Core Curriculum: Learning to Work Like a Mathematician

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Time for a change?

Perhaps it's time we stopped teaching mathematics. It has bad karma. Each of us knows at least one person who has been damaged by maths education; in some cases ourselves; in some cases, one of our students.

What an appalling situation!

We could blame the victim - *It's okay, you weren't cut out to do maths*. We could punish the victim - *We'll put you in a lower stream*. - the implication that you weren't 'born right' remains and you will be treated like an idiot by being fed the same stuff you have already failed only more slowly. Or we could look at our teaching practice.

Traditional (dare I say dominant) mathematics teaching practices derive from the needs of the industrial revolution. Around that time compulsory, universal education was introduced as a social reform and the purpose of mathematics in the curriculum was to train people who would sit at candle-lit, sloping desks wearing eye shades and using a dipping pen to fill in ledgers recording the transactions of industry. Such clerks had no need to question, explore, challenge, debate, deviate... - they had to follow the recipe and do it right - in the same way as...

> *Pythagoras observed that the Egyptians and Babylonians conducted each calculation in the form of a recipe that could be followed blindly. The recipes, which would have been passed down through generations, always gave the correct answer and so nobody bothered to question them or explore the logic underlying the equations. What was important for these civilizations was that a calculation worked - why it worked was irrelevant.*
>
> Singh, Simon (1997) *Fermat's Enigma*, Walker and Company, New York, P.7

Remind you of anything?

It was the Greeks (Pythagoras and friends) who took a different approach to mathematics *and mathematics teaching*. Read their manuscripts and you will find learning is more often a discussion between teacher and pupil. Teacher poses a question; student responds; teacher facilitates further thinking by building on the answer with another question; ...the adventure continues (and sometimes the student manages to extend the teacher's thinking).
Remind you of anything?

Perhaps its time to stop teaching mathematics (of the Egyptian variety) and start paying more attention to the Greek variety. After all, there is very little content in world mathematics curricula that wasn't know to the Greeks.

Perhaps it's time to stop teaching mathematics and start learning to work like a mathematician.

Working Mathematically

Does anyone believe that a professional mathematician bounces into the office in the morning, turns on the computer and delights in doing all the exercises on the left hand side of the screen - perhaps checking the answers on the back of the monitor? No doubt many students, having experienced 10 years of compulsory mathematics education, suspect there is an element of truth in the description. Many who have been asked what they think a mathematician does (if they think they aren't all dead people from history) say 'sums'.

On the other hand a Year 1 teacher was impressed by the insight of a student whose response was 'They solve the world's hardest problems'.

Around 1993/4 the Mathematics Task Centre Project found out what professional mathematicians do by asking some. To a person their response was, in one way or another:

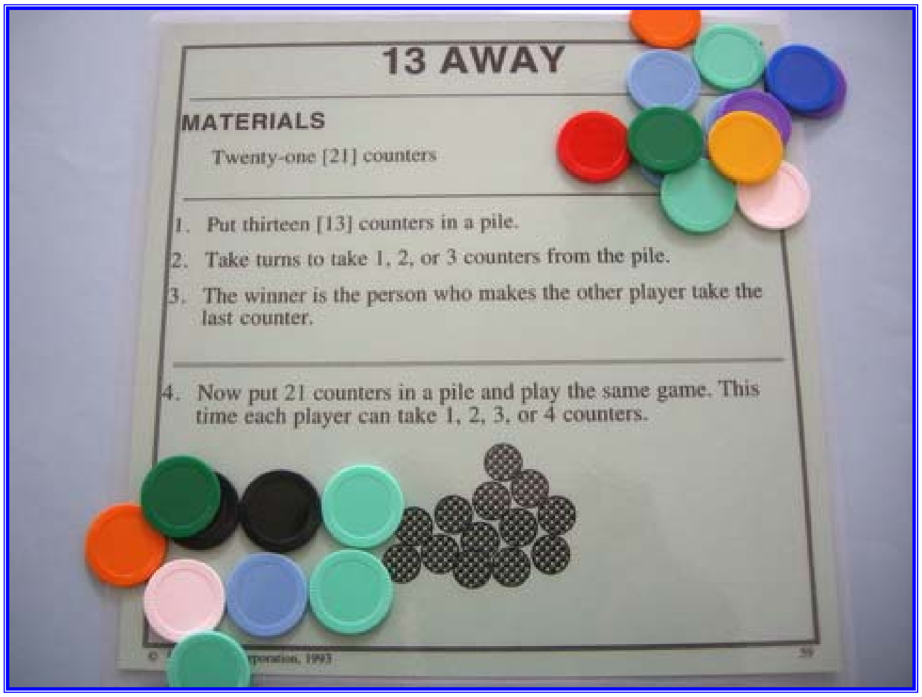
First give me an interesting problem.

Now there's a guide for developing mathematics curriculum.

I have to teach ... (fractions) ... tomorrow. First I need to find a problem that involves fractions then think about the teaching craft that will interest my students in it.

Note, a problem, not an unfamiliar situation requiring the application of a known procedure. Rather, something that by its nature doesn't have an obvious response or perhaps even a way to begin. So, if the core of our curriculum is to become learning to work like a mathematician, we have to find a whole bunch of problems and tease out the teaching craft that fascinates, captivates and absorbs the students in them.

13 Away



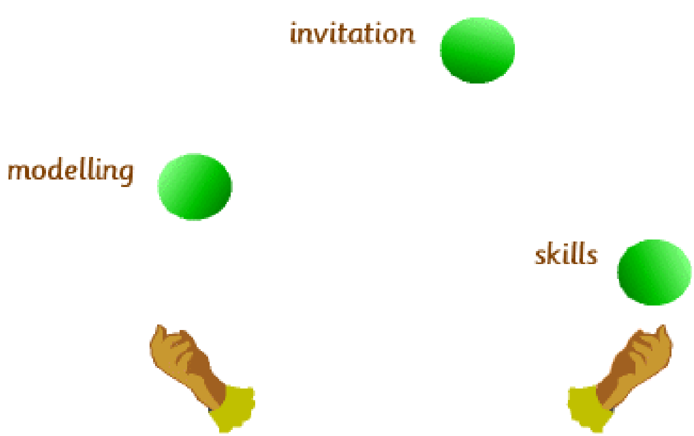
This is a problem from the Mathematics Task Centre. In this form it is an invitation to two students to work like a mathematician.

Students might be interested because of the colours, the counters, because it looks easy or doesn't have many words or because of the game context. For whatever reason, *they* have chosen the task and there is a problem implied - to find out if there is a strategy for winning.

First give me an interesting problem.

There is potential here for working like a mathematician, but the students are not likely to find it for themselves. These resources are teacher involving, not teacher excluding.

If you are going to learn to work like anything there are three major components.



Someone has to show you how to do it. You have to have the opportunity to do it yourself and you will need to practise related skills.

For example, learning to drive. At 16 a young person gets the learner's permit. Mum and dad could give them the car and the keys and leave them alone on the road for two years - an invitation to learn to drive. Or, mum and dad could say: *Mate, I have been doing this for 30 years. Sit here next to me for the next two years and I will show you how it's done.* - modelling how to drive. Or mum and dad could decide that the bit which trips up every other kid is the skill of reverse parking and spend two years practising (and no doubt testing) that.

Clearly none of these things alone will aid the learner driver to obtain their provisional licence. A successful learning to drive curriculum would involve juggling the time spent on each to find the appropriate balance for the student. So it is when the core curriculum of learning to work like a mathematician.

The teacher can interact at two points (at least) with students who are using the task above. The double line ruled across the card following Question 3 is one. It is optional, but it was the intent of the first task centre teachers in 1977 that when students reached this double line they were required to speak with the teacher. The part below the double line is usually a 'bit harder' so the teacher questions to review what has been discovered to date and to prepare the students for their next challenge.

The second is at the end of the card. Students saying *We've done this one Miss.* and being answered with *Good work, choose another one.* will learn that the objective is to get through as many tasks as possible. The teacher will soon hear *But we've done 'em all Miss. This is boring*. Instead, the teacher again questions, firstly to applaud what has been achieved and secondly to suggest that there is more that could be done with the problem. The students don't have to do it right now, but it does leave the door open to returning to the problem either as a pair, individually, or in a whole class investigation. A mathematician is never finished with a problem. They are only finished for now. Some day they, or another mathematician, will ask the question that sparks further investigation.

Three Lives of a Task

Another way teachers use tasks is to convert them to whole class investigations for the purpose of modelling how a mathematician works. The 238 tasks in the Mathematics Task Centre collection have been gathered over thirty years and each has been chosen because it has three lives - the invitation as a packaged hands-on challenge for two, the possibility of conversion to a whole class investigation to model Working Mathematically and the opportunity to become a deeper investigation for a pair led by an Investigation Guide written by the staff. A great deal of skill development happens in context through these lives and more is included as Tool Box lessons are added to the curriculum.

To convert 13 Away a teacher only needs to gather sets of counters - Poly Plug works well because the counters are quiet, contained and easily packed up.

Moving deliberately away from the front of the room (why?) the teacher gathers the students around two volunteers in a 'fishbowl' situation:

- *John and Mary I want you to help me explain today's challenge to the rest of us. Mary can you press out 13 plugs please...*
- *John convince me there are thirteen there to start with...*
- *Can anyone convince me another way...*
- *Have a chat with the person beside you and think of at least two ways you could convince someone there are 13...*
- *Did anyone think of looking at what was left in the red board?*

This sequence only takes two minutes, but it is a great informal teaching opportunity which not only shares mental arithmetic strategies, but validates the mathematician's question *Can I check it another way?*.

> *John and Mary, decide who goes first and on your turn you can remove 1, 2 or 3 counters from the pile. ... The person who removes the last counter loses.*
>
> *Okay everyone, that's how the game works. Now use your own plugs and find out what all you can about it.*

As hypotheses develop about how to win, record them (how about the students recording them on the whiteboard and owning them by adding their initials), discuss them and perhaps check them by organising class play offs. Don't forget that to test a game hypothesis you must assume you are playing against the world's best. There is no point claiming your hypothesis works provided your partner makes a mistake or fails to notice something.

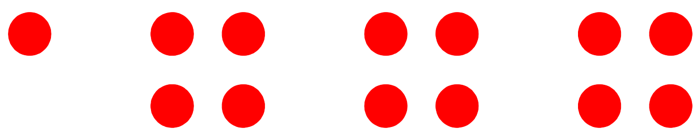
After an appropriate time students will discover that the winning strategy is to invite the other person to go first and then remove 1, 2 or 3 on your turn so that they face 9 counters then 5 counters then 1 counter. So, the mathematicians have solved the problem, but their work is not over. The result must be communicated to colleagues. It must be published so others can check the work and so this piece of mathematics can be a platform for the next problem.

How can we explain to someone else how they can play this game to win? Perhaps they want to play it against their parents to get an increase in pocket money.

The teacher now encourages reflection on the problem from the point of view of what someone else needs to know. This reprocessing consolidates, and often extends, learning. Working through students' suggested explanations encourages language development. Perhaps someone will suggest working backwards, trying every possible case along the lines of:

- If I leave you facing 1, I will win (because...).
- If I leave you facing 2 I will lose (because...).
- If I leave you facing 3 I will lose (because...).
- If I leave you facing 4 I will lose (because...).
- If I leave you facing 5 I will win (because...).
- ...

Perhaps someone will suggest making a model or drawing a diagram based on the counters:



I want you to take the last counter. So if you go first I keep removing counters to make up a four.

But why is it four? What's that got to do with the problem? And what happens if...?

What happens if the person who takes the last counter wins? The task card suggests changing the start number and the removal rules. What happens if each person can remove 2, 3, or 4, but never 1 counter?

If I tell you any number of counters and any counting rule can you tell me how to win?

And on another day, if we play the game of 33 Away on a calculator, where on our turn we have to subtract 1 or 2 or 3 and the person who has to make the screen zero (or less) loses, can we transfer our learning by asking: *Do I know a similar problem?*

So 13 Away can be an invitation to work like a mathematician or it can be an opportunity to model how a mathematician works, or both. Also, since it becomes clear that efficient use of skills related to multiplication (or division if you like) and subtraction (or addition if you like) are important for having the upper hand in the problem, students respond well to a follow up Toolbox Lesson introduced with something like:

The last couple of days while we have been exploring 13 Away it has been important to be skilful using our times tables. Today we are just going to practise those skills so they are sharper the next time we need them in a problem. Open your books to page...

Reviewing 13 Away

To find a place in a core curriculum published as learning to work like a mathematician, a problem like 13 Away must be more than a game and it must be more than fun. There are many lenses which could be passed over an investigation like this - policy, equity, learning theory, professional development, assessment... - but perhaps most importantly:

- Where do we see the best practice teaching craft that is likely to encourage happy, healthy, cheerful, productive, inspiring classrooms?
- How have we worked like a mathematician?

Learning Features

Our work is 100% professional development and over decades of exploring and gathering stories like these we have asked many teachers from many places to identify pedagogical features likely to encourage learning. The list to date is appended to this article and is offered as a support for your own planning.

- Which of the items on the list are built into the invitation form of 13 Away?
- Which of them have teachers built into the whole class modelling form of 13 Away?
- Can you put the list to work to interest students in a different problem?

How have we worked like a mathematician?

When mathematicians told us:

First give me an interesting problem.

they went on to detail the process of working with such a challenge. Their Working Mathematically process is also appended to this article. You are invited to apply it to 13 Away.

- In what ways have we worked like a mathematician?

What happens if...?

13 Away clearly passes every test the 'curriculum police' might require of it. But what happens if we have not one, but hundreds of problems like this integrated throughout a curriculum that balances invitation, modelling and skill practice to encourage learning to work like a mathematician?

Research and Teacher Stories suggest that the result is more success more often for teachers and students. For example the four year INISSS study (Improving Numeracy for Indigenous Secondary School Students) showed that, compared to a control group, students whose teachers developed Working Mathematically as their core curriculum had better problem solving skills, better content skills and better literacy in Year 10.

You can read the evidence for yourself at the Mathematics Task Centre:

- http://www.blackdouglas.com.au/taskcentre

in the Research & Stories link.

Also the Maths Tasks link, through a range of sub-links, documents years of collected classroom wisdom related to successfully using hands-on tasks. And you will find at least 80 more problems like 13 Away freely documented for your use in the Task Cameos link. And there's more ... but no steak knives.

In summary, there is no reason for mathematics to be taught the way it always has been.

Lesson Features

- a checklist for encouraging learning -

- ❑ application focus
- ❑ assessment opportunities
- ❑ builds on personal student experience
- ❑ communicating mathematics
- ❑ concept focus
- ❑ concrete materials
- ❑ concurrent teaching of topics
- ❑ differentiation for ability range
- ❑ easy to state/easy to start
- ❑ estimation
- ❑ first hand data
- ❑ game context
- ❑ group work
- ❑ history of mathematics
- ❑ home/school links
- ❑ inclusive
- ❑ informal or incidental learning
- ❑ interdisciplinary connections
- ❑ investigative process
- ❑ ....
- ❑ ....
- ❑ kinaesthetic
- ❑ links to learning theory
- ❑ mathematical modelling
- ❑ mixed ability
- ❑ multiple entry & exit points
- ❑ non-threatening
- ❑ open-ended
- ❑ outdoor
- ❑ ownership
- ❑ recording & publishing
- ❑ skill development in context
- ❑ social issues
- ❑ story shell
- ❑ technology (calculators)
- ❑ technology (software)
- ❑ physical involvement
- ❑ visual (visualisation)
- ❑ whole class
- ❑ working mathematically process
- ❑ ....
- ❑ ....

- This list has been constructed through discussion with teachers in many workshop situations.
- It is our attempt to build a common language to debate the features of a classroom more likely to engage students in mathematics learning.
- Is there a place for some of these features in every lesson you plan?
- Please 'play' with the list in professional learning and team planning situations and let us know what support it gives you.
- Contact Doug. Williams: doug@blackdouglas.com.au

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if …?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Break the problem into smaller parts
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- ...

If one way doesn't work I just start again another way.