Cats and bods

Purpose:

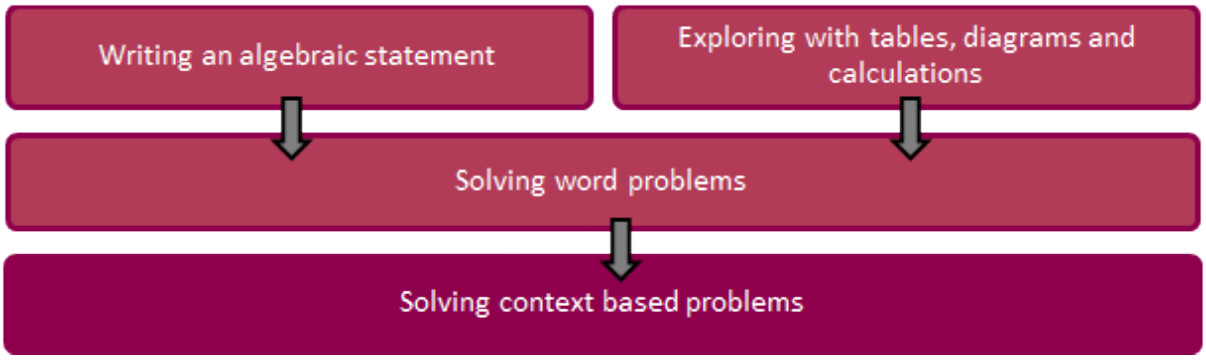
The purpose of this activity is to engage students in solving a problem involving the multiplicative strategies that form the basis of early algebraic investigation.

Achievement Objectives:

NA4-1: Use a range of multiplicative strategies when operating on whole numbers.
NA4-4: Apply simple linear proportions, including ordering fractions.
NA4-7: Form and solve simple linear equations.

Description of mathematics:

The background knowledge and skills that need to be established before and/or during this task are outlined in the diagram below:



Exploring with tables, diagrams and calculations
I used two pegs for each shirt and one peg for each hanky I hang on the line. If I used 10 pegs in all, what are the possibilities for how many shirts and hankies I hung?

Writing an algebraic statement to express information
I used two pegs for each shirt and one peg for each hanky I hang on the line. Express this as an algebraic statement.

Solving word problems
If I used two pegs for each shirt and one peg for each hanky I hang on the line, how many pegs are needed for five shirts and two hankies?

Word problems
I used two pegs for each shirt and one peg for each hanky I hang on the line, hanging twice as many shirts as hankies. If I used 24 pegs, how many shirts did I hang?

This activity may be carried out with step by step guidance, or by allowing the student to follow their own method of solution. The approach should be chosen in sympathy with students' skills and depth of understanding.

Activity:

In our household, there are twice as many cats as humans, with sixty legs in total.

How many cats are there to be fed?

The procedural approach

The student is able to represent the problem in such a way that leads to a solution.

Prompts from the teacher could be:

1. How could you represent this situation?
2. Could you draw the legs?
3. How many cat legs will there be for each human?
4. How many legs in total (cat and human) will there be for each human?
5. Use what you have found so far, to get to 60 legs in total.
6. How many cats are there when you have 60 legs in total?

T: Talk to me about your strategy here.

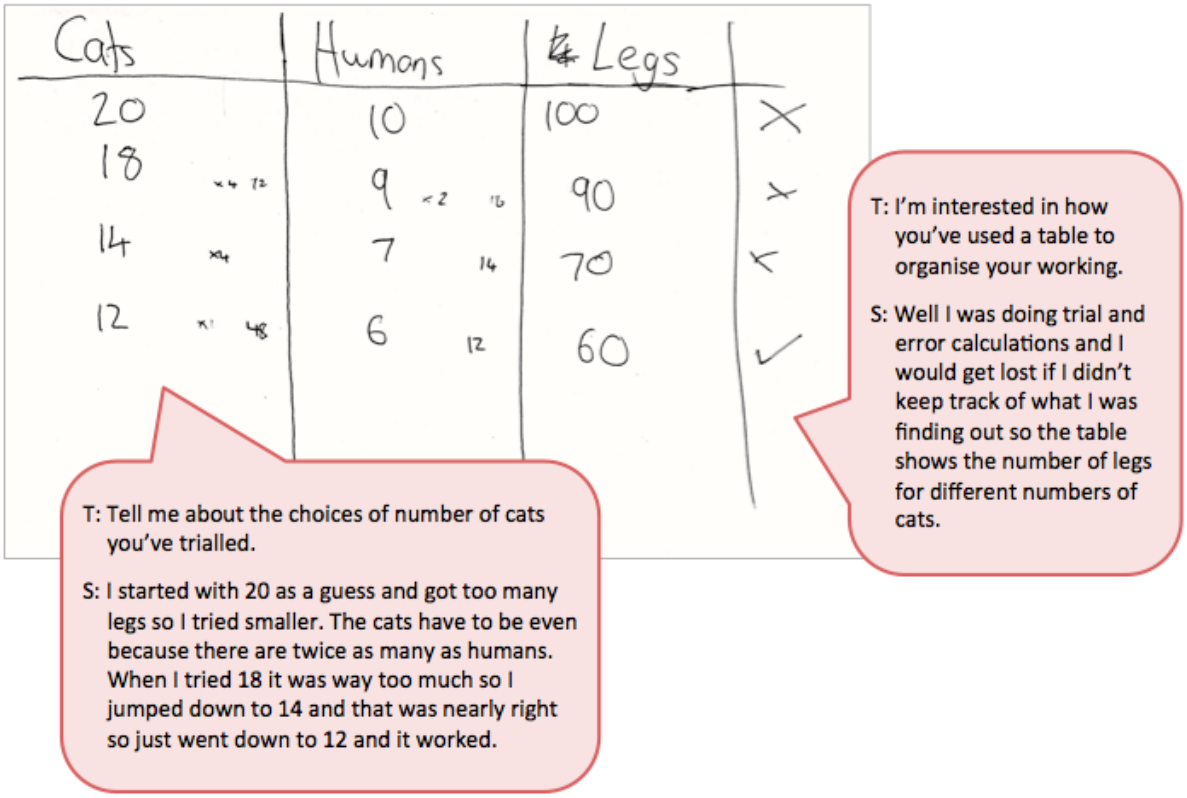
S: I prefer using pictures so I drew out all the c legs and h legs in sets of 2 cats for each human. I kept going until I got to 60.

The arithmetic approach

The student is able to explore, with appropriate calculations, to solve a problem based on a given pattern.

Prompts from the teacher could be:

1. How could you represent this situation?
2. How many legs in total (cat and human) will there be for each human?
3. Use what you have found so far, to get to 60 legs in total.
4. How many cats are there when you have 60 legs in total?



Cats	Humans	Legs	
20	10	100	✗
18	9	90	✗
14	7	70	✗
12	6	60	✓

T: I'm interested in how you've used a table to organise your working.

S: Well I was doing trial and error calculations and I would get lost if I didn't keep track of what I was finding out so the table shows the number of legs for different numbers of cats.

T: Tell me about the choices of number of cats you've trialled.

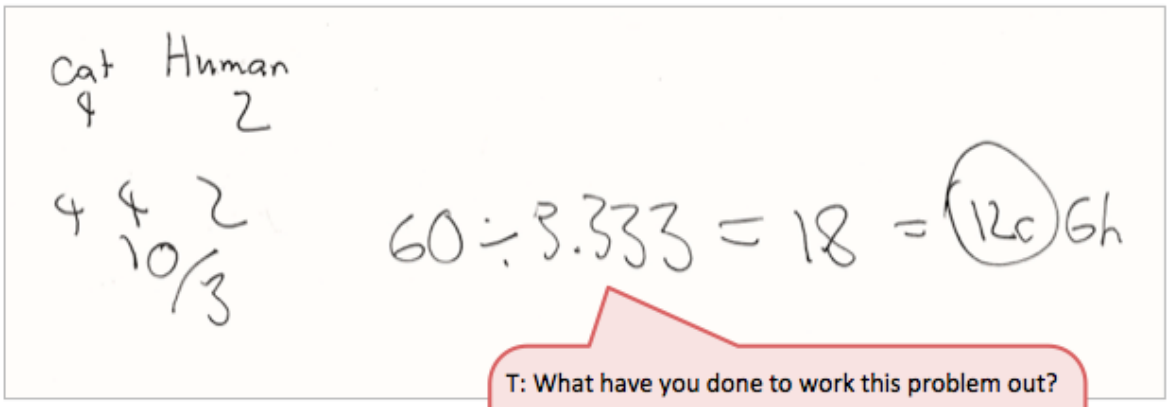
S: I started with 20 as a guess and got too many legs so I tried smaller. The cats have to be even because there are twice as many as humans. When I tried 18 it was way too much so I jumped down to 14 and that was nearly right so just went down to 12 and it worked.

The conceptual approach

The student is able to find the solution to a problem involving patterns.

Prompts from the teacher could be:

1. How could you represent this situation?
2. What pattern can you find between the number of legs and the total number of cats and humans?
3. What pattern can you find between the number of legs and the number of cats?
4. How many cats are there when you have 60 legs in total?



T: What have you done to work this problem out?

S: I found the average number of legs, which must be 3 and a third. I then divided 60 by this and got 18 animals. Then split 18 into 6 and twice as many, 12. So there are 12 cats.

Pattern between two variables (cats and humans) is described.