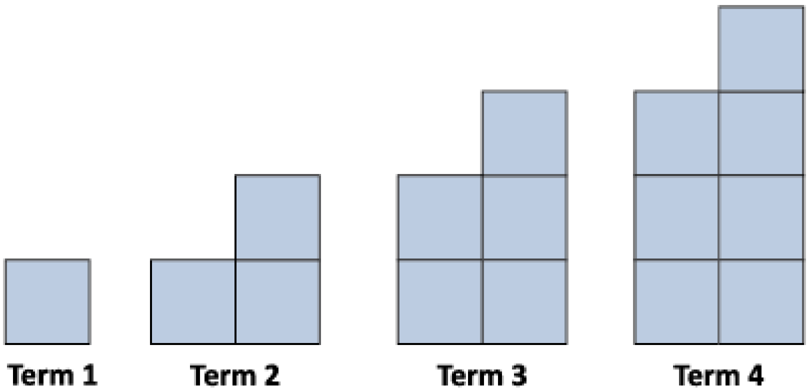
Linear Patterns

A pattern has regularity or consistency, that is there a predictable way in which the elements behave. In this unit the patterns of elements grow in a predictable way. The same number of tiles is added to the previous term to get the next, for every pair of consecutive terms. This type of pattern is called a linear pattern.
A progression in the way students process linear patterns is well established in the research. That progression is as follows:

1. Copy a spatial pattern with materials.
2. Identify change and extend the spatial pattern with materials.
3. Represent the relationship between term number and number of items and use recursive rules to predict further members.
4. Use function (direct) rules to connect term numbers with number of items.
5. Reverse (direct) rules to find the term number for a given number of items.

Here is a simple example of a linear pattern. As the pattern grows the number of tiles increases by two.



Phase 1: Copy a spatial pattern with materials.

At this phase students are able to copy a pattern that is shown to them but are unable to continue it. In this example, students could replicate the four terms shown but could not produce term 5.

Phase 2: Identify change and extend the spatial pattern with materials.

At this phase students are able to copy a pattern that is shown to them and can see the change that is happening between terms. They can use this knowledge to build the next term in the pattern. In this example, students could replicate the first four terms and then use tiles to create term 5.

Phase 3: Represent the relationship between term number and number of items and use recursive rules to predict further members.

At this phase students will be able to see the change between consecutive terms and predict how many items will be required for the next term without needing to use materials to make it. In this example, students could count the number of tiles required for teach of the first four terms, identify the change between them and use this to predict the number required for the fifth term, then the sixth term, etc… At this point students can create tables linking the term number with the number of items used.

Term	Number of square tiles
1	1
2	3
3	5
4	7
5	9

A recursive rule for the pattern is "add two tiles to the previous term to get the next term". The numbers in the right-hand column are all odd numbers.

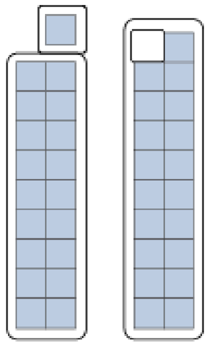
Phase 4: Use function (direct) rules to connect term numbers with number of items.

At this phase students are able to find a rule that gives the number of items required for any term number, without needing to know the number used for the previous term. Tables of values for several terms can be a useful tool to identify the change between terms, to help identify a general rule.

Term	Number of square tiles
1	1
2	3
3	5
4	7
5	9

2 x team number (n) – 1

The development of direct rules is supported by connecting spatial (figurative) reasoning and numeric reasoning. Looking for structure in a non-sequential term can help students to see the way the pattern is organised. Here is the tenth member of the odd numbers pattern. Students might notice that the figure is made up of a rectangle plus an extra square, or a rectangle minus a square.



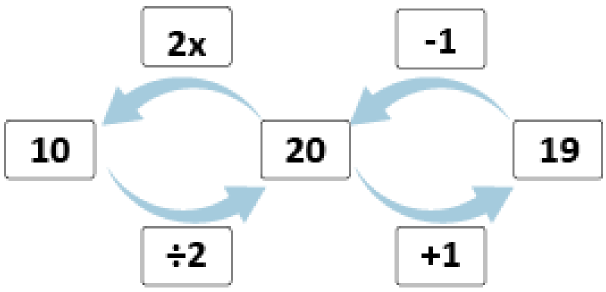
Connecting the rectangles to multiplication in arrays might lead to equations like 9 x 2 + 1 = 19, and 10 x 2 – 1 = 19 for the number of tiles.

Looking for that same spatial and numeric structure in other non-sequenced examples can support students to find and express rules such as “Multiply one less than the pattern number (term) by two, then add one,” or “Multiply the pattern number (term) by two then subtract one.”

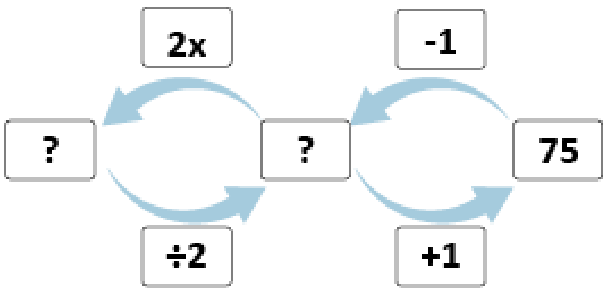
Phase 5: Reverse (direct) rules to find the term number for a given number of items.

At this phase students will be able to use their rule from the previous phase ‘backwards’ to find the term number that would require a given number of items.

Progression to the fifth phase involves the concept of inverse operations (operations that undo one another, e.g. division by five undoes multiplying by five.) Flowcharting is sometimes useful in supporting students to control the application of inverses. Taking the specific example of term ten, gives a flowchart like this:



A problem like, “If you had 75 square tiles what term in the pattern could you build?” can be solved using a flowchart.



The chain of operations, 75 + 1 = 76 and 76 ÷ 2 = 38, gives the missing term number, 38.