

Newsletter 83: June 2009

Introduction

This is the 83rd newsletter so I feel obliged to say something profound about the number 83. And the first thing that comes to mind is that Eratosthenes might have had his 83rd birthday before he died. After all, Mactutor

(www-history.mcs.st-andrews.ac.uk/Biographies/Eratosthenes.html) tells us that he was born in 276 BC and died 194 BC.

But the important thing about him is that he had a sieve. Now I know that many of you have sieves. My wife and I strain our rice using one. But Eratosthenes used his to sort out the primes from the other numbers. There's a nice animated version of this sieve at http://en.wikipedia.org/wiki/Sieve_of_Eratosthenes. You should check it out. I'll use it now to find all the primes up to 83. This is how it works.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83							

First put down all the numbers up to 83. I'll do that in a systematic way:

Now, given that 1 isn't a prime for technical reason, 2 is the first number that is a prime so I'll colour that cell yellow. But I will then colour black all cells that contain numbers that are multiples of 2. So I get:

	<mark>2</mark>	3		5	7	9	
11		13		15	17	19	
21		23		25	27	29	
31		33		35	37	39	
41		43		45	47	49	
51		53		55	57	59	
61		63		65	67	69	
71		73		75	77	79	
81		83					

The first number that is not black after 2 is therefore a prime number. So I'll colour this number, (3), in yellow and colour all the multiples of 3 that are not yet black, in black. This gives me:

	<mark>2</mark> <mark>3</mark>	5	7	
11	13		17	19
	23	25		29
31		35	37	
41	43		47	49
	53	55		59
61		65	67	
71	73		77	79
	83			

Right, now the next number after 3 that isn't black is 5 so 5 must be a prime. You know the drill now. Colour it in yellow and colour all other multiples of 5 in black. And keep going like this. The next number after 5 that isn't black I colour yellow and then black out all multiples of this number. And I keep going like this until I get all the numbers up to 83 coloured yellow or black. The yellow numbers are the primes up to and including 83.

So I did all that and found this:

	<mark>2</mark>	<mark>3</mark>	<mark>5</mark>	<mark>7</mark>		
<mark>11</mark>		<mark>13</mark>		<mark>17</mark>	<mark>19</mark>	
		<mark>23</mark>			<mark>29</mark>	
<mark>31</mark>				<mark>37</mark>		
<mark>41</mark>		<mark>43</mark>		<mark>47</mark>		
		<mark>53</mark>			<mark>59</mark>	
<mark>61</mark>				<mark>67</mark>		
<mark>71</mark>		<mark>73</mark>			<mark>79</mark>	
		<mark>83</mark>				

For what it's worth, 83 is prime and it's the 23rd prime. And the sieve has held on to all of the primes and let all the other numbers go.

But you might want to look up the Mactutor website to see why 11/83 is important and what it has to do with Eratosthenes anyway.

Unpacking the Maths: Comparing Fractions 3

In the last newsletter I discussed comparing 2/3 and 3/7 to see which one is bigger. In the end I did this by referring to the benchmark of 1/2. This was done by noting that 2/3 is bigger than $(1 \ 1/2)/3$ and so 2/3 is bigger than 1/2. In the same kind of way I found that 3/7 is smaller than 1/2. Hence I was able to see that 2/3 is bigger than 3/7.

But I'm interested in finding a way to compare **any** two fractions, for instance 37/62 and 491/758. The problem here is that it is not always easy to find a nice neat benchmark that we can compare both of our fractions to. So we need to use another method that is a little more complicated but uses three of the ideas that I used in the last newsletter. The first of these is the fraction wall, the second the simple fact that 1/99 is smaller than 2/99 which is smaller than 3/99 and so on, and the third is the fact that 3/4, 6/8, 12/16, 24/32, and 48/64 are all the same.



Let's look at the fraction wall from last time that showed 1/2 and 1/3 and 3/6 all together.

Now we know that 1/2 is bigger than 1/3 – we can easily see that from the fraction wall or from the fact that the denominator of 1/2 is smaller than the denominator of 1/3. But look at both of those numbers in comparison with the sixths in the fraction wall. 1/2 = 3/6 and 1/3 = 2/6. Obviously 3/6 is bigger than 2/6. And this gives us a general idea. If I can find a **denominator** that is the **same** for any two fractions that I am comparing, then it is going to be easy to see which fraction is bigger.

In another example, if I can show that 2/3 = 14/21 and 3/7 = 9/21, then clearly 14/21 is bigger than 9/21 and so 2/3 is bigger than 3/7. So the problem now is reduced to finding the denominator that is the same. This denominator is called the *common denominator*.

Let's find a simpler pair of fractions first. How about 2/3 and 3/4? Let's put them on a fraction wall.



From the fraction wall I can see that 2/3 = 8/12 and 3/4 = 9/12 and so 3/4 is bigger than 2/3.

But how do I know that 12 is the common denominator that I want here? And how did I know that 21 was the common denominator that I wanted for 2/3 and 3/7? What has 12 got to do with thirds and fourths? What has 21 got to do with thirds and sevenths?

Can we guess what common denominator I need to compare 2/3 and 5/8?

Let me take another direction for a moment. In what ways can the fraction 2/3 be written? From the doubling of the last newsletter we know that 2/3 = 4/6 = 8/12 = 16/32 and so on. But what is so special about doubling? Why not triple the top and bottom of the fraction? Is 2/3 equal to 6/9? Look at the fraction wall below. Clearly 2/3 does equal 6/9.

	2/3
	6/9

But what is so special about doubling or tripling? Isn't 2/3 = 8/12 or 10/15 or 22/33? Check this out using a fraction wall. It turns out that if we start with 2/3 and multiply both the numerator 2 and the denominator 3 by the same number then fraction at the end is exactly the same as 2/3. So 2/3 = 14/21.

But we can do that with any fraction. 3/7 = 6/14 = 33/77. But for our purposes 3/7 = 9/21. Once we have the same denominator it's easier to compare two fractions. And the trick to finding the common denominator is to look at the original two denominators and just multiply them together. So let's say we wanted to compare 5/11 and 9/20. Then we should aim for the common denominator of $11 \times 20 = 220$. To get 5/11 into 220ths just multiply the top and bottom of 5/11 by 20. This gives 100/220. Then multiply the top and bottom of 5/11 by 20. This gives 100/220. Then

The thing about common denominators is that they can be used for lots of other things. We'll get to that in the next newsletter.

Footnote

I know that cross product seems to have come in use for multiplication these days but there is another use of the word that helps to decide which of two fractions is bigger. Let's go with 2/3 and 3/7 as an example again. Let's write $2/3 \square 3/7$, in the box will either be less than or greater than sign. It would be nice to get rid of the denominators so first multiply both sides by 3. This gives $2 \square 9/7$. And then multiply both sides by 7 to get $14 \square 9$. Since 14 is bigger than 9, 2/3 is bigger than 3/7 or 2/3 > 3/7.

OK. Why does this work? You actually know all you need to know to decide this.

Professional Development Reading

Core Curriculum: Learning to Work Like a Mathematician by Doug Williams

The Numeracy Facilitators' Conference is held in February every year and is a chance for a lot of people involved in the NDP to get together, to share experiences and to listen to speakers from around the world. Doug Williams was someone who was invited over from Australia to give some workshops. He has written an article for the newsletter which is too big for us to include in total but you can access it at (http://www.nzmaths.co.nz/sites/default/files/DougWilliams.pdf). However, to give you some idea of what it contains we have included an abstract here.

Doug starts off by comparing the Ancient Egyptian, Babylonian and Greek approaches to education. First the quote of Singh:

Pythagoras observed that the Egyptians and Babylonians conducted each calculation in the form of a recipe that could be followed blindly. The recipes, which would have been passed down through generations, always gave the correct answer and so nobody bothered to question them or explore the logic underlying the equations. What was important for these civilizations was that a calculation worked - why it worked was irrelevant.

Singh, Simon (1997) Fermat's Enigma.

Then the quote of Williams:

It was the Greeks (Pythagoras and friends) who took a different approach to mathematics *and mathematics teaching*. Read their manuscripts and you will find learning is more often a discussion between teacher and pupil. Teacher poses a question; student responds; teacher facilitates further thinking by building on the answer with another question; ...the adventure continues (and sometimes the student manages to extend the teacher's thinking).

But this article is about how mathematicians work. In the past he had found out the answer by actually talking to mathematicians. It turns out that they don't just solve lists of equations, rather they try to solve problems.

To illustrate this, Doug states the 13 Away problem and analyses what can be done with it in class and how students can learn from this what mathematicians do and how they work. It also contains a more than strong hint that we should think more like a Greek than an Egyptian when it comes to our teaching. We recommend that you try the problem and then look at the full article.

13 Away Problem

Materials: 21 counters.

- 1. Put 13 counters in a pile
- 2. Take turns to take 1, 2, or 3 counter from the pile.
- 3. The winner is the person who makes the other player take the last counter.
- 4. Now play the same game with 21 counters. This time each player can take 1, 2, 3, or 4 counters.

Problem Solving

Years 1-3

Here is a 2 by 2 by 2 block made of cubes.



- 1. What fraction of the block are the red cubes?
- 2. What fraction of the block is a single white cube?

Here is a 2 by 2 by 4 block made of cubes.



- 3. What fraction of this block are red cubes?
- 4. What fraction of the block is a single white cube?
- 5. Is the fraction of the white cubes in Block 1 bigger than the fraction of the white cubes in Block 2?

<u>Years 4-6</u>

I have some one dollar coins and some 2 dollar coins. The one dollar coins are a quarter of all the coins I have. My grandma doubled the number of 1 dollar coins I have and grandpa doubled the number of 2 dollar coins. What fraction of my coins are the one dollar coins now?



<u>Years 7 -9</u>

A third of Frances' jelly beans were red and the rest blue.

- 1. Draw a diagram to show this.
- 2. Frances ate half of the blue jelly beans. What fraction of what Frances had left were the red jellybeans?
- 3. Frances was given some more blue jellybeans so that she had twice as many blue jelly beans as when she started. What fraction of the jelly beans that Frances now has are the red jellybeans?



Answers to Problem Solving Questions

<u>Years 1-3</u>

Q1. 4/8 or 1/2

Q2. 1/8

Q3. 8/16 or 1/2

Q4. 1/16

Q5. No, in both blocks the white cubes are half of the total number of cubes.

<u>Years 4 -6</u>

Still one quarter. Because the number of both \$1 and \$2 coins doubled the proportion stays the same.

Years 7-9

Q1. A diagram that shows one third red and two thirds blue. Either a fraction wall showing three equal parts or three sets each with the same number of jelly beans will show this.

Red	Blue	Blue	

Q2. One half. If half the blue ones are eaten then there are two equal groups one of red jelly beans and one with blue jelly beans. Half of the remaining jelly beans half are red.Q3. One fifth. If Frances has twice as many blue jelly beans as she started with, she will have 4 groups of blue jelly beans and 1 group of red jelly beans.

Red Blue Blue Blue Blue					
	Red	Blue	Blue	Blue	Blue

Classroom section

<u>Classroom Idea</u>: With the onset of winter you may wish to use some of these ideas to develop a mini cross-strand maths unit on the theme of Winter.

Strand	AU	ACTIVITY
Measurement	Measurement AO1	Measure ingredients for a winter recipe, e.g.
and Geometry	(Levels 1-4)	scones.
	Measurement AO1	Compare temperatures. Order pictures from cold to
	(Levels 1&2)	hot weather activities (Level 1)
		Measure using a thermometer (Level 2)
	Measurement AO1	Make and use an anemometer (wind meter).
	(Levels 3&4)	Solve problems involving sunrise & sunset times.
	Shape AO1 (Levels 1&2)	Draw a snowman using 2D shapes.
	Position and Orientation	Draw a plan of school and routes from one place to
	(Levels 3&4)	another suitable for a wet day.
	Transformation	Make a snowflake patterns using transformations.
	(Levels 1-4)	Design umbrella patterns using centre of rotation.
Number and	Number Knowledge	Order temperatures, rainfall amounts, wind gusts.
Algebra	(Levels 1&2 AO1)	
	Number Strategies	Counting activities using winter theme (hats, gloves,
	(Levels 1&2 AO1)	umbrellas, gumboots, scarves etc). (Level 1)
		Solve addition and subtraction problems using
		temperatures. (Level 2)
	Number Strategies and	Find the temperature range when the numbers
	knowledge (Level 4 AO2)	include negative degrees.
	Number Strategies	Adapting measurements for different quantities of
	(Levels 3&4)	winter recipes, e.g. soup, toasted sandwiches.
		Calculate energy use of appliances. Google search
		"appliance power usage" for energy use amounts.
	Number Strategies and	Calculate percentages of children who own
	knowledge (Level 4 AO3)	gumboots, raincoats, umbrellas.
Statistics	Statistical investigations	Survey, communicate and make conclusions about
	(Levels 1-4)	winter activities – e.g. sport choices, transport to
		school, leisure activities. Compare to summer.
	Statistical literacy	Interpret results of classmates' surveys and
	(Levels 1-4)	conclusions from winter activities survey.

Website Links

This year Maths Week is from the 9-16th of August. The Maths Week website has games, activities, and competitions. Last year an estimated 50 000 students and adults participated. You can register to take part by going to the website www.mathsweek.org.nz

Website updates

Survey

We are continuing to collect feedback about the changes made to the design of the website. There is a link to the survey available from the bottom of the home page of the website. If you would like a response to your feedback please either include your email address in your survey responses or email Andrew at andrew@nzmaths.co.nz.

Resources for the Resource Database

We are looking for submissions of resources to be added to the Resource Database. If you have a numeracy activity that you have designed and would like to share then you can do this by downloading a template from the link at the bottom of the home page of the website. This template includes all of the headings that need to be completed for resources to be included in the database. Authors' names can be included in their resource at their request.

National Standards for Literacy and Numeracy

The Ministry of Education has now released draft National Standards for literacy and numeracy in primary and intermediate schools. National Standards aim to lift achievement in literacy and numeracy (reading, writing, and mathematics) by being clear about what students should achieve and by when. Consultation on the National Standards will take place until the 3rd of July. To keep up to date with this development or for information on how to request a consultation pack visit the http://www.minedu.govt.nz/theMinistry/Consultation/NationalStandards.aspx.

Webinar

We are holding an online webinar on June 19th at 4pm (New Zealand time). The webinar will approximately an hour long and be on the topic of the National Standards for literacy and numeracy, with a focus on the standards for numeracy. The webinar will be presented by Vince Wright, one of the authors of the National Standards for numeracy and will feature online streaming video and a PowerPoint presentation, they will need an up to date operating system and browser and Adobe Flash Player (version 9 or higher). Participants will be able to ask Vince questions both before and during the webinar. We have 40 'seats' available for the webinar. While these will be allocated on a first in first served basis, we do ask that users from outside New Zealand give priority to those from within New Zealand as this webinar is more directly relevant to them. If you would like to participate in the webinar, please email Andrew at andrew@nzmaths.co.nz by June 17th.

Families – Maths with Marshmallows

On a cold, wet wintery day you may like to entertain your children with some marshmallow maths. The additional equipment needed is listed after the activity.

	5 – 7 year olds	8 - 10 year olds	11 – 13 year olds
Measuring weight	Find other things that feel as if they weigh about the same weight as the packet of marshmallows, are lighter, and are heavier than the packet.	Read how many grams the packet weighs. Find other items that weigh a similar amount, check your ideas on kitchen scales. (Kitchen scales).	Guess how many marshmallows are in the bag. Empty about half the bag and count them. Adjust your guess, count all the marshmallows.
Counting	Guess how many marshmallows are in the bag. Count them one by one.	Guess how many marshmallows are in the bag Arrange them in rows of 5, then skip count 5, 10, 15 etc	Read the packet for the serving size. How many marshmallows are in a serving? (Calculator)
Probability	Put 10 white and 5 pink marshmallows back in the bag. Which colour are you more likely to pick out? (White) Why? (There are more of them) Try different amounts. Count the number of pink marshmallows, and number of white marshmallows in the bag. What colour are you more likely to get from this bag?	If the bag has 3 white and 1 pink marshmallow left in it, what is the chance of picking out a pink one? (1 out of 4). Try different amounts of each colour and the same question.	If there are 8 white and 4 pink marshmallows, what is the chance of picking a pink one? (4 out of 12, or 1 out of 3) Try different amounts. How many pink and white marshmallows do you need to have a 2 out of 3 chance of picking a pink one?(2 pink 1 white or 4 pink 2 white etc) Try different amounts.
Measuring length	Measure how long objects are using marshmallows (e.g. wooden spoon, can opener, chopping board). Estimate how long objects are using marshmallows. Look for items longer or shorter than 10 marshmallows.	Take 10 marshmallows and arrange them in a line so they are touching, but make the longest line you can. Measure with a ruler. Make the shortest line you can with 10 marshmallows. Measure with a ruler. Explain how you can arrange them to make short and long lines. (Ruler)	Count the number of marshmallows across the centre of a plate, and the number around the rim of a plate. What is the relationship? (about 3 times longer around the rim) Try with different plates. Is the relationship the same? (Selection of different sized plates)
Making shapes	Arrange marshmallows to make shapes – squares, triangles, oval, pentagons. Count number of sides and corners.	Make a cube and a pyramid using marshmallows and toothpicks. (Toothpicks)	Using marshmallows and toothpicks make 3D shapes – cubes, pyramids, triangular prism (like a Toblerone). Build a 3D shape from a pentagon shape. (Toothpicks)
Marshmallow correct basic	vs can be made into fruit kebabs, rocky fact answers.	road or other recipes. Marshmall	ows can be given as prizes for