

Area of a regular pentagon

Purpose:

The purpose of this multi-level task is to engage students in a task that requires them to apply geometric properties of polygons and right angles triangle techniques in solving for the area of a polygon.

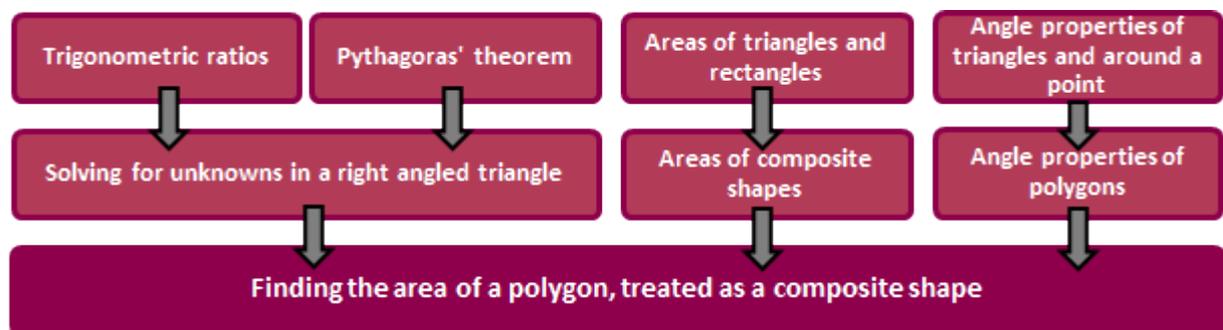
Achievement Objectives:

GM5-3: Deduce and use formulae to find the perimeters and areas of polygons and the volumes of prisms.

GM5-10: Apply trigonometric ratios and Pythagoras' theorem in two dimensions.

Description of mathematics:

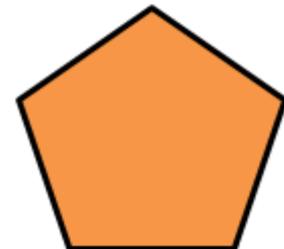
The background knowledge presumed for this task is outlined in the diagram below:



The task can be presented with graded expectations to provide appropriate challenge for individual learning needs.

Activity:

Task: By breaking down into triangles and/or rectangles, find the area of a regular pentagon of side length 10 cm.



The arithmetic approach

The student is able to apply geometric properties of polygons and to use trigonometric ratios to find an unknown area.

Prompts from the teacher could be:

1. Draw a pentagon.
2. Consider the way it could be divided up into right angled triangles and rectangles. Notice the symmetry of the shape. Mark out the rectangles and triangles you will calculate the areas of.
3. Label all the sides and angles that you know.
4. Consider how you will use trigonometry and Pythagoras to solve for the area of the triangles.
5. Use the rules that we have learnt in geometry to find angles that you will need to use.
6. Now work through finding sides of the triangles, to get their areas, etc...

T: How did you know that this angle was 54° ?

S: I used the rule for internal angles adding up to $(n-2)$ times 180. Then I halved my answer.

T: Tell me about your construction lines.

S: Well I worked everything out for half of the shape and doubled it to get the answer. I broke the half into rectangles I can subtract triangles from.

T: How did you know this length was 5cm?

S: Well, the line down the middle of the pentagon is a mirror line, so it must be half of the side length.

T: Is this pentagon to scale?

S: Yes, I wanted to see what the area looked like so I could imagine the answer. I can see a 10cm by 10 cm square will fit in easily so I knew the answer is more than 100cm^2 but it looks like less than 200cm^2 .

Handwritten work on grid paper:

$$\frac{4 \times 180}{5} = 108$$

$$90 - 54 = 36$$

$$108 - 36 = 72$$

$$23.9 \text{ cm}^2$$

$$9.5 \times 8.1 - 14.7 = 62.3$$

$$108 - 36 = 72$$

$$9.5$$

$$18$$

$$54$$

$$36$$

$$\frac{72}{162}$$

$$\frac{180}{162} = 1.11$$

Top: $\cos 36 = \frac{A}{10}$, $A = 10 \cos 36 = 8.1$
 $\sin 36 = \frac{O}{10}$, $O = 10 \sin 36 = 5.9$
 $\text{Area} = \frac{1}{2} \times 8.1 \times 5.9 = 23.9$

Bottom: $\tan 18 = \frac{3.1}{A}$, $A = \frac{3.1}{\tan 18} = 9.5$, $\Delta \text{Area} = \frac{1}{2} \times 3.1 \times 9.5 = 14.7$
Pentagon Area $= 2(23.9 + 14.7) = 77.2 \text{ cm}^2$

The procedural algebraic approach

The student is able to divide a regular polygon into composite triangles and rectangles and apply geometric properties of polygons and to use trigonometric ratios and Pythagoras' theorem to find an unknown area.

Prompts from the teacher could be:

1. Draw a pentagon.
2. Consider the way it could be divided up into right angled triangles and rectangles. Notice the symmetry of the shape. Mark out the rectangles and triangles you will calculate the areas of.
3. Consider how you will use trigonometry and Pythagoras to solve for the area of the triangles.

$(n-2) \times 180 = 3 \times 180$
 $= 540$
 $540 \div 5 = 108$
 $108 \div 2 = 54$
 $\cos 54 = \frac{A}{H}$
 $\cos 54 = \frac{A}{10}$
 $A = 10 \cos 54$
 $= 5.88$
 $10^2 = b^2 + 5.88^2$
 $b^2 = 10^2 - 5.88^2$
 $= 65.4256$
 $b = 8.09$
 $A = \frac{1}{2}bh = \frac{1}{2} \times 8.09 \times 5.88$
 $= 23.785$

$\cos 72 = \frac{A}{10}$
 $A = 10 \cos 72 = 3.09$
 $\sin 72 = \frac{O}{10}$
 $O = 10 \sin 72 = 9.51$
 $A = \frac{1}{2}bh = \frac{1}{2} \times 3.09 \times 9.51$
 $= 14.69$

$2 \times 14.69 + 2 \times 23.785 + 95.1 = 172.05$
 The area of the pentagon is 172 cm^2

T: I see in your working you've got different values for A.

S: Oh yeah, sometimes A is for adjacent when I'm doing trig stuff and sometimes it's area.

T: Tell me how you knew this angle was 72° .

S: Well the whole angle inside the pentagon is 108 and the triangle at the top takes off 36, because $90 - 54$ is 36, so $108 - 36$ gives 72° left over.

T: Tell me about the rounding of your final answer.

S: Well I rounded values at lots of stages so I'm only really sure about my answer to the nearest cm^2 .

The conceptual algebraic approach

The student is able to divide a regular polygon into composite triangles and rectangles and apply geometric properties of polygons and to use trigonometric ratios and Pythagoras' theorem to find an unknown area.

To encourage the use of algebra to form a general rule, the teacher can modify the task by calling the length of each side x , rather than 10 cm.

T: Tell me about the working on the side here.

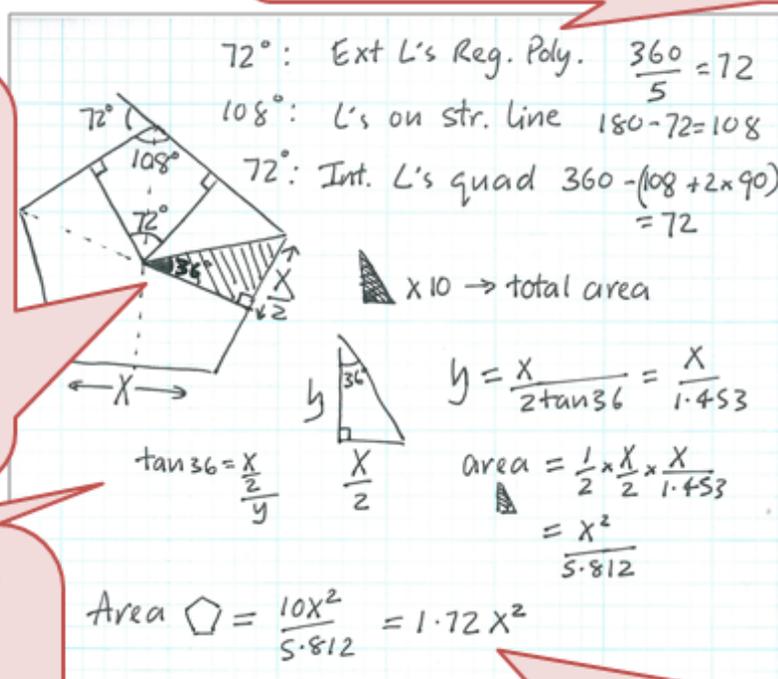
S: Well, to find the angle I needed inside one triangle I had to go back to use the rules we learned in geometry.

T: What did you do to get 36° ?

S: I used the symmetry of regular polygons. The shaded triangle is half of the diamond I worked the angles out on. 36° is half of 72° .

T: you've used trig here.

S: Yeah – for area of triangles I need base and height. I had the base and because it's a right angle triangle and I've got another angle I used tan to get the height.



T: I see you've given the total area in terms of x .

S: Yeah, I've made a general rule – whatever the side of the pentagon is I just put that in where x is.

T: So if x is 10 what would the answer be?

S: 10 squared is 100, so it would be 172cm^2 .