Newsletter 87: March 2010

Introduction

What's so special about 87?

If you are a cricket fan you will know that there is a rumour around that 87 is bad luck for Australian batsmen. And why wouldn't it be – after all it is only 13 less than 100 and we know how much bad luck 13 can bring. The theory is that many Australian batsmen get out on 87 and so you would want to tremble a bit if you were out there in the middle with a baggy green cap and 87 comes up against your name on the scoreboard. Personally I would be more than pleased to be in that position. Even in grade 99 cricket I never reached that score.

Staying on the topic of cricket, but looking at New Zealanders – when Chris Cairns retired he held the record for sixes hit in test cricket with, you guessed it – 87.

Actually 87 must be one of the most boring numbers – and not because it is associated in some way with Australian cricket. It doesn't have the sense even to be a prime. The only mildly interesting thing about it is that it equals (2x2) + (3x3) + (5x5) + (7x7). Or do you want to know that **87** Sylvia is one of the largest main-belt asteroids. It is a member of the Cybele group located beyond the core of the belt. Sylvia is remarkable for being the first asteroid known to possess more than one moon.

And even the Australian cricket rumour that was supposed to have been started by Keith Miller and kept alive by Richie Benaud, is patently false. Apparently more Australian batsmen have got out for 85, 86, 88 and 89 than have got out for 87! But if you are really interested in all this you can find a lot more in http://www.theage.com.au/news/sport/why-87-is-anything-but-australian-crickets-magic-number/2007/11/19/1195321694999.html?page=fullpage.

Let me assure you though that the rest of this newsletter is much more stimulating than the number 87.

More on fractions - Betweenness

Overseas research seems to suggest that a significant number of early secondary students do not know that there exist decimals between two 'close' decimals. We will tackle that problem in a later newsletter, but here we consider the case of the number of fractions that exist between two given fractions.

In last year's newsletter we spent a lot of time looking at fractions. We talked about what they mean, how to represent them, how to order them, how to add them and so on. Now we will use some of that machinery to look at what's between them. Stand by for a shock.

Before we get to shock you, however, we want to spend a minute on another subset of the real numbers: the whole numbers. Now the whole numbers are 0, 1, 2, 3, ... and so on for ever. Now between some whole numbers there are other whole numbers. For instance, there are clearly whole numbers between 3 and 59. But between some whole numbers there are no other whole numbers! Again, clearly there are no whole numbers between 2 and 3 or 67 and 68. And that is always the situation with two consecutive whole numbers.

The question we want to look at now is are there fractions that don't have any fractions between them?

Perhaps we had first better show that some fractions do have fractions between them. Let's think about 1/5 and 3/5. In the natural progression of things, 2/5 sits neatly between 1/5 and 3/5.

But what about 1/5 and 1/3? Is there another fraction in that gap? If so, how can we find it? To solve this problem we want to think back to common denominators. We can find a common denominator of 15 for these two fractions and we see that 1/5 = 3/15 and 1/3 = 5/15. But it's easy again to see that 4/15 lies between 3/15 and 5/15 and so between 1/5 and 1/3. No doubt that's not the only fraction you can see between 1/3 and 1/5.

What about 1/5 and 1/4 though? The common denominator shows us that 1/5 = 4/20 and 1/4 = 5/20. Is there an obvious number between 4/20 and 5/20? (4 ½)/20 might be possible if we could have a fraction as the numerator of a fraction. But what might 4 ½ represent here? How about the average of 4 and 5?

Think about the average of two numbers, ***any*** two numbers. Isn't the average of two numbers less than the biggest of the two and more than the smallest. So maybe we should take the average of 1/5 and 1/4?

$$\frac{\left(\frac{1}{5}+\frac{1}{4}\right)}{2}=\frac{\frac{9}{20}}{2}=\frac{9}{40}$$

We'll leave you to check the arithmetic. Of course we should have realised already that (4 ½)/20 was really 9/40. However, having sidetracked that we have discovered how to find, by averages, a fraction between any two different fractions.

But that's not the end of it. Take any two different fractions call them *a* and *b*. Then find their average, *c*. We know that *c* lies between *a* and *b*. OK but the average of *a* and *c* lies between *a* and *c* (and so between *a* and *b*). Then the average of *a* and the of the average *c* and *a* also lies inside *a* and *b*. But why stop there? Because we can do this indefinitely, we have suddenly discovered that there is ***an infinite number of fractions between any two fractions***!!

Did you know that? But do all the fractions between *a* and *b* come from averages?

Problem Solving

Years 1-3

1. How many fractions can you name between 0 and 1?
2. If that is easy for you, how many fractions can you name between 0 and 2?

Years 4 - 6

How many fractions can you name between one half and one?

Years 7- 9

How many fractions can you name between 1/4 and 1/3?

Answers to Problem Solving

There are an infinite number of fractions between ANY two numbers. The solutions given to these questions will depend on the range of fractions that the students can use.

Years 1 - 3

1. Students at this age should be able to identify that unit fractions (1/2, 1/4, 1/3, etc) are all less than 1. They should also be able to suggest some other common fractions such as 3/4. Some students will also identify other fractions that fall between 0 and 1. Any positive fraction for which the denominator (bottom number) is greater than the numerator (top number) falls between 0 and 1.
2. Students may be able to list several additional fractions to their previous list (all the fractions on their previous list should also be on this list). Any positive fraction for which the numerator is less than twice the denominator falls between 0 and 2.

Years 4 - 9

Students at this age should be able to list a number of fractions greater than 1/2 and less than 1. At a minimum they should know that 3/4 and 2/3 fall between them. Any positive fraction for which the numerator is more than half the denominator, but not more than the denominator falls between 1/2 and 1.

Years 7 – 9

Students at this age should be able to use common denominators to compare fractions. This is one way to find fractions between fractions. 1/4 = 6/24 and 1/3 = 8/24, so 7/24 falls between 1/4 and 1/3.

Students may also convert to decimals of percentages. 1/4 = 25% and 1/3 = 33.3%, so any percentage between can be included (26/100, 27/100…).

Students should know that once they have found one fraction they can write many equivalent fractions. 7/24 = 14/48 = 28/96…

Classroom section

There are lots of wonderful classrooms with attractive colourful wall displays to inspire and aid students' learning. Here are a few ideas of ways to include mathematics in the classroom displays.

Wall displays

- A long number line across the top of the white board, in tenths for older students
- Commercial Posters – times tables, height chart, transformation patterns
- School map and class timetable
- Teacher made and student made posters on current topics
- Graphs about the students in the class
- Photos of students doing fun maths activity e.g. constructing 3D shapes, orienteering, water activities

Mobiles

- 3 dimensional shapes

Maths Activity Table

- Jigsaw puzzles, tangrams
- Estimation activity, e.g. how many marbles in the jar
- Board games
- Did you know? maths facts
- Picture books with a maths theme
- Photocopies of puzzles e.g. mazes, magic squares
- Instructions and paper for origami
- Photocopies of nets for 3D shapes for students to construct

Website Links

nzmaths updates

The Numeracy Database is open for entry of 2010. See the help file for information on how to transfer students to their new classes for 2010
http://numeracydb.nzmaths.co.nz/Numeracy/NumeracyDB2/help.aspx.

A section of the website has been developed to provide information and resources to support the extension of high achieving students. You can access this section from the home page or at
http://www.nzmaths.co.nz/node/5845.

Families

Here are some activities for helping your child with their basic facts. There are more activities in the Families section of the website.

Practising facts to 10

Bean Addition:

The purpose of this activity is to help your child to learn the number facts to 5 and to 10.
You will need 10 Haricot Beans or similar. Paint these on one side using paint or nail polish.
What to do:

- Take 5 beans. Scatter the beans – some will land with the coloured side up.
- Talk about the combination. Eg. For 3 coloured beans and 2 plain beans:
  3 and 2 makes 5
  There are 3 coloured beans, how many more are needed to make 5?
  I've got the 3 coloured beans, you've got the 2 plain beans. How many is that altogether?
- Ask your child to show you the number fact using the fingers on one hand. For example, to show 3 coloured beans and 2 plain beans put 3 fingers up and 2 fingers down.
- When your child is quick with the facts to 5 then use 10 beans.

Addition and subtraction facts

I Spy – Addition

The purpose of this activity is to help your child to develop instant recall of addition facts to 20.
You will need a pack of cards with the picture cards removed. Ace counts as 1; 40 cards in total.
What to do:

- Deal out the cards in 10 rows of 4 or 5 rows of 8.
- Players take turns to challenge the others. For example, *I spy two cards that add to 12*
- Players look for 2 cards next to each other, horizontally, vertically or diagonally, that add to the number specified.
- The player that finds the combination collects the 2 cards. If the combination of cards cannot be found, the player who posed the 'I Spy' question takes the two cards.
- If the player made an error and there is no such combination of cards, nobody collects any cards and the next player takes their turn.
- As cards are removed, the remaining cards are rearranged to fill in the spaces.
- The winner is the player with the most cards once all the cards have been collected.

Multiplication facts

Adapt the I Spy - Addition game to use multiplication.