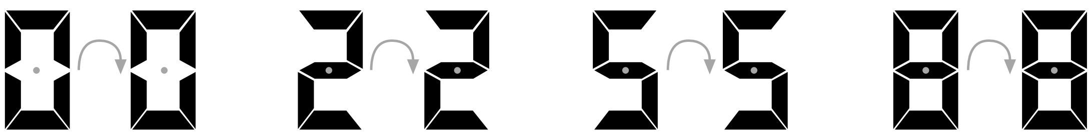






The digits 6 and 9 are not quite mirror images of each other.

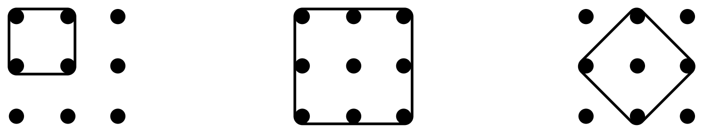
A similar analysis can be made of the rotational symmetry (turn symmetry) of each digit.



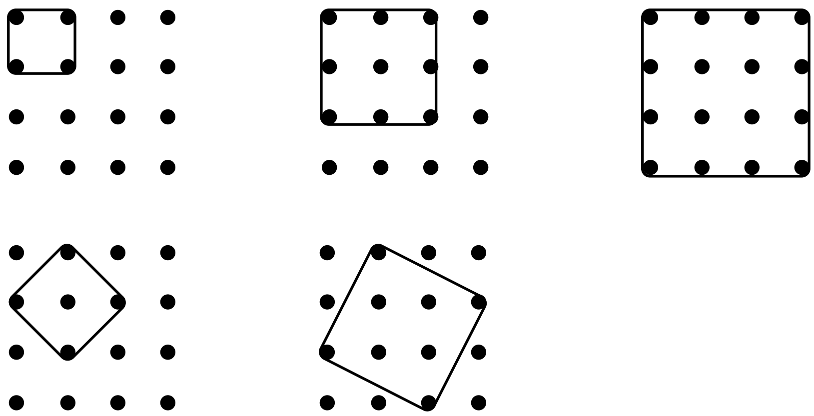
Each of the digits 0, 2, 5, and 8 has half-turn symmetry.

Problem Five

Diagonal squares are often overlooked by students. With a 3 x 3 geoboard, the possible squares are:



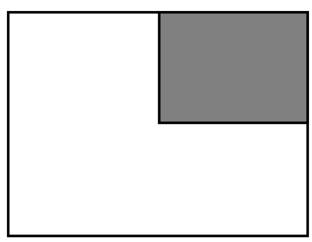
Moving to a 4 x 4 geoboard increases the number of possible squares. Students will need to adopt a systematic strategy, such as finding all the squares involving a corner pin and then finding all the squares involving a centre-side pin.



Page 8: Upside Down

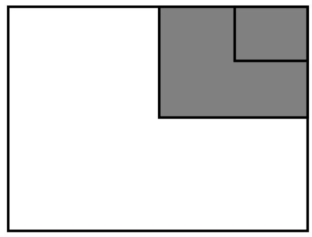
Problem One

Encourage students to visualise and use reasoning before they measure the rectangles, though this measuring can be used to confirm their thinking. They will have to use fractional knowledge in this problem.



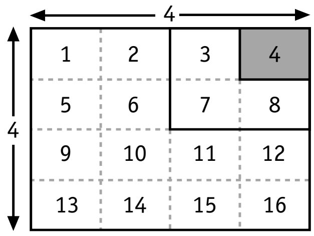
The shaded rectangle is a quarter of the large rectangle.

so:

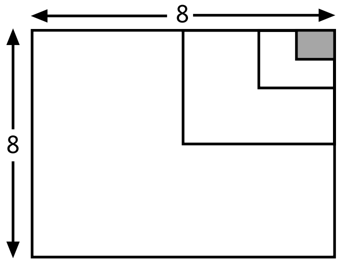


The smallest rectangle is a quarter of a quarter, which is one-sixteenth ($\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$).

This could be confirmed by fitting the small rectangle into the large one 16 times, as shown.



Using similar reasoning, a quarter of the small rectangle must be $^1/_4$ of $^1/_{16}$, which is $^1/_{64}$ (one sixty-fourth).



Problem Two

Sarah will receive $2 – $1.20 = 80c in change. Students will need to look for combinations of four coins that add to 80 cents. The results can be organised into a table to eliminate combinations that don't work and to avoid duplication.

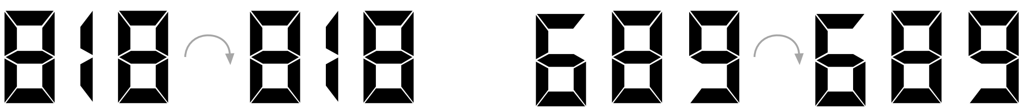
Coins				Number of coins
5c	10c	20c	50c	
2		1	1	4
	3		1	4
		4		4

Systematically working through by starting with combinations involving 5 cent coins will find all the possibilities.

Problem Three

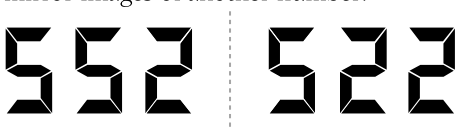
This problem uses rotational symmetry. All the numbers used in the expression are either:

- images of themselves after a half turn, for example:



or

- mirror images of another number.



The matching numbers are shown below.

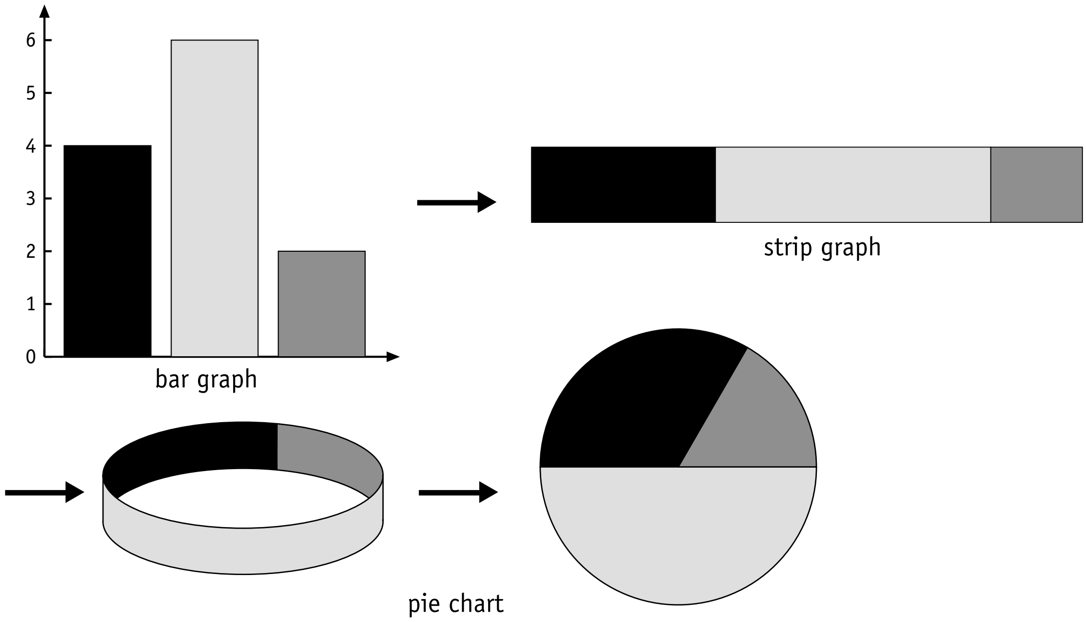
As shown: 552+818+255+689+181+986

Upside down: 986+181+689+552+818+255

The answers are the same because the same numbers are added.

Problem Four

One way to confirm that these graphs are showing the same data is to imagine how the bar graph could be transformed into a pie graph.



Another method is to apply fractional knowledge to the numbers involved. The bars have heights of 4, 6, and 2 respectively, so there are 4 + 6 + 2 = 12 items of data. The red bar has height 4, so it is $^4/_{12}$ or $^1/_3$ of the number of the data items. The blue has height 6, so it is $^6/_{12}$ or $^1/_2$ of the data items. The pie chart shows fractions of $^1/_3$, $^1/_2$, and $^1/_6$, so the representations match.

Page 9: Ps for Ever

Problem One

Students may need squared paper to draw the rectangles they are investigating. Their results can be organised in a table so that they can look for patterns and eliminate possibilities.

Length	Width	Perimeter	Area
1	1	4	1
1	2	6	2
1	3	8	3
1	4	10	4

Rectangles with two sides of 1 centimetre will always have a perimeter that is greater than the area. Investigating rectangles with lengths of 2 centimetres gives the following results:

Length	Width	Perimeter	Area
2	2	8	4
2	3	10	6
2	4	12	8
2	5	14	10

The perimeter is always four greater than the area.

Investigating rectangles with lengths of 3 centimetres gives:

Length	Width	Perimeter	Area
3	3	12	9
3	4	14	12
3	5	16	15
3	6	18	18

So, a 3 x 6 rectangle has equal perimeter and area. Similar reasoning will show that a 4 x 4 rectangle also works.

If students include decimal values for the side lengths, other solutions are possible, such as 2.5 x 10.

Problem Two

Sachin needs 19 kilograms of rice, but the mass of the bags does not allow him to buy the exact amount. Students will need to systematically list the ways in which Sachin could buy at least 19 kilograms and calculate the cost of each way.

Amount (bags)	Total amount	Cost	Total cost
7 x 3 kg	21 kg	7 x $2.30	$16.10
6 x 3 kg and 1 x 5 kg	23 kg	(6 x $2.30) + $3.25	$17.05
5 x 3 kg and 1 x 5 kg	20 kg	(5 x 2.30) + $3.25	$14.75
4 x 3 kg and 2 x 5 kg	22 kg	(4 x 2.30) + (2 x $3.25)	$15.70
4 x 5 kg	20 kg	4 x $3.25	$13 ✔

Ask students what is the cheapest way of buying other amounts of rice. For example, to buy 15 kilograms, three 5 kilogram bags are cheapest, but to buy 12 kilograms, four 3 kilogram bags are cheaper than three 5 kilogram bags. However, the cheapest way to buy 12 kilograms is one 3 kilogram bag and two 5 kilogram bags.

Problem Three

Students can work with sequential patterns in this problem. Use of relationships can simplify the problem.

For example, using a table gives:

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Number	1	2	3	4	5	6										

The number of each letter increases by one for each subsequent letter. P is the 16th letter, so there will be 16 of them. Z is letter number 26, so there will be 26 of them. The total number of letters is therefore:

$$1 + 2 + 3 + 4 + 5 + \ldots + 22 + 23 + 24 + 25 + 26$$

These numbers can be paired into numbers that add to 27 (that is, 1 + 26, 2 + 25, 3 + 24, 4 + 23, 5 + 22). There are 13 such pairs, so the total number of letters is 13 x 27 = 351.

Problem Four

Students need to work backwards from the total cost of postage in an organised way.

Letters	Postcards	Cost
6	0	$2.70 ✔

Three postcards can be sent for the cost of two letters, so other solutions involve decreasing the number of letters by two and increasing the number of postcards by three.

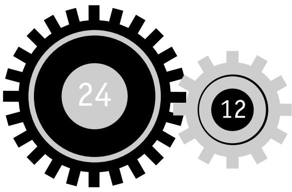
Letters	Postcards	Cost
4	3	$2.70
2	6	$2.70
0	9	$2.70

You can set similar problems by varying the total amount paid and the cost of sending each article. For example, "Isabella spent $6 on postage. Letters cost 40 cents to send, and postcards cost 25 cents. How many of each article could she have sent?"

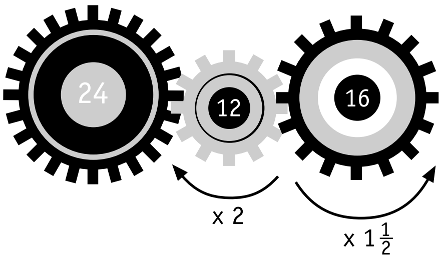
Page 10: Locomotive Magic

Problem One

In the gear train shown, cog B acts only as a transmitter. As cog A turns one turn anticlockwise, cog B will make two turns clockwise.



As cog B makes two turns clockwise, cog C will be driven to turn $1^1/_2$ turns anticlockwise. This happens because cog B has three-quarters the number of sprockets as cog C, so two turns of cog B make $1^1/_2$ turns of cog C.

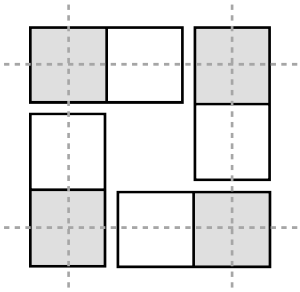


Therefore, in four turns of cog A, cog C will turn $1^1/_2$ as many turns. $4 \times 1^1/_2 = 6$ turns

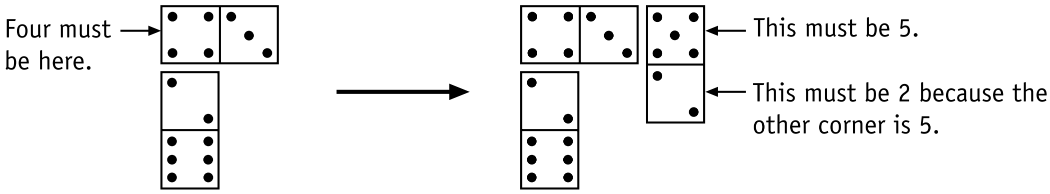
Gear train relationships are easily observed in construction toys. They occur in many everyday situations, particularly in cars, bicycles, and motor-driven machinery.

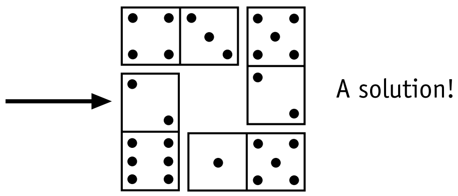
Problem Two

Many students will find making a paper copy of the dominoes useful so that they can move them around to find a solution. Students who know the way dominoes are numbered should be able to use this logically to find a solution.



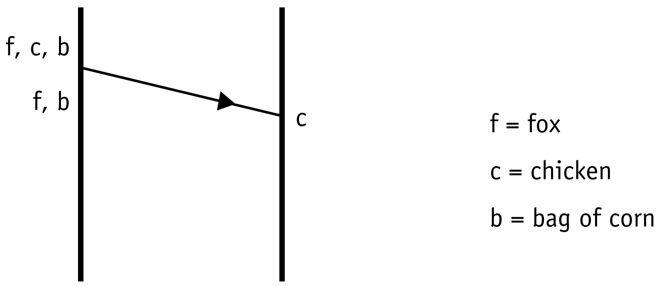
If the four sides each have 12 dots, this gives a sum of 4 x 12 = 48 dots. The dots in each corner square are counted twice to reach this 48-dot sum. In total, there are 28 dots on the four dominoes. The corner squares must account for the difference of 20 dots (48 – 28). Working on this premise, the only numbers that can go in the corner square are 5, 4, 5, and 6. Students can place the corner dots and then work out their solution knowing that the side total is 12 dots.



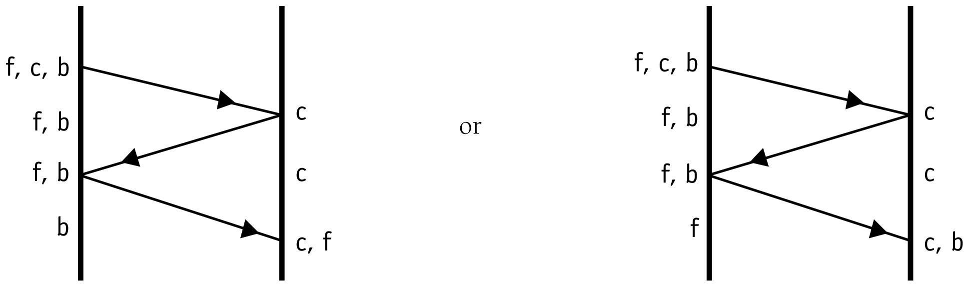


Problem Three

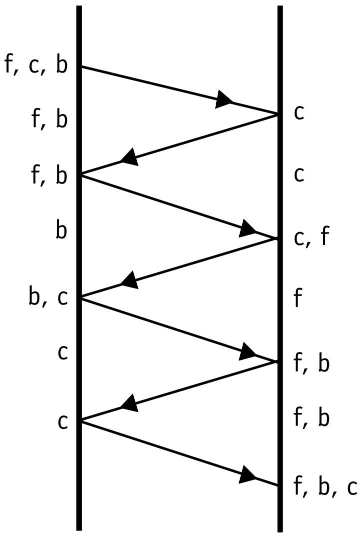
This traditional problem provides the ideal opportunity to use an act-it-out strategy. Students can assume the role of the characters and try out possible solutions. Alternatively, students could use counters for the characters or use some form of recording strategy. For example:



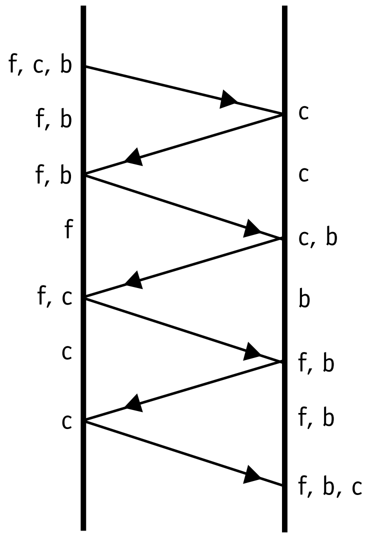
The only possible first move is to take the chicken across because all other trips leave a disastrous couple behind.



Two possible moves can occur in this next move: either the farmer returns to take the fox across or he returns to take the bag of corn across. The next move is to bring the chicken back as it cannot be left with either the fox or the bag of corn.



or



In either case, the chicken is brought back, the fox or the corn is taken across, and the farmer returns to pick up the chicken.

Problem Four

The problem can be acted out with cut-down plastic drink bottles used as the buckets.
A systematic recording strategy is needed to keep track of the quantities.

An important piece of reasoning is that the only way that 5 litres can be measured is if 2 litres are already in the 3 litre bucket or there is a way of pouring exactly 2 litres from a full 7 litre bucket.

Encourage students to use a structured recording system such as the one shown below:

7 L bucket ⟶ 7 L | 0 7 ⟵ means fill up the 7 L bucket
3 L bucket ⟶ 3 L | 0 0 ⟵ means nothing in the 3 L bucket

Below are two possible solution strategies that use this recording system.

1.

7 L	0	7	4	4	1	1	0	7	5
3 L	0	0	3	0	3	0	1	1	3

Empty the 3 L bucket.

Fill the 3 L bucket from the 7 L bucket.

Fill it again.

Fill the 3 L bucket, which already has 2 L of water in it.

2.

7 L	0	0	3	3	6	6	7	0	2	2	5
3 L	0	3	0	3	0	3	2	2	0	3	0

Fill the 3 L bucket.

Pour the remaining 2 L from the 3 L into the 7 L bucket.

Page 11: Challenging Times

Problem One

From 12.30 p.m. to 3.30 p.m. is a period of 3 hours. In this time, Stanley's watch loses 3 minutes. There are eight periods of 3 hours in a 24 hour day, so his watch will lose $8 \times 3 = 24$ minutes.

As an extension, other time problems could be posed. The difficulty of each problem will depend on the complexity of the rate involved. For example:
"Tania set her watch to the radio time at 12.00 p.m. When she checked later, she noticed that when the announcer said the time was 9.00 p.m., her watch showed 9.06 p.m. How many minutes would her watch gain over a 24 hour period?"

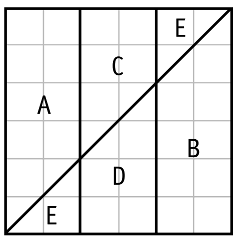
In this problem, the rate of time gain or loss is much more difficult. It can be represented as:

Time period	Time gain
9 hours	6 minutes
24 hours	?

In this case, the watch gains $^{2}/_{3}$ of a minute every hour. Students may deduce that in 3 hours, the watch will gain 2 minutes. There are 8 periods of 3 hours in a day, so the total number of minutes gained will be $8 \times 2 = 16$ minutes.

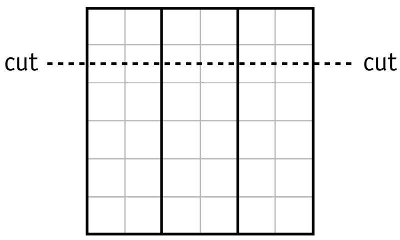
Problem Two

The concept of area is important in this problem. Consider the birthday cake being this size, with the cuts shown and each piece labelled:



Pieces A and B are the largest, with an area of 10 square units. There are 36 square units in total, so these pieces are each larger than a quarter (nine square units). Whā's cut does not work.

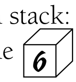
In order to make quarters, Whā must create areas of nine square units, using the area model shown below.



Each piece before Whā's cut is 12 square units. So three square units must be removed from each piece. This can be done by a horizontal cut as shown. In general, the cut needs to be $^{1}/_{4}$ of the way down each third.

Problem Three

For all the stacks to have the same totals, all the numbers must add to a multiple of three:
$3 + 4 + 5 + 7 + 10 + 1 + 9 + 6 + 9 = 54$.

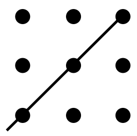
Dividing 54 by three gives the total needed for each stack: $54 \div 3 = 18$. The picture shows stack totals of 12, 18, and 24 respectively. To balance these, the 

will need to be shifted from the right stack to the left stack.

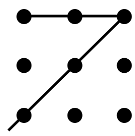
Problem Four

This type of problem is often referred to as a "Eureka" problem – it requires a flash of insight to solve. You could suggest to students that they "think outside the square"!

Only four connected lines are allowed and there are nine dots, so each line will need to pass through the maximum number of dots (an average of $2^1/_4$ dots). If we try to go through the maximum number of dots each time, we might get this:

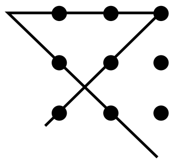


3 dots taken

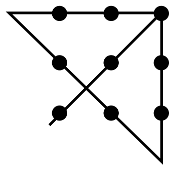


5 dots taken

At this point, the next line can go through only one dot. This will not be sufficient:



Extending the second line means the third line goes through two dots.

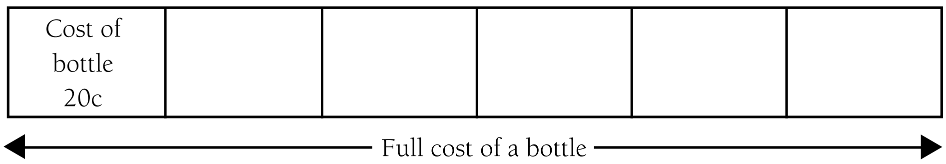


This leads to the final solution.

Page 12: Thirsty Work

Problem One

If the bottle is worth 20 cents, the cost of a full bottle is six times that: 6 x 20c = \$1.20.
This means that the soft drink is worth \$1 (\$1.20 – 0.20). Another way to think of the problem is to use fractions.

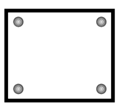


One-sixth of the cost of the full bottle is 20 cents, so the remaining five-sixths must cost 5 x 20c = \$1.

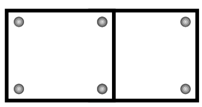
Students may enjoy similar ratio problems, such as: "There are half as many ducks as geese in the farmyard. Altogether there are 27 birds. How many geese are there?"

Problem Two

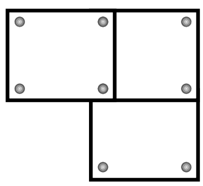
Students need to find which arrangement of the notices minimises the number of pins needed. Using sheets of paper with counters to represent the pins will help model the problem. Adding each new sheet by using the least number of pins leads to the best solution:



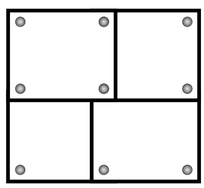
1 sheet
4 pins



2 sheets
6 pins

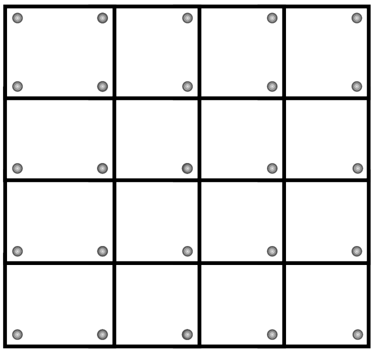


3 sheets
8 pins



4 sheets
9 pins

Continuing in this way, students will see that a rectangular arrangement will use the least number of pins.



16 notices
25 pins

Note that a rectangular arrangement may not be possible if there is a large variety in the size and shape of the notices.

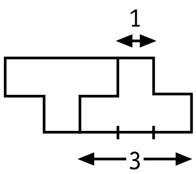
This problem can be extended to find the least number of pins for other arrangements of 16 newsletters, such as in groups of four or one central newsletter surrounded by groups of three.

Problem Three

Applying algebraic reasoning will help to simplify the problem. In particular, making a table allows the student to use number patterns.

Number of Ts	1	2	3	…	10
Perimeter	10	14	18	…	?

With each T that is added, the perimeter increases by 4 centimetres.



The diagram above shows the position of the 4 centimetres added each time.

Students can extend the table to find the perimeter values by adding four each time.

10 14 18 22 26 30 34 38 42 46

+ 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4

More advanced students will realise that they can multiply by four. For example,

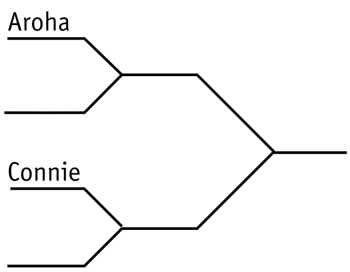
10 + (9 x 4) = 46

perimeter of first T — nine additions of four

Problem Four

Students will need to apply the clues logically and use a systematic recording strategy. Making name tags for the players can be useful when trying out possibilities.

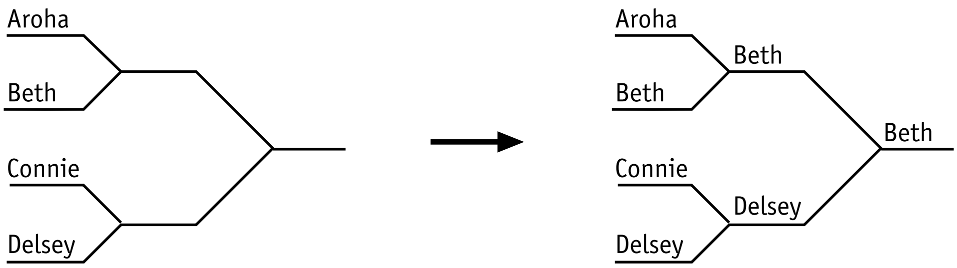
Here is one way of working through the problem:



Aroha did not play Connie.

This leaves two possible placements for the other players.

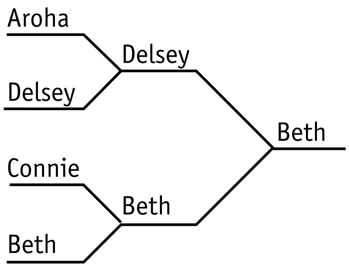
Possibility 1



The second clue is that Beth beat Delsey.

This is only possible if they both won their first round games and met in the final.

Possibility 2



Similarly, the only way that Beth can beat Delsey is if they meet in the final.

Page 13: Simple Solutions

Problem One

Students should observe a recursive (one after the other) pattern in the digits in this set of equations.

$$(1 \times 8) + 1 = 9$$
$$(12 \times 8) + 2 = 98$$
$$(123 \times 8) + 3 = 987$$
$$(1\ 234 \times 8) + 4 = 9\ 876$$

The next largest digit is put in the ones place, while the other digits are shifted one place left.

The next smallest digit is put in the ones place, while the other digits are shifted one place to the left.

As anticipated, the solution to the next equation follows the pattern:
(12 345 x 8) + 5 = 98 765.

Students can use two different strategies to predict the solution to (123 456 789 x 8) + 9.

1. Use recursion by continuing the set of equations:

$$(123\ 456 \times 8) + 6 = 987\ 654$$
$$(1\ 234\ 567 \times 8) + 7 = 9\ 876\ 543$$
$$(12\ 345\ 678 \times 8) + 8 = 98\ 765\ 432$$
$$(123\ 456\ 789 \times 8) + 9 = 987\ 654\ 321$$

2. Direct reasoning:

these digits are the same

(123 456 x 8) + 6 = 987 654

The answer has the same number of digits as the initial number, but starts with 9 and continues down.

Problem Two

Students will need to remember to include Mrs Brown as the seventh person when solving this problem. Some students may need equipment to model the problem. However, encourage students to use number facts to find solutions. These could be organised in a list to simplify the problem (see the table in the answers section).

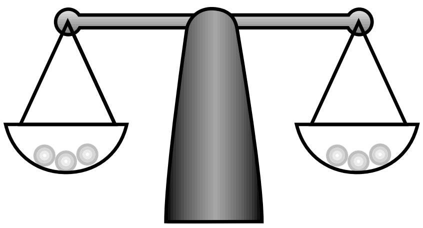
Students might investigate the patterns for various numbers of guests, for example, 24 people and 60 buns.

Problem Three

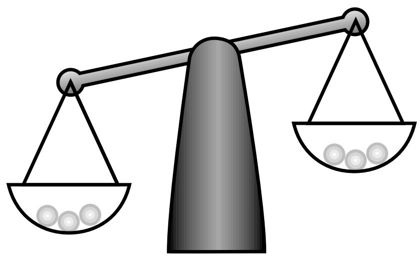
This problem assumes students are aware of how balance scales work. Their most likely experience with a balance will be the use of a see-saw. Students will know that when two people are of similar mass, the see-saw is well balanced and easy to operate. If one person is heavier, their end of the see-saw has a greater force downwards.

Janice's problem can be modelled with multilink cubes, with eight of one colour representing the marbles of equal mass and one cube of another colour representing the lighter marble. Students can then trial various weighing scenarios to test the effectiveness of each scenario in finding the lighter marble.

Janice's problem can be solved in just two weighings. The nine marbles are separated into three sets of three marbles. Two of these sets are placed on the scales. Either of two things will occur:

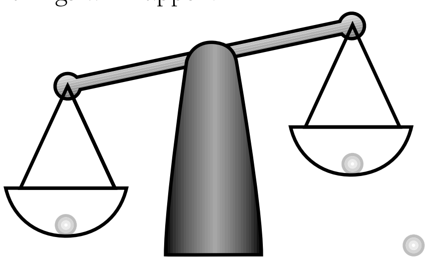


The scales balance, so the lighter marble is in the other set of three.

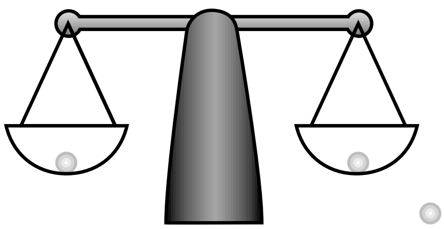


One set of three is lighter, so the light marble is in that set of three.

Put two of the marbles from the lighter set of marbles on the scales, one at each end. Either of two things will happen:

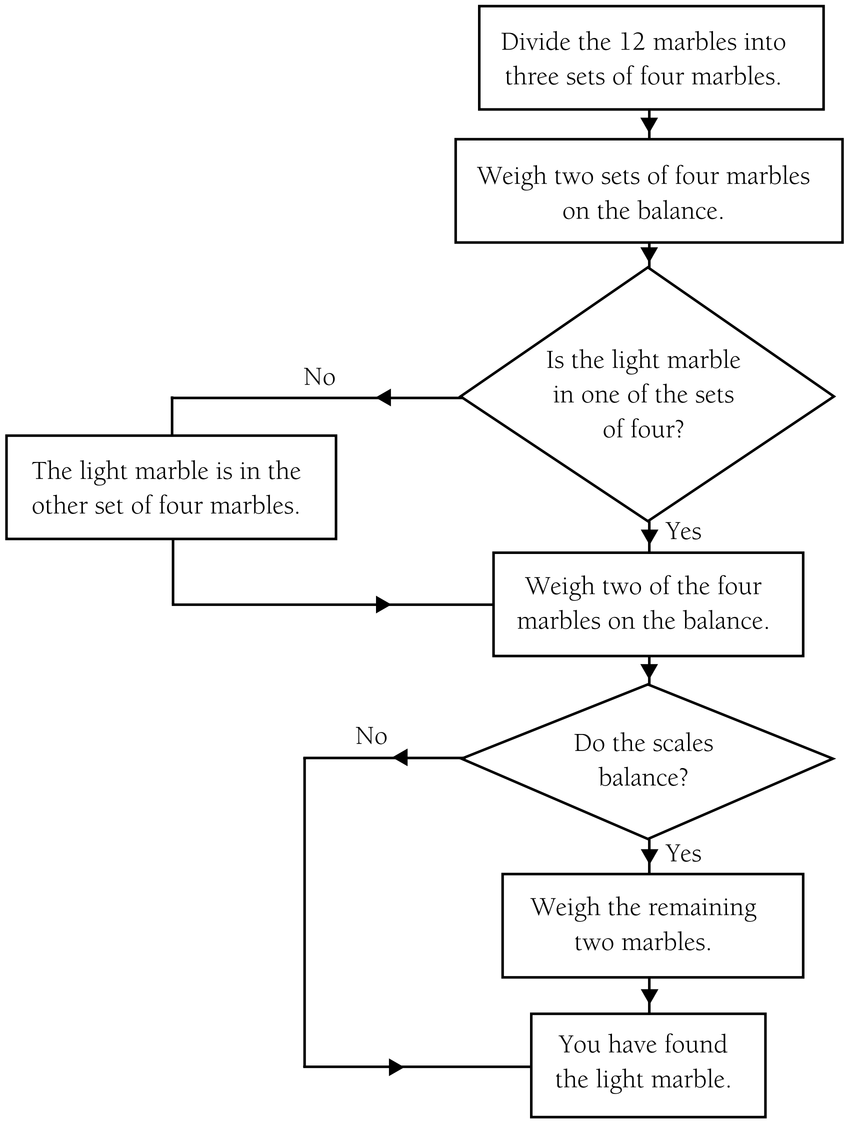


One of the marbles is heaver, so the other marble is the light one.



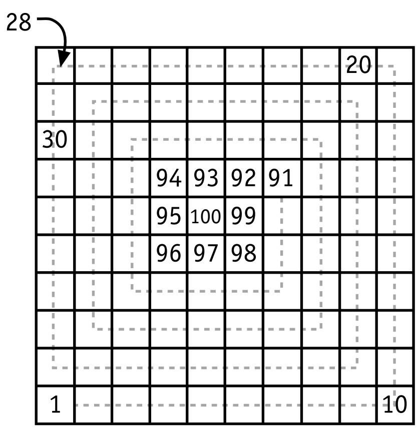
The scales balance, so the marble not on the scales is the lightest.

Students might like to investigate the minimum weighings needed for different numbers of marbles when one is light. For example, with 12 marbles, three weighings are needed. Split the 12 marbles into three sets of four and weigh two of the sets. This will tell which set of four marbles the lighter one is in. In a maximum of two more weighings, the light marble can be found. This process can be demonstrated in a flow chart.



Problem Four

This problem is ideal for applying the strategy of working backwards. It would be tempting to continue the number pattern as it spirals around, but finding the square that will have one in it can make the solution easier.



Following the spiral pattern will show you where one is. Note that the spiral back to the position of one does not involve counting. A path of descending digits can be traced without naming the digits. From the position of one, it is easy to establish the position of 10, 20, and 30, using the fact that each row and column of the grid has 10 squares. The number 28 can be found by working back from 30.

Students may enjoy inventing other number grid puzzles for someone else to solve. For example:

					1
					2
					3
					4
					5
				7	6

Which square will have the number 35 in it?

Page 14: Arrow Antics

Problem One

This problem can be likened to switches being open or closed. The early development of computers involved binary reasoning. Binary in this case means that only two possibilities exist: an arrow can be either up or down.

If binary numbers were used, 1 for up and 0 for down, the problem would look like this:

1 0 1 0 1 0 Picture One

1 1 1 0 0 0 Picture Two

This representation may make the problem easier for students who get confused with all the arrows. Another method is to put the arrows onto cards so that they can be moved. Here is one way of solving the problem:

1 0 1 0 1 0	↑ ↓ ↑ ↓ ↑ ↓	(Picture One)
1 0 1 1 0 0	↑ ↓ ↑ ↑ ↓ ↓	
1 1 0 1 0 0	↑ ↑ ↓ ↑ ↓ ↓	
1 1 1 0 0 0	↑ ↑ ↑ ↓ ↓ ↓	(Picture Two)

Students should be able to apply similar logic to find out how many adjacent arrow turns are needed to

change ↑ ↑ ↑ ↓ ↓ ↓ into ↓ ↓ ↓ ↑ ↑ ↑

Problem Two

The digital sums of multiples produce fascinating patterns.

In the case of multiples of nine, the pattern is:

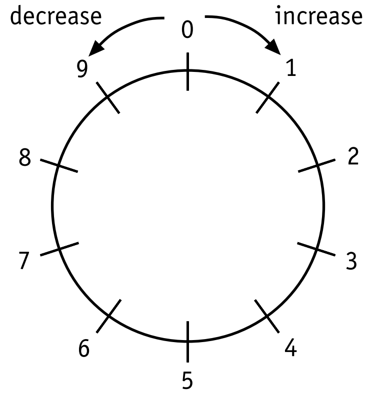
Multiples of 9	9	18	27	36	45	54	63	72	81	90	99	108
Digital sum	9	9	9	9	9	9	9	9	9	9	18→9	9

Ask students to explain why the digit sum for the first 10 multiples of nine is 9. They may notice that, because 9 is one less than 10, each addition of 9 (obviously up to 90) results in the tens digit increasing by one but also the ones digit decreasing by one.

With multiples of eight, we would expect that the ones digit will decrease by two and the tens digit will increase by one with each addition of eight. The digital sum should therefore decrease by one with each successive multiple.

Multiples of 8	8	16	24	32	40	48	56	64	72	80	...
Digital sum	8	7	6	5	4	12→3	11→2	10→1	9	8	...

The principle of digits increasing and decreasing is based on the idea of clock arithmetic.



In special cases like 48, the rule appears to break down. After 40 (8 x 5), increasing the tens digit by one would give 5_ (50 something). Decreasing the ones digit by two on the clock would result in eight. To compensate for the minus two, we get 50 – 2 = 48.

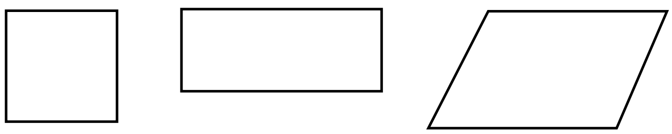
Similarly, with multiples of seven, the tens digit increases by one and the ones digit decreases by three. With the 21 → 28 transition, 21 → 31 (tens digit up by one) minus three gives 28.

Multiples of 7	7	14	21	28	35	42	49	56	63	70	...
Digital sum	7	5	3	10→1	8	6	13→4	11→2	9	7	...

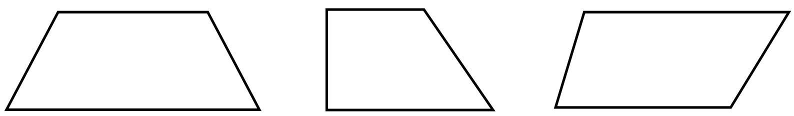
Students should now be able to anticipate the pattern for digital sums of multiples of six.

Problem Three

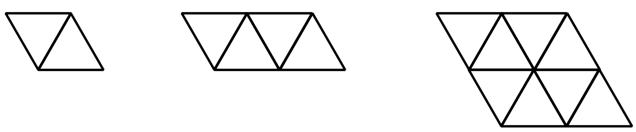
Students will need to know what parallelograms and trapezia are. A parallelogram is a four-sided polygon with two pairs of parallel sides. The shapes below are all parallelograms.



A trapezium is a four-sided polygon with only one pair of parallel sides. The shapes below are all trapezia.

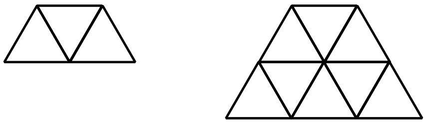


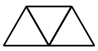
Students will need to use a systematic approach to find all the parallelograms and trapezia in the figure. They will find three sizes of parallelograms:

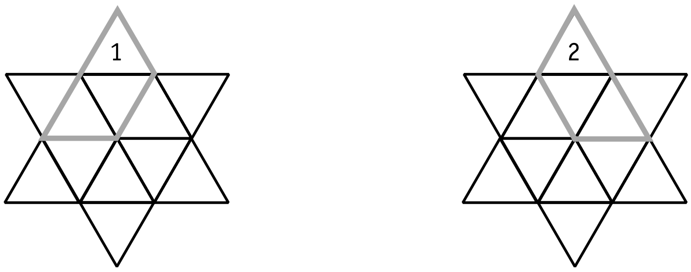


12 of this size   12 of this size   3 of this size

There are only two sizes of trapezia.

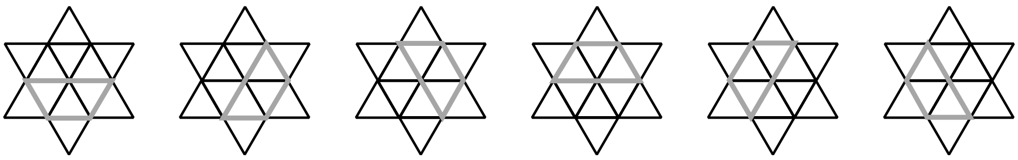


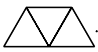
To work out the number of , each corner triangle can be dealt with separately.



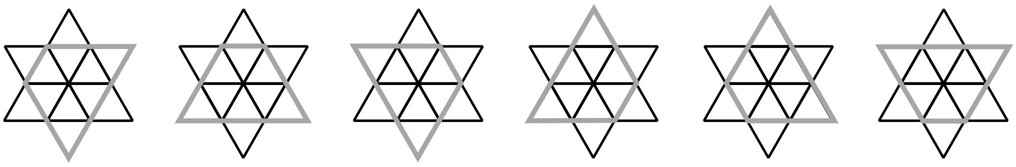
Two trapezia of this size are made using each corner triangle. This gives 6 x 2 = 12 in total.

Within the hexagon formed by the six internal triangles, there are six further trapezia of that size.



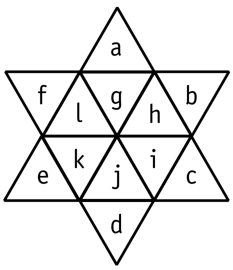
So there are 12 + 6 = 18 trapezia of the size .

Considering that two corner triangles each time must be used to make a large trapezium, there are six possibilities:



This gives a total of 18 + 6 = 24 trapezia.

A slow but thorough strategy is to label each triangle in the star and use the labels to name each shape as it is found.



A strategy for finding all the parallelograms is to start with all the parallelograms that involve triangle a, then b, and so on.

ag	aghi	aglk	aghijdkl		
bh	bhij	bhgl	bhijkelg		
ci	cijk	cihg	cijkhglf		
dj	djkl	djih			
ek	eklg	ekji			
fl	flgh	flkj			
gh	gl	hi	ij	jk	kl

Problem Four

There is a spotty bag on each side of the right-hand balance, so these could both be removed and the balance would be retained. Therefore one pink bag weighs the same as two blue bags.

Looking at the left-hand balance, if each pink bag were replaced with two blue bags, the balance would remain. This would mean that six blue bags weigh the same as two spotty bags. One spotty bag is therefore balanced by three blue bags.

Algebraically, the scales pictures could be represented as the equations $3p = 2s$ and $s + p = 2b + s$, where p is the mass of a pink bag, s the mass of a spotty bag, and b the mass of a blue bag. We can remove a spotty bag from each side of the right-hand balance, so now we have:

$3p = 2s$ and $p = 2b$

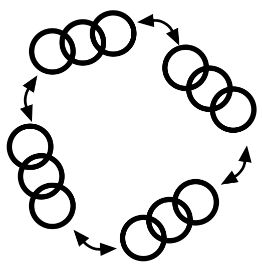
which can be simplified as

$$\begin{aligned} 3 \times 2b &= 2s \\ 6b &= 2s \\ 3b &= s. \end{aligned}$$

Page 15: Missing Links

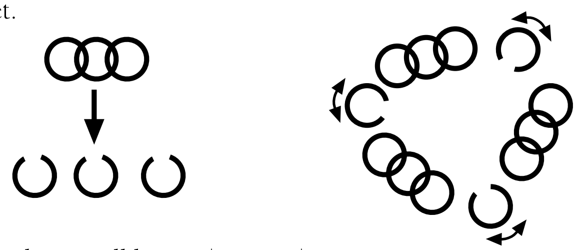
Problem One

An obvious strategy is for students to try to keep each length of chain intact.



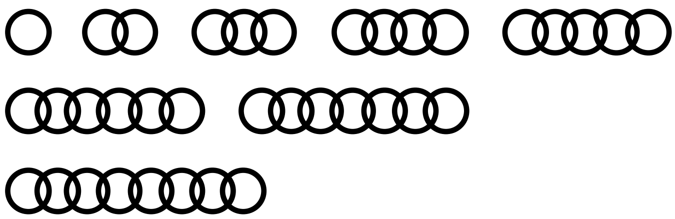
This joining will require four links to be opened and closed. That will cost 4 x \$1.20 = \$4.80, so it cannot be done within budget.

If the jeweller opens and closes all three links in one length of chain, the job can be done within budget.



The total cost will be 3 x \$1.20 = \$3.60.

As an extension, challenge students to join these lengths of chain into one bracelet for the least cost:



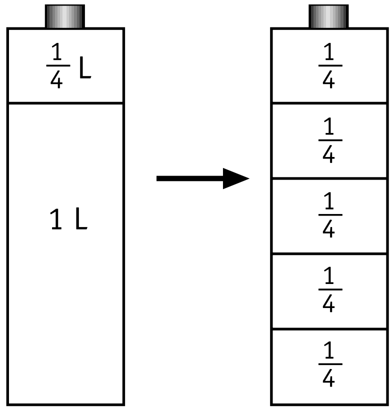
This can be done by opening and closing six links.

Problem Two

This problem explores the idea of common ratios or equivalent fractions. Students should know that an answer greater than one can only be obtained if the number being divided (the dividend) is greater than the divisor, for example, 8 ÷ 4 = 2, 6 ÷ 4 = 1.5, 3 ÷ 2 = 1.5.

1.25 L is a common capacity of soft-drink bottles, and some students may know it as "one and a quarter litres". A representation of this might be:

So $1\frac{1}{4}$ litres is five quarters.



One way of getting 1.25 is 5 ÷ 4. Students may know that 5 ÷ 2 = 2.5 or $2\frac{1}{2}$, so dividing five by twice two (four) will halve the answer (1.25).

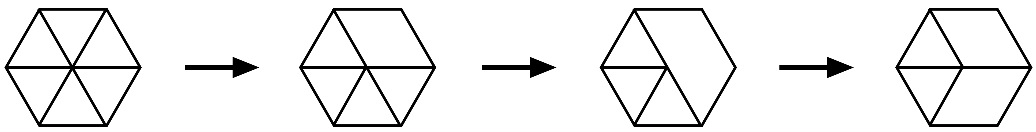
From this solution, an infinite number of other possibilities can be developed. For example:

$$\frac{\overset{\times 2}{5}}{\underset{\times 2}{4}} = \frac{10}{8} = 1.25 \qquad \frac{\overset{\div 2}{5}}{\underset{\div 2}{4}} = \frac{2.5}{2} = 1.25 \qquad \frac{\overset{\times 25}{5}}{\underset{\times 25}{4}} = \frac{125}{100} = 1.25$$

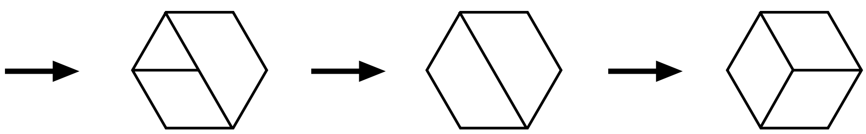
Problem Three

Some students may need to use pattern blocks to find the different ways of making hexagons.

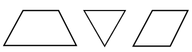
A systematic way of working is:



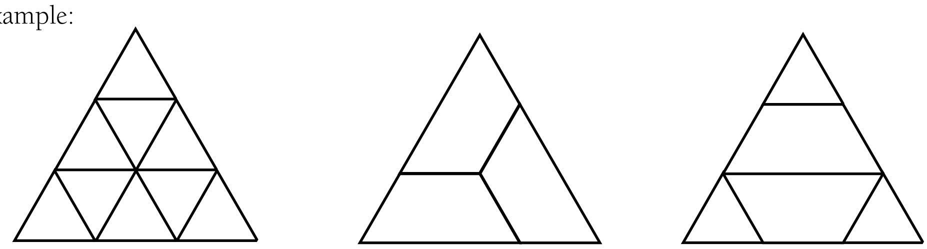
$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ $\quad\frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ $\quad\frac{1}{2} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ $\quad\frac{1}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6}$



$\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$ $\quad\frac{1}{2} + \frac{1}{2}$ $\quad\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

As an extension, students might explore how many ways a triangle could be filled using a number of these shapes: 

For example:



Problem Four

The first four equations in the pattern are:

4 x 44 = 176
4 x 444 = 1 776
4 x 4 444 = 17 776
4 x 44 444 = 177 776

In the pattern, there is always one more four in the second factor than there are sevens in the answer. Reversing this …

4 x ? = 1 777 776

six 4s ⟵ five 7s

leads to 4 x 444 444 = 1 777 776

Page 16: Darting for Cover

Problem One

Making an organised list is a useful strategy for finding all the possible outcomes.

3 + 3 + 3 + 3 = 12
3 + 3 + 3 + 5 = 14
3 + 3 + 3 + 9 = 18
3 + 3 + 5 + 5 = 16
3 + 3 + 5 + 9 = 20
3 + 3 + 9 + 9 = 24
3 + 5 + 5 + 5 = 18
3 + 5 + 5 + 9 = 22
3 + 5 + 9 + 9 = 26
3 + 9 + 9 + 9 = 30

All the possibilities with at least one dart landing on 3

5 + 5 + 5 + 5 = 20
5 + 5 + 5 + 9 = 24
5 + 5 + 9 + 9 = 28
5 + 9 + 9 + 9 = 32

All the remaining possibilities with at least one dart landing on 5 and no darts on 3

9 + 9 + 9 + 9 = 36

The only possibility involving all the darts on 9 with none on 3 or 5

These results could also be organised into a table to avoid duplications:

Dartboard Scores			
3	5	9	Total
4			12
3	1		14
3		1	18
2	1	1	20
.	.	.	.
.	.	.	.
.	.	.	.

Problem Two

This example belongs to a family of letter-for-number problems called cryptorithms.

A useful strategy in these problems is to put the letters and all the possible digits in a table so that possibilities can be eliminated.

	0	1	2	3	4	5	6	7	8	9
A										
B										
C										

Considering the A in the hundreds place of the top number, it is clear that this must stand for one because there is no hundreds digit in the answer.
This transforms the problem into:

$$\begin{array}{r} 1\ B\ 1 \\ -\quad C\ 1 \\ \hline 1\ B \end{array}$$

From this, we can use the ones column to see that B must be zero.

So the problem becomes:

$$\begin{array}{r} 1\ 0\ 1 \\ -\quad C\ 1 \\ \hline 1\ 0 \end{array}$$

This then shows us that C must be nine because 10 – C = 1.

Problem Three

Students may enjoy acting the problem out by forming groups of four and trying out various orders.

Another strategy is to solve a simpler problem. Starting with two people, A and B, there are two possible orders:

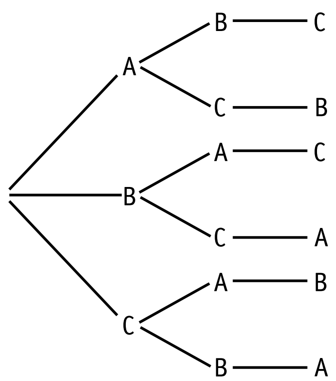
AB BA

With three people, A, B, and C, there are six possible orders:

ABC ACB BAC BCA CAB CBA

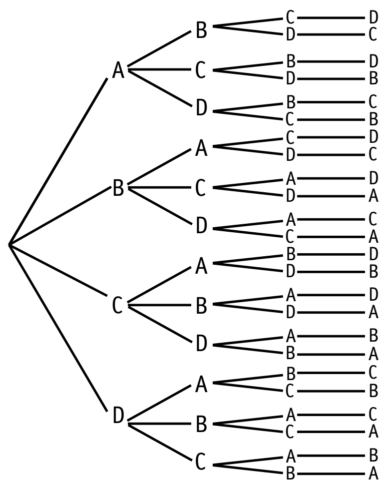
Note how the possibilities with A in front are exhausted, then all the possibilities with B in front, and then with C in front.

Students are unlikely to realise that the pattern involves multiplication unless these orders are organised as a tree diagram:



3 x 2 x 1 = 6 (called 3! or three factorial)

When another person is added, the tree diagram becomes:



4 x 3 x 2 x 1 = 24 (called 4! or four factorial)

The solution for the possible orders of five people is given by 5 x 4 x 3 x 2 x 1 = 120 (or 5!).

Problem Four

These types of logic problems can be modelled with a table or by using moveable labels of the people and sports. Using these models, the problem can be solved like this:

In Vijay's and Harley's favourite sports, all the players can kick the ball.

	Volleyball	Rugby	Soccer	Hockey	Netball
Sam					
Harley	✘			✘	✘
Rupert					
Vijay	✘			✘	✘
Narissa					

Harley Vijay
rugby soccer

Sam Rupert Narissa
volleyball hockey netball

No one's favourite sport begins with the same letter as their name.

	Volleyball	Rugby	Soccer	Hockey	Netball
Sam			✘		
Harley	✘			✘	✘
Rupert		✘			
Vijay	✘			✘	✘
Narissa					✘

Harley Vijay
rugby soccer

Sam Rupert Narissa
volleyball hockey No
netball

Narissa likes the sport where the ball is hit with a stick.

	Volleyball	Rugby	Soccer	Hockey	Netball
Sam			✘	✘	
Harley	✘			✘	✘
Rupert		✘		✘	
Vijay	✘			✘	✘
Narissa	✘	✘	✘	✓	✘

Harley Vijay
rugby soccer

Sam Rupert Narissa
volleyball netball hockey

Sam loves being the goal shoot in her sport.

	Volleyball	Rugby	Soccer	Hockey	Netball
Sam	✘	✘	✘	✘	✓
Harley	✘			✘	✘
Rupert		✘	✘	✘	✘
Vijay	✘			✘	✘
Narissa	✘	✘	✘	✓	✘

Harley rugby

Vijay soccer

Sam netball

Rupert volleyball

Narissa hockey

Both Harley's and Sam's favourite sports are played with a spherical ball.

	Volleyball	Rugby	Soccer	Hockey	Netball
Sam	✘	✘	✘	✘	✓
Harley	✘	✘	✓	✘	✘
Rupert	✓	✘	✘	✘	✘
Vijay	✘	✓	✘	✘	✘
Narissa	✘	✘	✘	✓	✘

Harley soccer

Vijay rugby

Sam netball

Rupert volleyball

Narissa hockey

Page 17: Over a Barrel

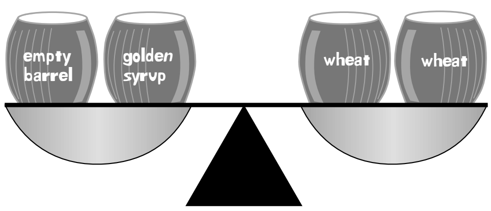
Problem One

Students may solve the problem by trial and improvement. This will involve trying numbers for the mass of the barrel and seeing if the resulting masses for wheat and golden syrup work.

Mass of barrel	Mass of wheat	Mass of golden syrup
20 kg	10 kg	20 kg
5 kg	25 kg	50 kg
15 kg	15 kg	30 kg
10 kg	20 kg	40 kg ✔

The mass of 10 kilograms for the barrel works because the mass of the golden syrup is twice the mass of wheat.

Another way to solve the problem is to use logical reasoning. If golden syrup is twice as heavy as wheat, this collection of barrels would have the same mass:



A barrel of golden syrup has a mass of 50 kilograms, and a barrel of wheat has a mass of 30 kilograms, so this can be represented as:

❑ + 50 = 30 + 30 where ❑ is the mass of an empty barrel.

So ❑ + 50 = 60

So ❑ = 10

Problem Two

Students should realise that because each equation involves multiplication of a number by itself and the answers are less than 100, ◯, △ , and □ must be less than 10.

They can then solve each equation by trial and improvement or by eliminating all the possibilities:

Number (n)	(n x n) + n
0	0
1	2
2	6
3	12
4	20 ✔
5	30 ✔
6	42
7	56
8	72
9	90 ✔

The numbers in the right-hand column show a pattern of differences that provides an interesting extension:

n	0	1	2	3	4	5	6	7	8	9	10
(n x n) + n	0	2	6	12	20	30	42	56	72	90	↑
	+ 2	+ 4	+ 6	+ 8	+ 10	+ 12	+ 14	+ 16	+ 18		

what is (10 x 10) + 10?

Problem Three

This problem is similar to **Problem One** on this page because it involves possibilities (that is, the number of kittens) and a constraint (there are 21 more legs than tails).

As with **Problem One**, students could use trial and improvement. However, a more efficient method might be to realise that each kitten has three more legs than tails (4 – 1 = 3). To get 21 more legs than tails would require seven kittens because 7 x 3 = 21.

Similarly, a kitten has two more legs than eyes, so 14 more legs than eyes means seven kittens as well because 7 x 2 = 14.

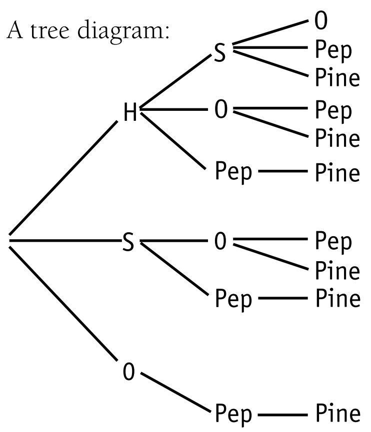
Problem Four

Combinations such as this can be solved in a variety of ways (see the notes on probability, *Answers and Teachers' Notes: Statistics*, Figure It Out, Level 3). These include:

- An organised list:

ham – salami – olives ham – salami – peppers ham – salami – pineapple ham – olives – peppers ham – olives – pineapple ham – peppers – pineapple	All possibilities with ham
salami – olives – peppers salami – olives – pineapple salami – peppers – pineapple	All possibilities with salami but no ham
olives – peppers – pineapple	Remaining possibility

- A tree diagram:



- Tables:

Ham with …

	Salami	Olives	Peppers	Pineapple
Salami				
Olives	✔			
Peppers	✔	✔		
Pineapple	✔	✔	✔	

Salami with …

	Olives	Peppers	Pineapple
Olives			
Peppers	✔		
Pineapple	✔	✔	

Olives with …

	Peppers	Pineapple
Peppers		
Pineapple	✔	

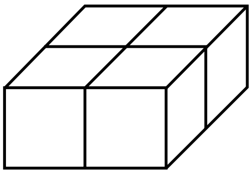
Page 18: Colourful Calculations

Problem One

This problem involves students finding the volumes of three cuboids (rectangular prisms). Some students may need to use multilink cubes to build the cuboids. This is a very slow process, and discussion should focus on more efficient ways to find the number of cubes in each shape.

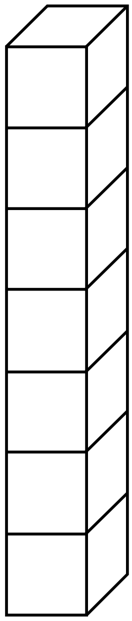
Many students are likely to apply an equal additions strategy. This involves finding the number of cubes in one layer or one column and repeatedly adding however many of these layers or columns are in the whole solid.

For example, cuboid **b** might be seen as:



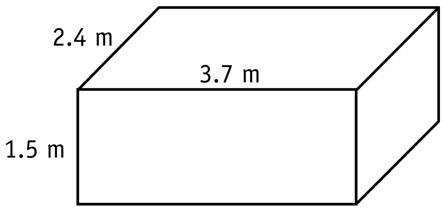
four cubes in each layer
4 + 4 + 4 + 4 + 4 + 4 + 4 = 28 cubes

or



7 cubes in each column
7 + 7 + 7 + 7 = 28 cubes

This equal addition of layers or columns is a step on the way to finding volume by multiplication, for example, finding the volume of cuboid **b** by saying 7 x 4 or 4 x 7. Finding volume by multiplication becomes much more important when the edges of the cuboid are not whole numbers. For example:



Volume = 1.5 x 3.7 x 2.4
= 13.32 m^3

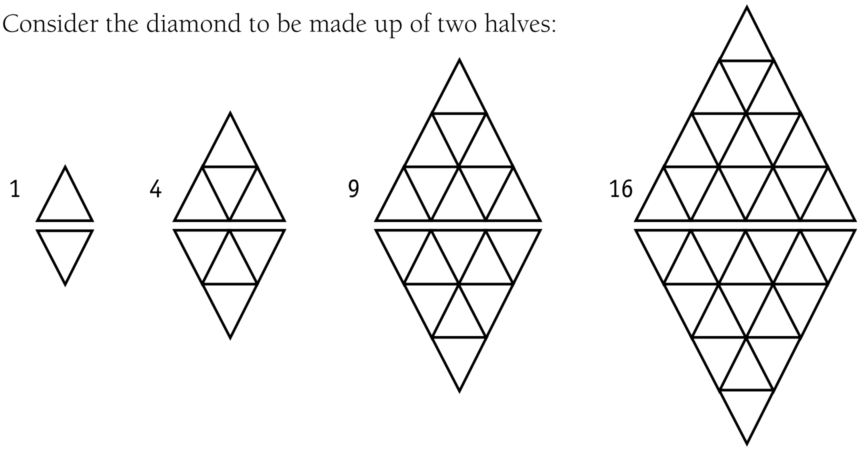
So, in **Problem One**, the volumes are:

a. 3 x 3 x 4 = 36 cubes
b. 7 x 2 x 2 = 28 cubes
c. 2 x 5 x 3 = 30 cubes

Problem Two

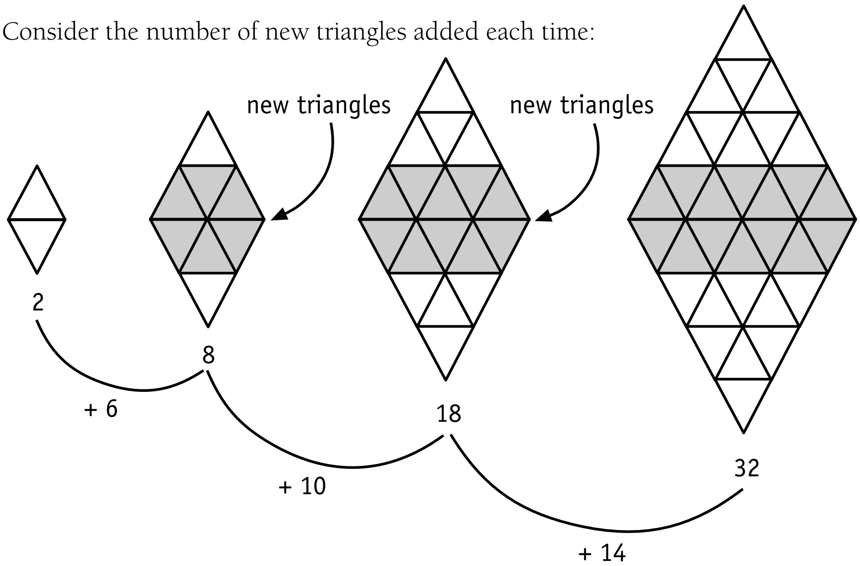
The number of triangles is increasing rapidly between one diamond and the next, so students will need to use patterns to make this problem easier to solve. Here are a number of possible approaches:

- Consider the diamond to be made up of two halves:



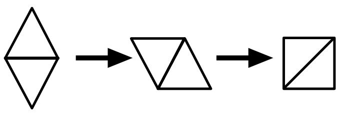
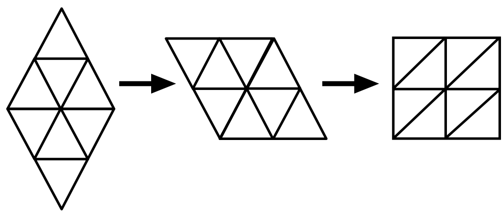
Students might recognise that the half numbers are square numbers, that is, 1 = 1 x 1, 4 = 2 x 2, 9 = 3 x 3, 16 = 4 x 4. So, the next half will be 5 x 5 = 25, and the whole shape should have 25 + 25 = 50 triangles.

- Consider the number of new triangles added each time:

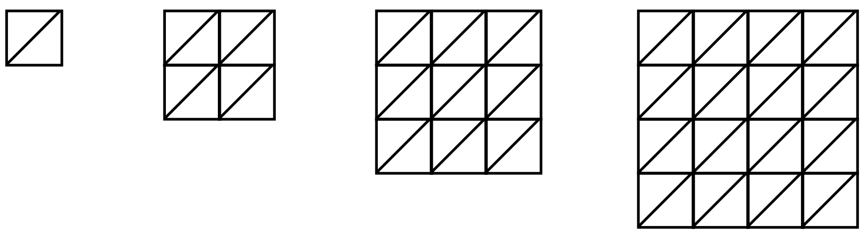


There are four more new triangles each time, 6 + 4 = 10, 10 + 4 =14, so for the next diamond, there will be 18 new triangles. 32 + 18 = 50, so the next diamond will have 50 triangles.

- The diamonds can be looked at as squares:
For example:

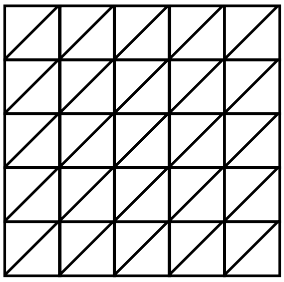
 and 

Each small square is made up of two triangles. So the pattern becomes:



1 square	4 squares	9 squares	16 squares
2 triangles	8 triangles	18 triangles	32 triangles

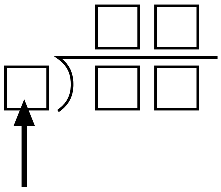
The next diamond can be transformed into this square:



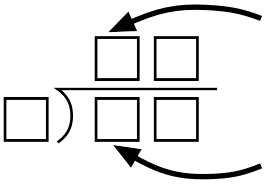
25 squares
50 triangles

Problem Three

Students should use their knowledge of division to limit the location of digits in the set.

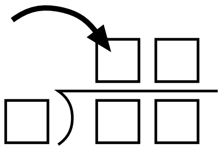


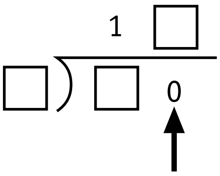
This digit cannot be 0 or 1. Division by 0 is undefined. Division by 1 would mean that the digits in the answer would be the same as those in the dividend.



This number must be four, five, or six times smaller than this number.

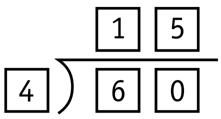
This digit must therefore be 1.





This number must be 0 because 10 cannot be the answer. (Only one 0 is available, and, in order to have an answer of 10, either the divisor or the dividend would need 0 as one of its digits.)

With this knowledge, students can try the combinations of locations for 4, 5, and 6 to find one that works:

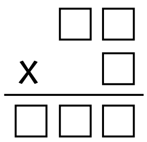


This works!

Students may enjoy making up their own problems of this type. This is easier than solving them. Have them start with a known addition, subtraction, multiplication, or division result in which each digit is unique. For example:

$$\begin{array}{r} 43 \\ \times\ 6 \\ \hline 258 \end{array}$$

and present it as



Arrange the digits 2, 3, 4, 5, 6, and 8, one in each box, to make a correct multiplication.

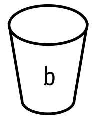
Problem Four

Students may solve this problem by trying various values for the number of blue marbles to see whether they work. For example:

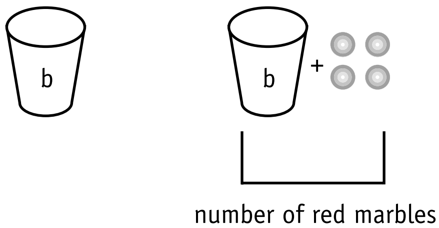
Blue	Red	Yellow	Green	Total
1	5	11	2	19
5	9	15	2	31
8	12	18	2	40
6	10	16	2	34
7	11	17	2	37 ✔

An approach that involves algebraic reasoning might be:

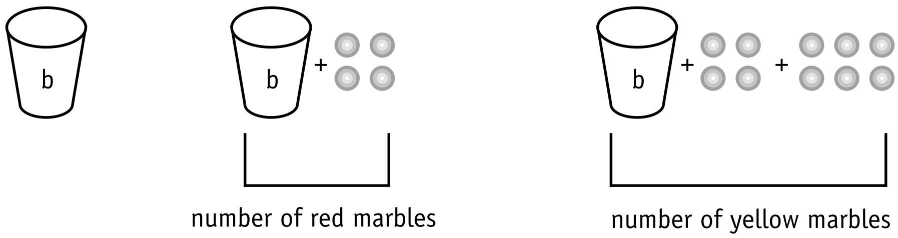
- Two of the 37 marbles are green, so the other 35 must be blue, red, or yellow.
- Let an empty cup represent the number of blue marbles we don't know:



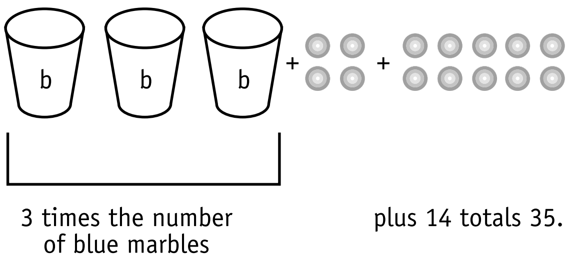
- There are four more red marbles than blue, so these amounts can be represented as:



- There are six more yellow marbles than red marbles, so all 35 marbles can be represented as:



- Collected, this means:



- Three times the number of blue marbles must equal 35 – 14 = 21.
  21 ÷ 3 = 7, so seven is the number of blue marbles.
- 7 + 4 = 11 gives the number of red marbles.
  11 + 6 = 17 gives the number of yellow marbles.
  We already know that there are two green marbles.

Page 19: Stretching Exercises

Problem One

Consecutive numbers are numbers that are adjacent in the integer number counting sequence, such as five and six, and 99 and 100. With each of problems **a**, **b**, and **c**, students could use trial and improvement by experimenting with various pairs of numbers. They could also apply reasoning:

a. If $\underbrace{\square}_{\text{number}} + (\underbrace{\square}_{\text{the next number}} + 1) = 373$, then $\square$ must be about half of 373 (slightly less).

Half of 373 is 373 ÷ 2 = 186.5, so $\square$ must be 186.

Check: 186 + 187 = 373 (It works.)

b. If $\square \times (\square + 1) = 306$, then $\square$ must be between 10 and 20 because 10 x 11 = 110 and 20 x 21 = 420.

In fact, $\square$ is likely to be close to 20 because 306 is closer to 420 than to 110.
Try: 19   19 x 20 = 380 ✗
18   18 x 19 = 342 ✗
17   17 x 18 = 306 ✔

Another method is to find $\sqrt{306}$. This is the number that when multiplied by itself gives 306. On a calculator, keying 306 $\boxed{\sqrt{\ }}$ gives 17.49 (rounded). 17.49 is very close to 17.5. The values for $\square$ and $\square$ + 1 are either side of 17.5, that is 17 and 18.

c. Knowing that 1.1 involves 0.1, which is $^1/_{10}$, and that 1.1 is another name for $^{11}/_{10}$ or 1 ÷ 10 gives the value of the consecutive numbers.

Problem Two

Organising the results in a table will help students find patterns:

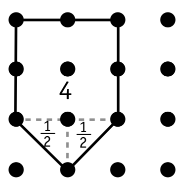
Date (Oct)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
David's lollies	48	45	42	39	36	33	30	27	24	21	18	15	12	9	6	3
Joanna's lollies	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3

Another way to look at the problem is to consider the difference between their numbers of lollies. On 1 October, David has 15 more lollies than Joanna. Each day they eat lollies, that difference is reduced by one. Therefore it will take 15 days for the difference to be reduced to zero.

Problem Three

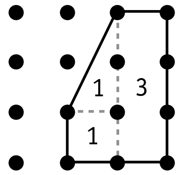
Students will need to use a systematic approach to finding the areas of the shapes they make. This will probably involve them dividing the shapes into smaller shapes with known area and finding the total of these areas.

For example, the area of the shape shown in this problem could be found by:

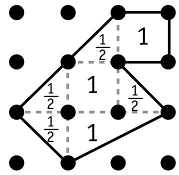


$4 + {}^1/_2 + {}^1/_2 = 5$ squares

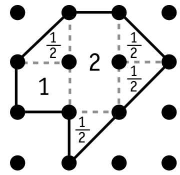
A variety of shapes can be made with the rubber band touching eight pegs with two pegs inside. These include:



3 + 1 + 1 = 5 squares



$^1/_2 + {}^1/_2 + {}^1/_2 + {}^1/_2 + 1 + 1 + 1$
= 5 squares



$^1/_2 + {}^1/_2 + {}^1/_2 + {}^1/_2 + 2 + 1$
= 5 squares

This problem is a special case of Pick's Theorem, which relates the area of a geoboard shape to the number of pins touching the perimeter of a shape and the number of pins it encloses.

Problem Four

Students may like to explore what differences and sums they can get using the numbers two and five. Obviously all multiples of two and five can be made easily.

$\boxed{5}\boxed{-}\boxed{2}\boxed{=}\boxed{3}$, so multiples of three can be made in this way.

$\boxed{2}\boxed{+}\boxed{5}\boxed{=}\boxed{7}$, so multiples of seven can be made in this way.

Therefore six could be made as $\boxed{5}\boxed{+}\boxed{5}\boxed{-}\boxed{2}\boxed{-}\boxed{2}$ or $\boxed{2}\boxed{+}\boxed{2}\boxed{+}\boxed{2}$, and eight could be made as $\boxed{5}\boxed{+}\boxed{5}\boxed{-}\boxed{2}$ or $\boxed{2}\boxed{+}\boxed{2}\boxed{+}\boxed{2}\boxed{+}\boxed{2}$ (if more than seven presses are allowed).

Students will enjoy the challenge of trying to get all the whole numbers up to 20 in the display. Here are some ways:

1 = 5 – 2 – 2
2 = 2
3 = 5 – 2
4 = 2 + 2
5 = 5

6 = 2 + 2 + 2 or 5 + 5 – 2 – 2 (as above)
7 = 5 + 2
8 = 2 + 2 + 2 + 2 or 5 + 5 – 2 (as above)
9 = 5 + 2 + 2 or 5 + 5 + 5 – 2 – 2 – 2
10 = 5 + 5

11 = 5 + 2 + 2 + 2
12 = 5 + 5 + 2
13 = 5 + 5 + 5 – 2
14 = 5 + 5 + 2 + 2
15 = 5 + 5 + 5

16 = 5 + 5 + 2 + 2 + 2
17 = 5 + 5 + 5 + 2
18 = 5 + 5 + 5 + 5 – 2
19 = 5 + 5 + 5 + 2 + 2
20 = 5 + 5 + 5 + 5

Page 20: Mixed Fruit

Problem One

There are two ways to establish the most likely outcomes. One way is to roll two dice many times and record the differences. This could be done using a tally chart like this:

0	1	2	3	4	5
𝍸 I	𝍸𝍸 II	𝍸 III	IIII	III	II

Provided that enough dice throws are recorded, this will give a strong indication of the frequency of these differences.

Another strategy is to use a systematic approach to find all the possible outcomes. A table is possibly the best way to do this.

		Dice one					
		1	2	3	4	5	6
Dice two	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

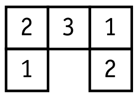
This table shows how each difference can be generated.

From the 36 possible outcomes when two dice are rolled, 10 outcomes give a difference of one, and eight outcomes give a difference of two.

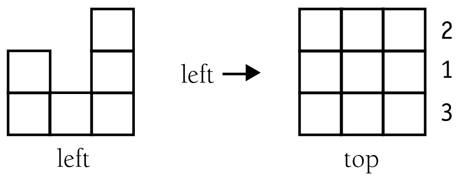
Problem Two

Students may choose to model this problem with multilink cubes.

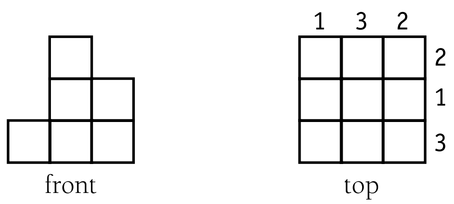
Encourage students to solve this problem by using each view to draw a bird's-eye plan, numbering the squares so that each number refers to the number of cubes in the column. For example:



In **Problem Two**, the bird's-eye view can be developed like this:

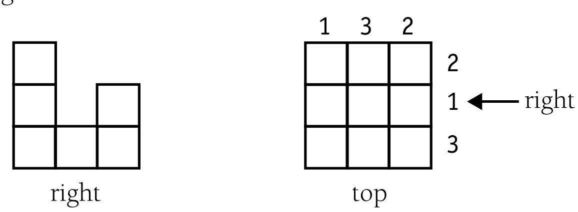


highest possible height of a column in each horizontal row

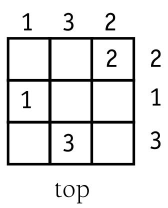


highest possible height of a column in each vertical row

The right view confirms the left view.



So, filling in the squares in a way that satisfies the maximum height with the smallest number of cubes gives this bird's-eye view:



So the model could be made with only six cubes.

If students assume that the model must be a connected whole, this diagram gives a minimum solution with eight cubes:

1	3	2	
		2	2
1	1	1	1
	3		3

Problem Three

Students could use trial and improvement to solve this problem. This method will be more efficient if students notice that an orange must cost 20 cents less than an apple. Trading an orange for an apple reduces the cost by 20 cents ($2.35 – $2.15).
Students could use a table to organise the possibilities:

Cost of apple	Cost of orange	3 apples + 2 oranges	2 apples + 3 oranges
50c	30c	$2.10	$1.90
60c	40c	$2.60	$2.40
55c	35c	$2.35 ✔	$2.15 ✔

Alternatively, students could use a pattern:

If five oranges cost $1.75, each orange must cost 35 cents because $1.75 ÷ 5 = $0.35.

Problem Four

A common property of the sets of five numbers is that they will have an average (mean) of 15 because 75 ÷ 5 = 15. Working with 15 as the central number gives many solutions. For example:

7 9 11 13 15 17 19 21 23

balances 23 balances 19 balances 11 balances 7

balances 17 balances 13

7 9 11 13 15 17 19 21 23

balances 21 balances 9

Other solutions that include 15 are:

7	9	15	21	23
7	11	15	19	23
7	13	15	17	23
9	11	15	19	21
9	13	15	17	21
11	13	15	17	19

Note that the balancing pairs of numbers add to 30. Another way is to consider two balancing pairs of numbers where the pairs add to 60 in a different way.

$$\underbrace{9 \qquad 11}_{20} \qquad \overset{15}{+} \qquad \underbrace{17 \qquad 23}_{40}$$

$$\underbrace{7 \qquad 13}_{20} \qquad \overset{15}{+} \qquad \underbrace{19 \qquad 21}_{40}$$

If 15 is not chosen as the central number, the others must be weighted to give an average of 15. For example:

9	11	13	19	23
7	9	17	19	23

Page 21: Talk About: One

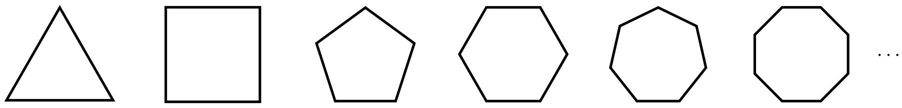
The Talk About pages on pages **21–24** are designed for small group discussions to be followed by a whole-class round-up.

Problem One

Give students about 5 minutes to discuss the similarities and differences of a triangle and a circle. Each group must have someone taking notes. You could let each group send out a "spy", who can report back ideas from other groups. Students may choose to focus on a range of characteristics:

Triangle	Circle
2-dimensional	2-dimensional
closed curve	closed curve
3 sides and vertices (corners)	limiting case with infinite sides and corners
3 lines of reflective symmetry (equilateral)	an infinite number of lines of symmetry
rotational symmetry of order 3 (equilateral)	rotational symmetry of infinite order
centre is where lines of symmetry intersect	centre is where lines of symmetry intersect
used in building because of its strength	used in machinery because of its smooth roll properties
tessellates with no gaps or overlaps	does not tessellate

As a result of the whole-class round-up, a number of questions may arise that, in turn, become the subject of a whole-class investigation. Students may debate the number of sides and vertices that a circle has. It can be thought of as a polygon that has an infinite number of sides and vertices. Students could make a table of regular polygons. From this, they will see that as the number of sides increases, the polygon becomes more like a circle:



This shows that a circle is the limiting case. The limiting case is the case which defines the limit (boundary, end point) of a process. In this instance, as the number of sides and vertices increases, the shape approaches the limiting case, a circle.

Problem Two

In this problem, students need to show that they understand the decimal system. If students consider the place values of whole numbers and decimals, they will see that division by 10, 100, or 1 000 causes the decimal point to move to the left.
For example, consider 250:

hundreds	tens	ones	.	tenths	hundredths	thousandths
2	5	0				

Dividing by 10 can be interpreted as "How many tens are there in ...?"
Dividing by 100 can be interpreted as "How many hundreds are there in …?"
Students can use a calculator to find out what happens to 250 as it is divided by 10, 100, and 1 000.

	hundreds	tens	ones	.	tenths	hundredths	thousandths
	2	5	0				
÷ 10		2	5	.	0		
÷ 100			2	.	5	0	
÷ 1 000			0	.	2	5	0

In the case of 25.0, 2.50, and 0.250, the zero has no effect because it is not acting as a place holder.

Students will see that there are $2\frac{1}{2}$ (2.5) hundreds in 250. Use pattern to show that 250 ÷ 1 000 is 0.250. Because 250 is a quarter of 1 000, there are $\frac{1}{4}$ (0.25) thousands in 250.

As a number is divided by 10, the value of each digit becomes one-tenth of its previous value. This effect is best described as the digits shifting one place to the right relative to the decimal point. With division by 100, the shift is two places, and with division by 1 000, it is three places.
So, consider the whole number 497:

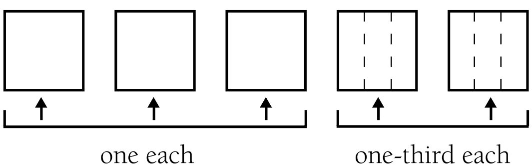
$$497 \div 10 = 49.7$$

$$497 \div 100 = 4.97$$

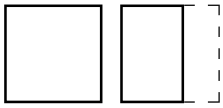
$$497 \div 1\,000 = 0.497$$

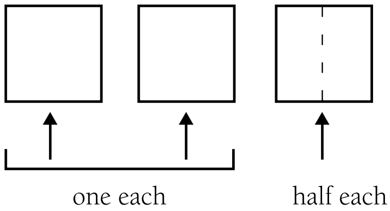
Problem Three

Students may wish to use a physical model of the problem, such as strips or squares of paper. An important feature is that the strips or squares used to represent the custard squares are of the same size. Using this idea:

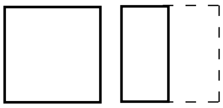


three people sharing five custard squares

So each person gets  $1^2/_3$ custard squares.



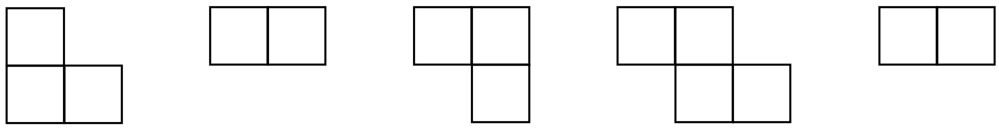
two people sharing three custard squares

So each person gets  $1^1/_2$ custard squares.

Students will see that two-thirds is greater than one half, and therefore the three people sharing five custard squares get more.

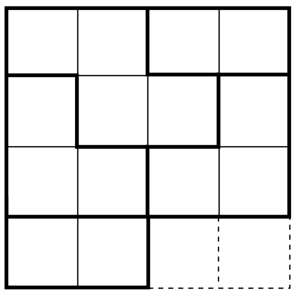
Problem Four

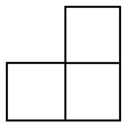
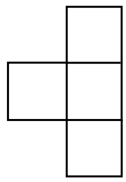
For the pieces to form a square, the total number of small squares forming the square must add to a square number (1, 4, 9, 16, 25, 36, …).

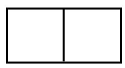
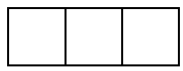


3 + 2 + 3 + 4 + 2 = 14

There are only 14 small squares, so two small squares must be added for the pieces to form a 4 x 4 square. Encourage students to add one square to two different pieces to make this work.
A number of solutions are possible, for example:



changing  to 

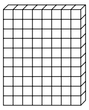
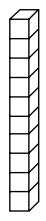
and  to 

will create a set of pieces that form a square.

Page 22: Talk About: Two

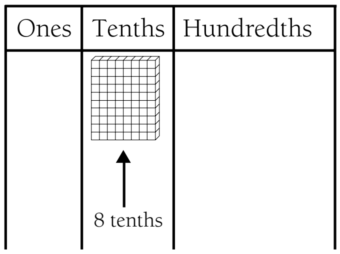
Problem One

This problem confronts a common error that because 79 is greater than eight, 0.79 must be greater than 0.8. This misconception occurs when students ignore the significance of the place of each digit.

Students could use place value blocks to model the two numbers. We assume firstly that  (the flat) is one,  is one-tenth because 10 of them make one, and  is one-hundredth because 100 of them make one.

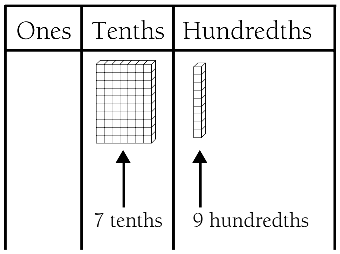
ones		tenths	hundredths
0	.	8	

can be modelled as:



ones		tenths	hundredths
0	.	7	9

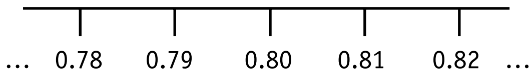
can be modelled as:



Students will see that one-hundredth must be added to 0.79 to make it 0.8.

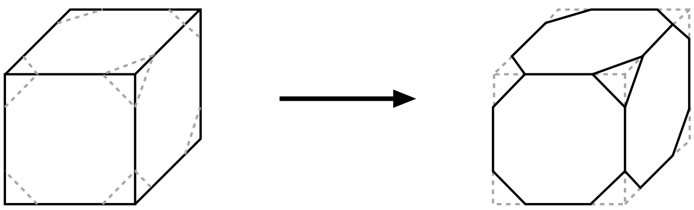
Students could be asked how many hundredths are in both numbers (79 for 0.79 and 80 for 0.8). If 0.8 were written as 0.80, it would be obvious that it is greater than 0.79.

On a tape measure, 79 centimetres comes before 0.8 metres.



Problem Two

Cutting the corners off a cube creates a polyhedron known as a truncated cube (truncated means cut off). A model made from Plasticine or a large potato can be used to help visualise the solid created.



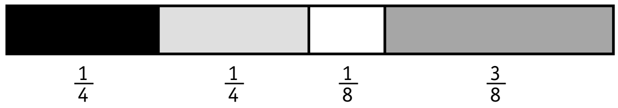
The original cube was made of six square faces, 12 edges, and eight vertices (corners).

A consequence of cutting the cube at each vertex is that each square becomes a regular octagon and an equilateral triangle is formed at each previous vertex.

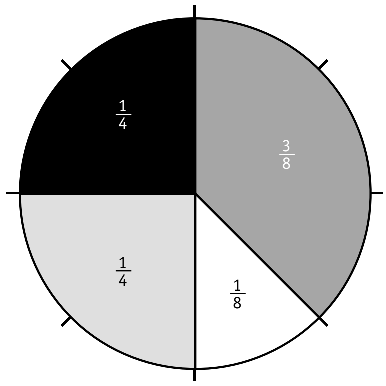
So the truncated cube has six octagonal faces and eight triangular faces, giving a total of 14 faces. For each previous vertex, there are now three vertices, so the number of vertices is 3 x 8 = 24. All of the original 12 edges of the cube are still present in truncated form. Added to them are the three sides of each of the eight triangles. So the number of edges is $\underbrace{12}_{\text{original}} + (3 \times \underbrace{8}_{\text{triangle sides}}) = 36$.

Problem Three

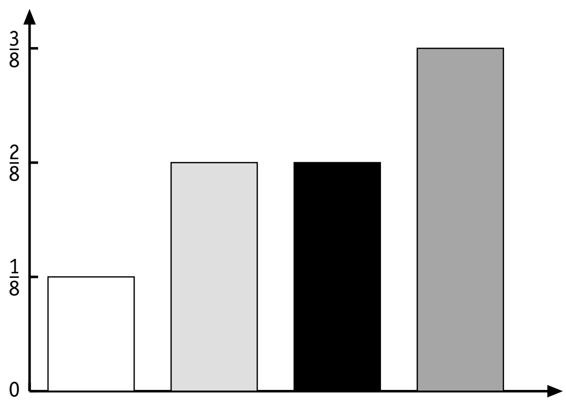
By inspecting the strip graph, students will see that half the data set is divided into two quarters, with the remainder being split into $^1/_8$ and $^3/_8$.



The pie graph also has these ratios.



All the lengths of the bars on the bar graph are different, so it cannot represent the same data set as the strip graph. Encourage students to draw an accurate bar graph of the data. For example:



Problem Four

One litre of water has a mass of 1 kilogram (1 000 grams), so students can find the mass of each empty bottle by taking the mass of the water from the total mass of the bottle.

In each case:

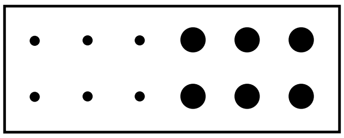
$$\begin{aligned} 550 - 500 &= 50 \text{ grams} \\ 1\,600 - 1\,500 &= 100 \text{ grams} \\ 4\,000 - 3\,000 &= 1\,000 \text{ grams} \\ &= 1 \text{ kilogram} \end{aligned}$$

The 3 litre bottle has a mass of 1 000 grams, so adding 1.5 litres of water (1 500 grams) will give it a total mass of 1 000 + 1 500 = 2 500 grams.

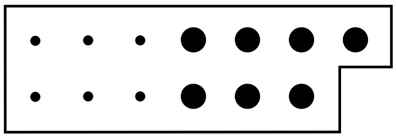
Page 23: Talk About: Three

Problem One

A model helps students visualise the effect of adding even and odd numbers. Even numbers are evenly divisible by two, and odd numbers have a remainder of one when divided by two.

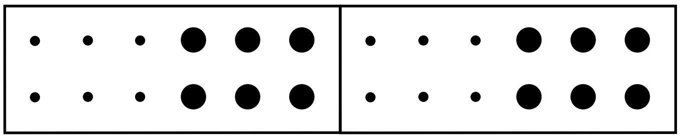


even numbers



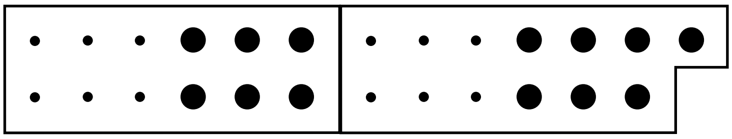
odd numbers

The effect of adding even numbers is:



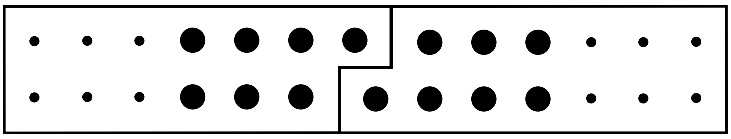
always even and divisible by two.

The effect of adding an odd and even number is:



always odd and has a remainder of one when divided by two.

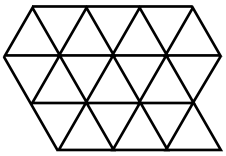
The effect of adding two odd numbers is:



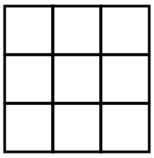
always even and divisible by two.

Problem Two

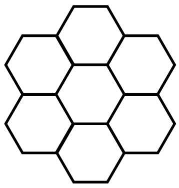
There are only three regular tessellations, that is, tessellations formed by the same regular polygon. They are:



equilateral triangles



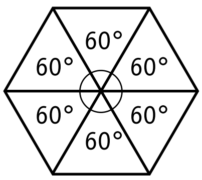
squares



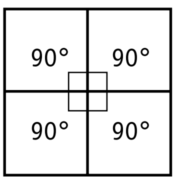
regular hexagons

So the statement is false.

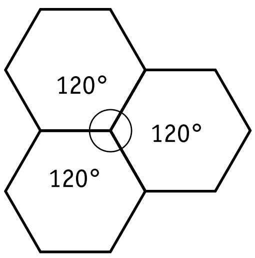
A key feature of tessellations is that the internal angles of shapes meeting at a vertex add to 360° (a full turn).



6 x 60° = 360°



4 x 90° = 360°



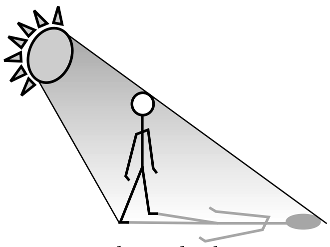
3 x 120° = 360°

By considering the internal angles of other regular polygons, students can see that the tessellations shown are the only possibilities from single regular polygons.

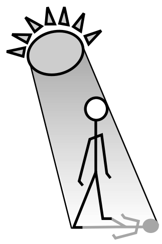
Sides	Internal angles	360º divisible by internal angle
3	60°	360° ÷ 60 = 6
4	90°	360° ÷ 90 = 4
5	108°	360° ÷ 108: not divisible
6	120°	360° ÷ 120 = 3
7	128.57°	not divisible
8	135°	360° ÷ 135: not divisible
9	140°	not divisible

Problem Three

Students may wish to predict what will happen to the length of their shadow as the day passes. They can then take measurements at different times of the day to check their prediction.
A reasoned explanation for the pattern should involve the height of the sun.

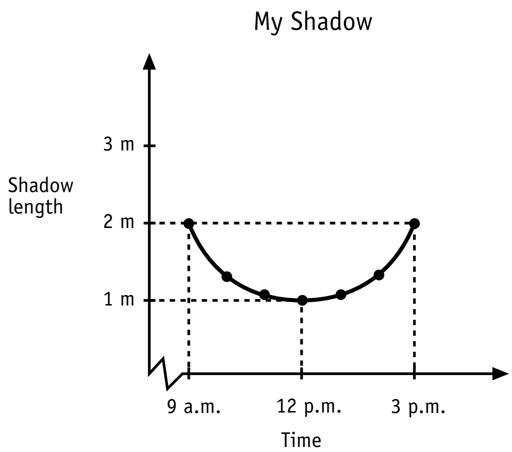


long shadow
Sun low at early morning and late afternoon



short shadow
Sun high at midday

So a graph of shadow length against time will look like this:



Problem Four

The average mass of students in years 5 and 6 is about 30–40 kilograms. This means that a class of 30 students will have a combined mass of between 30 x 30 = 900 kilograms, which is 0.9 tonnes, and 30 x 40 = 1 200 kilograms, which is 1.2 tonnes.

The live weight of a good-sized bull is between 450 and 650 kilograms. A prize show bull can have a mass as great as 1 tonne.

So a class of students is likely to have a combined mass much greater than the average bull.

Page 24: Talk About: Four

Problem One

Through this problem, students explore the idea of reciprocals. One is its own reciprocal because dividing by one has the same effect as multiplying by one.

Students will need to discuss what multiplying by a decimal less than one means. Multiplying by 0.5 is the same as finding a half of something. So 0.5 x 2 = 1 (because 1 is a half of 2), 0.5 x 10 = 5, and 0.5 x 18 = 9. Finding a half is the same as dividing by two.

Multiplying by 0.1 is the same as finding a tenth of something. So 0.1 x 3 = 0.3 (because one-tenth of three is three-tenths) and 0.1 x 15 = 1.5 (because one-tenth of 15 is fifteen-tenths, which is 1.5). Finding 0.1 of something is the same as dividing by 10.

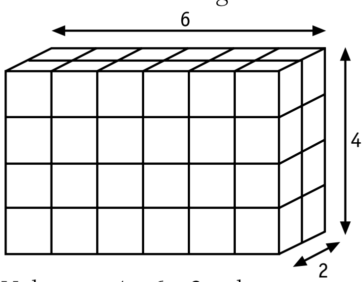
Dividing by four is like finding a quarter of something. Since 0.25 = $^1/_4$, multiplying by 0.25 has the same effect as dividing by four. For example:

8 x 0.25 = 2	8 ÷ 4 = 2
24 x 0.25 = 6	24 ÷ 4 = 6

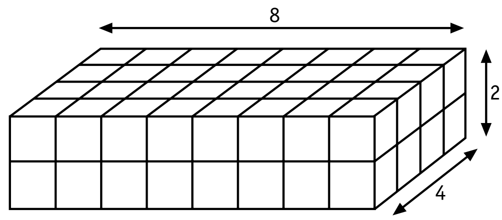
Problem Two

Finding the volumes of the cuboids shown is more difficult than previous problems involving whole numbers of cubes. Students will need to generalise the method for calculating the volume of a cuboid (length x width x height).

Many of them will need to go back to cube models to realise this:

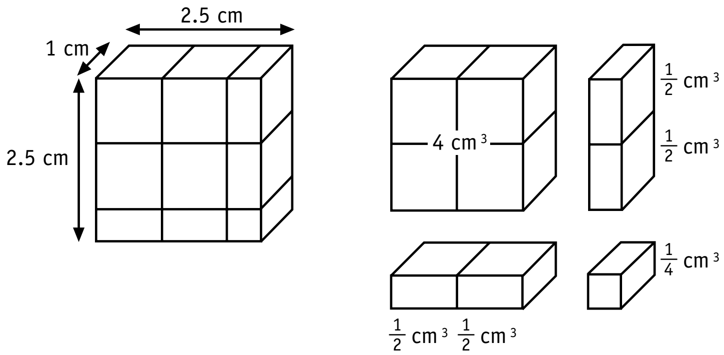


Volume = 4 x 6 x 2 cubes
= 48 cubes



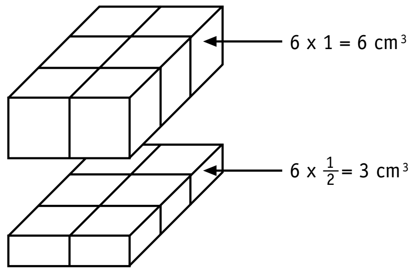
Volume = 2 x 8 x 4 cubes
= 64 cubes

The boxes shown on page **24** of the students' booklet have no lines to mark cubic centimetres. Putting these in may help some students visualise the dimensions of each cuboid.



Counting the cubes and parts of cubes verifies that multiplying the sides gives the volume: 2.5 x 2.5 x 1 = 6.25 cm^3.

Similarly, the volume of the other cuboid can be found. (See **Problem One** on page **18** of the students' booklet.)

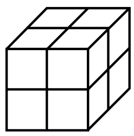


Volume = $1.5 \times 2 \times 3 = 9 \text{ cm}^3$

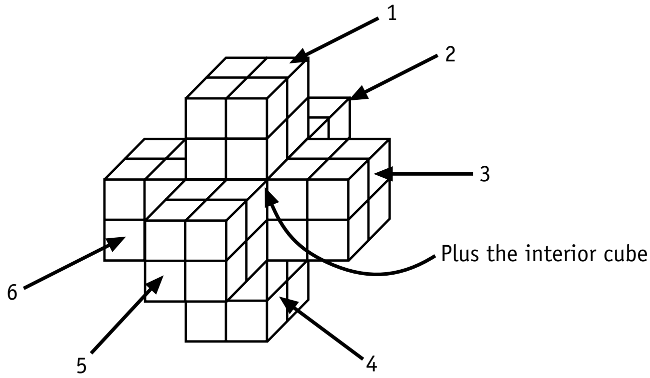
Problem Three

Encourage students to visualise the model without using cubes to build it. Students will have a variety of methods for working out the number of cubes in the model. For example:

The building is made up of $2 \times 2 \times 2$ cubes like this:

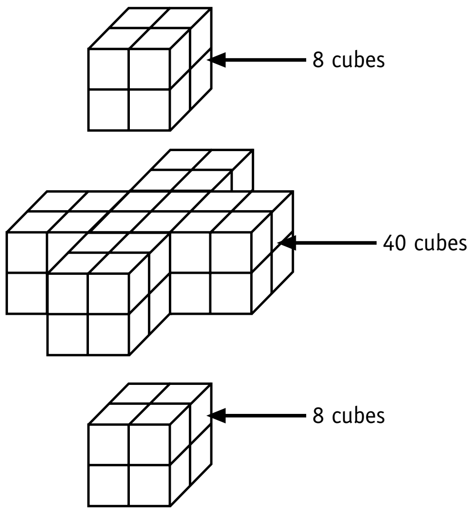


8 cubes



$7 \times 8 = 56$ cubes

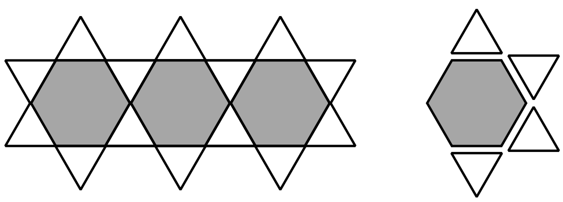
Alternatively, the model can be divided other ways:



8 + 40 + 8 = 56 cubes

Problem Four

This problem involves continuing a sequential pattern. Students can model it with hexagonal and triangular pattern blocks. As students build the pattern, they will see that for each new hexagon added, four triangles are added.



This can also be shown in a table:

Hexagons	1	2	3	4	5	…
Triangles	6	10	14	18	22	…

The table could be extended to 20 hexagons by adding four triangles each time.

Students who see repeated addition of four as multiplication by four will find more efficient ways to solve the problem, such as:

- $6 + (19 \times 4) = 82$

first hexagon

19 lots of 4 triangles

- $2 + (20 \times 4) = 82$
- $14 + (17 \times 4) = 82$

triangles in first 3 hexagons

17 lots of 4 triangles

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