

Newsletter 89: August 2010

## Introduction

Because this is the $89^{\text {th }}$ newsletter we need to say something about the number 89 . With a little bit of work you can show that it is a prime number. To do this you only have to check whether the numbers from 2 to 9 are factors or not. This is because the square root of 89 is a little more than 9 but less than 10 and if any number has a factor more than its square root it is forced to have one less than that square root. This way you can speed up checking a number to see if it is prime or not. In fact for 89 we only have to test $2,3,5$ and 7 as factors, because if $4,6,8$ or 9 were factors, then so would be 2 and 3 . But a quicker way is to Google 'prime numbers less than 1000 ' and see if 89 is listed - which it is.

Now you may think that the following fact is interesting: 89 and 11 add to 100 , and both 89 and 11 are primes. However, there are about 5 other such pairs of primes, so it's not as interesting as it might otherwise have been. But are there any pairs of primes that add to 1000 ?

But 89 is interesting because of at least two things: Route 89 in the USA and the TI 89 a Texas instruments calculator. If you happen to be driving around the USA, Route 89 is a good road to go on. If you are into nature, there are certainly some fascinating things to be seen along this north-south highway. In fact it is known as the National Parks Highway as it links seven national parks in its wanderings down the backbone of the Rockies. One section of US 89 links Flagstone, Arizona (close to the Grand Canyon) and Yellowstone National Park.

For today the TI 89 will serve to illustrate the incredible development of hand held machines over the last little while. Electronic calculators were long able to do the four operations on numbers. They progressed to being able to find values of trig functions and so on. But two quantum leaps have taken place recently: the ability to draw graphs and geometrical objects and the ability to do algebra. Students potentially now have a power in their hands that wasn't available to universities of the 1960s. You have to ask whether this should affect what and how we teach mathematics. Should a large amount of time still be spent on teaching basic skills when they could be assumed and higher order skills be taught instead?

But you might also think what an interesting year 1989 was. After all, the Berlin Wall fell; George H. W. Bush succeeded Ronald Reagan and became the $41^{\text {st }}$ President of the USA; Rain Man was the best picture at the Oscars; and the Simpsons first full length episode appeared on TV.

On the other hand, 1889 also wasn't all that boring. Coca Cola was incorporated that year and van Gogh painted Starry Night. And while you have to feel sorry for the people involved, there was an interesting truce in Samoa. For some reason a German boat fired on an American base there. America got two ships to square off with the Germans. But along came a great storm and sank the potential combatants. Without fire power peace had to be declared. Perhaps we don't see enough of that.

Going back further to 1789, we have the Mutiny on the Bounty; the first Thanksgiving Day in the States; and the storming of the Bastille.

Finally, without looking at your keyboard, can you say what you would have typed if you held down the Shift key and pressed the keys with 8 and 9 on them?

## Some thoughts on decimals

What are decimals? Well, the simple answer is that they are just a continuation of the whole numbers. Fibonacci discovered the work of the Arabs, Indians and Chinese at the start of the $13^{\text {th }}$ Century. He then came back to Europe to spread the word about a counting system that was a great improvement on what the Romans had, at least when it came to adding, subtracting, multiplying and dividing. This system was, of course, based on the number 10. So now we write numbers like 6482 , where the different places for the digits have different values. The number 6482 is really

$$
6 \times 10^{3}+4 \times 10^{2}+8 \times 10+2
$$

If we want to write bigger numbers we might have to use bigger powers of 10. But what about smaller numbers? Clearly with smaller powers of 10 we can cope with numbers in the hundreds and then numbers in the tens and then we are down to the digits, $1,2,3,4,5,6,7,8$ and 9.

It's worth noting here that 4 , for example can actually be written $4 \times 10^{\circ}$. There are two reasons for that. First the power of $10^{\circ}$ fits into the pattern of $3,2,1$, with the other powers of 10 that are decreasing from left to right:

$$
6 \times 10^{3}+4 \times 10^{2}+8 \times 10^{1}+2 \times 10^{0}
$$

And second it fits a more important pattern. Now $10^{5} / 10^{3}=10^{2}$. You can see this by writing the numerator as 100000 and the denominator as 1000. Cancelling gets 100 which is $10^{2}$. The general pattern here is $10^{\mathrm{a}} / 10^{\mathrm{b}}=$ $10^{a-b}$. You can do some more examples to help verify this. OK then, what is $10^{5} / 10^{5}$ ? Clearly it is 1 . But using the pattern we get $10^{5-5}$. So $1=10^{5-5}=10^{0}$ !

Fine, now, recall that 6482 is:

$$
6 \times 10^{3}+4 \times 10^{2}+8 \times 10^{1}+2 \times 10^{0}
$$

Then where might we want to go with powers of 10 . We've got down to zero so why not go negative? So what is

$$
6 \times 10^{3}+4 \times 10^{2}+8 \times 10^{1}+2 \times 10^{0}+5 \times 10^{-1}+3 \times 10^{-2}+5 \times 10^{-3} ?
$$

And that's not hard because $10^{-1}=1 / 10 ; 10^{-2}=1 / 10^{2}=1 / 100$ and $10^{-3}=1 / 10^{3}=1 / 1000$.
The problem though is that $6 \times 10^{3}+4 \times 10^{2}+8 \times 10^{1}+2 \times 10^{0}+5 \times 10^{-1}+3 \times 10^{-2}+5 \times 10^{-3}$ is a bit cumbersome. So we first drop the positive and zero powers of 10 to give $6482+5 \times 10^{-1}+3 \times 10^{-2}+5 \times 10^{-3}$. And the next step is to reduce the negative powers of 10 . To do this we put a '.' after the 2 , to warn where the negative powers start and then just write down the digits - just as we did for the positive powers. In the end then, $6 \times 10^{3}+4 \times 10^{2}+8 \times 10^{1}+2 \times 10^{0}+5 \times 10^{-1}+3 \times 10^{-2}+5 \times 10^{-3}$ reduces to something manageable and that something is
6482.535.

All of this is the logical outcome of using powers of 10 to write our numbers, rather than Is and Xs and Ls and Cs and things, like the Romans did.

So now we know what numbers like $0.391\left(0 \times 10^{0}+3 \times 10^{-1}+9 \times 10^{-2}+1 \times 10^{-3}\right), 0.566\left(0 \times 10^{0}+5 \times 10^{-1}+6 \times\right.$ $\left.10^{-2}+6 \times 10^{-3}\right)$, and $1.528\left(1 \times 10^{0}+5 \times 10^{-1}+2 \times 10^{-2}+8 \times 10^{-3}\right)$ are. More than that we have some more difficult ones - like these:

$$
\begin{aligned}
& 0.407\left(=0 \times 10^{0}+4 \times 10^{-1}+0 \times 10^{-2}+7 \times 10^{-3}\right) \\
& 0.4=0.400\left(=0 \times 10^{0}+4 \times 10^{-1}+0 \times 10^{-2}+0 \times 10^{-3}\right) ; \text { and } \\
& 0.041\left(=0 \times 10^{0}+0 \times 10^{-1}+4 \times 10^{-2}+1 \times 10^{-3}\right)
\end{aligned}
$$

If we really want to, we can write all of these as decimals as fractions:

$$
\begin{aligned}
& 0.407=4 / 10+0 / 100+7 / 1000=407 / 1000 \\
& 0.4=4 / 10+0 / 100+0 / 1000=400 / 1000=4 / 10 ; \text { and } \\
& 0.041=0 / 10+4 / 100+1 / 1000=41 / 1000
\end{aligned}
$$

Now the important thing to notice about decimals is that they are easy to order - much easier than the fractions. The basic rule is the same as that for whole numbers. First look at the multiples of the highest power of 10. The number with the biggest multiple of the highest power is the biggest.

So 6310 is bigger than 5999 because the 6 is bigger than the 5 .

And 631 is smaller than 5999 because 5999 has 5 times 1000, while 631 has no times 1000 .

Let's do a few decimals to show how it works. Which is bigger, 0.407 or 0.521 ? That's easy. We can see that they both have a multiple of $1 / 10$ but 0.521 has the biggest -5 . So 0.521 is bigger than 0.407 .

Which is bigger, 0.3 or 0.041 ? Here 0.3 is bigger even though it seems to have fewer active digits. Let's see why. $0.3=3 / 10$, while $0.041=0 / 10+4 / 100+1 / 1000=41 / 1000$. Notice that $3 / 10=30 / 100=300 / 1000$ and this is a bit bigger than $41 / 1000$ So 0.3 wins over 0.041 .

You'll notice that this has nothing to do with the number of digits in the decimal. Certainly 0.041 has more digits. But they are 'in the wrong place'. But that's not a surprise. After all 6000000000000000 is bigger than 99999, even though the first number has only one non-zero number. The important thing is where the 6 is here. And it's bang up against a much bigger power of 10 than any of the 9 s in 99999.

If you are worried about this then maybe it is easier to rewrite 0.3 as 0.300 . Adding the zeros adds nothing to the number. Now it should be quite clear that $0.3=0.300$ is bigger than 0.041 .

To compare decimals less than 1, then it's just a matter of comparing them with the same number of decimal places.

## Book Review: 'More Kiwi Conundrums' by Russell Dear

Russell was a secondary teacher for many years. During that time he contributed problems to The Southland Times, the Otago Daily Times and the New Zealand Science Monthly. He also contributed to this newsletter for a number of years. He has now retired and, apart from working on books like this, now spends a fair amount of his time in local Southland bands, watching birds, and travelling (largely in search of grandchildren).

Like its predecessor, 'Kiwi Conundrums', this book is a collection of things mathematical with an emphasis on mathematical problems. It consists of 16 chapters that provide more than 100 problems covering number, geometry, probability, and logic. Below there are samples of the problems and the other topics that Russell has collected together.

1. "What is so special about the numbers, 28; 180; 212; and 666?"
2. "Which is correct, A or B? A dromedary camel has A one hump; B two humps?"
3. In a chapter about various maths textbooks and their authors from the past, the following problem is stated:
"A man driving his geese to market was met by another who said, "Good morrow master with your 100 geese." Says he, "I have not 100 but if I had half as many as I have now and two geese and a half, beside the number I already have, I should have 100." How many had he?"
4. "The number of 50 c stamps needed to cover a square metre is (a) 100; (b) 500; (c) 1,500 ; (d) 15,000; (e) 150,000."
5. "While Jenny was in the shop she exchanged a one dollar coin for six other coins. On the way home she lost one of them down the back seat of the car. What is the probability that the lost coin was a 10 cent piece?"
6. While talking about several famous 'puzzlists' from the past, Russell quotes this problem from Rouse Ball:
"Use the digits 9, 8, 7, 6, 5, 4, 3, 2, 1, 0 , to express numbers whose sum is unity."
Unity means 1 . So you might need to find two equal fractions that use all of the digits and sum to 1.
7. And then there is a chapter on historical problems. Try this one from Early Greek times.
"A mule travelling with a donkey complained of being too heavily laden, and said, "If I gave you one of my sacks we would have the same number but if you were to give me one of yours I would have twice as many as you. How many sacks did each have?"
8. There is a neat solution to this one but you might get there by trial and error. It could be worthwhile first trying smaller values than 24.
"What is the smallest (natural) number with 24 divisors?"
9. "Imagine that you have a number of white cubes, all the same size. You take one of the cubes and on each face paint a black line from the middle of one edge to the middle of the opposite edge. You do this for all six faces of the cube. How many different cubes can you paint in this way so that they are distinguishable from one another?"
10. "There are a number of expressions or sayings that include the number one or associated word. See if you can fill in the missing words of the following.
(1) The ${ }^{* * * * *}$ cut is the deepest.
(2) ${ }^{* * *}$ for the money and two for the show.
(3) ${ }^{* * *}$ beats a King.
(4) ${ }^{* * *}$ flew over the cuckoo's nest."

There is something in this book for anyone who has some interest in maths, from a primary student right up to that student's grandmother. It is written with both humour and compassion - the compassion comes, when, after hours of trying to solve one of Russ' problems, you find the answer at the end of the chapter. (The reviewer has also been compassionate - see the end of this review.)
'More Kiwi Conundrums' is available from the author (russlyn@xtra.co.nz) for about \$23 postage paid in New Zealand.

## Answers

1. 28 - the number of days in the lunar cycle; 180 - the number of degrees ion a triangle; 212 - the boiling point of water in degrees Fahrenheit; 666 - the number of the Beast in the Book of Revelations.
2. A dromedary has two humps.
3. The man had 65 geese.
4. 1500 stamps are needed.
5. 7/12 (That's not so easy.)
6. $35 / 70+148 / 296$.
7. The mule has 7 sacks and the donkey 5 .
8. 360. 
1. 8. 
1. (1) first; (2) one; (3) Ace; (4) One.

## Problem Solving

These problems come from the high achieving students section of the website:
http://www.nzmaths.co.nz/activities-students

## Years 1-3

Betty has a window in her bedroom that looks like this. It has four panes of glass.

For her birthday she was given two big stickers with a ballet dancer on and two with horses. She wants to put a different sticker on each pane of glass. In how many ways can she do this?


Years 4-6

Sally is making spinners. She has divided two circles into 3 equal parts.


Now she is going to put $1 \mathrm{~s}, 2 \mathrm{~s}$ and 3 s stickers in the spaces on the circles. But she finds that she only has two 1s stickers, two 2s stickers, and two 3s stickers.

Where should she put the numbers so that when she spins the two spinners the total of 5 comes up as often as possible?

## Years 7-9

Sally is making spinners. As you can see from the diagram she has divided two circles into 4 equal parts.


Now she is going to put numbers in the spaces on the circles.
Where should she put the numbers so that when she spins the two spinners each possible total comes up equally often? Do this for 1 possible total; 2 possible totals; 4 possible totals; 8 possible totals; and 16 possible totals. Why only those numbers of totals?

## Answers to Problem Solving

## Years 1-3

There are 6 ways to arrange the stickers. Here is how you can work it out:


First take one of the ballet dancer stickers. Try putting it in the top left pane. Then there are three places the other ballet dancer sticker could go. Have a look at the picture here. The horses go in the spare panes.


Now let's try the first ballet dancer in the top right pane. We can't put the second one in the top left pane because that would be the same as one we have done above. So there are only two places that she can go.


Now try putting the first ballet dancer sticker in the bottom left position. As we can see there is now only one place the other one can go.

If we try to put the ballet dancer in the bottom right pane, the other sticker has to go in a position we have already shown.

So there are $3+2+1$ possible ways of decorating Betty's windows.

## Years 4-6

The only way that Sally can get a total of 5 is if there is a 2 on one spinner and a 3 on the other. If she has a 2 and a 3 on each of the spinners, then she'll get two possible totals of 5 . If she has two 2 s on one spinner and a 3 on the other, that will give her two ways of getting 5 too. However, if she has two 2 s on one spinner and two 3 s on the other she has found four ways to get a total of 5 .

It's not possible for her to find five ways of getting 5 though. So the spinners must look like this.


## Years 7-9

Note that because there are 4 spaces on one spinner and four spaces on the other, there are potentially $4 \times 4$ = 16 different totals. So the number of equally possible totals must be a factor of 16 .

This can be done in five kinds of ways.
First if there is only one possible total: put the same number in all four spaces on each spinner.
Second if there are just two possible totals: put the same number in all of the spaces on one of the spinners; put two different numbers on the other spinner.

Third if there are just four possible totals: put the same number in all of the spaces of one of the spinners; put four different numbers on the other spinner.

Fourth if there are just eight possible totals: put two different numbers on one of the spinners; put four carefully chosen different numbers on the other. Be careful that there are no unwanted repeated totals here. For instance, this will work:

(How careful do you have to be here?)
Fifth if there are sixteen possible totals: put four different numbers on one of the spinners; put four carefully chosen different numbers on the other. (How careful do you have to be?)

## Classroom section

## Design a game

Here is a template for a simple board game. The aim of the game is to move both of your counters in a zigzag from the start to the finish. The first player to get both counters to the finish wins.
Students input to the game

1. Students write questions for the ? squares.
2. Students write basic facts questions for the * squares.
3. Students write an answer page to the questions.

If a player lands on a ? square they turn over a ? card and answer the question. If the answer is correct they move forward 3 squares.
If a player lands on a* square they turn over the * card. The first player to answer the question correctly moves forward 3 squares.
Other variations that students might add to the game, for example, a bonus for throwing a six or landing on another counter, more * or ? squares, chance cards with bonuses and penalties (e.g. miss a turn). Students will be sure to come up with other variations of their own.


Students will need card and scissors to make copy of the game board, question cards and answers.

To play you will need: Board game, 2 counters of each player, Die, ? cards, * cards, answer page.

## What's new in maths?

nzmaths rebranding
TKI has recently launched new branding, and nzmaths is doing the same. Keep an eye on the site over the next week or so because we are going to launch a whole new look. The content will remain the same, but we think everyone is going to love the improvements to our site's design.


## Assessment Online

The Assessment Online site provides information on all areas of assessment including assessment practices, moderation and the alignment of assessment tools to the National Standards. This information is being updated on a regular basis. To receive the most recent updates sign-up for the assessment newsletter here: www.assessment.tki.org.nz

The link to sign up for the newsletter is on the right hand side of the page.


## Families

Dice games are an easy and fun way to do some maths at home with your child. Here are a few ideas that you could try.

## Dice Game - A Place Value Game

Equipment
2 dice (or more for older players)
1 counter (for decimal variation)

## How to play

Roll the dice and put them in order to make the highest number possible. If you roll a 4 and a 6, for example, your best answer would be 64. Using 3 dice, a roll of 3,5 and 2 should give you 532, and so on. Write down your answer, pass the dice, and challenge the next player to "Beat That!"
Play in rounds and assign a winner to each round.
Variations:
Try making the smallest number possible.
Include decimals by using a counter as the decimal point.

## Dice Game: Doubles addition

Equipment
4 dice
Paper and pencil to score

## How to play

The object of the game is score points by adding the doubles. The first player rolls the dice. They add any double numbers and that is their score of the round. For example, if a player rolled 4, 3, 6, 3 their score would be $3+3=6$. The next play has a turn. After 5 rounds the winner is the player with the highest score.

## Variations

Find the doubles and then add on the other dice numbers for your score, for example, 4, 3, 6, 3 is $3+3+4+6$ $=16$.

## Dice Game: Multiplication

## Equipment

2 dice
Paper and pencil to score

## How to play

The object of the game is score points by multiplying the numbers. The first player rolls the dice. They multiply the numbers to make the biggest number and this is their score of the round. For example, if a player rolled 4 and 56 their score would be $4 \times 5=20$. After 5 rounds the winner is the player with the highest score. Variations
Put stickers on one or both of the dice and change the numbers.

