Newsletter 86: November 2009

Introduction

What's so special about 86?

Is there anything interesting about 86? Well you might ask that of any number. Surely every number can't be interesting? Well actually it can. There's an old saying that says that every number is interesting. And there is a proof too, as you would expect in mathematics. It's a proof by contradiction. Let's assume that we have in our hand the smallest number that is not interesting. Then the very fact that it is the smallest uninteresting number makes it interesting. This contradicts our assumption that is it not interesting. So all numbers have to be interesting.

So let's delve a little into 86. What can we say about it? Well it's always nice to look at the factors of a number. The factors of 86 are 1, 2, 43 and 85. Is there anything interesting here? So it has four factors? How special is that? Are there any other numbers that have four factors?

Perhaps the easiest way to resolve this is to test out a few small cases. I've put a table of numbers and the numbers of their factors below.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of factors	1	2	2	3	2	4	2	4	3	4	2	6	2	4

Well, it looks as if 86 is not unique in having four factors. After all, 6, 8, 10 and 14 all have four factors. So what does 86 have in common with 6, 8, 10 and 14? Is there a clue there that enables us find ***all*** numbers that have four factors?

Back off a minute. Is 1 the only number with one factor? What numbers have two factors? What numbers have three factors? The list of questions is endless. It's not likely that we can ask or solve all of those questions here.

But surely 1 is unique in having only one factor. Whatever other number you choose it must have two factors – 1 and the number itself. So that makes it more interesting to think about numbers with only two factors. They have to have 1 and the number itself as factors. And these are surely called **prime numbers**. So we ought to be pretty quickly able to settle one of our questions above.

Oh and this is may be why 1 isn't considered a prime number in polite society. But it's not. One is cast out of the set of prime numbers because mathematicians are over neat. They discovered that numbers like 86 can be written only one way (if you forget about the order the numbers are written in) in terms of prime numbers. Clearly 86 = 2 x 43 (or 43 x 2 but the same prime numbers are used the same number of times). If we allowed 1 exclusive membership in the set of primes then we wouldn't have this nice property. We would be able to write 86 as 2 x 43 or as 1 x 2 x 43 or as 1 x 1 x 1 x 1 x 1 x 1 x 2 x 43. There wouldn't be just one way to write 86 as the product of its primes. Don't you think that that is a small sin to be expelled from being a prime?

So we now know about numbers that have one factor and numbers that have two factors. Three clearly comes next. This group of numbers includes 4 and 9. What do they have in common? Well they are squares. Does every square have three factors? The next square going up is $4^2 = 16$. I'll let you check that that is no good. What about $5^2 = 25$? Hmm that works. So what is going on here? How is 25 different to 16 but similar to 4 and 9? And would 36, 49, 64, 81, 100, 121 or 144 be the next to join the three factors club?

Of course, it's all a matter of primes again. The factors of the square of a prime are 1, the prime itself, and the prime squared. Is there any other way to get three factors?

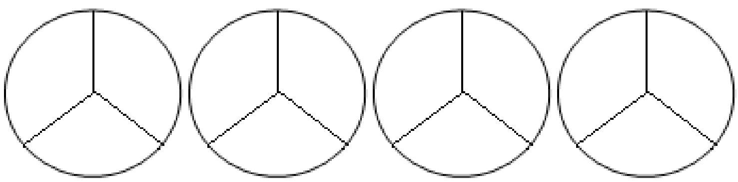
But we are really interested in four factors. And 86 = 2 x 43, a product of two ***different*** primes. If we have any product of two different primes will that give us four factors? Yes. If a number has four factors will it be the product of two different factors? Yes. ... er no. The number 8 is a counter-example. But is it the only one? And if it isn't the only one, what do all of the others look like?

And, finally, if you know what the prime factorisation of a number is, will you then be able to tell how many factors it has?

Unpacking the Maths – Dividing by Fractions

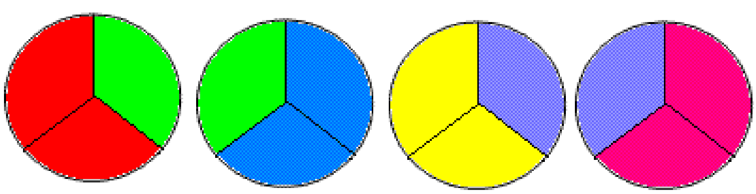
In our last newsletter we were multiplying fractions now we look at dividing by fractions. Dividing by fractions is a difficult concept and fits within Stage 8 of the Number Framework and level 4 of the New Zealand curriculum. Division can be regarded as a sharing model, for example, 12 lollies are shared between 4 people. Division can also be regarded as a measurement model that involves finding the number of equal sets that can be made, for example, 12 lollies need to be put into bags of 4, how many bags are there? The most useful model to use when teaching division by fractions is the measurement model.

To help make sense of division problems it is best to present problems within a context.
Josie has 4 pizzas and she cuts them into thirds. How many pieces does she have?
This word problem is asking how many thirds are in 4, or 4 ÷ 1/3 = □.



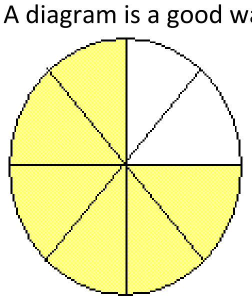
From the diagram we can count 12 thirds. So 4 ÷ 1/3 = 12. It is not a coincidence the answer is the result of multiplying 4 by 3. There are 3 thirds in 1 so 4 x 3 thirds is 12.

If each person at Josie's party eats 2/3 of a pizza, how many people will the 4 pizzas feed? This problem is asking how many two thirds are in 4, or 4 ÷ 2/3 = □.



In this diagram the regions of two-thirds are coloured. We can count 6 two-thirds.
We can also count 1 1/2 lots two thirds in each whole, so altogether there is 4 x 1 1/2 or 4 x 3/2 lots of two thirds or 6 lots of two thirds. 4 ÷ 2/3 = 6.

Let's try dividing a fraction by a fraction. Max cut a cake into eighths.
Now there is 3/4 of the cake left, how many slices are there?
This word problem is asking how many eighths are in 3/4 or 3/4 ÷ 1/8 = □.
A diagram is a good way to get started.



From the diagram we can see that 3/4 is the same as 6/8.   So 3/4 ÷ 1/8 = □ is the same as asking 6/8 ÷ 1/8 = □, which is 6.

Using Algorithms

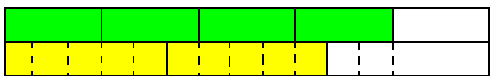
There are two different algorithms for solving division.

1. Lowest Common-Denominator Algorithm

We have already hinted at this in the previous example when we converted 3/4 to 6/8.
Let's try with 4/5 ÷ 2/3. It is asking how many two thirds are in 4/5.
The lowest common denominator for 5 and 3 is 15. So 4/5 becomes 12/15 and 2/3 becomes 10/15.

$\frac{12}{15} \div \frac{10}{15} = \frac{12/10}{1}$ or 12/10 or 1 2/10.

We can check this example with a diagram.



There is 1 two thirds (shown in yellow) in the 4/5 piece. The extra piece is equivalent to 2/10 of the two thirds piece, giving an answer of 1 2/10.

2. Invert and Multiply Algorithm

This algorithm says to invert the divisor and multiply so a/b ÷ c/d becomes a/b x d/c.
We have started many of the division with fraction problems by working out how many of the divisor is in 1.
You may have noticed the pattern that the number of the divisor in one is the divisor's inverse. For example, there are 3 thirds in one (3 and 1/3 are inverses), 1 1/2 lots of two thirds in one (3/2 and 2/3 are inverses).
In our very first example we solved 4 ÷ 1/3 using a diagram. Let's solve it using the algorithm:

$4 \div \frac{1}{3} = ?$

$4 \times 3 = 12$

Let's try with two fractions.

$\frac{3}{6} \div \frac{1}{8} = ?$

$\frac{3}{6} \times 8 = \frac{24}{6} = 4$

We can check this makes sense because we know 8 eighths make a whole. So in 3/6 (or a half) there would be 4 (half of 8).

Let's try an example with a divisor other than a unit fraction.

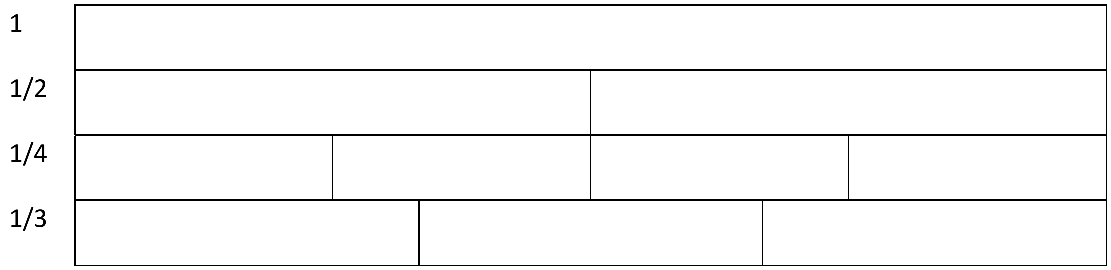
$\frac{4}{5} \div \frac{2}{3} = ?$

$\frac{4}{5} \times \frac{3}{2} = \frac{12}{10}$

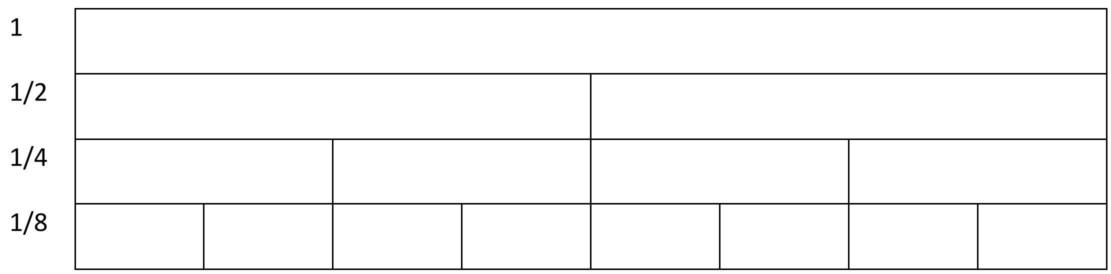
You can check this by looking at the earlier diagram, or using the lowest common-denominator algorithm.

Problem Solving

Years 1-3



1. Colour in 1/4.
2. What is bigger 1/4 or 1/3?
3. What is the same size as 1/2?



1. How many 1/8s are in 1/2?
2. What is bigger 3/4 or 5/8?
3. What is 2/4 + 2/8?
4. What is another way to write 2/4 + 2/8?

Years 4 - 6

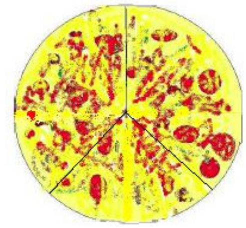
1. Honi and Joe shared equally 3/4 of a cake. How much of the cake they each get?

2. Mary buys 3 small pizzas for her family. There are four people in Mary's family. If they all have the same amount of pizza how much of a pizza do they each get?

3. Jack, Tom and Paul are going to share a chocolate bar. They divided it into equal parts. But then Paul gives half of his piece to Tom. How much does each boy eat?

4. Annie, Kate and Lauren are going to take turns on the school fair stall. They will share 2 1/2 hours equally. How long will each girl spend on the fair stall?

Years 7- 9



1. Mum cuts 4 pizzas into thirds. Write this as a number sentence. How many slices are there?
2. Four cakes were cut into fifths and everyone had 2 slices. How many people shared the cakes? Write this as a number sentence.
3. What gives a bigger answer 5 x 1/3 or 5 ÷ 1/3? Draw a diagram to show your answer.
4. Use a diagram to show that 5 ÷ 1/2 is the same as 5 x 2.

Answers to Problem Solving

Years 1- 3

1. Colour in.
2. 1/3
3. 2/4
4. 4
5. 3/4
6. 3/4 or 6/8
7. 1/2 + 1/4 or 2/4 + 1/4 or 4/8 + 2/8

Years 4 - 9

1. 3/8
2. 3/4
3. Jack gets 1/3, Paul gets 1/6 (half of 1/3). Tom get 4/6 (1/3 + 1/6 = 3/6 + 1/6 = 4/6)
4. 50 minutes. 2 1/2 hours is 150 minutes, divided by 3 = 50.

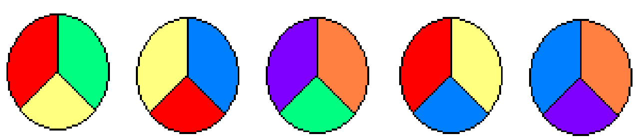
Years 7 - 9

1. 4 ÷ 1/3 = 12
2. 4 ÷ 2/5 = 10
3. 5 x 1/3 = 1 2/3

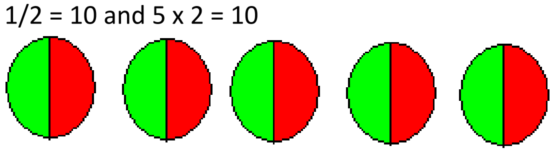
1/3	1/3	1/3

1/3	1/3

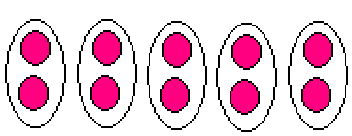
5 ÷ 1/3 = 15



4. 5 ÷ 1/2 = 10 and 5 x 2 = 10



There are 10 half pieces in 5 wholes.



5 groups of 2 equals 10.

Classroom section

As this is our last newsletter for the year here are some ideas for an end of year topic titled "Highlights on 2009". It is a based on the Statistical Investigation area of the Curriculum and gives students the opportunity to reflect on the highlights of the 2009 school year. The activity ideas cover a range from Level 1 to Level 4.

Statistical Investigations

Focus	Activity Ideas
Posing questions	• Work in groups to write questions to ask classmates. For example, which of these class trips was your favourite? • Work in pairs to pose questions suitable for an investigation. For example, which school sports teams do boys and girls belong to? • Discuss and trial different styles of questions. For example, open ended, multi-choice, Likert Scale. • Write a survey to investigate some aspect of School Life in 2009.
Planning an investigation	• Make decisions about the sample. For example, who to ask (selected group, random group, whole class, sample across the school), and how many people. • Select questions for their investigation and publish the survey. (ICT link is possible)
Gathering data	• Gather the data – interview style, observations, data squares, and survey forms for participants. • Collate the data – tally charts, spreadsheets.
Displaying data	• Construct accurate graphs with the correct use of titles, axis, labels, legends etc. • Select the most appropriate graph to display their results. • Use a computer program, for example Excel, to make graphs.
Results and Conclusions	• Make a conclusion based on the data. • Identify patterns, or trends in the data. • Analyse the data (range, mean, mode, median etc) using calculators, spreadsheet functions. • Written and oral presentation of conclusions. • Suggest further investigations based on the results.

Website Links for the Classroom

Online Maths Games

In the Links site we have a review of a website called http://nrich.maths.org/public/. Within this site is games page for students. You can access the students games page by cutting and pasting the following link: http://nrich.maths.org/public/search.php?search=All%20Games

Website Links

National Standards for Literacy and Numeracy

National Standards come into effect in English-medium schools with pupils in Years 1 to 8 in 2010. The standards set clear expectations that students need to meet in reading, writing and mathematics in the first eight years at school. The English-medium National Standards packs are being distributed to schools from 30 October 2009. These packs will initially be distributed to all English-medium schools, based on the following:

- boards of trustees in schools with students in Years 1 to 8 (three copies per board)
- primary and secondary principals (one copy each)
- Year 1 to 8 teachers (one copy each)
- Year 9 teachers (one reference copy each)
- primary and secondary schools (two reference copies per school).

Additional packs are free to order at Down the Back of the Chair. (Cut and paste this link http://thechair.minedu.govt.nz/servlet/Srv.Ecos_Signon?CN=12587&UC=MOENATSTDS&AC=A877804567019987) For further enquiries, phone the Down the Back of the Chair team on 0800 660 662. The National Standards are also available online on the New Zealand Curriculum website. (Cut and paste this link http://nzcurriculum.tki.org.nz/National-Standards/)

nzmaths updates

Links section

The links section of the website provides links to other websites that we think will be useful to teachers. We are looking for suggestions of websites we can add to our Links section. The websites must be free sites and contain no advertisements (including Google Ads). Please email the web address to andrew@nzmaths.co.nz.

Resources for the Resource Database

We are looking for submissions of resources to be added to the Resource Database. If you have a numeracy activity that you have designed and would like to share then you can do this by clicking to downloading this template (34KB) and completing all the sections. Email your completed activity to andrew@nzmaths.co.nz.

Feedback on newsletter

This year we have changed the format of the newsletter. We hope that you have found the newsletter useful and interesting. If you would like to comment on the newsletter or have any suggestions for the unpacking the maths section, classroom, or families pages please email frances@nzmaths.co.nz.

Families

As this is the last newsletter for 2009 here are some ideas for the holidays.

Planning an Outing

Children can be involved in planning a holiday activity. For example, working out the cost of admission to activities, planning the amount of food for the group, reading timetables, and looking at maps.

Games in the Car

Looking for numbers.

Younger children can keep an eye out for small numbers (e.g.1 - 9), they may be part of bigger numbers such as speed limits, distances to places, or licence plates.

Older children can look for numbers such as multiples of 5.

Looking for shapes.

Each person chooses a shape to look for and the first to see their shape wins. Or race each other to be the first to spot a circle then the winner chooses the next shape.

Activities on Holiday

Bingo, games with dice and counting, keeping tally scores.

Construction with old boxes

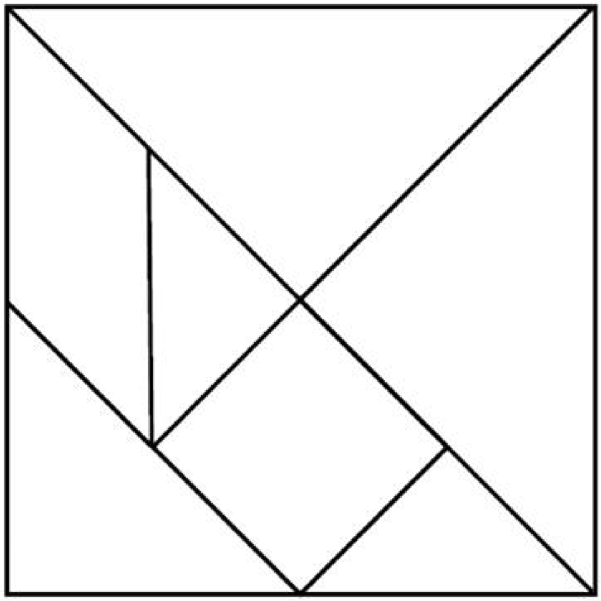
Cooking – reading recipes, measuring quantities

Practice telling the time

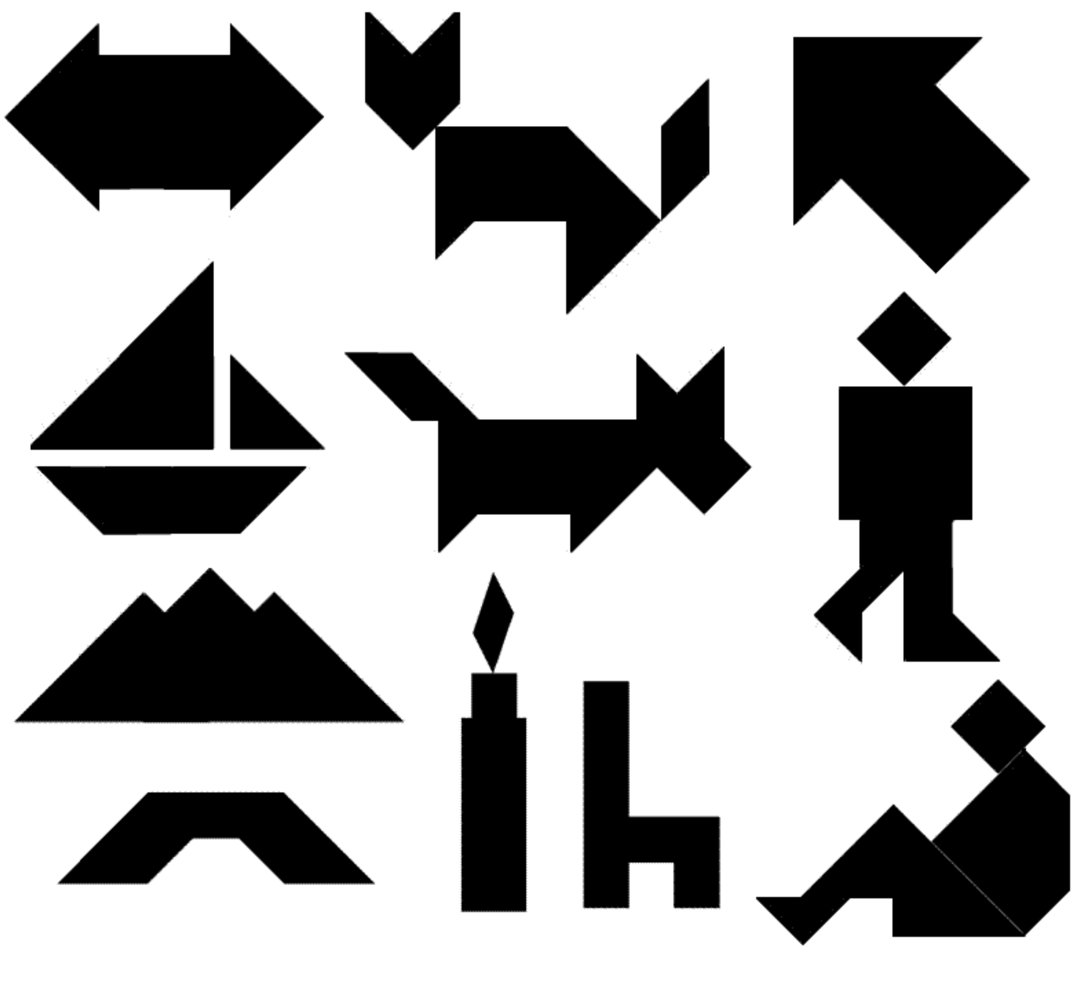
Read picture books with a maths theme

Tangrams: Here is a puzzle to give to your children.

Glue the square on the next page onto some card (old cereal boxes are good) then cut out the 7 pieces.



There are lots of websites with tangram puzzles. Here are some shapes that you can make using all 7 pieces with no overlapping. It is can be fun making your own shapes for other people to make.



The Human Calculator does Cube Roots

Recently, a man who is known as the 'human calculator' toured New Zealand. His speed of calculation is amazing, but some of his techniques are actually very simple and can be taught to children. One demonstration is that he is apparently able to give the cube root for 4 – 6 digit numbers almost instantly. It looks very impressive when seen 'live' but even I can do this one! There have been mathematicians who can *calculate* any cube root in this range, which is truly impressive, but the conditions around this one make it relatively straightforward.

The setup is as follows: someone is asked to enter a two-digit number into a calculator and then cube it. When the answer of say, 300 763 is revealed, the human calculator announces that the original number was 67, to the awe of those listening. However, this is more in the line of a nice trick than displaying actual mathematical ability as such. I have carried out this activity with Year 7 – 8 students, and believe that it is best suited for children at Stage 6 or higher to investigate.

I open the investigation by getting students to give me a few examples and, after dazzling them with my apparent prowess, let them know that they too will become human calculators after some inspection of data and some practice.

Firstly, I get them to create a good amount of examples on calculators and then create a public pool of their data (whiteboard or data show) by recording the two digit number and its cube. Students independently note the pattern of the last digits; the digits 0, 1, 4, 5, 6, and 9 remain the same, while 2 'swaps' with 8 and 3 'swaps' with 7. So, take a cube such as 39 304. The last digit of the cube root has to be 4. For 50 653, the last digit of the cube root is 7.

Now let us consider what happens with the tens digit. If the tens digit is 1, then the resulting cube must be at least 1 000, but cannot be larger than 8 000. If the tens digit is 2, the cube must be at least 8 000, but must be less than 27 000. These boundaries can be recognised as the cube of the digit X 1 000 up to the cube of the next digit X 1000. So, for 39 304, 39 is between 27 and 64 so 3 is the tens digit.

Combining these gives us an algorithm to follow; listen to the 'thousands' part of the number, this gives you the tens digit, then listen for the ones digit, this tells you the ones digit. You then announce the number as the cube root and look intelligent!

Examples: For 17 576, the tens digit must be 2 (17 is between 8 and 27) ones digit must be 6, number is 26. For 551 368, the tens digit must be 8 (551 is between 512 and 729) ones digit must be 2, so 82. Students have been exposed to some interesting mathematical ideas and processes while being highly engaged with the enjoyment of solving the question, "How does he do *that*?"

Bruce Moody, University of Waikato