Newsletter 84: August 2009

Introduction

Eighty-Four – The Intro

So you have guessed that this is the 84th newsletter. And what ancient mathematician will I drag out today? Well, as it happens, a very recent one. William, Bill, Tutte died in 2002 and was especially known to the combinatorialists of Australasia (myself included) as he attended a number of their annual conferences.

So that raises at least two questions. Who was Bill Tutte? What did he do? And what is combinatorics? (Alright, three questions then.)

Bill really had two important lives. While he was still a graduate student at Cambridge he was shoulder-tapped to go off to Bletchley Park, where he took a significant part in the breaking of the famous German Enigma code. After graduating in 1948, he moved to Canada and became one of the most important combinatorialists of the following 30 years.

Combinatorics deals with a whole host of problems that involve discrete objects. Counting sets of objects and looking at structures made up with dots and lines joining them, are two examples of the art. But the one example that is world famous these days is Sudoku. This is really a special case of a thing called a Latin Square. These are all objects that have the same numbers in each row and column but each of these numbers only occurs once in each row and column. They are particularly important in the design of experiments involving treatment of crops.

For more information on Bill, see http://en.wikipedia.org/wiki/W._T._Tutte and http://www.math.binghamton.edu/zaslav/Nytimes/+Science/+Math/+Obits/tutte.html .

But I also wanted to mention Bill because he was clearly a Bright Spark; one of those students who stands out for their interesting and exciting ideas. Someone at Cambridge obviously had pointed him out to the authorities for the decoding work. Now it's likely that every so often you will have a student who stands out too. What you may not know is that there is material on the web site for such students. It is the Bright Sparks section (http://www.nzmaths.co.nz/node/1841) and contains a number of interactive problems that they can work with on screen. This year we are writing supporting notes for both the students and their teachers so that they can both get more out of that section of our site. These should be available by the end of Term 3. You might also come across a Bright Spark that you don't know what to do with. If so you could get in touch with me at derek@nzmaths.co.nz. I'll respond within a day or so.

Unpacking the Maths – Addition and Subtraction Fractions

Here's a problem for you: What is the sum of 2 apples and 5 oranges?

If you think about that for a moment I think that you have to conclude that the answer is the same as 5 oranges and 2 apples but I'm not sure that you can get any further than that. You just can't add 2 apples to 5 oranges in any meaningful way except perhaps in your super market trolley.

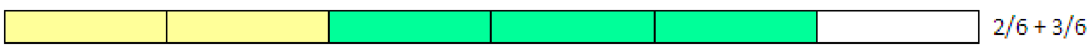
But if you really needed to add them you'd have to find some common currency – fruit perhaps. You could ask if you had 2 apples and 5 oranges, how many ***fruit*** would you have.

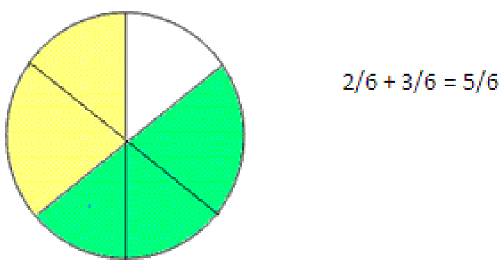
In the same vein I now want to ask: What is the sum of 1/3 and 3/7?

And perhaps that doesn't make any sense in the same way that adding apples and oranges (or even oranges and apples) doesn't make too much sense. But we did find a common denominator for apples and oranges so maybe there's hope for 1/3 and 3/7.

But it might be useful to try something a little easier first. How about we added 2/6 and 3/6? That shouldn't be too hard. Since 2/6 is 2 of something (sixths) and 3/6 is also 3 of the same thing, then adding them together should give us 5 of the common item. So

$$2/6 + 3/6 = 5/6.$$





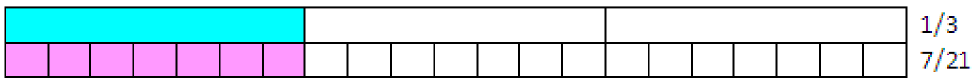
And that is surely the same for any two fractions where the denominator is the same. You can definitely make a diagram that will show that

$1/5 + 2/5 = 3/5$ or $1/6 + 4/6 = 5/6$ or $22/109 + 33/109 = 55/109$.

But I don't fancy my chances of cutting up a pizza into 109 pieces with any accuracy. I have trouble enough with thirds.

Getting back to 1/3 and 3/7 we know that we should be able to add them. They are after all numbers and you must be able to add two numbers no matter what they are. You can just get hold of a passing number line and mark out 1/3 and then move down the line a further 3/7 and that is the point that is the sum of 1/3 and 3/7. It might be hard to get the answer to any great accuracy though. Don't worry about that though, it would seem that all we have to do to get great accuracy is to put the two numbers into some common currency. What basket will both thirds and sevenths fit into?

The answer to that simply comes from the last newsletter where we found out about ***common denominators***. What common denominator do we have with thirds and sevenths? If you remember, if we multiply ***both*** the numerator and the denominator of 1/3 by 7 we get 7/21. (See the fraction wall below.)



And if we multiply ***both*** the numerator and the denominator of 3/7 by 3 we get 9/21. In the last newsletter we were only worried about using this device to ***compare*** 1/3 and 3/7. However, now the trick comes in handy to give 1/3 and 3/7 the common currency of twenty-firsts. In that common currency we can just add them, just as we added 2/6 and 3/6 or 1/5 and 2/5 etc., etc.

So,

1/3 + 3/7 = 7/21 + 9/21 = 16/21.

Oh and you can subtract fractions this way too. Have a go at 2/3 – 3/7.

So far we have been finding the common denominator by effectively multiplying the denominators. This worked well for thirds and sevenths. But in the example of 1/4 + 1/8 changing to a denominator of 32 (4 x 8) seems more complicated than is necessary. I guess what we would like to find when we are adding fractions is the smallest number that both denominators go into. This is called the least common multiple of the two denominators and will give us the lowest common denominator.

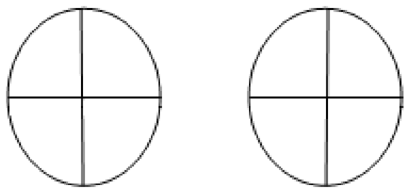
For thirds and sevenths the lowest common denominator is 21, but for quarters and eighths instead of 32 we could use 8. The lowest number which is a multiple of both 4 and 8 is 8. To change 1/4 to eighths we multiply by 2 because 2 x 4 = 8. We then multiply both the numerator and the denominator by 2 and get 2/8. 1/8 already has a denominator of 8, so we can simply add 2/8 and 1/8 to get 3/8.

For 1/6 and 3/4 the lowest common denominator is 12. It is the lowest number that both 4 and 6 multiply in to. To change 1/6 to twelfths we need to multiply by 2, as 6 x 2 = 12. We multiply both the numerator and the denominator in 1/6 by 2 and get 2/12. To change 3/4 to twelfths we need to multiply by 3, as 4 x 3 =12. We multiply both the numerator and the denominator in 3/4 by 3 and get 9/12. Now we can add 2/12 + 9/12 and get 11/12. Just like we can add apples and apples together.

Problem Solving

Years 1-3

1. At the party there were 2 cakes. Kate ate 3/4 of one and Jack ate 2/4, how much cake is that altogether? How much cake is left for Rose to eat?



2. At the party Kate ate 3/5 and Jack ate 4/5 of a cake. How much did they eat altogether? If Rose ate the same amount as Kate and finished off the cakes, how many cakes were there to start with?
3. At the party there were 3 cakes. Together Jack and Kate ate 4/3, if Kate ate 2/3 how much did Jack eat? How much was left for Rose?

Years 4-6

1. At the party there were 2 cakes. Kate ate 2/3 of one and Jack ate 5/6, how much cake is that altogether? How much cake is left for Rose to eat? Who ate the most cake?
2. At the party Kate ate 3/5 and Jack ate 7/10 of a cake. How much did they eat altogether? If Rose ate the same amount as Jack and finished off the cakes, how many cakes were there to start with? Who ate the most cake?
3. At the party there were 3 cakes. Kate ate 3/4 of one and together she and Jack ate 9/8. How much did Jack eat? How much was left for Rose? Who ate the most cake?

Years 7-9

1. If you can get 84 desks from 35 trees, how many desks can you get from 45 trees?
2. If you can make 84 cakes from 6 bags of flour, how many bags will you need to make 154 cakes?
3. Now 3 pet dinosaurs consume 84 trees in 2 hours. If I get 2 more dinosaurs how many trees an hour will I need to provide for them?

Answers to Problem Solving

Years 1-3

1. $3/4 + 2/4 = 5/4$. $2 = 8/4$. So Rose ate $8/4 - 5/4 = 3/4$.
2. $3/5 + 4/5 = 7/5$. $7/5 + 3/5 = 10/5 = 2$. There were two cakes.
3. $4/3 - 2/3 = 2/3$, so Jack ate $2/3$ of a cake. $3 - 4/3 = 9/3 - 4/3 = 5/3$.

Years 4-6

1a. $2/3 + 5/6 = 4/6 + 5/6 = 9/6$ or $1\ 1/2$.
1b. $2 = 12/6$. So Rose ate $12/6 - 9/6 = 3/6 = 1/2$.
1c. Kate ate $2/3 = 4/6$, Jack ate $5/6$, and Rose ate $3/6$. So Jack had the most.
2a. $3/5 + 7/10 = 6/10 + 7/10 = 13/10$.
2b. $13/10 + 7/10 = 20/10 = 2$. There were two cakes.
2c. Kate ate $3/5 = 6/10$, Jack and Rose both are $7/10$. Jack and Rose ate the most.
3a . $9/8 - 3/4 = 9/8 - 6/8 = 3/8$, so Jack ate $3/8$ of a cake.
3b. $3 - 9/8 = 24/8 - 9/8 = 15/8$.
3c. Kate ate $3/4 = 6/8$, Jack ate $3/8$, Rose ate $15/8$. Rose ate the most cake.

Years 7-9

1. 108 desks. You can simplify the numbers of 84 and 35 using common factors. Both 84 and 35 can be divided by 7. So 12 desks come from 5 trees. Since 45 trees is 9 times more than 5 trees, we multiply 12 by 9 to get 108 desks.

2. 11 bags. 6 bags of flour make 84 cakes. So 1 bag will make 14 cakes. 154 cakes is 84 + 70 cakes. Since 1 bag makes 14 cakes, then 70 cakes will take 5 bags ($70 \div 14 = 5$). So it takes 6 bags to make 84 cakes and 5 bags to make 70, so 11 bags altogether.

Or an alternative method is 12 bags of flour will make $2 \times 84 = 168$ cakes. So 12 bags is too many. But $6 \times 14 = 84$, so 1 bag of flour can make 14 cakes. So we must need 11 bags to make 154 cakes.

3. In one hour the 3 dinosaurs would eat 42 trees. Each dinosaur would eat $42 \div 3 = 14$ trees. Two more dinosaurs would eat $14 \times 2 = 28$ trees. So in one hour the 5 dinosaurs would eat $42 + 28 = 70$ trees. The problem can also be solved using fractions: The number of pet dinosaurs has increased by a factor of $5/3$. So the number of trees has to increase by that much too. So I need $5/3 \times 42 = 210/3 = 70$ trees.

Classroom section

Helping Children learn their basic facts

It is fundamental that students learn to instantly recall their basic facts. Knowing their basic facts allows students to develop mental strategies to solve calculations and problems.

It takes time for students to learn their basic facts. The best approaches to learning basic facts are based on understanding the facts, having lots of practice time with a variety of activities and are systematic so students move on to new facts as they are ready.

When students have an understanding of the concepts involved they are ready to practice instantly recalling the facts.

1. Practise with Number Cards.

   Make each student a set of 10 cards with numbers 1-10 on one side and 11-20 on the other side. A student will then stack the cards with numbers 1-10 facing up. Then the student flips through the numbers and adds a number e.g +2 to each number before placing the card at the bottom of the pile. Subtraction facts can be practiced by turning the pile over and subtracting the number e.g. 2 from each number 20 to 11. The 1-10 cards can be used to practise multiplication facts.

   Students should work through addition facts, subtraction facts and then multiplication facts. Within each set of facts students should only start on a new number when they have mastered the previous one.

2. Testing Instant Recall.

   A paper test with a table like the one below can be adapted for any number in the addition or multiplication facts, change the left hand side to 11-20 numbers to test subtraction. If a student can complete the table in less than 20 seconds on 3 occasions they are ready for the next number

3. Tracking Progress.

   Each student should have a record of what basic facts they are learning. It is also an easy way for teachers to track where each student is at. A sticker or stamp is placed by the number when students can answer the 10 related basic facts in less than 20 seconds.

Testing Table Example

+	2
7	
1	
3	
8	
5	
9	
10	
2	
4	
6	

Tracking Table Example

Addition	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Website Links

We have added 30 new activities to the Families section of the website. You can find a link to this section from the nzmaths homepage. The section has a numeracy focus and is arranged by stages of the Number Framework. Each activity is designed for families to do at home. The activities are designed to be short, fun and easy for families to do. Many of the new activities focus on basic facts and are in the Number Facts section. There are activities for children to use to practise their basic facts (for example, the Test Yourself activities and the Quick Recall cards). There are also games and puzzles (for example, bingo games, loopy games, and jigsaw puzzles).

Maths Week

Maths Week is nearly here, it starts on the 9th of August. The Maths Week website has games, activities, and competitions. Last year an estimated 50 000 students and adults participated. You can register to take part by going to the website www.mathsweek.org.nz

National Standards for Literacy and Numeracy

The main themes from the consultation process on the National Standards are available on the Themes of Consultation page. For more information on the National Standards visit the Ministry of Education National Standards page.

nzmaths updates

Early Childhood Education

A new section of the site has been developed to support early childhood educators to use opportunities that arise in everyday interactions with children to foster the development of mathematical thinking. You can find the material from the link on the front page of the site.

Resources for the Resource Database

We are looking for submissions of resources to be added to the Resource Database. If you have a numeracy activity that you have designed and would like to share then you can do this by clicking to downloading this template (34KB) and completing all the sections. Email your completed activity to andrew@nzmaths.co.nz.

Families

The ability to solve problems is an important part of mathematics. Here are 3 problems that can be solved by acting out the problem.

Squash on the Sofa

How many different ways can 3 people sit beside each other on the sofa?

You may want to write the seating arrangement down to keep track of the answer.

Shaking Hands

If everyone in your family shakes hands with everyone else in the family, how many handshakes are there?

Hint: It is easiest if one person starts by shaking hands with everyone, then the next person starts shaking hands with everyone they haven't shaken hands with already and so on.

Crossing the Hallway.

Pretend the hallway is a river that the family needs to cross. Only two people can cross the river at a time. Children can only cross the river with an adult. How many trips across the river will it take your family to get everyone to the other side?

Answers

Squash on the Sofa: 3 people can sit in 6 different ways: ABC, ACB, BAC, BCA, CAB, CBA. Or use the number trick for this problem is 3 x 2 x 1 = 6.

Shaking Hands : 3 people = 3 handshakes, 4 people = 6 handshakes, 5 people = 10 handshakes, 6 people = 15 handshakes

Crossing the Hallway: Answers will vary depending on the number of adults and children in your family.