

Newsletter No. 79

November 2008



As the song said, that's 'another year over and deeper in debt'. And in debt we are, to all of you who have contributed to the nzmaths website - at all levels and in all aspects. Those of you who have written the work units and others who have laid them out. There was all that formatting to check, all the technical stuff - diagrams, computer skills, original ideas - the millions of things that go to make up a successful website. Our thanks too for all your contributions to the newsletter.

And let me thank Russ Dear, in particular, who lays out the initial structure of the newsletter with items such as the Booke Review and the Endeavour of the month, which then gets filled in by others.

Andrew Tagg should also get a mention. Not only is he the person that tidies the newsletter up and puts it to bed but he also runs the nzmaths office, answers all your queries and does all that Jenny Ward and Frances Neill don't have time for.

Of course you knew that 79 is the number of episodes of Star Trek; that 79 is a prime (actually a twinned prime with its reverse 97 – there aren't a lot of these around); and that 79 weeks is the record number of weeks for a record to hold the Billboard number 1 hit by ... The King, Elvis Presley. And, as the year draws to an end, and I'm trying to sell my house in a falling market, and I'm planning of retiring, it's how old I feel.

But I'm not so old that I didn't think this was cute.



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What's new on nzmaths.co.nz

There is a new user survey available from the link on the front page of the website or by going directly to <u>http://www.nzmaths.co.nz/nzmaths6.htm</u>. We are collecting information on how users find information on the website as we are developing a new interface to much of the material on the site.

Keep an eye on the website over the summer as there will be a lot of changes happening between now and the end of January!

Diary Dates

The Le@rning Federation is conducting a pilot project called *Helping at home* to assist Australian and New Zealand parents. *Helping at home* has a range of digital activities suitable for primary school aged children. It also provides parents/caregivers with support to help their children by providing suggestions of how to use each activity, questions to ask children and further activities for families to do together. The digital activities are also available in the Learning Objects section of the nzmaths website (http://www.nzmaths.co.nz/Learning Objects/).

On the first day of Christmas

I know it's only just November but there'd be more than one person in your school counting down the days till Rudolf gets the call and the gang's all off. Now given that much of the known world is in deep recession at the moment and wondering how to keep up the mortgage payments, Santa's ho-hoing may be a little muted this year. However, I firmly believe that on the first day of Christmas my true love will send to me a partridge on a pear tree. What's more on the second day I'm in for a couple of turtle doves and another partridge in a pear tree. (You should be glad that you don't have to listen to me singing this.)

Now by Boxing Day I've already racked up three presents, if you don't count the pair of turtle doves as more than one present. What's more I'm starting to worry about what I'm going to do with all the partridges that I'm about to collect over the next 10 days. And where will I put them? Will this first pair of turtle doves along with the 10 further pairs be happy enough to fly round the garage all day? I can hardly let them out. My true love isn't going to be very happy if I take them down to the Botanical Gardens and let them go. Would they last a Dunedin winter anyway?

So it looks like I'm going to have to find homes for 12 partridges and 11 pairs of turtle doves. Can I work out ahead of time how many presents my true love is going to give me again this year? You know the list of things I'm going to get so you should be able to work it out. Take a little time out now to do the sums.

By the way, thanks to <u>http://en.wikipedia.org/wiki/Image:Streptopelia_turtur01.JPG</u>, I can show you what a turtle dove looks like. (Incidentally have you any idea what sort of mess 11 pairs of turtle doves can produce in your average single car garage?)



Actually the same web site will tell you that 62% of Europe's turtle dove population had disappeared between 1980 and 2005. I'm a little worried by this. Has my true love been overdoing the Chrissy presents? If you persist with the reading of the website you will also see that, in the same period, the grey partridge has declined in numbers by 79%. If my true love is stuffing them and mounting them on pear trees, then it's no wonder. Does the Green Party know about this? When you think about it, there aren't as many lords-a-leaping these days as there were when I was a boy and can you think of the last time you saw maids-a-milking? In Southland they've pretty well had to be replaced by milking machines!

I'm going to have to have a word with my true love before it's too late.

But I've gone off the track. Did you get 364? Did you know that my true love was dishing out one present for each day of the year minus one? Or as many steps as there are on the pyramid at Chichen Itza (see the September newsletter). So it looks like my true love has Mayan connections. I wonder how many maids-a-milking there are in Chichen Itza now. Were they used as human sacrifices do you think? And what would Mayans think of lords-a-leaping? Was that the secret weapon that the 100 Spaniards used to overcome the 100,000 Mayans? "Here senoras look at these pipers piping." And while they were falling over themselves with laughing the Spaniards crept up behind and woomp!

This is dreadful; my true love is killing off the major birds of Europe and has decimated the population of half of Mexico!

It doesn't bear thinking about. Let's get back to numbers. 364. That's just

12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1.

The carnage doesn't bear thinking about but suppose my true love gave me presents for 13 days. On the 13^{th} day she might give me 13 bankers bankrupting. No. I can't bear it. She's responsible for the current economical state of the western world! Think about numbers. Would I get 364 + 13 presents? No, hang on. The 13 bankers, like the 12 drummers drumming, is just one present. So it's 364 + 1. No, it can't be that because I'd have had 13 wretched partridges stuck in pear trees by then so it's at least 364 + 13 + 1. Hang it, there must be

13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1

by the 13th day.

Now I'm sure that sometime we looked at adding up strings of consecutive numbers like that. Didn't we see that it's the first plus the last divided by 2 and multiplied by the number of numbers? So for the original 12 days of Christmas we'd have (12 + 1)/2 by 12. That's 13 x 12/2 =78.

Oh!

There was a hint on a web site that said the answer wasn't 78. So is it 364 or 78 and why? Well on the second day I actually got 3 presents – one partridge thingy and two (*TWO*!) turtle whatsits. I don't know why I tried to convince you that two birds was one present. So I guess

the first day I got 1 present; the second day I got 3 presents; the third day I got (1 + 2 + 3) 6 presents; and so on and so on etc.

I'm right up with the play now. On day 4 I'm going to have to get 4 more presents, 4 whatsits whatsitting. So altogether that day I get 10 presents. Ah, the number of presents goes up by the number of days each time.

So altogether I actually get

1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55 + 66 + 78

and that is 364! (I think.)

But is there some way to add those numbers up without having to resort to counting all, counting on, the addition algorithm or in some other way upsetting the founders of the Numeracy Development Projects?

What are the subtotals?

1; 1 + 3 = 4; 1 + 3 + 6 = 10; then 20; then 35; then 56; then ... Is there a pattern here?

Maybe not but let me show you the Christmas Stocking Theorem.

												1												
											1		1											
										1		2		1										
									1		<mark>3</mark>		3		1									
								1		<mark>4</mark>		<mark>6</mark>		4		1								
							1		5		<mark>10</mark>		10		5		1							
						1		6		15		<mark>20</mark>		15		6		1						
					1		7		21		35		<mark>35</mark>		21		7		1					
				1		8		28		56		70		<mark>56</mark>		<mark>28</mark>		8		1				
			1		9		36		84		126		126		<mark>84</mark>		<mark>36</mark>		9		1			
		1		10		45		120		210		256		210		120		<mark>45</mark>		10		1		
	1		11		55		165		330		466		466		330		<mark>165</mark>		55		11		1	
1		12		66		220		495		796		932		796		495		<mark>220</mark>		<mark>66</mark>		12		1

Look! I've got a Mayan pyramid of numbers. The red numbers are the ones that I want to add. And the blue ones are the progressive sums. So now we can see that

1 + 3 + 6 + 10 + 15 = 35 and 1 + 3 + 6 + 10 + 15 + 21 = 56 and 1 + 3 + 6 + 10 + 15 + 21 + 28 = 84

And if you put in the next two rows of the Mayan pyramid, then you'll be able to read off the sum total 364 of presents.

And can you see the Christmas Stocking? The red numbers as far as you want to go form the leg and the toe is the sum. So you can see the leg of 1 + 3 + 6 + 10 + 15 and the toe of 35.

Now you can find the total number of presents for 13, 14 or any number of days that you like. Be nice if we could get a formula though.

If you want to find out more about the 12 days of Christmas have a look at http://www.aip.org/isns/reports/2002/058.html

Booke Review

The Aryabhatiya

Aryabhata wrote his Aryabhatiya around 500 CE when India was a world centre for art, science, literature and architecture. The work was written as a poem of just thirty-three verses (123 stanzas) written in Sanskrit. It is particularly important not only because it gives a summary of the mathematical and astronomical knowledge of the time but because it provided an impetus for further study. After an invocation to Brahma, for Aryabhata was a Hindu, the book is divided into three parts; on mathematics, time-reckoning and the sphere.

The work contains a number of inaccuracies. Although Aryabhata is only about three hours short in his estimate of the solar year and fairly accurate with his diameter of the earth, he is wildly off in his estimated size of the orbits of the sun, moon and planets. In the section on mathematics, formulae for areas of a triangle are correct but those for spheres and pyramids are not. His formulae for the sums of natural numbers, their squares and cubes are also correct. He calculated π to be 3.1416 but this figure had been around for some time and was known to the Greeks 300 years earlier. It seems he was aware of the alpha-numeric system of notation, a step towards the way we write numbers today, and had methods for solving linear and quadratic equations and simple simultaneous equations. There was also an introduction to sines. Like many writers of the era and earlier, Aryabhata does not explain how he arrived at his formulae and catalogue of rules for solving equations, although they may have been written elsewhere and lost.

The book contained an enormous amount of information in a small space. 500 years later, basing his thoughts on the book, a famous Arab mathematician was to say that Hindu mathematics offered two types of nuggets; common pebbles and costly crystals.

Solution to Last Month's Endeavour

Well, the pattern of primes 31, 331, 3331, 33331, 33331 does continue a little further. 3333331 and 33333331 are prime but 333333331 is not - it has a divisor 17.

Euler's formula n(n + 1) + 41, where $n \in W$, certainly generates a lot of primes but not, for example when n = 40 or 41. It does for n = 0 right up to 39 and overall the proportion of primes it generates is just under half of all the numbers it generates.

And I'd like to thank Derek Smith for his complete discussion of the last two Endeavour's.

This Month's Endeavour

Our Endeavour for November is another of our end-of-year multi-choice quizzes, we hope you enjoy it. The answers are in *Afterthoughts* below.

2008 End Of Year Quiz

- 1. What are rational numbers?
 - (a) Numbers that think for themselves
 - (b) Non-recurring decimals
 - (c) A fraction with numerator more than its denominator
 - (d) A fraction with numerator less than its denominator
 - (e) Decimals that recur or are of finite length.

2. Three dots in a plane can be mutually joined by at most three straight lines (when they form a triangle). They can also be joined by two lines (when they are in a straight line). What is the maximum number of distinct straight lines that can be drawn to connect eight dots in a plane?
(a) 16 (b) 24 (c) 28 (d) 32 (e) none of those.

- How many square numbers can be formed from permutations of any number of the digits 2, 3, 7 and 8?
 (a) 0
 (b) 1
 (c) 2
 (d) 3
 (e) 4.
- 4. If the area of a square is 169 sq.cm. what is its perimeter?
 (a) 13cm
 (b) 42.25cm
 (c) 26cm
 (d) 52cm
 (e) none of those.
- 5. A 'Happy' number is one where the sum of the squares of its digits, and the sum of the squares of the digits of the answer, and so on, eventually comes to 1. So, 921 is a happy because $9^2 + 2^2 + 1^2 = 86$, $8^2 + 6^2 = 100$ and $1^2 + 0^2 + 0^2 = 1$. Which of the following is/are happy? (a) 123 (b) 345 (c) 222 (d) 129 (e) 383
- 6. Fold a sheet of paper in half lengthways (with the crease parallel to one edge. Trim off the four corners. Now fold the paper in half again along a crease at right-angles to the first. Trim off the four corners. Repeat the process a few times. After each fold open the paper to the original sheet and you will find, as the process of folding and trimming is repeated, an increasing number of holes appearing in the original sheet. After one fold there are no holes, after two folds there is one, after three folds there are three. How many holes are there after five folds?

(a) 21 (b) 23 (c) 25 (d) 27 (e) 29

7. How many vertices (corners) does an octahedron have?
(a) 4 (b) 6 (c) 8 (d) 10 (e) 12.

- 8. Which early Greek mathematician wrote *The Method* that only came to light in 1906? Some of his other titles were *On Spirals*, *On the Measurement of the Circle* and *On the Sphere and Cylinder*.
 - (a) Archimedes
 - (b) Bendthenees
 - (c) Conon
 - (d) Dinocrates
 - (e) Euclid
- 9. How many prime divisors are there of one million and one?
 (a) none
 (b) one
 (c) two
 (d) three
 (e) none of those.
- 10. The sum of a two-digit number and the number obtained by reversing the order of its digits is a square number. How many such numbers are there?
 (a) 2 (b) 4 (c) 6 (d) 8 (e) 10.
- 11. Only three of the Platonic solids; tetrahedron (T), cube (C), octahedron (O), dodecahedron (D) and icosahedron (I) can be found in nature as crystals. Which are the three?
 - (a) (C), (D) and (I)
 - (b) (D), (I) and (O)
 - (c) (I), (O) and (T)
 - (d) (C), (O) and (T)
 - (e) (D), (O) and (T).
- 12. Newspapers occasionally print 'top five' or 'top ten' popular film lists. If the order is never the same in any two successive weeks and no film gains any lost popularity, for how many weeks, at most, can a top five list contain the same five films?
 - (a) 9 (b) 11 (c) 13 (d) 15 (e) none of those.
- 13. How often can you subtract fractions by the simple technique of subtracting their numerators and multiplying their denominators?
 - (a) Never
 - (b) For only one pair of fractions
 - (c) Sometimes
 - (d) Always
 - (e) None of those.
- 14. In how different many ways can you make up a dollar? Well, there's a dollar coin, that's one way; two fifty cent pieces, that's another
 - (a) 9 (b) 10 (c) 11 (d) 12 (e) none of those.

- 15. A drawer contains red socks and blue socks. When two socks are drawn at random from the drawer the probability they are both red is one half. How small can the number of socks in the drawer be?
 (a) 3 (b) 4 (c) 5 (d) 6 (e) 10.
- 16. Who acted the part of mathematician John Nash in the movie A Beautiful Mind?
 - (a) Russell Crowe
 - (b) Anthony Hopkins
 - (c) Colin Firth
 - (d) Heath Ledger
 - (e) Orson Welles.
- 17. How many capital letters in the alphabet have one or more lines of symmetry? (a) 11 (b) 12 (c) 13 (d) 14 (e) 15.
- 18. If D₆ is a standard set of dominoes with tiles ranging from [0,0] to [6,6] and there are 28 of them (you knew that!), how many dominoes would there be in D₈?
 (a) None of the following (b) 32 (c) 36 (d) 42 (e) 45.
- 19. What is a scalene triangle?
 - (a) A triangle drawn to scale
 - (b) One with all its sides of different length
 - (c) One with two of its sides the same length
 - (d) One with all of its sides the same length
 - (e) One with none of its sides of different length.
- 20. Here's another question about triangles. What are congruent triangles?
 - (a) Ones that are similar (in the mathematical sense, i.e. their angles match)
 - (b) Triangles that have at least one side of matching length
 - (c) Those that have at least one angle of matching size
 - (d) Triangles that fit exactly on top of one another
 - (e) Those that have the same area.

Afterthoughts

Answers to 2008 Quiz:

- 1. (e) Rational numbers are ones that can be expressed as fractions these either recur or are of finite length when expressed as decimals. (c) and (d) may be rational numbers but, then again, they may not.
- 2. (c) The answer is readily obtained by careful drawing but you might be able to derive a formula that will do the job (see *2 in Notes below).
- 3. (a) Write down the first ten square numbers. What are their last digits? No square number can have 2, 3, 7 or 8 as its last digit.
- 4. (d). If the area of the square is 169 sq.cm., the length of its side is 13cm and there are four of these that make up the perimeter. This is the question that was most often defaulted on by Year 7 students in our annual maths competitions.
- 5. (d) and (e)
- 6. The pattern goes 0, 1, 3, 9 but not 27. After five folds there are 21 holes, so the answer is (a).
- 7. (b)
- 8. (a)
- 9. (c). They are 101 and 9901.
- 10. (d). If the number is 10a + b then the sum mentioned is: 10a + b + 10b + a = 11a + 11b = 11(a + b).
 Since this is a square number and a and b are single digits, a + b = 11. Hence the pairs (a, b) can be (2, 9), (3, 8), (4, 7), 5, 6) and their reversals, i.e. there are eight solutions.
- 11. (d)
- 12. (b). The answer is readily obtainable by listing possibilities but you might be able to derive a formula that will do the job (see *12 in Notes below).

13. When you see, for example that

$${}^{3}{}_{4}{}_{-}{}^{2}{}_{3}{}_{=}{}^{1}{}_{12}$$
,
 ${}^{4}{}_{5}{}_{-}{}^{2}{}_{3}{}_{=}{}^{2}{}_{15}$,
 ${}^{4}{}_{5}{}_{-}{}^{1}{}_{2}{}_{=}{}^{3}{}_{10}$,
 ${}^{6}{}_{13}{}_{-}{}^{2}{}_{5}{}_{=}{}^{4}{}_{65}$,
 ${}^{5}{}_{11}{}_{-}{}^{1}{}_{3}{}_{=}{}^{4}{}_{33}$,

and

you'll realise that the answer is (c), sometimes! I'm sure you'll be able to write a few more fractions of your own which follow this odd rule of subtraction.

- 14. (c).
- 15. (b). Three red and one blue.
- 16. (a).
- 17. (e). A, B, C, D, E, H, I, K, M, O, T, V, W, X, Y. It does depend a bit on the font you use. The B and C here don't appear to have a line of symmetry but try them in a different font: B, C.
- 18. (e). Listing and counting will give you the answer readily enough but there is a formula which is not too difficult to determine (see *18 in Notes below).
- 19. (b).
- 20. (d). Congruent triangles have all the properties listed in answers (a) to (e) but are only defined by answer (d). I asked this question because someone I met recently, on hearing I was a teacher of mathematics, told me he'd never understood Euclidean geometry at school because he'd never known what congruent triangles were. He still didn't know. I told him!

Notes:

- *2. The formula is $\frac{1}{2}n(n-1)$, where n indicates the number of dots.
- *12. The formula is $\frac{1}{2}n(n-1) + 1$, where n indicates the number of films in the list.
- *18. $\frac{1}{2}(n+1)(n+2)$ gives the number of dominoes.