Curious Cubes

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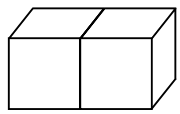
Exercise 1: Challenges using cubes

We are learning that shapes may be different but can still have the same properties.

Equipment: Multi-link cubes

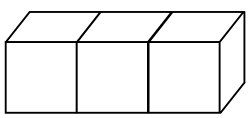
1) Here is one way of drawing of two connecting cubes:



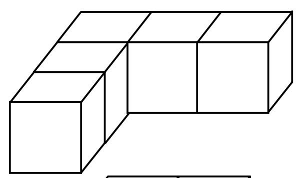
Identify and draw the: Front view, Top view, Left side view

2) Do the same as you did in question one for each of the shapes drawn below:

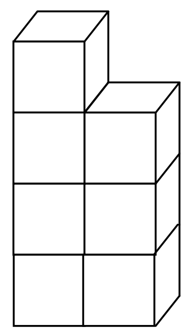
a)



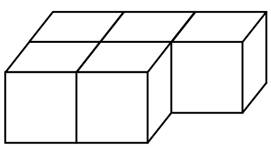
b)



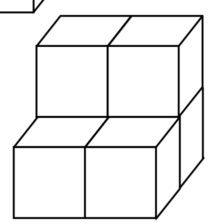
c)



d)



e)



3) Copy and complete the following table. Refer to the shapes drawn above.

Shape	Number of:		
	Edges	Faces	Vertices
a			
b			
c			
d			
e			

4) In what follows, use the answers you obtained in question 3.
Are any of the answers you obtained the same for different shapes?

Is there a relation between the number of edges, faces or vertices? You will need to explore other shapes in order to do this. It might be worth experimenting by keeping some of these numbers fixed and changing others.

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Exercise 2: Four-Cube Houses

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We are learning use isometric paper and to find all the different shapes that can be constructed with the same volume.

You will need multi-link cubes and isometric paper for this task.

1) Julie is building a new house. It will have four rooms. Using multi-link cubes make one possible design for her house. Draw your design on isometric paper.

2) Building regulations say that Julie cannot have any overhanging rooms. (This means a room on the second story without a room beneath it) Use multi-link cubes to explore all the different designs Julie could choose. Draw each of these on your isometric paper.

3) Are all your designs unique? How can you tell?

4) How do you know you have found all the possible combinations? Discuss.

4) To build this house it will cost Julie:
   - $10 000 for each unit square of land covered.
   - $6 000 per square of external wall
   - $8 000 per square of roof

Investigate: Which house will be cheapest to build? Which will be the most expensive?

5) Explore: How would Julie's designs change if her house could have 5 rooms?

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Exercise 3: Bigger cubes....

We are learning to discover the relationship between the dimensions of a 'cube' and its volume.

1) Mark wants to build an apartment complex that has the same width, height and length. If the complex was 3 rooms wide, how many rooms would the complex have?

2) What about if the complex was
    a) 4 rooms wide?
    b) 5 rooms wide?

3) If the complex had the following number of rooms, how wide would the complex be?
    a) 1000
    b) 343
    c) 400

4) Investigate the relationship between the width, height, length and the number of cubes needed to build an apartment.

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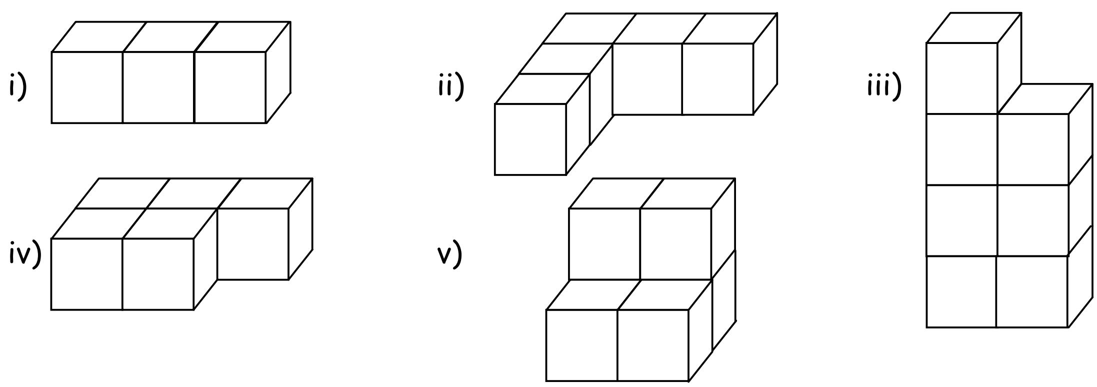
Exercise 4: Painted Cubes

We are investigating shapes that are made up of cubes and exploring their properties.

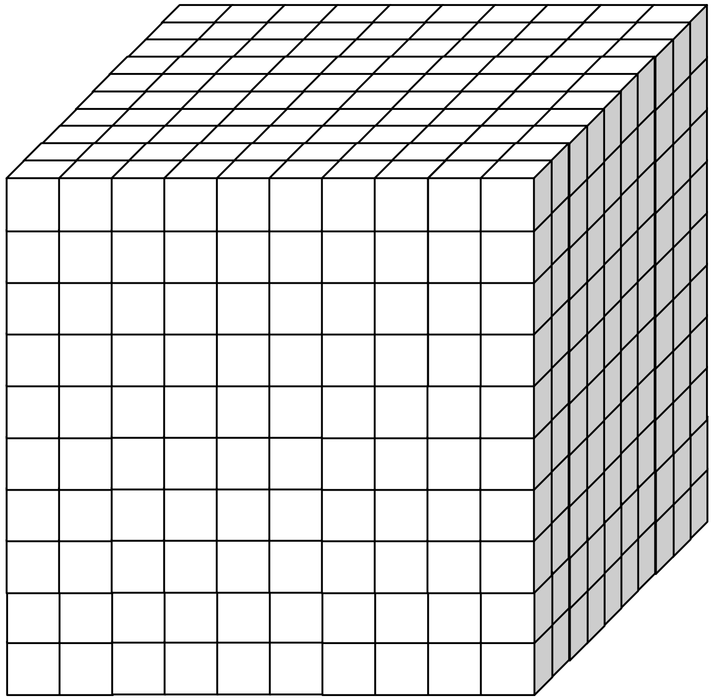
1) Imagine that each of the outsides of the solids drawn below was painted red, and then separated into separate cubes. How many cubes will have exactly:
   a) 5 red faces?
   b) 4 red faces?
   c) 3 red faces?
   d) 2 red faces?



2) Imagine one yellow cube. How many one-centimetre blue cubes are needed to completely cover the yellow cube and so as to produce a larger blue cube**?**

3) A 10 cm × 10 cm × 10 cm cube was painted blue and then separated into 1000 small centimetre cubes.
   Copy and complete the table showing how the 1000 cubes are painted.

Number of faces painted blue	Number of cm cubes
0	
1	
2	
3	
4	



4) Suppose each of the corner cubes is removed before painting. How many cubes with 0, 1, 2 and 3 faces painted will there be?

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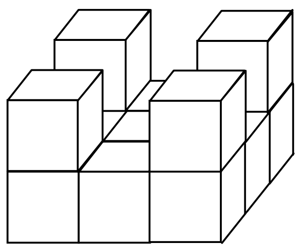
Exercise 5: Castles

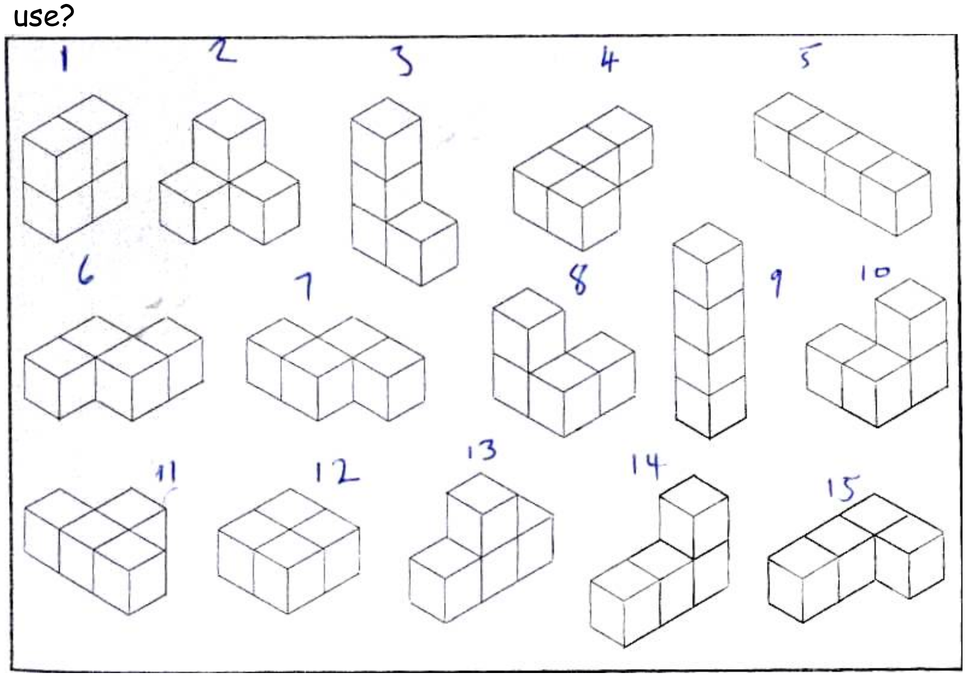
We are investigating shapes that are made up of cubes and exploring ways of calculating their volume.

Melanaite uses cubes to build castles like the one below. These all have a square base of cubes and one cube on the top of this square at each corner



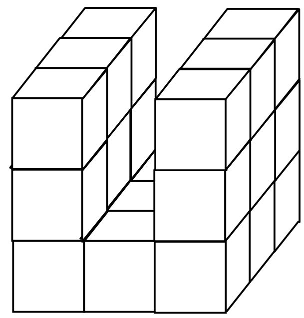
1) How many cubes did she use in the caste shown?

2) If she made a castle that was 4 cubes wide how many cubes would she use?



3) How many cubes would she use for a castle that was 10 cubes wide? Try to use two different strategies to calculate the number of blocks.

4) Repeat this investigation with the following starting castle based on a square of cubes.



5) Design your own 'starting' castle and use it to explore the number of cubes needed to make castles of different sizes. Draw diagrams to help.

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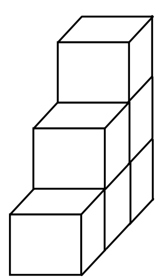
Exercise 6: Staircase

We are investigating shapes that are made up of cubes and exploring ways of calculating their volume.

1) Pita used 6 cubes to build a staircase with three steps. How many cubes will Pita use to make a 5-step staircase?



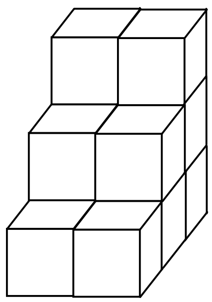
2) Complete the table

Number of steps	Number of cubes

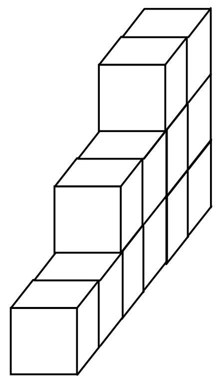
1	
2	
3	
4	
5	
6	
7	
8	

3) Pita explores two other design possibilities of staircases.
How do these change the number of cubes needed? Show your working and explain your reasoning.

a)



b)



4) Investigate: What happens to the number of cubes in a staircase which is twice as wide, twice as long and twice as tall as that in Q1?

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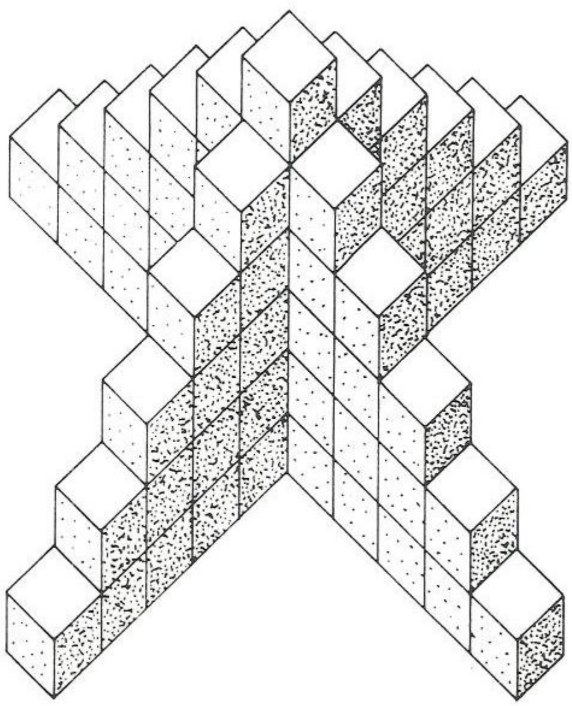
Exercise 7: Skeleton Tower

We are investigating shapes that are made up of cubes and exploring ways of calculating their volume.



1) Jacob uses cubes to build a tower like the one above.
   How many cubes did he need to build this tower?

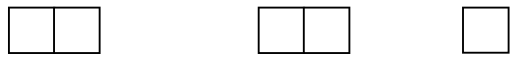
2) How many cubes would he use for a tower that was (a) 13 cubes wide and (b) 17 cubes wide? Try to use two different strategies to calculate the number of blocks.

3) Explain how you worked out your answer to question 2.

4) Find a formula for the number of cubes needed for a tower ***n*** cubes high?

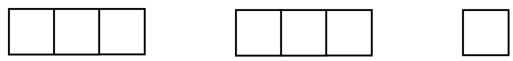
Exercise 1 Answers

1) front top left

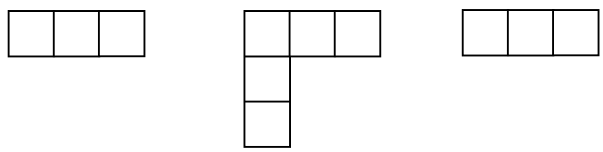


2) front top left

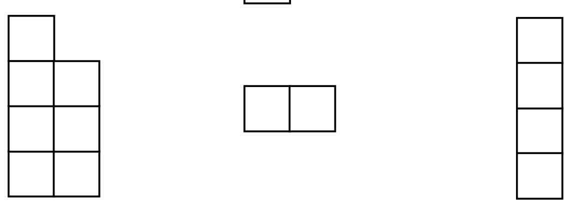
a)



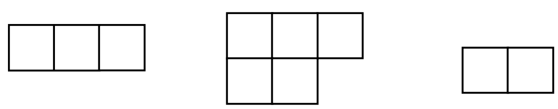
b)



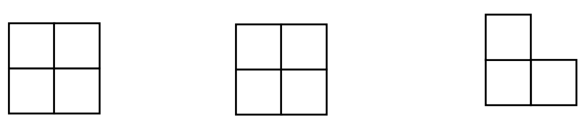
c)



d)



e)



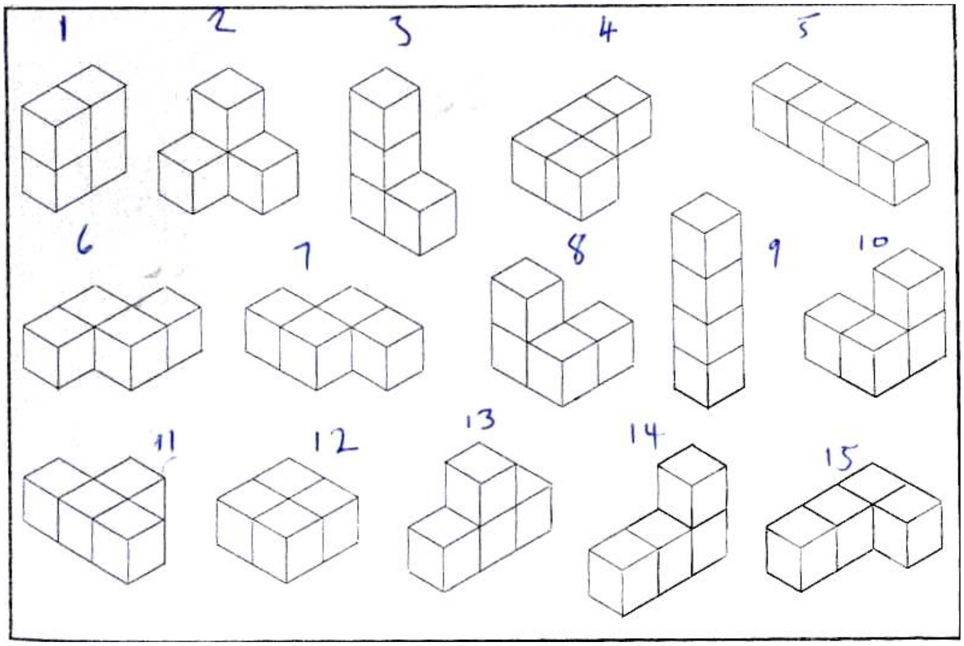
3)

Shape	Number of:		
	Edges	Faces	Vertices
a	12	6	8
b	18	8	12
c	18	8	12
d	18	8	12
e	18	8	12

6) Yes.
Number vertices – number of edges – number of faces = 2 (or equivalent)

Exercise 2 Answers

1) & 2) The fifteen possible designs are drawn below:



3) Combinations which are simple rotations around a vertical axis are not counted as they are not considered unique.

4) Any 'reasonable' explanation with justification. (i.e. the above comment.)

5) From left to right.

Shape	Floors $10 000 ea	External wall $6 000 ea	Roof $8 000 ea	Total cost
1	20 000	72 000	16 000	108 000
2	30 000	72 000	24 000	126 000
3	20 000	84 000	16 000	120 000
4	40 000	60 000	32 000	132 000
5	40 000	60 000	32 000	132 000
6	40 000	60 000	32 000	132 000
7	40 000	60 000	32 000	132 000
8	30 000	72 000	24 000	126 000
9	10 000	96 000	8 000	114 000
10	30 000	72 000	24 000	126 000
11	40 000	60 000	32 000	132 000
12	40 000	48 000	32 000	120 000
13	30 000	60 000	24 000	114 000
14	30 000	72 000	24 000	126 000
15	40 000	60 000	32 000	132 000

Cheapest houses: Shapes 1
Most expensive houses: Shapes 4, 5, 6, 7, 11, 15

7) Students should be encouraged to present their work using diagrams and be encouraged to describe the strategy they used. The approach taken, rather than the precise numeric solution is the important thing here.

Exercise 3 Answers

1) 27

2) a) 64 (b) 125

3) a) 10 (b) 7 (c) impossible

4) When width = length = height then the volume = width^3
Also Width = $\sqrt[3]{volume}$

Exercise 4 Answers

1)

Red Faces (no.) Shape	5	4	3	2
i	2	1	0	0
ii	2	3	0	0
iii	1	3	3	0
iv	1	3	1	0
v	0	2	2	2

2) 26

3)

Number of faces painted blue	Number of cm cubes
0	512
1	384
2	96
3	8
4	0

4)

Number of faces painted blue	Number of cm cubes
0	512
1	384
2	72
3	24
4	0

Exercise 5 Answers

1) 13 (2) 20 (3) 10

4) 21, 28, 118

5) Student's own solutions with clear diagrams and calculations required.

Exercise 6 Answers

1) 15

2)

Number of steps	Number of cubes
1	1
2	3
3	6
4	10
5	15
6	21
7	28
8	36

3)
a)

Number of steps	Number of cubes
1	2
2	6
3	12
4	20
5	30
6	42
7	56
8	72

b)

Number of steps	Number of cubes
1	2
2	6
3	12
4	20
5	30
6	42

7	56
8	72

Both of these designs double the amount of blocks required.
(A diagram may be drawn to illustrate this.)

4) 8 times as many at each step.(Tables or diagrams may be drawn to support this conclusion.)

Exercise 7 Answers

1) 66

2) (a) 71 (b) 153

3) clear description required

4) "number of cubes = $n(2n-1)$" or equivalent