AC
EA
AA
AM
AP

Generalising from number

Alphabet soup

We are learning to use the language of mathematics, and to record in symbols instead of words

Exercise 1

What to do

Write the following phrases using symbols instead of words

1) two more than a number

(2) three less than a number

3) multiply some number by two

(4) treble a number

5) divide the number by two

(6) subtract five from the number

7) quadruple the number

(8) divide the number by ten

9) square the number

(10) find the square root of the number

11) halve the number

(12) add seven to the number

13) reduce the number by six

(14) increase the number by five

15) double the number

(16) a third of a number

17) find a quarter of the number

(18) divide the number by three

19) find ten times the number

(20) divide some number by four

21) find the reciprocal of the number

22) Write five phrases of your own, and get a partner to try writing them with symbols

23) Write list of the different words you can use to indicate that you should be:
(a) Adding
(b) Subtracting
(c) Multiplying
(d) Dividing

At the end of the next exercises, make sure you update your lists, adding any new words you meet. Be prepared to talk about your list of words at the next session.

Exercise 2

What to do

Write the following phrases using symbols instead of words

1) add one number to another number
2) find the difference between two numbers
3) multiply the two numbers
4) divide one number by the other
5) square the first number then add the second
6) double the first number then add one
7) add one to the first number then add the second number
8) find the square root of the number, then subtract two
9) find the sum of two numbers
10) add the two numbers then subtract five
11) add half of one number to double the other
12) halve the difference between the two numbers
13) treble the number then add the number you started with
14) add the two numbers then double them
15) Write five problems of your own, and try them out on someone else in the class

Convention: using brackets in mathematics

1) Brackets are used in mathematics to keep things together

For example $(n - 1) \times 4$ says we subtract one from our number first, then we multiply this answer by four

2) We also use brackets to tell us that this bit of the problem should be done before other bits

Exercise 3

What to do

Find the answers to these number problems by working out the bit in brackets first. Show all working.

1) $4 + (7 + 9)$ (2) $(3 \times 2) + 6$ (3) $4 + (2 \times 4)$ (4) $5 + (6 - 4)$
5) $2 \times (3 + 6)$ (6) $(2 + 4) \times (6 - 1)$ (7) $(3 + 5) \div (4 - 2)$ (8) $87 - (9 \times 9)$
9) Try these problems. Show your working.

(a) $16 - (2 \times 4)$ (b) $16 + (2 \times 4)$

Compare the way you have written this problem down to the way others in your group have written them. Did anyone write it differently?

Discuss how mathematicians usually record this problem with your teacher at the next session.

Exercise 4

What to do

Write out in words what these symbol phrases say to do

1)	$a + b$	(2)	$n + 2$	(3)	$n \times 3$
4)	$a - 3$	(5)	$a \times b$	(6)	$(n + 1) \times 2$
7)	$\frac{a}{b}$	(8)	$\frac{n}{2}$	(9)	$(a + 2) + (b + 3)$
10)	n^2	(11)	$\sqrt{x}$	(12)	$\sqrt{n} + 1$
13)	$a \times ½$	(14)	$¼ \times x$	(15)	$n \div 6$
16)	$x \times y$	(17)	$n^2 \times 2$	(18)	$3 \times n$
19)	$\frac{1}{a}$	(20)	$\frac{n+1}{4}$	(21)	$n^2 + 1$
22)	$(a + 3) \times 4$	(23)	$a \times 5 \div 3$	(24)	$x + y - 1$

Pair challenge

Work on this challenge with one or two other people, and discuss with them how you can address the problem

1) When we use multiplications, we know that changing the order of the numbers does not change the answer, so $2 \times 3 = 6$ and $3 \times 2 = 6$.

   When we record multiplications shown with letters we need to make sure it is clear that we realise that this rule still works. Your challenge is to invent a way to record multiplications like $a \times 3 \times b$ and $b \times a \times 3$ so they look like they end up with the same 'answer'. Your method should also work for phrases like $b \times a$ and $2 \times a \times 4$.

2) When we use divisions, we know that halving a number and dividing by two gets the same answer, so half of eight is four, and eight divided by two is four. When recording with letters we also want to show that we understand these two problems get the same answer. The second part of your challenge is to invent a way to write fractions and divisions so they look like they give the same answer

Be prepared to explain what you have invented, and the problems you had to tackle in trying to answer this challenge at the next teaching session.

Exercise 5

What to do

Write the following phrases using symbols instead of words. Remember to use the conventions for recording multiplications and divisions

1) double a number
(2) halve a number
3) multiply a number by six
(4) double a number and add one
5) add two numbers, then double them
6) add two numbers, then halve them
7) add one to a number, then double it
8) add two numbers, then subtract four
9) double a number, then multiply this by another number
10) multiply the product of two numbers by six
11) multiply the sum of two numbers by three
12) divide a number by ten
13) find one tenth of a number
14) find the reciprocal of each of two numbers, then add the two together
15) add three to a number, then multiply the answer by four
16) multiply a number by eight, then add five

Exercise 6

What to do

Write out in words what these symbol phrases say to do

1) $2x + 5$
(2) $3n - 2$
(3) $a + b + 4$

4) $2(a + 4)$
(5) $2(a + b)$
(6) $3(1 - x)$

7) $\frac{a-b}{2}$
(8) $\frac{b+1}{4}$
(9) $\frac{2a}{3}$

10) $2ab$
(11) $3xy$
(12) $3x + 5$

13) $\frac{5x}{4}$
(14) $\frac{1}{x} + 2$
(15) $\frac{4x}{y}$

16) $3x^2$
(17) $\sqrt{x+2}$
(18) $2\sqrt{n}$

19) Make up five of your own, and explain what each piece of algebra is saying has to be done to the number or numbers

Exercise 7

The number line below shows that a number has been placed on the number line. Copy the number line into your book.

On the number line in your book, show where you would find the following numbers

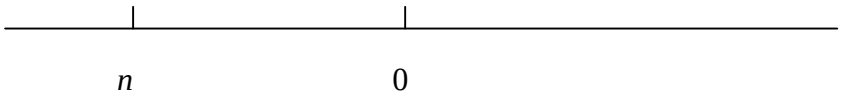
1) $2n$ (2) $3n$ (3) $\frac{n}{2}$

4) Explain why you put these numbers where you did

Copy the number line again. This time show the following numbers on it

5) $n + 1$ (6) $n - 1$ (7) $n + 5$

8) Explain why you put these numbers where you did



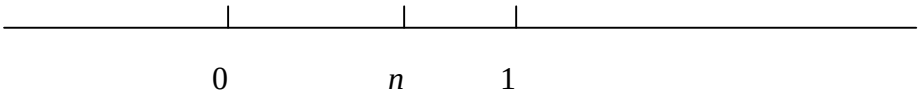
The number line above also shows that a number has been placed on the number line. Copy the number line into your book.

9) What can you say about the number?

On the number line in your book, show where you would find the following numbers

10) $2n$ (11) $n + 1$ (12) $\frac{n}{2}$

13) $n - 2$ (14) Explain why you put these numbers where you did



The number line above shows another number on the number line. Copy the number line into your book.

15) What can you say about the number?

On the number line in your book, show where you would find the following numbers

16) $n + 1$ (17) $2n$ (18) $-n$

19) $n - 1$ (20) $\frac{n}{2}$ (21) $\frac{2n}{3}$

22) Explain why you put these numbers where you did

Alphabet Soup
Answers

Exercise 1

1)	$n + 2$	(2)	$n - 3$	(3)	$2n$
4)	$3n$	(5)	$n \div 2$	(6)	$n - 5$
7)	$4n$	(8)	$n \div 10$	(9)	n^2
10)	$\sqrt{n}$	(11)	$n \times ½$	(12)	$n + 7$
13)	$n - 6$	(14)	$n + 5$	(15)	$2n$
16)	$n \times \frac{1}{3}$	(17)	$n \times ¼$	(18)	$n \div 3$
19)	$n \times 10$	(20)	$n \div 4$	(21)	$\frac{1}{n}$

You may be asked. In this case the way you have written the problems should not change – only the letter you have used.

Exercise 2

In these answers, the letters *a* and *b* have been used for the two different numbers. You may have used two other letters – like *x* and *y* or *m* and *n*

1)	$a + b$	(2)	$a - b$	(3)	$a \times b$
4)	$a \div$ b	(5)	$a^2 +$ b	(6)	$a \times 2 + 1$
7)	$a + 1 +$ b	(8)	$\sqrt{a} - 2$	(9)	$a + b$
10)	$a + b - 5$	(11)	$a \times ½ + b \times 2$	(12)	$(a - b) \div 2$
13)	$a \times 3 + a$	(14)	$(a + b) \times 2$		

Exercise 3

1) $4 + (7 + 9) = 4 + 16$
$= 20$

(2) $(3 \times 2) + 6 = 6 + 6$
$= 12$

3) $4 + (2 \times 4) = 4 + 8$
$= 12$

(4) $5 + (6 - 4) = 5 + 2$
$= 7$

5) $2 \times (3 + 6) = 2 \times 9$
$= 18$

(6) $(2 + 4) \times (6 - 1) = 6 \times 5$
$= 30$

7) $(3 + 5) \div (4 - 2) = 8 \div 2$
$= 4$

(8) $87 - (9 \times 9) = 87 - 81$
$= 6$

9) Discuss what you have found out and how you think these problems should be recorded with your teacher

Exercise 4

For this exercise, it is possible that you have used different words that mean the same thing. If you are not sure if your words mean the same, check your answers wit your teacher next session

1)	add two numbers	(2)	add two to a number
3)	multiply a number by three	(4)	subtract three from a number
5)	multiply two different numbers	(6)	add one to a number, then double
7)	divide one number by another	(8)	divide a number by two
9)	add two to one number, and three to another, then find the sum of both		
10)	square the number	(11)	take the square root of the number

12) take the square root of the number, then add one
13) halve the number
(14) quarter the number
15) divide the number by six
(16) multiply two different numbers
17) square the number then multiply that by two
18) find three times the number
(19) find the reciprocal of the number
20) add one to a number, then divide this by four
21) square the number then add one
22) add three to then number, then multiply this by four
23) multiply the number by five, then divide this by three
24) add two numbers then subtract one

Pair challenge

Your answers to this challenge are to be discussed with your group and your teacher

Exercise 5

In this exercise the letters x and y have been used to show two different numbers, you may have used other letters

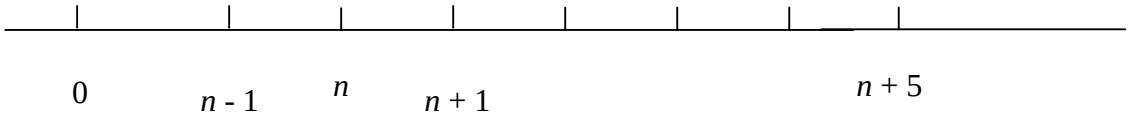
1) $2x$
(2) $\frac{x}{2}$ or $\frac{1}{2}x$
(3) $6x$
4) $2x + 1$
(5) $2(x + y)$
(6) $\frac{1}{2}(x + y)$ or $\frac{x + y}{2}$
7) $2(x + 1)$
(8) $x + y - 4$
(9) $2xy$
10) $6xy$
(11) $3(x + y)$
(12) $\frac{x}{10}$
13) $\frac{x}{10}$
(14) $\frac{!}{x} + \frac{1}{y}$
(15) $4(x + 3)$
16) $8x + 5$

Exercise 6

1) double the number then add five
2) treble the number then subtract two
3) add two different numbers, then add four
4) add four to the number then double it
5) add two different numbers, then double it
6) subtract the number from one, then multiply this by three
7) subtract the two numbers, then halve it
8) add one to the number, then divide this by four
9) double the number then divide this by three
10) multiply the two numbers then double them
11) multiply the two numbers then treble them
12) treble the number then add five
13) multiply the number by five, then divide this by four
14) find the reciprocal of the number then add two
15) multiply the number by four, then divide this by the other number
16) square the number then multiply this by three
17) add two to the number than take the square root of this
18) take the square root of the number then double this

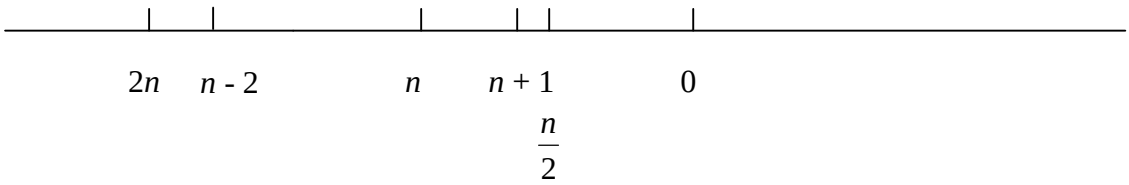
Exercise 7

4) In placing $2n$, this number is double n, so it has to go twice as far from zero as n. $3n$ has to be three times as far from zero as n. $\frac{n}{2}$ is another way of writing half n, so it is only half as far as n is away from zero



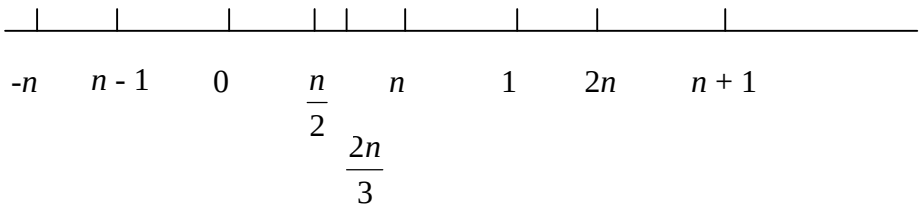
8) $n + 1$ is one bigger than n , so it goes to the right of n. (Exactly where does not really matter – its your decision.) Once $n + 1$ is located, the size of a unit has been defined, so $n - 1$ needs to go an equal distance to the left of n. $n + 5$ then needs to go five units to the right of n

9) The number is less than zero (so is negative)



14) The reasons for number placement explained above still hold for this number line

15) the number is less than one (so can be written as a fraction or a decimal. It is further than halfway, so is bigger than a half, and we can work out exactly what number it is if we want to, as we can measure the gap from zero to n and the gap from zero to one and make a fraction by dividing these measurement. (Try it)



22) As the size of the unit is already known, where numbers like $n + 1$ are placed can be worked out exactly, by measuring