Area of a regular pentagon

Purpose:

The purpose of this multi-level task is to engage students in a task that requires them to apply geometric properties of polygons and right angles triangle techniques in solving for the area of a polygon.

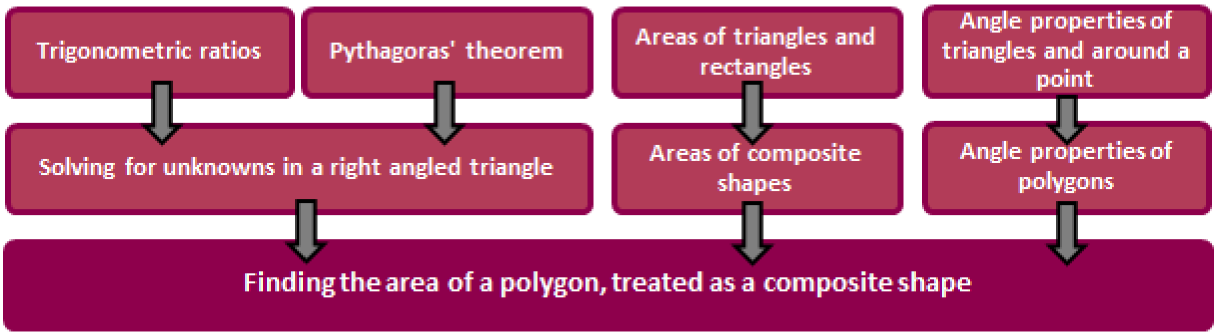
Achievement Objectives:

GM5-3: Deduce and use formulae to find the perimeters and areas of polygons and the volumes of prisms.

GM5-10: Apply trigonometric ratios and Pythagoras' theorem in two dimensions.

Description of mathematics:

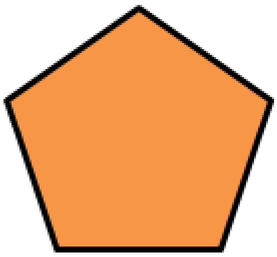
The background knowledge presumed for this task is outlined in the diagram below:



The task can be presented with graded expectations to provide appropriate challenge for individual learning needs.

Activity:

Task: By breaking down into triangles and/or rectangles, find the area of a regular pentagon of side length 10 cm.

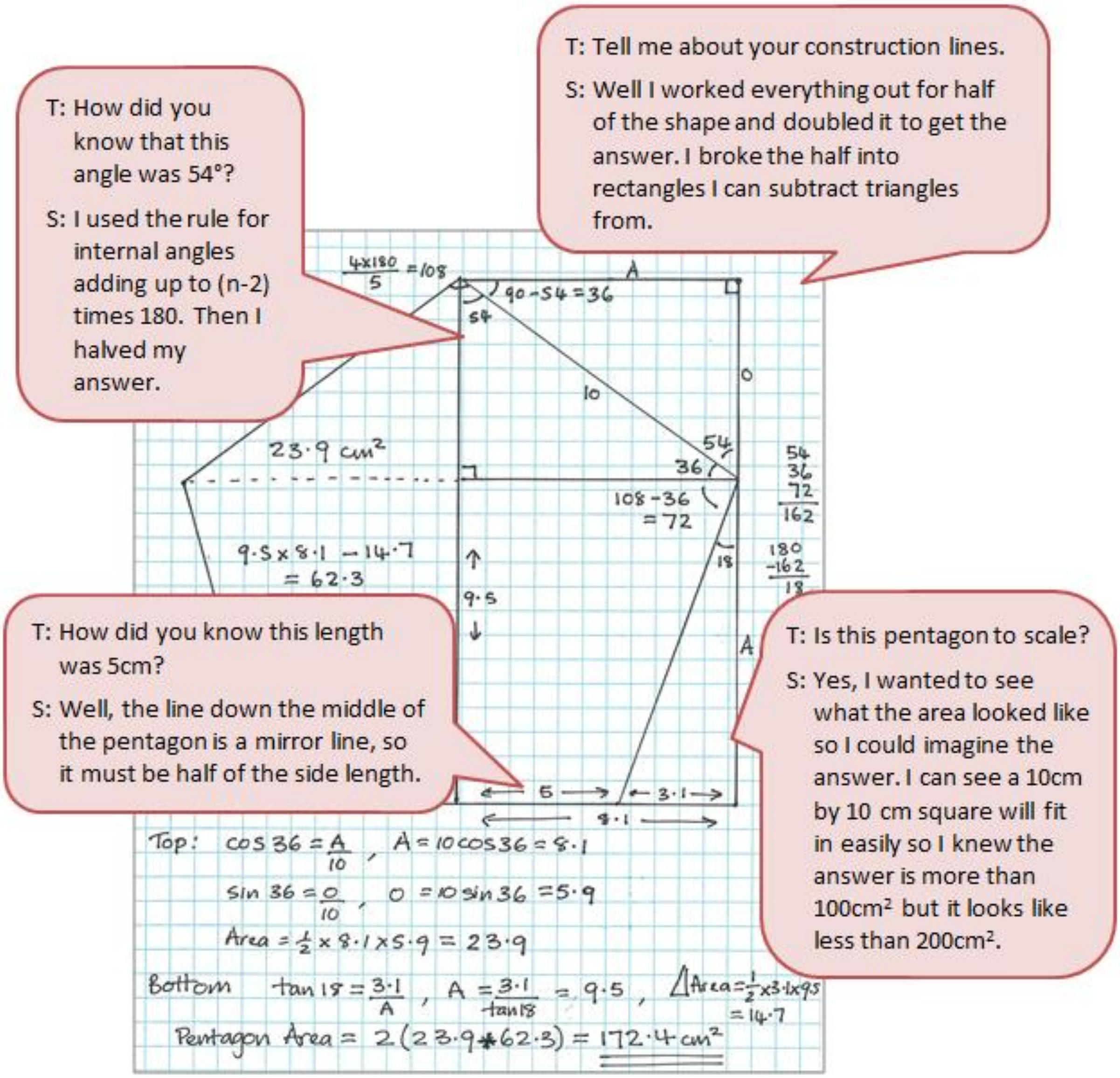


The arithmetic approach

The student is able to apply geometric properties of polygons and to use trigonometric ratios to find an unknown area.

Prompts from the teacher could be:

1. Draw a pentagon.
2. Consider the way it could be divided up into right angled triangles and rectangles. Notice the symmetry of the shape. Mark out the rectangles and triangles you will calculate the areas of.
3. Label all the sides and angles that you know.
4. Consider how you will use trigonometry and Pythagoras to solve for the area of the triangles.
5. Use the rules that we have learnt in geometry to find angles that you will need to use.
6. Now work through finding sides of the triangles, to get their areas, etc…

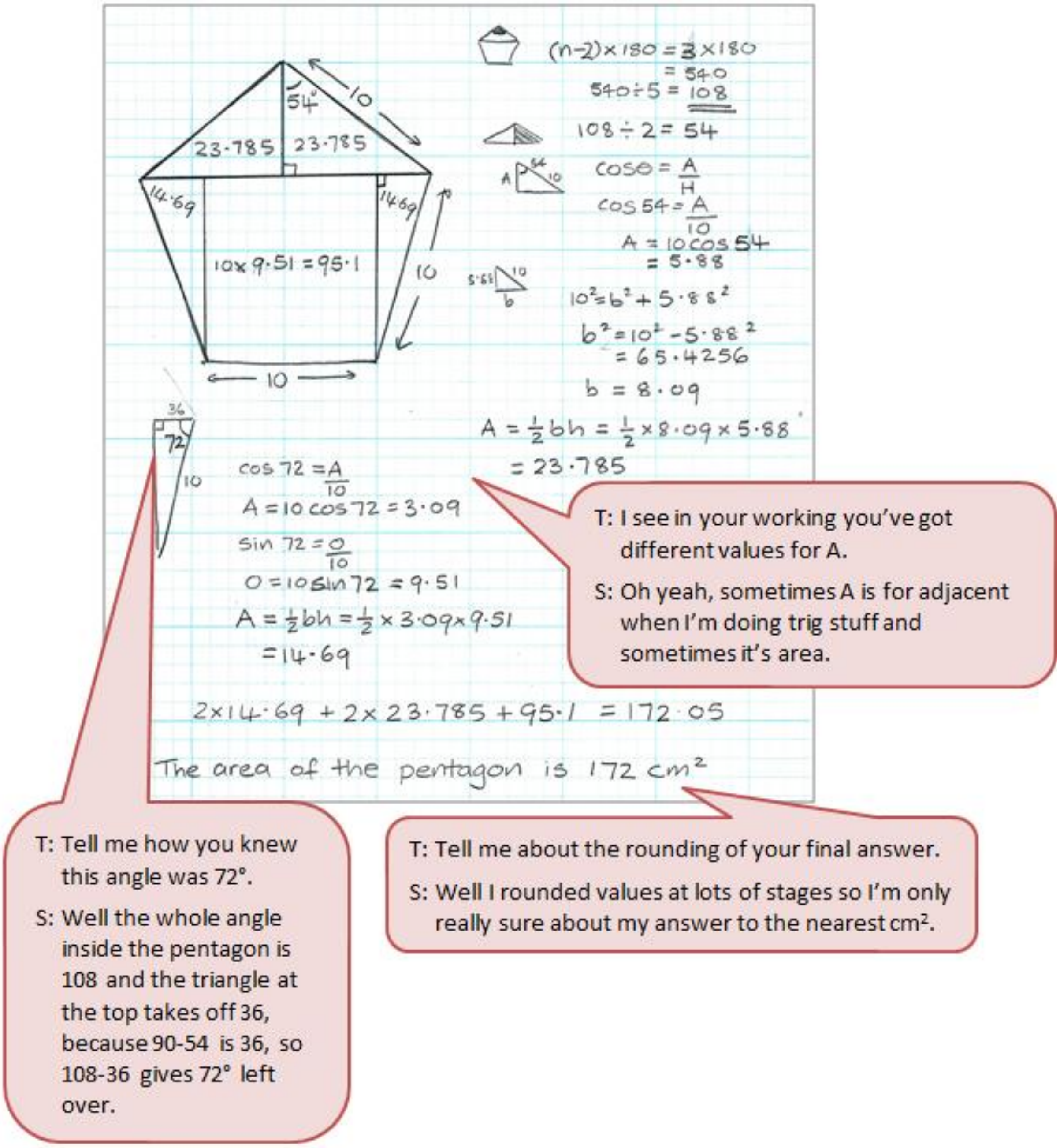


The procedural algebraic approach

The student is able to divide a regular polygon into composite triangles and rectangles and apply geometric properties of polygons and to use trigonometric ratios and Pythagoras' theorem to find an unknown area.

Prompts from the teacher could be:

1. Draw a pentagon.
2. Consider the way it could be divided up into right angled triangles and rectangles. Notice the symmetry of the shape. Mark out the rectangles and triangles you will calculate the areas of.
3. Consider how you will use trigonometry and Pythagoras to solve for the area of the triangles.



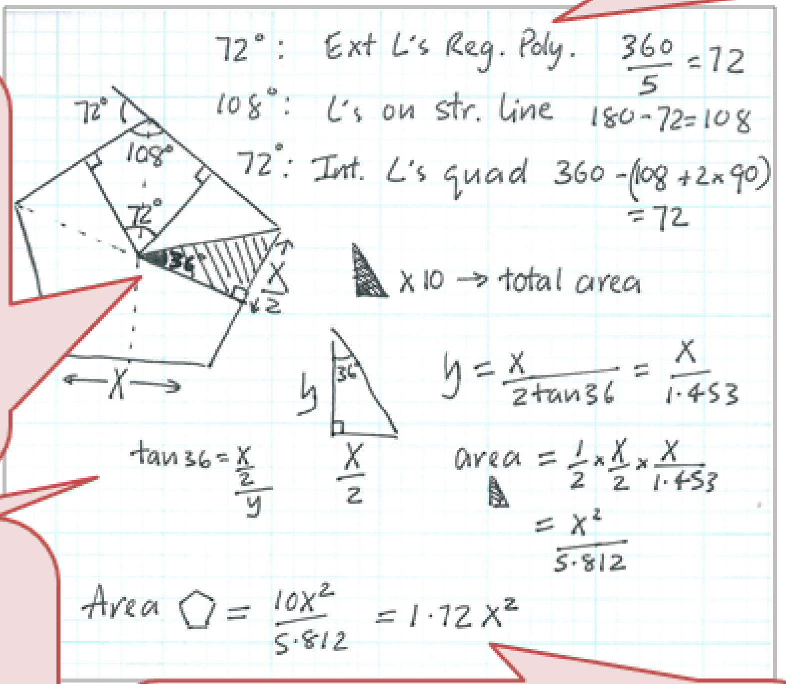
The conceptual algebraic approach

The student is able to divide a regular polygon into composite triangles and rectangles and apply geometric properties of polygons and to use trigonometric ratios and Pythagoras' theorem to find an unknown area.

To encourage the use of algebra to form a general rule, the teacher can modify the task by calling the length of each side x, rather than 10 cm.

T: Tell me about the working on the side here.

S: Well, to find the angle I needed inside one triangle I had to go back to use the rules we learned in geometry.



T: What did you do to get 36°?

S: I used the symmetry of regular polygons. The shaded triangle is half of the diamond I worked the angles out on. 36° is half of 72°.

T: you've used trig here.

S: Yeah – for area of triangles I need base and height. I had the base and because it's a right angle triangle and I've got another angle I used tan to get the height.

T: I see you've given the total area in terms of x.

S: Yeah, I've made a general rule – whatever the side of the pentagon is I just put that in where x is.

T: So if x is 10 what would the answer be?

S: 10 squared is 100, so it would be 172cm².