Newsletter 91: December 2010

Introduction

This is the final nzmaths newsletter for 2010. It is also the 91st we have produced for the website. You can have a look at some of the old newsletters on this page:
http://nzmaths.co.nz/newsletter

As you are no doubt aware, 91 is a very interesting and important number. A quick search on Wikipedia (http://en.wikipedia.org/wiki/91_%28number%29) will very quickly tell you that 91 is:

- The atomic number of protactinium, an actinide.
- The code for international direct dial phone calls to India
- In cents of a U.S. dollar, the amount of money one has if one has one each of the coins of denominations less than a dollar (penny, nickel, dime, quarter and half dollar)
- The ISBN Group Identifier for books published in Sweden.

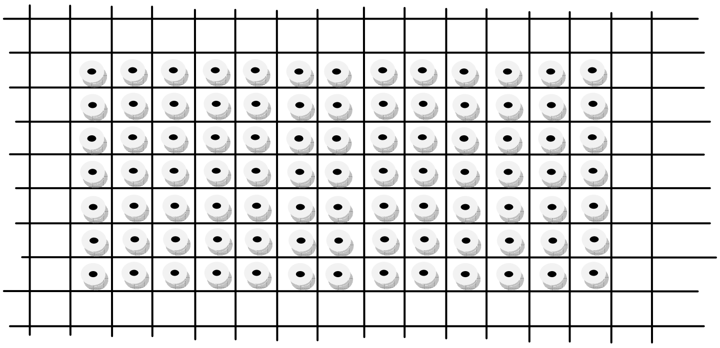
In more mathematically related trivia, 91 is:

- the twenty-seventh distinct semiprime.
- a triangular number and a hexagonal number, one of the few such numbers to also be a centered hexagonal number, and it is also a centered nonagonal number and a centered cube number. It is a square pyramidal number, being the sum of the squares of the first six integers.
- the smallest positive integer expressible as a sum of two cubes in two different ways if negative roots are allowed (alternatively the sum of two cubes and the difference of two cubes): $91 = 6^3+(-5)^3 = 43+33$.
- the smallest positive integer expressible as a sum of six distinct squares: $91 = 1^2+2^2+3^2+4^2+5^2+6^2$. The only other ways to write 91 as a sum of distinct squares are: $91 = 1^2+4^2+5^2+7^2$ and $91 = 1^2+3^2+9^2$.
- a repdigit in base 9 (111).

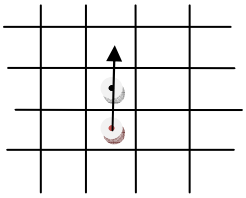
Best wishes from the team at nzmaths for a safe and happy holiday season. Our first newsletter for 2011 will be out in February.

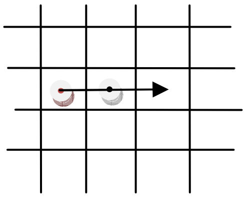
91 stones and Sole Survivor

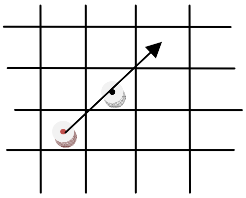
It should come as no surprise that 91 = 7 × 13. Here it is below, sitting on a grid of squares.



Having given you that, I can talk about the solitaire game Sole Survivor. The aim of the game is to remove all but one of the stones according to the following rules:





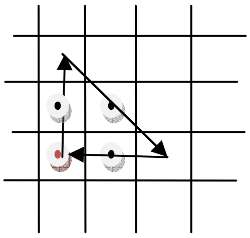


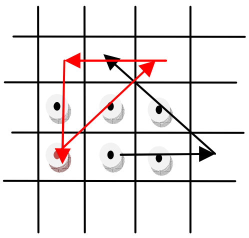
In the first diagram, the red stone can jump over the black stone provided there is an empty square where the red stone can land. In that case the black stone is removed and the red stone continues from the new square.

In the second diagram, the same things happens only this time the jump is a diagonal one. In the third diagram you now can guess what happens.

So we have three ways to get rid of a stone. But I'll allow three more removing steps. The obvious ones are the three moves that are opposite to the ones above, namely: vertically down, horizontally to the right, and diagonally from top right to bottom left.

So the first question is can you remove all of the stones except one, using the six removal methods you now know? Now, just about here, unless you have played a lot with this kind of game, you'll be totally overcome. Just to get you started, I've shown how you might do some smaller blocks of stones.





The red stone in the 2 × 2 block can remove all of the others by following the arrows round. In the second diagram there I've done the 2×3 case. If you first move along the black lines and then along the red ones, you should see that the red stone is a sole survivor here too.

At this point, I'd recommend you try some more small blocks. Before long you might be able to show that blocks in a 2×4 shape, a 2×5 shape and even a 2×63 shape, can be reduced to one stone. From there you might move on to find you can eventually do 3×25, in fact 3 by anything. Can you see how I'm building things up? What about 4 by anything and 5 by anything and 6 by anything and 7×13?

There may be at least two things to be learnt from this. First, in a number of situations, such as this game, it's often a good strategy to start small and to work up to where you are hoping to go. A lot of mathematics works this way. If you can do some small cases you start to get a feel for how the problem works in general and, with luck, you can eventually get the larger problem out.

Another thing that comes out of this is that using small cases you can build up to bigger cases using the method of the smaller cases. For instance, here, if you can do the 3×25 case, you might have done it by reducing the 3×25 block to a 3×24 block to 3×23 block, and so on. There is a set of removals that crops of one line of three stones on one end of the block. It turns out that it you master this, with a little more work you'll be able to reduce the 7×13 block to a 7×10 block. Before you know it all you'll be down to a 7×4 block or even a 4×4 block. Maybe you will know at this point how to remove all but one of the stones in a 4×4 block. If you don't, then you should think about the very smallest case I did above.

A natural question for a mathematician at this point is, is it is possible to reduce a $m \times n$ block, where m and n are ***any*** whole numbers? Actually they will have seen something that is even more restrictive. In all the cases above, the final stone can be made to end up on the square where it started. Can this be done for any $m \times n$ block that can be reduced to one stone? Oh, and how many of these blocks have the property that any stone we like can be the last man standing and finally stands on the square it began on?

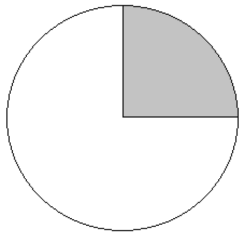
If you want to play with this game you might look in the Bright Sparks section of this web site (http://nzmaths.co.nz/bright-sparks). It is there under the name Sole Survivor. The Bright Sparks section is for able students and there are a few problems that have been animated there to make them a little more interesting. Maybe I'll talk about some more of these other problems later. However, if you have a student or two who show more than reasonable ability in maths, it might be a good idea to encourage them to look at these problems.

Comparing Fractions

In this last newsletter for the year I want to go back over comparing fractions again. This will largely be a summary of material that I have already written but I thought that it might be useful to put all this together in one place. So I plan to cover the following ways of comparing fractions:

- Diagrams
- Unit fractions
- Numerator/denominator
- Benchmarks
- Common denominator
- Percentages/decimals

As I go I'll look at the strengths and weaknesses of each and make some overall comments on strategies in comparing fractions.



Fractions are probably introduced gradually with an emphasis on the practical and with use of concrete materials. So it is often a simple step from fractions to the diagrammatic representation of fractions. Enter pie diagrams, which lead to or come from sharing pieces of pie or pizzas. In the diagram a quarter of a pizza is shown shaded.

This way to model a fraction is a good start and useful to get a feel for the part and the whole and see relative sizes of small fractions. But as the fractions become more complicated the circularity becomes a bit of a barrier. For instance, it is hard to get a feeling for the difference between 1/15 and 1/16, especially if they are displayed on two adjacent circles. It is harder to compare fractions such as 3/11 and 5/16, even if they are placed on the same circle.

The non-linearity of the circle is the thing that sometimes makes things difficult. So it is worth then thinking about showing fractions by shading sections of rectangles. Although still not brilliant for comparing some fractions, rectangular diagrams can be reasoned out more easily. Students might be encouraged to draw a rectangle and then halve it and halve the subsequent halves until they have 16ths. So they should be able to get 5/16 reasonably accurately. Elevenths are a problem but 11 of them fit equally into the rectangle so a rough guess might be made in an effort to compare 3/11 and 5/16.

But let's step back a step. Unit fractions, ones with a numerator of 1 can be decided by sight, but this might require a diagram to lay the groundwork. To show 1/15 on a diagram you divide the whole into 15 equal pieces. To show 1/16 on a diagram you divide the whole into 16 equal pieces. Because you are getting more pieces for the 1/16 division of the same cake or pizza or whatever, than you will get for the 1/15 division, 1/15 is bigger than 1/16. This actually is anti-intuitional in the sense that the students have know from way back that 16 is bigger than 15. It takes a while for us to get used to 1/15 being bigger than 1/16. But the logic of dividing something between 15 or 16 people is undeniable. So gradually students will get to know how to order unit fractions based on the idea that a bigger denominator means a smaller fraction.

Of course, life isn't so easy when you get away from unit fractions. Some things *are* easy though. If the denominators are the same then we get back to the good old organising provided by whole numbers. So 5/7 is bigger than 3/7. If the numerators are the same, again things are straightforward. We just have to think of what happens to the unit fractions. So given that 1/15 is bigger than 1/16, then 7/15 is bigger than 7/16.

But how do we compare fractions that are not quite so obliging? Sometimes benchmarks are useful. Simple fractions such as 1/3, 1/2, 3/4 and 1 are most usefully employed here. The usual benchmark approach is to look at something like 7/15 and 9/16 and notice that 7 is less than 7.5 and since 7/15 is less than 7.5/15, 7/15 is less than a half. On the other hand, 8/16 is a half and so 9/16 is bigger than 1/2. This means that 9/16 is bigger than 7/15.

There's a sense in which this and unit fractions together can sometimes work backwards. For instance, look at 6/7 and 19/20. Notice that it takes 1/7 to get 6/7 up to 1 and 1/20 to get 19/20 to 1. Now 1/20 is smaller than 1/7 and so 19/20 is closer to 1 than 6/7. Consequently 19/20 is bigger than 6/7.

Naturally, though, you have to be flexible in your benchmark. For instance, which is bigger, 7/15 or 6/16? Well we know that 6/15 is bigger than 6/16, so 7/15 is bigger than 6/16. Here we've kind of used 6/16 as our benchmark.

There will be occasions when none of the above strategies will work too well and so we need a general method or two that will work with any two arbitrary numbers. The common denominator is perhaps the first arithmetic approach to this and other problems in fractions. This takes time and effort to teach and learn but pays off immediately when we want to compare or add or subtract fractions, as well as in the future when algebra requires combinations like $1/(x – 1) + 2/(x + 1)$. Provided the common denominator algorithm has been mastered, it eventually provides a way to compare any two fractions, even two as ugly as 4573/8771 and 460/878.

But a step along the way can speed up the process. Consider 4573/8771 and 460/878. (You might like to guess which is the biggest.) What we are faced with is computing (4573 × 878)/(8771 × 878) and (460 × 8771)/(878 × 8771). What a depressing thought that is. Mind you we don't need to do all those calculations. Clearly whatever (8771 × 878) is, it's the same as (878 × 8771). So all we are really interested in is (4573 × 878) and (460 × 8771). Since (4573 × 878) = 4015094 and (460 × 8771) = 4034660, and 4034660 > 4015094, then 460/878 is bigger than 4573/8771. When the fractions are smaller you can probably do this operation, which some people call cross multiplication, in your head.

Comparing fractions can be hard but comparing decimals is considerably easier as we don't have two competing numbers in the notation. So we could note that 4573/8771 = 0.521377 = 52.1377% and 460/878 = 0.523918 = 52.3918% and so find that 460/878 is the bigger fraction. If your student can divide quickly, this is a viable way for them to compare most of the fractions that you are likely to ask them. But if you don't have a calculator you don't need to go all the way in this process, you can stop at the first decimal place where there is a difference. And someone who is good at arithmetic can eye-ball simpler situations than the one above. 83/101 is clearly about 0.83 while 63/80 starts off as 0.7... So 83/101 is bigger than 63/80.

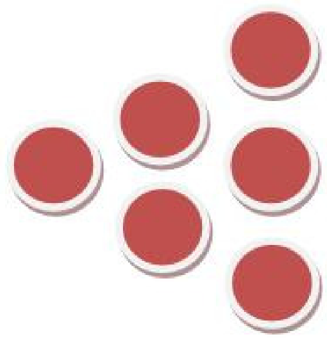
One of the things that separates students with mathematical ability from the rest in comparing fractions, is not so much that they can do all of the operations above on fractions but rather that they are flexible in their choice of strategy. Some fractions are easier to compare, say, by a common denominator method than they are by using a benchmark. For example, 3/4 and 7/8 are easily changed to eighths and since 6/8 < 7/8, 3/4 < 7/8. The more able students will pick up on this and make their life easier. Less able students may well have a method for comparing fractions but they are likely to use it every time they need to compare two fractions regardless of the fractions that are being considered.

Problem Solving

These problems come from the high achieving students section of the website:
http://www.nzmaths.co.nz/activities-students

Years 1-3

In the diagram below, there are 6 counters. Alyssa thinks they look like a fish swimming to the left.



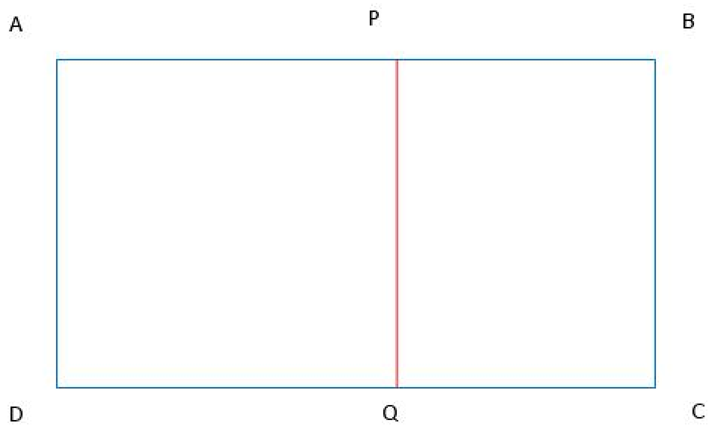
She wonders if she can make it swim to the right. Yes, she can. But what is the smallest number of counters that she needs to move to make the fish swim to the right?

Years 4 - 6

I'm thinking of a 4-digit number. If I add its four digits together I get 34. How many different numbers are there that I could possibly be thinking of?

Years 7- 9

Jenny drew the rectangle ABCD below that has one side of length 10 and the other $10\sqrt{3}$.

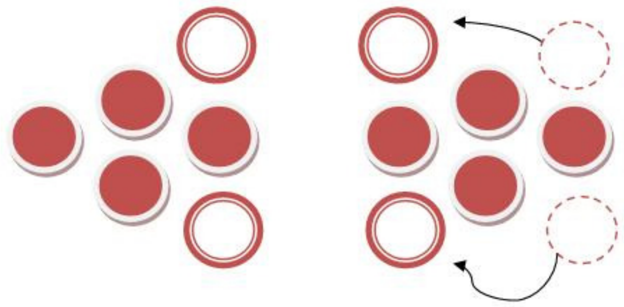


She then drew in the line PQ to make a rectangle with C and B.
If the ratios AB : BC and BC : QC are equal, what is the length of BQ?

Answers to Problem Solving

Years 1 - 3

Alyssa manages to make the fish swim in the opposite direction by moving just two counters.



She took the two counters and moved them as in the diagram. Now she has the fish swimming to the right!

Years 4 - 6

(i) It's first important to establish what the four digits could be. What four digits when added together give a total of 34?

Well, suppose that one of the digits is 6. Then the other three add to $34 - 6 = 28$. But $3 \times 9 = 27$, so no three digits exist that can add to 28. So none of the original digits is 6. In fact none of them is smaller than 6 for the same reason.

Now suppose that one of the digits is 7. Because the other digits add up to 27, they must all be 9s. So here I had to have 7, 9, 9, 9.

Now suppose that one of the digits is 8. Because the other digits add up to 26, it can easily be seen that one is 8 and the others are 9s. So here I had to have 8, 8, 9, 9.

Clearly they are not all 9 ($4 \times 9 = 36 > 35$).

(ii) So what numbers can be made up with one 7 and three 9s?

There are just **four** of them. They are 9997, 9979, 9799, 7999.

(iii) So what numbers can be made up with two 8s and two 9s?

There are just **six** of them. They are 9988, 9898, 9889, 8998, 8989, 8899.

(iv) The answer to the question is therefore 10.

Years 7 – 9

We know that $AB : BC = BC : QC$

$BC = AB/\sqrt{3}$, so $QC = BC/\sqrt{3} = 10/\sqrt{3}$

By Pythagoras' Theorem, $BQ2 = BC2 + QC2$

$BQ2 = 102 + (10/\sqrt{3})2$

$BQ2 = 102 + 102/3$

$BQ2 = 102 \times 4/3$

$BQ2 = 400/3$

So $BQ = 20/\sqrt{3}$.

What's new on nzmaths?

NZC and Standards section

This section has been updated and now includes supporting material for all achievement objectives up to level 8: http://nzmaths.co.nz/nzc-and-standards

There is also a glossary of mathematics and statistics terms: http://www.nzmaths.co.nz/glossary

A collection of Illustrations of the National Standards for Mathematics are being developed. This section currently contains illustrations of six mathematics tasks. Each task includes annotated student work samples illustrating aspects of several standards. These can be found in the NZC and Standards section of the website, or at this link: http://nzmaths.co.nz/national-standards-illustrations

2009 Numeracy Research Compendium

The 2009 Numeracy research compendium is now available online. It includes 16 papers relating to a variety of aspects of the Numeracy Development Projects: http://nzmaths.co.nz/node/7516

Learning Objects

We have 5 new learning objects on the website. Two of them are to support students in filling the gaps in their basic facts. Because the Learning Objects remember your known facts you need to choose a username and password the first time you use them. Two use place value equipment to help illustrate addition and subtraction using place value, and the final new learning object allows the user to use place value equipment to explore numbers in bases other than 10. They can all be found in the Learning Objects section of the website: http://nzmaths.co.nz/digital-learning-objects

Addition and Subtraction Facts: http://www2.nzmaths.co.nz/LearningObjects/BF/AdditionBF.htm
Multiplication and Division Facts: http://www2.nzmaths.co.nz/LearningObjects/BF/MultiplicationBF.htm
Place Value Addition: http://www2.nzmaths.co.nz/LearningObjects/PV/BasesMulti_add.html
Place Value Subtraction: http://www2.nzmaths.co.nz/LearningObjects/PV/BasesMulti_sub.html
Bases: http://www2.nzmaths.co.nz/LearningObjects/PV/BasesMulti_multi.html

We have plenty of other new material on the way so do remember to keep checking the website for new updates!

Classroom section

Position and Orientation in the Playground

Here are some ideas to add to a unit based on Position and Orientation achievement objectives. Units of work for this strand are available under the "Units of Work" heading on the homepage of nzmaths.co.nz. These activities are based in the school playground and at Level One and Two involve the use of video cameras and digital cameras.

Level	AO	SLO	Activity
Level One	1	Follow and give instructions using left & right directions & turns.	Students work in pairs to follow and give instructions for movement around the playground. Use left and right directions and quarter and half turns.
		Describe movement using directions and positional language.	Video a student moving through the adventure playground. Play it back and ask students to describe the movement, for example, running forward and swinging under the bar.
	2	Use positional language.	Photograph students in the playground. Ask students to label the photos using positions, for example, James is in the tunnel.
Level Two	1	Draw a simple map including a key.	Ask students to complete a simple map of the school playground area including a key to show features such as rubbish bins and outdoor seating
	2	Match a view with a map position.	Take photos of school features and ask students to match the view with the position on the map it was taken from.
		Identify the compass direction of the view using a map.	Ask students to take photos of school views and ask other students to decide what direction (N,S, E, W) the photographer was facing.
Level Three	1	Use a co-ordinate system to identify the location of objects.	Lay string on the adventure playground to form grid squares and label them (letters on one axis, numbers on the other axis). Ask students to identify objects in given grid squares and identify what grid reference an object is in.
		Describe a pathway using distances and directions.	Using rulers and trundle wheels to measure distances, and left and right directions ask students to give instructions for a pathway through an adventure playground course or a fitness lap in the school.
Level Four	1	Use a scale map, and compasses to follow instructions.	Provide students with a scale map of the school, trundle wheels, and compasses. Prepare an orienteering course for them to follow.
		Write instructions involving distances and compass directions.	Ask students to develop an orienteering course for other students.

Families

Activities about Position and Orientation

Here are some ideas for helping your child develop skills about position and orientation.

Following left and right instructions

Pretend you are a robot and ask your child to walk behind and give you instructions using left and right directions. Ask them to instruct you to move from one room in the house to another room.

Swap over and give instructions for your child to follow.

Making a Fire Escape Plan

Draw a simple scale map of your house. Older children can do this map with a little help. Ask your child to draw the quickest and safest escape pathways out of each room in the house. Ask them to give instructions for the pathway they drew.

Reading maps

Next time you are planning to drive somewhere give your child a street map or a map from google maps and ask them to help you plan your route. Help them to read the index to find your street and destination. Draw on the map the best route to take.

Maps in the Community

Look for opportunities for children to read maps, for example in shopping malls, museums, parks. There may be community orienteering courses or activities that you can take part.

Play BattleShips

Battleships is a good game for helping children use the grid references shown on some maps. It reinforces the convention that the horizontal value is given first and the vertical value second. That is, (5,6) refers to the location 5 units across and 6 units up. Each player has a grid (10 by 10 squares is good), which they mask from the other player. Each player connects some points on the grid to represent battleships. A destroyer ship is two adjacent points and a cruiser is three adjacent points. Each player marks 2 destroyer and one cruise ship on their grid. Players take turns to have shots at each other's battleships by calling out a co-ordinate. If the call matches a battle ship the player the says "hit", if not, they call "miss". A battleship is sunk when all of the co-ordinates representing that battleship are discovered. The first player to sink all of their opponent's battleships is the winner.